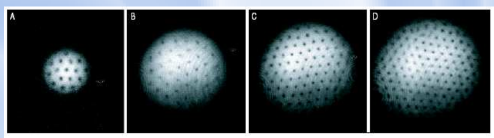
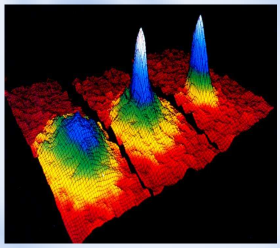
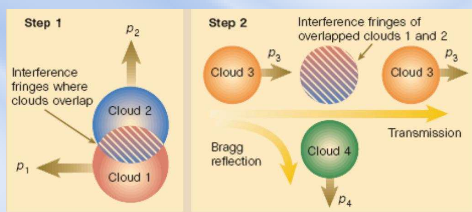
Mean-Field Effects







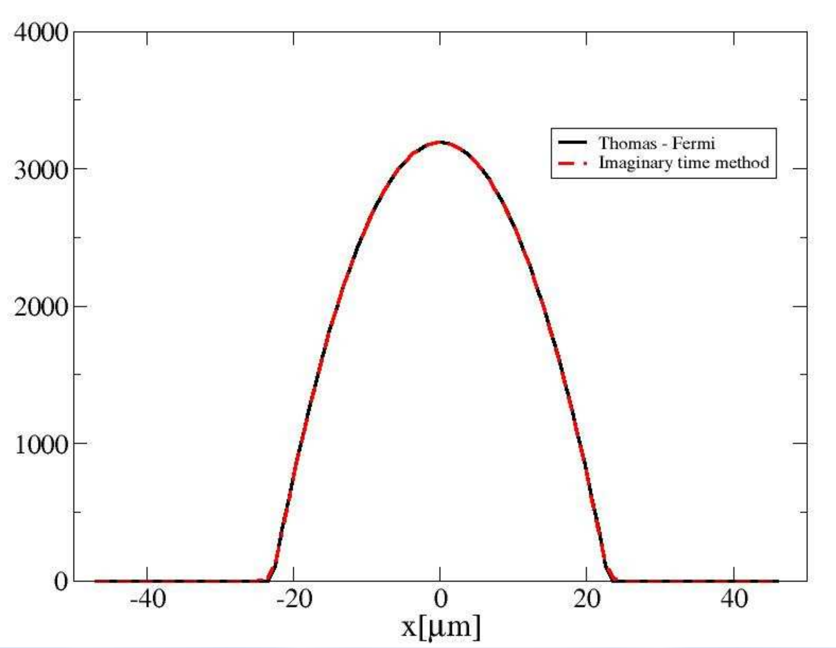
Theoretical model

Grossa-Pitaevskii Equation (NRS)

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{-\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\hbar V(x,y,z,t)\psi+NU_0|\psi|^2\psi$$

Atom-atom interactions $$U_0=\frac{4\pi a_0\hbar^2 N}{m}$$

Harmonic Potential $V(x,y,z,t)$



5 Imaginary time method

$$i\partial_t|\Psi\rangle = H|\Psi\rangle$$

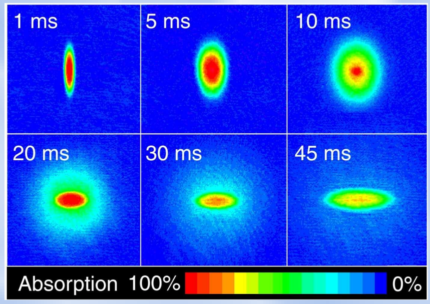
$$|\Psi^{(0)}\rangle = \sum_i \alpha_i|\Psi_i\rangle, \qquad \text{where} \qquad H|\Psi_i\rangle = E_i|\Psi_i\rangle$$

$$|\Psi^{(1)}\rangle = e^{-\tau H}|\Psi^{(0)}\rangle = \sum_i \alpha_i e^{-\tau\cdot E_i}|\Psi_i\rangle$$

$$|\Psi^{(2)}\rangle = \frac{|\Psi^{(1)}\rangle}{\sqrt{\langle\Psi^{(1)}|\Psi^{(1)}\rangle}} = \frac{\sum_i \alpha_i e^{-\tau\cdot E_i}|\Psi_i\rangle}{\sqrt{\sum_i |\alpha_i|^2 e^{-2\tau\cdot E_i}}}$$

$$= \frac{\alpha_0|\Psi_0\rangle}{\sqrt{\sum_i |\alpha_i|^2 e^{-2\tau\cdot(E_i-E_0)}}} + \frac{\sum_{i\geq 0} \alpha_i e^{-\tau\cdot(E_i-E_0)}|\Psi_i\rangle}{\sqrt{\sum_i |\alpha_i|^2 e^{-2\tau\cdot(E_i-E_0)}}}$$

Free expansion



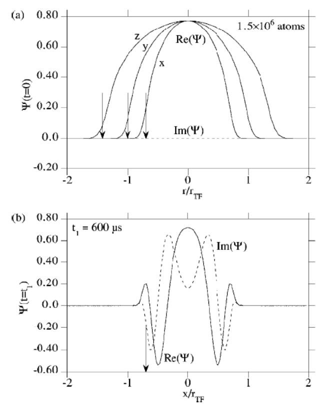


FIG. 3. (a) Cuts along the x, y and z axes of the parent condensate wave function $\Psi(x,y,z,t=0)$ for $N=1.5\times 10^6$ atoms in a trap with harmonic frequencies of 84, 59.4, and 42 Hz in the respective x, y, and z directions. The arrows show the TF radii $r_{TF}(i)$ in the $i=x,y,z$ directions. The curves labeled "x," "y," and "z," respectively represent $\mathrm{Re}[\Psi(x,0,0,0)]$, $\mathrm{Re}[\Psi(0,y,0,0)]$, and $\mathrm{Re}[\Psi(0,0,z,0)]$; $\mathrm{Im}[\Psi(x,y,z,0)]$ is identically zero for each case. (b) Cuts along the x axis of $\mathrm{Re}[\Psi(x,0,0,t=t_1)]$ and $\mathrm{Im}[\Psi(x,0,0,t=t_1)]$ for $t_1=600\ \mu s$.

Free expansion: variational approach

$$\mathcal{L} = \frac{i}{2}\hbar\left(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\right) - \frac{\hbar^2}{2m}|\nabla\psi|^2 + V(r)|\psi|^2 + \frac{2\pi a\hbar^2}{m}|\psi|^4,$$

$$\psi(x,y,z,t) = A(t)\prod_{\eta=x,y,z} e^{-\frac{[\eta-\eta_0(t)]^2}{2w_\eta^2} + i\eta\alpha_\eta(t) + i\eta^2\beta_\eta(t)}.$$

$$L = \langle \mathcal{L} \rangle = \int_{-\infty}^{\infty} \mathcal{L}\, d^3\vec{r}.$$

$$\ddot{w}_x + \lambda_x^2\nu^2 w_x = \frac{\hbar^2}{m^2 w_x^3} + \sqrt{\frac{2}{\pi}}\frac{a\hbar^2 N}{m^2 w_x^2 w_y w_z},$$

Rotations and quantized vortices in Bose superfluids

Complex order parameter: $\Psi = n^{1/2} e^{iS}$

n : density
S : phase

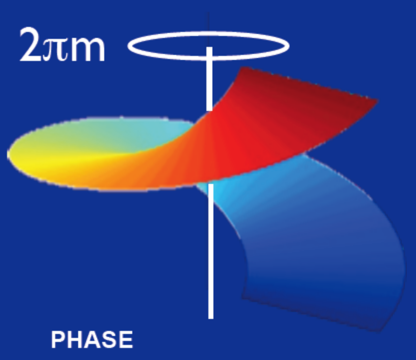
Velocity field : $v = (\hbar / m)\nabla S$

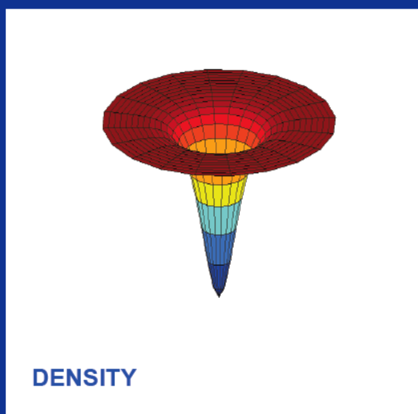
which implies: $\nabla \times v = 0$

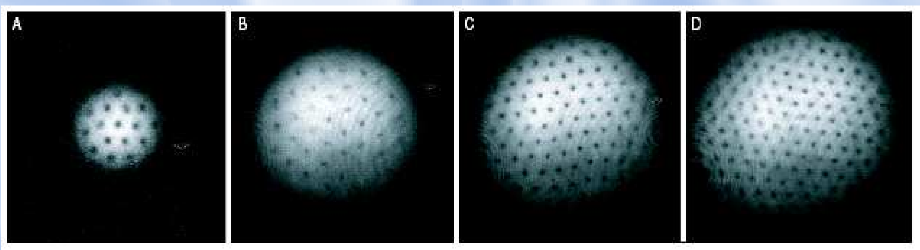
A superfluid has an irrotational velocity field

BEC vortex

- Topologically nontrivial state
- Low density core
- Screw-like phase dislocation
- Quantized circulation






A
B
C
D

Vortices observed at:

- JILA-Boulder
- ENS-Paris
- MIT
- Oxford

Produced with different techniques:

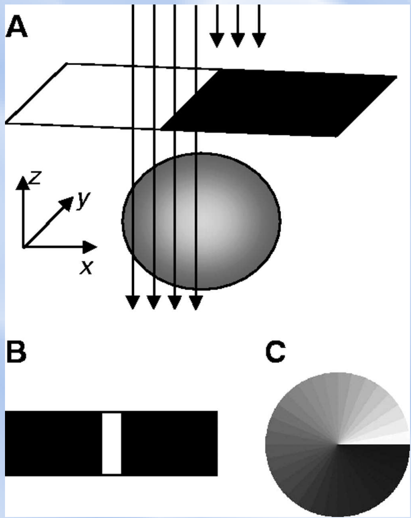
- Phase imprinting, rotating laser spoon, rotating magnetic trap, rotating thermal cloud, selective evaporation, decay of solitons, etc.

A lot of physical questions:

- Nucleation mechanisms.
- Observation of density and phase.
- Stability, decay, precession.
- Shape and dynamics of a single vortex.
- Formation and dynamics of vortex lattices.
- Fast rotating condensates and giant vortices.
- Coreless vortices and textures in spinor condensates.
- Interaction with thermal atoms, solitons, surface modes.
- Vortex rings, vortex-antivortex pairs, etc.

A lot of theoretical papers !!

Phase imprinting method



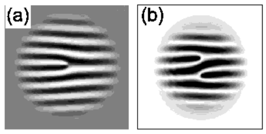


Figure: Interference pattern of the vortex state (5 ms after phase imprinting): (a) superimposed with a plane wave of $k = 1.8\,\mu\mathrm{m}^{-1}$ moving vertically in the plane of the figure;

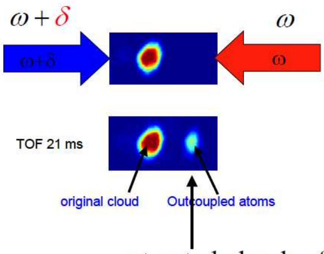
(b) 5 ms after applying Bragg pulses transferring momentum to a part of the condensate (along the vertical axis in the plane of the figure). The ideal case of equal splitting of the whole condensate has been assumed. The incident pulse was focused at the trap center and had a sharp step in the intensity profile ($\delta = 0.04$).

J. Denschlag et al., Science 287, 97 -101 (2000)

Published by AAAS

Science AAAS

Bragg spectroscopy of the momentum distribution: principle



extracted cloud reflects $\mathcal{P}(p_z)$

Energy and momentum conservation:
Bragg outcoupling resonant with atoms of momentum p_z (Doppler effect):

$$\delta = 2\frac{\hbar k_L^{\,2}}{M} + 2k_L\frac{p_z}{M}$$

By scanning δ one can measure the momentum distribution $\mathcal{P}(p_z)$

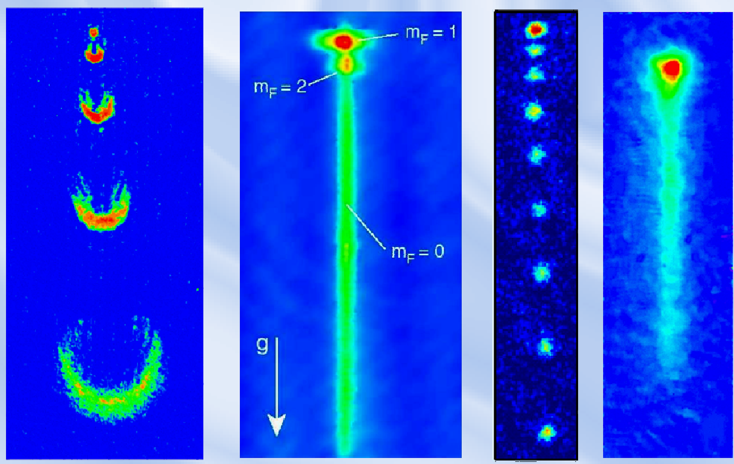
cf MIT and NIST

15

Atom lasers

Atom laser gallery

Height:
5, 2, 0.5, 1 mm



MIT '97 Munich '99 Yale '98 NIST '99

Theoretical model

Grossa-Pitaevskii Equation (NRS)

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + \hbar V(x, y, z, t)\psi + NU_0|\psi|^2 \psi$$

Atom-atom interactions $U_0 = \dfrac{4\pi a_0 \hbar^2 N}{m}$

Harmonic Potential $V(x, y, z, t)$

Uniwersytet Warszawski Wydział Fizyki

Nonlinear Optics versus BEC

Puls propagation

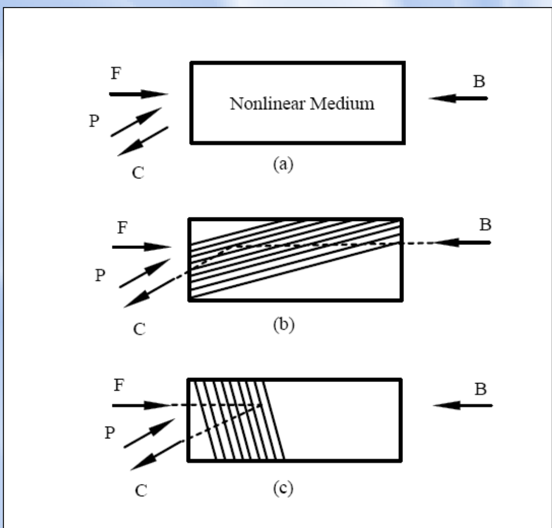
$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + i\frac{\gamma_{xx}}{2}\frac{\partial^2 A}{\partial x^2} + i\frac{\gamma_{yy}}{2}\frac{\partial^2 A}{\partial y^2} + i\gamma_{NL}|A|^2 A$$

BEC evolution

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + V(x,y,z)\psi + i\frac{U_0}{\hbar}|\psi|^2\psi$$

Uniwersytet Warszawski — Wydział Fizyki

Four Wave Mixing



$$P_{NL} \propto E_F E_B E_P^*$$

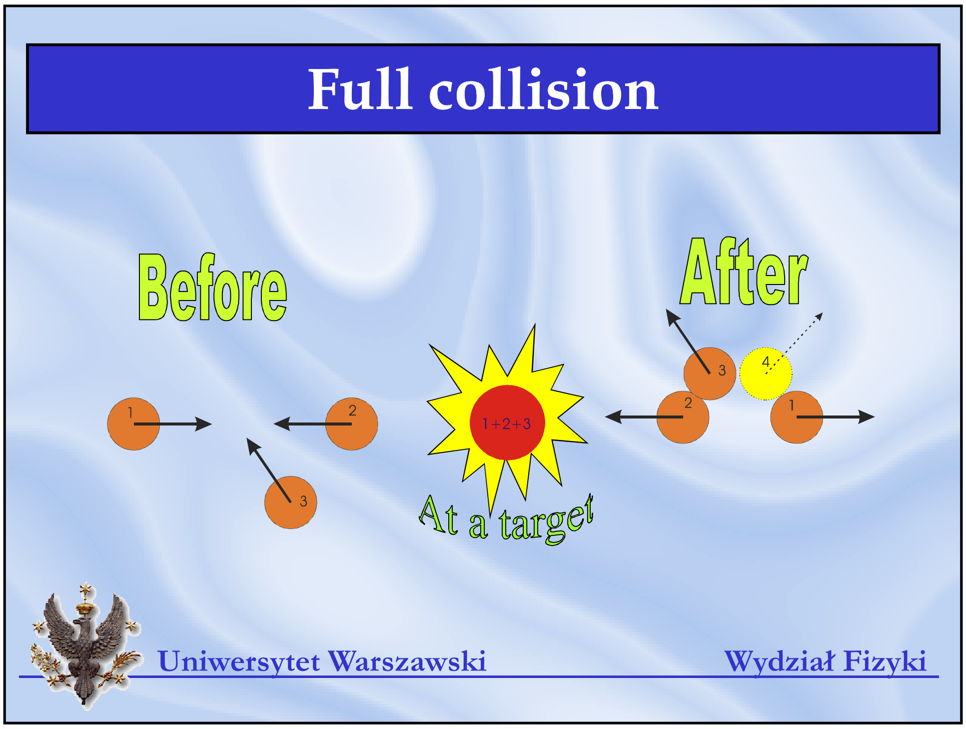
$$E_F \leftrightarrow e^{i(k\cdot r-\omega t)}$$

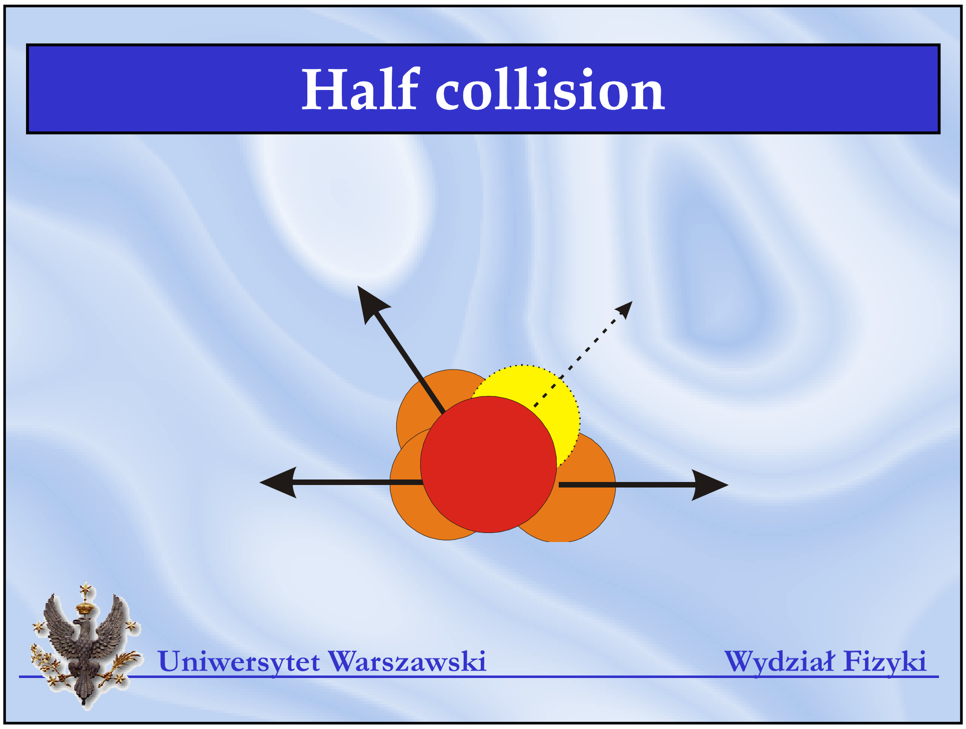
$$E_B \leftrightarrow e^{i(-k\cdot r-\omega t)}$$

$$E_P \leftrightarrow e^{i(\kappa\cdot r-\omega t)}$$

$$E_P^* \leftrightarrow e^{i(-\kappa\cdot r+\omega t)}$$

$$P_{NL} \leftrightarrow e^{i(-\kappa\cdot r-\omega t)}$$


Full collision
Before
After
1
2
3
1+2+3
4
At a target
Uniwersytet Warszawski
Wydział Fizyki


Half collision
Uniwersytet Warszawski
Wydział Fizyki

Resonant conditions

- Conservation of energy
- Conservation of momentum

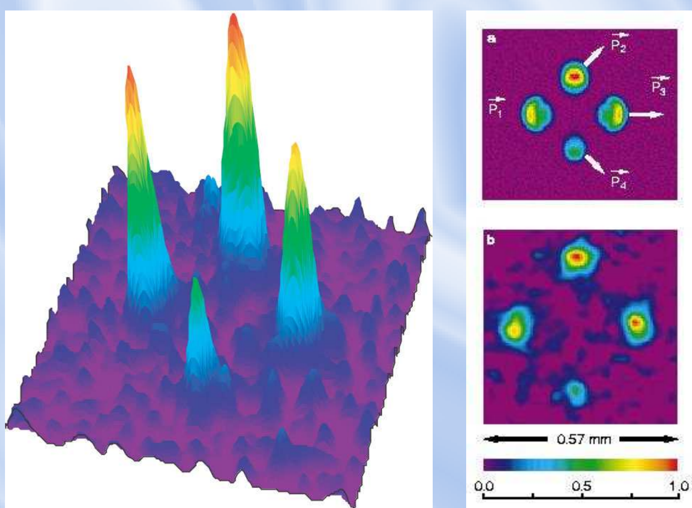
$$E = \frac{\hbar^2 k^2}{2m}$$

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4$$

4 particle process

$$k_1^2 + k_2^2 = k_3^2 + k_4^2$$

Four Wave Mixing



(W.D. Phillips, Nature, 1999)

Qualitative agreement

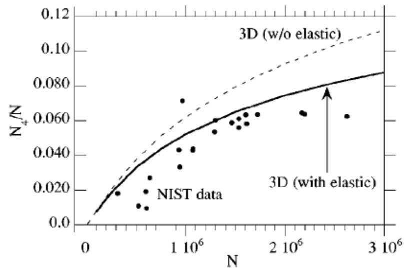


FIG. 13. Fraction of atoms in the 4WM output wave packet N_4/N versus the total number of initial atoms N calculated in 3D without and with inclusion of elastic scattering loss as discussed in Sec. II E. The dots are experimental data [2]. The trap is the same as in Fig. 3. The Bragg pulses are applied 600 μs after the trapping potential is turned off.

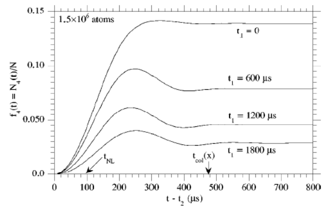


FIG. 8. Comparison of $N_4(t)/N$ versus t-t_2 for 1.5×10^6 atoms. The different curves show cases where the Bragg pulses are applied at t_1=0, 600, 1200, and 1800 μs after the trapping potential is turned off ($t_2 \approx t_1$). The trap is the same as in Fig. 3.

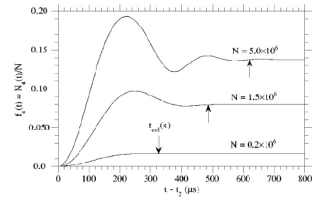


FIG. 7. Comparison of $N_4(t)/N$ versus t-t_2 for 0.2×10^6, 1.5×10^6, and 5.0×10^6 atoms. The trap is the same as in Fig. 3. The Bragg pulses are applied 600 μs after the trapping potential is turned off.

Simple model

$$i\hbar\frac{\partial\Psi(\vec r,t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\Delta+V+U_0N|\Psi|^2\right)\Psi(\vec r,t).$$

After we apply Bragg pulses

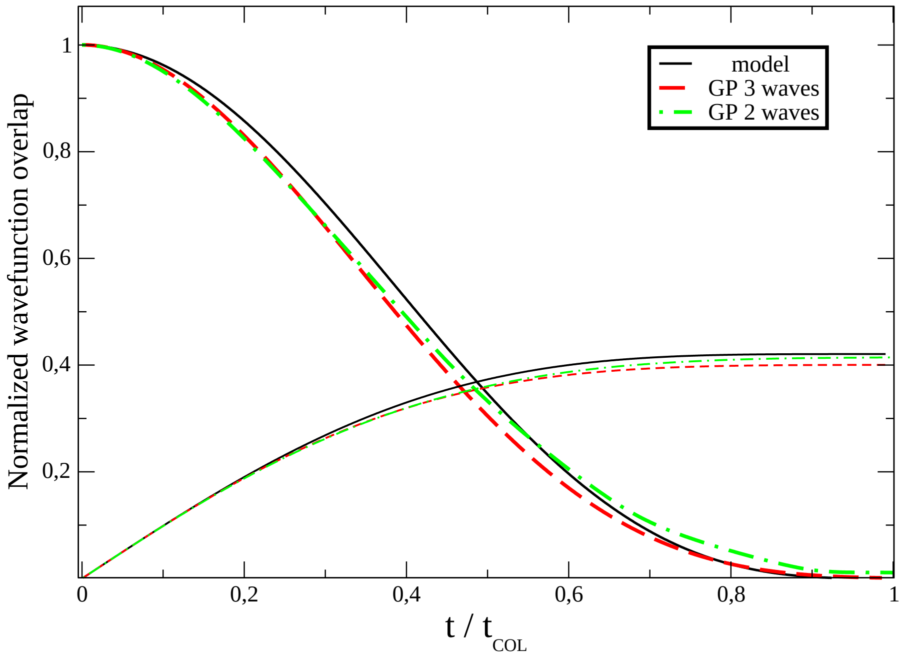
$$\Phi(\vec r,t_1)=\Psi(\vec r,t_1)\sum_{i=1}^{3} f_i^{1/2}e^{i\vec P_i\cdot\vec r/\hbar}.$$

. . . and separate using SVA

$$\left(\frac{\partial}{\partial t}+\frac{\hbar\vec k_4}{m}\cdot\vec\nabla-\frac{i\hbar}{2m}\Delta\right)\Phi_4 = -\frac{iNU_0}{\hbar}\left(|\Phi_4|^2+2\sum_{i=1}^{3}|\Phi_i|^2\right)\Phi_4 - \frac{2iNU_0}{\hbar}\Phi_1\Phi_2^*\Phi_3e^{i\Delta\omega t}$$

$$\tau_{NL}=(U_0N|\Psi_m|^2/\hbar)^{-1}$$

$$\tau_{col}=2r_{TF}/v$$



Full phase matching case

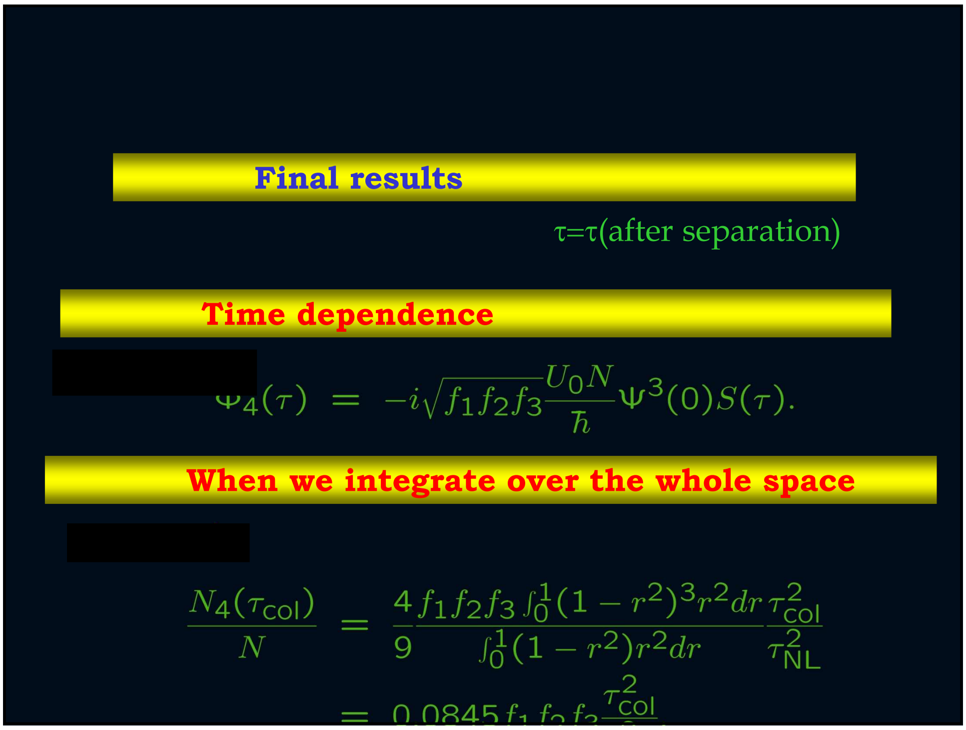
$$\frac{\partial \Phi_4}{\partial t} = -2ig(t)\sqrt{f_1 f_2 f_3}\frac{U_0 N}{\hbar}\Psi^3(0).$$

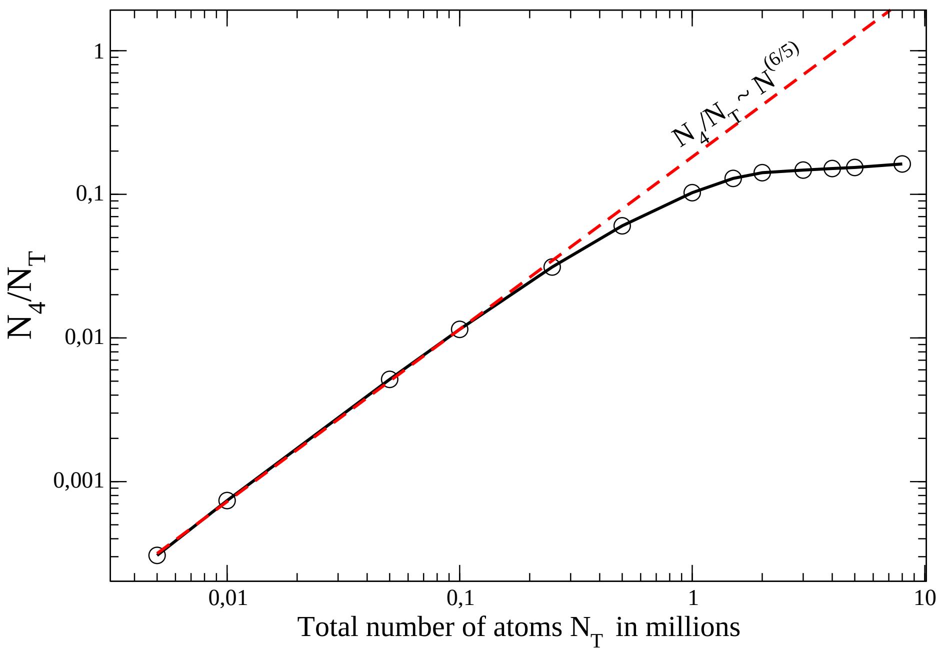
Simple model

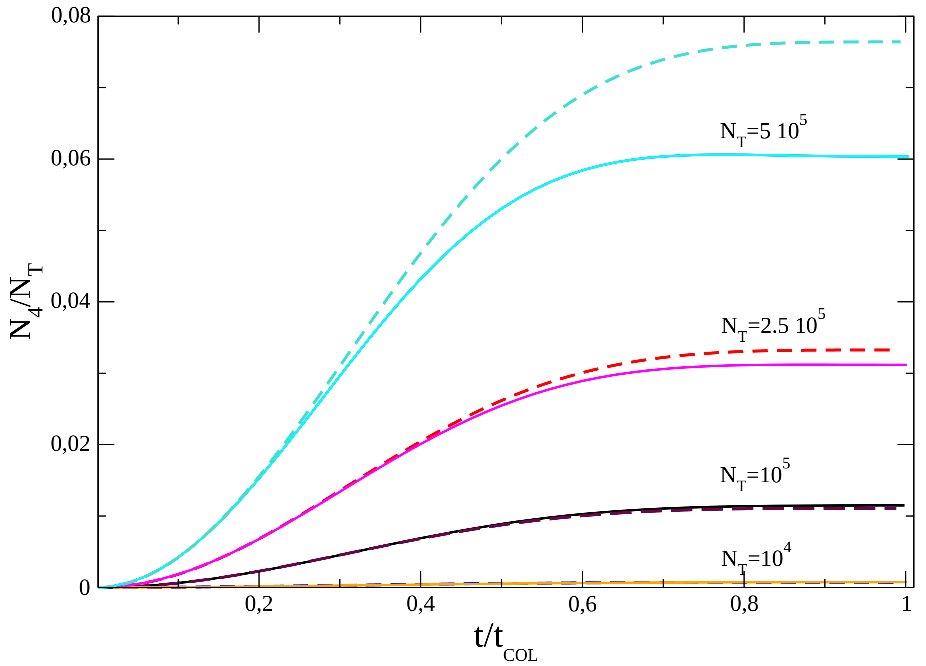
$$g(t) = \left(1 - \frac{t}{\tau_{\mathrm{col}}}\right)^2$$

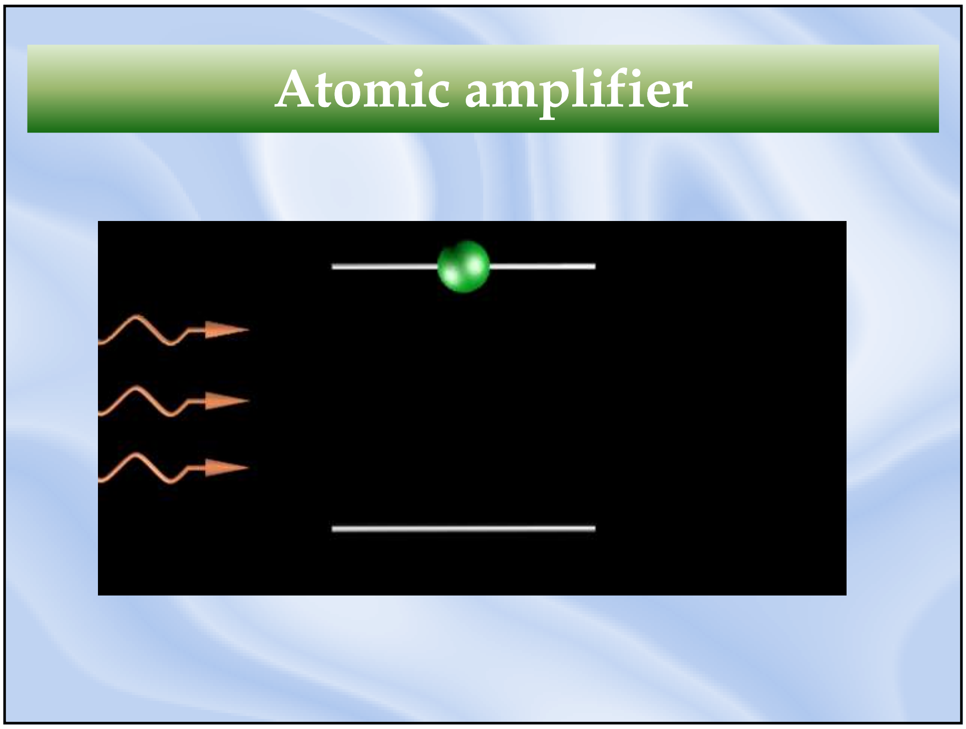
$$t \le \tau_{\mathrm{col}}$$

$$t > \tau_{\mathrm{col}} \Rightarrow g(t) = 0$$

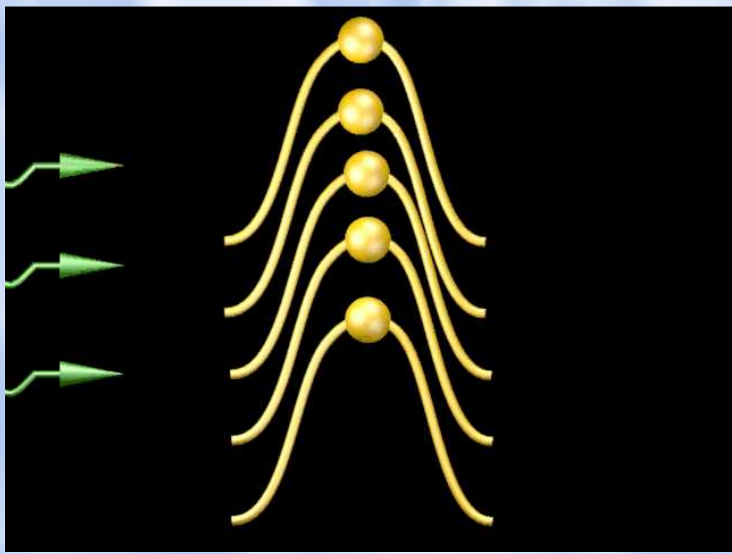

Final results
τ=τ(after separation)
Time dependence
[redacted]
$\Psi_4(\tau) = -i\sqrt{f_1 f_2 f_3}\frac{U_0 N}{\hbar}\Psi^3(0)S(\tau).$
When we integrate over the whole space
[redacted]
$\frac{N_4(\tau_{col})}{N} = \frac{4}{9}\frac{f_1 f_2 f_3 \int_0^1 (1-r^2)^3 r^2 dr}{\int_0^1 (1-r^2) r^2 dr}\frac{\tau_{col}^2}{\tau_{NL}^2}$
$= 0.0845 f_1 f_2 f_3 \tau_{col}^2$


1
0,1
0,01
0,001
N_4/N_T
$N_4/N_T \sim N^{(6/5)}$
0,01
0,1
1
10
Total number of atoms N_T in millions

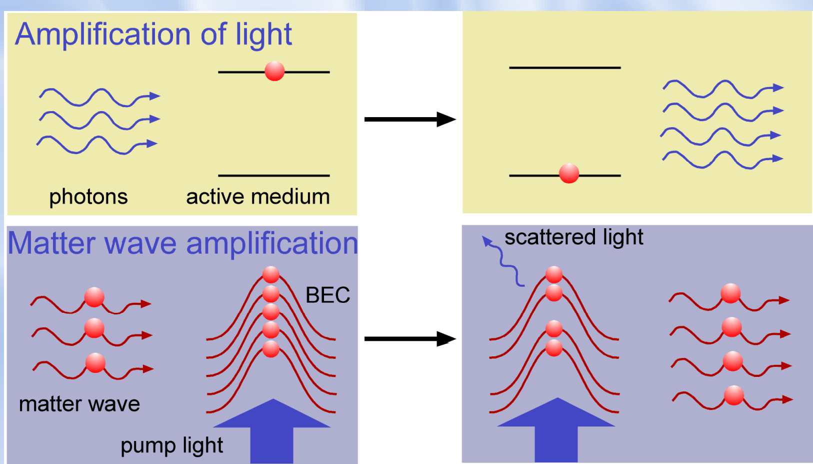

0,08
0,06
0,04
0,02
0
0,2
0,4
0,6
0,8
1
N_4/N_T
t/t_{COL}
N_T=5 10^5
N_T=2.5 10^5
N_T=10^5
N_T=10^4


Atomic amplifier

Atomic amplifier



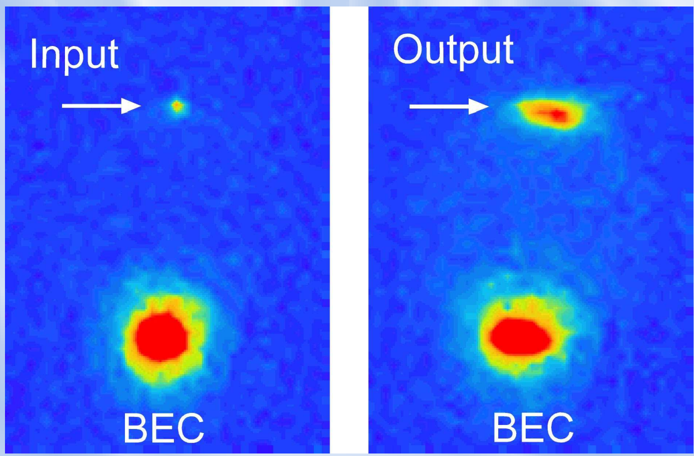
Atomic Amplifier



INNOWACYJNA GOSPODARKA
NARODOWA STRATEGIA SPÓJNOŚCI

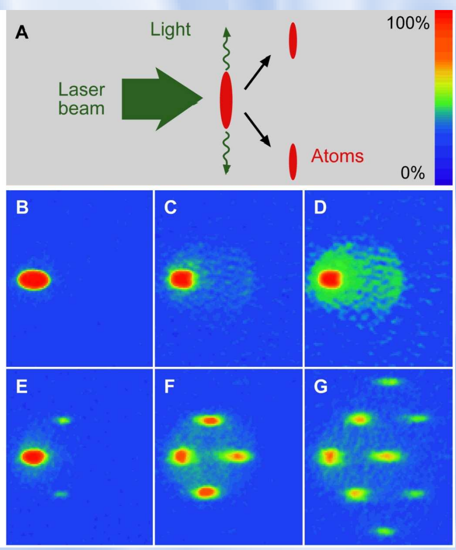
UNIA EUROPEJSKA
EUROPEJSKI FUNDUSZ ROZWOJU REGIONALNEGO

Amplifier



Ketterle (Nature, 1999)

Supperradiant Rayleigh scattering



(W. Ketterle, Science 1999)

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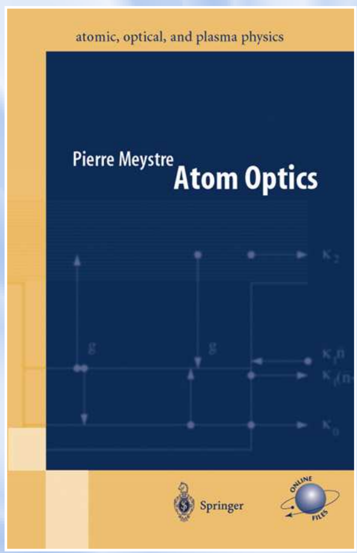
New field …


atomic, optical, and plasma physics
Pierre Meystre
Atom Optics
Springer