True Glue

Note on gluon-fusion production of pseudo-goldstone higgs

(last compiled November 3, 2007)

1 Top Sector and Higgs Mass

We consider a set-up with some (equal) number of left- and right-handed top quarks. The mass matrix is general non-diagonal:

$$\mathcal{L} = -\mathcal{M}_{ij}(\tilde{v})\overline{t_{L,i}}t_{R,j} + \text{h.c.} \tag{1.1}$$

and depends on the higgs vev $\tilde{v}$. We denote the top mass eigenstates by t_n. The rotation matrices

$$t_{L,i} = L_{in}t_{L,n} \qquad t_{R,i} = R_{in}t_{R,n} \tag{1.2}$$

are such that

$$L_{in}\mathcal{M}_{ij}R_{jm} = m_n\delta_{mn} \quad L_{in}\mathcal{M}_{ik}\mathcal{M}_{jk}L_{jm} = m_n^2\delta_{mn} = R_{in}\mathcal{M}_{ki}\mathcal{M}_{kj}R_{jm} \tag{1.3}$$

For simplicity we ignored all phases so that the rotation matrices are diagonal. Similarly, if we have additional b-quarks, we will have the matrices L^b_{in} and R^b_{in} that rotate the bottom mass matrix to diagonal; $\mathcal{L} = \mathcal{M}^b_{ij}\overline{B_{L,i}}B_{R,j} + \text{h.c.}$, $L^b_{in}\mathcal{M}^b_{ij}R^b_{jm} = m^b_n\delta_{mn}$. If there are some exotic (e.g. charge 5/3) they will have their rotation matrices too.

Every kid knows that cancellation of the quadratic divegence requires that the trace of the mass matrix squared does not depend on the higgs vev:

$$\frac{\partial}{\partial\tilde{v}}\text{Tr}M^\dagger M = 0 \tag{1.4}$$

We can also reformulate the condition in the diagramatic way. Consider the higgs couplings to the top quarks

$$\mathcal{L} = -y_{nm}h\overline{t_{L,n}}t_{R,m} + \frac{1}{2}k_{nm}h^2\overline{t_{L,n}}t_{R,m} + \mathcal{O}(h^3) + \text{h.c.} \tag{1.5}$$

We also denote the diagonal coupling as $k_n \equiv m_n k_{nn}$. The coupling matrices are related to the mass matrices via:

$$y_{nm} = \frac{\partial}{\partial\tilde{v}}M(\tilde{v}) \quad k_{nm} = \frac{\partial^2}{\partial\tilde{v}^2}M(\tilde{v}) \tag{1.6}$$

Tops have this annoying habit of contributing to the higgs mass. We obtain

$$\begin{aligned}-i\Delta m_h^2 = \quad & -4N_c\int\frac{d^4k}{(2\pi)^4}\frac{k^2(|y_{nm}|^2+|y_{mn}|^2)+m_nm_m(y_{nm}y_{mn}+y^*_{nm}y^*_{mn})}{(k^2-m_n^2)(k^2-m_m^2)} \\ & +4N_c\sum_n k_n\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_n^2)}\end{aligned} \tag{1.7}$$

Evaluating this in dimensional regularization

$$\begin{aligned}\Delta m_h^2 &= \frac{N_c}{8\pi^2}\frac{\Gamma(2-d/2)}{d/2-1}\Bigg\{ \\ &+ \sum_{n\neq m}\frac{(|y_{nm}|^2+|y_{mn}|^2)(m_n^d-m_m^d)+m_n m_m(y_{nm}y_{mn}+y^*_{nm}y^*_{mn})(m_n^{d-2}-m_m^{d-2})}{m_n^2-m_m^2} \\ &+ \sum_n m_n^{d-2}(d|y_{nn}|^2+(d/2-1)(y_{nn}^2+{y^*_{nn}}^2)-2k_n)\Bigg\}\end{aligned} \tag{1.8}$$

The cancellation of quadratic divergences (the pole at $d=2$ in the above) requires the relation

$$\sum_{n,m} y_{nm}^2 = \sum_n k_n \tag{1.9}$$

The log divergence of the higgs mass can be read off from the coefficient of the $1/\epsilon$ pole at $d=4$

$$\Delta m_h^2 \to \frac{N_c}{4\pi^2}\log\Lambda\left(4\sum_{n,m}|y_{nm}|^2 m_n^2+\sum_{n,m}(y_{nm}y_{mn}+y^*_{nm}y^*_{mn})m_n m_m-2\sum_n k_n m_n^2\right) \tag{1.10}$$

2 Glue Glue Higgs

Here i review the lowest order result for the glue-glue-higgs amplitude, as they stand in ref. [4]. The one-loop amplitude from a quark of mass m_n coupled to the higgs via y_{nn} is given by:

$$\mathcal{M} = \frac{\alpha_s}{4\pi}\delta_{ab}K_{ab}(\epsilon,p))M_f(\theta)\frac{y_{nn}}{m_n} \tag{2.1}$$

where K_{ab} depends on the gluon polarization and momenta, while $M_f(\theta)$ is the form factor depending on the scattering angle. As long as as $m_h^2 \ll 4m_n^2$ for all n, the only thing of interest here is the factor y_{nn}/m_n that is sensitive. In the SM it is given by $1/v$ where $v_{EW}=174\,\mathrm{GeV}$. In order to determine the deviation of the higgs production amplitude from the SM prediction we shall compute the quantity:

$$R^{1/2} = \frac{\mathcal{M}_{ggh}}{\mathcal{M}_{ggh}^{SM}} \approx v_{EW}\sum_n \frac{y_{nn}}{m_n} \tag{2.2}$$ e.r

In the basis where the mass matrix M is diagonal we can rewrite:

$$\frac{y_{nn}}{m_n} = \mathrm{Tr}(yM^{-1}) = \mathrm{Tr}\frac{\partial M(\tilde v)}{\tilde v}M^{-1}(\tilde v)) = \mathrm{Tr}\frac{\partial\log M(\tilde v)}{\partial\tilde v} = \frac{\partial \mathrm{Tr}\log M(\tilde v)}{\partial \tilde v} \tag{2.3}$$

Because of the trace, we can in fact use any basis to evaluate it. Furthermore, using $\mathrm{Tr}\log = \log\det$ we get

$$R = v_{EW}^2\left(\frac{\partial}{\partial\tilde v}\log\det M(\tilde v)\right)^2 \tag{2.4}$$ e.rsmart

This smart formula has been known since forever, but i dont know who discovered it.

In the following we calculate R in some specific models.

3 4D Models

Superminimal SU(3) model

Consider the top sector of the simplest little higgs [2]. On the left side it has one doublet $q_{L,i}$ and one singlet T_L^c. On the right side it has two singlets $t_{R,i}^c$. So there are just two top states and one massless chiral bottom.

The heavy top has the noble task of cutting of the quadratic divergences created by the SM top. To this end the lefties are collected into an $SU(3)$ triplet:

$$\Psi_L = \begin{pmatrix} t_L \\ b_L \\ T_L^c \end{pmatrix} \tag{3.1}$$

The yukawas obey SU(3) that is only softly broken by mass terms

$$-\mathcal{L}_{yuk} = \lambda_1 \overline{\Psi}_L \Phi t_{R,1}^c + f\lambda_2 \overline{T}_L^c t_{R,2}^c + \text{h.c.} \tag{3.2}$$

where the higgs boson is embedded in the triplet as

$$\Phi = \begin{pmatrix} \sin\left(\frac{\tilde{v}+h}{f}\right) \\ 0 \\ \cos\left(\frac{\tilde{v}+h}{f}\right) \end{pmatrix} \tag{3.3}$$

Here $\tilde{v}$ is the higgs vev, and h is a canonically normalized higgs field. Note that $\tilde{v} \neq v_{EW}$; instead $v_{EW} = f\sin(\tilde{v}/f)$.

To get the bottom mass we would need to add b_R and write down the normal SM yukawa interaction with the higgs doublet. This would reintroduce quadratic divergences, but since bottom yukawa is small naturalness is not screwed. In the following we will ignore the bottom mass whatsoever.

Inserting the higgs we get the mass terms for the tops

$$-f^{-1}\mathcal{L}_m = \lambda_1 \sin x \bar{t}_L t_{R,1}^c + \lambda_1 \cos x \overline{T_L^c} t_{R,1}^c + \lambda_2 \overline{T}_L^c t_{R,2}^c + \text{h.c.} \tag{3.4}$$

Here, there and everywhere $\sin x \equiv \sin(\tilde{v}/f)$. The mass terms can be written in a matrix for:

$$(\bar{t}_L, \overline{T}_L^c) M \begin{pmatrix} t_{R,1}^c \\ t_{R,2}^c \end{pmatrix} \qquad M = f\begin{pmatrix} \lambda_1 \sin x & 0 \\ \lambda_1 \cos x & \lambda_2 \end{pmatrix} \tag{3.5}$$

One can check that the trace of the mass matrix squared does not depend on $\tilde{v}$, which means there are no quadratic divergences to the higgs mass parameters at one loop. The determinant of the mass matrix is $\lambda_1\lambda_2 \sin x$ so that, using eq. (2.4) we find the ratio

$$R = \cos^2(\tilde{v}/f) = 1 - \frac{v^2}{f^2} \tag{3.6}$$

Quite simple. In particular, $R < 1$ so that the higgs production is always diminished. Furthermore, measuring the higgs production cross section provides a simple handle on the scale f.

In order to gain more confidence in this result we rederive it straight from eq. (2.2). To this end, we first rotate to the basis of eigenstates *before* EW breaking:

$$
\begin{aligned}
t^c_{R,1} &= \cos r t^c_R + \sin r T^c_R \qquad \sin r = \frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2}} \\
t^c_{R,2} &= -\sin r t^c_R + \cos r T^c_R
\end{aligned}
\tag{3.7}
$$

which yields

$$
(\bar{t}_L, \overline{T}^c_L) M \begin{pmatrix} t^c_R \\ T^c_R \end{pmatrix} \quad M = f \begin{pmatrix} \lambda_1 \cos r \sin x & \lambda_1 \sin r \sin x \\ \lambda_1 \cos r (\cos x - 1) & \lambda_T + \lambda_1 \sin r (\cos x - 1) \end{pmatrix} \tag{3.8}
$$

where $\lambda_T = \sqrt{\lambda_1^2+\lambda_2^2}$. One top quark is vector-like – it has a mass non-zero mass M_{T_c} before EW breaking

$$
M_{T_c} \equiv f \sqrt{\lambda_1^2 + \lambda_2^2} \tag{3.9}
$$

The other eigenvalue before EW breaking is zero.

Next, we find the rotation matrices L and R that diagonalize the top mass matrix. We do it numerically working to order v^2/f^2 (note that this allows us to determine the lowest eigenvalue to order v^3/f^3) After EW breaking the top quark gets the mass

$$
m_t \approx v \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}} + \mathcal{O}(v^3/f^2) \tag{3.10}
$$

Now compute the couplings of the tops to the higgs. We replace $\sin(\tilde{v}/f) \to \sin(\tilde{v} + h/f)$ and expand the sines and cosines to quadratic order in h. The we expand in powers of v/f up to quadratic order. At the end of the day we find:

$$
\begin{aligned}
y_{tt} &= -\frac{\lambda_1 \lambda_2}{(\lambda_1^2+\lambda_2^2)^{1/2}} \left(1 - \frac{v^2}{f^2} \frac{\lambda_1^4 - \lambda_1^2 \lambda_2^2 + \lambda_2^4}{2(\lambda_1^2+\lambda_2^2)^2} \right) \\
y_{TT} &= \frac{v}{f} \frac{\lambda_1^2 \lambda_2^2}{(\lambda_1^2+\lambda_2^2)^{3/2}} \\
y_{Tt} &= \frac{v}{f} \frac{\lambda_1 \lambda_2^3}{(\lambda_1^2+\lambda_2^2)^{3/2}} \\
y_{tT} &= -\frac{\lambda_1^2}{(\lambda_1^2+\lambda_2^2)^{1/2}} \left(1 - \frac{v^2}{f^2} \frac{2\lambda_2^4 - \lambda_1^2 \lambda_2^2}{2(\lambda_1^2+\lambda_2^2)^2} \right)
\end{aligned}
\tag{3.11}
$$

Similarly, we can compute the couplings of two higgses to top quarks. The diagonal couplings are given by

$$
\begin{aligned}
k_t &= \lambda_1^2 \frac{v^2}{f^2} \frac{\lambda_2^4}{(\lambda_1^2+\lambda_2^2)^2} \\
k_T &= \lambda_1^2 \left(1 - \frac{v^2}{f^2} \frac{\lambda_2^4}{(\lambda_1^2+\lambda_2^2)^2} \right)
\end{aligned}
\tag{3.12}
$$

Using these couplings we can compute the coefficient of the log givergence

$$
\Delta m_h^2 \to -f^2 \lambda_1^2 \lambda_2^2 \frac{N_c}{4\pi^2} \log \Lambda^2 \tag{3.13}
$$

In the ratio R, all the dependence on yukawa couplings cancels out. What remains is

$$R^{1/2} \approx 1 - \frac{1}{2}x^2 \tag{3.14}$$

which indeed is the beginning of $\cos x$. Fine.

Minimal spinorial SO(5) model

On the left side it has two doublets $q_{L,i}$ one singlet top T_L^c and one singlet bottom B_L^c. On the right side it has two top singlets $t_{R,i}^c$, two bottom singlets $b_{R,i}^c$ one doblet Q_R. This makes three top and three bottom states and no exotics. The fields are collected into $SO(5)$ spinors as follows

$$\Psi_L = \begin{pmatrix} q_{L,1} \\ -iB_L^c \\ -iT_L^c \end{pmatrix} \qquad \Psi_R = \begin{pmatrix} Q_R \\ ib_{R,1}^c \\ it_{R,1}^c \end{pmatrix} \tag{3.15}$$

The higgs is also embedded in an $SO(5)$ fundamental as $\Phi_M = f(0,0,0,\sin(h/f),\cos(h/f))$ Yukawas have $SO(5)$ global symmetry, which is softly broken by the mass terms

$$-\mathcal{L}_{yuk} = y\overline{\Psi}_L\Gamma^M\Phi_M\Psi_R + f\lambda_q\overline{q}_{L,2}Q_R + f\lambda_t\overline{T}_L^c t_{R,2}^c + f\lambda_b\overline{B}_L^c b_{R,2}^c + \text{h.c.} \tag{3.16}$$

Look in appendix to admire the gamma matrices for SO(5).

Inserting the higgs we get the mass terms for the tops

$$\begin{aligned} -f^{-1}\mathcal{L}_{top} = \quad & y\sin x\bar{t}_{L,1}t_{R,1}^c - y\sin x\overline{T}_L^c T_R + y\cos x\bar{t}_{L,1}T_R + y\cos x\overline{T}_L^c t_{R,1}^c \\ & +\lambda_q\bar{t}_{L,2}T_R + \lambda_t\overline{T}_L^c t_{R,2}^c \end{aligned} \tag{3.17}$$

The bottom mass terms are the same with $t \to b$ everywhere. In the matrix form they read

$$(\bar{t}_{L,1}, \bar{t}_{L,2}, \overline{T_L^c})M \begin{pmatrix} t_{R,1}^c \\ t_{R,2}^c \\ T_R \end{pmatrix} \tag{3.18}$$

$$M = f\begin{pmatrix} y\sin x & 0 & y\cos x \\ 0 & 0 & \lambda_q \\ y\cos x & \lambda_t & -y\sin x \end{pmatrix} \tag{3.19}$$

One can check that the trace of the mass matrix squared does not depend on x, which means there are no quadratic divergences to the higgs mass parameters at one loop. In fact, this model is supersoft – logarithmic divergences vanish as well. Somebody tell me why.

The determinant is again simple: $\lambda_q\lambda_t y\sin x$. Thus the ratio from eq. (2.4) is again

$$R = \cos^2 x \tag{3.20}$$

Again we recompute it perturbatively using higgs couplings. We rotate to the basis of eigenstates *before* EW breaking:

$$\begin{aligned} t_{R,1}^c &= \cos r_t t_R^c + \sin r_t T_R^c \qquad \sin r_t = \frac{y}{\sqrt{\lambda_t^2 + y^2}} \\ t_{R,2}^c &= -\sin r_t t_R^c + \cos r_t T_R^c \\ t_{L,1} &= \cos l t_L + \sin l T_L \qquad \sin l = \frac{y}{\sqrt{\lambda_q^2 + y^2}} \\ t_{L,2} &= -\sin l t_L + \cos l T_L \end{aligned} \tag{3.21}$$

which yields

$$(\bar{t}_L, \overline{T}_L, \overline{T^c_L}) M \left(\begin{array}{c} t^c_R \\ T^c_R \\ T_R \end{array} \right) \tag{3.22}$$

$$M = f \left(\begin{array}{ccc} y\cos r_t \cos l \sin x & y\cos l(\cos x - 1) & y\sin r_t \cos l \sin x \\ y\cos r_t \sin l \sin x & \lambda_Q + y\sin l(\cos x - 1) & y\sin r_t \sin l \sin x \\ y\cos r_t(\cos x - 1) & -y\sin x & \lambda_T + y\sin r_t(\cos x - 1) \end{array} \right) \tag{3.23}$$

where $\lambda_Q = \sqrt{\lambda_q^2 + y^2}$, $\lambda_T = \sqrt{\lambda_t^2 + y^2}$. The bottom mass matrix is alike with $t \to b$ everywhere where possible. In this model we have a vector-like doublet with $m_Q = \lambda_Q f$, a heavy top singlet with $m_{T_c} = \lambda_T f$, and a heavy bottom singlet $m_{B_c} = \lambda_B f$, up to corrections from EW breaking. The top and bottom mass are given by, at lowest order

$$\begin{aligned} m_t &= \frac{y\lambda_q\lambda_t}{\sqrt{\lambda_q^2 + y^2}\sqrt{\lambda_t^2 + y^2}} v + \mathcal{O}(v^3/f^2) \\ m_b &= \frac{y\lambda_q\lambda_b}{\sqrt{\lambda_q^2 + y^2}\sqrt{\lambda_b^2 + y^2}} v + \mathcal{O}(v^3/f^2) \end{aligned} \tag{3.24}$$

Note that the only parameter breaking custodial symmetry is the splitting between λ_t and λ_b. To obtain the experimental value for m_t/m_b we need $\lambda_t \gg \lambda_b$ and also $\lambda_b \ll y$. This implies the existence of a relatively light vector-like bottom state, with $m_{B_c} \approx yf$.

Higgs couplings. Expanding in powers of v/f we find (here due to lack of space we display only lowest order):

$$\begin{array}{lll} y_{tt} = \dfrac{y\lambda_q\lambda_t}{\sqrt{\lambda_q^2 + y^2}\sqrt{\lambda_t^2 + y^2}} & y_{QQ} = \frac{v}{f}\frac{\lambda_q^2\lambda_t^2 y^2}{(\lambda_q^2+y^2)^{3/2}(\lambda_q^2-\lambda_t^2)} & y_{TT} = -\dfrac{v}{f}\dfrac{\lambda_q^2\lambda_t^2 y^2}{(\lambda_t^2+y^2)^{3/2}(\lambda_q^2-\lambda_t^2)} \\ & y_{tT} = \frac{\lambda_q y^2}{(\lambda_q^2+y^2)^{1/2}(\lambda_t^2+y^2)^{1/2}} & y_{Qt} = \dfrac{\lambda_t y^2}{(\lambda_q^2+y^2)^{1/2}(\lambda_t^2+y^2)^{1/2}} \\ & y_{QT} = \frac{y^3}{(\lambda_q^2+y^2)^{1/2}(\lambda_t^2+y^2)^{1/2}} & y_{TQ} = -y \\ & y_{tQ} = \mathcal{O}(v/f) & y_{Tt} = \mathcal{O}(v/f) \end{array} \tag{3.25}$$

$$\begin{aligned} k_t &= \frac{v^2}{f^2}\frac{\lambda_q^2\lambda_t^2 y^2(\lambda_q^2\lambda_t^2 - y^4)}{(\lambda_t^2+y^2)^2(\lambda_q^2+y^2)^2} \\ k_Q &= y^2 \\ k_T &= y^2 \end{aligned} \tag{3.26}$$

The ratio R is found to be

$$R^{1/2} \approx 1 - \frac{1}{2}\frac{v^2}{f^2} \tag{3.27}$$

in concordance with the yield of the smart formula.

Minimal fundamental SO(5) model

Here comes the model of ref. [3]. On the left side it has one doublet q_L one singlet T^c_L and one exotic doublet $\tilde{Q}_L$. On the right side it has two singlets $t^c_{R,i}$ and one exotic doublet $\tilde{Q}_R$. This makes three top states, one massless chiral bottom and one exotic junk. The lefties are collected into an $SO(5)$ fundamental

$$\Psi_L = \begin{pmatrix} \frac{1}{\sqrt{2}}(b_L + \chi_L) \\ -\frac{i}{\sqrt{2}}(b_L - \chi_L) \\ \frac{i}{\sqrt{2}}(t_L - \tilde{T}_L) \\ \frac{1}{\sqrt{2}}(t_L + \tilde{T}_L) \\ T^c_L \end{pmatrix} \tag{3.28}$$

The higgs boson is embedded in the $SO(5)$ fundamental as

$$\Phi = f\left[0,0,0,\sin\left(\frac{\tilde{v}+h}{f}\right), \cos\left(\frac{\tilde{v}+h}{f}\right)\right]^T \tag{3.29}$$

Yukawas have $SO(5)$ global symmetry, which is softly broken by the mass terms

$$-\mathcal{L}_{yuk} = \lambda_1 \overline{\Psi}_L \Phi t^c_{R,1} + f\lambda_2 \overline{T}^c_L t^c_{R,2} + f\lambda_\chi \overline{\tilde{Q}_L}\tilde{Q}_R + \text{h.c.} \tag{3.30}$$

Inserting the higgs we get the mass terms for the tops

$$-f^{-1}\mathcal{L}_m = \frac{1}{\sqrt{2}}\lambda_1 \sin x \bar{t}_L t^c_{R,1} + \frac{1}{\sqrt{2}}\lambda_1 \sin x \overline{\tilde{t}_L} t^c_{R,1} + \lambda_1 \cos x \overline{T^c_L} t^c_{R,1} + \lambda_2 \overline{T}^c_L t^c_{R,2} + \lambda_\chi \overline{\tilde{T}_L}\tilde{T}_R + \text{h.c.} \tag{3.31}$$

The matrix:

$$(\bar{t}_L, \overline{T}^c_L, \overline{\tilde{T}_L}) M \begin{pmatrix} t^c_{R,1} \\ t^c_{R,2} \\ \tilde{T}_R \end{pmatrix} \qquad M = f \begin{pmatrix} \frac{1}{\sqrt{2}}\lambda_1 \sin x & 0 & 0 \\ \lambda_1 \cos x & \lambda_2 & 0 \\ \frac{1}{\sqrt{2}}\lambda_1 \sin x & 0 & \lambda_\chi \end{pmatrix} \tag{3.32}$$

One can check that the trace of the mass matrix squared does not depend on x, which means there are no quadratic divergences to the higgs mass parameters at one loop. We dont have λ_3 of [3] because it reintroduces quadratic divergences. It would change nothing in the gluegluehiggs amplitude, though.

The determinant is const $\cdot \sin x$, which is starting to become boring. The R ratio is again

$$R = \cos^2 x \tag{3.33}$$

Twin Higgs

With the twin higgs we retreat to two-top models. We consider the left-right symmetric twin higgs. The higgs doublet H together with its twin $\tilde{H}$ makes up a fundamental **4** of the approximate $SU(4)$ symmetry. The higgs boson is embedded in the those higgses as

$$H \to \begin{pmatrix} 0 \\ \sin\left(\frac{\tilde{v}+h}{f}\right) \end{pmatrix} \qquad \tilde{H} \to \begin{pmatrix} 0 \\ \cos\left(\frac{\tilde{v}+h}{f}\right) \end{pmatrix} \tag{3.34}$$

On the left side there is an $SU(2)_L$ doublet $Q_L = (t_L, b_L)$ and singlets T^c_L, B^c_L. On the right side there is an $SU(2)_R$ doublet $Q_R = (t^c_{R,2}, b^c_{R,2})$ and singlets $t^c_{R,1}$, $b^c_{R,1}$. The tops get a mass from the interactions

$$-\mathcal{L} = yH^\dagger \overline{Q_L} T^c_R + y\tilde{H}^\dagger \overline{Q_R} T^c_L + \lambda f \overline{T}^c_L T^c_R + \text{h.c.} \tag{3.35}$$

These interactions are invariant under twin parity acting as $H \longleftrightarrow \tilde{H}$, $Q_L \longleftrightarrow Q_R$, $T^c_L \longleftrightarrow t^c_{R,1}$.

Inserting the higgs we get the mass terms for the tops

$$-f^{-1}\mathcal{L}_m = y \sin x \bar{t}_L t^c_{R,1} + y \cos x \overline{T^c_L} t^c_{R,2} + \lambda \overline{T}^c_L t^c_{R,1} + \text{h.c.} \tag{3.36}$$

The mass terms can be written in a matrix form:

$$(\bar{t}_L, \overline{T}^c_L) M \begin{pmatrix} t^c_{R,1} \\ t^c_{R,2} \end{pmatrix} \qquad M = f \begin{pmatrix} 0 & y \sin x \\ y \cos x & \lambda \end{pmatrix} \tag{3.37}$$

One can check that the trace of the mass matrix squared does not depend on x, which means there are no quadratic divergences to the higgs mass parameters at one loop. The determinant is const $\cdot \sin x \cos x$. Finally something new. The R ratio is

$$R = \frac{\cos^2(2\tilde{v}/f)}{\cos^2 \tilde{v}/f} = \frac{(1 - 2v^2/f^2)^2}{1 - v^2/f^2} \approx 1 - 3v^2/f^2 \tag{3.38}$$

The cross section, though different than before, is again reduced, in fact more reduced than before.

We play the diagrammatic game again. We first rotate to the basis of eigenstates before EW breaking:

$$\begin{aligned} t^c_{R,1} &= \cos r t^c_R + \sin r T^c_R \qquad \sin r = \frac{\lambda}{\sqrt{\lambda^2 + y^2}} \\ t^c_{R,2} &= -\sin r t^c_R + \cos r T^c_R \end{aligned} \tag{3.39}$$

$$(\bar{t}_L, \overline{T}^c_L) M \begin{pmatrix} t^c_R \\ T^c_R \end{pmatrix} \qquad M = f \begin{pmatrix} y \cos r \sin x & y \sin r \sin x \\ y \cos r (1 - \cos x) & \lambda \sin r + y \cos r \cos x \end{pmatrix} \tag{3.40}$$

One top quark is vector-like – it has a mass non-zero mass M_{T_c} before EW breaking

$$M_T \equiv f\sqrt{\lambda^2 + y^2} \tag{3.41}$$

The other eigenvalue before EW breaking is zero. After EW breaking the other top quark gets the mass

$$m_t \approx v \frac{y^2}{\sqrt{\lambda^2 + y^2}} + \mathcal{O}(v^3/f^2) \tag{3.42}$$

The couplings of the tops to the higgs:

$$\begin{aligned} y_{tt} &= \frac{y^2}{(\lambda^2 + y^2)^{1/2}} \left(1 - \frac{v^2}{f^2} \frac{y^4 + 8\lambda^2 y^2 + 4\lambda^4}{2(\lambda^2 + y^2)^2}\right) \\ y_{TT} &= -\frac{v}{f} \frac{y^4}{(\lambda^2 + y^2)^{3/2}} \\ y_{Tt} &= \frac{v}{f} \frac{\lambda y (\lambda^2 + 2y^2)}{(\lambda^2 + y^2)^{3/2}} \\ y_{tT} &= \frac{\lambda y}{(\lambda^2 + y^2)^{1/2}} \left(1 - \frac{v^2}{f^2} \frac{\lambda^4 - 4y^4}{2(\lambda^2 + y^2)^2}\right) \end{aligned} \tag{3.43}$$

Similarly, we can compute the couplings of two higgses to top quarks. The diagonal couplings are given by

$$\begin{aligned} k_t &= y^2 \frac{v^2}{f^2} \frac{y^2(2\lambda^2 + y^2)}{(\lambda^2 + y^2)^2} \\ k_T &= y^2 \left(1 - \frac{v^2}{f^2} \frac{y^2(2\lambda^2 + y^2)}{(\lambda^2 + y^2)^2}\right) \end{aligned} \tag{3.44}$$

Using these couplings we can compute the coefficient of the log givergence

$$\Delta m_h^2 \to -f^2 y^4 \frac{N_c}{4\pi^2} \log \Lambda^2 \tag{3.45}$$

The ratio R turns out to be

$$R^{1/2} \approx 1 - \frac{3}{2} x^2 \tag{3.46}$$

which indeed matches $\cos 2x / \cos x$. Fine.

5D inspired fundamental SO(5) model

The following model is designed to reproduce the zero-mode level and the first excited level of the top sector in 5D pseudo-goldstone higgs models with the top embedded in the fundamental representation. On the left side it has two doublets $q_{L,i}$ one singlet T_L^c and one exotic doublet $\tilde{Q}_L$. On the right side it has two singlets $t_{R,i}^c$, one doublet Q_R and one exotic doublet $\tilde{Q}_R$. This makes four top states, one massless chiral bottom, and big vector-like bottom and one vector-like exotic. The fields are collected into an $SO(5)$ fundamental as follows

$$\Psi_L = \begin{pmatrix} \frac{1}{\sqrt{2}}(b_L + \chi_L) \\ -\frac{i}{\sqrt{2}}(b_L - \chi_L) \\ \frac{i}{\sqrt{2}}(t_{L,1} - \tilde{T}_L) \\ \frac{1}{\sqrt{2}}(t_{L,1} + \tilde{T}_L) \\ T_L^c \end{pmatrix} \qquad \Psi_R = \begin{pmatrix} \frac{1}{\sqrt{2}}(B_R + \chi_R) \\ -\frac{i}{\sqrt{2}}(B_R - \chi_R) \\ \frac{i}{\sqrt{2}}(T_R - \tilde{T}_R) \\ \frac{1}{\sqrt{2}}(T_R + \tilde{T}_R) \\ t_{R,2}^c \end{pmatrix} \tag{3.47}$$

The big difference with the previous case is that the heavy states fill the full SO(5) fundamental representation.

The higgs is also embedded in an $SO(5)$ fundamental as $\Phi = f(0,0,0,\sin(h/f),\cos(h/f))$ Yukawas have $SO(5)$ global symmetry, which is softly broken by mass terms:

$$\begin{aligned} -\mathcal{L}_{yuk} = & \quad y_1 \overline{\Psi}_L \Phi t_{R,1}^c + y_2 \overline{T_L^c} \Phi^\dagger \Psi_R \\ & f\lambda_1 \overline{q}_{L,1} Q_R + f\lambda_2 \overline{q}_{L,2} Q_R + f\lambda_\chi \overline{\tilde{Q}_L} \tilde{Q}_R + \text{h.c.} \end{aligned} \tag{3.48}$$

Some mass terms allowed by gauge symmetries are omitted, as they would violate softness, as e.g. $T_L^c t_{R,2}^c$.

Inserting the higgs we get the mass terms for the tops

$$\begin{aligned} -f^{-1}\mathcal{L}_m = & \; y_1 \sin x \overline{t}_{L,1} t_{R,1}^c + y_1 \sin x \overline{\tilde{T}_L} t_{R,1}^c + \sqrt{2} y_1 \cos x \overline{T_L^c} t_{R,1}^c \\ & + y_2 \sin x \overline{T_L^c} T_R + y_2 \sin x \overline{T_L^c} \tilde{T}_R + \sqrt{2} y_2 \cos x \overline{T_L^c} t_{R,2}^c \\ & + \lambda_1 \overline{t}_{L,1} T_R + \lambda_2 \overline{t}_{L,2} T_R + \lambda_\chi \overline{\tilde{T}_L} \tilde{T}_R + \text{h.c.} \end{aligned} \tag{3.49}$$

The matrix:

$$(\bar t_{L,1}, \bar t_{L,2}, T^c_L, \overline{\tilde T_L}) M \begin{pmatrix} t^c_{R,1} \\ t^c_{R,2} \\ T_R \\ \tilde T_R \end{pmatrix} \qquad M = f \begin{pmatrix} y_1 \sin x & 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ \sqrt{2} y_1 \cos x & \sqrt{2} y_2 \cos x & y_2 \sin x & y_2 \sin x \\ y_1 \sin x & 0 & 0 & \lambda_\chi \end{pmatrix} \tag{3.50}$$

One can check that the trace of the mass matrix squared does not depend on h, which means there are no quadratic divergences to the higgs mass parameters at one loop.

Surprise, the determinant is different than in the previous model and is proportional to $\sin 2x$ rather than $\sin x$. In consequence, the R-ration becomes:

$$R = \frac{\cos 2x}{\cos x} \tag{3.51}$$

It is still always smaller than one (expect close to $x = \pi/2$ but in this region one of the vector-like tops becomes light and this computation is invalid).

4 5D Models

We move to 5D models of gauge-higgs unification. Before going to concrete and steel models, we prove some remarkably useful formulae.

The 5D action for a fermion charged under a gauge group reads

$$S_5 = \int d^4x \int_0^L dy \sqrt{g} \left\{ \overline{\Psi} (i \Gamma^N (D_N - i g_5 A_N) - M) \Psi \right\} . \tag{4.1}$$

We assume the warped background $ds^2 = a^2(y) dx^2 - dy^2$. Furthermore, we assume that the fifth component acquires a vev:

$$\langle A_5 \rangle = T^{\hat a} \frac{a^{-2}(y)}{\left(\int_0^L a^{-2} \right)^{1/2}} \tilde v \tag{4.2}$$

where we singled out one generator $T^{\hat a}$ along which the vev resides. The quadratic action in that background reads

$$S_5 = \int d^4x \int_0^L dy \left\{ i a^3 \overline{\Psi} \gamma^\mu \partial_\mu \Psi - a^4 \overline{\Psi} (\gamma^5 \hat D_y - M) \Psi \right\} . \tag{4.3}$$

where $\hat D_y = \hat\partial_y - i g_5 \langle A_5 \rangle$ and $\hat \partial_y = \partial_y + 2a'/a$.

We expand the fermions into the KK mass eigenstates

$$\Psi_L(x,y) = f_{L,n}(\tilde v, y) \Psi_{L,n}(x) \qquad \Psi_R(x,y) = f_{R,n}(\tilde v, y) \Psi_{R,n}(x) \tag{4.4}$$

The profiles satisfy the equations of motion:

$$\begin{aligned} \left(\hat D_5 + M \right) f_{R,n}(\tilde v, y) &= m_n a^{-1} f_{L,n}(\tilde v, y) \\ \left(-\hat D_5 + M \right) f_{L,n}(\tilde v, y) &= m_n a^{-1} f_{R,n}(\tilde v, y) \end{aligned} \tag{4.5}$$

and boundary conditions appropriate for the model. Furthermore, there are the orthonormality/normalization condition:

$$\int_0^L a^3(y) f_{L,n}^\dagger(\tilde{v}, y) f_{L,m}(\tilde{v}, y) = \int_0^L a^3(y) f_{R,n}^\dagger(\tilde{v}, y) f_{R,m}(\tilde{v}, y) = \delta_{nm} \tag{4.6}$$

and the completeness relation

$$\sum_n f_{L,n}^{a}{}^\dagger(\tilde{v}, y) f_{L,n}^b(\tilde{v}, z) = \sum_n f_{R,n}^{a}{}^\dagger(\tilde{v}, y) f_{R,n}^b(\tilde{v}, z) = \delta_{ab} a^{-3}(y) \delta(y - z) \tag{4.7}$$

where a, b are indices of the representation of Ψ. With this expansion, the quadratic fermionic action become diagonal in the KK basis:

$$S_5 = \int d^4x \left\{ \overline{\Psi}_n(x) [i\gamma^\mu \partial_\mu - m_n] \Psi_n(x) \right\} . \tag{4.8}$$

We define the higgs field as the (y-independent) oscillation around $\tilde{v}$:

$$A_5 \to \frac{a^{-2}(y)}{\left[\int_0^L a^{-2}\right]^{1/2}} T^{\hat{a}} (\tilde{v} + h(x)) \tag{4.9}$$

The normalization factors are to make the higgs $h(x)$ canonically normalized in the 4D theory.

The yukawa coupling of the higgs to the fermionic eigenstates read:

$$y_{nm} = -i g_5 \frac{\int_0^L a^2(y) f_{L,n}^\dagger(\tilde{v}, y) T^{\hat{a}} f_{R,n}(\tilde{v}, y)}{\left[\int_0^L a^{-2}\right]^{1/2}} \tag{4.10}$$ e.hyc

We are interested in the contribution of the fermionic eigenstates to the ggh amplitude. In the following we assume that all the fermionic eigenstates are heavier than the higgs and we can use the result in the decoupling limit. This assumption is OK for a top quark tower, as long as the higgs is lighter than the top, but it is certainly not valid for a bottom quark tower!. For the latter separate treatment is needed.

We compute:

$$R^{1/2} = v \mathrm{Re} \left\{ \sum_n \frac{y_{nn}}{m_n} \right\} = -i \frac{g_5 v}{\left[\int_0^L a^{-2}\right]^{1/2}} \mathrm{Re} \left\{ \sum_n m_n^{-1} \int_0^L a^2(y) f_{L,n}^\dagger(\tilde{v}, y) T^{\hat{a}} f_{R,n}(\tilde{v}, y) \right\} \tag{4.11}$$

We now use the following trick. We can rewrite the equations of motion in the integral form as:

$$\begin{aligned} f_{R,n}(\tilde{v}, y) &= \Omega(y) a^{-2}(y) e^{-My} \left(f_{R,n}(\tilde{v}, 0) + m_n \int_0^y a(y') e^{My'} \Omega^{-1}(y') f_{L,n}(\tilde{v}, y') \right) \\ f_{L,n}(\tilde{v}, y) &= \Omega(y) a^{-2}(y) e^{My} \left(f_{L,n}(\tilde{v}, 0) - m_n \int_0^y a(y') e^{-My'} \Omega^{-1}(y') f_{R,n}(\tilde{v}, y') \right) \end{aligned} \tag{4.12}$$

where

$$\Omega(y) = e^{ig_5 \int_0^y \langle A_5 \rangle} \tag{4.13}$$

Inserting this into the expression for $R^{1/2}$ and using the completeness relation we derive:

$$R^{1/2} = -ig_5 v \left[\int_0^L a^{-2} \right]^{1/2} \mathrm{Re} \sum_n m_n^{-1} f_{L,n}^\dagger(\tilde{v}, 0) T^{\hat{a}} f_{R,n}(\tilde{v}, 0) \tag{4.14}$$

To clean up, we introduce the scale f:

$$f = \frac{\sqrt{2}}{g_5 \left[\int_0^L a^{-2} \right]^{1/2}} \tag{4.15}$$

Furthermore, a trained eye may notice that the sum is related to the propagator in 5D position space. More precisely, we have

$$P^{ab}(p^2, y, z) = \sum_n f_n^a(\tilde{v}, y) f_n^{b*}(\tilde{v}, z) \frac{i(\not{p} + m_n)}{p^2 - m_n^2} \tag{4.16}$$

Thus the ratio is given by the following simple expression

$$R^{1/2} = \frac{v}{f} \mathrm{Re}\ \mathrm{Tr}[\sqrt{2} T^{\hat{a}} P_{LR}(0,0,0)] = -\frac{v}{f} \mathrm{Re}\ \mathrm{Tr}[\sqrt{2} T^{\hat{a}} P_{RL}(0,0,0)] \tag{4.17}$$

All we have to do is to compute the UV brane-to-brane propagator of the 5D fermion (in the gauge-higgs background) and take the zero momentum limit. As simple as that. Holography is dripping from this formula.

How to compute the propagators is explained in [5]. Here we apply those methods to the specific 5D models.

SO(5) Spinorial Light-Shadow Model

We start with the light-shadow spinorial model, that includes two $SO(5)$ spinor multiplets mixed by IR mass terms. The light multiplet is denoted by Q, while the shadow multiplet is denoted as S. They have bulk masses M_Q and M_S. The **4** represenation decomposes under $SU(2)_L \times SU(2)_R$ as $4 = (2,1) \oplus (1,2)$ The top and bottom quarks are embedded as

$$Q = (q,\ q^c) = (t,\ b,\ t^c,\ b^c) \qquad S = (\tilde{q},\ \tilde{q}^c) = (\tilde{t},\ \tilde{b},\ \tilde{t}^c,\ \tilde{b}^c) \tag{4.18}$$

The UV boundary conditons are analogous:

$$\begin{aligned} f_R^t(0) &= 0 \qquad & f_L^{t^c}(0) &= 0 \\ f_R^{\tilde{t}}(0) &= 0 \qquad & f_L^{\tilde{t}^c}(0) &= 0 \end{aligned} \tag{4.19}$$

The IR boundary conditions are different though. In the presence of IR brane mass terms,

$$\mathcal{L} = -\delta(L) \tilde{M} a_L^4 \overline{q}_L \tilde{q}_R - \delta(L) \tilde{M}_c a_L^4 \overline{q}_R^c \tilde{q}_L^c + \mathrm{h.c.}, \tag{4.20}$$

they read

$$\begin{aligned} f_R^t(L) &= \tilde{M} f_R^{\tilde{t}}(L) & \qquad f_L^{t^c}(L) &= -\tilde{M}_c f_L^{\tilde{t}^c}(L) \\ f_L^{\tilde{t}}(L) &= -\tilde{M}^* f_L^t(L) & \qquad f_R^{\tilde{t}^c}(L) &= \tilde{M}_c^* f_R^{t^c}(L) \end{aligned} \tag{4.21}$$

We write down the propagator ansatz that is compatible with these boundary conditions

$$\begin{aligned} \hat{P}_{LL}^{<ta} &= C_{M_Q}(y) c_L^{<ta}(z) & \qquad \hat{P}_{LL}^{>ta} &= \bar{C}_{M_Q}(y) c_L^{>ta}(z) + \tilde{M}\bar{S}_{M_Q}(y) c_L^{>\tilde{t}a}(z) \\ \hat{P}_{LL}^{<t^ca} &= S_{M_Q}(y) c_L^{<t^ca}(z) & \qquad \hat{P}_{LL}^{>t^ca} &= \bar{S}_{M_Q}(y) c_L^{>t^ca}(z) - \tilde{M}_c\bar{C}_{M_Q}(y) c_L^{>\tilde{t}^ca}(z) \\ \hat{P}_{LL}^{<\tilde{t}a} &= C_{M_S}(y) c_L^{<\tilde{t}a}(z) & \qquad \hat{P}_{LL}^{>\tilde{t}a} &= \bar{S}_{M_S}(y) c_L^{>\tilde{t}a}(z) - \tilde{M}^*\bar{C}_{M_S}(y) c_L^{>ta}(z) \\ \hat{P}_{LL}^{<\tilde{t}^ca} &= S_{M_S}(y) c_L^{<\tilde{t}^ca}(z) & \qquad \hat{P}_{LL}^{>\tilde{t}^ca} &= \bar{C}_{M_S}(y) c_L^{>\tilde{t}^ca}(z) + \tilde{M}_c^*\bar{S}_{M_S}(y) c_L^{>t^ca}(z) \end{aligned} \tag{4.22}$$

$$\begin{aligned} \hat{P}_{RR}^{<ta} &= S_{-M_Q}(y) c_R^{<ta}(z) & \qquad \hat{P}_{RR}^{>ta} &= \bar{S}_{-M_Q}(y) c_R^{>ta}(z) + \tilde{M}\bar{C}_{-M_Q}(y) c_R^{>\tilde{t}a}(z) \\ \hat{P}_{RR}^{<t^ca} &= C_{-M_Q}(y) c_R^{<t^ca}(z) & \qquad \hat{P}_{RR}^{>t^ca} &= \bar{C}_{-M_Q}(y) c_R^{>t^ca}(z) - \tilde{M}_c\bar{S}_{-M_Q}(y) c_R^{>\tilde{t}^ca}(z) \\ \hat{P}_{RR}^{<\tilde{t}a} &= S_{-M_S}(y) c_R^{<\tilde{t}a}(z) & \qquad \hat{P}_{RR}^{>\tilde{t}a} &= \bar{C}_{-M_S}(y) c_R^{>\tilde{t}a}(z) - \tilde{M}^*\bar{S}_{-M_S}(y) c_R^{>ta}(z) \\ \hat{P}_{RR}^{<\tilde{t}^ca} &= C_{-M_S}(y) c_R^{<\tilde{t}^ca}(z) & \qquad \hat{P}_{RR}^{>\tilde{t}^ca} &= \bar{S}_{-M_S}(y) c_R^{>\tilde{t}^ca}(z) + \tilde{M}_c^*\bar{C}_{-M_S}(y) c_R^{>t^ca}(z) \end{aligned} \tag{4.23}$$

The $c(z)$'s are determined by the gluing conditions:

$$\begin{aligned} \Omega(L)\hat{P}_{LL}^{<}(z,z) &= \hat{P}_{LL}^{>}(z,z) \\ \Omega(L)\partial_y\hat{P}_{LL}^{<}(y,z)|_{y=z} &= \partial_y\hat{P}_{LL}^{>}(y,z)|_{y=z} - ia_z^{-3}e^{-Mz}\bar{\Omega}^{-1}(z) \\ \Omega(L)\hat{P}_{RR}^{<}(z,z) &= \hat{P}_{RR}^{>}(z,z) \\ \Omega(L)\partial_y\hat{P}_{RR}^{<}(y,z)|_{y=z} &= \partial_y\hat{P}_{RR}^{>}(y,z)|_{y=z} - ia_z^{-3}e^{Mz}\bar{\Omega}^{-1}(z) \end{aligned} \tag{4.24}$$

which lead to two systems of equations with eight variables.

At this point i decide that i've done enough at let Wolfram do the rest. I only asked him the final expression for

$$\mathrm{Tr}[\sqrt{2}T^{\hat{a}}P_{LR}(p^2,0,0)] = \frac{1}{2}P_{LR}^{tt^c}(p^2,0,0) + \frac{1}{2}P_{LR}^{\tilde{t}\tilde{t}^c}(p^2,0,0) \tag{4.25}$$

He quoteth:

$$\begin{aligned} &\tfrac{\sin(\tilde{v}/f)}{4}\Big\{|1-\tilde{M}\tilde{M}_c^*|^2\cos(\tilde{v}/f) + (1-|\tilde{M}\tilde{M}_c|^2)(S_{M_Q}(L)S_{-M_Q}(L) - S_{M_S}(L)S_{-M_S}(L))\Big\} \\ &\qquad /\Big\{[C_{M_S}(L)S_{-M_Q}(L) + |\tilde{M}|^2 e^{2(M_Q-M_S)L}C_{M_Q}(L)S_{-M_S}(L)] \\ &\qquad \cdot[C_{-M_S}(L)S_{M_Q}(L) + |\tilde{M}_c|^2 e^{2(M_S-M_Q)L}C_{-M_Q}(L)S_{M_S}(L)] \\ &\quad -\sin^2(\tilde{v}/2f)[|1-\tilde{M}\tilde{M}_c^*|^2 + (1-|\tilde{M}\tilde{M}_c|^2)(S_{M_Q}(L)S_{-M_Q}(L) - S_{M_S}(L)S_{-M_S}(L))] \\ &\qquad +\sin^4(\tilde{v}/2f)|1-\tilde{M}\tilde{M}_c^*|^2\Big\} \end{aligned} \tag{4.26}$$

Looks complicated, but the limit $p^2 \to 0$ is simple as a wire:

$$\mathrm{Tr}[\sqrt{2}T^{\hat{a}}P_{LR}(0,0,0)] = \frac{\cos(\tilde{v}/f)}{\sin(\tilde{v}/f)} \tag{4.27}$$

independently of the all the top sector parameters.

Thus the ratio is found to be

$$R = \cos^2(\tilde{v}/f) \tag{4.28}$$

just like in the 4D spinorial model.

SO(5) Spinorial Top-Bottom Model

Here is a semi-realistic model that properly accounts for the top-bottom mass splitting and has enough structure to produce a correct EW breaking vacuum (the light-shadow mechanism [6] is operational).

We introduce two 5D quarks in the spinorial representation:

$$Q_1 = (q_1,\ q_1^c) = (t_1,\ b_1,\ t_1^c,\ b_1^c) \qquad Q_2 = (q_2,\ q_2^c) = (t_2,\ b_2,\ t_2^c,\ b_2^c) \tag{4.29}$$

The IR boundary conditions are the same for the two:

$$q_{R,i}(L) = q^c_{L,i}(L) = 0 \tag{4.30}$$

For the UV boundary conditions we first try:

$$q_{R,i}(0) = t^c_{L,1}(0) = b^c_{R,1}(0) = t^c_{R,2}(0) = b^c_{L,2}(0) = 0 \tag{4.31}$$

This would give us two zero mode doublets $q_{L,i}$ and one zero-mode top singlet $t^c_{L,1}$ and one zero-mode top singlet $b^c_{L,1}$. Clearly, we should get rid of one doublet zero mode. This can be done economically by introducing a UV brane RH fermion $\tilde{q}_R$ that mixes with the two doublets:

$$\mathcal{L} = i\delta(0)\bar{\tilde{q}}_R\gamma^\mu\partial_\mu\tilde{q}_R - \delta(0)\Lambda^{1/2}\left\{\theta_1\bar{q}_{L,1}\tilde{q}_R - \theta_2\bar{q}_{L,2}\tilde{q}_R + \text{h.c.}\right\} \tag{4.32}$$

We take the limit $\Lambda \to \infty$. In this limit the combination that couples to the boundary fermion has vanishing *left* chirality on the UV brane, while for the orthogonal combination the *right* chirality vanish there.

In the end, the profiles satisfy the following UV boundary conditions

$$\begin{aligned}
\theta_2 f^{q_1}_{R,n}(0) - \theta_1 f^{q_2}_{R,n}(0) &= 0 \\
\bar{\theta}_1 f^{q_1}_{L,n}(0) + \bar{\theta}_2 f^{q_2}_{L,n}(0) &= 0 \\
f^{t_1^c}_{L,n}(0) = f^{t_2^c}_{R,n}(0) &= 0 \\
f^{b_1^c}_{R,n}(0) = f^{b_2^c}_{L,n}(0) &= 0
\end{aligned} \tag{4.33}$$

There are two parameters that break the custodial symmetry: $M_1 - M_2$ and $\theta_1 - \theta_2$. Both can be used to generate the top-bottom mass splitting.

The quantization condition for the top tower

$$\begin{aligned}
&\theta^2 C_{-M_1}(L)S_{M_1}(L)C_{M_2}(L)S_{-M_2}(L) \\
&+ \left(S_{M_1}(L)S_{-M_1}(L) - \sin^2(\tilde{v}/2f)\right)\left(C_{M_2}(L)C_{-M_2}(L) - \sin^2(\tilde{v}/2f)\right) = 0
\end{aligned} \tag{4.34}$$

Here $\theta^2 = |\theta_1/\theta_2|^2$.

We need to compute

$$\mathrm{Tr}[\sqrt{2}T^{\hat{a}}P_{RL}(p^2,0,0)] = \frac{1}{2}\left\{P_{RL}^{t_1 t_1^c} + + P_{RL}^{t_1^c t_1} + P_{RL}^{t_2 t_2^c}\right\} \tag{4.35}$$

We find

$$\begin{aligned} &\tfrac{1}{4}\sin(\tilde{v}/f)\left\{S_{M_1}(L)S_{-M_1}(L) - S_{M_2}(L)S_{-M_2}(L) + \cos(\tilde{v}/f)\right\} \\ &\quad / \left\{\theta^2 C_{-M_1}(L)S_{M_1}(L)C_{M_2}(L)C_{-M_2}(L)\right. \\ &+ \left.\left(S_{M_1}(L)S_{-M_1}(L) - \sin^2(\tilde{v}/2f)\right)\left(S_{M_2}(L)S_{-M_2}(L) - \cos^2(\tilde{v}/2f)\right)\right\} \end{aligned} \tag{4.36}$$

In the limit $p^2 \to 0$:

$$\mathrm{Tr}[\sqrt{2}T^{\hat{a}}P_{LR}(0,0,0)] = \frac{\cos(\tilde{v}/f)}{\sin(\tilde{v}/f)} \tag{4.37}$$

again...

The bottom profiles

$$\begin{aligned} \hat{f}_L^{b_1}(y) &= -\alpha_{b_1}\theta_1 S_{M_1}(y) + \alpha_{b_2}\bar{\theta}_2 C_{M_1}(y) \\ \hat{f}_R^{b_1}(y) &= \alpha_{b_1}\theta_1 C_{-M_1}(y) + \alpha_{b_2}\bar{\theta}_2 S_{-M_1}(y) \\ \hat{f}_L^{b_2}(y) &= -\alpha_{b_1}\theta_2 S_{M_2}(y) - \alpha_{b_2}\bar{\theta}_1 C_{M_2}(y) \\ \hat{f}_R^{b_2}(y) &= \alpha_{b_1}\theta_2 C_{-M_2}(y) - \alpha_{b_2}\bar{\theta}_1 S_{-M_2}(y) \\ \hat{f}_L^{b_1^c}(y) &= \alpha_{b_1^c} C_{M_1}(y) \\ \hat{f}_R^{b_1^c}(y) &= \alpha_{b_1^c} S_{-M_1}(y) \\ \hat{f}_L^{b_2^c}(y) &= -\alpha_{b_2^c} S_{M_2}(y) \\ \hat{f}_R^{b_2^c}(y) &= \alpha_{b_2^c} C_{-M_2}(y) \end{aligned} \tag{4.38}$$

Solving the boundary conditions for the constants we find

$$\begin{aligned} \alpha_{b_1} &= -i\alpha_{b_2^c}\frac{1}{\sin(\tilde{v}/2f)\cos(\tilde{v}/2f)}\frac{\bar{\theta}_2}{|\theta_1|^2}\frac{C_{M_1}(L)S_{-M_1}(L)C_{-M_2}(L)S_{M_2}(L)}{\cos^2(\tilde{v}/2f) - S_{M_1}(L)S_{-M_1}(L)} \\ \alpha_{b_2} &= i\alpha_{b_2^c}\frac{1}{\sin(\tilde{v}/2f)\cos(\tilde{v}/2f)}\frac{\theta_1}{|\theta_1|^2}C_{-M_2}(L)S_{M_2}(L) \\ \alpha_{b_1^c} &= \alpha_{b_2^c}\frac{\bar{\theta}_2}{\bar{\theta}_1}\frac{C_{-M_2}(L)S_{M_2}(L)}{\cos^2(\tilde{v}/2f) - S_{M_1}(L)S_{-M_1}(L)} \end{aligned} \tag{4.39}$$

and the quantization condition for the bottom tower

$$\begin{aligned} &C_{M_1}(L)S_{-M_1}(L)C_{-M_2}(L)S_{M_2}(L) \\ &+\theta^2\left(S_{M_2}(L)S_{-M_2}(L) - \sin^2(\tilde{v}/2f)\right)\left(C_{M_1}(L)C_{-M_1}(L) - \sin^2(\tilde{v}/2f)\right) = 0 \end{aligned} \tag{4.40}$$

The higgs couplings to the lightest bottom quark can be computed from eq. (4.10)

$$y_{bb} = |\alpha_{b^c}|^2\frac{S_{M_2}(L)C_{-M_2}(L)}{v}\left[1 - \frac{\sin^2(\tilde{v}/2f) - S_{M_2}(L)S_{-M_2}(L)}{\cos^2(\tilde{v}/2f) - S_{M_1}(L)S_{-M_1}(L)}\right] \tag{4.41}$$

Approximately,

$$y_{bb} \approx |\alpha_{b^c}|^2\frac{S_{M_2}(L)}{v}\frac{\cos(\tilde{v}/f)}{\cos^2(\tilde{v}/2f)} + \mathcal{O}(m_b^2) \tag{4.42}$$

The constant in front can be calculated from normalization. Assuming that the m_b/m_t hierarchy come from the smallness of θ_1/θ_2 we find $|\alpha_{b_c}|^2 \approx \cos^2(\tilde{v}/f)/\int a^{-1}e^{-2M_2 y} + \mathcal{O}(m_b^2)$. Then

$$\frac{y_{bb}}{m_b} \approx v^{-1}\cos(\tilde{v}/f) + \mathcal{O}(m_b^2) \tag{4.43}$$

so that

$$\sum_{n>0} \frac{y^b_{nn}}{m^b_n} = \mathcal{O}(m_b^2) \tag{4.44}$$

Thus, the bottom tower contribution to the gluon-gluon-higgs amplitude is strongly suppressed.

SO(5) Fundamental Top-Bottom Model

We introduce two 5D quarks Q_1, Q_2 in the fundamental representation **5**, with X charge 2/3 and $-1/3$

$$Q_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi + b_1 \\ i(\chi - b_1), \\ t_1 + \tilde{t} \\ i(t_1 - \tilde{t}) \\ \sqrt{2}t^c \end{pmatrix} \qquad Q_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho + t_1 \\ i(\rho - t_1), \\ b_1 + \tilde{b} \\ i(b_1 - \tilde{b}) \\ \sqrt{2}b^c \end{pmatrix} \tag{4.45}$$

The upper four in the multiplet can be collected into bi-doublets: $(q_1, \tilde{q}_1)$ and $(q_2, \tilde{q}_2)$. The IR boundary conditions are:

$$q_{R,i}(L) = \tilde{q}_{R,i}(L) = 0 \qquad t^c_L(L) = b^c_L(L) = 0 \tag{4.46}$$

For the UV boundary conditions we again use the trick with a boundary doublet. In the end, the fermions satisfy the following UV boundary conditions:

$$\begin{aligned} \theta_2 q_{1,R}(0) - \theta_1 q_{2,R}(0) &= 0 \\ \bar{\theta}_1 q_{1,L}(0) + \bar{\theta}_2 q_{2,L}(0) &= 0 \\ t^c_L(0) = b^c_L(0) &= 0 \\ \tilde{q}_{1,L}(0) = \tilde{q}_{2,L}(0) &= 0 \end{aligned} \tag{4.47}$$

The top profiles satisfying UV boundary conditions (n suppressed):

$$\begin{aligned} \hat{f}^{t_1}_L(y) &= -\alpha_{t_1}\theta_1 S_{M_1}(y) + \alpha_{t_2}\bar{\theta}_2 C_{M_1}(y) \\ \hat{f}^{t_1}_R(y) &= \alpha_{t_1}\theta_1 C_{-M_1}(y) + \alpha_{t_2}\bar{\theta}_2 S_{-M_1}(y) \\ \hat{f}^{t_2}_L(y) &= -\alpha_{t_1}\theta_2 S_{M_2}(y) - \alpha_{t_2}\bar{\theta}_1 C_{M_2}(y) \\ \hat{f}^{t_2}_R(y) &= \alpha_{t_1}\theta_2 C_{-M_2}(y) - \alpha_{t_2}\bar{\theta}_1 S_{-M_2}(y) \\ \hat{f}^{\tilde{t}}_L(y) &= -\alpha_{\tilde{t}} S_{M_1}(y) \\ \hat{f}^{\tilde{t}}_R(y) &= \alpha_{\tilde{t}} C_{-M_1}(y) \\ \hat{f}^{t^c}_L(y) &= -\alpha_{t^c} S_{M_1}(y) \\ \hat{f}^{t^c}_R(y) &= \alpha_{t^c} C_{-M_1}(y) \end{aligned} \tag{4.48}$$

The quantization condition for the top tower

$$C_{-M_1}(L)\left[\theta^2 C_{-M_1}(L)S_{M_1}(L)S_{-M_2}(L)+C_{-M_2}(L)\left(S_{M_1}(L)S_{-M_1}(L)-\frac{1}{2}\sin^2(\tilde{v}/f)\right)\right]=0 \tag{4.49}$$

For gluon-fusion purposes we need to compute

$$\mathrm{Tr}[\sqrt{2}T^{\hat{a}}P_{RL}(p^2,0,0)]=\frac{1}{\sqrt{2}}\left\{-P_{RL}^{t^c t_1}+P_{RL}^{t^c\tilde{t}}\right\} \tag{4.50}$$

We find

$$\begin{aligned}&-\tfrac{1}{4}\sin(2\tilde{v}/f)C_{-M_2}(L)\\ &/\left[C_{-M_2}(L)S_{M_2}(L)S_{-M_1}(L)+\theta^2 C_{-M_1}(L)\left(S_{M_2}(L)S_{-M_2}(L)-\tfrac{1}{2}\sin^2(\tilde{v}/f)\right)\right]\end{aligned} \tag{4.51}$$

In the limit $p^2\to 0$:

$$\mathrm{Tr}[\sqrt{2}T^{\hat{a}}P_{RL}(0,0,0)]=\frac{\cos(\tilde{v}/f)}{\sin(\tilde{v}/f)} \tag{4.52}$$

So that

$$\sum_n \frac{y^t_{nn}}{m_t}=v^{-1}\cos(\tilde{v}/f) \tag{4.53}$$

The bottom profiles

$$\begin{aligned}
\hat{f}^{b_1}_L(y) &= -\alpha_{b_1}\theta_1 S_{M_1}(y)+\alpha_{b_2}\bar{\theta}_2 C_{M_1}(y)\\
\hat{f}^{b_1}_R(y) &= \alpha_{b_1}\theta_1 C_{-M_1}(y)+\alpha_{b_2}\bar{\theta}_2 S_{-M_1}(y)\\
\hat{f}^{b_2}_L(y) &= -\alpha_{b_1}\theta_2 S_{M_2}(y)-\alpha_{b_2}\bar{\theta}_1 C_{M_2}(y)\\
\hat{f}^{b_2}_R(y) &= \alpha_{b_1}\theta_2 C_{-M_2}(y)-\alpha_{b_2}\bar{\theta}_1 S_{-M_2}(y)\\
\hat{f}^{\tilde{b}}_L(y) &= -\alpha_{\tilde{b}} S_{M_2}(y)\\
\hat{f}^{\tilde{b}}_R(y) &= \alpha_{\tilde{b}} C_{-M_2}(y)\\
\hat{f}^{b^c}_L(y) &= -\alpha_{b^c} S_{M_2}(y)\\
\hat{f}^{b^c}_R(y) &= \alpha_{b^c} C_{-M_2}(y)
\end{aligned} \tag{4.54}$$

The quantization condition for the bottom tower

$$C_{-M_2}(L)\left[C_{-M_2}(L)S_{M_2}(L)S_{-M_1}(L)+\theta^2 C_{-M_1}(L)\left(S_{M_2}(L)S_{-M_2}(L)-\frac{1}{2}\sin^2(\tilde{v}/f)\right)\right]=0 \tag{4.55}$$

Solving for the constants in the bottom sector:

$$\begin{aligned}
\alpha_{b_1} &= i\alpha_{b^c}\frac{\sqrt{2}}{\sin(\tilde{v}/f)\cos(\tilde{v}/f)}\frac{\bar{\theta}_2}{|\theta_1|^2}\frac{S_{-M_1}(L)C_{-M_2}(L)S_{M_2}(L)}{C_{-M_1}(L)}\\
\alpha_{b_2} &= -i\alpha_{b^c}\frac{\sqrt{2}}{\sin(\tilde{v}/f)\cos(\tilde{v}/f)}\frac{\theta_1}{|\theta_1|^2}C_{-M_2}(L)S_{M_2}(L)\\
\alpha_{\tilde{b}} &= -i\alpha_{b^c}\frac{\sin(\tilde{v}/f)}{\sqrt{2}\cos(\tilde{v}/f)}
\end{aligned} \tag{4.56}$$

The higgs couplings to the lightest bottom quark can be computed from eq. (4.10)

$$y_{bb} = |\alpha_{b^c}|^2 \frac{S_{M_2}(L) C_{-M_2}(L)}{v \cos(\tilde{v}/f)} \tag{4.57}$$

The constant in front can be calculated from normalization. Assuming that the m_b/m_t hierarchy come from the smallness of θ_1/θ_2 we find $|\alpha_{b_c}|^2 \approx \cos^2(\tilde{v}/f)/\int a^{-1} e^{-2M_2 y} + \mathcal{O}(m_b^2)$. Then

$$\frac{y_{bb}}{m_b} \approx v^{-1} \cos(\tilde{v}/f) + \mathcal{O}(m_b^2) \tag{4.58}$$

so that

$$\sum_{n>0} \frac{y^b_{nn}}{m^b_n} = \mathcal{O}(m_b^2) \tag{4.59}$$

Thus, the bottom tower contribution to the gluon-gluon-higgs amplitude is strongly suppressed.

Appendix A SO(5) generators

a.g

Appendix A.1 Spinorial

The smallest $SO(5)$ representation is the **4** spinorial.

The five 4x4 gamma matrices of SO(5) can be written in a convenient basis as:

$$\Gamma^a = \begin{bmatrix} 0 & \sigma^a \\ \sigma^a & 0 \end{bmatrix} \quad \Gamma^4 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \Gamma^a = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{A.1}$$

From that one can construct the 10 generators:

$$T^a_L = \frac{1}{2}\begin{bmatrix} \sigma^a & 0 \\ 0 & 0 \end{bmatrix} \qquad T^a_R = \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & \sigma^a \end{bmatrix} \tag{A.2}$$

$$T^a_C = \frac{i}{2\sqrt{2}}\begin{bmatrix} 0 & \sigma^a \\ -\sigma^a & 0 \end{bmatrix} \qquad T^4_C = \frac{1}{2\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{A.3}$$

The T_3s of $SU(2)_L \times SU(2)_R$ are non-diagonal in this basis. Thus, we can easily see how $SU(2)_L \times SU(2)_R$ quantum numbers are embedded in **4**:

$$q = \begin{bmatrix} q_{+0} \\ q_{-0} \\ q_{0+} \\ q_{0-} \end{bmatrix} \qquad \mathbf{4} = (\mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}) \tag{A.4}$$

The wilson exponential $\omega = \exp(i T^{\hat{a}}_C h^{\hat{a}})$ for $h = h^{\hat{4}}$

$$\omega = \begin{bmatrix} \cos(h/2\sqrt{2}) & i\sin(h/2\sqrt{2}) \\ i\sin(h/2\sqrt{2}) & \cos(h/2\sqrt{2}) \end{bmatrix} \tag{A.5}$$

Appendix A.2 Fundamental

Here comes fundamental $SO(5)$ representation **5**.

The 10 generators of the **fundamental representation** in a convenient basis:

$$\begin{aligned}
T^a_{L,ij} &= -\frac{i}{2}\left[\frac{1}{2}\epsilon^{abc}(\delta^b_i\delta^c_j - \delta^b_j\delta^c_i) + (\delta^a_i\delta^4_j - \delta^a_j\delta^4_i)\right] \qquad a = 1\dots 3 \\
T^a_{R,ij} &= -\frac{i}{2}\left[\frac{1}{2}\epsilon^{abc}(\delta^b_i\delta^c_j - \delta^b_j\delta^c_i) - (\delta^a_i\delta^4_j - \delta^a_j\delta^4_i)\right] \qquad a = 1\dots 3 \\
T^{\hat{a}}_{C,ij} &= -\frac{i}{\sqrt{2}}\left[\delta^{\hat{a}}_i\delta^5_j - \delta^{\hat{a}}_j\delta^5_i)\right] \qquad \hat{a} = 1\dots 4
\end{aligned} \tag{A.6}$$

The generators are normalized as $\mathrm{Tr}T^\alpha T^\beta = \delta^{\alpha\beta}$ (not 1/2, silly). T^a_L and T^a_R generate the $SO(4) \equiv SU(2)_L \times SU(2)_R$ subgroup of $SO(5)$ and $T^{\hat{a}}_C$ are the coset generators. The commutators are:

$$[T^a_L, T^b_L] = i\epsilon^{abc}T^c_L \qquad [T^a_R, T^b_R] = i\epsilon^{abc}T^c_R \qquad [T^a_L, T^b_R] = 0 \tag{A.7}$$ e.so5cr

$$[T^a_C, T^b_C] = \frac{i}{2}\epsilon^{abc}(T^c_L + T^c_R) \qquad [T^a_C, T^4_C] = \frac{i}{2}(T^a_L - T^a_R) \tag{A.8}$$

$$[T^a_{L,R}, T^b_C] = \frac{i}{2}\left(\epsilon^{abc}T^c_C \pm \delta^{ab}T^4_C\right) \qquad [T^a_{L,R}, T^4_C] = \mp\frac{i}{2}T^a_C \tag{A.9}$$

The T_3s of $SU(2)_L \times SU(2)_R$ are non-diagonal in this basis:

$$T^3_L = -\frac{i}{2}\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad T^3_R = -\frac{i}{2}\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A.10}$$

We can express the 5-vector of $SO(5)$ as a combination of eigenvectors.

$$Q = \frac{1}{\sqrt{2}}\begin{pmatrix} q_{++} + q_{--} \\ iq_{++} - iq_{--} \\ q_{+-} + q_{-+} \\ iq_{+-} - iq_{-+} \\ \sqrt{2}q^c \end{pmatrix} \tag{A.11}$$

where $\pm$ denotes $\pm 1/2$. Thus **5** contains a bifundamental and a singlet under $SU(2)_L \times SU(2)_R$:

$$\begin{matrix} \begin{bmatrix} q_{+-} & q_{++} \\ q_{--} & q_{-+} \end{bmatrix} & \qquad & q^c \\ (2, \overline{2}) & \qquad & 1 \end{matrix} \tag{A.12}$$

The wilson exponential $\omega = \exp(iT^{\hat{a}}_C h^{\hat{a}})$

$$\omega_{ij} = \delta_{ij} + \left(\delta^{\hat{a}}_i\delta^{\hat{b}}_j + \delta^5_i\delta^5_j\delta^{\hat{a}\hat{b}}\right)\frac{h^{\hat{a}}h^{\hat{b}}}{h^2}\left(\cos(h/\sqrt{2}) - 1\right) + \left(\delta^{\hat{a}}_i\delta^5_j - \delta^{\hat{a}}_j\delta^5_i\right)\frac{h^{\hat{a}}}{h}\sin(h/\sqrt{2}) \tag{A.13}$$ e.fwe

where $h = (h^{\hat{a}} h^{\hat{a}})^{1/2}$. For $h = h^{\hat{4}}$

$$T_C^4 = -\frac{i}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix} \qquad e^{ihT_C^4} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos(h/\sqrt{2}) & \sin(h/\sqrt{2}) \\ 0 & 0 & 0 & -\sin(h/\sqrt{2}) & \cos(h/\sqrt{2}) \end{bmatrix} \tag{A.14}$$

We also show how these matrices operate in the basis (q_{+-}, q_{-+}, q^c):

$$T_C^4 = \frac{1}{2} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \qquad e^{ihT_C^4} = \begin{bmatrix} \frac{1+\cos(h/\sqrt{2})}{2} & \frac{1-\cos(h/\sqrt{2})}{2} & -i\frac{\sin(h/\sqrt{2})}{\sqrt{2}} \\ \frac{1-\cos(h/\sqrt{2})}{2} & \frac{1+\cos(h/\sqrt{2})}{2} & i\frac{\sin(h/\sqrt{2})}{\sqrt{2}} \\ -i\frac{\sin(h/\sqrt{2})}{\sqrt{2}} & i\frac{\sin(h/\sqrt{2})}{\sqrt{2}} & \cos(h/\sqrt{2}) \end{bmatrix} \tag{A.15}$$

One last handy formula:

$$f = f_L^a T_L^a + f_R^a T_R^a + f_C^{\hat{a}} T_C^{\hat{a}} = -\frac{i}{2} \begin{bmatrix} 0 & f_L^3 + f_R^3 & -f_L^2 - f_R^2 & f_L^1 - f_R^1 & \sqrt{2} f_C^1 \\ -f_L^3 - f_R^3 & 0 & f_L^1 + f_R^1 & f_L^2 - f_R^2 & \sqrt{2} f_C^2 \\ f_L^2 + f_R^2 & -f_L^1 - f_R^1 & 0 & f_L^3 - f_R^3 & \sqrt{2} f_C^3 \\ -f_L^1 + f_R^1 & -f_L^2 + f_R^2 & -f_L^3 + f_R^3 & 0 & \sqrt{2} f_C^4 \\ -\sqrt{2} f_C^1 & -\sqrt{2} f_C^2 & -\sqrt{2} f_C^3 & -\sqrt{2} f_C^4 & 0 \end{bmatrix} \tag{A.16}$$

Appendix A.3 Adjoint

The following in size is the antisymmetric tensor representation **10**.

The antisymmetric tensor Φ transforms as $\Phi \to \Omega \Phi \Omega^T$. We can represent the tensor as:

$$\Phi = \Phi_L^a T^a + \Phi_R^a T^a + \Phi_C^{\hat{a}} T^{\hat{a}} \tag{A.17}$$

where T are $SO(5)$ generators in the fundamental representation. From the commutation relations (A.7) it is clear that Φ_L is an $SU(2)_L$ triplet, Φ_R is an $SU(2)_R$ triplet and Φ_C is an $SU(2)_L \times SU(2)_R$ bifundamental. More precisely, the $SU(2)_L \times SU(2)_R$ quantum numbers of various components are

$$(\mathbf{3}, \mathbf{1}): \quad \Phi_L^1 \pm i\Phi_L^2 \to (\pm 1, 0) \quad \Phi_L^3 \to (0, 0) \tag{A.18}$$

$$(\mathbf{1}, \mathbf{3}): \quad \Phi_R^1 \pm i\Phi_R^2 \to (0, \pm 1) \quad \Phi_R^3 \to (0, 0) \tag{A.19}$$

$$(\mathbf{2}, \mathbf{2}): \quad \Phi_C^1 \pm i\Phi_C^2 \to (\pm 1/2, \pm 1/2) \quad \Phi_C^3 \pm i\Phi_C^4 \to (\pm 1/2, \mp 1/2) \tag{A.20}$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{2}, \mathbf{2}) \tag{A.21}$$

References

aa [1] A. Falkowski, "About the holographic pseudo-Goldstone boson," Phys. Rev. D **75**, 025017 (2007) [arXiv:hep-ph/0610336].

sc [2] M. Schmaltz, “The simplest little Higgs,” JHEP **0408**, 056 (2004) [arXiv:hep-ph/0407143].

bary [3] R. Barbieri, B. Bellazzini, V. S. Rychkov and A. Varagnolo, “The Higgs boson from an extended symmetry,” arXiv:0706.0432 [hep-ph].

anbebu [4] C. Anastasiou, S. Beerli, S. Bucherer, A. Daleo and Z. Kunszt, “Two-loop amplitudes and master integrals for the production of a Higgs boson via a massive quark and a scalar-quark loop,” JHEP **0701**, 082 (2007) [arXiv:hep-ph/0611236].

eip [5] notes: *Everyone into Position*

aasusy [6] A. Falkowski, “Pseudo-goldstone higgs from five dimensions,” arXiv:0710.4050 [hep-ph].