ERRATUM TO "EXCITATION SPECTRUM OF INTERACTING BOSONS IN THE MEAN-FIELD INFINITE-VOLUME LIMIT" ANNALES HENRI POINCARÉ, ONLINE FIRST, DOI: 10.1007/S00023-013-0302-4

JAN DEREZIŃSKI AND MARCIN NAPIÓRKOWSKI

ABSTRACT. We correct the assumptions on the interaction potential v in [1]. We also present a careful discussion of the periodization of a potential–the dicussion contained in [1] was not quite precise.

1. CONDITIONS ON POTENTIALS

In the first section of [1] we stated that the *interaction potential* v is an even real function on $\mathbb{R}^d$ satisfying the following assumptions:

(1) $v \geq 0$,
(2) $v \in L^1(\mathbb{R}^d)$,
(3) $\hat{v} \geq 0$,
(4) $\hat{v} \in L^1(\mathbb{R}^d)$,

where $\hat{v}$ denotes the Fourier transform given by $\hat{v}(\mathbf{p}) = \int\limits_{\mathbb{R}^d} v(\mathbf{x})\, \mathrm{e}^{-\mathrm{i}\mathbf{p}\mathbf{x}}\, \mathrm{d}\mathbf{x}$. Unfortunately, these assumptions seem not sufficient for the proof of the main result of [1]. However, all the arguments of that paper are correct if we replace the condition (4) by the condition

$$(4') \qquad \text{there exists } C \text{ such that, for } L \geq 1, \quad \frac{1}{L^d} \sum_{\mathbf{p} \in \frac{2\pi}{L}\mathbb{Z}^d} \hat{v}(\mathbf{p}) \leq C.$$

Note that, even though (4) and (4′) are closely related, neither of these conditions implies the other one.

Some readers may complain that (4′) looks somewhat complicated. Therefore, we give yet another condition, which looks easier and which implies (4′):

$$(4'') \qquad \text{there exists } C \text{ and } \mu > d \text{ such that} \quad |\hat{v}(\mathbf{p})| \leq C(1 + |\mathbf{p}|)^{-\mu}.$$

In fact, (2) implies that $\hat{v}$ is continuous and (4″) implies that $\hat{v} \in L^1$. Then an easy argument involving Riemann sums and the Lebesgue Dominated Convergence Theorem yields

$$\lim_{L\to\infty} \frac{1}{L^d} \sum_{\mathbf{p} \in \frac{2\pi}{L}\mathbb{Z}^d} \hat{v}(\mathbf{p}) = (2\pi)^{-d} \int \hat{v}(\mathbf{p}) \mathrm{d}\mathbf{p}. \qquad (1.1)$$

Clearly, (4′) is an immediate consequence of (1.1).

2. PERIODIZATION OF POTENTIALS

One of the concepts used in our paper [1] is the *periodization* of a potential. Below we would like to give a discussion of this concept which is somewhat more careful from the one contained in [1].

Suppose that $v \in L^1(\mathbb{R}^d)$ and $L > 0$. Then the following formula

$$v^L(\mathbf{x}) = \sum_{\mathbf{n}\in\mathbb{Z}^d} v(\mathbf{x}+\mathbf{n}L) \tag{2.1}$$

defines a function on $\mathbb{R}^d$ periodic with respect to the lattice $L\mathbb{Z}^d$ and satisfying $v^L \in L^1\big([-L/2, L/2[^d\big)$, which will be called the *periodization of v with period L* (see [2], sect. 4.2.1).

Suppose now in addition that

$$\sum_{\mathbf{p}\in\frac{2\pi}{L}\mathbb{Z}^d} |\hat{v}(\mathbf{p})| < \infty. \tag{2.2}$$

Then the *Poisson summation formula* shows that

$$v^L(\mathbf{x}) = \frac{1}{L^d} \sum_{\mathbf{p}\in\frac{2\pi}{L}\mathbb{Z}^d} \mathrm{e}^{\mathrm{i}\mathbf{p}\mathbf{x}}\,\hat{v}(\mathbf{p}). \tag{2.3}$$

(See, e.g. [3], Thm. 2.4 or [2], Sect. 4.2.2).

In [1] we used (2.3) to define v^L. Strictly speaking, this was not a mistake, at least under the conditions (1), (4′) (which clearly imply (2.2) for all L), since then the definitions (2.3) and (2.1) are equivalent. However, one can argue that the definition (2.1) is more natural and slightly more general and thus we should have used it in [1].

In [1] we wrote that $v^L(\mathbf{x}) \to v(\mathbf{x})$ as $L \to \infty$. The meaning of that statement can be the following: if $v \in L^1(\mathbb{R}^d)$ and I is a compact subset of $\mathbb{R}^d$, then $v^L \to v$ in $L^1(I)$. In fact, let $I \subset [-L_0/2, L_0/2[^d$. Then for $L > L_0$ we have

$$\int_I |v(\mathbf{x}) - v^L(\mathbf{x})| \mathrm{d}\mathbf{x} \leq \sum_{n\in\mathbb{Z}^d\setminus\{\mathbf{0}\}} \int_I |v(\mathbf{x}+nL)| \mathrm{d}\mathbf{x} \xrightarrow[L\to\infty]{} 0.$$

Note also that $v \in L^1(\mathbb{R}^d)$, $v \geq 0$ and (2.1) imply immediately that

$$v(\mathbf{x}) \geq 0 \Rightarrow v^L(\mathbf{x}) \geq 0.$$

We use this fact in the proof of Lemma 4.1 of [1].

References

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(J. Dereziński) Dept. of Math. Methods in Phys., Faculty of Physics, University of Warsaw, Hoza 74, 00-682 Warszawa, Poland

E-mail address: `Jan.Derezinski@fuw.edu.pl`

(M. Napiórkowski) Dept. of Math. Methods in Phys., Faculty of Physics, University of Warsaw, Hoza 74, 00-682 Warszawa, Poland

E-mail address: `Marcin.Napiorkowski@fuw.edu.pl`