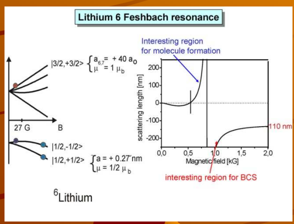
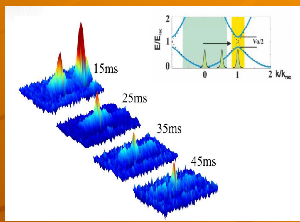
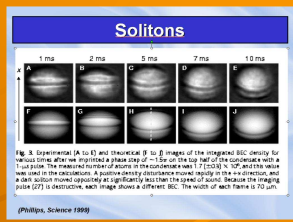
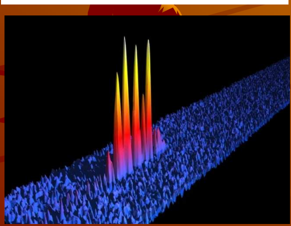
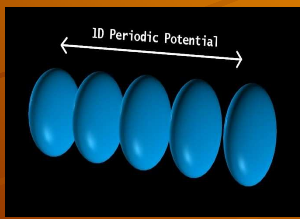
Solitons in BEC

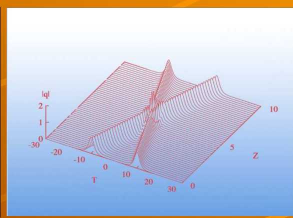












SOLITON COMMUNICATIONS

What is a Soliton | Soliton based Optical Communications | Profile of A.Hasegawa | What we do

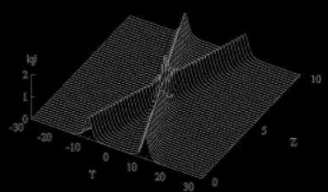
Executive Consultant
AKIRA HASEGAWA

▷ What is a Soliton

Soliton is a very stable pulse-shaped wave in nonlinear and dispersive media.

▷ Optical Soliton

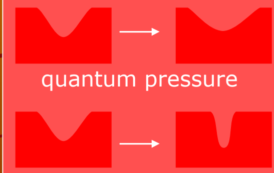
The Optical Soliton was discovered by Dr. Akira Hasegawa in 1973 and is the only stable optical pulse in fibers, thus is most suitable for ultra-high speed optical information carrier.

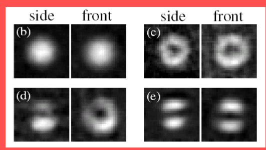


Soliton in Collision

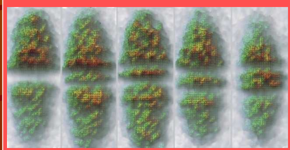
Solitons in BEC

dark solitons $g>0$ filled solitons

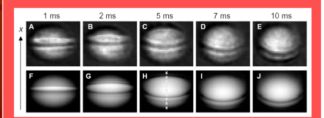




B. P. Anderson et al., PRL **86**, 2926 (2001)

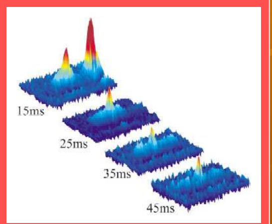


S. Burger et al., PRL **83**, 5198 (1999)



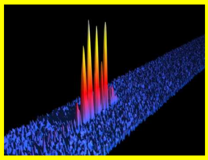
J. Denschlag et al., Science **287**, 97 (2000)

gap solitons
"negative mass"

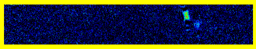


B. Eiermann et al. PRL **92**, 230401(2004)

$g<0$ bright solitons



K.S. Strecker et al., Nature **417**, 150 (2002)



L. Khaykovich et al., Science **296**, 1290 (2002)

$N_{Soliton} < 10^4$

quasi-1D regime

↕

collapse for $E_{int} > E_{radial}$

Theoretical model

Grossa-Pitaevskii Equation (NRS)

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\hbar V(x,y,z,t)\psi + NU_0|\psi|^2\psi$$

Atom-atom interactions

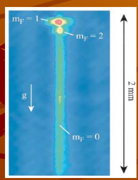
$$U_0 = \frac{4\pi a_0\hbar^2 N}{m}$$

Harmonic Potential

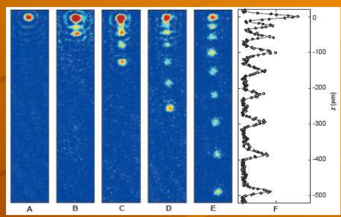
$$V(x,y,z,t)$$

Atom Lasers

- Atom laser: a coherent beam or pulse of atoms.



Max Planck Quantum Optics: Outcoupling via spin flip (2001)



Yale: Gravity-induced tunneling through a lattice (1998)

How about attractive nonlinearity?

Phase imprinting

$$\Psi_k(x)=\sqrt{n_0}\left(i\cdot\frac{v_k}{c_s}+\sqrt{1-\frac{v_k^2}{c_s^2}}\cdot\tanh\left[\frac{x-x_k}{l_0}\sqrt{1-\frac{v_k^2}{c_s^2}}\right]\right),$$

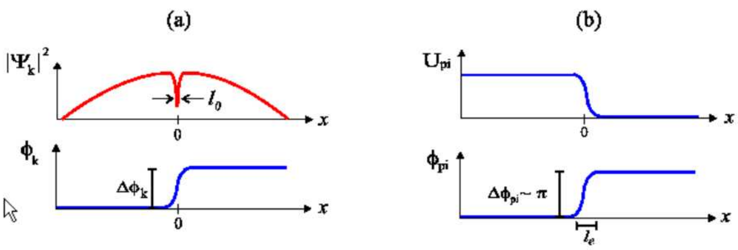
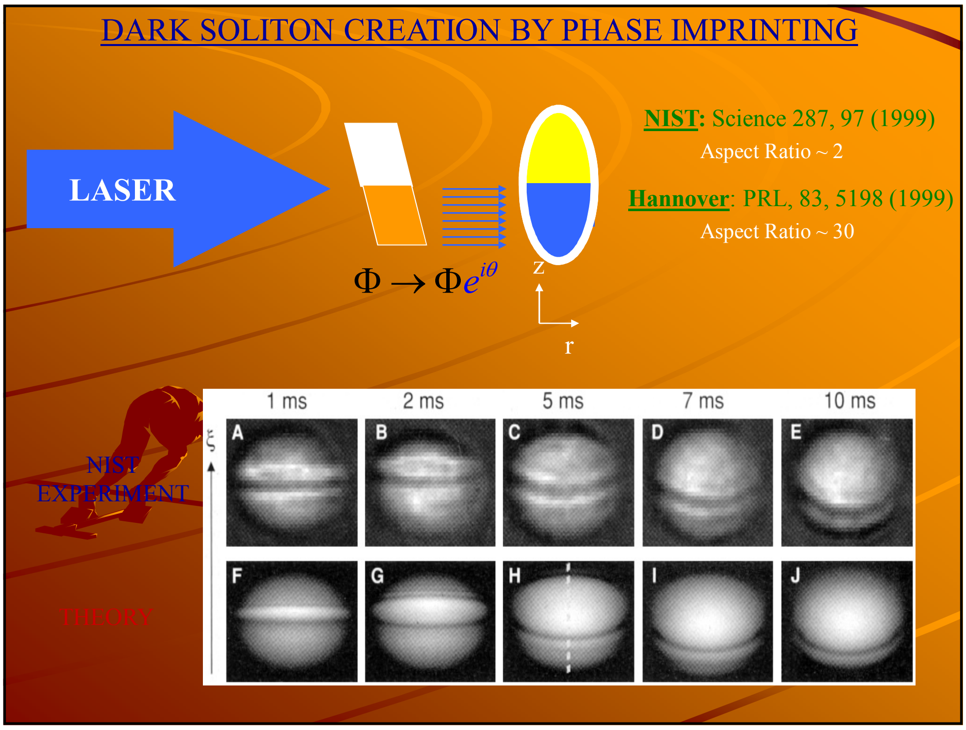
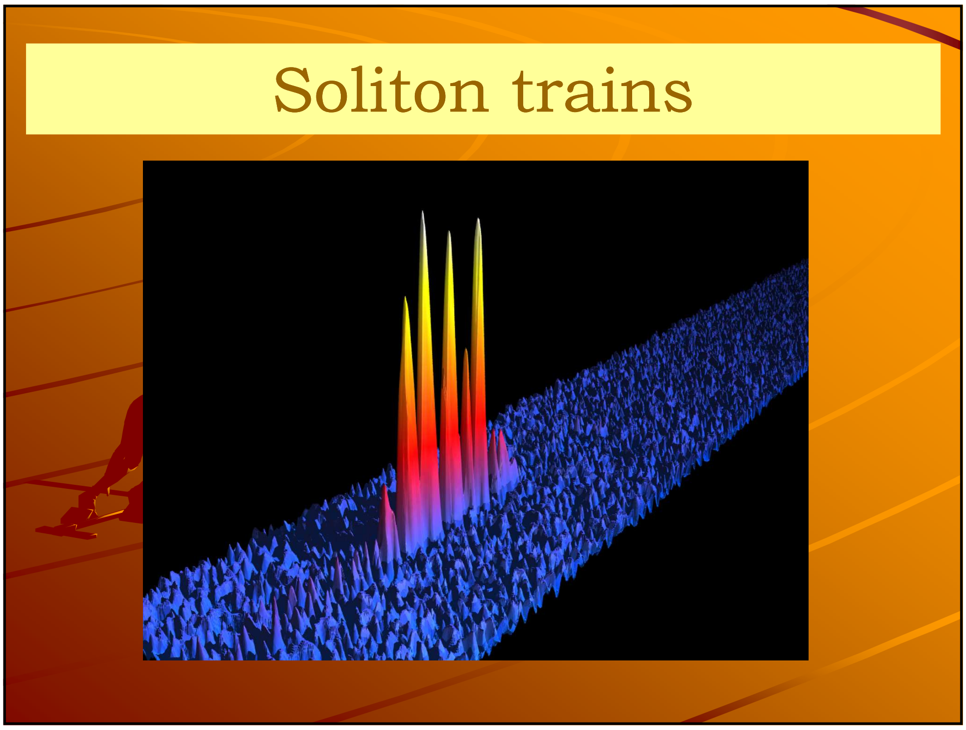
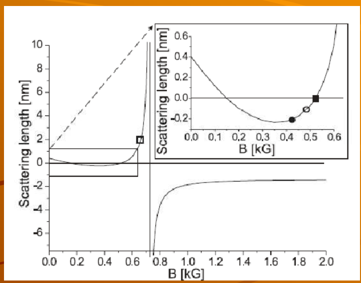


Figure 1: *(a) Density and phase distribution of a dark soliton state with $\Delta\phi_k \sim \pi$. The density minimum has a width of $\sim l_0$. (b) Phase imprinting potential, U_{pi}, and associated phase distribution.*


DARK SOLITON CREATION BY PHASE IMPRINTING
LASER
$\Phi \rightarrow \Phi e^{i\theta}$
z
r
NIST: Science 287, 97 (1999)
Aspect Ratio ~ 2
Hannover: PRL, 83, 5198 (1999)
Aspect Ratio ~ 30
NIST EXPERIMENT
THEORY
1 ms
2 ms
5 ms
7 ms
10 ms
A
B
C
D
E
F
G
H
I
J
ξ


Soliton trains



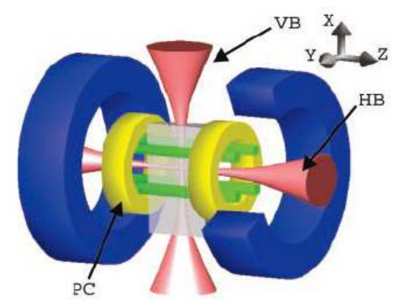
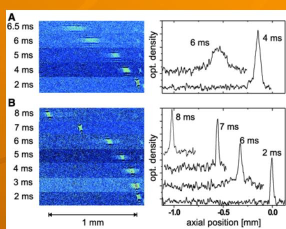
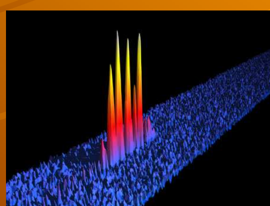


Fig. 1. Experimental setup for soliton production. ^{7}Li atoms are evaporatively cooled in a Ioffe-Pritchard magnetic trap and transferred into a crossed optical dipole trap in state $|F = 1, m_F = 1\rangle$ where they Bose condense. Magnetic tuning of the scattering length to positive, zero, and negative values is performed with the two pinch coils (PC). Switching-off the vertical trapping beam (VB) allows propagation of a soliton in the horizontal 1D waveguide (HB). Absorption images of solitons and BECs are recorded on a charge-coupled device camera in the x, z plane.

Observations of Bright Solitons

- Bright soliton in a waveguide
  - Rolls down potential hill
    - Ecole normale, Paris (2002)
- Bright soliton train
  - 4 or 5 solitons oscillate and interact in harmonic trap
    - Rice University (2002)



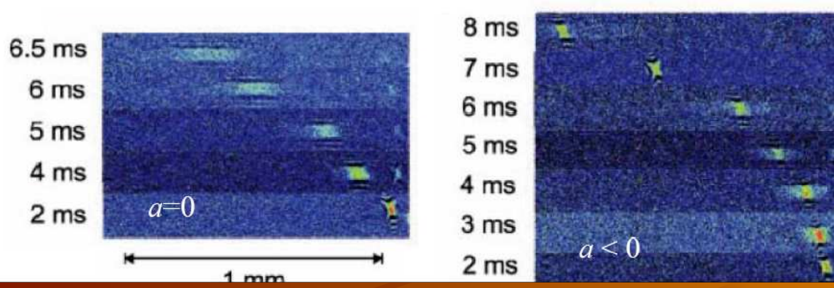


Manipulating interactions : soliton formation

One prepares a lithium 7 condensate with repulsive interaction and place it in a quasi-1D geometry (ENS):

$\hbar\omega_{\perp} \square\ k_B T, \mu$

One switches interactions towards a=0 (ideal gas) or a<0 (attractive)



Formation of soliton trains in Bose-Einstein condensates by temporal Talbot effect

Krzysztof Gawryluk,[1] Mirosław Brewczyk,[1] Mariusz Gajda,[2] and Jan Mostowski[2]
[1] Instytut Fizyki Teoretycznej, Uniwersytet w Białymstoku, ulica Lipowa 41, 15-424 Białystok, Poland
[2] Instytut Fizyki PAN, Aleja Lotników 32/46, 02-668 Warsaw, Poland
(Dated: May 26, 2005)

We study the recent observation of formation of matter-wave soliton trains in Bose-Einstein condensates. We emphasize the role of the box-like confinement of the Bose-Einstein condensate and find that there exist time intervals for the opening the box that support the generation of real solitons. When the box-like potential is switched off outside the existing time windows, the number of peaks in a train changes resembling missing solitons observed in the experiment. Our findings indicate that a new way of generating soliton trains in condensates through the temporal, matter-wave Talbot effect is possible.

letters to nature

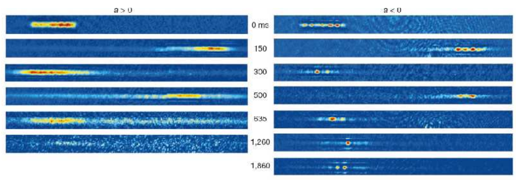


Figure 3 Comparison of the propagation of repulsive condensates with atomic solitons. The images are obtained using destructive absorption imaging, with a probe laser detuned 27 MHz from resonance. The magnetic field is reduced to the desired value before switching off the end caps (see text). The times given are the intervals between turning off the end caps and probing (the end caps are on for the $t = 0$ images). The axial dimension of each image frame corresponds to 1.28 mm at the plane of the atoms. The amplitude of oscillation is ~370 μm and the period is 310 ms. The $a > 0$ data correspond to 630 G, for which $a \approx 10a_0$, and the initial condensate number is ~3×10^5. The $a < 0$ data correspond to 547 G, for which $a \approx -3a_0$. The largest soliton signals correspond to ~5,000 atoms per soliton, although significant image distortion limits the precision of number measurement. The spatial resolution of ~10 μm is significantly greater than the expected transverse dimension $l_r \approx 1.5$ μm.

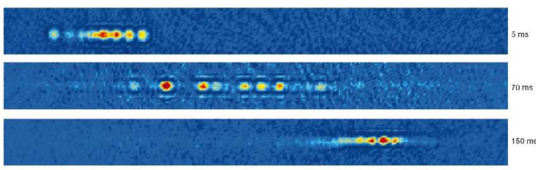
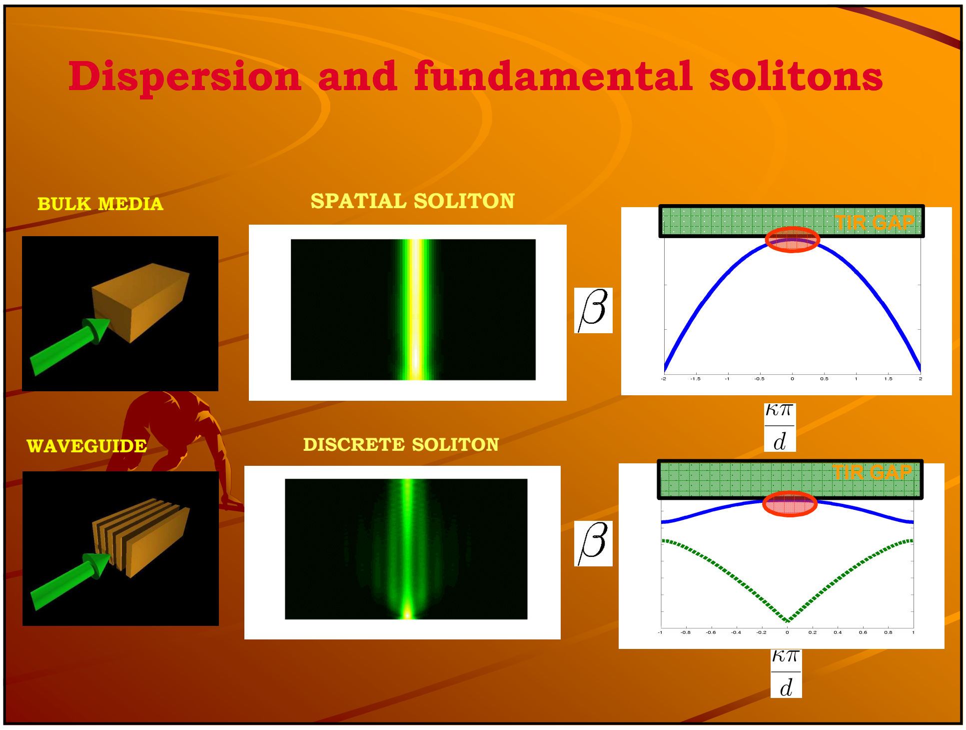
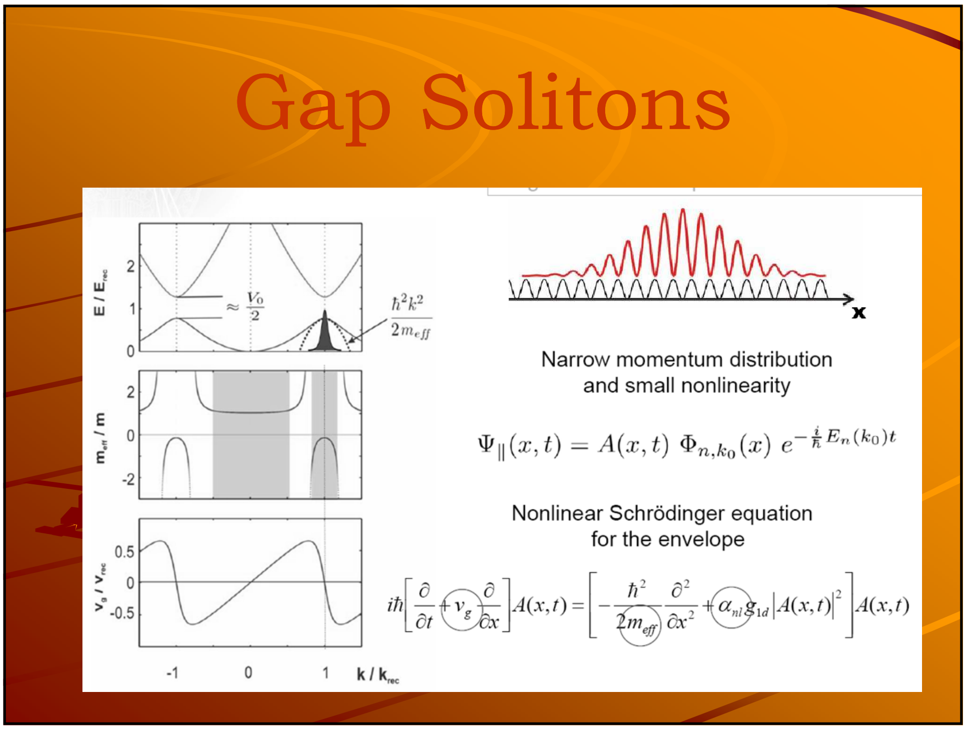
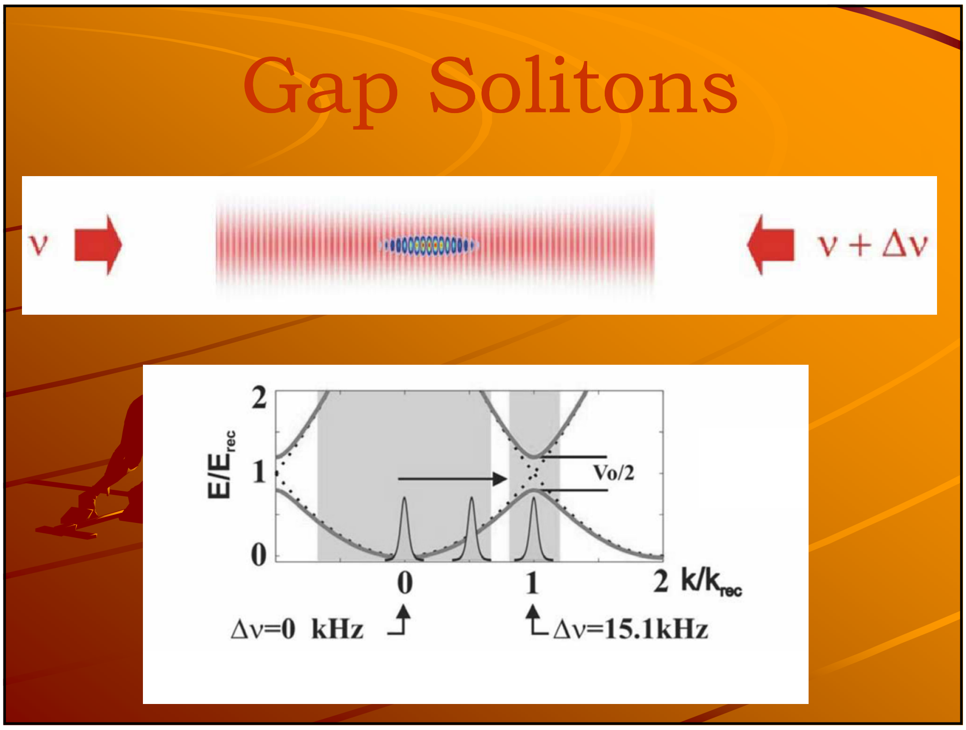


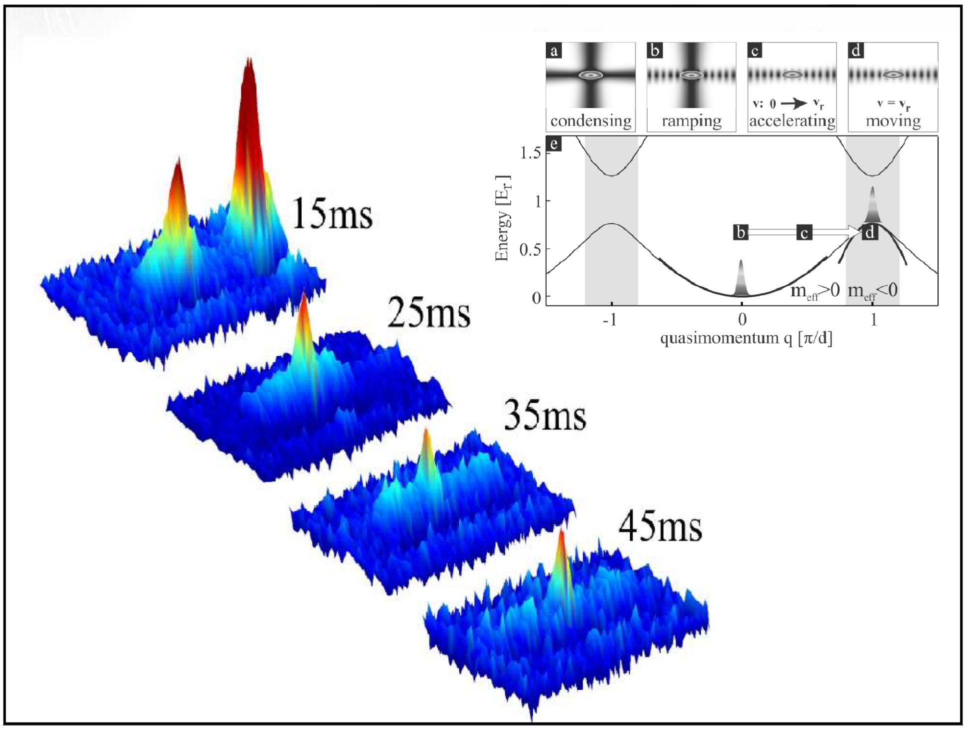
Figure 4 Repulsive interactions between solitons. The three images show a soliton train near the two turning points and near the centre of oscillation. The spacing between solitons is compressed at the turning points, and spread out at the centre of the oscillation. A simple model based on strong, short-range, repulsive forces between nearest-neighbour solitons indicates that the separation between solitons oscillates at approximately twice the trap frequency, in agreement with observations. The number of solitons varies from image to image because of shot to shot experimental variations, and because of a very slow loss of soliton signal with time. As the axial length of a soliton is expected to vary as $1/N$ (ref. 11), solitons with small numbers of atoms produce particularly weak absorption signals, scaling as N^2. Trains with missing solitons are frequently observed, but it is not clear whether this is because of a slow loss of atoms, or because of sudden loss of an individual soliton.





Narodowe Laboratorium Technologii Kwantowych


Gap Solitons
ν
ν + Δν
E/E_rec
2
1
0
Vo/2
0
1
2 k/k_rec
Δν=0 kHz
Δν=15.1kHz


a
b
c
d
condensing
ramping
accelerating
moving
e
Energy [E_r]
quasimomentum q [π/d]
m_eff>0 m_eff<0
15ms
25ms
35ms
45ms

Variational approach

equation of motion $$i\frac{\partial u}{\partial z} = -\frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u$$

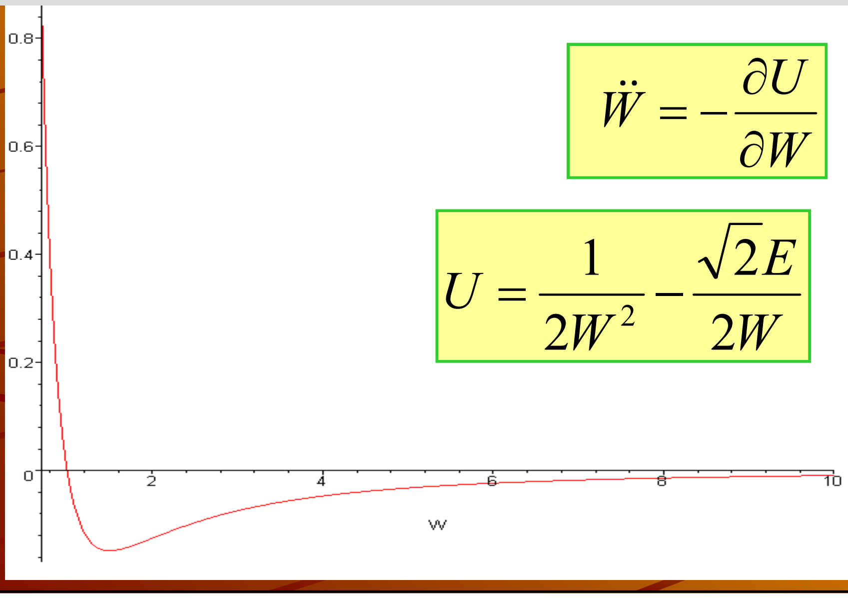
Lagrangian density $$L = i\left(u_z u^* - u u_z^*\right) - |u_x|^2 - |u|^4$$

Ansatz $$u = A(z)\exp\left(-\frac{x^2}{2W(z)^2} + ib(z)x^2 + i\phi(z)\right)$$

$$L = \frac{\sqrt{\pi}}{4W} A^2\left(4\phi' W^2 + b' W^4 + 1 + W^{-2} - A^2\sqrt{2}W^2 + b^2 W^4\right)$$

reduced dynamics $$\ddot{W} = \frac{1}{W^3} - \frac{\sqrt{2}E}{2}\frac{1}{W^2}$$

Stability in 1D



$$\ddot{W} = -\frac{\partial U}{\partial W}$$

$$U = \frac{1}{2W^2} - \frac{\sqrt{2}E}{2W}$$

Stability in > 1D

equation of motion

$$i\frac{\partial u}{\partial z} = -\frac{1}{2}\Delta_{\perp}u + |u|^2 u$$

Lagrangian Density

$$L = i\left(u_z u^* - u u_z^*\right) - |u_x|^2 - |u_y|^2 - |u|^4$$

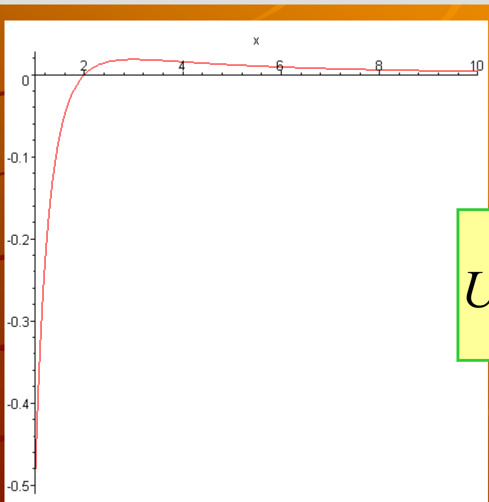
2D

$$U = \frac{A}{W^2}$$

3D

$$U = \frac{A}{2W^2} - \frac{A}{W^3}$$

Stability in 3D



$$\ddot{W} = -\frac{\partial U}{\partial W}$$

$$U = \frac{1}{2W^2} - \frac{\sqrt{2}}{2W^3}$$

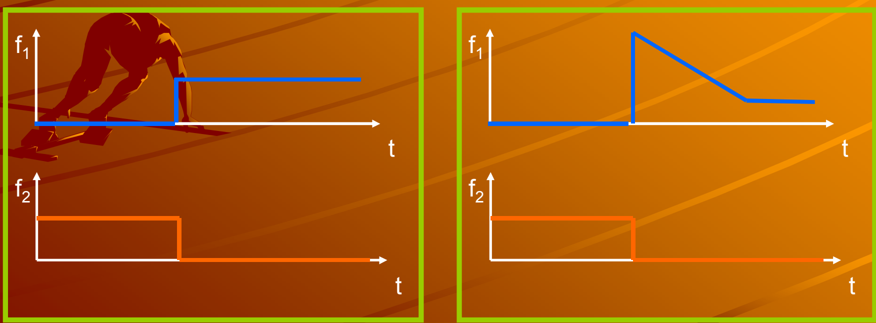
INNOWACYJNA GOSPODARKA
NARODOWA STRATEGIA SPÓJNOŚCI

UNIA EUROPEJSKA
EUROPEJSKI FUNDUSZ
ROZWOJU REGIONALNEGO

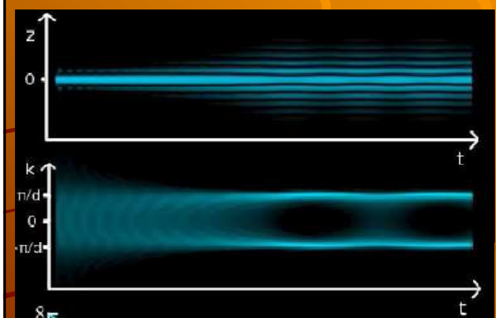
Gap Solitons

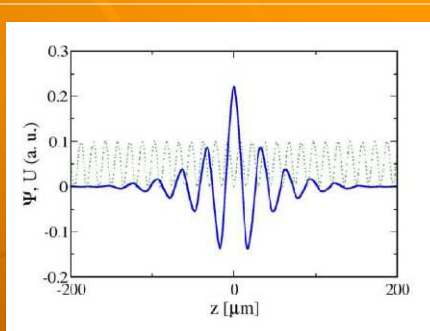
$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\Delta + U(\mathbf{r},t) + \frac{4\pi a\hbar^2}{m}|\Psi|^2\right]\Psi,$$

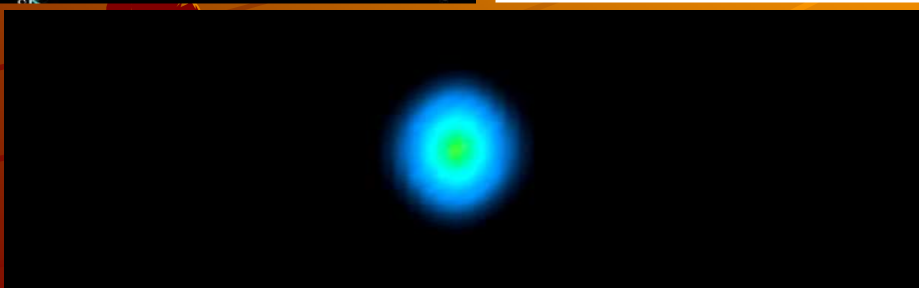
$$U(\mathbf{r},t) = f_1(t)\varepsilon\sin^2\left(\frac{2\pi z}{\lambda}\right) + \frac{m}{2}\left[\omega_\perp^2\varrho^2 + f_2(t)\omega_z^2 z^2\right]$$



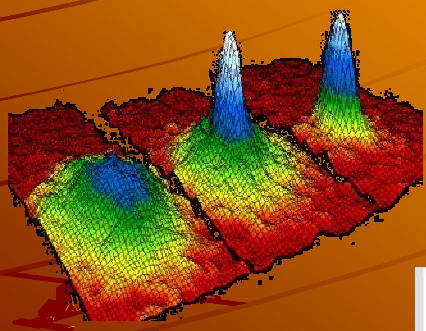
Autoformacja

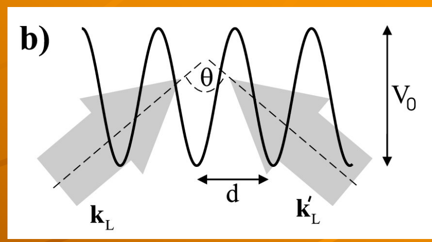


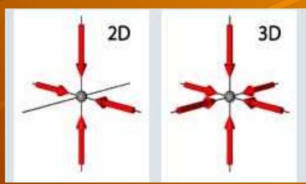


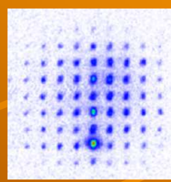


BEC in the lattice





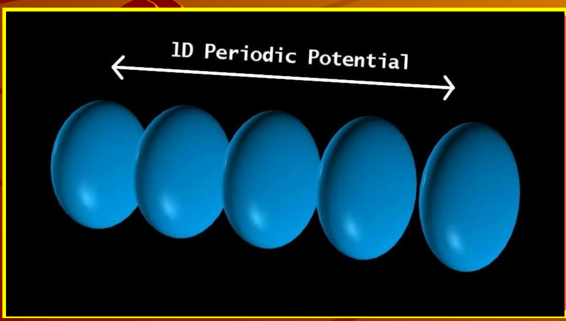


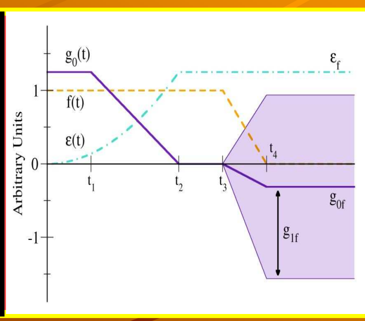


3D + lattice

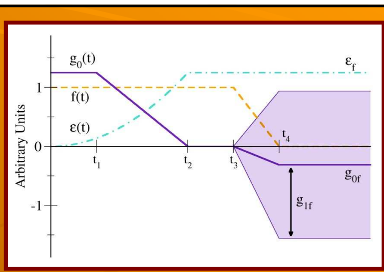
$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\Delta\psi + U(\mathbf{r},t) + g(t)|\psi|^2\psi$$

$$U(\mathbf{r},t) = \varepsilon(t)(1-\cos(2z)) + f(t)\left[\frac{\omega_\perp^2}{2}\rho^2 + U_0(z)\right]$$

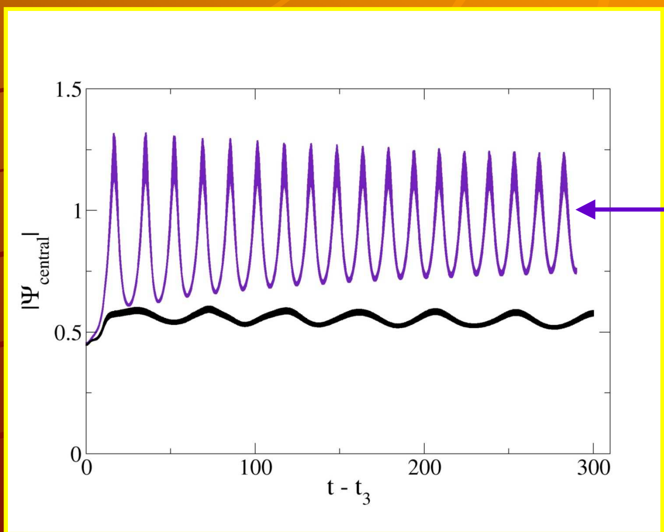




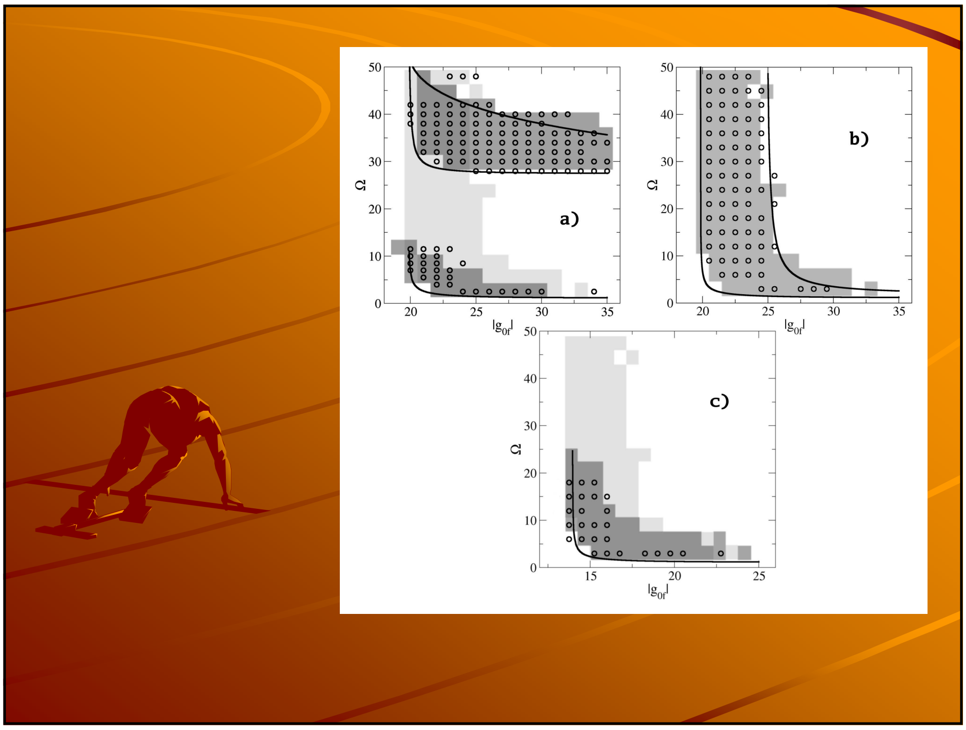
Dynamics

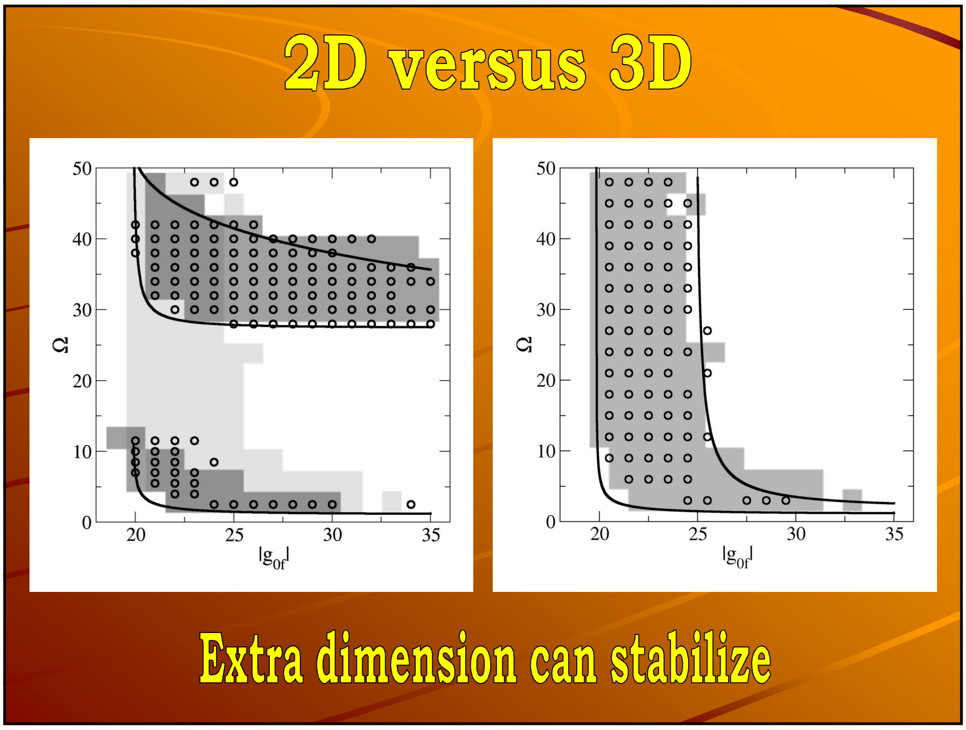


Stability of the solution



fully 3D


a)
b)
c)
Ω
$|g_{0f}|$


2D versus 3D
Ω
$|g_{0f}|$
Extra dimension can stabilize