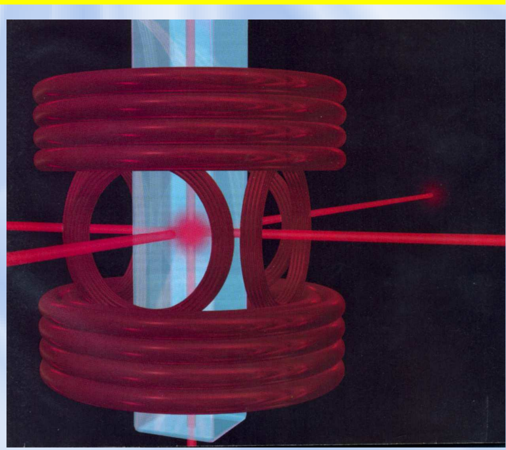
INNOWACYJNA GOSPODARKA
NARODOWA STRATEGIA SPÓJNOŚCI

UNIA EUROPEJSKA
EUROPEJSKI FUNDUSZ
ROZWOJU REGIONALNEGO

Lecture 2:
Cooling and Trapping towards BEC



Some pre-history

- Keppler observing comets tails
- 1875 Crookes - demostartion
- 1901 Lebiediev and Nicols
- 1933 Frish reflection of atoms
- 1962 Asharian intensity gradient

Narodowe Laboratorium Technologii Kwantowych

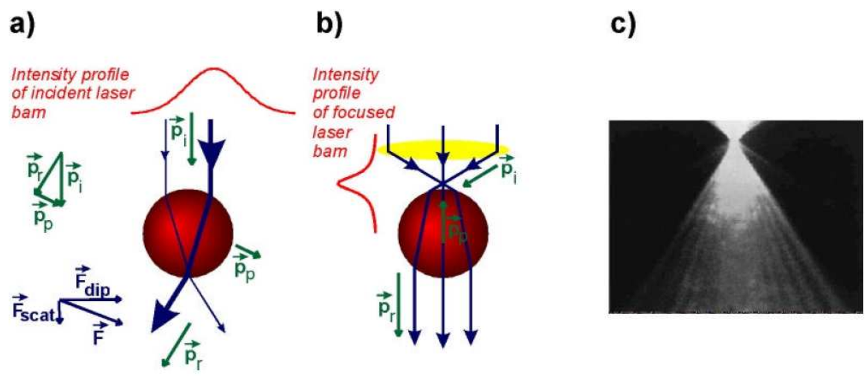
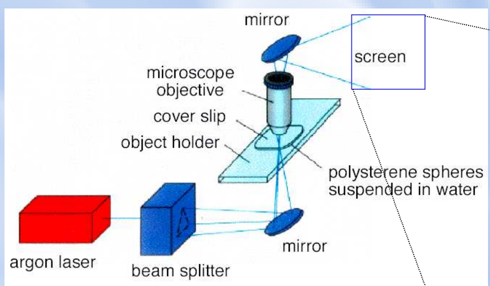
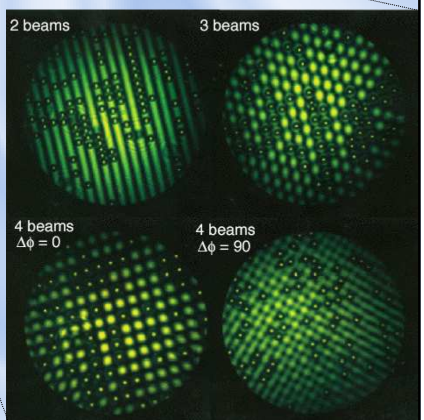


Fig. 3. Optical forces on a dielectric sphere: a) The sphere is placed off-axis in a collimated laser beam with Gaussian transverse profile. The light is refracted such that the incoming momentum, $\bar{p}_i$, is changed to $\bar{p}_r$. The difference, $\bar{p}_p$, is picked up by the particle and felt as a force, $\bar{F}$, pushing the sphere towards the beam axis ($\bar{F}_{dip}$) and along the propagation direction ($\bar{F}_{scat}$); b) The sphere is placed near a focused beam. Refraction now results in a force towards the focus; c) Photograph of a 10 micron glass sphere trapped in water with green light from an argon laser using the set-up shown in b). The picture is a fluorescence image taken using a green blocking, red transmitting filter. (Photograph courtesy of S. Chu).

Dipole traping of macroscopic objects

4 μm polyester spheres in water [M. Burns et al., Science **249**, 749 (1990)]





- spheres dragged to the high intensity regions(red detuning)
- water cools down particles
- red light from Ar^+ laser seen by Rayleigha scattering

Other examples – *optical tweezers*

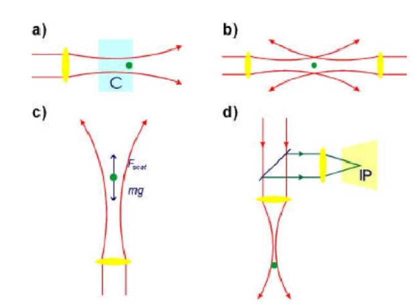


Fig. 27. Trapping of micron sized particles. a) A particle in a cell (C) is attracted to the centre of the weakly focused laser beam by the dipole force and pushed along to the end of the cell by the scattering force. b) The scattering forces from two counter propagating beams cancel and the particle is held in a two-beam optical trap. c) The scattering force is counterbalanced by gravity in the levitation trap. d) The particle is trapped in three dimensions by the dipole force from a tightly focused laser beam. This configuration known as optical tweezers is typically obtained by sending the trapping light through an optical microscope. The fluorescence from the sample can be viewed in the image plane (IP).

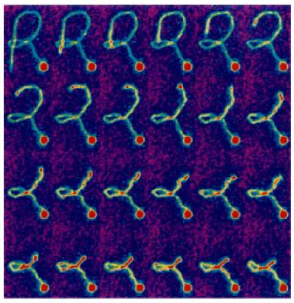


Fig. 28. The relaxation of a strand of fluorescently stained DNA in a concentrated solution of unstained molecules. One end of the DNA is attached to a micron sized polystyrene sphere, which is held by optical tweezers (bright red spot). The series of images shows how the DNA relaxes along its own path demonstrating the tube-like motion enforced by the entangled polymers in the solution. (Figure courtesy of S. Chu).

BOX 1

Absorption and emission of near-resonant light

In the simplest laser cooling scenario, the laser is tuned close to resonance with a transition from the ground state to a single excited state which decays by spontaneously emission back to the ground state. If the laser is exactly on resonance and there is no spontaneous emission, the atom undergoes cycles of absorption and stimulated emission, and the population oscillates between the ground and excited state at the frequency, ω_R, known as the *Rabi frequency* (as illustrated by the blue curve in Fig. B1a - see also Box 3). Spontaneous emission interrupts these Rabi oscillations by resetting the excited state component of the atomic state to zero (the red curve in Fig. B1a, simulated using a Monte-Carlo wavefunction). If one averages over many atoms, then the net effect of spontaneous emission is to damp the Rabi oscillations towards a steady-state excited state population (the green curve in Fig. B1a). The steady-state fraction, f_e, increases with intensity, approaching 1/2 for high intensity (Fig. B1b). This saturation effect sets the maximum value of the spontaneous light force.

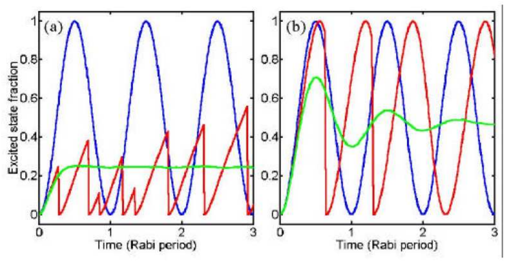


Fig. B1. Simulations of a two-level atom in a resonant laser field. With no spontaneous emission (blue), the atomic population oscillates between the ground and excited state at the Rabi frequency. Spontaneous emission interrupts the Rabi cycle and resets the excited state probability to zero (red). By averaging over many atoms (10000 in this example) one obtains the steady-state fraction, f_e (green). (a) At the saturation intensity corresponding to $\omega_R = \Gamma/\sqrt{2}$, $f_e = 1/4$. (b) At high intensity, $\omega_R = 3.3\,\Gamma$, f_e approaches 1/2.

BOX 2

Classical field picture.

An alternative description of the light force on atoms, in particular a more accurate picture of the dipole or gradient force, is provided by considering the interaction between the electric field $\boldsymbol{E} = \boldsymbol{E}_0 \cos(\omega_L t + \varphi)$ and induced atomic dipole moment $\boldsymbol{d}$. The interaction energy is given by

$$U = -\boldsymbol{d} \cdot \boldsymbol{E}$$

If we ignore spontaneous emission for the moment, the atomic dipole behaves similarly to any classical oscillator, e.g. a mass on a spring, with a oscillation frequency determined by the energy level spacing $\omega_0 = (E_e - E_g)/\hbar$ where E_g and E_e are the ground and excites states energies respectively. The laser acts as an external driving field. If the driving frequency ω_L is less than ω_0 (referred to as red detuning) the atomic dipole oscillates in phase. In this case, the interaction energy U is negative and the atom is attracted towards maximum intensity. In contrast, if the driving frequency ω_L is greater than ω_0 (blue detuning) the interaction energy is positive and the atom is repelled from maximum intensity. Exactly on resonance, the dipole moment and the electric field are orthogonal so the net force should be zero, however, in this case spontaneous emission is also important.

To include the effect of spontaneous emission we write that

$$d = E_0\left(\alpha' \cos(\omega_L t) - \alpha'' \sin(\omega_L t)\right)$$

which states that the atomic dipole moment d oscillates with an amplitude proportional to that of the driving field, E_0, and at the same frequency but not necessary in phase, α' and α'' play the roles of the real and imaginary parts of the atomic 'refractive' index (in analogy to Sec III A).

The time-averaged force is given by the gradient of the potential and can be written as:

$$F = \langle -\nabla U \rangle = -\frac{1}{2}\alpha' \nabla E_0^2 - \frac{1}{2}\alpha'' E_0^2 \nabla\varphi$$

where $\alpha' E_0$ and $\alpha'' E_0$ are the steady state in-phase and quadrature components of the induced atomic dipole moment. The force has two components: The first term, proportional to the gradient of the intensity,

$$F_{dip} = -\frac{1}{2}\alpha' \nabla E_0^2$$

is the gradient force or dipole force, and can be positive or negative depending on the sign of the detuning as discussed above; and the second term, proportional to the gradient of the phase,

$$F_{scat} = -\frac{1}{2}\alpha'' E_0^2 \nabla\varphi$$

is the radiation pressure force or scattering force.

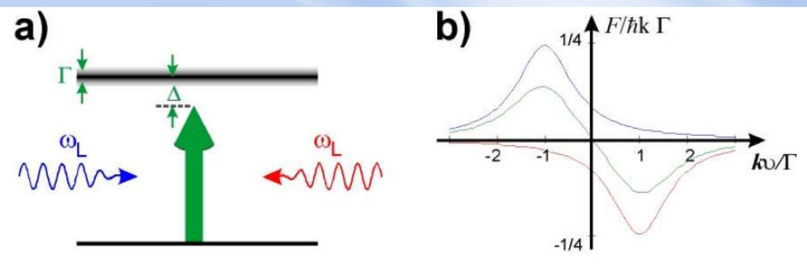


Fig. 5. One-dimensional Doppler cooling: (a) The frequency of the standing laser field ω_L is detuned by an amount Δ below the resonance with a transition to an excited state, which has a linewidth, Γ, equal to the inverse of the natural lifetime of the excited state; (b) Each of the counter-propagating beams exerts a force with a Lorentzian velocity dependence (red and blue curves). For an intensity equal to the saturation intensity the maximum force corresponds to one photon momentum transferred every four natural lifetimes. The green curve shows the combined force from the two beams, displaying the viscous damping around $\upsilon = 0$.

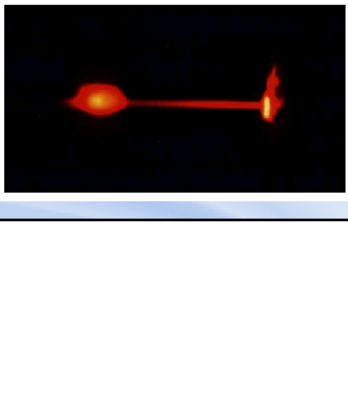
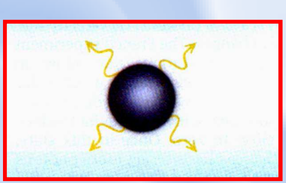
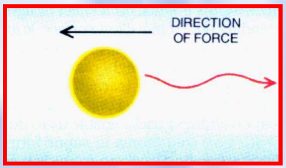
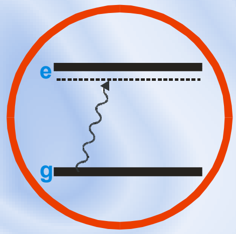


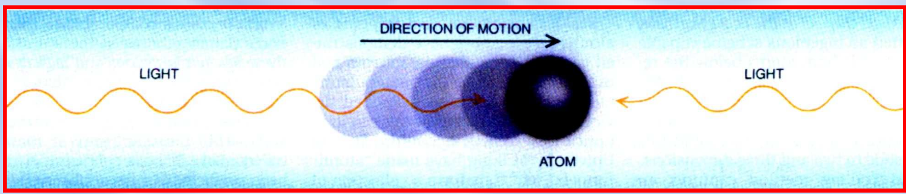
Fig. 6. Photograph showing atoms confined in optical molasses (right). The atomic beam, originating from the nozzle to the left, is slowed with a counter-propagating laser beam. The atoms are cooled further in optical molasses. The distance from the nozzle to the optical molasses is 5 cm. (Photograph courtesy of S. Chu).

Doppler cooling



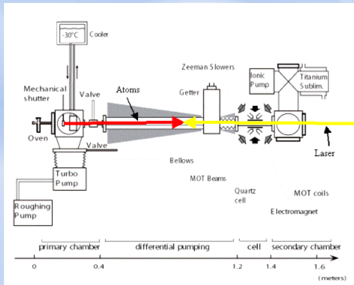


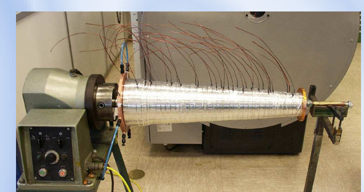




$$k_B T = \frac{\hbar\gamma}{4}\left(\frac{2\Delta}{\gamma} + \frac{\gamma}{2\Delta}\right)$$

Zeeman slower



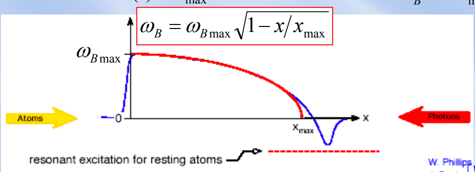


Field gradient

$\rightarrow$ max. velocity $\upsilon_{max} \Rightarrow$ max. dystans

$$x(t) = \upsilon_{\max} t - \tfrac{1}{2} a t^2 \qquad \upsilon(x) = \upsilon_{\max}\sqrt{1 - x/x_{\max}}$$
$$\Rightarrow$$
$$\upsilon(t) = \upsilon_{\max} - at \qquad \max\ \omega_B = k\upsilon_{\max}$$

$$\omega_B = \omega_{B\max}\sqrt{1 - x/x_{\max}}$$



[W. Phillips and H. Metcalf, Phys. Rev. Lett. 48, 596 (1982)
J. Prodan et al., Phys. Rev. Lett. 49, 1149

Subdoppler cooling

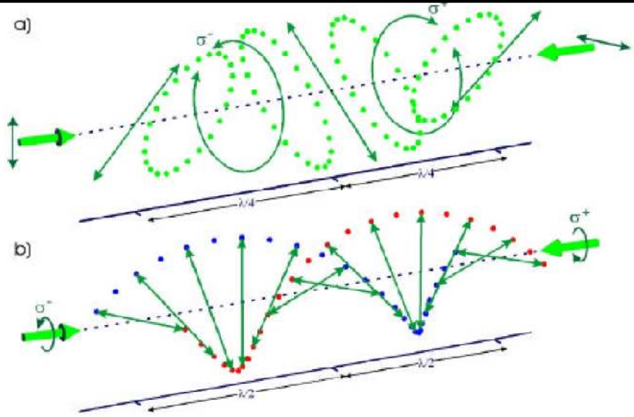
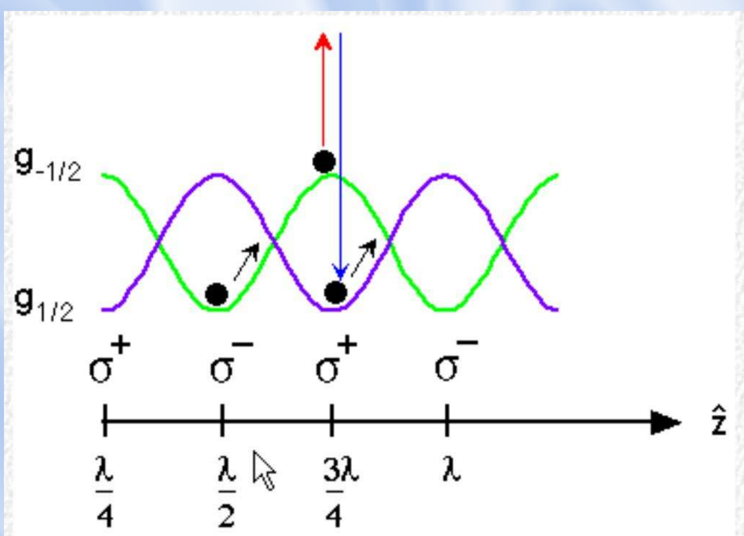


Fig. 7. The two field configurations considered in polarisation gradient cooling: (a) The two beams are both linearly polarised, but in orthogonal directions (lin⊥lin). The polarisation of the field varies from linear through circular to the opposite linear and back again over a distance of λ/2; (b) The two beams are circularly polarised in opposite directions (σ^+–σ^-). The electric field is linearly polarised everywhere, but the direction of polarisation rotates around the propagation direction with a pitch of λ/2.

Sisyphus cooling



Subdoppler cooling

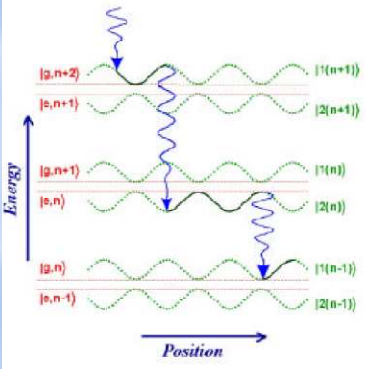


Fig. B5. *Sisyphus cooling of an atom in a standing wave: Near the nodes of the field, the energies are unshifted and the dressed wave functions coincide with the atomic wave functions, while near the anti-nodes, the wave functions are mixed. Spontaneous emission occurs preferentially where the dressed state has most excited state character, i.e., from the anti-nodes of states* $|1(n)\rangle$ *and the nodes of states* $|2(n)\rangle$*. The solid curve represents the motion of a slow atom through the standing wave. The net effect of spontaneous emission is such that the atom travels uphill more than downhill.*

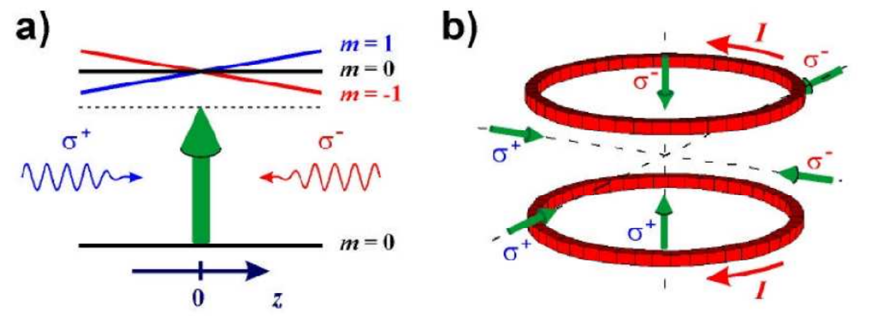


Fig. B4. *The magneto-optic trap (MOT): (a) An atom with a J = 0 to J' = 1 transition is placed in a linearly varying magnetic field* $B_z(z) = b\,z$*. For an atom with a positive z-coordinate, the* σ^- *beam, which drives the* $\Delta m = -1$ *transition and propagate to the right is closer to resonance, than the* σ^+ *beam which drives the* $\Delta m = +1$ *transition and propagates to the left. The net force pushes the atom towards* $z = 0$*; (b) The three-dimensional generalisation uses two coaxial coils with opposing currents and three orthogonal standing* $\sigma^+-\sigma^-$ *waves.*

Applications

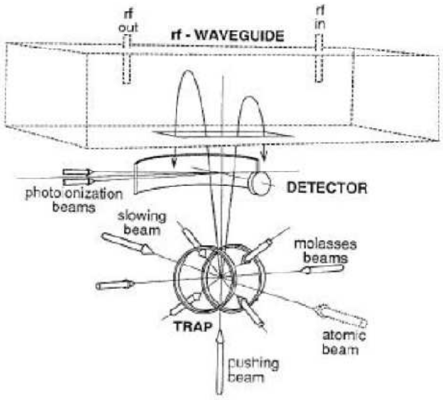


Fig. 22. The experimental set-up used to produce a cold-atom fountain. The main components are the magneto-optical trap, the rf interaction region, and the detection region.

Magnetic trap

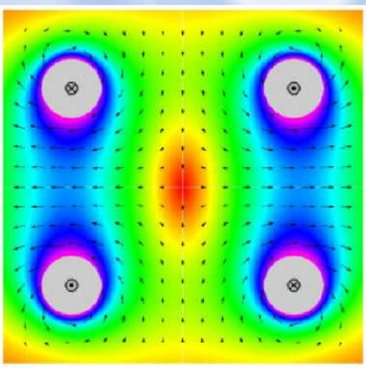


Fig. 15. Magnetic trapping can take place in the zero-point of the magnetic field created by two current loops (grey, out of the plane of the paper) in the anti-Helmholtz coil configuration. The colours indicate the magnitude of the magnetic field with red corresponding to B = 0. The arrows indicate the direction and magnitude of the field.

Time Orbiting Potential

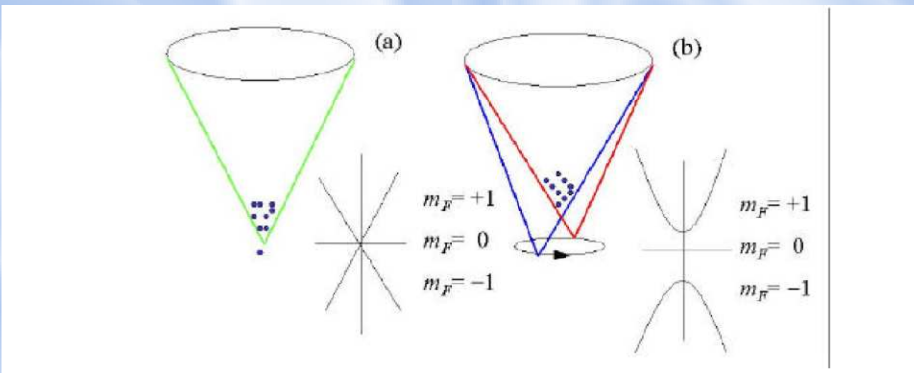
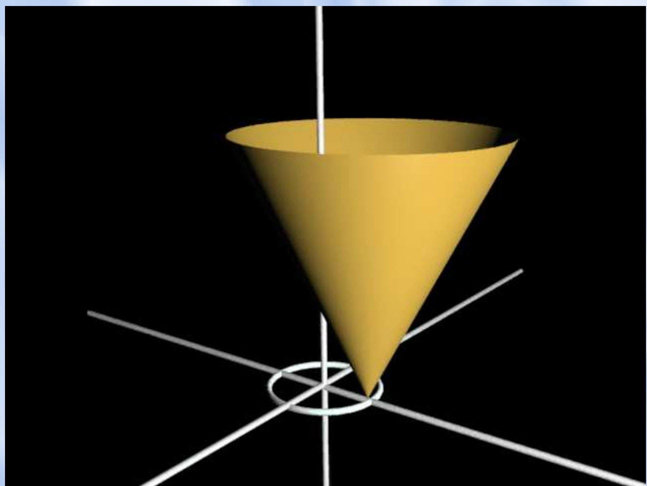


Fig. 16. (a) The linear magnetic trap showing the lossy zero-point of the magnetic field. (b) The TOP trap: An oscillatory magnetic field is applied to rotate the zero-field position. If the rotation period is much faster than the oscillation period of trapped atoms, the time-orbiting potential (TOP) is parabolic with no zero-crossing, so the spin-flip loss rate is significantly reduced.

T.O.P



All optical trap

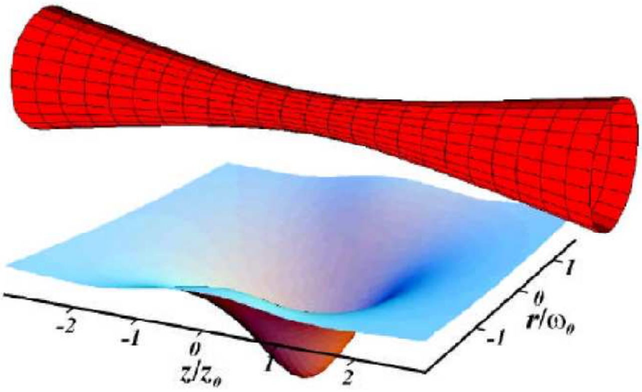
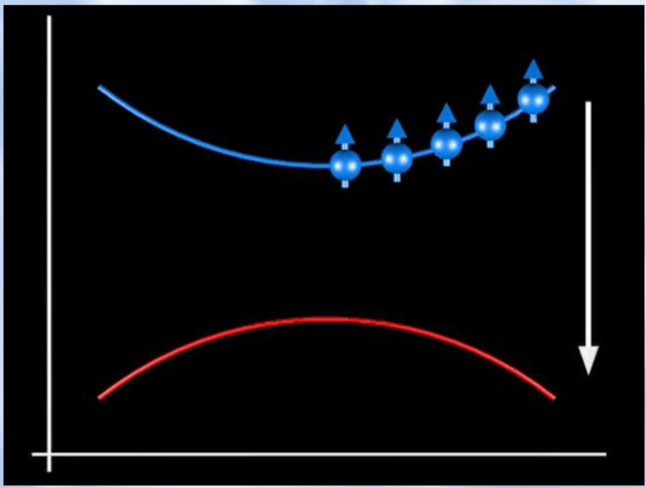


Fig. 17. The optical dipole trap: The potential energy of an atom is lowered in the presence of a strong light field tuned below resonance.

Evaporative cooling

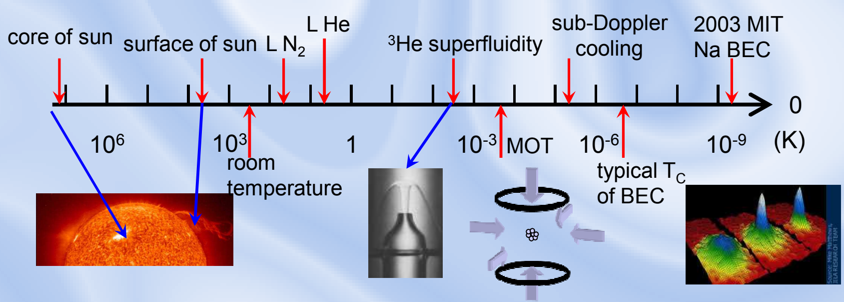
Evaporative colling



Optical Scalpel

Temperature Landmark

To appreciate something is a good motivation to learn something!

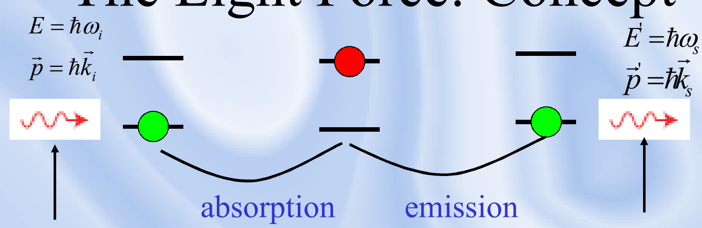


Laser cooling and trapping of atom is a breakthrough to the exploration of the ultracold world. A 12 orders of magnitude of exploration toward absolute zero temperature from room temperature !!!

Useful References

- Books,
  - H. J. Metcalf & P. van der Straten, "Laser cooling and trapping"
  - C. J. Pethick & H. Smith ,"Bose-Einstein condensation in dilute gases"
  - P. Meystre, "Atom optics"
  - C. Cohen-Tannoudji, J. Dupont-Roc & G. Grynberg "Atom-Photon interaction"
- Review articles
  - V. I. Balykin, V. G. Minogin, and V. S. Letokhov, "Electromagnetic trapping of cold atoms" , *Rep. Prog. Phys.* **63** No 9 (September 2000) 1429-1510.
  - V S Letokhov, M A Ol'shanii and Yu B Ovchinnikov Quantum Semiclass. Opt. 7 No 1 (February 1995) 5-40 "Laser cooling of atoms: a review"

The Light Force: Concept



Photon posses energy and momentum !

An exchange of momentum energy between photon and

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Force on atom

Net momentum exchange from the photon to atom

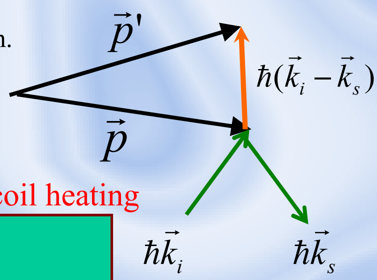
Energy and Momentum Exchange between Atom and Photon

- Photon posses momentum and energy.
- Atom absorbs a photon and re-emit another photon.

$$\Delta \vec{p} = \vec{p}' - \vec{p} = \hbar(\vec{k}_i - \vec{k}_s)$$

$$\Delta K = K' - K = \frac{(\vec{p}'^2 - \vec{p}^2)}{2m} = \hbar(\vec{k}_i - \vec{k}_s)\cdot\vec{v} + \frac{\hbar^2(\vec{k}_i - \vec{k}_s)^2}{2m}$$

always positive, recoil heating



Criteria of laser cooling

If $\langle (\vec{k}_i - \vec{k}_s)\bullet\vec{v}\rangle_{avg} < 0$ the momentum decrease, and if

$\left|\langle (\vec{k}_i - \vec{k}_s)\cdot\vec{v}\rangle_{avg}\right| > \left\langle \frac{\hbar(\vec{k}_i - \vec{k}_s)^2}{2m} \right\rangle_{avg}$ the kinetic energy decrease,

where avg stands for averaging over photon scattering events.

A laser cooling scheme is thus an arrangement of an atom-ph interaction scheme that satisfy the above criteria!

The Light force : quantum mechanics

- **Ehrenfest theorem,** the quantum-mechanical analogue of Newton's second law,

$$\vec{F} = \frac{d\langle\vec{p}\rangle}{dt} = -\langle \nabla V(\vec{r},t)\rangle = m\frac{d^2}{dt^2} < \vec{r} >$$

$$\hat{H} = \frac{\vec{p}^2}{2m} + V(\vec{r},t),$$

where $V(\boldsymbol{r},t)$ is the interaction potential.

- **Interaction potential**: for an atom interacting with the laser field, $\hat{V} = -\vec{d}\bullet\vec{E}$, where ***d*** is atomic dipole moment operator.

- **Semi-classical treatment of atomic dynamics:**
  - Atomic motion is described by the averaged velocity
  - EM field is treat as a classical field
  - Atomic internal state can be described by a density matrix which is determined by the optical Bloch equation

INNOWACYJNA GOSPODARKA
NARODOWA STRATEGIA SPÓJNOŚCI

UNIA EUROPEJSKA
EUROPEJSKI FUNDUSZ
ROZWOJU REGIONALNEGO

Validity of semi-classical treatment

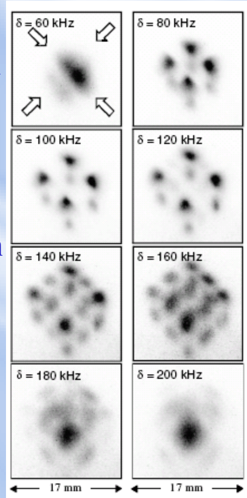
- Momentum width Δp is large compared with photon momentum $\hbar \boldsymbol{k}$.

$$\hbar k / \Delta p << 1$$ **an upper bound on**

- Atom travel over a distance smaller than the optical wavelength during internal relaxation time. (Internal variables are fast components and variation of atomic motion is slow components in density matrix of atom $\rho(\boldsymbol{r},\boldsymbol{v},t)$)

$$v\Gamma^{-1} << \lambda, \quad \text{or } kv/\Gamma << 1$$ **an lower bound on**

- Two conditions are compatible only if $\dfrac{\hbar^2 k^2/2m}{\hbar\Gamma} << 1$
- If the above conditions is not fullified, full quantum-mechanical treatment is needed. e.g. Sr narrow-line cooling, $\Gamma = 2\pi\times 7.5\text{kHz} \sim \omega_r = \hbar^2 k/2m = 2\pi\times 4.7\text{kHz}$



J. Dalibard & C. Cohen-Tannoudhi, J. Phys. B. 18,1661,1985
T.H. Loftus et.al. PRL 93, 073001,2004

The light force for a two-level atom

$$U = \langle V \rangle = -\langle \vec{d} \rangle \bullet \vec{E}$$

$$\vec{F} = -\nabla U = \langle \vec{d} \rangle \bullet \nabla \vec{E}$$

$$\vec{E} = \hat{e} E_0(\vec{r}) \cos(\omega t + \phi(\vec{r}))$$

$$\langle \vec{d} \rangle = Tr(\rho \vec{d}) = \rho_{12}\vec{d}_{21} + \rho_{21}\vec{d}_{12} = \vec{d}_{12}(\sigma_{12}e^{i\omega t} + \sigma_{21}e^{-i\omega t}) = 2\vec{d}_{12}(u\cos\omega t - v\sin\omega t)$$

Where $\boldsymbol{d}_{12} = \boldsymbol{d}_{21}$ are assumed to be real and we have introduced the Bloch vectors u,v, and w.

$$u = \frac{1}{2}(\sigma_{12} + \sigma_{21})$$

$$v = \frac{1}{2i}(\sigma_{12} - \sigma_{21})$$

$$w = \frac{1}{2}(\rho_{22} - \rho_{11})$$

Remark: dipole moment contain
in phase and in quadrature components with incident field.

ρ_{ij} (or σ_{ij})can be determined by the optical Bloch equation of atomic de

Narodowe Laboratorium Technologii Kwantowych

Optical Bloch equation

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \left\{\frac{d\rho}{dt}\right\} \qquad \left(\frac{d\rho_{ii}}{dt}\right)_{spon} = -\Gamma\rho_{ii}, \left(\frac{d\rho_{ij}}{dt}\right)_{spon} = -\frac{\Gamma}{2}\rho_{ij}$$

Incoherent part due to spontaneous emission or others relaxation processes

$$\frac{d\rho_{11}}{dt} = \Gamma\rho_{22} + \frac{i\Omega}{2}(\sigma_{21} - \sigma_{12})$$

$$\frac{d\rho_{22}}{dt} = -\Gamma\rho_{22} + \frac{i\Omega}{2}(\sigma_{12} - \sigma_{21})$$

$$\frac{d\sigma_{12}}{dt} = -\left(\frac{\Gamma}{2} + i\delta\right)\sigma_{12} + \frac{i\Omega}{2}(\rho_{22} - \rho_{11})$$

$\rho_{11} + \rho_{22} = 1$, *where*

$\hbar\Omega(\vec{r}) = -dE_0(\vec{r});\ \sigma_{12} = \rho_{12}\exp(-i\omega t);\ \delta = \omega - \omega_0$

steady state solution

$$\rho_{22} = \frac{s_0/2}{1+s_0+(2\delta/\Gamma)^2};\ \sigma_{21} = \frac{i\Omega}{2(\Gamma/2 - i\delta)\left(1+\frac{s_0}{1+(2\delta/\Gamma)^2}\right)}$$

$$S_0 = I/I_{sat};\ I_{sat} = \frac{h\pi c\Gamma}{3\lambda^3}$$

I_{sat} ~ 1-10 mW/cm^2 for alkali atom

Two types of forces

Without loss of generality, choose $\phi(\vec{r} = 0) = 0$

At $\boldsymbol{r}$ =0,
$$(\nabla\vec{E})_j = e_j(\cos\omega t \nabla E_0 - \sin\omega t E_0 \nabla\phi)$$
$$\langle d_j \rangle = 2(\vec{d}_{12})_j (u\cos\omega t - v\sin\omega t)$$

Take average over one optical cycle

$$\vec{F} = \sum_j (\langle d_j \rangle \nabla E_j)_{avg} = (\hat{e} \bullet \vec{d}_{12})(u\nabla E_0 + vE_0\nabla\phi)$$

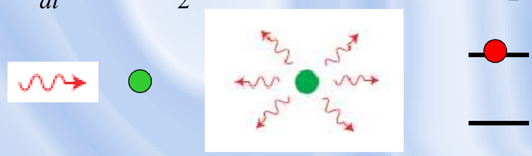
$$\vec{F} = \vec{F}_{dip} + \vec{F}_{rp} = \left(\frac{d\nabla E_0(\vec{r})}{2}\right)(\sigma_{12} + \sigma_{21}) + \nabla\phi\left(\frac{dE_0(\vec{r})}{2}\right)i(\sigma_{12} - \sigma_{21})$$

dipole force or
gradient force
a reactive force
Origin of optical trapping

radiation pressure or
spontaneous emission force
a dissipative force
Origin of optical cooling

Spontaneous emission force

From $\frac{d\rho_{11}}{dt} = \Gamma\rho_{22} + \frac{i\Omega}{2}(\sigma_{21} - \sigma_{12})$ for steady-state $\frac{i\Omega}{2}(\sigma_{12} - \sigma_{21}) = -\Gamma\rho_{22}$



Decay rate, Γ

For a plane wave $\phi(\vec{r}) = -\vec{k} \bullet \vec{r}$; $\nabla\phi = -\vec{k}$; $\nabla E_0 = 0$!!

$$\vec{F}_{rp} = \hbar\vec{k}\Gamma\rho_{22} = \hbar\vec{k}R_{sp}$$

$$R_{sp}(\delta) = \Gamma\rho_{22} = \frac{\Gamma}{2}\frac{S_0}{1 + S_0 + (2\delta/\Gamma)^2}$$

,where R_{sp} is the flourescence rate.

Max deceleration $a = \frac{\Gamma\hbar k}{2m} \approx 50000\ g$, for Na D_2 line

Dipole Force in a standing wave

- A standing wave has an amplitude gradient, but not a phase gradient. So only the dipole force exists.

$$E(\vec{r}, t) = \hat{e}_x E_0 \cos kz \cos\omega t$$

$$\vec{F}_{dip} = -\frac{\hbar\delta}{4}\frac{\nabla(\Omega^2)}{\delta^2 + \Gamma^2/4 + \Omega^2/2}$$

Where s_0 is the saturation parameter for each of the two beams that form

For δ<0 (red detuning), the force attracts atom toward high intensity regions.
For δ>0 (blue detuning), the force repels atom away from high intensity regions.

$$F_{dip} = -\nabla U$$

$$U = \frac{\hbar\delta}{2}\ln\left[1 + \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4}\right]$$

INNOWACYJNA GOSPODARKA
NARODOWA STRATEGIA SPÓJNOŚCI

UNIA EUROPEJSKA
EUROPEJSKI FUNDUSZ
ROZWOJU REGIONALNEGO

Velocity dependent force

Atom with velocity ***v*** experiences a Doppler shift ***k•v***.

$$\vec{F}_{rp} = \hbar\vec{k}\,\frac{\Gamma}{2}\,\frac{s_0}{1+s_0+(2(\delta-\vec{k}\bullet\vec{v})/\Gamma)^2}$$

The velocity range of the force is significant for atoms with velocity suc
detunings keeps them within one linewidth considering the power broad

$$\left|\delta - \vec{k}\bullet\vec{v}\right| \le \frac{\Gamma}{2}\sqrt{1+s_0}$$

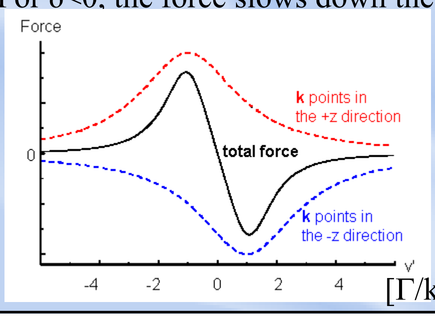
Doppler Cooling

$$\vec{F} = \vec{F}_+ + \vec{F}$$

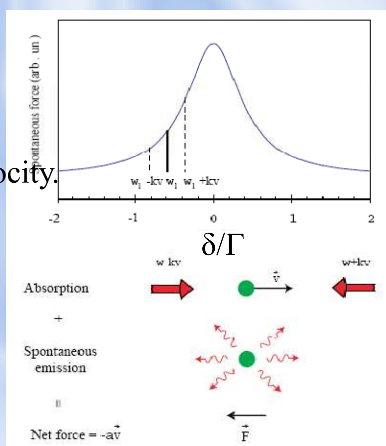
$$\vec{F}_\pm = \pm\frac{\hbar\vec{k}\Gamma}{2}\,\frac{s_0}{1+s_0+[2(\delta\mp kv)/\Gamma]^2}$$

$$F \approx \frac{8\hbar k^2\delta s_0\vec{v}}{\Gamma(1+s_0+(2\delta/\Gamma)^2)^2} \equiv -\beta\vec{v}, if\,(\frac{kv}{\Gamma})^4 << 1$$

For δ<0, the force slows down the velocity.



[Γ/k]



δ/Γ

Narodowe Laboratorium Technologii Kwantowych

Doppler Cooling limit

- Doppler cooling : cooling mechanism; Recoil heating : heating mechanism
- Temperature limit is determined by the relation that cooling rate is equal to heating rate.
- Recoil heating can be treat as a random walk with momentum step size ħk.

$$\langle p_x^2 \rangle = \hbar^2 k^2 \frac{\Gamma}{2} \frac{s_0}{1 + s_0 + (2\delta/\Gamma)^2}$$

$$\langle \dot{E}_{heat} \rangle = \frac{\langle p_x^2 \rangle}{2m} = -\langle \dot{E}_{cool} \rangle = -\vec{F} \bullet \vec{v} = \beta v^2$$

$$\frac{m \langle v^2 \rangle}{2} = \frac{k_B T}{2}$$

$$k_B T = \frac{-\hbar\Gamma}{4} \frac{1 + s_0 + (2\delta/\Gamma)^2}{2\delta/\Gamma}$$

For low intensity $s_0<<1$

$$k_B T = -\frac{\hbar\Gamma}{2}\left(\frac{\Gamma}{2\delta} + \frac{2\delta}{\Gamma}\right)$$

Minimum temperature

$$k_B T_D = \frac{\hbar\Gamma}{2}, when, \delta = -\Gamma/2$$

T_D ~ 100-200 μK for alkali atom