• Examples of the $BZ$ sector
• Couplings of unparticles to the SM
• Deconstruction of unparticles
• Spontaneous symmetry breaking with unparticles and Higgs boson physics
• UnCosmology
  – The equation of state for unparticles
  – Freeze-out and thaw-in
  – BBN constraints
• Summary
The scenario

\[ \begin{array}{ccc}
UV & CFT : BZ fields & SM fields \\
\downarrow & \downarrow & \downarrow \\
M_U & \frac{1}{M_U^k} O_{BZ} O_{SM} \\
\downarrow & \downarrow & \downarrow \\
\Lambda_U & \text{transmutation scale} & \\
\downarrow & & \\
IR & \text{fixed point} & cU \frac{\Lambda_U^{d_{BZ} - d_U}}{M_U^k} O_U O_{SM}
\end{array} \]

where \( k = d_{SM} + d_{BZ} - 4 \). \( d_{SM} \) and \( d_{BZ} \) are canonical dimensions of \( O_{SM} \) and \( O_{BZ} \), respectively, while \( d_U \) is the scaling dimension (the same as the mass dimension in this case) of \( O_U \):

\[ O_U(x) \rightarrow O'_U(x') = s^{-d_U} O_U(x) \quad \text{with} \quad 1 < d_U < 2 \quad \text{for} \quad x \rightarrow x' = sx \]

An example of matching between \( O_{BZ} \) and \( O_U \):

- \((\bar{q}q)\) in QCD \( \iff \) \( M \propto (\bar{q}q) \) mesons in the chiral non-linear model
Examples of the $BZ$ sector

- Banks & Zaks (1982): SU(3) YM with $n$ massless fermions in e.g. fundamental representation

$$\beta(g) = - \left( \beta_0 \frac{g^3}{16\pi^2} + \beta_1 \frac{g^5}{(16\pi^2)^2} + 3 \text{ loops} \cdots \right)$$

$$\beta_0 = 11 - \frac{2}{3}n \quad \beta_0(n_0) = 0 \quad n_0 = 16.5$$
$$\beta_1 = 102 - \frac{38}{3}n \quad \beta_1(n_1) = 0 \quad n_1 \approx 8.05$$

If $n_1 < n < n_0$ (so $\beta_0 > 0$ & $\beta_1 < 0$) then keeping $\beta_0$ and $\beta_1$ one gets

$$\beta(g_{IR}) = 0 \quad \text{for} \quad \frac{g_{IR}^2}{16\pi^2} = - \frac{33 - 2n}{306 - 38n}$$

Conclusions:

- If $g = g_{IR}$, then the low-energy theory is scale invariant with small anomalous scaling
- For $n \lesssim n_0$, the theory remains perturbative, so the continuous spectrum doesn’t emerge.
Couplings of unparticles to the SM

Assumptions:

- $\mathcal{O}_U$ in neutral under the SM gauge group
- $\text{dim}(\mathcal{O}_{SM}) \leq 4$

$$\mathcal{L}_{\text{int}} = c_U \frac{\Lambda_U^{d_{BZ}-d_U}}{M_U^k} \mathcal{O}_U \mathcal{O}_{SM} \propto \left( \frac{\Lambda_U}{M_U} \right)^k \Lambda_U^{4-d_{SM}-d_U} \mathcal{O}_U \mathcal{O}_{SM} \quad \text{for} \quad k = d_{SM} + d_{BZ} - 4$$

- Scalar unparticles $\mathcal{O}_U$: $\propto \Lambda_U^{2-d_U} H^\dagger H \mathcal{O}_U$ for $d_{SM} = 2$
- Spinor unparticles $\mathcal{O}_U^s$: $\propto \Lambda_U^{5/2-d_U} \bar{\nu}_R \mathcal{O}_U^s$ for $d_{SM} = 3/2$
Deconstruction of unparticles

Källen-Lehman representation of the Feynman propagator:

\[ i \Delta_{F}^{U}(p^2) = \int d^4x e^{ipx} \langle 0| T\{\mathcal{O}_{U}(x)\mathcal{O}_{U}(0)\}|0 \rangle = \int_{0}^{\infty} \frac{dm^2}{2\pi} \rho(m^2) \frac{i}{p^2 - m^2 + i\varepsilon} \]

with \( \rho_{U}(m^2) = A_{dU} \theta(m^2)(m^2)^{d_{U}-2} \). Deconstruction (Stephanov’07):

\[ \mathcal{O}_{U} \rightarrow \sum_{n=0}^{\infty} F_n \varphi_n \quad \text{with} \quad m_n^2 = \Delta^2 n \]

Then

\[ i \Delta_{F}^{U}(p^2) = \int d^4x e^{ipx} \langle 0| T\{\mathcal{O}_{U}(x)\mathcal{O}_{U}(0)\}|0 \rangle = \sum_{n=0}^{\infty} \frac{iF_n^2}{p^2 - m_n^2 + i\varepsilon} \]

if \( F_n^2 = \frac{A_{dU}}{2\pi} \Delta^2 (m_n^2)^{d_{U}-2} \) then

\[ i \frac{A_{dU}}{2\pi} \sum_{n=0}^{\infty} \frac{(m_n^2)^{d_{U}-2}}{p^2 - m_n^2 + i\varepsilon} \Delta^2 \rightarrow i \frac{A_{dU}}{2\pi} \int \frac{(m^2)^{d_{U}-2}dm^2}{p^2 - m^2 + i\varepsilon} = \int \frac{dm^2}{2\pi} \rho(m^2) \frac{i}{p^2 - m^2 + i\varepsilon} \]
So, the undeconstructed result has been confirmed. Now, let’s focus on the non-trivial phase:

$$\text{Im} \left\{ \sum_{n=0}^{\infty} \frac{F_n^2}{p^2 - m_n^2 + i\varepsilon} \right\} = -\sum_n F_n^2 \pi \delta(p^2 - m_n^2) \rightarrow \Delta \rightarrow 0 - \frac{A_d u}{2} \theta(p^2)(p^2) du dU - 2$$

So, each peak becomes lower as $F_n^2 \sim \Delta^2 \rightarrow 0$, but their density increases.

- Each mode $\varphi_n$ breaks the scale invariance.
- In the limit

$$\lim_{N \rightarrow \infty} \sum_{n=0}^{N}$$

the scale invariance is recovered.
The deconstruction for $t \rightarrow u O_U$ decay

\[ i \frac{\lambda}{\Lambda_{dU}} \tilde{u} \gamma_\mu (1 - \gamma_5) t \partial^\mu O_U \rightarrow i \frac{\lambda}{\Lambda_{dU}} \tilde{u} \gamma_\mu (1 - \gamma_5) t \sum_{n=0}^{\infty} F_n \partial^\mu \varphi_n \]

\[ \downarrow \]

\[ \Gamma(t \rightarrow u \varphi_n) = \frac{\lambda^2}{\Lambda_{dU}^2} \frac{m_t E_u^2}{2\pi} F_n^2 \quad \text{with} \quad E_u = \frac{m_t^2 - m_n^2}{2m_t} \quad \text{and} \quad F_n^2 = \frac{A_{dU}}{2\pi} \Delta^2 (m_n^2)^{dU - 2} \]

Number of states $|\varphi_n\rangle$ in the interval $(E_u, E_u + dE_u)$: $dN = dE_u \frac{2m_t}{\Delta^2}$

\[ \downarrow \]

\[ \frac{d\Gamma}{dE_u} = \frac{2m_t}{\Delta^2} \Gamma(t \rightarrow u + \varphi_n) = \frac{\lambda^2}{\Lambda_{dU}^2} A_{dU} \frac{m_t^2}{2\pi^2} E_u^2 (m_t^2 - 2m_t E_u)^{(dU - 2)} \theta(m_t - 2E_u) \]

The same as the Georgi’s result!
Spontaneous symmetry breaking with unparticles and Higgs boson physics

(Delgado, Espinosa, Quiros’07)

\[ UV : \quad \frac{1}{M_u^{d_{BZ}-2}} |H|^2 \mathcal{O}_{BZ} \]

\[ \Downarrow \]

\[ IR : \quad c_u \left( \frac{\Lambda_u^{d_{BZ}-d_u}}{M_u^{d_{BZ}-2}} |H|^2 \right) \mathcal{O}_u \equiv \kappa_u |H|^2 \mathcal{O}_u \]

Deconstruction (\( \mathcal{O}_u \rightarrow \sum_n F_n \varphi_n, \ m_n^2 = \Delta^2 n \)) \( \Rightarrow \)

\[ V_{\text{tot}} = m^2 |H|^2 + \lambda |H|^4 + \delta V \]

for

\[ \delta V = \frac{1}{2} \sum_{n=0}^{\infty} m_n^2 \varphi^2 + \kappa_u |H|^2 \sum_{n=0}^{\infty} F_n \varphi_n \]

\[ \langle \varphi_n \rangle = -\frac{k_u u^2 F_n}{m_n^2} \quad \text{for} \quad \langle |H|^2 \rangle = v^2, \quad F_n^2 = \frac{A_{du}}{2\pi} \Delta^2 (m_n^2)^{d_u-2} \]

So,

\[ \langle \mathcal{O}_u \rangle = \sum_{n=0}^{\infty} F_n \langle \varphi_n \rangle \rightarrow -k_u u^2 A_{du} \frac{2}{2\pi} \int_0^\infty \frac{dm^2}{(m^2)^3 - du} = -\infty \]
• The IR divergence!

• A possible regularization $\delta V' = \zeta |H|^2 \sum \varphi_n^2$ is not scale invariant.

Since the scaling invariance is anyway violated by the vacuum expectation value $\neq 0$ through $|H|^2 O_\mathcal{U}$ so we adopt

$$\delta V' = \zeta |H|^2 \sum \varphi_n^2$$

as the IR regulator. Then

$$v_n = \langle \varphi_n \rangle = -\frac{\kappa_\mathcal{U}v^2}{2(m_n^2 + \zeta v^2)} F_n$$

The minimization for $H$ reads:

$$m^2 + \lambda v^2 + \kappa_\mathcal{U} \sum F_n v_n + \zeta \sum v_n^2 = 0$$

Inserting $v_n$ one gets in the continuum limit ($\Delta \to 0$):

$$m^2 + \lambda v^2 - \lambda_\mathcal{U}(\mu^2)^2 - d_\mathcal{U}v^2(d_\mathcal{U}-1) = 0$$
for $\lambda_{U} \equiv \frac{d_{U}}{4} \zeta^{d_{U} - 2} \Gamma(d_{U} - 1) \Gamma(2 - d_{U})$ and $(\mu_{U}^{2})^{2 - d_{U}} \equiv \kappa_{U}^{2} \frac{A_{d_{U}}}{2\pi}$

$$V_{\text{eff}} = m^{2} |H|^{2} - \frac{2^{d_{U} - 1}}{d_{U}} \lambda_{U}(\mu_{U}^{2})^{2 - d_{U}} |H|^{2d_{U}} + \lambda |H|^{4}$$

Even if $m^{2} = 0$ one can get the vacuum expectation value $\neq 0$ ($\Lambda_{U}$ provides the scale):

$$v^{2} = \left(\frac{\lambda_{U}}{\lambda}\right)^{\frac{1}{2 - d_{U}}} \mu_{U}^{2} \text{ for } \mu_{U}^{2} = \left(\frac{A_{d_{U}}}{2\pi}\right)^{\frac{1}{2 - d_{U}}} \left(\frac{\Lambda_{U}^{2}}{M_{U}^{2}}\right)^{\frac{d_{SM} - 2}{2 - d_{U}}} \Lambda_{U}^{2}$$
The equation of state for unparticles

The trace anomaly of the energy momentum tensor for a gauge theory with massless fermions:

$$\theta_{\mu} = \frac{\beta}{2g} N [F_{a\mu\nu} F_{a\mu\nu}]$$  \hspace{1cm} (1)

where $\beta$ denotes the beta function and $N$ stands for the normal product.

Non-trivial IR fixed point at $g = g_*$, so in the IR we assume

$$\beta = \gamma (g - g_*) , \quad \gamma > 0$$

in which case the running coupling reads

$$g(\mu) = g_* + c\mu^\gamma; \quad \beta[g(\mu)] = \gamma c\mu^\gamma$$

where $c$ is an integration constant and $\mu$ is the renormalization scale.
From the thermal average of (1) choosing the renormalization scale \( \mu = T \) and using \( \langle \theta^\mu_\mu \rangle = \rho U - 3p_U \), we get

\[
\rho U - 3p_U = \frac{\beta}{2g_*} \langle N [F^\mu_\nu F^\mu_\nu] \rangle = AT^{4+\gamma}
\]

\[
\rho U - 3p_U = AT^{4+\gamma}
\]

\[
\downarrow
\]

\[
\rho U = \sigma T^4 + A \left(1 + \frac{3}{\gamma}\right) T^{4+\gamma} \quad \text{and} \quad p_U = \sigma \frac{T^4}{3} + \frac{A}{\gamma} T^{4+\gamma}
\]

where \( \sigma \) is an integration constant.

\[
\downarrow
\]

\[
\rho U = \frac{1}{3} \rho U \left(1 - B \rho U^{\gamma/4}\right) \quad \text{for} \quad B \equiv \frac{A}{\sigma^{1+\gamma/4}}
\]

One can expect that \( A \propto \Lambda_U^{-\gamma} \), therefore we obtain

\[
\rho_{NP} = \frac{\pi^2}{30} T^4 \times \begin{cases} 
    g_{\text{IR}} + f \left(\frac{T}{\Lambda_U}\right)^\gamma & \text{for} \quad T \lesssim \Lambda_U \\
    g_{\text{BZ}} & \text{for} \quad T \gtrsim \Lambda_U
\end{cases}
\]
where \( g_{\text{BZ}} = 2(n_c^2 - 1 + \frac{7}{8}n_c n_f) \) for \( SU(n_c) \) with \( n_f \) flavours in the \( \text{BZ} \) sector.

• From the continuity at \( T = \Lambda_{\mathcal{U}} \), the constant \( f \) could be determined: \( f = g_{\text{BZ}} - g_{\text{IR}} \).

• We will assume \( g_{\text{BZ}} \sim g_{\text{IR}} \).
\[ \rho_{\text{NP}} = \frac{\pi^2}{30} T^4 \times \begin{cases} 
 g_{\text{IR}} + f \left( \frac{T}{\Lambda_u} \right)^\gamma & \text{for } T \lesssim \Lambda_u 
 g_{\text{BZ}} & \text{for } T \gtrsim \Lambda_u 
 \end{cases} \]

Deconstruction (Stephanov’07):

\[ O_u \rightarrow \sum_{n=0}^{\infty} F_n \varphi_n \quad \text{with} \quad m_n^2 = \Delta_n^2 \]

The above result fits the following guess for the effective number of degrees of freedom:

\[ g_u(T) \propto \frac{\int_T^2 dM^2 \rho(M^2) \theta(\Lambda_u^2 - M^2)}{\int_0^{\Lambda_u^2} dM^2 \rho(M^2)} \]

where \( \rho(M^2) \propto (M^2)^{(d_u - 2)} \). Then

\[ g_u(T) \propto \left( \frac{T}{\Lambda_u} \right)^{2(d_u - 1)} \]

\[ \Rightarrow \] In the presence of just one unparticle operator one can argue that \( \gamma = 2(d_u - 1) \).
- Freeze-out and thaw-in

- **Brief history of the Universe in the presence of unparticles (no mass-gap).**

  - \( T \gg M_U \): the \( BZ \) sector in form of massless particles (no unparticles yet), thermal equilibrium with the SM is maintained (assumption), so \( T = T_{BZ} = T_{SM} \)

  - \( T \lesssim M_U \):
    - The \( BZ \) sector starts to decouple, as the average energy is no longer sufficient to create mediators.
    - However, the thermal equilibrium may still be maintained (\( T = T_{BZ} = T_{SM} \)) depending on the strength of effective couplings between the SM and the extra sector (which at higher temperature, \( T \gtrsim \Lambda_U \), is made of the \( BZ \) matter, while below \( \Lambda_U \) of unparticles).

Let’s denote by \( T_f \) the decoupling temperature at which

\[
\Gamma(SM \leftrightarrow NP) \simeq H
\]

where \( H \) is the Hubble parameter

\[
H^2 = \frac{8\pi}{3M_{Pl}^2} \rho_{\text{tot}}(T) \quad \text{for} \quad \rho_{\text{tot}} = \rho_{SM} + \rho_{NP}
\]
There are 2 interesting cases:

- $M_U > T_f > \Lambda_U$:
  - $T_f$ is determined by the condition
    \[
    \Gamma(SM \leftrightarrow BZ) \simeq H
    \]
  - For $T > T_f$ the SM and the $BZ$ sectors evolve in thermal equilibrium, but even for $T < T_f$ their temperatures remain equal ($T = T_{BZ} = T_{SM}$) since $\Lambda_U > v$.

- $\Lambda_U > T_f$:
  - Till $T = \Lambda_U$ the SM and unparticles still have the same temperature.
  - For $\Lambda_U \gtrsim T \gtrsim T_f$ still the equilibrium is maintained (assumption, in general this depends on $d_U$). The decoupling temperature $T_f$ must be now determined by
    \[
    \Gamma(SM \leftrightarrow O_U) \simeq H
    \]
  - Till $T \sim v$ temperatures of SM and unparticles remain equal, at $T \sim v$ they split.

$\implies$ The unparticle cosmic background should be there.
♣ The Banks-Zaks phase.

\[ \mathcal{L}_{\text{BZ}} = \frac{1}{M_U} (H^\dagger H) (\bar{q}_{\text{BZ}} q_{\text{BZ}}) \]

Then

\[ \Gamma_{\text{BZ}} \propto \frac{T^3}{M_{U}^2} \quad \text{and} \quad H \propto \frac{T^2}{M_{Pl}} \quad \implies \quad \text{decoupling for} \quad T \lesssim T_{f-BZ} \]

♣ The unparticle phase.

\[ \mathcal{L}_{U} = c_{U} \frac{\Lambda_{U}^{d_{BZ}-d_{U}}}{M_{U}^{k}} O_{U} O_{SM} \quad \text{for} \quad k = d_{SM} + d_{BZ} - 4 \]

The most relevant operators for scalar unparticles are

\[ \mathcal{L}_{s} = c_{U}^{(s)} \frac{\Lambda_{U}^{1-d_{U}}}{M_{U}} (H^\dagger H) O_{U}, \quad \mathcal{L}_{f} = c_{U}^{(f)} \frac{\Lambda_{U}^{3-d_{U}}}{M_{U}^{3}} (\bar{\ell} H e) O_{U}, \quad \mathcal{L}_{v} = c_{U}^{(v)} \frac{\Lambda_{U}^{3-d_{U}}}{M_{U}^{3}} (B_{\mu \nu} B^{\mu \nu}) O_{U} \]

\[ \mathcal{L}_{s} \quad \implies \quad \Gamma_{U} \propto \frac{\Lambda_{U}^{3} M_{U}^{2}}{(T/\Lambda_{U})^{2d_{U}-3}} \quad \text{and} \quad H \propto \frac{T^2}{M_{Pl}} \quad \implies \quad T_{f-U} \]
\[ \frac{\Gamma_u}{H} \propto T^{2d_U - 5} \implies \begin{cases} \text{decoupling for } T < T_{f - U} & d_U > \frac{5}{2} \\ \text{decoupling for } T > T_{f - U} & d_U < \frac{5}{2} \end{cases} \]

**Figure 1:** Regions of \((M_U, \Lambda_U)\) for various scenarios of decoupling for \(d_U = 3/2\).

**Figure 2:** Regions of \((M_U, \Lambda_U)\) for various scenarios of decoupling for \(d_U = 3\).

<table>
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<th>decoupling in the unparticle phase</th>
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BBN constraints

Big-Bang Nucleosynthesis $\Rightarrow \Delta N_\nu = -0.37^{+0.10}_{-0.11}$ $\Rightarrow$ upper limit for $g_{IR}$

- Assume freeze-out above the EW scale ($T_f > v = 246$ GeV, $\mathcal{L} \propto H^\dagger H O_U$)

\[
\rho U(T_{BBN}) = g_{IR} \frac{\pi^2}{30} T_{BBN}^4 \left[ \frac{g_{SM}^{(s)}(T_{BBN})}{g_{SM}^{(s)}(T = v)} \right]^{4/3}
\]

$\downarrow$

$g_{IR} \lesssim 4.3$ at $4\sigma$

To be compared with e.g. $g_{BZ} = 2(n_c^2 - 1 + \frac{7}{8}n_cn_f)$, for $n_c = 3$ and $n_f = 10$, $g_{BZ} \simeq 60$.

- Assume freeze-out below $T_{BBN}$ ($T_{f-U} < T_{BBN}$, $\mathcal{L} \propto B_{\mu\nu}B^{\mu\nu}O_U$)

\[
g_{IR} = \frac{7}{4} \Delta N_\nu \quad \Rightarrow \quad g_{IR} \lesssim 0.05 \text{ at } 4\sigma
\]
Summary

• Intensive activity on unparticles (∼ 200 citations of the first Georgi’s paper)

• Interesting and exotic phenomenology

• Unparticles could be deconstructed

• Troubles with IR divergences

• Cosmological consequences
  – Rough arguments for the equation of state for unparticles: 
    \[ p_U = \frac{1}{3} \rho_U \left[ 1 - B \rho_U^{\delta/4} \right] \]
  – Rough arguments for the energy density for unparticles "derived": 
    \[ \rho_{NP} = \frac{\pi^2}{30} T^4 \times \left\{ \frac{g_{IR} + (g_{BZ} - g_{IR}) \left( \frac{T}{\Lambda_U} \right)^\gamma}{g_{BZ}} \right\} \]
    for \( T \lesssim \Lambda_U \)
    for \( T \gtrsim \Lambda_U \)
  – Unparticles in equilibrium: freeze-out and thaw-in.
  – BNN bounds on the number of degrees of freedom for unparticles.
Experimental constraints


\[
\mathcal{L}_{\text{Uff}} = \frac{c_V}{M_{Zd_u-1}} \bar{f} \gamma_\mu f \mathcal{O}_U^\mu + \frac{c_A}{M_{Zd_u-1}} \bar{f} \gamma_\mu \gamma_5 f \mathcal{O}_U^\mu + \frac{c_{S1}}{M_Z} \bar{f} \Phi f \mathcal{O}_U + \frac{c_{S2}}{M_Z} \bar{f} \gamma_\mu f \partial^\mu \mathcal{O}_U \\
+ \frac{c_{P1}}{M_Z} \bar{f} \Phi_5 f \mathcal{O}_U + \frac{c_{P2}}{M_Z} \bar{f} \gamma_\mu \gamma_5 f \partial^\mu \mathcal{O}_U.
\]

Here the coefficients have been scaled to a common mass, chosen as the $Z$-boson mass $M_Z$, so that the only unknown quantities are the dimensionless coupling constants $c_i$. 
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Jochum van der Bij and S. Dilcher:


The model:

- Extra-dimensional ($\delta$) scalars neutral under the SM gauge group

\[
\phi(x, y) = \frac{1}{\sqrt{2L}\delta/2} \sum_{\vec{k}} \phi_{\vec{k}}(x)e^{i2\pi\vec{k}\cdot\vec{y}}
\]

- Extra terms in the scalar potential

\[
V(H, \phi) = \cdots - \frac{\lambda_1}{8}(2f_1\phi - |H|^2)
\]
Similarities:

- The continuous mass spectrum e.g. for \( s \to \infty \): \( \rho(s) \sim s^{-3+\delta/2} \)

Differences

- In HEIDI only scalars, while unparticles could have any spin

- Van der Bij and Dilcher don’t assume scale invariance of the extra sector

- In HEIDI interactions between the SM and the extra scalars assumed to be renormalizable

- Van der Bij and Dilcher claim that only for \( 0 < \delta < 1 \) there is no tachyons in the scalar spectrum, so the potential is stable (\( 1 < d_U < 2 \))