Information geometric foundations of quantum theory

Ryszard Paweł Kostecki

Doctoral dissertation

University of Warsaw
Faculty of Physics
Institute of Theoretical Physics
Thesis Advisor: prof. dr hab. Jerzy Lewandowski

Warszawa
February 2013

Kogóż to z nas tonący nie wiózł wrak?
Któż z nas zaprzeczyć może że ułomny?
Kogóż nie łudził oślepiony ptak?
Kogóż w bezludzie nie wiódł pies bezdomny?

A przecież wciąż przyciąga strefa ogrodzona
I ogrodzona nie bez celu – trzeba wierzyć
To nie my w Zonie, to nam odebrana Zona
Nam ją niepewnym, ale własnym krokiem mierzyć
Póki nadziei gorycz wreszcie nie pokona.

Dlatego, mimo druty, wieże i strażnice
Tam chcemy dotrzeć, gdzie nam dotrzeć zabroniono
Bezużyteczne, śmieszne posiąść tajemnice
Byleby jeszcze raz gorączką tęsknot płonąć
Nim podmuch jakiś strzepnie chwiejne potylice.

Droga okrężna może być i oszukańcza,
Może nas wiedzie szalbierz chciwy paru groszy,
Lecz lepsze to, niż śmierć na wapniejących szańcach
U progu granic niewidzialnych i aproszy,
Gdzie ziewa żołnierz tak podobny do skazańca.

Po zatopionych dawno droga to dolinach;
Pod płytką wodą nieczytelne czasu grypsy:
Szlak po ikonach, rękopisach, karabinach
Nad którym wiosło kreśli plusk Apokalipsy;
Nie po nas płacz – i nie po przodkach – płacz po synach.

Więc prawda, którą znaleźć nam to pusty pokój
Gdzie nagle dzwonią wyłączone telefony?
Serdeczna krew snująca w martwym się potoku,
Bezsilny gniew na obojętność Nieboskłonu
I magia słów, co chronić ma od złych uroków?

Więc prawda, którą znaleźć nam to stół z kamienia,
Z którego przedmiot modłów spadł nietknięty dłonią?
W stukocie kół transportu – Aria Beethovena?
Bezdenna toń, a nad bezdenną tonią
Twarz własna co przegląda się w przestrzeniach?

Z tonących – komu los nie zesłał tratw?
Z ułomnych – zdrowia nie przywrócił komu?
Gdy oślepiony ptak odnalazł ślad
I pies bezdomny siadł na progu domu...

Jacek Kaczmarski,
Stalker.

Acknowledgments

Ὁ βίος βραχύς,
ἡ δὲ τέχνη μακρή,
ὁ δὲ καιρὸς ὀξύς,
ἡ δὲ πεῖρα σφαλερή,
ἡ δὲ κρίσις χαλεπή.

Ἱπποκράτης,
Αποφθέγματα

Dziękuję wszystkim bliskim i przyjaciołom, którzy na różne sposoby wspierali mnie na tej niepewnej i trudnej drodze. A także wszystkim tym, z którymi dzieliłem przez ten czas swoje życie, w odysei kolejnych miejsc zamieszkania, podróży, pokoi doktoranckich. Kończąc niniejszym długi proces edukacji, dziękuję też nauczycielom z liceum, którzy wsparli wówczas mój rozwój, oraz mojej mamie za miłość i poświęcenie.

I would like to especially thank Professor Christopher Isham, for his kind interest in my work and inspiring discussions that gave me the courage for iconoclastic research in the foundations of quantum theory, Professor Jerzy Lewandowski, for unconditional support of my long march despite its shift from quantum gravity and topos theory to quantum foundations and information geometry, and Professor Stanisław Lech Woronowicz, for generously introducing me to the world of modular theory and noncommutative integration, what became a turning point at the mathematical level of my work. I also express my gratitude to Professors Krzysztof Gawlikowski, Zoya Morokhoeva and Stanisław Zapaśnik, who have strongly influenced my perspective at variety and relative intersubjective character of cognitive categories of logic, space, time, change, causality, identity, relation, etc. The thought style of semiotic cultural anthropology, furnished with the perspectives of [1617, 660, 565], was guiding my roaming through samsara of concepts and mathematical structures.

I would like to thank also Marcus Appleby, Benjamin Bahr, Vyacheslav Belavkin, Ingemar Bengtsson, Paolo Bertozzini, Peter Bryant, Ariel Caticha, Rafał Demkowicz–Dobrzański, Jan Dereziński, Andreas Döring, Paweł Duch, Marco Favretti, Christopher Fuchs, Paolo Gibilisco, Frank Hellmann, Peter J. Huber, Roman S. Ingarden, Anna Jenčová, Wojciech Kamiński, Jacek Kopeć, Andrzej Kossakowski, Marcin Kościelecki, Marcin Kotowski, Michał Kotowski, Stanisław Lem, Władysław A. Majewski, Józef Mazurewicz, Nicolas Menicucci, Ryszard Mrugała, Włodzimierz Natorf, Tomohiro Ogawa, Izumi Ojima, Dmitriĭ Pavlov, Aleksander Pełczyński, Arkady Plotnitsky, Gonzalo Reyes, Zofia Rosińska, Carlos C. Rodríguez, John Skilling, Lee Smolin, Rafael Sorkin, Rob Spekkens, Julian Srebrny, John Stachel, Ray Streater, Jamie Vicary, Michael Wright, and Marek Zawadowski for discussions and/or correspondence.

Many thanks for help and/or hospitality to: Piotr Achinger, Barbara Baranowska, John Barrett, Paolo Bertozzini, Grażyna Bleja, Roman Bleja, Dorota Czarnocka–Cieciura, Łukasz Cincio, Łukasz Degórski, Bianca Dittrich, Olaf Dreyer, Cecilia Flori, Frank Hellmann, Shahrzad Haddadan, Ewa Infeld, Roman Kalyakin, Ekaterina Kamlovskaya, Grzegorz Kapustka, Magdalena Kapustka, Iwona Kołodziejska–Degórska, Tomasz Kołodziejski, Tatiana Kostecka, Aleksander Kubica, Marie La Palme–Reyes, Fotini Markopoulou, Daniel McCarthy, Aneta Pękała, Gonzalo Reyes, Piotr Sołtan, Jadwiga Smulko, Grzegorz Smulko, Janina Spicha–Konarzewska, Jamie Vicary, Joanna Zielińska.

During my research I was partially supported by the *Mistrz* grant 2007-2010 of Fundacja na rzecz Nauki Polskiej, *Quantum Geometry and Quantum Gravity* short visit grant 1955 of European Science Foundation, *Quantum Geometry and Quantum Gravity* exchange grant 2706 of European Science Foundation, *182/N* QGG/2008/0 grant of Ministerstwo Szkolnictwa Wyższego i Nauki, and *N N202* 343640 grant of Narodowe Centrum Nauki.

I would like also to acknowledge the open access archives for scientific texts: arXiv (arxiv.org), Project Euclid (projecteuclid.org), Numdam (numdam.org), Math-Net (mathnet.ru), and *especially* Library Genesis (libgen.org). My research would be hardly possible without the possibilities provided by these sources of knowledge, as well as without a little help from my friends, who supplied me with the missing texts, which were hidden in the dark corners of distant libraries or behind the paywalls of evil corporations.

The key inspiration for this work were the writings of Edwin Jaynes, Stanisław Lem, and Oswald Spengler. Among other important inspirations I wish to acknowledge the works of Jean Baudrillard, Nikolaĭ Berdyaev, Ludwik Fleck, Witold Gombrowicz, Alexander Grothendieck, Jacek Kaczmarski, Antoni Kępiński, Krzysztof Kieślowski, Jiddu Krishnamurti, Bill Lawvere, Andreĭ Tarkovskiĭ.

Yet, only silence is the sound of an angel.

Contents

Abstract **7**

Streszczenie **9**

List of symbols **11**

1 Introduction **13**
- 1.1 Mathematical framework . . . 14
- 1.2 Conceptual motivations . . . 18
- 1.3 Postulates . . . 23
- 1.4 Conceptual framework . . . 25
- 1.5 Published and joint work . . . 27
- 1.6 Conventions . . . 27

2 Towards new foundations **29**
- 2.1 Noncommutative integration . . . 30
- 2.2 Quantitative meaning . . . 37
- 2.3 Summary of results . . . 48
- 2.4 Open problems . . . 52

3 Quantum information geometry **55**
- 3.1 Information models . . . 55
  - 3.1.1 Statistical and quantum models . . . 55
  - 3.1.2 Markovian categories . . . 58
- 3.2 Information distances . . . 60
  - 3.2.1 $\mathfrak{f}$-distances . . . 61
  - 3.2.2 Bregman functionals . . . 64
  - 3.2.3 Bregman distances . . . 71
  - 3.2.4 γ-distances . . . 76
- 3.3 Smooth information geometries . . . 88
  - 3.3.1 Information manifolds . . . 88
  - 3.3.2 Smooth $\mathfrak{f}$- and γ-geometries . . . 94
  - 3.3.3 Exponential models . . . 105

4 Quantum information dynamics **109**
- 4.1 Quantum inference . . . 110
  - 4.1.1 Quantum entropic instruments and effects . . . 112
  - 4.1.2 Von Neumann–Lüders rules . . . 115
  - 4.1.3 Jaynes–Mitchell source theory . . . 124
  - 4.1.4 Information geometric renormalisations . . . 130
- 4.2 Quantum evolution . . . 133
  - 4.2.1 Banach–Lie–Poisson structure . . . 135
  - 4.2.2 Extended liouvillean instruments . . . 139
  - 4.2.3 Transition distances . . . 149
  - 4.2.4 Quantum space-times . . . 155

A W^*-algebras and integration **163**
A.1 Algebras and functionals . . . 163
A.1.1 C^*-algebras . . . 164
A.1.2 Functionals . . . 166
A.1.3 Representations . . . 170
A.1.4 von Neumann algebras . . . 171
A.1.5 Barycentric decompositions . . . 172
A.2 Modular theory . . . 175
A.2.1 Kubo–Martin–Schwinger condition . . . 176
A.2.2 Tomita–Takesaki modular theory . . . 177
A.2.3 Relative modular theory . . . 180
A.2.4 Canonical representation and bimodules . . . 184
A.2.5 Classification of W^*-algebras . . . 188
A.3 Automorphisms and their representations . . . 191
A.3.1 Derivations . . . 193
A.3.2 Standard liouvilleans . . . 194
A.3.3 Crossed products . . . 196
A.3.4 Canonical core algebra . . . 201
A.4 Noncommutative integration . . . 203
A.4.1 Noncommutative Radon–Nikodým type theorems . . . 205
A.4.2 Integration relative to a trace . . . 209
A.4.3 Operator valued weights . . . 211
A.4.4 Integration relative to a weight . . . 215
A.4.5 Canonical integration . . . 219
A.4.6 Integration relative to a measure . . . 225
A.4.7 Canonical commutative integration . . . 235

B C^∞-geometries **247**
B.1 Riemannian and affine geometries . . . 247
B.2 Norden–Sen and Eguchi geometries . . . 249
B.3 Hessian manifolds . . . 251

References **255**

Index **295**

Abstract

This thesis is concerned with a new approach to the mathematical foundations of quantum theory. On the basis of noncommutative integration theory and quantum information geometry, a construction of nonlinear quantum kinematics and dynamics is proposed, defined in terms of geometric and algebraic objects that are completely independent of Hilbert spaces.

The starting point is the definition of spaces of quantum states as sets of positive, finite, normal functionals on noncommutative W^*-algebras. These spaces form a quantum generalisation of sets of finite positive measures on localisable Boole algebras. Geometric structures are introduced by generalised quantum relative entropies, defined as two-point nonsymmetric functions of distance on spaces of quantum states. In particular, under suitable differentiability conditions, generalised relative entropies determine uniquely the corresponding riemannian metrics and pairs of affine connections on these spaces. Two classes of generalised relative entropies are especially important from the point of view of information theory, namely those which are monotone with respect to completely positive linear maps between state spaces, and those that satisfy the generalised pythagorean theorem. A detailed review of known results on these two classes is provided in this work. Next, using the noncommutative L_p spaces, a canonical family of generalised quantum relative entropies which belong to both classes is constructed. This result generalises earlier results to the form independent of the choice of representation. Moreover, it is shown that for the finite dimensional case, this family satisfies the smooth version of the generalised pythagorean theorem, with respect to the metrics and connections that are induced by these entropies.

The approach to quantum dynamics proposed in this work starts with the notion of an instrument, understood as a map between two state spaces conditioned upon the set of control parameters. Two classes of instruments are considered: active and passive. Their goal is to generalise the roles that are played in the standard formalism of quantum mechanics by quantum measurement and unitary evolution, respectively.

As a nontrivial example of active instruments, maps that maximise the constrained generalised relative entropies are considered. Such instruments are interpreted as quantum analogues of procedures of statistical inference. It is shown that, under some additional conditions, such instruments determine the semi-spectral measures representing the results of measurements. This is an analogue of the same property of completely positive linear instruments considered in the standard quantum information theory. Under some further conditions, these measures are supported by the states corresponding to superselection sectors that are specified by a uniquely determined commutative algebra. This solves the problem of choice of 'preferred' Hilbert space basis associated with the description of measurement results. Moreover, it is proved that the standard quantum mechanical description of measurement in terms of the von Neumann–Lüders rule is a special case of the entropic instrument. This result is a quantum analogue of the derivation of Bayes' rule as a special case of the maximisation of relative entropy with constraints.

The discussion of nontrivial examples of passive instruments starts by considering a generalisation of symplectic structure to the manifold of quantum states, induced by the commutator of noncommutative W^*-algebra. A specific class of instruments, which are predualisations of such one-parameter weakly-$\star$ continuous, but not necessarily unitary, groups of automorphisms of W^*-algebras that determine the hamiltonian vector fields on the state spaces, is characterised. Next, it is shown how the geometric structures on the state spaces can be expressed in terms of operators acting on the fibres of a Hilbert space bundle over a state space. In particular, the time-dependent multi-point correla-

tion functions are constructed to include corrections to the unitary evolution which come from the geometric structure of the state space. The class of passive instruments representing this evolution is constructed too. These results are based on the theory of unbounded perturbations of unbounded generators of unitary evolutions that represent one-parameter automorphisms of W^*-algebras.

In addition, it is analysed in what sense and how the quantum mechanical notion of a time-dependent transition amplitude can be expressed and generalised in terms of the proposed approach. A preliminary discussion is also provided regarding the possibility of a nonperturbative formulation of the renormalisation of quantum dynamical models using the quantum geometry of state spaces.

A procedure for constructing of space-times, understood as 'emergent' objects, obtained directly from the geometry of quantum states, is also proposed. This procedure is based on the transformation of a quantum riemannian metric to a lorentzian form by a 'complex rotation' with respect to a global vector field of a temporal evolution determined by a passive instrument. As a nontrivial (albeit somewhat restricted) model, a space of quantum states, which recovers the Schwarzschild space-time is constructed.

A separate chapter is dedicated to the discussion of the conceptual meaning of mathematical structures and results that are considered and obtained in this work. The discussion is based on the conceptual framework of information theory, probability theory, and the bayesian approach to statistical inference theory. The relationships of the proposed approach to the standard (Hilbert space based), logico-algebraic (orthocomplemented poset based), algebraic (C^*-algebra based) and operational (semi-spectral measure based) approaches are extensively studied.

Constructions provided in this work are based on a number of results obtained in recent years in the theories of operator algebras and quantum information geometry. These include: construction of the canonical theory of noncommutative integration and noncommutative L_p spaces (Falcone–Takesaki), construction of the Lie–Poisson manifold structure on preduals of W^*-algebras (Odzijewicz–Ratiu), construction of a smooth manifold structure on faithful state spaces over arbitrary W^*-algebras based on noncommutative Orlicz spaces (Jenčová), construction of the theory of perturbation of liouvilleans by unbounded operators (Dereziński–Jakšić–Pillet), and construction of the bundle of Hilbert spaces over preduals of W^*-algebras (Odzijewicz–Sliżewska). This work is their first application to the problems of foundations of quantum theory and quantum geometry.

This thesis also includes an extensive appendix, which contains a systematic exposition of notions and results of the theory of noncommutative integration on W^*-algebras. It is a unique elaboration of this research area, covering many results that are otherwise scattered across the literature of the subject.

Streszczenie

Tematem tej rozprawy jest nowe podejście do problemu matematycznych podstaw teorii kwantowej. W oparciu o niekomutatywną teorię całkowania oraz kwantową geometrię informacji, zaproponowana została konstrukcja nieliniowej kwantowej kinematyki i dynamiki, w której podstawową rolę odgrywają obiekty geometryczne i algebraiczne całkowicie niezależne od przestrzeni Hilberta.

Punktem wyjścia pracy jest zdefiniowanie przestrzeni stanów kwantowych jako zbiorów dodatnich, skończonych, normalnych funkcjonałów na niekomutatywnych W^*-algebrach, co stanowi kwantowe uogólnienie zbiorów skończonych dodatnich miar na lokalizowalnych algebrach Boole'a. Struktury geometryczne zostają wprowadzone poprzez dwupunktowe niesymetryczne funkcje odległości na tych przestrzeniach, będące uogólnieniem kwantowej względnej entropii. Rozmaite dodatkowe warunki nałożone na przestrzenie stanów i uogólnione entropie prowadzą do różnych specyficznych geometrii. W szczególności, przy odpowiednich warunkach różniczkowalności, uogólnione względne entropie determinują jednoznacznie riemannowską metrykę i parę afinicznych koneksji na przestrzeni stanów kwantowych. Szczególnie istotne, z punktu widzenia teorii informacji, są dwie klasy uogólnionych względnych entropii: entropie monotoniczne ze względu na całkowicie dodatnie liniowe odwzorowania przestrzeni stanów, oraz entropie spełniające uogólnione twierdzenie Pitagorasa. W pracy przeprowadzony jest szczegółowy przegląd znanych wyników dotyczących obydwu klas. Następnie zostaje skonstruowana, w oparciu o niekomutatywne przestrzenie L_p, kanoniczna rodzina uogólnionych kwantowych względnych entropii należących do przecięcia tych dwóch klas. Wynik ten uogólnia znane wcześniej konstrukcje do postaci niezależnej od wyboru reprezentacji. Prócz tego wykazane zostaje, że dla skończenie wymiarowej sytuacji otrzymana rodzina spełnia różniczkową wersję uogólnionego twierdzenia Pitagorasa, względem metryki i koneksji indukowanych przez te entropie.

Zaproponowane w pracy podejście do kwantowej dynamiki wychodzi od pojęcia instrumentu, jako odwzorowania między dwoma przestrzeniami stanów, zależnego od zbioru parametrów kontrolnych. Rozważane są dwie klasy instrumentów: aktywne i pasywne, mające za zadanie uogólnić role pełnione w standardowym formalizmie mechaniki kwantowej przez, odpowiednio, kwantowy pomiar oraz ewolucję unitarną.

W roli nietrywialnego przykładu aktywnych instrumentów zaproponowane zostają nieliniowe odwzorowania maksymalizujące uogólnione kwantowe entropie z więzami. Instrumenty te są zinterpretowane jako kwantowe odpowiedniki procedur statystycznego wnioskowania. Pokazane zostaje, że, przy pewnych dodatkowych założeniach, instrumenty te determinują miary półspektralne reprezentujące wyniki pomiarów, co stanowi odpowiednik tejże własności posiadanej przez całkowicie dodatnie liniowe instrumenty rozważane w standardowej kwantowej teorii informacji. Ponadto, przy pewnych dodatkowych warunkach, nośnikami tych miar są stany odpowiadające sektorom superselekcji zadanym przez jednoznacznie określoną komutatywną algebrę, co rozwiązuje problem wyboru 'preferowanej' bazy w przestrzeni Hilberta związanej z opisem wyników pomiaru. Ponadto udowodnione zostaje, że standardowy kwantowomechaniczny opis pomiaru przy pomocy reguły von Neumanna–Lüdersa jest szczególnym przypadkiem entropowego instrumentu. Wynik ten stanowi kwantowy odpowiednik wyprowadzenia reguły Bayesa jako szczególnego przypadku maksymalizacji względnej entropii z więzami.

Punktem wyjścia do dyskusji nietrywialnych przykładów nieliniowych pasywnych instrumentów jest rozpatrzenie uogólnienia struktury symplektycznej na rozmaitość stanów kwantowych, indukowanej przez komutator niekomutatywnej W^*-algebry. Scharakteryzowana zostaje klasa instrumentów, które są predualizacją jednoparametrowych słabo-$\star$ ciągłych, ale niekoniecznie unitarnych, grup automor-

fizmów W^*-algebr, oraz determinują hamiltonowskie pola wektorowe na przestrzeniach stanów. Następnie zostaje pokazane w jaki sposób struktury geometryczne na przestrzeni stanów mogą zostać wyrażone pod postacią operatorów działających na włóknach wiązki przestrzeni Hilberta nad nią. W szczególności, skonstruowane zostają zależne od czasu wielopunktowe funkcje korelacji uwzględniające poprawki do unitarnej ewolucji pochodzące od struktury geometrycznej przestrzeni stanów, a także klasa pasywnych instrumentów reprezentujących tę ewolucję. Konstrukcja ta opiera się na teorii nieograniczonych perturbacji nieograniczonych generatorów unitarnych ewolucji reprezentujących jednoparametrowe automorfizmy W^*-algebr.

Prócz tego, zanalizowano w jakim sensie i w jaki sposób kwantowomechaniczne pojęcie zależnej od czasu amplitudy przejścia daje się wyrazić i uogólnić w ramach badanego podejścia. Wstępnej dyskusji poddane są także możliwości nieperturbacyjnego sformułowania renormalizacji modeli kwantowej dynamiki opartej o kwantową geometrię przestrzeni stanów.

W pracy zaproponowana została też procedura konstrukcji czasoprzestrzeni jako 'emergentnych' obiektów, otrzymywanych bezpośrednio z geometrii przestrzeni stanów kwantowych. Procedura ta opiera się na transformacji kwantowej riemannowskiej metryki do postaci lorentzowskiej poprzez 'zespolony obrót' względem globalnego pola wektorowego ewolucji czasowej zadanej pasywnym instrumentem. W roli nietrywialnego (choć posiadającego pewne ograniczenia) modelu, skonstruowana zostaje przestrzeń stanów kwantowych, która odtwarza czasoprzestrzeń Schwarzschilda.

Osobny rozdział pracy poświęcony jest dyskusji znaczenia pojęciowego badanych struktur matematycznych i otrzymanych wyników, w oparciu o aparat pojęciowy teorii informacji, teorii prawdopodobieństwa, oraz teorii wnioskowania statystycznego w ujęciu bayesowskim. Szeroko przedyskutowane zostały związki i różnice zaproponowanego podejścia z podejściami: standardowym (opartym na przestrzeniach Hilberta), logiczno-algebraicznym (opartym na częściowych porządkach z ortokomplementacją), algebraicznym (opartym na C^*-algebrach), oraz operacyjnym (opartym na miarach półspektralnych).

Przeprowadzone w pracy konstrukcje opierają się o szereg rezultatów otrzymanych w ostatnich kilkunastu latach w teorii algebr operatorów i w kwantowej geometrii informacji. Są to między innymi: konstrukcja kanonicznej teorii niekomutatywnego całkowania i niekomutatywnych przestrzeni L_p (Falcone–Takesaki), konstrukcja struktury rozmaitości Lie–Poissona na predualach W^*-algebr (Odzijewicz–Ratiu), konstrukcja struktury gładkiej rozmaitości na stanach wiernych na dowolnych W^*-algebrach oparta na niekomutatywnych przestrzeniach Orlicza (Jenčová), konstrukcja teorii perturbacji liouvilleanów przez nieograniczone operatory (Dereziński–Jakšić–Pillet), konstrukcja wiązki przestrzeni Hilberta nad predualami W^*-algebr (Odzijewicz–Sliżewska). Niniejsza praca stanowi pierwsze ich zastosowanie w problemach podstaw teorii kwantowej i kwantowej geometrii.

Rozprawa zawiera także obszerny dodatek zawierający systematyczne opracowanie pojęć i rezultatów teorii niekomutatywnego całkowania na W^*-algebrach. Jest to jedyne takie opracowanie w literaturze przedmiotu, uwzględniające wiele rezultatów rozproszonych w trudno dostępnych publikacjach.

List of symbols

(...) żaden symbol graficzny nie wyraża myśli dokładniej,
niż słowo lub zdanie, którym opisano znaczenie tego symbolu!

Adam Wiśniewski-Snerg,
Jednolita teoria czasoprzestrzeni[1]

P^x	38 209
E	41 110 114
$\mathcal{M}$	55 56 57
$\mathcal{S}$	56 167
$T_\star$	58
D	60
$\mathfrak{f}$	61 63
$\mathfrak{f}^c$	61
$D_\mathfrak{f}$	61 62 63
χ^2	61
epi	65
efd	65 66
$\partial\Psi$	65
$\Psi^{\mathbf{L}}$	65
efc	66
$\mathfrak{D}^{\mathrm{G}}_+$	66
$\mathfrak{D}^{\mathrm{G}}$	66
$\mathfrak{D}^{\mathrm{F}}$	66
$\widetilde{D}_\Psi$	67
W_Ψ	68 71
$\widetilde{\mathfrak{P}}^\Psi_C$	69
$\hat{\mathfrak{P}}^\Psi_C$	70
$(\ell,\ell^@)$	72
D_Ψ	72 74
$\mathbf{L}_\Psi$	72 73
Θ_Ψ	73
add	73
Ξ_Ψ	73
adc	73
$\bar{D}_\Psi$	73 74
$\bar{\mathfrak{P}}^\Psi_C$	75
$\mathfrak{P}^\Psi_K$	76
D_γ	76 78 79
$\mathfrak{f}_\gamma$	77
ℓ_γ	78
$\tilde{\ell}_\gamma$	78
Ψ_γ	81
$c(\omega,h)$	85
ω^h	85
$\widetilde{\phi^h}$	86
$\mathfrak{P}^\gamma_C$	87 88 96
$\mathcal{S}_0$	89 167
$\mathbf{T}_p\mathcal{M}$	89 247
$\mathbf{T}_p\ell_\gamma$	89 92
Υ	90 93
Υ_1	90
Υ_ϕ	93
L_Υ	90 93
$\mathbf{g}^{\mathrm{FRJ}}$	94
∇^γ	95
$\mathbf{g}^\mathfrak{f}$	95
$\mathfrak{c}$	97
$\mathfrak{h}$	97 98
$\nabla^\mathfrak{f}$	95
$\mathbf{g}^\mathfrak{h}$	97 98
$\mathfrak{I}^\mathfrak{h}_\rho$	97
$\mathfrak{F}$	98
$\mathbf{g}^{\mathrm{BKM}}$	99
$\mathbf{g}^\gamma$	101
d_{Bures}	101 150
$\mathcal{M}_{\mathrm{exp}}$	105 107
$\mathbf{S}_{\mathrm{GS}}$	105
$\mathbf{S}_{\mathrm{vN}}$	106
$\mathfrak{I}$	109
$\mathfrak{M}^\mho$	110
Λ	112
$\mathfrak{P}^{D,\Lambda}_\mathcal{Q}$	112
$\Game$	113
$\mathfrak{P}^{D,\Lambda}_\Game$	113
$\mathfrak{P}^\gamma_\Game$	113
$\mathfrak{y}$	132
ad	135 137
$\mathrm{C}^\infty_{\mathrm{F}}$	135
$\{\cdot,\cdot\}$	135 136 137
$\mathbf{d}f$	135
$\mathfrak{X}_k$	135 136 137
ϖ	135
w^h_t	137
Ad	137
h^α	139
$\mathcal{HM}$	139
K^α_ω	140
$\mathfrak{a}$	140
$\mathbf{A}$	140
$K^{\alpha,x}$	141
α^x	141
$\mathcal{L}(\omega,t)$	145
$\mathfrak{I}_\mathcal{L}$	145
$(\bigotimes^n\mathcal{H})\mathcal{M}$	147
$\mathrm{TP}_\mathcal{H}$	149
TP_{R}	149
$\mathrm{TP}_{\mathrm{CU}}$	149
TP	149
DeQuant	155
$\hat{\mathfrak{g}}$	155 156
$\mathcal{C}$	164
$\mathbb{I}$	164 234
x^*	164
$\mathcal{C}^o$	165
Aut	165
$\mathcal{C}_\alpha$	165
$\leq$	165 168 226
$(\cdot)^+$	166
Proj	166
$(\cdot)^{\mathrm{sa}}$	166
$(\cdot)^{\mathrm{asa}}$	166
$(\cdot)^{\mathrm{uni}}$	166
$(\cdot)^{\mathbf{B}}$	166 225
$(\cdot)_\star$	167 225
$(\cdot)_0$	167
$(\cdot)_1$	167
$\mathcal{N}$	167
$\mathfrak{B}(\mathcal{H})$	168
$\mathfrak{m}_\omega$	168
$\mathcal{W}$	168 228
$\mathcal{W}_0$	168 228
$\mathfrak{n}_\phi$	169
supp	169 169 178
$\ll$	169 236
$\lvert\phi\rvert$	170
π	170
π_ω	170 171
Ω_ω	170
$\mathcal{H}_\omega$	170 171
$\langle\cdot,\cdot\rangle_\omega$	170
$[\cdot]_\omega$	171 171
$\mathcal{N}^\bullet$	171
$\mathfrak{Z}_\mathcal{N}$	172
face	173
co	173
$\mathcal{C}^\alpha_\infty$	176
Lin	178
J_ω	178 179 179
Δ_ω	178 179 179
σ^ω	178 179 223
$\mathcal{N}_{\sigma^\omega}$	179
K_ω	179
$\mathcal{H}^\natural_\Omega$	180
$\mathcal{H}^\natural$	180
ξ_π	180
$\xi^\pi_\natural$	180
$J_{\phi,\omega}$	181
$\Delta_{\phi,\omega}$	181 182 188
$V_{\phi,\omega}$	182

[1] «(...) no graphic symbol does express thoughts more accurately than a word or phrase that describes the meaning of this symbol!» Adam Wiśniewski-Snerg, *Uniform theory of spacetime* [1874].

$[\phi : \omega]_t$	182 182 184 208 223
$\frac{\psi}{\phi^\bullet}$	183 188 224
$J_{\mathcal{N}}$	186
$\pi_{\mathcal{N}}$	186
$\mathfrak{L}$	187 224
$\mathfrak{R}$	187 224
$\mathfrak{d}$	188
α	191
$\eth$	193
V_α	195
K^α	195
$\tilde{\mu}^G$	197
$\mathrm{C_c}$	197
π_α	198
u_G	198
$\rtimes_\alpha$	198
$\hat{G}$	198
π_{σ^ψ}	199
$u_{\mathbb{R}}$	199
ϕ^λ	186 200 201 219 220 224
$\hat{\mathcal{N}}$	200
$\mathcal{N}(t)$	201
$\mathcal{H}(t)$	202
$\widetilde{\mathcal{N}}$	202
$\widetilde{\mathcal{H}}$	202
aff	205 206
$\mathscr{M}(\mathcal{N})$	205
τ_h	206 206
ϕ_h	207
$L_p(\mathcal{N}, \tau)$	209 210
$\mathscr{M}(\mathcal{N}, \tau)$	210
$\mathfrak{G}_p$	211 211
$\mathcal{N}^{\mathrm{ext}}$	211
$\tilde{\tau}$	212
$\tilde{\phi}$	212
$\tilde{\tau}_x$	213
T	59 60 213
$\mathfrak{n}_T$	213
$\mathfrak{m}_T$	214
$\mathcal{E}$	213
$\hat{T}$	214 215
$\hat{\phi}$	214 215 220
h_ϕ	216
$\mathscr{M}^p(\hat{\mathcal{N}}, \tau_\psi)$	216
$L_p(\mathcal{N}, \psi)$	217 218
$L_p(\mathcal{N}, \psi^\bullet)$	218
$[\![\cdot, \cdot]\!]_\psi$	218
$L_p(\mathcal{N})$	186 219 222 222
grad	221
$\mathscr{M}^p(\widetilde{\mathcal{N}}, \widetilde{\tau})$	221
sup	226
inf	226
$\mathcal{A}$	227
$\mathrm{sp_S}$	228
μ	228
$L_p(\mathcal{A})$	229 229 236
$L_p(\mathcal{A}, \mu)$	229 229
χ	230
$\mathcal{A}^\mu$	230
$\mho$	232
$\mho^0$	232
$\tilde{\mu}$	233
Meas^+	233
null	233
$\mho^{\tilde{\mu}}$	233
$L_p(\mathcal{X}, \mho(\mathcal{X}), \tilde{\mu})$	233
$\mathcal{A}_{\tilde{\mu}}$	234
$\mho_{\mathrm{Borel}}(\mathcal{X})$	238
$\mathrm{Meas}^+_\star$	238
$\mathrm{Rad}(\cdot)^+$	239
$\mathrm{sp_G}$	240
C	240
$\mathbf{T}^\circledast_q\mathcal{M}$	247
$\mathbf{g}$	247 248
$d_{\mathbf{g}}$	247
∇	248
$\mathbf{t}^\nabla$	248
R^∇	248
$\mathfrak{T}^\nabla$	248
Γ^∇	249
κ^∇	249
$\exp^\nabla$	249
$\nabla^\dagger$	249
$\widetilde{\nabla}^\vartheta$	250
$(\ell, \ell^\dagger)$	251
$\mathbf{g}^\Phi$	252
$\mathbf{g}^\Psi$	252

Apart from this the following notation is used: i, e, π, ℕ, ℝ, ℂ, ℤ are standard, and the convention $\mathbb{R}^+ = [0, \infty[$ is used; id denotes identity morphism in the category of objects of a given kind, dom denotes domain, cod denotes codomain, ran denotes image, $\mathrm{span}_{\mathbb{K}}$ denotes linear span in the vector space over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, int denotes interior of a topological space, **Set** denotes the category of sets and functions; given a small category $\mathscr{C}$, $\mathrm{Ob}(\mathscr{C})$ denotes the set of all objects in $\mathscr{C}$, while $\mathrm{Hom}_{\mathscr{C}}(X, Y)$ denotes the set of all arrows between $X, Y \in \mathrm{Ob}(\mathscr{C})$; $\wp(\mathcal{X}) := \mathrm{Hom}_{\mathbf{Set}}(\mathcal{X}, \{0, 1\})$ is the power set of a set $\mathcal{X}$; $\#(\mathcal{X})$ denotes the cardinality of a set $\mathcal{X}$ if $\mathcal{X}$ is finite and $+\infty$ otherwise; $\oplus$ and $\otimes$ denote direct product and tensor product, respectively, that are defined dependently of the category of objects under consideration; $\boldsymbol{\delta}_{ij}$ denotes Kronecker's delta, while $\boldsymbol{\delta}(x - y)$ denotes Dirac's delta; G denotes group, g denotes group element, while e denotes neutral element of a group; $\mathcal{H}$ and $\mathcal{K}$ denote Hilbert spaces, while ξ, ζ denote Hilbert space elements; sp denotes spectrum of an operator on a Hilbert space, while $\int^\oplus$ denotes direct integral in von Neumann's [1813] sense.

Chapter 1

Introduction

Nie mogę ci powiedzieć, kiedy rojenie zmieniło mi się w hipotezę, bo sam nie wiem. Postawiłem na niezdecydowanie, wiesz przecież. Odkrycie moje jest fizyką, należy do fizyki, ale do takiej, której nikt nie zauważył, bo droga wiodła przez tereny ośmieszone, wyzute z wszelkich praw. Trzeba było przecież zacząć od myśli, że słowo może stać się ciałem, że zaklęcie m o ż e się zmaterializować – trzeba było dać nura w ten absurd, wejść w związki zakazane, aby dostać się na drugi brzeg, tam gdzie już oczywiście jest równoważność informacji i masy.

Stanisław Lem,
Profesor A. Dońda[1]

The aim of this thesis is to investigate the possibility of developing new mathematical and conceptual foundations for quantum theory, based on quantum information geometry on preduals of W^*-algebras instead of spectral theory of operators on Hilbert spaces. Apart from recent tendencies to develop various information theoretic approaches to foundations of quantum *mechanics* (understood mathematically as the theory established by von Neumann [1800]), we are strongly motivated by the problem of lack of nonperturbative foundations of predictive quantum field theory and the problem of mathematical and conceptual unification of general relativity with quantum theory. The impressive resistance of the last two problems, despite many attempts to solve them, make us believe that the general mathematical and conceptual formulation of quantum *theory* still waits to be developed. We consider this problem as requiring essentially different perspective and treatment than the problems of: (i) providing information theoretic reconstruction of quantum mechanics; (ii) constructing some sort of quantisation of general relativity theory; (iii) further development of effective perturbative quantum field theories. The main intention underlying this text is to use the conceptual insights from information theory and bayesian approach to statistics, as well as mathematical tools of quantum information geometry *without* relying on the framework of Hilbert spaces, in order to propose a foundational approach to quantum theory, which would be intrinsically geometric and nonlinear, allowing for a new perspective on the problems of nonperturbative predictive quantum theory and quantum space-time geometry.

[1] «I cannot tell you when it was that my reverie became a hypothesis, for I do not know this myself. You know that I have chosen indecision. My discovery is physics, belongs to physics, but physics overlooked as the road to it led through the lands derided, deprived of all laws. For it springs from the thought that a word may become flesh, that a spell m a y hatch into matter—one had to plunge into this absurdity, and enter the forbidden relationships to emerge on the other side, where information and mass are, obviously, equivalent.» Stanisław Lem, *Professor A. Dońda* [1056].

In what follows we will present the summary of key mathematical and conceptual ideas of our approach. Conceptual aspects will be further discussed in detail in Section 2, while mathematical aspects will be further discussed in the rest of this work. The detailed discussion of the mathematical contents of this thesis, including the new results and open problems, is given in Section 2.3. The purpose of this Chapter is to explain underlying motivations and insights.

1.1 Mathematical framework

The idea to quantify the distance between two probability distributions dates back to Laplace and Gauß. Such quantification was found necessary in order to decide, on the basis of a given evidence, which among a *given set* of abstract probability distributions, provides the 'best' fit to this evidence. The notion of 'best' was defined as *minimal* in a given *distance*, under constraints that represent the problem under consideration. In particular, Laplace's principle of minimum variance distance [443] produces median as a 'best' unconstrained estimate, while the Gauß principle of least square distance [616] produces expectation as a 'best' unconstrained estimate.[2]

Consideration of *distances* on the sets of probability distributions may suggest to investigate geometric properties of these sets by means of some suitable distances. However, despite the use of differential geometry in the study of spaces of thermodynamic parameters (initiated already by Riemann [1441], who calculated the Riemann curvature tensor of some thermodynamic variables), this idea has waited long for its time. The geometric considerations of the spaces of probabilities were initiated by Rao's [1397] and Jeffreys' [875] observation that Fisher's matrix [562] $\mathbf{g}_{ij}(\theta)$ determines a riemannian metric on the space of probability densities. The systematic development of geometric study of families of probability distributions has begun with the works of Chencov [312, 313, 314, 315, 316, 317], who (among other results) introduced a class of affine connections on the space of probability measures, generalised pythagorean theorem for relative entropies (considered as nonsymmetric distances), and provided a characterisation of the class of riemannian metrics and affine connections that are monotone in the category of probabilistic models with coarse grainings as arrows.

The main mathematical structure of resulting theory of *information geometry* are nonsymmetric distance functions (relative entropies), which can play the role of generalisations of squared euclidean (or Hilbert space) distance because they satisfy nonlinear nonsymmetric generalisations of pythagorean and cosine theorems. The Taylor expansion of relative entropy at a given point allows to define additional local differential geometric structures, such as the above riemannian metric.

Chencov's revolutionary idea was that the geometric properties of spaces of probability distributions, together with their transformation properties encoded in the structure of a suitable category, should be considered as a basis for foundations of statistical inference theory[3]. The idea that forms the basis of this thesis is to apply the same foundational perspective to quantum theory. However, this immediately leads to two problems. First is to find a suitable generalisation of mathematical framework that would fit this purpose. As discussed in details in the following Sections and Chapters, Hilbert spaces «present a picture of structural monotony» [1166], and cannot serve for this purpose. Second problem is to deal with an ocean of conceptual quandaries associated with variety of different conflicting interpretations of statistical inference theory and quantum theory. These two problems are strongly entangled. In particular, a misguided preference of some conceptual setting might lead to inevitable preference of some mathematical structures, and conversely.

The crucial mathematical input, allowing us to use a suitable generalisation of information geometry for quantum *foundational* purposes, comes from the noncommutative integration theory on W^*-algebras. As opposed to the commutative setting, for which there is almost complete equivalence between the approaches based on the notion of measure and the approaches based on the notion of integral, this is no longer the case in the noncommutative setting.

[2]Both minimisation problems are stated in terms of the space of estimated functions and not the space of probability measures. However, the modern approach to variational optimisation problems based on duality (see e.g. [1467, 1061, 172]) allows to consider these problems interchangeably (see e.g. [34]).

[3]Later, but independently, the similar idea was proposed by Amari [36, 37, 38, 39, 40].

We begin with postulating that quantum theoretic kinematics should be constructed as a generalisation of mathematical framework of probability theory, with measurable spaces $(\mathcal{X},\mho(\mathcal{X}))$ replaced by noncommutative W^*-algebras $\mathcal{N}$, probabilistic models $\mathcal{M}(\mathcal{X},\mho(\mathcal{X}),\tilde{\mu})\subseteq L_1(\mathcal{X},\mho(\mathcal{X}),\tilde{\mu})_1^+$ replaced by the quantum models[4] $\mathcal{M}(\mathcal{N})\subseteq L_1(\mathcal{N})^+\cong\mathcal{N}_\star^+$, and additional structures associated with probabilistic models (geometry, estimators) replaced by the structures on $\mathcal{M}(\mathcal{N})$. This setting can be also thought of as a generalisation and replacement of the Hilbert space based kinematics of quantum mechanics, with elements of $\mathcal{M}(\mathcal{N})$ generalising the elements of a Hilbert space $\mathcal{H}$ and the density operators on $\mathcal{H}$, and with the quantum relative entropy functional[5] $D(\cdot,\cdot)$ on $\mathcal{M}(\mathcal{N})$ replacing the quantitative and geometric role played by the scalar product $\langle\cdot,\cdot\rangle$ on $\mathcal{H}$. From the conceptual point of view, formulation of the new mathematical foundations of quantum theory on the basis of integration theory on W^*-algebras and geometry of spaces of these integrals is aimed at removing all mathematical reasons for consideration of Hilbert spaces and measure spaces on the foundational level. In such case, the spectral and semi-spectral measures acting from measurable spaces $(\mathcal{X},\mho(\mathcal{X}))$ to spaces $\mathfrak{B}(\mathcal{H})^+$ of positive bounded operators on $\mathcal{H}$ become irrelevant too, and the same applies to probability distributions, arising from traces of spectral and semi-spectral measures with density operators.

The passage from Hilbert spaces to W^*-algebras is not a simple generalisation, because it changes the roles played by the noncommutative algebra and by the two point functional acting on what is to be considered the space of quantum states. In the former setting the noncommutative algebra consists of operators in a specific representation, while the scalar product is responsible for defining the expectations of elements of an algebra and for defining the geometric structures on the state space. On the contrary, in the latter setting the noncommutative W^*-algebra $\mathcal{N}$ is abstract (so it does not consist of operators), the expectations of elements of this algebra are defined by quantum states, $\phi:\mathcal{N}\ni x\mapsto\phi(x)\in\mathbb{C}$, without invoking any two point functional, while the quantum relative entropy $D(\cdot,\cdot)$ is used exclusively to provide the notion of a distance and to define other geometric structures on the spaces $\mathcal{M}(\mathcal{N})$.

In particular, under suitable differentiability conditions imposed on $\mathcal{M}(\mathcal{N})$ and D, the second and third order Taylor expansions of $D(\cdot,\cdot)$ determine a riemannian metric $\mathbf{g}$ on $\mathcal{M}(\mathcal{N})$ and a pair of torsion-free affine connections $(\nabla,\nabla^\dagger)$, respectively. These objects satisfy the characteristic property of the Norden–Sen geometries[6], namely

$$\mathbf{g}(\mathbf{t}_c^\nabla(u),\mathbf{t}_c^{\nabla^\dagger}(v))=\mathbf{g}(u,v)\quad\forall u,v\in\mathbf{T}\mathcal{M}(\mathcal{N}),\tag{1.1}$$

and for all smooth curves $c:\mathbb{R}\to\mathcal{M}(\mathcal{N})$, where $\mathbf{t}_c^\nabla$ denotes a ∇-parallel transport along c. This is a generalisation of the riemannian geometry. On the other hand, for every model $\mathcal{M}(\mathcal{N})$ one can consider a quantum distance

$$D_{1/2}(\phi,\psi)=\frac{1}{2}\|\xi_{\pi_\mathcal{N}}(\phi)-\xi_{\pi_\mathcal{N}}(\psi)\|^2_{L_2(\mathcal{N})},\tag{1.2}$$

where $\xi_{\pi_\mathcal{N}}:\mathcal{N}_\star^+\to L_2(\mathcal{N})^+$ is a canonical representation in the canonical Hilbert space $L_2(\mathcal{N})$.[7] The space $L_2(\mathcal{N})$ is an operator algebraic generalisation of the Hilbert–Schmidt space $\mathfrak{G}_2(\mathcal{H})$, the association $\mathcal{N}\to L_2(\mathcal{N})$ is functorial over the category of W^*-algebras, while the representation $\xi_{\pi_\mathcal{N}}$ is a generalisation of the map $\mathfrak{G}_1(\mathcal{H})^+\ni\rho\mapsto\sqrt{\rho}\in\mathfrak{G}_2(\mathcal{H})^+$, where $\mathfrak{G}_1(\mathcal{H})$ is the space of all trace class operators on $\mathcal{H}$. These properties are quite impressive. However, the main result that requires to consider $D(\cdot,\cdot)$ as an important *geometric* structure on $\mathcal{M}(\mathcal{N})$ is the generalised pythagorean equation,

$$D(\phi,\mathfrak{P}^D_\mathcal{Q}(\psi))+D(\mathfrak{P}^D_\mathcal{Q}(\psi),\psi)=D(\phi,\psi)\quad\forall(\phi,\psi)\in\mathcal{Q}\times\mathcal{M}(\mathcal{N}),\tag{1.3}$$

[4] Which are the subsets of the positive cones $\mathcal{N}_\star^+$ of Banach preduals $\mathcal{N}_\star$ of W^*-algebras $\mathcal{N}$, which is equivalent to saying the $\mathcal{M}(\mathcal{N})$ are sets of finite positive normal functionals on W^*-algebras.

[5] Strictly speaking, the *quantum distance* is a map $D(\cdot,\cdot):\mathcal{N}_\star^+\times\mathcal{N}_\star^+\to[0,\infty]$ such that $D(\phi,\psi)=0\iff\phi=\psi$, while the *quantum relative entropy* is a map $(-D)(\cdot,\cdot):\mathcal{N}_\star^+\times\mathcal{N}_\star^+\to[-\infty,0]$. See Section 3.2 for more discussion. In this Section we will use these notions interchangeably, meaning D in all cases.

[6] See Section B.2.

[7] See Section A.2.4.

where $\mathcal{Q} \subseteq \mathcal{M}(\mathcal{N})$ is a closed convex set such that

$$\mathfrak{P}^D_{\mathcal{Q}}(\psi) := \arg\inf_{\omega\in\mathcal{Q}} \{D(\omega,\psi)\} \tag{1.4}$$

exists and is unique. This property is satisfied by a large family of distances $D(\cdot,\cdot)$, see Sections 3.2.2 and 3.2.3. Under suitable additional conditions, (1.3) can be restated in local differential geometric terms of (1.1) as a claim that the nonlinear projection of $\psi \in \mathcal{M}(\mathcal{N})$ along ∇-geodesic onto $\nabla^\dagger$-autoparallel $\nabla^\dagger$-affine submanifold $\mathcal{Q} \subseteq \mathcal{M}(\mathcal{N})$ coincides with the constrained relative entropic projection (1.4).[8] On the other hand, the generalised pythagorean theorem (1.3) for (1.2) turns to the pythagorean theorem in the Hilbert space $L_2(\mathcal{N})$ with (1.4) turning to the orthogonal projection onto convex closed subset of this Hilbert space.

These properties show that the quantum information geometry based on quantum distances $D(\cdot,\cdot)$ over quantum models $\mathcal{M}(\mathcal{N})$ allows for a sort of "re-unification" of the riemannian geometry with the geometry of Hilbert spaces. This striking feature leads us to consider the geometries of $\mathcal{M}(\mathcal{N})$ defined by $D(\cdot,\cdot)$ as the main structural element for the underlying kinematic setting of new foundations of quantum theory. Like riemannian geometry, quantum information geometry naturally carries infinitesimal and local nonlinear structures, but, like in the Hilbert space geometry, individual points have their own internal structure.

Apart from the above structures, quantum models $\mathcal{M}(\mathcal{N})$ are naturally equipped with one more geometric structure. It is the Poisson bracket $\{\cdot,\cdot\}$, which is induced by the commutator of the W^*-algebra $\mathcal{N}$, and which equips $\mathcal{N}_\star^{\mathrm{sa}}$ and some of its subspaces with the Banach–Lie–Poisson manifold structure (see Section 4.2.1). Hence, while $D(\cdot,\cdot)$ is a generalisation of the scalar product and is able to reproduce riemannian metric and affine connections (which allows to study Killing vector fields, geodesic flows, etc.), $\{\cdot,\cdot\}$ provides a generalisation of the symplectic form (which allows to study hamiltonian vector fields, Poisson flows, etc.). While one can postulate various quantum distances $D(\cdot,\cdot)$ on a given quantum model $\mathcal{M}(\mathcal{N})$, the Poisson bracket $\{\cdot,\cdot\}$ on $\mathcal{M}(\mathcal{N})$ is determined uniquely by $\mathcal{N}$. In this sense, $\{\cdot,\cdot\}$ is a part of quantum geometry of $\mathcal{M}(\mathcal{N})$, but is not an independent elements of construction of quantum kinematics. For commutative W^*-algebras $\mathcal{N}$, the BLP structure trivialises, while the quantum information geometry based on $D(\cdot,\cdot)$ reduces to the corresponding information geometry of statistical models $\mathcal{M}(\mathcal{X},\mho(\mathcal{X}),\tilde{\mu})$, where $L_\infty(\mathcal{X},\mho(\mathcal{X}),\tilde{\mu}) \cong \mathcal{N}$. On the other hand, the above setting allows to recover the Kähler geometry of pure states (Hilbert space vectors) modulo phase in a special case, for a specific choice of D which admits an extension of the derived riemannian metric to pure states (see [470, 1672, 1322]). In this sense, the geometry of quantum mechanics and the geometry of probability theory (called «geometrostatistics» by Kolmogorov, cf. [320]) are just the special cases of quantum geometry of $\mathcal{M}(\mathcal{N})$.[9]

We also propose to consider quantum theoretic dynamics as a direct generalisation of the statistical inference theory, with various rules of inference, conditioning, and causality replaced by the general setting of *instruments*. An instrument is defined as a function from the set of 'control parameters' to the space of mappings $\mathcal{M}_1(\mathcal{N}) \to \mathcal{M}_2(\mathcal{N})$ between two quantum models $\mathcal{M}_1(\mathcal{N}), \mathcal{M}_2(\mathcal{N}) \subseteq \mathcal{N}_\star^+$. These mappings are not assumed to be linear or completely positive. Instruments provide a generalisation of various forms of quantum dynamics and conditioning, including the description of "quantum measurement" and unitary evolution. We investigate various instruments that are dependent on the geometric structure of quantum models $\mathcal{M}(\mathcal{N})$. We are especially concerned with instruments that are generated by constrained minimisation (1.4) of the quantum distance $D(\cdot,\cdot)$ and with instruments that arise from predualisation of automorphisms $\{\alpha_t \mid t \in \mathbb{R}\} \subseteq \mathrm{Aut}(\mathcal{N})$ of W^*-algebras $\mathcal{N}$. They provide the key examples of 'active' and 'passive' classes of instruments, respectively.[10]

---

[8] See Section B.3.

[9] We will use term 'quantum information geometry' to speak of geometries on $\mathcal{M}(\mathcal{N})$ arising from $D(\cdot,\cdot)$, and we will speak of 'quantum geometry' to refer to all geometric structures on $\mathcal{M}(\mathcal{N})$, including those that arise from $D(\cdot,\cdot)$ and those that arise from $\{\cdot,\cdot\}$.

[10] The idea of two different and independent temporal evolutions, *external*—governed by maximisation of entropy, and *internal*—governed by the energy, each one associated with its own notion of time, is due to Kępiński [966, 967, 968]. Our concept of active and passive instruments is intended as a suitable generalisation of this idea (we were also inspired

An instrument is said to be ‘active’ if it is used for the purposes of inductive inference conditioned on given set of epistemic constraints. Thus, it is a ‘rule of conditioning’ of information states. Bayes’ rule and the von Neumann–Lüders rules are the most standard examples of conditioning rules. (Recently it has been shown that Bayes’ rule is an entropic instrument in the above sense [962, 1609, 1822, 292, 633]. In Sections 4.1.2 and 4.2.3 we prove that the same is true for the von Neumann–Lüders rules. This justifies consideration of entropic instruments as nonlinear active instruments.) Moreover, as we show in Section 4.1.1, under some additional conditions, active instruments determine the semi-spectral measures representing the results of measurements. This is an analogue of the same property of completely positive linear instruments considered in the standard quantum information theory. Under some further conditions, these measures are supported by the states corresponding to superselection sectors that are specified by a uniquely determined commutative algebra. This provides a specific solution to the problem of choice of ‘preferred’ Hilbert space basis associated with the description of measurement results.

An instrument is called ‘passive’ if it is used for the purpose of defining the referential system of causality, understood as a transformation of the information states that is independent of any action of active instruments. This provides a generalisation of the role played by the unitary evolution of density operators in quantum mechanics and the predualised action $\{\alpha^t_\star : \mathcal{N}^+_\star \to \mathcal{N}^+_\star \mid t \in \mathbb{R}\}$ of the weakly-$\star$ continuous group of automorphisms of a W^*-algebra $\mathcal{N}$ in the algebraic approach. This predualisation works also in the commutative case, and one can apply it to automorphisms of an algebra $L_\infty(\mathcal{X}, \mho(\mathcal{X}), \tilde{\mu})$ induced by automorphisms of an underlying boolean algebra $\mho(\mathcal{X})/\mho^{\tilde{\mu}}(\mathcal{X})$ (see e.g. [591]) or its sample-space representation. So, the passive instruments can be also considered as a specific method of modelling causality in the sense of statistical inference theory (see e.g. [1286] for a review). The presence of the BLP structure, which is a generalisation of the symplectic structure, leads us to analyse its role in the construction of passive instruments. We characterise the class of predualised automorphisms $\alpha_\star$ that are also Poisson flows of $\{\cdot, \cdot\}$, and show that under certain conditions hamiltonian vector fields of these flows can be expressed by generators of unitary evolutions in the Hilbert space bundle over $\mathcal{M}(\mathcal{N})$. Using this correspondence, we construct a new family of nonlinear passive instruments, which arise from the additional perturbations by one forms on the BLP manifold. Trivialisation of the BLP structure for commutative W^*-algebras shows an important difference between quantum theory and statistical inference theory: in former causality may be analysed in terms of hamiltonians and their perturbations, while in the latter this is impossible. Yet, both have the same conceptual meaning.

Quantum distances $D(\cdot, \cdot)$ play a crucial role in construction of active instruments that implement the nonlinear inductive inference procedures, generalising widely the setting of Bayes’ and von Neumann–Lüders’ rules. In infinitesimal approximation, they determine riemannian metrics and affine connections acting on the tangent bundle of the model $\mathcal{M}(\mathcal{N})$. On the other hand, the BLP structure $\{\cdot, \cdot\}$ plays a crucial role in construction of passive instruments that implement the nonlinear causality systems, generalising the unitary evolutions. It is genuinely related to the cotangent bundle of the model $\mathcal{M}(\mathcal{N})$. In this sense, two independent geometric structures on the quantum models $\mathcal{M}(\mathcal{N})$ play the key role in the construction of nonlinear examples of two independent forms of dynamics on models $\mathcal{M}(\mathcal{N})$.

The question regarding the relationship between those two structures leads us to the final step of our investigations, which amounts to proposing that the *space-times* should be thought of as objects that emerge from information geometric kinematics and causal (passive) dynamics, while the space-time trajectories (not necessarily causal) should arise from the active dynamics. We propose to implement this general postulate by changing the signature of the riemannian metric induced by $D(\cdot, \cdot)$ to lorentzian form in a way that makes the global vector field of a given passive (causal) evolution timelike.

---

by Savvidou’s work [1515, 1516] on two different time evolutions in the Isham–Linden approach to consistent quantum histories). This terminology stems from observation that entropic evolution can be regarded as *active*, because it describes the change of knowledge and represents a *decision*, while energy-governed evolution (such as unitary evolution) can be considered as passive, because it describes invariance properties of knowledge and represents the criteria of judging the quality of decision (e.g. whether a given active instrument is causal or not).

1.2 Conceptual motivations

Let us move to the discussion of conceptual justification of this specific choice of mathematical structure and its relationship to quantum mechanics. Our starting point is an opinion that the predictive content of quantum theory should be independent of any other theory, so it should rely neither on any 'classical theory' nor on any additional (and chosen *ad hoc*) methods of statistical inference used for probabilistic estimation. This leads us to part with two foundational principles of the orthodox approach to quantum mechanics: quantisation[11] and spectral representation.

The idea of quantisation was originally proposed [464, 467] as a method of construction of quantum theoretic models that relies on some additional ('classical') theory, which is a subject of quantisation procedure. But there are many inequivalent variants of this procedure,[12] and most of them are in some sense mathematically ambiguous,[13] while those that are not ambiguous have a seriously restricted applicability. In addition, the 'classical' theories subjected to quantisation are quite often devoid of any predictive content (e.g. most of nonabelian Yang–Mills theories), so they are also *ad hoc*. Finally, quantisation seems awkward from the conceptual point of view, if one considers quantum theory as more fundamental than 'classical' theory (or if one considers quantum theory as an analogue of probability theory)—why more fundamental theory should rely on the less fundamental one?

On the other hand, the purpose of spectral representation [1794, 1797, 1813] is to construct a probability measure supported over the subsets of sample space that are indexed by eigenvalues of a self-adjoint operator or a commutative algebra of self-adjoint operators. Hence, its use is based on the assumption, or leads to a conclusion, that the predictive validity of quantum theory has to rely on other additional theories, namely probability theory and statistical estimation theory. Moreover, it also reflects a belief (or a postulate) that eigenvalues of self-adjoint operators are mutually independent «in principle observable» [767], which (from the frequentist statistical point of view) justifies using them as labels for subsets of sample space. However, the eigenvalues of self-adjoint operators *are not* directly observable or measurable: they are abstract symbols, while this what is measured consists of values of various experimental variables.[14] The relationship between them is *not* direct, as it relies on the choice of *ad hoc* conventions of statistical inference. The idealistic character of the assumption of "in principle observability" was criticised in length in the literature on operational approach to quantum theory (see Section 2.2 for an extended discussion), while the idealistic character of the assumption of mutual independence of eigenvalues comes under criticism in the literature on renormalisation theory[15]. Eigenvalues, as well as any other abstract mathematical concept, such as expectation values or higher order correlation functions, can be used only to *estimate* the values of some functions that are, more or less directly, related to some selective choice of what is considered as experimental 'data'. Finally, if the mathematical formalism of kinematics of quantum theory can be shown to be a strict

[11] More precisely, quantisation is not a *principle* of von Neumann formalisation of quantum mechanics. However, it became the main method of practical quantum theoretic model construction, and is widely considered to be a constitutive component of an orthodox approach to quantum mechanics.

[12] Such as Dirac's and other canonical quantisations, the Feynman–Kac and other path integral quantisations, the Kirillov–Konstant–Souriau and other geometric quantisations, the Weyl–Wigner–Groenewold–Moyal quantisation, etc. See e.g. [626] for a partial review.

[13] E.g. from the van Hove theorem [1776] it follows that it is impossible to preserve the whole 'classical' Poisson bracket algebra under 'canonical quantisation procedure', defined as replacing the Poisson brackets between smooth functions on the phase space by commutators between the corresponding operators on the Hilbert space. Moreover, the choice of a particular subalgebra is arbitrary. In general, this means that the equivalent 'classical' Poisson algebras of smooth functions on the phase (= sample) space lead to unitarily inequivalent 'canonically quantised' noncommutative algebras of operators on a Hilbert space. See also [1589] and references therein.

[14] In our opinion, the role of eigenvalues in foundations of quantum mechanics is quite similar to the role of 'virtual particles' in the perturbative approach to quantum field theory: they are abstract symbols devoid of any *direct* relationship to epistemic evidence, but are kept due to their applicability in calculations within available mathematical formalism (in fact of lack of alternative to it) *and* due to the ontological conceptual content they carry. In the case of perturbative quantum field theory the corresponding formalism is given by the Fok–Cook Hilbert space representation, while the conceptual content is the ontological interpretation in terms of particles.

[15] In principle, introduction of an epistemically motivated cut-off on unbounded 'ideal' (and predictively unreliable) self-adjoint operators leads to a mutual dependence of eigenvalues of a predictive, renormalised, operator. (A similar problem, but encountered for expectation values, will be analysed in details in Section 4.1.3.)

generalisation of formalism of probability theory, then the predictive applicability of former should require no further translation to the latter.

In this sense, denying the principles of quantisation and spectral representation we aim at removing dependence of quantum theory on auxiliary and, in our opinion, foundationally irrelevant theories: classical mechanics and probability theory.

In order to explain better the reasons behind, and consequences of, the conceptual change we propose, let us take a short comparative look at the mathematical and conceptual foundations of quantum mechanics and of frequentist statistics. The origins of both theories seem to be heavily influenced by the postulate that the full content of a predictive theory can be *reduced* to manipulation with tables (matrices) of experimental data (numbers). Most clearly these ideas were expressed by the founders of the modern formulations of these theories: Heisenberg [767] postulated[16] that all experimental data should be in principle reducible in terms of real eigenvalues of the matrices with complex valued entries, while Fisher [561, 562] postulated that all experimental data should be in principle reducible in terms of sufficient statistics over sample spaces. These theories have been elevated to their nowadays orthodox mathematical formalisms in the monographs of von Neumann [1800] and Kolmogorov [972], respectively. The latter text established the abstract measure theory as a framework for probability theory (following earlier ideas of [1623]), while the former one established the abstract Hilbert space theory as a framework for quantum mechanics (following earlier ideas of [1794]). However, none of these books provided solid *conceptual* grounds for frameworks provided there.[17] This has fruitfully contributed to long debates on the interpretations of probability theory and quantum theory. More importantly, none of these books have offered general and unambiguous methods of quantitative model construction and conditioning[18] that would be based on the presented formalisms, and would reflect the quantitative descriptions and constraints of the practical (experimental) situations under consideration in a unique way. This has led to long debates on *the correct* methods of statistical inference and *the correct* methods of construction of 'interacting' quantum theories (and quantum field theories in particular).

It became soon clear that the idea underlying the original postulates of Fisher and Heisenberg is idealistic (metaphysical) and, more importantly, it is invalid in most of situations of *practical predictive concern*. In quantum theory this has been shown ultimately by Wintner [1873] and Wielandt [1848], who proved that the Born–Jordan–Dirac–Heisenberg commutation relations cannot be represented by bounded operators in Hilbert space. In statistical theory this has been shown by Darmois [428], Koopman [974] and Pitman [1345], who proved that exponential families are unique probabilistic models that admit sufficient statistics whose dimension remains bounded with the increase of the sample size. Thus, the experimental data alone are insufficient to determine the structure of, as well as quantitative predictions drawn from quantum and statistical models. In order to construct the model and to draw inferences from it, some additional principles are necessary. This leads to the problems of justification of the choice of specific principles[19] and reevaluation of the conceptual and mathematical foundations of the respective theories in the light of the key role played by these principles for predictive verifiability of those theories.

[16] As it was made precise by Born and Jordan [208, 207].

[17] [972] explicitly states that measure theoretic foundations of probability theory are not justified by frequencies, but also does not give any preference to a subjective, or other, point of view. [1800] contains allusions to several different interpretations, including ontological idealism (while speaking about "quantum systems" as ontological objects), subjectivism (while speaking of "quantum measurements" as provided by consciousness), and operationalism (while associating projections with experimental filters).

[18] [972] introduced conditional expectations as a method of conditioning, while [1800] introduced projection postulates (as shown by Nakamura and Umegaki [1226], a version of projection postulate, given by the weak Lüders' rule, is a noncommutative analogue of a conditional expectation). However, the practical applicability of both methods is strongly restricted: most of epistemic situations under consideration *do not* provide a type of information that can be handled by these tools. For example, the information specified by means of average values of some experimental quantities, estimated theoretically by means of expectation, cannot be recast using a projection on a closed convex subset of a Hilbert space.

[19] For example, in quantum information theory one deals with the problem of construction of a specific semi-spectral measure corresponding to a given epistemic situation (see Section 2.2), while in effective quantum field theory one deals with the problem of choice of the renormalisation method encoding known epistemic constraints.

However, the pragmatic success of several models based on adaptation of *ad hoc* additional principles has overshadowed these problems. As a result, the failure of initial postulates has not led to downfall of the theories built upon them, despite clear further indications of the foundational crisis. In quantum theory the problem of multitude of different unitarily inequivalent Hilbert space representations of a given abstract algebra generated by commutation relations was discovered[20], together with the closely related problem of ambiguity of construction of 'interacting dynamics'. The former discovery implied that the eigenvalues of operators, which are quantities dependent on the choice of a specific representation, cannot be given absolute (universal, objective, ontological) status, while the latter was indicated by the widespread appearance of infinities in quantum field theory. In statistical theory the problems of lack of general criteria of model selection, optimal estimation, and the choice among different inequivalent statistical tests and methods of introducing causality, arose. For the wide range of cases, the solution that have been adapted to *pragmatically* cure this issue was: in quantum theory—to stay within one Hilbert space representation (provided by Schrödinger [1534] and Heisenberg–Born–Jordan [767, 208, 207] in the finite dimensional case and by Fok [567] and Cook [372] in the infinite dimensional case) and to construct predictive dynamical models by means of perturbative expansions; in statistical theory—to stay within the range of asymptotic applicability of exponential and normal models (with sample size assumed to be infinite), and to use such tools as Fisher's maximum likelihood principle [561], or, more generally, Le Cam's local asymptotic normality theory [1043, 1044]. The unsolved critical issue of lack of general methods of control over the quality of these approximations were swept under the carpet by conceptual downgrading the problem of construction of quantum dynamics and the problem of choice of statistical inference procedure that corresponds to a *given* epistemic situation to the status of *technical* (and no longer *foundational*) issues, governed by the set of *pragmatic* rules.

As a result, these two fields of research are plagued by a plethora of *ad hoc* techniques which lack strict mathematical derivation from the single set of underlying principles. At the level of practical applications, asymptotic techniques[21] (in frequentist statistics) and perturbative techniques[22] (in quantum field theory) still play the main role. This is a dark *predictive dynamical* side of the coin,

[20]See Footnote 3 in Section 2.1 for details.

[21]According to Weil, «although we know that statistical mathematics have been of considerable importance to science (and in particular to biological sciences), it needs to be mentioned that statistical books amount in fact to a collection of recipes and precepts we would like to believe to be well chosen. Being written in a highly algebraic form, sometimes using logarithms, exponentials and integrals, they all have the prestige of mathematical exactitude to the untrained eye while the so-called demonstrations statistics are wrapped in, even highly sophisticated, most often make no sense for the mathematician and are simply made of more or less convincing heuristic considerations» [1831] (engl. transl. in: [1879]). The average level of mathematical rigour in statistics has greatly improved since Weil wrote this judgement (see [1821, 1044, 1045, 317, 821, 673, 1327, 1642, 40, 1046, 186, 1722, 1597]), yet the choice of specific mathematical structures and methods corresponding to a given epistemic situation is still based on «more or less convincing heuristic considerations», and there is also a large gap between this what can be strictly proved and this what is applied to a given epistemic situation. In fact, most of mathematical results apply only to the asymptotic case of infinite number of data points, which is devoid of practical meaning from the *strict* point of view. For example, Le Cam notes that «limits theorems 'as n tends to infinity' are logically devoid of content about what happens at any particular n. All they can do is to suggest certain approaches whose performance must then be checked on the case at hand. Unfortunately the approximation bounds we could get were too often too crude and cumbersome to be of any practical use» [1046]. The routinely used *vague* point of view is based upon using *ad hoc* conventions that relate the asymptotic results with nonasymptotic situations: «in statistics no 'general' theory has yet been formulated that would investigate from a single point of view the properties of estimates, tests and other inference procedures and would be accepted by the majority specialists. A fundamental concept such as optimality is usually defined *ad hoc* in each special case, depending on the nature of the problem» [1596].

[22]As Borchers notes, the lagrangean approach to quantum field theory «has the disadvantage that the expressions which appear in this theory have only formal meaning. Up to now there is no convincing scheme which puts the formal expressions onto a solid and consistent mathematical basis. The existing perturbation and renormalization theory does not, in most cases, indicate anything about the quality of the approximation. Therefore, only comparison with experiment can indicate the quality of the Lagrange function and the approximation» [199]. The comparison of this remark with quotes in footnote 21 leaves a noticeable impression. It is quite striking to observe that two seemingly distinct fields of scientific inquiry share a strong structural similarity of the large variety of *ad hoc* methods mixed with a language of specialised, but rigorously undefined, concepts and semi-mathematical results, which hide the lack of conceptually clear *and* mathematically strict principles. In fact, this observation was a motivation for a conceptual reverse engineering of relationships of these two theories carried in Chapter 1.

bright side of which is a mathematical clarity of the *abstract kinematical* frameworks for probability theory and quantum mechanics that were provided and turned into orthodoxy by Kolmogorov and von Neumann, respectively. Persistence of this division despite all its negative consequences indicates that this division is necessary in order to satisfy some implicit but very important postulate. For the purpose of identification of it, let us summarise the main classes of problems that are encountered at the predictive side of the above two theories:

(1) the arbitrariness of choice of a particular statistical inference method that is applied to a particular experimental situation. This choice might be suggested by intuition, tradition, or some *convention*, but is not derived from any general principle that would explicitly take into consideration the available knowledge that defines a given experimental situation. This situation is based on the frequentist interpretation, which equips probability distributions with ontological meaning, which forbids from considering them as carriers of epistemic knowledge, encoded and processed directly by means of some conventions. As a result, the division of inductive inference theory to probability theory and statistical inference theory is enforced, with probability theory keeping the ideal 'ontological' look and statistical inference handling all epistemic contents by means of *ad hoc* methods. See [870] for an extensive critical discussion concerning these issues, and [960, 1043, 1358, 959, 862] for various examples of spectacular failures of standard *ad hoc* methods;

(2) the arbitrariness of construction of *predictive* ('effective', 'interacting') quantum dynamics. Quantum mechanics does not provide any prescription how to construct hamiltonians that would directly encode the quantitative constraints imposed by experimentally verifiable temporal relationships of experimental control-and-response parameters. There are various, inequivalent methods of 'quantisation' of 'classical' hamiltonians or lagrangeans, which lack common conceptual or mathematical foundation and are, in general, ambiguous. The results of use of these techniques often do not correspond directly to particular quantitative results and descriptions of experimental procedures under consideration (they do not define *predictive* dynamics), hence they have to be modified using additional *ad hoc* procedures (with more or less *ad hoc* choice of parameters), like cut-offs, parameter fitting, regularisation, or renormalisation. These procedures lead to serious problems of conceptual and mathematical character. In particular, they might move the operators between different unitarily inequivalent representations. In such case, the originally postulated lagrangean function or hamiltonian operator, far from being unique, has also no experimental meaning, because it cannot be justified by the reference to quantitative results of experimental procedures neither in terms of spectral representation, nor in terms of the expectation values or higher correlation moments. Thus, the choice of a hamiltonian might be suggested by intuition or tradition, but is not derived from any general principle (or explicitly specified class of principles) which in *some* clearly defined sense would optimally encode the available information about given experimental situation (or a class of them);

(3) the arbitrariness of relationship between the choice of specific method of model construction, statistical inference procedure, and the assumptions regarding the causal structure that is considered as responsible for the changes of experimental data (see [1618, 1285, 1182, 1286] for a detailed discussion);

(4) the arbitrariness of relationship between unitary (Born–Jordan–Dirac–Schrödinger) temporal evolution and nonunitary (von Neumann–Lüders) temporal evolution. Quantum mechanics does not give any definite prescription how to relate these two forms of temporal evolution of density operators. This conflict is sometimes rephrased by saying that «the border between the field of application of Schrödinger's equation and the one of the projection postulate is not well defined» [100] (see also [183]), or that «no formulation of a projection postulate tells us exactly at which point [of time parameter of the unitary evolution] to apply it» [1700]. This sometimes leads to denial of the structure associated with the nonunitary evolution as an attempt to solve this conflict, see e.g. [252]. However, one should note that the above elements of mathematical

structure of quantum mechanics do not give any argument which favours unitary continuous temporal structure over nonunitary discrete temporal structure. Such preference may come only from some additional assumption (interpretation), which can be usually identified as a belief that quantum mechanics is a theory in some sense analogous to classical mechanics, as opposed to probability theory.

Moreover, when it comes to assertion of predictive verifiability of dynamical quantum theoretic models, all above problems combine into a new quality, due to:

(5) the arbitrariness (and *a priori* postulated necessity) of division of a knowledge describing (*defining*) a given experimental situation into a part that is subjected to probabilistic statistical inference and a part that is subjected to quantum theoretical dynamics.[23] This perspective is based on *a priori* division of actual scientific inquiry into two different parts: "ideal/ontic theories" and "statistical/epistemic estimation". As a result, the predictive application of quantum theoretic formalism becomes split into: (i) the *ad hoc* methods of probabilistic statistical estimation (inductive inference), which aims at selecting the unique semi-spectral measure and unique density operator that fit the specified quantitative results (see e.g. [777, 807, 809, 269, 427] and discussion in Section 2.2), (ii) the abstract Hilbert space based dynamical model construction, based on *ad hoc* procedures. The orthodox conceptual perspective considers dynamical model of some theory as something that can be arbitrarily postulated ("let us consider the following langrangean/hamiltonian of an interacting system..." and "let us postulate the following initial state of an interacting system..."), and later, in face of experimental evidence, it can be either dismissed or arbitrarily changed. As a result, "ideal/ontic theories" do not fit *optimally* the experimental data, and require some fine tuning, which is more or less arbitrary (and is provided either on the level of statistical estimation, or on the level of perturbative construction of 'interacting' hamiltonian). This shows strong contrast between the idealistic principles of a theory and the pragmatic procedures of its application (and is also an illustration of Duhem's thesis [507, 508]). From the pragmatic (or renormalisation-minded) point of view, the *ad hoc* hamiltonians/langrangeans serve only as a starting reference point for some other procedures whose aim is to encode the particular experimental evidence.

In this context, let us note that the idea of "true probability distribution", whose best incomplete description is provided by means of frequentist statistic, is an analogue of the idea of "classical theory", whose incomplete description is provided by means of quantum operators. The former idea is based on consideration of the notion of an 'elementary event' (modelled by an element of a sample space) as a fundamental ontic entity, which is quantified by means of a statistic, while the latter idea is grounded in consideration of the notion of an 'elementary property' (modelled by an element of a phase space) as a fundamental ontic entity, which is quantified by means of a function on a phase space that is further subjected to quantisation. In this sense the notion of a statistic in frequentist statistics and the notion of an observable in quantum mechanics carry strong ontological undertones.[24] Note also that, while quantum mechanics does not rely formally on "classical theories" and quantisation on the level of foundations, the spectral representation reintroduces the ontological interpretation by relying on frequentist statistics over the sample space associated with eigenvalues. As a specific but crucial consequence, this leads to interpretation of the procedure of perturbation of 'quantised' hamiltonians as construction of the models of 'interaction' of some ontic entities, and not as providing the methods of inference conditioned upon epistemic control parameters.

Thus, while it is disputable whether the quantum theory and statistical inference theory are currently the prisoners of ontological postulates or of a lack of alternative mathematical framework (probably both), it seems for us that the above mathematical and conceptual problems require more than to postulate some new mathematical setting. It is essential to relate the structure of this setting with

[23] One can consider this sentence as nonontological reading (or the practical meaning) of the so-called "Bohr–Heisenberg cut".

[24] It is quite ironic, as compared with the postulates of Fisher and Heisenberg. Yet, it is a common problem of all reductionisms.

some specific postulates regarding the dynamical model construction and verifiability of predictions in face of epistemic (experimental) evidence.

1.3 Postulates

The conceptual layer of our approach (as summarised in the main postulate below) provides an interpretation of the underlying mathematical structure that is aimed at two goals: eliminating the ontological assumptions imposed on the formalisms of quantum mechanics and statistical inference, and clarifying the relationship between the predictive verifiability of dynamical models and their dependence on the conventions used for the purpose of their construction.

Our main postulate is to regard quantum theoretic kinematics and dynamics as generalisations of, respectively, probability theory *and* statistical inference to a regime where the underlying object of inquiry is not a set of measurable sets, but an integrable noncommutative algebra, with nonlinear quantum kinematics determined by the quantum relative entropy, and nonlinear quantum dynamics determined by two independent notions of evolution, provided by active and passive instruments. The corresponding conceptual postulate is to regard quantum theory as a strictly epistemic theory of information and intersubjective inference[25], as opposed to ontic theories of substance its action (such as classical mechanics and general relativity).

The specific interpretation of the mathematical elements of our framework is:

1. **Algebras**. The W^*-algebras $\mathcal{N}$ are viewed as the collections of abstract 'elementary qualities', which are *intersubjectively regarded* as basic entities of a theoretical inquiry, and are subjected to quantification by integration. The integrals on $\mathcal{N}$ are given by quantum states $\phi \in \mathcal{N}_\star^+$, and the corresponding quantifications are given by (not necessarily normalised) expectations $\phi : \mathcal{N}^{\mathrm{sa}} \mapsto \phi(x) \in \mathbb{R}$. This is in strong analogy to viewing abstract boolean algebras as collections of abstract 'elementary events', which are subjected to quantification in terms of measures. (If these measures are normalised, then the corresponding quantifications are probabilities.) See Sections 2.1 and 2.2 for a thorough discussion of these issues. Elements of $\mathcal{N}$ *are not* regarded as "observables".

2. **Models**. Quantum models $\mathcal{M}(\mathcal{N})$ are viewed as sets of quantitative descriptions of epistemic[26] knowledge about 'abstract elementary qualities' in $\mathcal{N}$. The passage from abstract notion of 'quantification' to more specific term, given in the previous sentence, is conditional upon additional requirement. It states that an application of $\mathcal{M}(\mathcal{N})$ for the purpose of description of some epistemic situation has to be based on explicit specification of a set theoretic bijection between $\mathcal{M}(\mathcal{N})$ and the set of all those possible quantitative descriptions of this situation which are *intersubjectively regarded* as relevant. In other words, our semantic requirement of the quantum model construction is: everything, that is considered to be an intersubjectively relevant quantitative knowledge that *defines* a given epistemic situation, should be bijectively translatable (= encodable and decodable) in terms of $\mathcal{M}(\mathcal{N})$.[27] This is independent of the language

[25] In a sense similar, to some extent, to statistical mechanics and thermodynamics, when considered from the perspective of Jaynes [856, 858, 871, 859, 860, 863, 867], Ingarden [832, 834, 825, 826, 831], and others [1725, 1517, 1726, 658, 659]. From another point of view, one could say that our approach regards quantum theory as a framework for epistemic information processing. This resembles some similarity with the viewpoints of approaches of Srinivas [1619, 1620] and Abramsky–Coecke [3]. However, apart from other differences, we allow a large variety of inference rules, which are not required to be linear or local.

[26] Our preference to the adjective 'epistemic' instead of 'experimental' stems from information theoretic and inference theoretic perspective we adopt. According to it, quantum theory provides mathematical and conceptual framework for processing *any* quantifiable knowledge. To some extent, this can be traced back to Hartley [756], who postulated that information should be measured regardless of its semantic content. However, we impose an additional *semantic* constraint, requiring that this knowledge has to be intersubjective (the meaning of this term is discussed onwards).

[27] The idea that the probability density function should be constructed as a result of maximisation of an absolute entropy functional subjected to constraints that are expressed exclusively in terms of experimentally defined parameters was introduced by Gibbs [627] and was further developed by Jaynes [856, 863, 864, 870] (see also [651, 1844]). As opposed to Jaynes, we allow arbitrary intersubjective conventions of model construction.

used for the description of epistemic situation, an intentional subject of concern of experiments, experimental setting handled, etc.

3. **Geometry**. The meaning we assign to defining quantum theoretic kinematics in terms of quantum information geometry stems from the general purpose of any kinematics as a framework for quantitative analysis of the changes of states provided by dynamics, with a special emphasis on initial and final states. In the case of information dynamics, the initial states are called 'priors', while the finals states are called 'posteriors'.[28] The specific choice of geometric structure on a quantum model is an explicit declaration of *intersubjective criteria* of *relative relevance* (*distinguishability*[29]) that are used for the purpose of selection of priors on the basis of available epistemic evidence[30] and for the purpose of judging the quality of inference on $\mathcal{M}(\mathcal{N})$ by comparison of posteriors with some reference (e.g. some state or a subset of states). The main issue at stake here is that the prior selection and posterior quality judgement, which are pre- and post-inferential procedures, respectively, require explicit specification of the criteria of 'best'/'optimal' fit of the quantum state to some standard of reference. This is implemented by the quantum distance $D(\cdot,\cdot)$, as well as by the derived notions, such as the riemannian metric $\mathbf{g}$. Note that quantum distance $D(\cdot,\cdot)$ on $\mathcal{M}(\mathcal{N})$ is irrelevant for construction of a model $\mathcal{M}(\mathcal{N})$ as a whole (this is provided already by the Item 2 above). Putting it into a slogan: quantum models can be *constructed*; quantum states can be *distinguished*.

4. **Dynamics**. The active instrument defines the *intersubjective convention* regarding inference procedure. It can be considered as an 'information theoretic force'[31]. On the other hand, the passive instrument defines the *intersubjective convention* regarding causality. It encodes the assumptions about transformations of the epistemic evidence used for the purpose of model construction. By itself, it carries no predictive content, but it serves as a reference for judgement about inferences provided in terms of active instruments. In this sense, it is an 'information theoretic reference frame of motion'.[32] For this reason, we will use the phrases 'causality system' and 'referential evolution' interchangeably with the notion of 'passive instrument'.

   The post-inferential judgement of quality of inference may be provided with respect to a quantum state selected (:= distinguished) on the base of some epistemic evidence. But, as discussed in Section 4.2.3, it can be also provided with respect to a passive instrument. Thus, instead of postulating a hamitonian/lagrangean of a "free evolution" of a "noninteracting system" and further perturbing it by means of an "interaction" (with *indefinite* methods and criteria of quantification of quality of this approximation with respect to the available epistemic constraints), we postulate that one should specify the referential evolution (passive instrument) $\mathfrak{I}_{\mathrm{pass}}$, the specific choice of $D(\cdot,\cdot)$ used for comparison, and then impose constraints on the allowed range of active instruments (quantum inference procedures) by means of D and $\mathfrak{I}_{\mathrm{pass}}$. The concept of transition distance, introduced in Section 4.2.3, is intended to serve this purpose. In other words, we postulate that the individual acts of inference (represented by active instruments) should be considered as relative to an intersubjective choice of the causality system, which describes the motion of information states when no specific evidence and no active instrument is given.

[28] The entropic instrument that uses a prior *measure* as an input and produces an effect *measure* as an output is a generalisation of this idea. (See Item 4 below and Section 4.1.1 for definitions of these notions.) Actually, this suggests to construct a category of such generalised mappings, but we leave it as a topic for future research. See also Footnote 5 in Chapter 4.

[29] «this geometry does not describe the dynamics, or evolution, of a quantum system; rather it places limits on our ability to distinguish one state from another through measurements» [241].

[30] We use term 'epistemic evidence' an 'quantitative epistemic knowledge' interchangeably.

[31] This interpretation is especially suggestive in the case of entropic instruments $\mathfrak{P}^{D,\Lambda}_{\mathfrak{D}}$. In this case the prior measure Λ can be interpreted as an 'information theoretic mass' (see Section 4.1.1).

[32] This interpretation is especially suggestive for any instrument defined as a predualisation of automorphism of $\mathcal{N}$ (thus, a causality system defined strictly at the algebraic level as a group of self-transformations of a set $\mathcal{N}$ of 'abstract qualities') which coincides with the Poisson flow of a globally defined hamiltonian vector field (see Section 4.2.2). All unitary evolutions of the finite dimensional parametric quantum models belong to this class of instruments.

5. **Observables**. The downfall of the principle of "in principle observability" of elements of spectra of self-adjoint operators on a Hilbert space, as well as regarding the elements of an abstract W^*-algebra as 'elementary qualities' leads us to dissolve the quantum mechanical notion of "observable" into a kinematic notion of 'estimable'[33] and dynamic notion of an 'effect'. The 'estimable' is defined as an arbitrary function on an information model that can be used in order to provide estimation (:= prior selection or posterior quality judgement) using the available epistemic evidence. In particular, every element of an abstract W^*-algebra $\mathcal{N}$ determines a *linear* estimable by means of $\hat{x} : \mathcal{M}(\mathcal{N}) \ni \phi \mapsto \phi(x) \in \mathbb{C}$. Yet, one can consider arbitrary nonlinear estimables on $\mathcal{M}(\mathcal{N})$ as well. Whenever $\mathcal{M}(\mathcal{N})$ is equipped with a smooth manifold structure, then the algebra of smooth functions on $\mathcal{M}(\mathcal{N})$ might be of particular concern. A detailed analysis of this class of nonlinear estimables and their relationships with the Banach–Lie–Poisson structure $\{\cdot,\cdot\}$ on $\mathcal{M}(\mathcal{N})$, as well as an extensive discussion of the relationship of these objects with an orthodox setting of quantum mechanical "observables" is due to Bóna [188, 190] (he uses the term "generalised observable", cf. also [1167]). On the other hand, the dynamic notion of an 'effect' is an "estimable-like" quantification of the posterior of an inference procedure that is uniquely determined by this procedure. This should be opposed to the *relative* quantification of this posterior by means of $D(\cdot,\cdot)$. More precisely, an 'effect' is defined as a function on $\mathcal{M}(\mathcal{N})$ such that it is uniquely determined by a given active instrument for any choice of constraints for this instrument. In Section 4.1.1 we introduce and analyse the nontrivial class of effects defined by the entropic instruments. We regain the relationship with the quantum mechanical setting by showing that, under some conditions, entropic effects determine the corresponding unique semi-spectral measures, and this procedure is structurally analogous to determination of semi-spectral measures by completely positive linear instruments.

6. **Space-times**. The notion of space-time does not belong to foundations of quantum mechanics. However, one of our main motivations for developing new foundations of quantum theory is to recover space-times from quantum theory as emergent objects (which is opposite to the idea of "quantisation of space-time"). The mathematical aspects of this problem are discussed in Sections 4.2.4 and 2.4. From the conceptual point of view, construction of emergent space-times amounts to regarding the lorentzian metric as an object encoding the conventions about the *local* criteria of optimal estimation (infinitesimal measure of distinguishability of information states) and the conventions about the *global* system of causality. This can be summarised by saying that: (i) two space-time points are distinguishable to a given extent *because* they represent the states of information that are distinguishable to a given extent; (ii) the space-time notion of causality, provided by space-time metric, is determined by the information theoretic notion of causality, provided by the passive instrument. Hence, the space-time causality encodes *intersubjective convention* regarding the 'causal factuality' and 'causal counterfactuality' in the same way as quantum information geometry encodes intersubjective conventions regarding 'verifiability' and 'falsifiability'. As a result, each epistemic situation determines in principle its own space-time. Different space-times can be related to each other whenever their corresponding epistemic subjects, as well as intersubjective conventions used for the purpose of their construction, can be related.

1.4 Conceptual framework

Note that we do not aim at avoiding a convention-dependent character of quantum dynamics/inductive inference and its predictive verification. Our intention is to get rid of nonoptimality and intersubjectively incoherent ad hockery. Following Spengler [1617] and Fleck [565], we do not expect that anything stronger than intersubjective coherence in the above sense can be provided (the ontological postulates serve the same purpose, after all, just without acclaiming it). In this sense, intersubjective bayesianism,

[33] We do not call it an 'estimator' in order to stress that its interpretation differs from, and is independent of, frequentist statistics.

as we define it here, can be understood as placed inside an intersection of two epistemic nominalisms: moderate conventionalism and moderate operationalism, where 'moderate' means 'lacking reductionist overtones'.

This perspective is strongly influenced by the ideas of Jaynes [858, 860, 863, 870]. Like earlier founders of modern subjective bayesian approach to statistical theory, he denied identification of probability with frequency. However, he argued that the statistical inference needs to be constrained more strongly than by individual subjective judgements. The point of view proposed by Jaynes was a specific information theoretic interpretation of probability distributions, according to which they are theoretical objects that serve for the purpose of encoding and decoding information regarding *intersubjectively* communicable and verifiable knowledge. Jaynes attempted to justify the uniqueness of some methods of statistical inference on the base of various arguments that were shown to be indefinite (see [1741, 1742]). Our own point of view is that, while scientific inference is definitively intersubjective and not personal, there is no absolute basis allowing to justify any unique rules of inductive inference, so one necessarily deals with intersubjective conventions. The reconsideration of information geometric approach to foundations of quantum theory from this perspective, leads to an interesting return of Poincaré's ideas [1350, 1351]: the choice of quantum geometry is conventional, so is the emergent space-time geometry. However, this is not a subjective convention, because the criteria of predictive verifiability have intersubjective character.

So far we have not defined the semantic term 'intersubjective'. This would require us to coin some specific metaphysics (not necessarily of ontological type), and argue in favour of it. But we consider such attempts as arbitrary, hence irrelevant for the foundations (as opposed to applications). Instead of this, we will try to specify the minimal conditions required for regarding some semantic setting as providing necessary and sufficient meaning to 'intersubjectivity' in the context of *scientific inquiry*. The starting point is an observation, which follows Heraclitus, that any particular individual instance of an experimental situation is never reproducible. Only abstract *schematic* [339] descriptions of experimental situations can be reproduced, so only these abstract descriptions can be used as a basis for *scientific* inquiry. Hence, the postulate of reproducibility forces to use some abstract language of description of experimental situations. A *scientific fact* is an abstract statement of the above language. Next, the scientific requirement of an *intersubjective* reproducibility implies that the criteria of providing intersubjective meaning to sentences of this abstract language *are already given*. Finally, the last necessary condition for the scientific inference is a strict separation between abstract language used for the purpose of schematic description and communication of experimental situations and an abstract language used for the purpose of theoretic inference. As discussed in details by Fleck [565], scientific facts (and more generally, whole epistemic layer of scientific inquiry) can evolve similarly to scientific theories, and these two evolutions are mutually dependent. The above postulate of strict separation warrants that there exists an intersubjective class of reference (community of users) for which the division between experiment and theory is kept clear and fixed. Our interpretation amounts to saying that: (1) an "information" and "information dynamics" is a theoretic quantification of knowledge and its changes[34]; (2) this knowledge and its changes refer to the scientific facts expressed in terms of an abstract language used for description of an epistemic layer of inquiry; (3) the quantification in (1) and the relationship in (2) are provided accordingly to intersubjectively shared conventions, and under strict separation of scientific inquiry into epistemic and theoretic layers.

In this context, let us now consider the conceptual aspect of our main postulate, as stated at the beginning of Section 1.3. By the phrase 'strictly epistemic' we mean a requirement that a given theory can be subjected to predictive quantitative verification in terms of the available epistemic evidence without invoking any external concepts, notions, and theories, and without assigning ontological status to any of its elements.[35] By 'experimental evidence' we understand sentences of some abstract lan-

[34] Quantification amounts to turning knowledge and its changes into *evidence*.

[35] «no verification is possible unless the relevant inference method is an integral part of the theory» [1677]. Note that we postulate a judgement of the quality of inference to be a part of a theory (in line with the methods of model construction, prior selection, and inference). In this sense, every theory constructed within our approach should carry its own criteria of predictive verifiability.

guage which is suitable for intersubjective communication of quantitative descriptions of experimental designs, actions, and results, but does not involve any model construction, inference, or verification.

Interpreting inductive inference procedures as referring to something that is of abstract and intersubjective character draws a difference between the conceptual setting of our approach and the personalistic ('subjective') bayesianism [440, 441, 1514, 442, 534, 603, 297, 844, 605, 604, 298, 1707, 606] on one side and rationalistic ('objective') bayesianism [961, 874, 376, 858, 1726, 148, 868, 870] on the other. Similarly to subjective bayesianism, we consider the choice of particular method of inductive inference as *in principle* arbitrary. Similarly to objective bayesianism, we consider the *scientific inference* as a specific case of inductive inference theories, in which the constraint of *intersubjective reproducibility* of the subject of inference should play main role.

Our implementation of both insights (which we tentatively call 'intersubjective bayesianism') stems from observations of Spengler [1617], Granet [660], Fleck [564, 565, 566], and Chwistek [339]. According to them, the subject of concern of scientific inquiry at *experimental* level is neither a content of an individual subjective experience, nor an ontic "material reality", but a structuralised system of abstract ideas, which is shared with other people educated and cognitively prepared in the similar way, and which frames the range of individual perceptions and thought style that can be subjected to intersubjective communication and predictive verification.

1.5 Published and joint work

The new perspective and main definitions contained in the above Introduction, and in Sections 3.1.1 and 4.1.1 were partially published in [992, 995]. The discussion of some of the conceptual issues connected with this new mathematical approach was provided in [994]. The new results contained in Sections 3.2.3 and 3.2.4 were published in [993]. The content of Section 4.1.2 is a result of the joint work with Wojciech Kamiński [924][36]. The quantum Schwarzschild space-time model of Section 4.2.4 is a joint work with Paweł Duch [506].

1.6 Conventions

The following conventions are used: 1) the symbols denote the same mathematical entities throughout all work; 2) the special syntactic forms used in text are: ***definitions***, «citations», 'notions subjectable to strict definition', "vague notions", and *attention markers*; 3) the mathematical style of text formatting (definition/theorem/proof) is used only for stating essentially new mathematical results; 4) whenever possible, we refer to original works containing results that are discussed or used; 5) the folk attributions of surnames to mathematical concepts are changed to historically correct ones whenever there is a definite evidence for the latter, and the concepts with attribution of three or four surnames to it are turned to an acronym after first use, while in the case of more authors we use only the descriptive naming of objects;[37] 6) for the Latin transliteration of the Cyrillic script (in references and surnames) we use the following modification of the system GOST 7.79-2000 B: ц = c, ч = ch, х = kh, ж = zh, ш = sh, щ = shh, ю = yu, я = ya, ё = ë, ъ = ', ь = ', ы = y, э = e', й = ĭ, with an exception that surnames beginning with Х are transliterated to H.[38]

[36]The idea to use the generalised pythagorean equation as a method of proof, as well as some calculations suggesting the correctness of this idea, emerged in our joint discussion with Frank Hellmann and Carlos Guedes.

[37]The only definite exception that we have made consciously is the abstract notion of a 'Hilbert space'. It was invented (and named) by von Neumann. Hilbert invented only its special case, an $\boldsymbol{\ell}_2$ space, while an $L_2(\mathbb{R}, \mathrm{d}\lambda)$ space was invented by Riesz. Yet, the standard naming convention is too influential to stand up against it. More generally, we have adopted the sequential adaptation of rules: (1) strong relevance of temporal priority, (2) weak relevance of folk popularity, (3) weak avoidance of acronyms. As a borderline example, we speak of the "Morse–Transue–Nakano–Luxemburg norm", abbreviated to the "MTNL norm", despite 5 years of difference between [1192, 1228] and [1108] due to wide popularity of the term "Luxemburg norm" (enforced by [998, 1400]), but we speak of the "Csiszár–Morimoto $\mathfrak{f}$-distance" despite 3 years of difference between [380, 1184] and the Ali–Silvey paper [31]. On the contrary to this partially restrictive *naming* system, the *references* are made as complete as it is reasonably possible.

[38]This is required for agreement with the widespread practice to transliterate Холево as Holevo, etc.

References

Я мысленно вхожу в ваш кабинет;
Здесь те, кто был, и те, кого уж нет,
Но чья для нас не умерла химера;
И бьётся сердце, взятое в их плен...

Максимилиан А. Волошин

[1] Aarnes J.F., 1969, *Physical states on a C^*-algebra*, Acta Math. **122**, 161.
[2] Aarnes J.F., 1970, *Quasi-states on C^*-algebras*, Trans. Amer. Math. Soc. **149**, 601.
[3] Abramsky S., Coecke B., 2004, *A categorical semantics of quantum protocols*, in: *Proceedings of the 19th Annual IEEE Symposium on Logic in Computer Science: LICS 2004*, IEEE Computer Society, p.415. arXiv:quant-ph/0402130.
[4] Abramsky S., Coecke B., 2009, *Categorical quantum mechanics*, in: Engesser K., Gabbay D.M., Lehmann D. (eds.), *Handbook of quantum logic and quantum structures: quantum logic*, Elsevier, Amsterdam, p. 261. arXiv:0808.1023.
[5] Accardi L., 2006, *Axioms for quantum probability*, in: Bożejko M., Młotkowski W., Wysoczański J. (eds.), *Quantum probability*, Banach Center Publications **73**, Instytut Matematyki Polskiej Akademii Nauk, Warszawa, p.15. Available at: art.torvergata.it/bitstream/2108/57184/1/Axioms%20for%20Quantum%20Probability.pdf.
[6] Aczél J., 1984, *Measuring information beyond communication theory – why some generalized information measures may be useful, others not*, Aequat. Math. **27**, 1.
[7] Aczél J., 1987, *Characterizing information measures: approaching the end of an era*, in: Bouchon–Mennier B., Yager R.R. (eds.), *Uncertainty in knowledge-based systems*, LNCS **286**, Springer, Berlin, p.359.
[8] Aczél J., Daròczy Z., 1975, *On measures of information and their characterizations*, Academic Press, New York.
[9] Aerts D., 1981, *The one and the many: towards a unification of the quantum and the classical description of one and many physical entities*, PhD Thesis, Vrije Universiteit Brussel, Brussel.
[10] Aerts D., 1982, *Description of many separated physical entities without the paradoxes encountered in quantum mechanics*, Found. Phys. **12**, 1131.
[11] Aerts D., 2009, *Quantum axiomatics*, in: Engesser K., Gabbay D.M., Lehmann D. (eds.), *Handbook of quantum logic and quantum structures: quantum logic*, Elsevier, Amsterdam, p.79.
[12] Aerts D., Daubechies I., 1978, *Physical justification for using tensor product to describe two quantum systems as one joint system*, Helv. Phys. Acta **51**, 661.
[13] Aerts D., van Steirteghem B., 2000, *Quantum axiomatics and a theorem of M.P. Solèr*, Int. J. Theor. Phys. **39**, 497.
[14] Al-Rashed M.H.A., Zegarliński B., 2007, *Noncommutative Orlicz spaces associated to a state*, Studia Math. **180**, 199.
[15] Al-Rashed M.H.A., Zegarliński B., 2011, *Noncommutative Orlicz spaces associated to a state II*, Lin. Alg. Appl. **435**, 2999.
[16] Alaoğlu L., 1940, *Weak topologies of normed linear spaces*, Ann. Math. **41**, 252.
[17] Alber Ya.I., 1996, *Metric and generalized projection operators in Banach spaces: properties and applications*, in: Kartsatos A.G. (ed.), *Theory and applications of nonlinear operators of accretive and monotone type*, Dekker, New York, p.15.
[18] Alber Ya.I., 1998, *Generalized projection, decompositions, and the pythagorean-type theorem in Banach spaces*, Appl. Math. Lett. **11**, 115.
[19] Alber Ya.I., 2007, *Young–Fenchel transformation and some new characteristics of Banach spaces*, in: Jarosz K. (ed.), *Function Spaces: Fifth Conference on Function Spaces, May 16-20, 2006, Southern Illinois University, Edwardsville, Illinois*, Contemp. Math. **435**, American Mathematical Society, Providence, p.1.
[20] Alber Ya.I., Butnariu D., 1997, *Convergence of Bregman projection methods for solving consistent convex feasibility problems in reflexive Banach spaces*, J. Optim. Theor. Appl. **92**, 33.
[21] Alberti P.M., 1983, *A note on the transition probability over C^*-algebras*, Lett. Math. Phys. **7**, 25.
[22] Alberti P.M., Uhlmann A., 1984, *Transition probabilities on C^*- and W^*-algebras*, in: Baumgärtel H., Laßner G., Pietsch A., Uhlmann A. (eds.), *Proceedings of the second international conference on operator algebras, ideals, and their applications in theoretical physics (Leipzig 1983)*, Teubner, Stuttgart, p.5. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh84a.pdf.
[23] Alberti P.M., Uhlmann A., 2000, *On Bures distance and $*$-algebraic transition probability between inner derived positive linear forms over W^*-algebras*, Acta Appl. Math. **60**, 1. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/AU00.pdf.
[24] Aleksandrov A.D., 1940, *Additive set-functions in abstract spaces*, Matem. Sb. N.S. **8**, 307.
[25] Aleksandrov A.D., 1941, *Additive set-functions in abstract spaces*, Matem. Sb. N.S. **9**, 563.
[26] Aleksandrov A.D., 1943, *Additive set-functions in abstract spaces*, Matem. Sb. N.S. **13**, 169. mathnet.ru:msb6177.
[27] Alfsen E.M., 1963, *Order theoretic foundations of integration*, Math. Ann. **149**, 419.
[28] Alfsen E.M., 1971, *Compact convex sets and boundary integrals*, Springer, Berlin.
[29] Alfsen E.M., Schultz F.W., 1976, *Non-commutative spectral theory for affine function spaces on convex sets*, Mem. Amer. Math. Soc. **172**, Providence.
[30] Alfsen E.M., Schultz F.W., 2003, *Geometry of state spaces of operator algebras*, Birkhäuser, Basel.
[31] Ali S.M., Silvey S.D., 1966, *A general class of coefficients of divergence of one distribution from another*, J. Roy. Stat. Soc. B **28**, 131.
[32] Alicki R., 1995, *Comment on "Reduced dynamics need not be completely positive"*, Phys. Rev. Lett. **75**, 3020.

[33] Alicki R., Majewski W.A., 1990, *On a general characterization of nonlinear quantum mechanical maps*, Phys. Lett. A **148**, 69.
[34] Altun Y., Smola A., 2006, *Unifying divergence minimization and statistical inference via convex duality*, in: Lugosi G., Simon H.U. (eds.), *Learning theory. 19th Annual Conference on Learning Theory, COLT 2006, Pittsburgh, PA, USA, June 22-25, 2006, Proceedings*, LNCS **4005**, Springer, Berlin, p.139.
[35] Amari S.-i., 1959, unpublished.
[36] Amari S.-i., 1980, *Theory of information spaces – a differential–geometrical foundation of statistics*, Post Res. Assoc. Appl. Geom. Mem. Report **106**.
[37] Amari S.-i., 1982, *Differential geometry of curved exponential families – curvatures and information loss*, Ann. Statist. **10**, 357. euclid:aos/1176345779.
[38] Amari S.-i., 1982, *Geometrical theory of asymptotic ancillarity and conditional inference*, Biometrika **69**, 1.
[39] Amari S.-i., 1983, *A foundation of information geometry*, Electr. Commun. Japan A **66**, 1.
[40] Amari S.-i., 1985, *Differential–geometrical methods in statistics*, Lecture Notes in Statistics **28**, Springer, Berlin.
[41] Amari S.-i., 1987, *Dual connections on the Hilbert bundles of statistical models*, in: Dodson C.T.J. (ed.), *Geometrization of statistical theory*, Univ. Lancaster Dept. Math. Publ., Lancaster, p.123.
[42] Amari S.-i., 2009, *α-divergence is unique, belonging to both f-divergence and Bregman divergence classes*, IEEE Trans. Inf. Theor. **55**, 4925.
[43] Amari S.-i., 2009, *Information geometry and its applications: convex function and dually flat manifold*, in: Nielsen F. (ed.), *Emerging trends in visual computing*, LNCS **5416**, Springer, Berlin, p.75.
[44] Amari S.-i., Cichocki A., 2010, *Information geometry of divergence functions*, Bull. Polon. Acad. Sci. Tech. Sci. **58**, 183.
[45] Amari S.-i., Kumon M., 1988, *Estimation in the presence of infinitely many nuisance parameters: geometry of estimating functions*, Ann. Statist. **16**, 1044.
[46] Amari S.-i., Nagaoka H., 1993, *Jōhō kika no hōhō*, Iwanami Shoten, Tōkyō (engl. transl. rev. ed.: 2000, *Methods of information geometry*, American Mathematical Society, Providence).
[47] Amemiya I., 1953, *A genaralization of Riesz–Fischer's theorem*, J. Math. Soc. Japan **5**, 353. euclid:jmsj/1261415457.
[48] Andai A., 2003, *Monotone riemannian metrics on density matrices with nonmonotone scalar curvature*, J. Math. Phys. **44**, 3675. arXiv:math-ph/0305060.
[49] Andai A., 2009, *On the geometry of generalized gaussian distributions*, J. Multivar. Anal. **100**, 777. arXiv:0706.0606.
[50] Andō T., 1969, *Banachverbände und positive Projektionen*, Math. Z. **109**, 121.
[51] Andō T., 1978, *Topics on operator inequalities*, Lecture notes, Hokkaidō University, Sapporo.
[52] Andō T., 1979, *Concavity of certain maps on positive definite matrices and applications to Hadamard products*, Lin. Alg. Appl. **26**, 203.
[53] Andō T., Choi M.-D., 1986, *Non-linear completely positive maps*, in: Nagel R., Schlotterbeck U., Wolff M.P.H. (eds.), *Aspects of positivity in functional analysis*, North-Holland, Amsterdam, p.3.
[54] Antoine J.-P., 1966, *Formalisme de Dirac et problèmes de symétrie en mécanique quantique*, Ph.D. thesis, Université Catholique de Louvain, Louvain.
[55] Antoine J.-P., 1969, *Dirac formalism and symmetry problems in quantum mehcanics I. General Dirac formalism*, J. Math. Phys. **10**, 53.
[56] Antoine J.-P., 1969, *Dirac formalism and symmetry problems in quantum mehcanics II. Symmetry problems*, J. Math. Phys. **10**, 2276.
[57] Antonne L., 1902, *Sur l'hermitien*, Rend. Circ. Mat. Palermo **16**, 104.
[58] Ara P., Goldstein D., 1993, *A solution to the matrix problem for Rickart C^*-algebras*, Math. Nachr. **164**, 259.
[59] Araki H., 1962, *Einführung in die axiomatische Quantenfeldtheorie I, II*, lecture notes, Eidgenössische Technische Hochschule, Zürich.
[60] Araki H., 1963, *A lattice of von Neumann algebras associated with the quantum theory of a free Bose field*, J. Math. Phys. **4**, 1343.
[61] Araki H., 1964, *On the algebra of all local observables*, Prog. Theor. Phys. **32**, 844.
[62] Araki H., 1964, *Type of von Neumann algebra associated with free field*, Prog. Theor. Phys. **32**, 956.
[63] Araki H., 1964, *Von Neumann algebras of local observables for free scalar field*, J. Math. Phys. **5**, 1.
[64] Araki H., 1968, *Multiple time analyticity of a quantum statistical state satisfying the KMS boundary condition*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **4**, 361. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=4&iss=2&rank=7.
[65] Araki H., 1969, *Gibbs states of a one dimensional quantum lattice*, Commun. Math. Phys. **14**, 120. euclid:cmp/1103841726.
[66] Araki H., 1969, *Local quantum theory I*, in: Jost R. (ed.), *Local quantum theory*, Academic Press, New York, p.65.
[67] Araki H., 1972, *Bures distance function and a generalization of Sakai's noncommutative Radon–Nikodym theorem*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **8**, 335. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=8&iss=2&rank=6.
[68] Araki H., 1973, *Expansionals in Banach algebras*, Ann. Sci. École Norm. Sup. 4 ème sér. **6**, 67. numdam:ASENS_1973_4_6_1_67_0.pdf.
[69] Araki H., 1973, *Golden–Thompson and Peierls–Bogoliubov inequalities for a general von Neumann algebra*, Commun. Math. Phys. **34**, 167.
[70] Araki H., 1973, *Relative hamiltonian for faithful normal states of von Neumann algebra*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **9**, 165. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=9&iss=1&rank=9.
[71] Araki H., 1974, *On the equivalence of the KMS condition and the variational principle for quantum lattice systems*, Commun. Math. Phys. **38**, 1. euclid:cmp/1103859963.
[72] Araki H., 1974, *Some properties of the modular conjugation operator of von Neumann algebras and a non-commutative Radon–Nikodym theorem with a chain rule*, Pacific J. Math. **50**, 309. euclid:pjm/1102913224.
[73] Araki H., 1976, *Introduction to relative hamiltonian and relative entropy*, in: Guerra F., Robinson D.W., Stora R. (eds.), *Les methodes mathematiques de la theorie quantique des champs, Marseille, 23-27 juin 1975*, Colloques Internationaux C.N.R.S. **248**, Éditions du C.N.R.S., Paris, p.782.
[74] Araki H., 1976, *Positive cone, Radon–Nikodym theorems, relative hamiltonian and the Gibbs condition in statistical mechanics*, in: Kastler D. (ed.), *C^*-algebras and their applications to statistical mechanics and quantum field theory*, North-Holland, Amsterdam.
[75] Araki H., 1976, *Recent developments in the theory of operator algebras and their significance in theoretical physics*, in: *Convegno sulle algebre C^* e loro applicazioni in fisica teoria (12-18 Marzo 1975, Convegno sulla teoria degli operatori*

indice e teoria K (13-18 Ottobre 1975), del Istituto Nazionale di Alta Matematica, Roma, Symposia Mathematica **20**, Academic Press, New York, p.395.
[76] Araki H., 1976, *Relative entropy for states of von Neumann algebras I*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **11**, 809. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=11&iss=3&rank=9.
[77] Araki H., 1977, *Relative entropy for states of von Neumann algebras II*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **13**, 173. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=13&iss=1&rank=8.
[78] Araki H., 1978, *On KMS states of a C^*-dynamical system*, in: Araki H., Kadison R.V. (eds.), *C^*-algebras and applications to physics. Proceedings of Second Japan–USA seminar, Los Angeles, April 18-22, 1977*, LNM **650**, Springer, Berlin, p.66.
[79] Araki H., 1986, *A continuous superselection rule as a model of classical measuring apparatus in quantum mechanics*, in: Gorini V., Frigerio A. (eds.), *Fundamental aspects of quantum theory*, Plenum, New York, p.23.
[80] Araki H., 1987, *Recent progress on entropy and relative entropy*, in: Mebkhout M., Sénéor R. (eds.), *Proceedings of VIIIth International Congress on Mathematical Physics*, World Scientific, Singapore, p.354.
[81] Araki H., 1993, *Ryoshiba no suri*, Iwanami Shoten, Tōkyō (engl. transl. 1999, *Mathematical theory of quantum fields*, Int. Ser. Monogr. Phys. **101**, Oxford University Press, Oxford).
[82] Araki H., Elliott G., 1973, *On the definition of C^* algebras*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **9**, 93. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=9&iss=1&rank=4.
[83] Araki H., Haag R., 1967, *Collision cross sections in terms of local observables*, Commun. Math. Phys. **4**, 77. euclid:cmp/1103839810.
[84] Araki H., Masuda T., 1982, *Positive cones and L_p-spaces for von Neumann algebras*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **18**, 339. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=18&iss=2&rank=11.
[85] Araki H., Miyata H., 1968, *On KMS boundary condition*, Publ. Res. Inst. Math. Sci. Kyōto Univ. Ser. A **4**, 373. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=4&iss=2&rank=8.
[86] Araki H., Raggio G.A., 1982, *A remark on transition probability*, Lett. Math. Phys. **6**, 237.
[87] Araki H., Sewell G.L., 1977, *KMS conditions and local thermodynamical stability of quantum lattice systems*, Commun. Math. Phys. **52**, 103. euclid:cmp/1103900491.
[88] Araki H., Woods E.J., 1963, *Representations of the canonical commutation relations describing a non-relativistic infinite free Bose gas*, J. Math. Phys. **4**, 637.
[89] Araki H., Woods E.J., 1968, *A classification of factors*, Publ. Res. Inst. Math. Sci. Kyōto Univ. Ser. A **4**, 51.
[90] Arazy J., 1975, *The isometries of C_p*, Israel J. Math. **22**, 247.
[91] Arveson W., 1976, *An invitation to C^*-algebras*, GTM **39**, Springer, Berlin.
[92] Arwini K.A., Dodson C.T.J., 2008, *Information geometry: near randomness and near independence*, LNM **1953**, Springer, Berlin.
[93] Aschbacher W., Jakšić V., Pautrat Y., Pillet C.-A., 2006, *Topics in non-equilibrium quantum statistical mechanics*, in: Attal s., Joye A., Pillet C.-A. (eds.), *Open quantum systems III: recent developments*, LNM **1882**, Springer, Berlin, p.1.
[94] Atkinson C., Mitchell A.F.S., 1981, *Rao's distance measure*, Sankhyā **43**, 345.
[95] Ay N., Jost J., Lê H.V., Schwachhöfer L., 2012, *Information geometry and sufficient statistics*, arXiv:1207.6736.
[96] Ayupov Sh.A., Chilin V.I., Abdullaev R.Z., 2012, *Orlicz spaces associated with a semi-finite von Neumann algebra*, Comment. Math. Univ. Carolinae **53**, 519. arXiv:1108.3267.
[97] Baaj S., Jungl P., 1983, *Théorie bivariante de Kasparov et opérateurs non bornés dans les C^*-modules hilbertiens*, Compt. Rend. Acad. Sci. Paris **296**, 875.
[98] Bacciagaluppi G., 2008, *The statistical interpretation according to Born and Heisenberg*, in: Joas C., Lehner C., Renn J. (eds.), *HQ-1: conference on the history of quantum physics*, Max-Planck-Institut für Wissenschaftsgeschichte, Berlin, p.269.
[99] Bade W.G., 1971, *The Banach space $C(S)$*, Aarhus Universitet, Lecture Notes No. **26**, Aarhus.
[100] Bähtold M., 2008, *Five formulations of the quantum measurement problem in the frame of the standard interpretation*, J. Gen. Phil. Sci. **39**, 17.
[101] Balian R., Alhassid Y., Reinhardt H., 1986, *Dissipation in many-body systems: a geometrical approach based on information theory*, Phys. Rep. **131**, 1.
[102] Banach S., 1922, *Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales*, Fund. Math. **3**, 133.
[103] Banach S., 1929, *Sur les fonctionnelles linéaires*, Studia Math. **1**, 211, 223.
[104] Banach S., 1931, *Teoria operacyj, Tom I, Operacje liniowe*, Wydawnictwo Kasy Mianowskiego, Warszawa (fr. transl. rev. ed., 1932, *Théorie des opérations linéaires*, Monografie Matematyczne **1**, Waszawa 1932. Available at: webs.um.es/joseori/documentos/analisisFuncional/preliminares/Banach-book.pdf; engl. transl. 1987, *Theory of linear operations*, North-Holland, Amsterdam).
[105] Banaschewski B., 1976, *The duality between M-spaces and compact Hausdorff spaces*, Math. Nachr. **75**, 41.
[106] Banerjee A., Guo X., Wang H., 2005, *On the optimality of conditional expectation as a Bregman predictor*, IEEE Trans. Inf. Theor. **51**, 2664.
[107] Banks D., 1993, *A conversation with I.J. Good*, Statist. Sci **11**, 1.
[108] Bannier U., 1987, *Allgemein kovariante algebraische Quantenfeldtheorie und Rekonstruktion von Raum-Zeit*, Ph.D. Thesis, Hamburg University, Hamburg.
[109] Barchielli A., Gregoratti M., 2009, *Quantum trajectories and measurements in continuous time. The diffusive case*, LNP **782**, Springer, Berlin.
[110] Barchielli A., Lupieri G., 2006, *Instruments and mutual entropies in quantum information*, in: Bożejko M., Młotkowski W., Wysoczański J. (eds.), *Quantum probability*, Banach Center Publications **73**, Instytut Matematyki Polskiej Akademii Nauk, Warszawa, p.65. arXiv:quant-ph/0412116.
[111] Barndorff-Nielsen O.E., 1978, *Information and exponential families in statistical theory*, Wiley, New York.
[112] Barndorff-Nielsen O.E., 1988, *Parametric statistical models and likelihood*, Lecture Notes in Statistics **50**, Springer, Berlin.
[113] Barndorff-Nielsen O.E., Gill R.D., Jupp P.E., 2003, *On quantum statistical inference (with discussion)*, J. Roy. Statist. Soc. B **65**, 775. arXiv:quant-ph/0307189, arXiv:quant-ph/0307191.
[114] Barnett N.S., Carone P., Dragomir S.S., Sofo A., 2002, *Approximating Csiszár f-divergence by the use of Taylor's formula with integral reminder*, Math. Ineq. Appl. **5**, 417. Available at: rgmia.org/papers/Csiszar/ACfDTFIRInt2.pdf.
[115] Barnum H., Duncan R., Wilce A., 2010, *Symmetry, compact closure and dagger compactness for categories of convex operational models*, arXiv:1004.2920.
[116] Barnum H., Wilce A., 2009, *Ordered linear spaces and categories as frameworks for information-processing characterizations*

of quantum and classical theory, arXiv:0908.2354.
[117] Barr M., 1979, *∗-autonomous categories*, LNM **752**, Springer, Berlin.
[118] Barut A.O., Rączka R., 1977, *Theory of group representations and applications*, Polskie Wydawnictwo Naukowe, Warszawa (2nd rev. ed., 1980).
[119] Basu A., Shioya H., Park C., 2011, *Statistical inference: the minimum distance approach*, Chapman & Hall, London.
[120] Bauer H., 1958, *Minimalstellen von Funktionen und Extremalpunkte*, Arch. Math. **9**, 389.
[121] Bauer H., 1961, *Schilowsche Rand und Dirichletsches Problem*, Ann. Inst. Fourier Grenoble **11**, 89.
[122] Baumgärtel H., 1995, *Operatoralgebraic methods in quantum field theory*, Akademie Verlag, Berlin.
[123] Baumgärtel H., Wollenberg M., 1992, *Causal nets of operator algebras: mathematical aspects of algebraic quantum field theory*, Akademie Verlag, Berlin.
[124] Bauschke H.H., 2003, *Duality for Bregman projections onto translated cones and affine subspaces*, J. Approx. Theor. **121**, 1.
[125] Bauschke H.H., Borwein J.M., 1997, *Legendre functions and the method of random Bregman projections*, J. Conv. Anal. **4**, 27. Available at: www.emis.de/journals/JCA/vol.4_no.1/j86.ps.gz.
[126] Bauschke H.H., Borwein J.M., 2001, *Joint and separate convexity of the Bregman distance*, Stud. Comp. Math. **8**, 23. Available at: people.ok.ubc.ca/bauschke/Research/c04.ps.
[127] Bauschke H.H., Borwein J.M., Combettes P.L., 2001, *Essential smoothness, essential strict convexity, and Legendre functions in Banach spaces*, Commun. Cont. Math. **3**, 615.
[128] Bauschke H.H., Borwein J.M., Combettes P.L., 2003, *Bregman monotone optimization algorithms*, Soc. Industr. Appl. Math. J. Contr. Optim. **42**, 596.
[129] Bauschke H.H., Combettes P.L., 2003, *Constructions of best Bregman approximations in reflexive Banach spaces*, Proc. Amer. Math. Soc. **131**, 3757.
[130] Bauschke H.H., Lewis A.S., 2000, *Dykstra's algorithm with Bregman projections: a convergence proof*, Optimization **48**, 409.
[131] Bayer W., Ochs W., 1973, *Quantum states with maximum information entropy I, II*, Z. Naturf. **28a**, 693.
[132] Bayes T., 1763, *An essay towards solving a problem in the doctrine of chances*, Phil. Trans. Roy. Soc. London **53**, 370 (reprinted in: 1958, Biometrika **45**, 293).
[133] Belinfante J.G.F., 1976, *Transition probability spaces*, J. Math. Phys. **17**, 285.
[134] Bell J.L., 1996, *Logical reflection on the Kochen–Specker theorem*, in: Clifton R. (ed.), *Perspectives on quantum reality*, Kluwer, Dordrecht, p.227.
[135] Beltiţă D., Ratiu T.S., 2005, *Symplectic leaves in real Banach Lie–Poisson spaces*, Geom. Funct. Anal. **15**, 753.
[136] Beltiţă D., Ratiu T.S., Tumpach A.B., 2008, *Induced and coinduced Banach Lie–Poisson spaces and integrability*, J. Funct. Anal. **225**, 1225.
[137] Beltrametti E.G., Cassinelli G., 1981, *The logic of quantum mechanics*, Addison-Wesley, Reading.
[138] Bendat J., Sherman S., 1955, *Monotone and convex operator functions*, Trans. Amer. Math. Soc. **79**, 58.
[139] Bengtsson I., Życzkowski K., 2006, *Geometry of quantum states: an introduction to quantum entanglement*, Cambridge University Press, Cambridge.
[140] Benoist R.W., Marchand J.-P., 1979, *Statistical inference in coupled quantum systems*, Lett. Math. Phys. **3**, 93.
[141] Benoist R.W., Marchand J.-P., Wyss W., 1979, *A note on relative entropy*, Lett. Math. Phys. **3**, 169.
[142] Benoist R.W., Marchand J.-P., Yourgrau W., 1977, *Statistical inference and quantum mechanical measurement*, Found. Phys. **7**, 827 (addendum: 1978, Found. Phys. **8**, 117).
[143] Beran L., 1985, *Orthomodular lattices, algebraic approach*, Reidel, Dordrecht.
[144] Beran R., 1977, *Minimum Hellinger distance estimates for parametric models*, Ann. Statist. **5**, 445.
[145] Berberian S.K., 1972, *Baer ∗-rings*, Springer, Berlin (2nd ed. 2011).
[146] Bercher J.-F., 1995, *Développement de critères de nature entropique pour la résolution des problèmes inverses linéaires*, Ph.D. thesis, Université de Paris-Sud, Orsay.
[147] Berezanskiĭ Yu.M., 1965, *Razlozhenie po sobstvennym funkciyam samosopryazhennykh operatorov*, Naukova Dumka, Kiev (engl. transl. 1968, *Expansions in eigenfunctions of selfadjoint operators*, American Mathematical Society, Providence).
[148] Berger J.O., 1985, *Statistical decision theory and bayesian analysis*, 2nd ed., Springer, Berlin.
[149] Beukers F., Huijsmans C.B., de Pagter B., 1983, *Unital embedding and complexification of f-algebras*, Math. Z. **183**, 131.
[150] Bezhanishvili G., 2010, *Stone duality and Gleason covers through de Vries duality*, Topol. Appl. **157**, 1064. Available at: sierra.nmsu.edu/gbezhani/deVries.pdf.
[151] Bhatia R., 1997, *Matrix analysis*, Springer, Berlin.
[152] Bhattacharyya A., 1943, *On a measure of divergence between two statistical populations defined by their probability distributions*, Bull. Calcutta Math. Soc. **35**, 99.
[153] Bhattacharyya A., 1943, *On discrimination and divergence*, in: **?**, *Proceedings of 29th Indian Scientific Congress*, part **III**, p.13.
[154] Bhattacharyya A., 1946, *On a measure of divergence between two multinomial populations*, Sankhyā **7**, 401.
[155] Bhattacharyya A., 1946, *On some analogues to the amount of information and their uses in statistical estimation*, Sankhyā **8**, 1.
[156] Biane P., 1995, *Calcul stochastique non-commutatif*, in: Biane P., Durrett R. (eds.), *Lectures on probability theory. Ecole d'eté de probabilités de Saint-Flour XXIII–1993*, LNM **1608**, Springer, Berlin, p.1.
[157] Bichteler K., 1998, *Integration: a functional approach*, Birkhäuser, Basel.
[158] Birke L., Fröhlich J., 2002, *KMS, etc.*, Rev. Math. Phys. **14**, 829. arXiv:math-ph/0204023.
[159] Birkhoff G., 1938, *Dependent probabilities and spaces* (*L*), Proc. Nat. Acad. Sci. U.S.A. **24**, 154.
[160] Birkhoff G., 1940, *Lattice theory*, American Mathematical Society, Providence.
[161] Birkhoff G., Pierce R.S., 1956, *Lattice-ordered rings*, An. Acad. Brasil. Ciênc. **28**, 41.
[162] Birkhoff G., von Neumann J., 1936, *The logic of quantum mechanics*, Ann. Math. **37**, 823.
[163] Birnbaum Z.W., Orlicz W., 1931, *Über die Verallgemeinerung des Begriffes der zueinander konjugierten Potenzen*, Studia Math. **3**, 1. Available at: matwbn.icm.edu.pl/ksiazki/sm/sm3/sm311.pdf.
[164] Bishop E., de Leeuw K., 1959, *The representation of linear functionals by measures on sets of extreme points*, Ann. Inst. Fourier Grenoble **9**, 305.
[165] Bishop R.L., Crittenden R.J., 1964, *Geometry of manifolds*, Academic Press, New York.
[166] Blackadar B., 2005, *Operator algebras: theory of C^*-algebras nad von Neumann algebras*, Springer, Berlin.
[167] Blackwell D.A., 1951, *Comparison of experiments*, in: Neyman J. (ed.), *Proceedings of the Second Berkeley Symposium on*

Mathematical Statistics and Probability, University of California Press, Berkeley, p.93. euclid:bsmsp/1200500222.
[168] Blackwell D.A., 1953, *Equivalent comparisons of experiments*, Ann. Math. Statist. **24**, 265.
[169] Blackwell D.A., Girshick M.A., 1954, *Theory of games and statistical decisions*, Wiley, New York.
[170] Bloch F., 1932, *Zur Theorie des Austauschproblems und der Remanenzerscheinung der Ferromagnetika*, Z. Phys. **74**, 295.
[171] Bochner S., 1933, *Integration von Funktionen, deren Werte die Elemente eines Vektorraumes sind*, Fund. Math. **20**, 262.
[172] Boţ R.I., 2010, *Conjugate duality in convex optimization*, Springer, Berlin.
[173] Bodiou G., 1964, *Théorie dialectique des probabilités*, Gauthier–Villars, Paris.
[174] Bogachev V.I., 2007, *Measure theory*, Vol.1-2, Springer, Berlin.
[175] Bogolyubov N.N., 1961, *Kvazisrednie v zadachakh statisticheskoĭ mekhaniki*, D-**871**, Ob'yedinyennyĭ Institut Yadernykh Issledovaniĭ, Laboratoriya Teoreticheskoĭ Fiziki, Dubna (germ. transl. 1962, *Quasimittelwerte in Problemen der Statistischen Mechanik I, II*, Physik. Abhandl. Sowjetunion **6**, 1, 229).
[176] Bogolyubov N.N., Logunov A.A., Oksak A.I., Todorov I.T., 1987, *Obshhie principy kvantovoĭ teorii polya*, Nauka, Moskva (2nd ed. 2005; engl. transl. 1990, *General principles of quantum field theory*, Dordrecht, Kluwer).
[177] Bogolyubov N.N., Logunov A.A., Todorov I.T., 1969, *Osnovy aksiomaticheskogo podkhoda v kvantovoĭ teorii polya*, Nauka, Moskva (engl. transl. 1975, *Introduction to axiomatic quantum field theory*, Benjamin, Reading).
[178] Böhm A., 1964, *The rigged Hilbert space in quantum physics*, ICTP Publ. **64/9**, International Center for Theoretical Physics, Trieste.
[179] Böhm A., 1966, *Rigged Hilbert space and mathematical description of physical systems*, Physica A **236**, 485.
[180] Böhm A., 1979, *Quantum mechanics*, Springer, Berlin (2nd ed. 1986).
[181] Böhm A., Gadella M., 1989, *Dirac kets, Gamov vector, and Gelfand triplets*, LNP **348**, Springer, Berlin.
[182] Böhm A., Uncu H., Komy S., 2009, *A brief survey of the mathematics of quantum physics*, Rep. Math. Phys. **64**, 5.
[183] Bohm D., Hiley B., 1993, *The undivided universe: an ontological interpretation of quantum theory*, Routledge & Kegan, London.
[184] Bohnenblust F., 1940, *An axiomatic characterisation of L_p spaces*, Duke Math. J. **6**, 627.
[185] Bohnenblust F., Kakutani S., 1941, *Concrete representation of abstract (M)-spaces*, Ann. Math. **42**, 1025.
[186] Bomze I.M, 1990, *A functional analytic approach to statistical experiments*, Longman, Harlow.
[187] Bomze I.M, 2003, *On the structure of undominated statistical experiments: sufficiency, invariance, and optimality in unbiased estimation*, Tatra Mt. Math. Publ. **26**, 255.
[188] Bóna P., 1991, *Quantum mechanics with mean-field backgrounds*, Faculty of Mathematics and Physics, Physics Preprint No. **Ph10-91**, Department of Theoretical Physics, Comenius University, Bratislava. Available at: sophia.dtp.fmph.uniba.sk/∼bona/preprint%20Ph10-91.html.
[189] Bóna P., 1993, *On nonlinear quantum mechanics*, in: Kowalski O., Krupka D. (eds.), *Differential geometry and its applications*, Slezská Univerzita, Opava, p.185. Available at: sophia.dtp.fmph.uniba.sk/∼bona/NLQM-Opava1992.pdf.
[190] Bóna P., 2000, *Extended quantum mechanics*, Acta Phys. Slov. **50**, 1. arXiv:math-ph/9909022.
[191] Bonsall F.F., Ducan J., 1973, *Complete normed algebras*, Springer, Berlin.
[192] Boole G., 1847, *The mathematical analysis of logic*, Cambridge.
[193] Boole G., 1854, *An investigation of the laws of thought on which are founded the mathematical theories of logic and probabilities*, Macmillan, London.
[194] Borchers H.-J., 1960, *Über die Mannigfaltigkeit der interpolierenden Felder zu einer kausalen S-Matrix*, Nuovo Cim. **15**, 784.
[195] Borchers H.-J., 1962, *On the structure of the algebra of field operators*, Nuovo Cim. **24**, 214.
[196] Borchers H.-J., 1983, *C^*-algebras and automorphism groups*, Commun. Math. Phys. **88**, 95.
[197] Borchers H.-J., 1983, *Symmetries and covariant representations*, in: Blanchard P., Streit L. (eds.), *Dynamics and processes. Proceedings of the Third Encounter in Mathematics and Physics, held in Bielefeld, Germany Nov. 30 – Dec. 4, 1981*, LNM **1031**, Springer, Berlin, p.1.
[198] Borchers H.-J., 1998, *Half-sided translations and the type of von Neumann algebras*, Lett. Math. Phys. **44**, 283.
[199] Borchers H.-J., 2000, *On revolutionizing of quantum field theory with Tomita's modular theory*, J. Math. Phys. **41**, 3604. Available as E. Schrödinger Inst. Math. Phys. preprint 731.
[200] Borel É., 1898, *Leçons sur la théorie des fonctions*, Gauthier–Villars, Paris (4th ed. 1950).
[201] Borel É., 1905, *Remarques sur certaines questions de probabilité*, Bull. Soc. Math. France **33**, 123.
[202] Borel É., 1909, *Les probabilités dénomerables et leurs applications arithmétiques*, Rend. Circ. Math. Palermo **27**, 247.
[203] Born M., 1926, *Das Adiabatenprinzip in der Quantenmechanik*, Z. Phys. **40**, 167.
[204] Born M., 1926, *Quantenmechanik der Stoßvorgänge*, Z. Phys. **38**, 803.
[205] Born M., 1926, *Zur Quantenmechanik der Stoßvorgänge*, Z. Phys. **37**, 863 (engl. trans. 1983, *On the quantum mechanics of collisions*, in: Wheeler J.A., Żurek W.H. (eds.), *Quantum theory and measurement*, Princeton University Press, Princeton).
[206] Born M., Heisenberg W., 1928, *La méchanique des quanta*, in: *Électrons et photons. Rapports et discussions du cinquième conseil de physique tenu à Bruxelles du 24 am 29 Octobre 1927 sons les auspices de l'Institut International de Physique Solvay*, Gauthier-Villars, Paris, p.143 (engl. transl. 2008, *Quantum mechanics*, in: Bacciagaluppi G., Valentini A., *Quantum theory at crossroads: reconsidering the 1927 Solvay conference*, Cambridge University Press, Cambridge, p.407, arXiv:quant-ph/0609184).
[207] Born M., Heisenberg W., Jordan P., 1926, *Zur Quantenmechanik II*, Z. Phys. **35**, 557.
[208] Born M., Jordan P., 1925, *Zur Quantenmechanik*, Z. Phys. **34**, 858.
[209] Borwein J.M., 1993, *On the failure of maximum entropy reconstruction for Fredholm equations and other infinite systems*, Math. Progr. **61**, 251.
[210] Borwein J.M., Lewis A.S., 1991, *Convergence of best entropy estimates*, Soc. Industr. Appl. Math. J. Optim. **1**, 191.
[211] Borwein J.M., Lewis A.S., 1991, *Duality relationships for entropy-like minimization problems*, Soc. Industr. Appl. Math. J. Contr. Optim. **29**, 325.
[212] Borwein J.M., Lewis A.S., 1991, *Moment-matching and best entropy estimation*, J. Math. Anal. Appl. **185**, 596.
[213] Borwein J.M., Lewis A.S., 1991, *On the convergence of moment problems*, Trans. Amer. Math. Soc. **325**, 249.
[214] Borwein J.M., Lewis A.S., 1992, *Decomposition of multivariate functions*, Can. J. Math. **44**, 463.
[215] Borwein J.M., Lewis A.S., 1992, *Partially-finite convex programming, part I: quasi relative interiors and duality theory*, Math. Prog. **57**, 15.
[216] Borwein J.M., Lewis A.S., 1992, *Partially-finite convex programming, part II: explicit lattice models*, Math. Prog. **57**, 49.
[217] Borwein J.M., Lewis A.S., 1993, *Partially-finite programming in L_1 and the existence of maximum entropy estimates*, Soc. Industr. Appl. Math. J. Optim. **3**, 248.

[218] Borwein J.M., Lewis A.S., 1993, *A survey of convergence results for maximum entropy methods*, in: Mohammad-Djafari A., Demoments G. (eds.), *Maximum entropy and bayesian methods*, Kluwer, Dordrecht, p.39.
[219] Borwein J.M., Lewis A.S., 1994, *Strong rotundity and optimization*, Soc. Industr. Appl. Math. J. Optim. **4**, 146.
[220] Borwein J.M., Lewis A.S., 2000, *Convex analysis and nonlinear optimization: theory and examples*, Springer, Berlin (2005, 2nd rev. ed.).
[221] Borwein J.M., Lewis A.S., Noll D., 1996, *Maximum entropy spectral analysis using derivative information. Part I: Fisher information and convex duality*, Math. Oper. Res. **21**, 442.
[222] Borwein J.M., Lewis A.S., Nussbaum R.D., 1994, *Entropy minimization, DAD problems and doubly stochastic kernels*, J. Funct. Anal. **123**, 264.
[223] Borwein J.M., Limber M.A., 1996, *On entropy maximization via convex programming*, preprint, available at: old-web.cecm.sfu.ca/projects/MomEnt+/ent_max.ps.gz.
[224] Borwein J.M., Vanderwerff J.D., 2001, *Convex functions of Legendre type on Banach spaces*, J. Conv. Anal. **8**, 569.
[225] Borwein J.M., Vanderwerff J.D., 2010, *Convex functions: constructions, characterizations and counterexamples*, Cambridge University Press, Cambridge.
[226] Borwein J.M., Vanderwerff J.D., 2010, *Fréchet–Legendre functions and reflexive Banach spaces*, J. Conv. Anal. **17**, 915.
[227] Borwein J.M., Zhu Q.J., 2005, *Techniques of variational analysis*, Springer, Berlin.
[228] Bös W., 1978, *A classification for selfdual cones in Hilbert space*, Arch. Math. **30**, 75.
[229] Bostelmann H., 2000, *Lokale Algebren und Operatorprodukte am Punkt*, PhD thesis, Georg-August-Universtität Göttingen. Available at: webdoc.sub.gwdg.de/diss/2000/bostelmann/produkte.pdf.
[230] Bostelmann H., 2005, *Operator product expansions as a consequence of phase space properties*, J. Math. Phys. **46**, 082304.
[231] Bostelmann H., 2005, *Phase space properties and the short distance structure in quantum field theory*, J. Math. Phys. **46**, 052301. arXiv:math-ph/0409070.
[232] Bostelmann H., D'Antoni C., Morsella G., 2009, *Scaling algebras and pointlike fields: A nonperturbative approach to renormalization*, Commun. Math. Phys. **285**, 763. arXiv:0711.4237.
[233] Bourbaki N., 1938, *Sur les espaces de Banach*, Compt. Rend. Acad. Sci. Paris **206**, 1701.
[234] Bourbaki N., 1952, 1956, 1959, 1963, 1969, *Intégration*, Ch.I-IV, V, VI, VII-VIII, IX, Actualités Scientifiques et Industrielles **1175**, **1244**, **1306**, **1364**, **1343**, Hermann, Paris.
[235] Bourbaki N., 1972, *Groupes et algèbres de Lie*, Ch.III, Hermann, Paris.
[236] Boyd S., Vandenberghe L., 2004, *Convex optimization*, Cambridge University Press, Cambridge.
[237] Bratteli O., 1986, *Derivations, dissipations and group actions on C^*-algebras*, LNM **1229**, Springer, Berlin.
[238] Bratteli O., Robinson D.W., 1976, *Unbounded derivations of C^*-algebras II*, Commun. Math. Phys. **46**, 11.
[239] Bratteli O., Robinson D.W., 1976, *Unbounded derivations of von Neumann algebras*, Ann. Inst. Henri Poincaré A **25**, 139.
[240] Bratteli O., Robinson D.W., 1979, 1981, *Operator algebras and quantum statistical mechanics*, Vol.1-2, Springer, Berlin (2nd ed., 1987, 1997).
[241] Braustein S.L., Caves C.M., 1994, *Statistical distance and the geometry of quantum states*, Phys. Rev. Lett. **72**, 3439.
[242] Bregman L.M., 1967, *Relaksacionnyĭ metod nakhozdheniya obshheĭ tochki vypuklykh mnozhestv i ego primenenie dla zadach vypuklogo programmirovaniya*, Zh. vychestl. matem. matem. fiz. **7**, 620. (engl. transl.: 1967, *The relaxation method for finding common points of convex sets and its application to the solution of problems in convex programming*, USSR Comput. Math. Math. Phys. **7**, 200).
[243] Broise M.M., 1966, *Sur les isomorphismes de certaines algèbres de von Neumann*, Ann. Sci. École Norm. Sup. **83**, 91.
[244] Broniatowski M., Keziou A., 2006, *Minimization of ϕ-divergences on sets of signed measures*, Studia Sci. Math. Hungar. **43**, 403. arXiv:1003.5457.
[245] Brønsted A., 1964, *Conjugate convex functions in topological vector spaces*, Mat. Fys. Medd. Dansk. Vid. Selsk. **34**, 1.
[246] Brønsted A., Rockafellar R.T., 1965, *On the subdifferentiability of convex functions*, Proc. Amer. Math. Soc. **16**, 605. Available at: www.math.washington.edu/∼rtr/papers/rtr-Subdifferentiability.pdf.
[247] Brown J.R., 1965, *Doubly stochastic measures and Markov operators*, Michigan Math. J. **12**, 367.
[248] Brown L.D., 1986, *Fundamentals of statistical exponential families with applications in statistical decision theory*, Institute of Mathematical Statistics, Hayward.
[249] Bub J., 1977, *Von Neumann's projection postulate as a probability conditionalization rule in quantum mechanics*, J. Phil. Logic **6**, 381.
[250] Bub J., 1979, *Conditional probabilities in non-boolean possibility structures*, in: Hooker C.A. (ed.), *The logico-algebraic approach to quantum mechanics*, Vol.2, Reidel, Dordrecht, p.209.
[251] Bub J., 1979, *The measurement problem in quantum mechanics*, in: Toraldo Di Francia G. (ed.), *Problems in the foundations of physics*, North-Holland, Amsterdam, p.71.
[252] Bub J., 1992, *Quantum mechanics without the projection postulate*, Found. Phys. **22**, 737.
[253] Bub J., Pitowsky I., 2009, *Two dogmas about quantum mechanics*, in: Saunders S. et al. (eds.), *Everett 50, Proceedings of the conferences held in Oxford and the Perimeter Institute*, Oxford University Press, Oxford.
[254] Buchholz D., D'Antoni C., Fredenhagen K., 1987, *The universal structure of local algebras*, Comm. Math. Phys. **111**, 123.
[255] Buchholz D., Haag R., 2000, *The quest for understanding in relativistic quantum physics*, J. Math. Phys. **41**, 3674. arXiv:hep-th/9910243.
[256] Bugajska K., Bugajski S., 1973, *The lattice structures of quantum logics*, Ann. Inst. Henri Poincaré **19**, 333.
[257] Bugajski S., 1976, *Joint probabilities in quantum mechanics*, Unpublished typescript, Uniwersytet Śląski, Katowice.
[258] Bunce L.J., Hamhalter J., 1994, *Jauch–Piron states on von Neumann algebras*, Math. Z. **215**, 491.
[259] Bunce L.J., Wright J.D.M., 1992, *Complex measures on projections in von Neumann algebras*, J. London Math. Soc. **46**, 269.
[260] Bunce L.J., Wright J.D.M., 1992, *The Mackey–Gleason problem*, Bull. Amer. Math. Soc. **26**, 288. arXiv:math/9204228.
[261] Bunce L.J., Wright J.D.M., 1994, *The Mackey–Gleason problem for vector measures on projections in von Neumann algebras*, J. London Math. Soc. **49**, 133.
[262] Burbea J., Rao R.C., 1982, *Entropy differential metric, distance and divergence measures in probability spaces: a unified approach*, J. Multivar. Anal. **12**, 575.
[263] Burbea J., Rao R.C., 1984, *Differential metrics in probability spaces*, Prob. Math. Statist. **3**, 241.
[264] Burdet G., Combe P., Nencka H., 2001, *On real hilbertian info-manifolds*, in: Sollich P., Hughston L.P., Streater R.F., Coolen A.C.C. (eds.), *Disordered and complex systems (King's College, London, UK, 10-14 July 2000)*, AIP Conf. Proc. **553**, AIP, Melville, p.153.
[265] Bures D.J.C., 1969, *An extension of Kakutani's theorem on infinite product measures to the tensor product of semiinfinite*

W^* *algebras*, Trans. Am. Math. Soc. **135**, 194.
[266] Burg J., 1975, *Maximum entropy spectral analysis*, Ph.D. thesis, Stanford University, Stanford.
[267] Burkholder D.L., 1961, *Sufficiency in the undominated case*, Ann. Math. Statist. **32**, 1191.
[268] Busch P., Grabowski M., Lahti P.J., 1989, *Some remarks on effects, operations, and unsharp measurements*, Found. Phys. Lett. **2**, 331.
[269] Busch P., Grabowski M., Lahti P.J., 1995, *Operational quantum physics*, LNPm **31**, Springer, Berlin. (1997, 2nd corr. print.).
[270] Busch P., Lahti P.J., Mittelstaedt P., 1991, *The quantum theory of measurement*, LNPm **2**, Springer, Berlin. (1996, 2nd rev. ed.).
[271] Butnariu D., Byrne C., Censor Y., 2003, *Redundant axioms in the definition of Bregman functions*, J. Conv. Anal. **10**, 245. Available at: math.haifa.ac.il/dbutnaru/publications/bbc.ps.
[272] Butnariu D., Censor Y., Reich S., 1997, *Iterative averaging of entropic projections for solving stochastic convex feasibility problems*, Comput. Optim. Appl. **8**, 21.
[273] Butnariu D., Iusem A.N., 2000, *Totally convex functions for fixed point computation and infinite dimensional optimization*, Kluwer, Dordrecht.
[274] Butnariu D., Iusem A.N., Zălinescu C., 2003, *On uniform convexity, total convexity and convergence of the proximal point and outer Bregman projection algorithms in Banach spaces*, J. Conv. Anal. **10**, 35.
[275] Butnariu D., Resmerita E., 2006, *Bregman distances, totally convex functions, and a method for solving operator equations in Banach spaces*, Abstr. Appl. Anal. **2006**, 84919. Available at: www.emis.de/journals/HOA/AAA/Volume2006/84919.pdf.
[276] Cai L., Hansen F., 2010, *Metric-adjusted skew information: convexity and restricted forms of superadditivity*, Lett. Math. Phys. **93**, 1.
[277] Calderón A.P., 1964, *Intermediate spaces and interpolation, the complex method*, Studia Math. **24**, 113.
[278] Campbell L.L., 1985, *The relation between information theory and the differential geometry approach to statistics*, Inf. Sci. (NY) **35**, 199.
[279] Campbell L.L., 1986, *An extended Čencov characterization of the information metric*, Proc. Amer. Math. Soc. **98**, 135.
[280] Canderolo D., Volčič A., 2002, *Radon–Nikodým theorems*, in: Pap E. (ed.), *Handbook of measure theory*, Vol.1, North-Holland, Amsterdam, p.249.
[281] Cantoni V., 1975, *Generalized "transition probability"*, Comm. Math. Phys. **44**, 125. euclid:cmp/1103899296.
[282] Cantoni V., 1977, *Intrinsic geometry of the quantum-mechanical phase space, hamiltonian systems and correspondence principle*, Rend. Accad. Naz. Lincei, **62**, 628.
[283] Cantoni V., 1977, *The riemannian structure on the states of quantum-like systems*, Comm. Math. Phys. **56**, 189. euclid:cmp/1103901145.
[284] Cantoni V., 1982, *Generalized transition probability, mobility and symmetries*, Commun. Math. Phys. **87**, 153.
[285] Carathéodory C., 1938, *Entwurf für eine Algebraisierung des Integralbegriffs*, Sitzungsber. Math.-Naturw. Kl. Bayer. Akad. Wiss. München **1938**, 27.
[286] Carathéodory C., 1956, *Mass und Integral und ihre Algebraisierung*, Lehrbücher und Monographien aus dem Gebiete der exakten Wissenschaften, Math. Reihe, Bd. **10**, Birkhäuser, Basel (engl. transl. 1963, *Algebraic theory of measure and integration*, Chelsea, New York; 2nd ed. 1986).
[287] Carleman T., 1923, *Sur les équations intégrales singulières à noyau réel et symmétrique*, Uppsala Universitets Årsskrift, Uppsala.
[288] Carrillo M.D., 2002, *Daniell integral and related topics*, in: Pap E. (ed.), *Handbook of measure theory*, Vol.1, North-Holland, Amsterdam, p.503.
[289] Cartan H., 1922, *Sur une généralisation de la notion de courbure de Riemann et les espaces à torsion*, Compt. Rend. Acad. Sci. Paris **174**, 593.
[290] Cartan H., 1923, *Sur les variétés à connexion affine et la théorie de la relativité généralisée*, Ann. École Norm. Sup. **40**, 325 (engl. transl. 1986, *On manifolds with an affine connection and the theory of general relativity*, Bibliopolis, Napoli).
[291] Cartan H., 1937, *Sur la mesure de Haar*, Compt. Rend. Acad. Sci. Paris **211**, 759.
[292] Caticha A., Giffin A., 2006, *Updating probabilities*, in: Mohammad-Djafari A. (ed.), *Bayesian inference and maximum entropy methods in science and engineering*, AIP Conf. Proc. **872**, 31. arXiv:physics/0608185.
[293] Cattaneo G., Dalla Chiara M.L., Giuntini R., Paoli F., 2009, *Quantum logic and nonclassical logics*, in: Engesser K., Gabbay D.M., Lehmann D. (eds.), *Handbook of quantum logic and quantum structures: quantum logic*, Elsevier, Amsterdam, p.127.
[294] Cattaneo G., Laudisa F., 1994, *Axiomatic unsharp quantum theory (from Mackey to Ludwig and Piron)*, Found. Phys. **24**, 631.
[295] Cattaneo G., Nisticò G., 1991, *Axiomatic foundations of quantum physics: critiques and misunderstandings. Piron's question-proposition system*, Int. J. Theor. Phys. **30**, 1293.
[296] Cattaneo G., Nisticò G., 1993, *A model of Piron's preparation-question structures in Ludwig's selection structures*, Int. J. Theor. Phys. **32**, 407.
[297] Caves C.M., Fuchs C.A., Schack R., 2001, *Quantum probabilities as bayesian probabilities*, Phys. Rev. A **65**, 022305. arXiv:quant-ph/0106133.
[298] Caves C.M., Fuchs C.A., Schack R., 2006, *Subjective probability and quantum certainty*, arXiv:quant-ph/0608190.
[299] Cecchini C., 1978, *On two definitions of measurable and locally measurable operators*, Bollettino U.M.I. A **15**, 526. Available at: dmitripavlov.org/scans/cecchini.pdf.
[300] Cecchini C., 1984, *Non commutative integration and conditioning*, in: Accardi L., Frigerio A., Gorini V. (eds.), *Quantum probability and applications to the quantum theory of irreversible processes. Proceedings of the International Workshop held at Villa Mondragone, Italy, September 6-11, 1982*, LNM **1055**, Springer, Berlin, p.76.
[301] Cecchini C., 1985, *Non commutative L^p spaces and K.M.S. functions*, in: Accardi L., von Waldenfels W. (eds.), *Quantum probability and applications II. Proceedings of a workshop held in Heidelberg, West Germany, October 1-5, 1984*, LNM **1136**, Springer, Berlin, p.136.
[302] Cecchini C., 1986, *Noncommutative integration for states on von Neumann algebras*, J. Oper. Theor. **15**, 217.
[303] Cecchini C., 1988, *Some noncommutative Radon–Nikodym theorems for von Neumann algebras*, in: Accardi L., von Waldenfels W. (eds.), *Quantum probability and applications III. Proceedings of a conference held in Obserwolfach, FRG, January 25-31, 1987*, LNM **1303**, Springer, Berlin, p.52.
[304] Cena A., 2003, *Geometric structures on the non-parametric statistical manifold*, PhD thesis, Università di Milano, Milano.
[305] Cena A., Pistone G., 2007, *Exponential statistical manifold*, Ann. Inst. Stat. Math. **59**, 27.

[306] Censor Y., Lent A., 1981, *An iterative row-action method for interval convex programming*, J. Optim. Theory Appl. **34**, 321.
[307] Censor Y., Lent A., 1987, *Optimization of "*$\log x$*" entropy over linear equality constraints*, Soc. Industr. Appl. Math. J. Contr. Optim. **35**, 921.
[308] Censor Y., Zenios S., 1997, *Parallel optimisation: theory, algorithms and applications*, Oxford University Press, Oxford.
[309] Černý J., Brunovský P., 1984, *A note on information without probability*, Inf. Contr. **25**, 134.
[310] Chamseddine A.H., Connes A., 1997, *The spectral action principle*, Commun. Math. Phys. **186**, 731. arXiv:hep-th/9606001.
[311] Chen G., Teboulle M., 1993, *Convergence analysis of a proximal-like minimization algorithm using Bregman functions*, Soc. Industr. Appl. Math. J. Optim. **3**, 538.
[312] Chencov N.N., 1964, *Geometriya "mnogoobraziya" raspredeleniĭ veroyatnosteĭ*, Dokl. AN SSSR **158**, 543.
[313] Chencov N.N., 1965, *Kategorii matematicheskoĭ statistiki*, Dokl. AN SSSR **164**, 511.
[314] Chencov N.N., 1966, *K sistematicheskoĭ teorii eksponencial'nykh semeĭstv raspredeleniĭ veroyatnosteĭ*, Teor. Ver. Premen. **11**, 483.
[315] Chencov N.N., 1968, *Nesimmetrichnoe rasstoyanie mezhdu raspredeleniyami veroyatnosteĭ, entropiya i teorema Pifagora*, Mat. Zametki **4**, 323. (engl. transl. 1968, *Nonsymmetrical distance between probability distributions, entropy and the theorem of Pythagoras*, Math. Notes **4**, 686.).
[316] Chencov N.N., 1969, *Obshhaya teoriya statisticheskogo vyvoda*, Mat. Zametki **5**, 635 (engl. transl. 1969, *General theory of statistical decisions*, Math. Notes **5**, 380).
[317] Chencov N.N., 1972, *Statisticheskie reshayushhie pravila i optimal'nye vyvody*, Nauka, Moskva (engl. transl.: 1982, *Statistical decision rules and optimal inference*, American Mathematical Society, Providence).
[318] Chencov N.N., 1978, *Algebraic foundation of mathematical statistics*, Math. Operationsforsch. Statist., Ser. Statist. **9**, 267.
[319] Chencov N.N., 1987, *Pochemu L_1-podkhod i chto za gorizontom*, appendix to: Devroye L., Györfi L., *Neparametricheskoe ocenivanie plotnosti. L_1-podkhod*, Mir, Moskva, p.348 (engl. transl. 2011, *Why L_1 view and what is next?*, Kybernetika **47**, 840).
[320] Chencov N.N., 1990, *The unfathomable influence of Kolmogorov*, Ann. Statist. **18**, 987.
[321] Cheng S.Y., Yau S.T., 1982, *The real Monge-Ampére equation and affine flat structures*, in: *Proceedings of the 1980 Beijing symposium of differential geometry and differential equations*, Gordon & Breach, New York, p.339.
[322] Chernoff H., 1952, *A measure of asymptotic efficiency for tests of a hypothesis based on the sum of observations*, Ann. Math. Statist. **23**, 493. Available at: omega.albany.edu:8008/Classics/Chernoff52.pdf.
[323] Ching W.-M., 1969, *Non-isomorphic non-hyperfinite factors*, Canad. J. Math. **21**, 1293.
[324] Choi M.-D., 1974, *A Schwarz inequality for positive linear maps on C^*-algebras*, Illinois J. Math. **18**, 565.
[325] Choi M.-D., Ruskai M.B., Seneta E., 1993, *Equivalence of certain entropy contraction coefficients*, Lin. Alg. Appl. **208**, 29.
[326] Choquet G., 1956, *Existence des représentiations intégrales au moyen des points extrémaux dans les cônes convexes*, Compt. Rend. Acad. Sci. Paris **243**, 699.
[327] Choquet G., 1956, *Existence des représentiations intégrales dans les cônes convexes*, Compt. Rend. Acad. Sci. Paris **243**, 736.
[328] Choquet G., 1956, *Existence unicite des représentiations intégrales au moyen des points extrémaux dans les cônes convexes*, Sém. Bourbaki **139**, 33. numdam:SB_1956-1958__4__33_0.
[329] Choquet G., 1956, *Unicité des représentiations intégrales au moyen des points extrémaux dans les cônes convexes reticules*, Compt. Rend. Acad. Sci. Paris **243**, 555.
[330] Choquet G., 1960, *Le théorème de représentation intégrales dans les ensemble convexes compacts*, Ann. Inst. Fourier Grenoble **10**, 333.
[331] Christensen E., 1972, *Non-commutative integration for monotone sequentially closed C^*-algebras*, Math. Scand. **31**, 171.
[332] Christensen E., 1978, *Generators of semigroups of completely positive maps*, Comm. Math. Phys. **62**, 167.
[333] Christensen E., 1982, *Measures on projections and physical states*, Comm. Math. Phys. **86**, 529.
[334] Christensen E., Evans D.E., 1979, *Cohomology of operator algebras and quantum dynamical semigroups*, J. London Math. Soc. **20**, 258.
[335] Christoffel E.B., 1869, *Über die Transformation der homogenen Differentialausdrücke zweiten Grades*, J. rein. angew. Math. B **70**, 46.
[336] Chruściel P.T, Galloway G.J., Pollack D., 2010, *Mathematical general relativity: a sampler*, Bull. Amer. Math. Soc. **47**, 567.
[337] Chruściński D., Jamiołkowski A., 2004, *Geometric phases in classical and quantum mechanics*, Birkhäuser, Boston.
[338] Chruściński D., Marmo G., 2009, *Remarks on the GNS representation and the geometry of quantum states*, Open Sys. Inf. Dyn. **16**, 157. arXiv:0810.1660.
[339] Chwistek L., 1935, *Granice nauki. Zarys logiki i metodologii nauk ścisłych*, Książnica/Atlas, Lwów/Warszawa (2nd rev. ed./engl. transl. 1948, *The limits of science: outline of logic and of the methodology of the exact sciences*, Harcourt Brace, New York).
[340] Cichocki A., Amari S.-i., 2010, *Families of alpha- beta- and gamma- divergences: flexible and robust measures of similarity*, Entropy **12**, 1532. Available at: www.mdpi.com/1099-4300/12/6/1532/pdf.
[341] Clarke F.H., 1987, *Optimisation and nonsmooth analysis*, Society for Industrial and Applied Mathematics, Philadelphia (1990, 2nd. ed).
[342] Clarke F.H., Ledyaev Yu.S., Stern R.J., Wolenski P.R., 1998, *Nonsmooth analysis and control theory*, Springer, Berlin.
[343] Coecke B., 2012, *The logic of quantum mechanics – take II*, arXiv:1204.3458.
[344] Coecke B., Moore D.J., Wilce A., 2000, *Operational quantum logic: an overview*, in: Coecke B., Moore D.J., Wilce A., *Current research in operational quantum logic: algebras, categories, languages*, Kluwer, Dordrecht, p.1. arXiv:quant-ph/0008019.
[345] Cohen J.E., Iwasa Y., Rautu G., Ruskai M.B., Seneta E., Zbaganu G., 1993, *Relative entropy under mappings by stochastic matrices*, Lin. Alg. Appl. **179**, 211.
[346] Cohen L., 1966, *Can quantum mechanics be formulated as a classical probability theory?*, Phil. Sci. **33**, 317.
[347] Cohen L., 1973, *Joint probability distributions in quantum mechanics*, in: Hooker C.A. (ed.), *Contemporary research in the foundations and philosophy of quantum theory: proceedings of a conference held at the University of Western Ontario, London, Canada*, Reidel, Dordrecht, p.66.
[348] Cohen L., Margenau H., 1968, *Probabilities in quantum mechanics*, in: Bunge M. (ed.), *Quantum theory and reality*, Springer, Berlin, p.71.
[349] Combes F., 1966, *Étude des représentations tracées d'une C^*-algèbre*, Compt. Rend. Acad. Sci. Paris **262**, 114.

[350] Combes F., 1967, *Étude des poids définis sur une C^*-algebre*, Compt. Rend. Acad. Sci. Paris **265**, 340.
[351] Combes F., 1968, *Poids sur une C^*-algèbre*, J. Math. Pures Appl. 9^e série **47**, 57.
[352] Combes F., 1971, *Poids associé à une algèbre hilbertienne à gauche*, Comp. Math. **23**, 49. numdam:CM_1971__23_1_49_0.
[353] Combes F., 1971, *Poids associé à une algèbre hilbertienne généralisée*, Compt. Rend. Acad. Sci. Paris Sér. A **270**, 33.
[354] Combes F., 1971, *Poids et espérances conditionnelles dans les algèbres de von Neumann*, Bull. Soc. Math. France **99**, 73.
[355] Combes F., Delaroche C., 1975, *Groupe modulaire d'une espérance conditionelle dans une algèbre de von Neumann*, Bull. Soc. Math. France **103**, 385.
[356] Connes A., 1972, *Groupe modulaire d'une algèbre de von Neumann*, Compt. Rend. Acad. Sci. Paris Sér. A **274**, 523.
[357] Connes A., 1973, *Sur le théorème de Radon–Nikodym pour le poids normaux fidèles semifinies*, Bull. Soc. Math. France 2-ème Sér. **97**, 253.
[358] Connes A., 1973, *Une classification des facteurs de type III*, Ann. Sci. École Norm. Sup. 4 ème sér. **6**, 133. numdam:ASENS_1973_4_6_2_133_0.
[359] Connes A., 1974, *Caractérisation des espaces vectoriels ordonnés sous-jacents aux algèbres de von Neumann*, Ann. Inst. Fourier Grenoble **24**, 121. numdam:AIF_1974__24_4_121_0.
[360] Connes A., 1975, *On hyperfinite factors of type III_0 and Krieger's factors*, J. Funct. Anal. **18**, 318.
[361] Connes A., 1975, *Outer conjugacy classes of automorphisms of factors*, Ann. Sci. École Norm. Sup. **8**, 383.
[362] Connes A., 1976, *Classification of injective factors. Cases II_1, II_∞, III_λ, $\lambda \neq 1$*, Ann. Math. **104**, 73.
[363] Connes A., 1977, *Periodic automorphisms of the hyperfinite factor type II_1*, Acta Sci. Math. Szeged **39**, 39.
[364] Connes A., 1980, *Correspondences*, unpublished manuscript. Available at: www.fuw.edu.pl/~kostecki/scans/connes1980.pdf.
[365] Connes A., 1980, *On a spatial theory of von Neumann algebras*, J. Func. Anal. **35**, 153.
[366] Connes A., 1982, *Classification des facteurs*, in: Kadison R. (ed.), *Operator algebras and applications (Kingston, Ontario, 1980)*, Proceedings of the Symposia in Pure Mathematics **38**, Part 2, American Mathematical Society, Providence, p.43.
[367] Connes A., 1985, *Factors of type III_1, property L'_λ and closure of inner automorphisms*, J. Op. Theor. **14**, 189.
[368] Connes A., 1994, *Noncommutative geometry*, Academic Press, Boston.
[369] Connes A., Størmer E., 1978, *Homogeneity of the state space of factors of type III_1*, J. Funct. Anal. **28**, 187.
[370] Connes A., Takesaki M., 1974, *Flots des poids sur les facteurs de type III*, Compt. Rend. Acad. Sci. Paris Sér. A **278**, 945.
[371] Connes A., Takesaki M., 1977, *The flow of weights on factors of type III*, Tōhoku Math. J. **29**, 473.
[372] Cook J.M., 1953, *The mathematics of second quantization*, Trans. Amer. Math. Soc. **74**, 222.
[373] Cooke R.M., Hilgevoord J., 1981, *A new approach to equivalence in quantum logic*, in: Beltrametti E., van Fraasen B.C. (eds.), *Current issues in quantum logic*, Plenum, New York.
[374] Cover T.M., Thomas J.A., 1991, *Elements of information theory*, Wiley, New York (2nd ed., 2006).
[375] Cox R.T., 1946, *Probability, frequency and reasonable expectation*, Am. J. Phys. **14**, 1.
[376] Cox R.T., 1961, *The algebra of probable inference*, John Hopkins University Press, Baltimore.
[377] Cramér H., 1946, *A contribution to the theory of statistical estimation*, Skandinavisk Aktuarietidskrift **29**, 85.
[378] Cressie N., Read T.R.C., 1984, *Multinomial goodness-of-fit tests*, J. Roy. Statist. Soc. B **46**, 440.
[379] Crooks G.E., Sivak D.A., 2011, *Measures of trajectory ensemble disparity in nonequilibrium statistical dynamics*, J. Stat. Mech. (to appear). Available at: threeplusone.com/pubs/technote/CrooksTechNote009-TrajDiv.pdf.
[380] Csiszár I., 1963, *Eine informationstheorische Ungleichung und ihre Anwendung auf den Beweis der Ergodizität von Markoffschen Ketten*, Magyar Tud. Akad. Mat. Kutató Int. Közl. **8**, 85.
[381] Csiszár I., 1966, *A note on Jensen's inequality*, Stud. Sci. Math. Hungar. **1**, 227.
[382] Csiszár I., 1967, *Information-type measures of difference of probability distributions and indirect observation*, Stud. Sci. Math. Hungar. **2**, 229.
[383] Csiszár I., 1975, *I-divergence geometry of probability distributions and minimization problems*, Ann. Prob. **3**, 146.
[384] Csiszár I., 1978, *Information measures: a critical survey*, in: Kozesnik J. (ed.), *Transactions of the seventh Prague conference on information theory, statistical decision functions and random processes*, Springer, Berlin, p.73.
[385] Csiszár I., 1984, *Sanov property, generalized I-projection and a conditional limit theorem*, Ann. Prob. **12**, 768.
[386] Csiszár I., 1991, *Why least squares and maximum entropy? An axiomatic approach to inference for linear inverse problems*, Ann. Stat. **19**, 2032.
[387] Csiszár I., 1994, *Maximum entropy and related methods*, in: Lachout P., Víšek J.Á. (eds.), *Transactions of twelfth Prague conference on information theory, statistical decision functions and random processes*, Academy of Sciences of the Czech Republic, Institute of Information Theory and Automation, Prague, p.58.
[388] Csiszár I., 1995, *Generalized projections for nonnegative functions*, Acta Math. Hung. **68**, 161.
[389] Csiszár I., 1996, *Maxent, mathematics and information theory*, in: Hanson K.M., Silver R.N. (eds.), *Maximum entropy and bayesian methods: proceedings of the fifteenth international workshop on maximum entropy and bayesian methods*, Kluwer, Dordrecht, p.35.
[390] Csiszár I., 2003, *Information projections revisited*, IEEE Trans. Inf. Theor. **49**, 1474.
[391] Csiszár I., 2008, *Axiomatic characterizations of information measures*, Entropy **10**, 261. Available at: www.mdpi.com/1099-4300/10/3/261/pdf.
[392] Csiszár I., Fischer J., 1962, *Informationsentfernungen im Raum der Wahrscheinlichkeitsverteilungen*, Magyar Tud. Akad. Mat. Kutató Int. Közl. **7**, 159.
[393] Csiszár I., Gamboa F., Gassiat E., 1999, *MEM pixel correlated solutions for generalized moment and interpolation problems*, IEEE Trans. Inf. Theor. **45**, 2253.
[394] Csiszár I., Matúš F., 2003, *Information projections revisited*, IEEE Trans. Inf. Theor. **49**, 1474.
[395] Csiszár I., Matúš F., 2008, *On minimization of entropy functionals under moment constraints*, in: *Proceedings of the 2008 IEEE International Symposium on Information Theory*, IEEE Information Theory Society, Washington, p.2101.
[396] Csiszár I., Matúš F., 2009, *On minimization of multivariate entropy functionals*, in: *Proceedings of 2009 IEEE Information Theory Workshop on Networking and Information Theory*, IEEE Information Theory Society, Washington, p.96.
[397] Csiszár I., Matúš F., 2012, *Generalized minimizers of convex integral functions, Bregman distance, pythagorean identities*, Kybernetika **48**, 637. arXiv:1202.0666.
[398] Csiszár I., Matúš F., 2012, *Minimization of entropy functionals revisited*, in: *Proceedings of the 2012 IEEE International Symposium on Information Theory*, IEEE Information Theory Society, Washington, p.150.
[399] Csiszár I., Shields P.C., 2004, *Information theory and statistics: a tutorial*, Found. Trends Commun. Inf. Theor. **1**, 417.
[400] Cuculescu I., Oprea A.G., 1994, *Noncommutative probability*, Kluwer, Dordrecht.
[401] Cudia D.F., 1964, *The geometry of Banach spaces. Smoothness*, Trans. Amer. Math. Soc. **110**, 284.

[402] Czachor M., 1993, *Aspects of nonlinear quantum mechanics*, Ph.D. thesis, Centrum Fizyki Teoretycznej Polskiej Akademii Nauk, Warszawa.
[403] Czachor M., 1995, *Elements of nonlinear quantum mechanics (II): triple bracket generalization of quantum mechanics*, arXiv:quant-ph/9501008.
[404] Czachor M., 1996, *Lie–Nambu and Lie–Poisson structures in linear and nonlinear quantum mechanics*, Acta Phys. Polon. B **27**, 2319.
[405] Czachor M., 1996, *Nonlinear Schrödinger equation and two-level atoms*, Phys. Rev. A **53**, 1310. arXiv:quant-ph/9501007.
[406] Czachor M., 1997, *Nambu-type generalization of the Dirac equation*, Phys. Lett. A **225**, 1. arXiv:quant-ph/9601015.
[407] Czachor M., 1998, *Nonlocal-looking equations can make nonlinear quantum dynamics local*, Phys. Rev. A **57**, 4122. arXiv:quant-ph/9708052.
[408] Czachor M., 1999, *Lie-Nambu and beyond*, Int. J. Theor. Phys. **38**, 475. arXiv:quant-ph/9711054.
[409] Czachor M., Doebner H.-D., 2002, *Correlation experiments in nonlinear quantum mechanics*, Phys. Lett. A **301**, 139. arXiv:quant-ph/0110008.
[410] Czachor M., Kuna M., 1998, *Complete positivity of nonlinear evolution: a case study*, Phys. Rev. A **58**, 128. arXiv:quant-ph/9708029.
[411] Czachor M., Kuna M., Leble S.B., Naudts J., 2000, *Nonlinear von Neumann-type equations*, in: Doebner H.-D. et al. (eds.), *Trends in quantum mechanics*, World Scientific, Singapore, p.209. arXiv:quant-ph/9904110.
[412] Czachor M., Marciniak M., 1998, *Density matrix interpretation of solutions of Lie-Nambu equations*, Phys. Lett. A **239**, 353. arXiv:quant-ph/9707013.
[413] Czelakowski J., 1974, *Logics based on partial boolean σ-algebras (I)*, Studia Log. **33**, 371.
[414] Czelakowski J., 1975, *Logics based on partial boolean σ-algebras (II)*, Studia Log. **34**, 69.
[415] Dacunha-Castelle D., Gamboa F., 1990, *Maximum entropie et problème des moments*, Ann. Inst. Henri Poincaré B **26**, 567.
[416] Dähn G., 1968, *An attempt at an axiomatic formulation of quantum mechanics and more general theories IV*, Commun. Math. Phys. **9**, 192.
[417] Dähn G., 1972, *The algebra generated by physical filters*, Commun. Math. Phys. **28**, 109.
[418] Dalla Chiara M.L., Giuntini R., 1999, *Łukasiewicz' theory of truth, from the quantum logical point of view*, in: Woleński J., Köhler E. (eds.), *Alfred Tarski and the Vienna Circle*, Kluwer, Dordrecht, p.127.
[419] Dalla Chiara M.L., Giuntini R., 2002, *Quantum logics*, in: Gabbay D., Guenthner F. (eds.), *Handbook of philosophical logic*, Vol.6, Kluwer, Dordrecht, p.129. arXiv:quant-ph/0101028.
[420] Dalla Chiara M.L., Giuntini R., Greechie R., 2004, *Reasoning in quantum theory: sharp and unsharp quantum logics*, Kluwer, Dordrecht.
[421] D'Andrea A.B., De Lucia P., Morales P., 1991, *The Lebesgue decomposition theorem and the Nikodym convergence theorem on orthomodular poset*, Atti Sem. Mat. Fis. Univ. Modena **34**, 137.
[422] Daniell P.J., 1918, *A general form of the integral*, Ann. Math. **19**, 279.
[423] Daniell P.J., 1919, *Integrals in an infinite number of dimensions*, Ann. Math. **21**, 30.
[424] Daniell P.J., 1920, *Further properties of the general integral*, Ann. Math. **21**, 203.
[425] Daniell P.J., 1920, *Stieltjes derivatives*, Bull. Amer. Math. Soc. **26**, 444.
[426] Daniell P.J., 1921, *Integral products and probability*, Am. J. Math. **43**, 143.
[427] D'Ariano G.M., Giovanetti V., Perinotti P., 2006, *Optimal estimation of quantum observables*, J. Math. Phys. **47**, 022102. arXiv:quant-ph/0510187.
[428] Darmois G., 1935, *Sur les lois de probabilite a estimation exhaustive*, C.R. Acad. Sci. Paris **260**, 1265.
[429] Darmois G., 1945, *Sur les limites de la dispersion des certaines estimations*, Rev. Inst. Intern. Statist. **13**, 9.
[430] Davies E.B., 1972, *Example related to the foundations of quantum theory*, J. Math. Phys. **13**, 39.
[431] Davies E.B., 1976, *Quantum theory of open systems*, Academic Press, New York.
[432] Davies E.B., 1979, *Generators of dynamical semigroups*, J. Funct. Anal. **34**, 421.
[433] Davies E.B., 1980, *One-parameter semigroups*, Academic Press, London.
[434] Davies E.B., 1980, *Uniqueness of the standard form of the generator of a quantum dynamical semigroup*, Rep. Math. Phys. **17**, 249.
[435] Davies E.B., Lewis J.T., 1970, *An operational approach to quantum probability*, Commun. Math. Phys. **17**, 239.
[436] Davis C., 1963, *Notions generalizing convexity for functions defined on spaces of matrices*, Proc. Amer. Math. Soc. Symp. **7**, 187.
[437] Dawid A.P., 1975, *Discussion of paper by B. Efron*, Ann. Statist. **3**, 1231.
[438] Dawid A.P., 1977, *Further comments on a paper by Bradley Efron*, Ann. Statist. **5**, 1249.
[439] de Fermat P., Pascal B., 1654, *Correspondance*, reprinted in: Henry Ch., Tannery P. (eds.), 1894, *Œuvres de Fermat*, Vol.2, Gauthier–Villars, Paris.
[440] de Finetti B., 1931, *Sul significato soggestivo della probabilità*, Fund. Math. **17**, 298.
[441] de Finetti B., 1937, *Prevision: ses lois logiques, ses sources subjectives*, Ann. Inst. Henri Poincaré **7**, 1. (engl. transl. 1964, *Foresight: its logical laws, its subjective sources*, in: Kyburg H.E. Jr., Smokler H.E. (eds.), *Studies in subjective probability*, Wiley, New York, p.93).
[442] de Finetti B., 1970, *Teoria delle probabilità*, Einaudi, Torino (engl. transl.: 1974, *Theory of probability: a critical introductory treatment*, Vol.1-2, Wiley, New York.
[443] de Laplace P.-S., 1774, *Mémoire sur la probabilité des causes par les évènements*, Mém. Acad. Roy. Sci. Paris (Savants Étrangers) **6**, 621. (reprinted in: 1986, Stat. Sci. **1**, 359.).
[444] De Lucia P., Dvurečenskij A., 1993, *Yoshida–Hewitt decompositions of Riesz-space valued measures on orthoalgebras*, Tatra Mt. Math. Publ. **3**, 101.
[445] de Muynck W.M., 2002, *Foundations of quantum mechanics: an empiricist approach*, Kluwer, Dordrecht.
[446] de Pagter, 2007, *Non-commutative Banach function spaces*, in: Boulabiar K., Buskes G., Triki A. (eds.), *Positivity*, Birkhäuser, Basel, p.197.
[447] De Pierro A.N., Iusem A.N., 1986, *A relaxed version of Bregman's method for convex programming*, J. Optim. Theor. Appl. **51**, 421.
[448] Decarreau A., Hilhorst D., Lemaréchal C., Navaza J., 1992, *Dual methods in entropy maximization. Application to some problems in crystallography*, Soc. Industr. Appl. Math. J. Optim. **2**, 173.
[449] Dedekind R., 1872, *Stetigkeit und die Irrationalzahlen*, Vieweg, Braunschweig (engl. transl: 1901, *Continuity and irrational numbers*, in: Dedekind R., *Essays on the theory of numbers*, Open Court, Chicago, p.1).

[450] Della Pietra S., Della Pietra V., Lafferty J., 2002, *Duality and auxiliary functions for Bregman distances*, Technical report CMU-CS-01-109R, School of computer science, Carnegie Mellon University, Pittsburgh.
[451] Dereziński J., Jakšić V., Pillet C.-A., 2003, *Perturbation theory of W^*-dynamics, liouvilleans and KMS-states*, Rev. Math. Phys. **15**, 447.
[452] Destouches-Février P., 1951, *La structure des théories physiques*, Presses Universitaires de France, Paris.
[453] Destouches-Février P., 1951, *Logique et theories physiques*, in: *Congrès International de Philosophie des Sciences, Paris 1949*, Vol.2, Actualités Scientifiques et Industrielles **1134**, Hermann, Paris, p.45.
[454] Destouches-Février P., 1952, *Les prévisions et la methode triondulatorie*, J. Phys. Rad. **13**, 605.
[455] Dhillon I.S., Tropp J.A., 2007, *Matrix nearness with Bregman divergences*, Soc. Industr. Appl. Math. J. Matrix Anal. Appl. **29**, 1120.
[456] Dieks D., 1988, *Overlap and distinguishability of quantum states*, Phys. Lett. A **126**, 303.
[457] Dieks D., Veltkamp P., 1983, *Distance between quantum states, statistical inference and the projection postulate*, Phys. Lett. A **97**, 24.
[458] Diepenbrock F.R., 1971, *Charakterisung einer allgemeineren Bedingung als Dominiertheit mit Hilfe von lokalisierten Maßen*, Ph.D. thesis, Universität Münster, Münster.
[459] Dieudonné J., 1940, *Topologies faibles dans les espaces vectoriels*, Compt. Rend. Acad. Sci. Paris **211**, 94.
[460] Dieudonné J., 1942, *La dualité dans les espaces vectoriels topologiques*, Ann. Sci. École Norm. Sup. **59**, 107.
[461] Digernes T., 1974, *Poids dual sur un produit croisé*, Compt. Rend. Acad. Sci. Paris **278**, 937.
[462] Digernes T., 1975, *Duality for weights on covariant systems and its applications*, PhD thesis, University of California, Los Angeles.
[463] Dillen F., Nomizu K., Vracken L., 1990, *Conjugate connections and Radon's theorem in affine differential geometry*, Monatsh. Math. **109**, 221.
[464] Dirac P.A.M., 1925, *The fundamental equations of quantum mechanics*, Proc. Roy. Soc. London A **109**, 642.
[465] Dirac P.A.M., 1926, *On the theory of quantum mechanics*, Proc. Roy. Soc. London A **112**, 661.
[466] Dirac P.A.M., 1927, *The physical interpretation of the quantum dynamics*, Proc. Roy. Soc. London A **113**, 621.
[467] Dirac P.A.M., 1930, *The principles of quantum mechanics*, Clarendon Press, Oxford.
[468] Dittmann J., 1993, *On the riemannian geometry of finite dimensional state space*, Sem. Sophus Lie **3**, 73.
[469] Dittmann J., 1994, *Some properties of the riemannian Bures metric on mixed states*, J. Geom. Phys. **13**, 203.
[470] Dittmann J., 1995, *On the riemannian metric on the space of density matrices*, Rep. Math. Phys. **36**, 309.
[471] Dittmann J., 1999, *Note on explicit formulae for the Bures metric*, J. Phys. A **32**, 2663. arXiv:quant-ph/9808044.
[472] Dittmann J., 1999, *The scalar curvature of the Bures metric on the space of density matrices*, J. Geom. Phys. **31**, 16. arXiv:quant-ph/9810012.
[473] Dittmann J., 2000, *On the curvature of monotone metrics and a conjecture concerning the Kubo-Mori metric*, Lin. Alg. Appl. **315**, 83. arXiv:quant-ph/9906009.
[474] Dittmann J., Uhlmann A., 1999, *Connections and metrics respecting purification of quantum states*, J. Math. Phys. **40**, 3246. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/DU99.pdf.
[475] Diu B., 1983, *Statistical inference and ditance between sdtates in quantum mechanics*, Ann. Inst. Henri Poincaré A **38**, 167.
[476] Dixmier J., 1949, *Les anneaux d'opérateurs de classe finie*, Ann. Sci. École Norm. Sup. **66**, 209.
[477] Dixmier J., 1950, *Les fonctionnelles linéaires sur l'ensemble des opérateurs bornés d'un espace de Hilbert*, Ann. Math. **51**, 387.
[478] Dixmier J., 1951, *Sur certains espaces considérés par M.H. Stone*, Summa Brasil. Math. **2**, 151.
[479] Dixmier J., 1951, *Sur la réduction des anneaux d'operateurs*, Ann. École. Norm. Sup. **68**, 185.
[480] Dixmier J., 1952, *Algèbres quasi-unitaires*, Comment. Math. Helv. **26**, 275.
[481] Dixmier J., 1953, *Formes linéaires sur un anneau d'opérateurs*, Bull. Soc. Math. France **81**, 9. numdam:BSMF_1953__81__9_0.
[482] Dixmier J., 1954, *Sur les anneaux d'opérateurs dans les espaces hilbertiens*, Compt. Rend. Acad. Sci. Paris **238**, 439.
[483] Dixmier J., 1957, *Les algèbres d'opérateurs dans l'espace hilbertien (algèbres de von Neumann)*, Gauthier–Villars, Paris. (2nd. rev. ed., 1969; engl. trans., 1981, *von Neumann algebras*, North-Holland, Amsterdam).
[484] Dixmier J., 1958, *Sur la relation* $i(PQ - QP) = 1$, Comp. Math. **13**, 263.
[485] Dixmier J., 1964, *Les C^*-algèbres et leurs représentations*, Gauthier Villars, Paris. (2nd. rev. ed., 1969; engl. transl. 1977, *C^*-algebras*, North-Holland, Amsterdam).
[486] Dixmier J., Lance E.C., 1969, *Deux noveaux facteurs de type II_1*, Invest. Math. **7**, 226.
[487] Dodds P.G., 1969, *The Riesz space structure of an abelian W^*-algebra*, Ph.D. thesis, California Institute of Technology, Pasadena. Available at: www.fuw.edu.pl/∼kostecki/scans/dodds1969.pdf.
[488] Dodds P.G., 1974, *The order dual of an abelian von Neumann algebra*, Austr. Math. Soc. **18**, 153.
[489] Dodds P.G., Dodds T.K., 1995, *Some aspects of the theory of symmetric operator spaces*, Quaest. Math. **18**, 47.
[490] Dodds P.G., Dodds T.K., de Pagter B., 1989, *Non-commutative Banach function spaces*, Math. Z. **201**, 583.
[491] Dodds P.G., Dodds T.K., de Pagter B., 1992, *Fully symmetric operator spaces*, Integr. Eq. Op. Theor. **15**, 941.
[492] Dodds P.G., Dodds T.K., de Pagter B., 1993, *Non-commutative Köthe duality*, Trans. Amer. Math. Soc. **339**, 717.
[493] Donald M.J., 1986, *On the relative entropy*, Commun. Math. Phys. **105**, 13. euclid:cmp/1104115254.
[494] Donald M.J., 1987, *Free energy and relative entropy*, J. Stat. Phys. **49**, 81.
[495] Donald M.J., 1987, *Further results on the relative entropy*, Math. Proc. Cambridge Phil. Soc. **101**, 363.
[496] Donald M.J., 1990, *Relative hamiltonians which are not bounded from above*, J. Funct. Anal. **91**, 143.
[497] Donald M.J., 1991, *Continuity of relative hamiltonians*, Commun. Math. Phys. **136**, 625.
[498] Donoghue W., 1974, *Monotone matrix functions and analytic continuation*, Springer, Berlin.
[499] Doplicher S., Kastler D., Robinson D., 1966, *Covariance algebras in field theory and statistical mechanics*, Commun. Math. Phys. **3**, 1.
[500] Drechsler W., 1984, *Poincaré gauge theory, gravitation, and transformation of matter fields*, Fortschr. Phys. **32**, 449.
[501] Drechsler W., 1992, *Quantized fiber dynamics for extended elementary objects involving gravitation*, Found. Phys. **22**, 1041.
[502] Drechsler W., Tuckey P.A., 1996, *On a quantum and parallel transport in a Hilbert bundle over spacetime*, Class. Quant. Grav. **13**, 611. arXiv:gr-qc/9506039.
[503] Driessler W., 1975, *Comments on lightlike translations and applications in relativistic quantum field theory*, Commun. Math. Phys. **44**, 133.
[504] Driessler W., 1977, *On the type of local algebras in quantum field theory*, Commun. Math. Phys. **53**, 295.
[505] Dubrovin B.A., Novikov S.P., Fomenko A.T., 1979, *Sovremennaya geometriya*, Nauka, Moskva. (engl. transl.: 1984, *Modern*

geometry: methods and applications, Springer, Berlin).
[506] Duch P., Kostecki R.P., 2011, *Quantum Schwarzschild space-time*, arXiv:1110.6566.
[507] Duhem P., 1894, *Quelques réflexions au sujet de la physique expérimentale*, Rev. Quest. Sci 2eme ser. **36**, 179.
[508] Duhem P., 1906, *La théorie physique, son objet, et sa structure*, Chevalier et Rivière, Paris (engl. transl. 1954, *The aim and structure of physical theory*, Princeton University Press, Princeton).
[509] Duistermaat J.J., 2001, *On hessian riemannian structures*, Asian J. Math. **5**, 79.
[510] Duncan A., Janssen M., 2012, *(Never) mind your p's and q's: von Neumann versus Jordan on the foundation of quantum theory*, arXiv:1204.6511.
[511] Dvurečenskij A., 1993, *Gleason's theorem and its applications*, Kluwer, Dordrecht.
[512] Dye H.A., 1952, *The Radon–Nikodým theorem for finite rings of operators*, Trans. Amer. Math. Soc. **72**, 243.
[513] Dyson F.J., 1949, *The radiation theories of Tomonaga, Schwinger, and Feynman*, Phys. Rev. **75**, 486.
[514] Earman J., 2008, *Superselection rules for philosophers*, Erkenntnis **69**, 377.
[515] Ebanks B., Sahoo P., Sander W., 1998, *Characterizations of information measures*, World Scientific, Singapore.
[516] Eckart C., 1993, *Operator calculus and the solution of the equations of motion of quantum dynamics*, Phys. Rev. **28**, 711.
[517] Eckstein J., 1993, *Nonlinear proximal point algorithms using Bregman functions, with applications to convex programming*, Math. Oper. Res. **18**, 202.
[518] Edwards C.M., 1970, *The operational approach to algebraic quantum theory. I*, Commun. Math. Phys. **16**, 207.
[519] Edwards C.M., 1971, *Classes of operations in quantum theory*, Commun. Math. Phys. **20**, 26.
[520] Efron B., 1975, *Defining the curvature of a statistical problem (with applications to second order efficiency)*, Ann. Statist. **3**, 1189.
[521] Efron B., 1978, *Controversion in the foundations of statistics*, Am. Math. Monthly **85**, 231.
[522] Eguchi S., 1983, *Second order efficiency of minimum contrast estimators in a curved exponential family*, Ann. Statist. **11**, 793. euclid:aos/1176346246.
[523] Eguchi S., 1985, *A differential geometric approach to statistical inference on the basis of contrast functionals*, Hiroshima Math. J. **15**, 341. euclid:hmj/1206130775.
[524] Eguchi S., 1992, *Geometry of minimum contrast*, Hiroshima Math. J. **22**, 631.
[525] Eijndhoven S.J.L., de Graaf J., 1986, *A mathematical introduction to Dirac's formalism*, North-Holland, Amsterdam.
[526] Einstein A., 1905, *Zur Elektrodynamik bewegter Körper*, Ann. Phys. **322**, 891.
[527] Einstein A., 1915, *Zur allgemeinen Relativitätstheorie*, Sitzungsber. Preuss. Akad. Wiss. Berlin **1915**, 831.
[528] Einstein A., 1916, *Die Grundlage der allgemeinen Relativitätstheorie*, Ann. Phys. **354**, 769.
[529] Einstein A., Grossmann M., 1913, *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation*, Z. Math. Phys. **62**, 225.
[530] Einstein A., Podolsky B., Rosen N., 1935, *Can quantum-mechanical description of physical reality be considered complete?*, Phys. Rev. **47**, 777.
[531] Ekeland I., Temam R., 1974, *Analyse convexe et problèmes variationnels*, Gauthier–Villars, Paris. (engl. transl.: 1976, *Convex analysis and variational problems*, North–Holland, Amsterdam).
[532] Elliott G.A., 1975, *On the Radon–Nikodym derivative with a chain rule in a von Neumann algbera*, Canad. Math. Bull. **18**, 661.
[533] Emch G.G., 1972, *Algebraic methods in statistical mechanics and quantum field theory*, Wiley, New York.
[534] Emch G.G., 1998, *On the need to adapt de Finetti's probability interpretation to QM*, in: Alicki R., Bożejko M., Majewski W.A. (eds.), *Quantum probability*, Banach Center Publications **43**, Instytut Matematyki Polskiej Akademii Nauk, Warszawa, p.157.
[535] Evans D.E., 1980, *A review of semigroups of completely positive maps*, in: Osterwalder L. (ed.), *Mathematical problems in theoretical physics*, Springer, Berlin.
[536] Evans D.E., 1984, *Quantum dynamical semigroups*, Acta Appl. Math. **2**, 333.
[537] Evans D.E., Kawahigashi Y., 1998, *Quantum symmetries on operator algebras*, Clarendon Press, Oxford.
[538] Fack T., Kosaki H., 1986, *Generalized s-numbers of τ-measurable operators*, Pacific J. Math. **123**, 269.
[539] Falcone A.J., 1996, *Operator valued weights, L^2-von Neumann modules and their relative tensor products*, PhD thesis, University of California, Los Angeles.
[540] Falcone A.J., 2000, *L^2-von Neumann modules, their relative tensor products and the spatial derivative*, Illinois J. Math. **44**, 407. Available at: euclid:ijm/1255984848.
[541] Falcone A.J., Takesaki M., 1999, *Operator valued weights without structure theory*, Trans. Amer. Math. Soc. **351**, 323. Available at: www.ams.org/journals/tran/1999-351-01/S0002-9947-99-02028-0/S0002-9947-99-02028-0.pdf.
[542] Falcone A.J., Takesaki M., 2001, *The non-commutative flow of weights on a von Neumann algebra*, J. Funct. Anal. **182**, 170. Available at: www.math.ucla.edu/∼mt/papers/QFlow-Final.tex.pdf.
[543] Fannes M., Verbeure A., 1977, *Correlation inequalities in quantum statistical mechanics and their application in the Kondo problem*, Commun. Math. Phys. **46**, 253.
[544] Favretti M., 2005, *Isotropic submanifolds generated by the maximum entropy principle and Onsager reciprocity relations*, J. Funct. Anal. **227**, 227.
[545] Favretti M., 2005, *Lagrangian submanifolds generated by the maximum entropy principle*, Entropy **7**, 1.
[546] Favretti M., 2007, *The information geometry description of inferential scattering*, preprint.
[547] Feldman D., Wilce A., 1993, *σ-additivity in manuals and orthoalgebras*, Order **10**, 383.
[548] Fell J.M.G., 1969, *An extension of Mackey's method to Banach *-algebraic bundles*, Memoirs of the American Mathematical Society **90**, American Mathematical Society, Providence.
[549] Fell J.M.G., 1977, *Induced representations and Banach $*$-algebraic bundles*, LNM **582**, Springer, Berlin.
[550] Fell J.M.G., Doran R.S., 1988, *Representations of $*$-algebras, locally compact groups, and Banach $*$-algebraic bundles*, Vol.1-2, Academic Press, San Diego.
[551] Fell J.M.G., Kelley J.L, 1952, *An algebra of unbounded operators*, Proc. Nat. Acad. Sci. U.S.A. **38**, 592.
[552] Fenchel W., 1949, *On conjugate convex functions*, Canadian J. Math. **1**, 73. Available at: math.ca/cjm/v1/cjm1949v01.0073-0077.pdf.
[553] Février P., 1937, *Les relations d'incertitude de Heisenberg et la logique*, Compt. Rend. Acad. Sci. Paris **204**, 481.
[554] Feynman R.P., 1948, *Space-time approach to non-relativistic quantum mechanics*, Rev. Mod. Phys. **20**, 367.
[555] Feynman R.P., 1949, *Space-time approach to quantum electrodynamics*, Phys. Rev. **76**, 769.
[556] Feynman R.P., 1951, *An operator calculus having applications in quantum electrodynamics*, Phys. Rev. **84**, 108.
[557] Feynman R.P., Hibbs A., 1965, *Quantum mechanics and path integration*, Wiley, New York.

[558] Fick E., Sauermann G., 1990, *The quantum statistics of dynamical processes*, Springer, Berlin.
[559] Fischer A., 2010, *Quantization and clustering with Bregman divergences*, J. Multivar. Anal. **101**, 2207.
[560] Fischer E., 1907, *Sur la convergence en moyenne*, Compt. Rend. Acad. Sci. Paris **144**, 1022.
[561] Fisher R.A., 1922, *On the mathematical foundation of theoretical statistics*, Phil. Trans. Roy. Soc. London Ser. A **222**, 309.
[562] Fisher R.A., 1925, *Theory of statistical estimation*, Proc. Cambridge Phil. Soc. **22**, 700.
[563] Flattot A., 2010, *Notes on non-commutative integration*, preprint, available at: www.math.sc.edu/∼flattot/re/Notes_noncommutative_int.pdf.
[564] Fleck L., 1929, *Zur Krise der "Wirklichkeit"*, Naturwiss. **17**, 425 (engl. transl.: 1986, *On the crisis of "reality"*, in: Cohen R.S., Schnelle T. (eds.), *Cognition and fact: materials on Ludwik Fleck*, Reidel, Dordrecht, p.47).
[565] Fleck L., 1935, *Entstehung und Entwicklung einer wissenschaftlichen Tatsache. Einführung in die Lehre vom Denkstil und Denkkollektiv*, Schwabe, Basel. (engl. transl. 1979, *Genesis and development of the scientific fact*, Chicago University Press, Chicago).
[566] Fleck L., 1936, *Zagadnienie teorii poznawania*, Przegl. Filozof. **39**, 3 (engl. transl.: 1986, *The problem of epistemology*, in: Cohen R.S., Schnelle T. (eds.), *Cognition and fact: materials on Ludwik Fleck*, Reidel, Dordrecht, p.79).
[567] Fok V.A., 1932, *Konfigurationsraum und zweite Quantelung*, Z. Phys. **75**, 622; **76**, 952.
[568] Forte B., 1969, *Measures of information: the general axiomatic theory*, Rev. franç. d'inf. res. opér., sér. rouge **3**, 63.
[569] Forte B., 1984, *Entropies with and without probabilities: applications to questionnaires*, Inf. Proc. Manag. **20**, 397.
[570] Foulis D.J., 1962, *A note on orthomodular lattices*, Portug. Math. **21**, 65.
[571] Foulis D.J., 1989, *Coupled physical systems*, Found. Phys. **7**, 905.
[572] Foulis D.J., 1999, *A half century of quantum logic – what have we learned?*, in: Aerts D., Pykacz J. (eds.), *Quantum structures and the nature of reality: the indigo book of Einstein meets Margritte*, Kluwer, Dordrecht, p.?.
[573] Foulis D.J., Randall C.H., 1979, *Tensor products of quantum logics do not exist*, Notices Amer. Math. Soc. **26**, 557.
[574] Foulis D.J., Randall C.H., 1981, *Emipirical logic and tensor products*, in: Neumann H. (ed.), *Interpretations and foundations of quantum theory*, Bibliographisches Institut Wissenschaftsverlag, Mannheim, p.9.
[575] Foulis D.J., Randall C.H., 1981, *What are quantum logics and what ought they to be?*, in: Beltrametti E.G., van Fraasen B.C. (eds.), *Current issues in quantum logic*, Plenum, New York, p.35.
[576] Fréchet M., 1906, *Sur quelques points du calcul fonctionnel*, Rend. Circ. Matem. Palermo **22**, 1.
[577] Fréchet M., 1907, *Sur les ensembles de fonctions et les opérations linéaires*, Compt. Rend. Acad. Sci. Paris **144**, 1414.
[578] Fréchet M., 1910, *Les ensembles abstraits et le calcul fonctionnel*, Rend. Circ. Mat. Palermo **30**, 1.
[579] Fréchet M., 1911, *Sur la notion de différentielle*, Comp. Rend. Acad. Sci. Paris **152**, 845, 1050.
[580] Frechét M., 1915, *Définition de l'intégrale sur un ensemble abstrait*, Compt. Rend. Hebd. Séanc. l'Acad. Sci. **160**, 839.
[581] Frechét M., 1915, *Sur l'intégrale d'une fonctionelle étendue à un ensemble abstrait*, Bull. Soc. Math. France **43**, 248. numdam:BSMF_1915__43__248_1.
[582] Fréchet M., 1922, *Familles additives et fonctions additives d'ensembles abstraits*, Enseign. Math. **22**, 113.
[583] Fréchet M., 1923, *Des familles et fonctions additives d'ensembles abstraits*, Fund. Math. **4**, 329. Available at: matwbn.icm.edu.pl/ksiazki/fm/fm4/fm4128.pdf.
[584] Fréchet M., 1923, *L'analyse générale et ensembles abstraits*, Revue de métaphysique et de monde, 1.
[585] Fréchet M., 1925, *Sur la notion de différentielle dans l'analyse générale*, Compt. Rend. Acad. Sci. Paris *180*, 806.
[586] Fréchet M., 1926, *Sur la notion de différentielle dans l'analyse générale*, Ann. Sci. École Norm. Sup. **42**, 293.
[587] Frechét M., 1943, *Sur l'extension des certaines évaluations statistiques au cas de petits énchantillous*, Rev. Inst. Intern. Statist. **11**, 183.
[588] Fredenhagen K., 1985, *On the modular structure of local algebras and observables*, Commun. Math. Phys. **97**, 79.
[589] Fredenhagen K., Haag R., 1987, *Generally covariant quantum field theory and scaling limits*, Commun. Math. Phys. **108**, 91.
[590] Fremlin D.H., 1974, *Topological Riesz spaces and measure theory*, Cambridge University Press, Cambridge.
[591] Fremlin D.H., 2000, 2001, 2002, 2003, 2008, *Measure theory*, Vol.1-5, Torres Fremlin, Colchester.
[592] Freudenthal H., 1936, *Teilweise geordnete Moduln*, Proc. Roy. Acad. Amsterdam **39**, 641.
[593] Friedrichs K.O., 1951, *Mathematical aspects of quantum field theory I, II*, Commun. Pure Appl. Math. **4**, 161.
[594] Friedrichs K.O., 1951, *Mathematical aspects of quantum field theory III, IV*, Commun. Pure Appl. Math. **5**, 1, 349.
[595] Friedrichs K.O., 1953, *Mathematical aspects of quantum field theory*, Interscience, New York.
[596] Friedrichs K.O., 1953, *Mathematical aspects of quantum field theory V*, Commun. Pure Appl. Math. **6**, 1.
[597] Frigyik B.A., Srivastava S., Gupta M.R., 2008, *Functional Bregman divergence and bayesian estimation of distributions*, IEEE Trans. Inf. Theor. **54**, 5130.
[598] Frigyik B.A., Srivastava S., Gupta M.R., 2008, *An introduction to functional derivatives*, Technical report **2008-0001**, Dept. Electr. Eng. Univ. Washington, Seattle. Available at: www.ee.washington.edu/research/guptalab/publications/functionalDerivativesIntroduction.pdf.
[599] Frölicher A., Kriegl A., 1988, *Linear spaces and differentiation theory*, Wiley, Chisterter.
[600] Fronsdal C., 1959, *Completion and embedding of the Schwarzschild solution*, Phys. Rev. **116**, 778.
[601] Fubini G., 1903, *Suelle metriche definite da una forma hermitiana*, Atti Inst. Veneto **6**, 501.
[602] Fuchs C.A., 1995, *Distinguishability and accessible information in quantum theory*, Ph.D. thesis, University of New Mexico, Albuquerque. arXiv:quant-ph/9601020.
[603] Fuchs C.A., 2001, *Quantum foundations in the light of quantum information*, in: Gonis A., Turchi P.E.A. (eds.), *Decoherence and its implications in quantum computational information transfer*, IOS Press, Amsterdam, p.38. arXiv:quant-ph/0106166.
[604] Fuchs C.A., 2003, *Notes on a Paulian idea: foundational, historical, anecdotal and forward-looking thoughts on the quantum*, Växjö University Press, Växjö. arXiv:quant-ph/0105039.
[605] Fuchs C.A., 2003, *Quantum mechanics as quantum information, mostly*, J. Mod. Opt. **50**, 987. arXiv:quant-ph/0205039.
[606] Fuchs C.A., 2010, *Qbism, the perimeter of quantum bayesianism*, arXiv:1003.5209.
[607] Fujiwara I., 1952, *Operator calculus of quantized operator*, Progr. Theor. Phys. **7**, 433.
[608] Fukamiya M., 1952, *On a theorem of Gelfand and Naimark and the B^*-algebras*, Kumamoto J. Sci. **1**, 17.
[609] Fukamiya M., Misonou M., Takeda Z., 1954, *An order and commutativity of B^*-algebras*, Tōhoku Math J. **6**, 89.
[610] Gamboa F., 1989, *Méthode du maximum d'entropie sur la moyenne et applications*, Ph.D. thesis, Université de Paris-Sud, Orsay.
[611] Gamboa F., Gassiat E., 1991, *Maximum d'entropie et problème des moments cas multidimensionnel*, Prob. Math. Statist. **12**, 67.
[612] Gamboa F., Gassiat E., 1997, *Bayesian methods and maximum entropy for ill-posed inverse problems*, Ann. Statist. **25**,

328.
[613] Gâteaux M.R., 1913, *Sur les fonctionnelles continues et les fonctionelles analytiques*, Compt. Rend. Acad. Sci. Paris **157**, 325.
[614] Gâteaux M.R., 1919, *Fonctions d'une infinité de variables indépendantes*, Bull. Soc. Math. France **47**, 70.
[615] Gâteaux M.R., 1922, *Sur les fonctionnelles continues et les fonctionelles analytiques*, Bull. Soc. Math. France **50**, 1.
[616] Gauss C.F., 1809, *Theoria motus corporum coelestium in sectionibus conicis solem ambientium*, Perthes et Besser, Hamburg. (engl. transl. 1963, *Theory of the motion of the heavenly bodies moving about the sun in conic sections*, Dover, New York).
[617] Gel'fand I.M., 1939, *On normed rings*, Dokl. Akad. Nauk SSSR **23**, 430.
[618] Gel'fand I.M., 1940, *Ideale und primäre Ideale in normierten Ringen*, Matem. Sbornik (N.S.) **9**, 41.
[619] Gel'fand I.M., 1941, *Normierte Ringe*, Matem. Sbornik (N.S.) **9**, 3.
[620] Gel'fand I.M., Naĭmark M.A., 1943, *On the imbedding of normed rings into the ring of operators in Hilbert space*, Matem. Sbornik (N.S.) **12**, 197.
[621] Gel'fand I.M., Naĭmark M.A., 1948, *Normirovannye kol'ca s involyucieĭ i ikh predstavleniya*, Izv. Akad. Nauk SSSR Ser. Mat. **12**, 445.
[622] Gel'fand I.M., Raĭkov D.A., 1943, *Neprivodimye unitarnye predstavleniya lokal'no bikompaktnykh grupp*, Matem. Sbornik (N.S.) **13**, 301.
[623] Gel'fand I.M., Raĭkov D.A., Shilov G.E., 1946, *Kommutativnye normirovannye kol'ca*, Usp. Mat. Nauk **1**, 48.
[624] Gel'fand I.M., Shilov G.E., 1941, *Über verschiedene Methoden der Einführung der Topologie in die Menge der maximalen Ideale eines normierten Ringes*, Matem. Sbornik (N.S.) **9**, 25.
[625] Gel'fand I.M., Vilenkin N.Ya., 1961, *Obobshhennye funkcii, vypusk 4. Nekotorye primeneniya garmonicheskogo analiza. Osnashhennye gilbertovy prostranstva*, Gosudarstvennoe Izdatel'stvo Fiziko-Matematicheskoĭ Literatury, Moskva (engl. transl. 1964, *Generalized functions, volume 4. Some applications of harmonic analysis. Rigged Hilbert spaces*, Academic Press, New York).
[626] Giachetta G., Mangiarotti L., Sardanashvily G., 2005, *Geometric and algebraic topological methods in quantum mechanics*, World Scientific, Singapore.
[627] Gibbs J.W., 1902, *Elementary principles in statistical mechanics, developed with especial reference to the rational foundation of thermodynamics*, Yale University Press, New Heaven.
[628] Gibilisco P., Isola T., 1999, *Connections on statistical manifolds of density operators by geometry of non-commutative L^p spaces*, Infin. Dim. Anal. Quant. Prob. Relat. Topics **2**, 169. Available at: www.mat.uniroma2.it/∼isola/research/preprints/GiIs01.pdf.
[629] Gibilisco P., Isola T., 2001, *Monotone metrics on statistical manifolds of density matrices by geometry of non-commutative L^2-spaces*, in: Sollich P., Hughston L.P., Streater R.F., Coolen A.C.C. (eds.), *Disordered and complex systems (King's College, London, UK, 10-14 July 2000)*, AIP Conf. Proc. **553**, AIP, Melville, p.129. Available at: www.mat.uniroma2.it/∼isola/research/preprints/GiIs02.pdf.
[630] Gibilisco P., Isola T., 2003, *Wigner–Yanase information on quantum state space: the geometric approach*, J. Math. Phys. **44**, 3752. Available at: /www.mat.uniroma2.it/∼isola/research/preprints/GiIs05.pdf.
[631] Gibilisco P., Pistone G., 1998, *Connections on non-parametric statistical manifolds by Orlicz space geometry*, Inf. Dim. Anal. Quant. Prob. Relat. Top. **1**, 325.
[632] Gibilisco P., Pistone G., 1999, *Analytical and geometrical properties of statistical connections in information geometry*, in: Beghi A., Finesso L., Picci G. (eds.), *Mathematical Theory of Networks and Systems (Padova, Italy, July 6-10 1998)*, Il Poligrafo, Padova, p.811.
[633] Giffin A., 2009, *Maximum entropy: the universal method for inference*, PhD thesis, State University of New York, Albany. arXiv:0901.2987.
[634] Giles R., 1974, *A non-classical logic for physics*, Studia Logica **33**, 397.
[635] Giles R., 1976, *Łukasiewicz logic and fuzzy set theory*, Int. J. Man-Mach. Stud. **67**, 313.
[636] Giles R., 1977, *A nonclassical logic for physics*, in: Wójcicki R. (ed.), *Selected papers on Łukasiewicz sentential calculi*, Ossolineum, Wrocław, p.13.
[637] Giry M., 1982, *A categorical approach to probability theory*, in: Banaschewski B. (ed.), *Categorical Aspects of Topology and Analysis Proceedings of an International Conference Held at Carleton University, Ottawa, August 11-15, 1981*, LNM **915**, Springer, Berlin, p.68.
[638] Givant S., Halmos P., 2009, *Introduction to boolean algebras*, Springer, New York.
[639] Gleason A.M., 1957, *Measures on the closed subspaces of a Hilbert space*, J. Math. Mech. **6**, 885.
[640] Gleason A.M., 1958, *Projective topological spaces*, Illinois J. Math. **2**, 223.
[641] Glimm J., Kadison R.V., 1960, *Unitary operators in C^*-algebras*, Pac. J. Math. **10**, 547.
[642] Godefroy G., 1989, *Existence and uniqueness of isometric preduals: a survey*, in: Bor-Luh Lin (ed.), *Workshop on Banach space theory: proceedings of a research workshop held July 5–25, 1987*, Contemp. Math. **85**, Amer. Math. Soc., Providence, p.131.
[643] Gokhberg I.C., Kreĭn M.G., 1965, *Vvedenie v teoriyu lineĭnykh nesamosopryazhennykh operatorov v gilbertovom prostranstve*, Nauka, Moskva (engl. transl. 1969, *Introduction to the theory of nonselfadjoint operators in Hilbert space*, American Mathematical Society, Providence).
[644] Goldstein D., 1995, *Polar decomposition in Rickart C^*-algebras*, Publ. Mat. **39**, 5.
[645] Goldstein S., 2002, *Niekomutatywne przestrzenie L^p*, preprint, available at: math.uni.lodz.pl/∼goldstei/download/papers/np_lp.pdf.
[646] Goldstein S., Phan V.T., 2000, *L^p spaces for C^*-algebras with a state*, Int. J. Theor. Phys. **39**, 687.
[647] Goldstine H.H., 1941, *Linear functionals and integrals in abstract spaces*, Bull. Amer. Math. Soc. **47**, 615.
[648] Golodec V.Ya., 1972, *Uslovnye ozhidaniya i modulyarnye avtomorfizmy neĭmanovskikh algebr*, Funkc. Anal. Prilozh. **6**, 68. (engl. transl. 1972, *Conditional expectation and modular automorphisms of von Neumann algebras*, Funct. Anal. Appl. **6**, 231).
[649] Gomes A.P., 1946, *Introduçãoao estudo duma noção de funcional em espaços sem pontos*, Portug. Mat. **5**, 1.
[650] Good I.J., 1950, *Probability and the weighing of evidence*, Griffin, London.
[651] Good I.J., 1963, *Maximum entropy for hypothesis formulation, especially for multidimensional contingency tables*, Ann. Math. Statist. **34**, 911.
[652] Gordon J.P., Louisell W., 1966, *Simultaneous measurements of non-commuting observables*, in: Kelley P., Lax M., Tannenwold B. (eds.), *Physics of quantum electronics*, McGraw–Hill, New York, p.833.
[653] Gorini V., Frigerio A., Kossakowski A., Sudarshan E.C.G., 1978, *Properties of quantum markovian master equations*, Rep.

Math. Phys. **13**, 149.
[654] Gorini V., Kossakowski A., Sudarshan E.C.G., 1976, *Completely positive dynamical semigroups of N-level systems*, J. Math. Phys. **17**, 821.
[655] Grabert H., 1982, *Projection operator techniques in non-equilibrium statistical mechanics*, Springer, Berlin.
[656] Grabowski M., 1989, *What is an observable?*, Found. Phys. **19**, 923.
[657] Grabowski M., Ingarden R.S., 1989, *Mechanika kwantowa. Ujęcie w przestrzeni Hilberta*, Państwowe Wydawnictwo Naukowe, Warszawa.
[658] Grandy W.T. Jr., 1987, 1988, *Foundations of statistical mechanics*, Vol.1-2, Reidel, Dordrecht.
[659] Grandy W.T. Jr., 2008, *Entropy and the time evolution of macroscopic systems*, Oxford University Press, Oxford.
[660] Granet M., 1934, *La pensée chinoise*, La Renaissance du livre, Paris.
[661] Gårding L., Wightman A.S., 1954, *Representations of the anticommutation relations*, Proc. Natl. Acad. Sci. USA **40**, 617.
[662] Gårding L., Wightman A.S., 1954, *Representations of the commutation relations*, Proc. Natl. Acad. Sci. USA **40**, 622.
[663] Grasselli M.R., 2001, *Classical and quantum information geometry*, PhD thesis, King's College, London.
[664] Grasselli M.R., 2001, *Dual connections in nonparametric classical information geometry*, arXiv:math-ph/0104031.
[665] Grasselli M.R., 2004, *Duality, monotonicity and the Wigner–Yanase–Dyson metrics*, Inf. Dim. Anal. Quant. Prob. Relat. Top. **7**, 215. arXiv:math-ph/0212022.
[666] Grasselli M.R., 2008, *Dual connections in nonparametric classical information geometry*, Ann. Inst. Stat. Math. DOI:**10.1007/s10463-008-0191-3**.
[667] Grasselli M.R., Streater R.F., 2000, *Quantum information manifolds for ϵ-bounded forms*, Rep. Math. Phys. **46**, 325.
[668] Grasselli M.R., Streater R.F., 2001, *On the uniqueness of the Chentsov metric in quantum information geometry*, Inf. Dim. Anal. Quant. Prob. Rel. Top. **4**, 173. arXiv:math-ph/0006030.
[669] Grasselli M.R., Streater R.F., 2001, *The uniqueness of the Chentsov metric*, in: Sollich P., Hughston L.P., Streater R.F., Coolen A.C.C. (eds.), *Disordered and complex systems (King's College, London, UK, 10-14 July 2000)*, AIP Conf. Proc. **553**, AIP, Melville, p.165. Available at: www.math.mcmaster.ca/∼grasselli/bkmshort.pdf.
[670] Graudenz D., 1994, *On the space-time geometry of quantum systems*, arXiv:gr-qc/9412013.
[671] Graudenz D., 1996, *The quantum gauge principle*, arXiv:hep-th/9604180.
[672] Greechie R.J., Gudder S.P., 1973, *Quantum logics*, in: Hooker C.A. (ed.), *Contemporary research in the foundations and philosophy of quantum theory: proceedings of a conference held at the University of Western Ontario, London, Canada*, Reidel, Dordrecht, p.143.
[673] Grenander U., 1981, *Abstract inference*, Wiley, New York.
[674] Grinbaum A., 2004, *Le rôle de l'information dans la théorie quantique (the significance of information in quantum theory)*, Ph.D. thesis, l'École Polytechnique, Paris. arXiv:quant-ph/0401007.
[675] Gross L., 1972, *Existence and uniqueness of physical ground states*, J. Funct. Anal. **10**, 52.
[676] Grothendieck A., 1955, *Réarrangements de fonctions et inégalités de convexité dans les algèbres de von Neumann munies d'une trace*, Sém. Bourbaki **113**, 1.
[677] Gudder S., Marchand J.-P., Wyss W., 1979, *Bures distance and relative entropy*, J. Math. Phys. **20**, 1963.
[678] Gudder S.P., 1965, *Spectral methods for a generalized probability theory*, Trans. Amer. Math. Soc. **119**, 428.
[679] Gudder S.P., 1966, *Uniqueness and existence properties of systems of bounded observables*, Pacif. J. Math. **19**, 81.
[680] Gudder S.P., 1968, *Joint distribution of observables*, J. Math. Mech. **18**, 325.
[681] Gudder S.P., 1970, *Axiomatic quantum mechanics and generalized probability theory*, in: Bharucha-Reid A.T. (ed.), *Probabilistic methods in applied mathematics*, Vol.2, Academic Press, New York, p.53.
[682] Gudder S.P., 1973, *Convex structures and operational quantum mechanics*, Commun. Math. Phys. **29**, 249.
[683] Gudder S.P., 1977, *Four approaches to axiomatic quantum mechanics*, in: Price W., Chissick S. (eds.), *The uncertainty principle and foundations of quantum mechanics: a fifty years' survey*, Wiley, New York, p.247.
[684] Gudder S.P., 1979, *A Radon–Nikodym theorem for *-algebras*, Pacific J. Math. **80**, 141.
[685] Gudder S.P., 1979, *Stochastic methods in quantum mechanics*, North–Holland, New York.
[686] Gudder S.P., 1979, *A survey of axiomatic quantum mechanics*, in: Hooker C.A. (ed.), *The logico-algebraic approach to quantum mechanics. Volume II: contemporary consolidation*, p.323.
[687] Gudder S.P., 1980, *Statistical inference in quantum mechanics*, Rep. Math. Phys. **17**, 265.
[688] Gudder S.P., 1981, *Comparison of the quantum logic, convexity, and algebraic approaches to quantum mechanics*, in: Neumann H. (ed.), *Interpretations and foundations of quantum theory*, Bibliographisches Institut Wissenschaftsverlag, Mannheim, p.125.
[689] Gudder S.P., 1986, *Partial Hilbert spaces and amplitude functions*, Ann. Inst. Henri Poincaré **45**, 311.
[690] Gudder S.P., 1988, *Quantum probability*, Academic Press, Boston.
[691] Gudder S.P., 2004, *Noncommutative probability and applications*, in: Rao M.M. (ed.), *Real and stochastic analysis: new perspectives*, Birkhäuser, Basel, p.199.
[692] Gudder S.P., Boyce S., 1970, *A comparison of the Mackey and Segal models for quantum mechanics*, Int. J. Theor. Phys. **3**, 7.
[693] Gudder S.P., Hudson R., 1978, *A noncommutative probability theory*, Trans. Amer. Math. Soc. **245**, 1.
[694] Gudder S.P., Marchand J.-P., 1972, *Noncommutative probability on von Neumann algebras*, J. Math. Phys. **13**, 799.
[695] Gudder S.P., Mullikin H.C., 1973, *Measure theoretic convergences of observables and operators*, J. Math. Phys. **14**, 234.
[696] Gudder S.P., Pulmannová S., 1987, *Transition amplitude spaces*, J. Math. Phys. **28**, 376.
[697] Guenin M., 1966, *Axiomatic formulations of quantum theories*, J. Math. Phys. **7**, 271.
[698] Guenin M., Misra B., 1963, *On the von Neumann algebras generated by field operators*, Nuovo Cim. **30**, 1272.
[699] Guichardet A., 1974, *Systèmes dynamiques non commutatifs*, Astérisque **13-14**, Société Mathématique de France, Paris.
[700] Gunson J., 1972, *Physical states on quantum logics*, Ann. Inst. Henri Poincaré Ser. A **17**, 295. numdam:AIHPA_1972__17_4_295_0.
[701] Guz W., 1980, *A non-symmetric transition probability in quantum mechanics*, Rep. Math. Phys. **17**, 385.
[702] Györfi L., Nemetz T., 1978, *f-dissimilarity: a generalization of the affinity of several distribution*, Ann. Inst. Statist. Math. **30**, 105.
[703] Gzyl H., 1994, *The method of maximum entropy*, World Scientific, Singapore.
[704] Haag R., 1955, *On quantum field theories*, Kong. Dansk. Vidensk. Mat.-fys. Medd. **29**, 1.
[705] Haag R., 1959, *Discussion des «axiomes»et des propriétes asymptotiques d'une théorie des champs locale avec particules composés*, in: Deheuvels R., Michel L. (eds.), *Les problèmes mathématiques de la théorie quantique des champs. Lille 3-8 Juin, 1957*, Centr. Natl. Rech. Sci., Paris, p.151 (orig. engl. ver. publ. 2010, *Discussion of the 'axioms' and the asymptotic*

properties of a local field theory with composite particles, Eur. Phys. J. H **35**, 243).
[706] Haag R., 1992, *Local quantum physics*, Springer, Berlin. (1996, 2nd rev. enlarg. ed.).
[707] Haag R., Hugenholtz N.M., Winnink M., 1967, *On the equilibrium states in quantum statistical mechanics*, Commun. Math. Phys. **5**, 215.
[708] Haag R., Kastler D., 1964, *An algebraic approach to quantum field theory*, J. Math. Phys. **5**, 848.
[709] Haag R., Kastler D., Trych-Pohlmeyer E.B., 1974, *Stability and equilibrium states*, Commun. Math. Phys. **38**, 173.
[710] Haag R., Narnhofer H., Stein U., 1984, *On quantum field theories in gravitational background*, Commun. Math. Phys. **94**, 219.
[711] Haag R., Ojima I., 1996, *On the problem of defining a specific theory within the family of local quantum physics*, Ann. Inst. Henri Poincaré **64**, 385.
[712] Haag R., Schroer B., 1962, *Postulates of quantum field theory*, J. Math. Phys. **3**, 248.
[713] Haag R., Trych-Pohlmeyer E.B., 1977, *Stability properties of equilibrium states*, Commun. Math. Phys. **56**, 214.
[714] Haagerup U., 1973, *The standard form of von Neumann algebras*, Preprint Ser. 1973 No. **15**, Københavns Universitet Matematisk Institut, København.
[715] Haagerup U., 1975, *Normal weights on W^*-algebras*, J. Funct. Anal. **19**, 302.
[716] Haagerup U., 1975, *The standard form of von Neumann algebras*, Math. Scand. **37**, 271.
[717] Haagerup U., 1976, *Operator valued weights and crossed products*, in: *Convegno sulle algebre C^* e loro applicazioni in fisica teoria (12-18 Marzo 1975, Convegno sulla teoria degli operatori indice e teoria K (13-18 Ottobre 1975), del Istituto Nazionale di Alta Matematica, Roma*, Symposia Mathematica **20**, Academic Press, New York, p.241.
[718] Haagerup U., 1977, *An example of a weight with type III centralizer*, Proc. Amer. Math. Soc. **62**, 278.
[719] Haagerup U., 1978, *On the dual weights for crossed products of von Neumann algebras I: removing separability conditions*, Math. Scand. **43**, 99.
[720] Haagerup U., 1978, *On the dual weights for crossed products of von Neumann algebras II: application of operator valued weights*, Math. Scand. **43**, 119.
[721] Haagerup U., 1979, *L^p-spaces associated with an aribitrary von Neumann algebra*, in: *Algèbres d'opérateurs et leurs applications en physique mathématique (Proc. Colloques Internationaux, Marseille 20-24 juin 1977)*, Colloques Internationaux C.N.R.S. **274**, Éditions du C.N.R.S., Paris, p.175. Available at: dmitripavlov.org/scans/haagerup.pdf.
[722] Haagerup U., 1979, *Operator valued weights in von Neumann algebras I*, J. Funct. Anal. **32**, 175.
[723] Haagerup U., 1979, *Operator valued weights in von Neumann algebras II*, J. Funct. Anal. **33**, 339.
[724] Haagerup U., 1987, *Connes' bicentralizer problem and uniqueness of the injective factor of type III_1*, Acta Math. **158**, 95.
[725] Haagerup U., Junge M., Xu Q., 2010, *A reduction method for noncommutative L_p-spaces and applications*, Trans. Amer. Math. Soc. **362**, 2125.
[726] Haagerup U., Rosenthal H.P., Sukochev F.A., 2003, *Banach embedding properties of noncommutative L^p-spaces*, Memoirs Amer. Math. Soc. **163** no. 776, American Mathematical Society, Providence.
[727] Haar A., 1933, *Der Maßbegriff in der Theorie der kontinuierlichen Gruppen*, Ann. Math. **34**, 147.
[728] Habil E., 1993, *Orthoalgebras and noncommutative measure theory*, Ph.D. thesis, Kansas State University.
[729] Hadjisavvas N., 1981, *Distance between states and statistical inference in quantum theory*, Ann. Inst. Henri Poincaré A **38**, 167.
[730] Hadjisavvas N., 1982, *On Cantoni's generalized transition probability*, Commun. Math. Phys. **83**, 43.
[731] Hadjisavvas N., 1986, *Metrics on the set of states of a W^*-algebra*, Lin. Alg. Appl. **84**, 281.
[732] Hahn H., 1927, *Über lineare Gleichungssysteme in linearen Räumen*, J. Reine Angew. Math. **157**, 214.
[733] Hájek A., 1997, *Mises redux' – redux: fifteen arguments against finite frequentism*, Erkenntnis **45**, 209.
[734] Haldane J.B.S., 1951, *A class of efficient estimators of a parameter*, Bull. Int. Statist. Inst. **33**, 234.
[735] Hall D., Wightman A.S., 1957, *A theorem on invariant analytic functions with applications to relativistic quantum field theory*, Kong. Dansk. Vidensk. Selsk., Mat.-fys. Medd. **31**, 1.
[736] Halmos P.R., 1950, *Measure theory*, Van Nostrand, New York.
[737] Halmos P.R., Savage L.J., 1949, *Applications of the Radon–Nikodym theorem to the theory of sufficient statistics*, Ann. Math. Statist. **20**, 225.
[738] Halpern H., 1972, *Unitary implementation of automorphism groups of von Neumann algebras*, Commun. Math. Phys. **25**, 253.
[739] Halvorson H., 2004, *Remote preparation of arbitrary ensembles and quantum bit commitment*, J. Math. Phys. **45**, 4920. arXiv:quant-ph/0310001.
[740] Halvorson H., 2007, *Algebraic quantum field theory (with an appendix by Michael Müger)*, in: Butterfield J., Earman J. (eds.), *Handbook of the philosophy of physics*, p.731. arXiv:math-ph/0602036.
[741] Hamhalter J., 2002, *Noncommutative phenomena in measure theory on operator algebras*, Rend. Instit. Mat. Univ. Trieste **34**, 19. Available at: www.dmi.units.it/∼rimut/volumi/34/03-hamhalter.ps.gz.
[742] Hamhalter J., 2003, *Quantum measure theory*, Kluwer, Dordrecht.
[743] Hamhalter J., 2004, *States and structure of von Neumann algebras*, Int. J. Theor. Phys. **43**, 1561.
[744] Hamhalter J., 2007, *Quantum structures and operator algebras*, in: Engesser K., Gabbay D.M., Lehmann D. (eds.), *Handbook of quantum logic and quantum structures: quantum structures*, Elsevier, Amsterdam, p.285.
[745] Hansen F., 2006, *Characterization of symmetric monotone metrics on the state space of quantum systems*, Quant. Inf. Comput. **6**, 597. arXiv:math-ph/0601056.
[746] Hansen F., 2008, *Metric adjusted skew information*, Proc. Nat. Acad. Sci. U.S.A. **105**, 9909. arXiv:math-ph/0607049.
[747] Hansen F., Pedersen G.K., 1982, *Jensen's inequality for operators and Löwner's theorem*, Math. Ann. **258**, 229.
[748] Hardegree G.M., 1977, *Reichenbach and the logic of quantum mechanics*, Synthese **35**, 3.
[749] Hardegree G.M., 1981, *Some problems and methods in formal quantum logic*, in: Beltrametti E.G., van Fraasen B.C. (eds.), *Current issues in quantum logic*, Plenum Press, New York, p.209.
[750] Hardegree G.M., Frazer P.J., 1981, *Charting the labyrinth of quantum logics: a progress report*, in: Beltrametti E.G., van Fraasen B.C. (eds.), *Current issues in quantum logic*, Plenum Press, New York, p.53.
[751] Harremoës P., Tishby N., 2007, *The information bottleneck revisited or how to choose a good distortion measure*, in: *Proceedings of the 2007 IEEE International Symposium on Information Theory*, IEEE Information Theory Society, Washington, p.566.
[752] Hartigan J.A., 1964, *Invariant prior distributions*, Ann. Math. Statist. **35**, 836.
[753] Hartigan J.A., 1965, *The asymptotically unbiased prior distribution*, Ann. Math. Statist. **36**, 1137.
[754] Hartigan J.A., 1967, *The likelihood and invariance principles*, J. Roy. Statist. Soc. **29**, 533.

[755] Hartigan J.A., 1967, *Representation of similarity matrices by trees*, J. Am. Stat. Assoc. **62**, 1140.
[756] Hartley R.V.L., 1928, *Transmission of information*, Bell Sys. Tech. J. **7**, 535.
[757] Hasegawa H., 1993, *α-divergence of the non-commutative information geometry*, Rep. Math. Phys. **33**, 87.
[758] Hasegawa H., 1995, *Noncommutative extension of information geometry*, in: Belavkin V.P., Hirota O., Hudson R.L. (eds.), *Proceedings of an International Workshop on Quantum Communications and Measurement, July 11-16, 1994, in Nottingham, England*, Plenum, New York, p.327.
[759] Hasegawa H., 1997, *Exponential and mixture families in quantum statistics: dual structure and unbiased parameter estimation*, Rep. Math. Phys. **39**, 49.
[760] Hasegawa H., 2003, *Dual geometry of the Wigner–Yanase–Dyson information content*, Inf. Dim. Anal. Quant. Prob. Relat. Top. **6**, 413.
[761] Hasegawa H., Petz D., 1997, *Non-commutative extension of information geometry II*, in: Hirota O. et al. (eds.), *Quantum communication, computing and measurements*, Plenum, New York, p.109. Available at: www.renyi.hu/∼petz/pdf/75.pdf.
[762] Hasegawa M., Perlman M.D., 1974, *On the existence of a minimal sufficient subfield*, Ann. Statist. **2**, 1049 (corr. 1975, Ann. Statist. **3**, 1371).
[763] Hausdorff F., 1914, *Grundzüge der Mengenlehre*, von Veit, Leipzig (engl. transl.: 1962, *Set theory*, Chelsea, New York).
[764] Havrda J., Chárvat F., 1967, *Quantification method of classification processes. The concept of structural α-entropy*, Kybernetika **3**, 30. Available at: www.kybernetika.cz/content/1967/1/30/paper.pdf.
[765] Hegerfeldt G.C., Kraus K., Wigner E.P., 1968, *Proof of the fermion superselection rule without the assumption of time-reversal invariance*, J. Math. Phys. **9**, 2029.
[766] Heims S.P., Jaynes E.T., 1962, *Theory of gyromagnetic effects and some related magnetic phenomena*, Rev. Mod. Phys. **34**, 143.
[767] Heisenberg W., 1925, *Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen*, Z. Phys. **33**, 879 (engl. transl. 1968, *Quantum-theoretical re-interpretation of kinematical and mechanical relations*, in: van der Waerden B.L. (ed.), *Sources of quantum mechanics*, Dover, Toronto, p.261).
[768] Heisenberg W., 1926, *Schwankungserscheinungen in der Quantenmechanik*, Z. Phys. **40**, 501.
[769] Heisenberg W., 1927, *Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik*, Z. Phys. **43**, 172.
[770] Hellinger E., 1909, *Neue Begründung der Theorie quadratischen Formen von unendlich vielen Veränderlichen*, J. Reine Angew. Math. **36**, 210.
[771] Hellinger E., Toeplitz O., 1910, *Grundlagen für eine Theorie der unendlichen Matrizen*, Math. Ann. **69**, 289.
[772] Hellwig K.E., Kraus K., 1969, *Pure operations and measurements*, Commun. Math. Phys. **11**, 214.
[773] Hellwig K.E., Kraus K., 1970, *Operations and measurements II*, Commun. Math. Phys. **16**, 142.
[774] Hellwig K.E., Kraus K., 1971, *Formal description of measurements in local quantum field theory*, Phys. Rev. D **1**, 566.
[775] Helstrom C.W., 1967, *Minimum mean-square error estimation in quantum statistics*, Phys. Lett. A **25**, 101.
[776] Helstrom C.W., 1968, *The minimum variance of estimates in quantum signal detection*, IEEE Trans. Inf. Theor. **14**, 234.
[777] Helstrom C.W., 1976, *Quantum detection and estimation theory*, Academic Press, New York.
[778] Hempel C.G., 1945, *Review of Philosophic foundations of quantum mechanics by Hans Reichenbach*, J. Symb. Log. **10**, 97.
[779] Henle M., 1970, *Spatial representation of groups of automorphisms of von Neumann algebras with properly infinite commutants*, Commun. Math. Phys. **19**, 273.
[780] Hepp K., 1972, *Quantum theory of measurement and macroscopic observables*, Helv. Phys. Acta **45**, 237.
[781] Hessenberg G., 1917, *Vektorielle Begründung der Differentialgeometrie*, Math. Ann. **78**, 187.
[782] Hiai F., 2010, *Matrix analysis: matrix monotone functions, matrix means, and majorization*, Interd. Inf. Sci. **16**, 139.
[783] Hiai F., Mosonyi M., Petz D., Bény C., 2011, *Quantum f-divergences and error correction*, Rev. Math. Phys. **23**, 691. arXiv:1008.2529.
[784] Hiai F., Ohya M., Tsukada M., 1981, *Sufficiency, KMS condition and relative entropy in von Neumann algebras*, Pacific J. Math. **96**, 99.
[785] Hiai F., Ohya M., Tsukada M., 1983, *Sufficiency and relative entropy in $*$-algebras with applications in quantum systems*, Pacific J. Math. **107**, 117.
[786] Hiai F., Petz D., 2000, *The semicircle law, free random variables, and entropy*, American Mathematical Society, Providence.
[787] Hilbert D., 1904, *Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen I*, Nachr. Königl. Gesselsch. Wissensch. Göttingen Math.-phys. Kl. **1904**, 49.
[788] Hilbert D., 1905, *Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen II*, Nachr. Königl. Gesselsch. Wissensch. Göttingen Math.-phys. Kl. **1905**, 213.
[789] Hilbert D., 1906, *Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen III, IV, V*, Nachr. Königl. Gesselsch. Wissensch. Göttingen Math.-phys. Kl. **1906**, 157, 307, 439.
[790] Hilbert D., 1910, *Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen VI*, Nachr. Königl. Gesselsch. Wissensch. Göttingen Math.-phys. Kl. **1910**, 355.
[791] Hilbert D., 1915, *Die Grundlagen der Physik*, Konigl. Gesell. Wiss. Göttingen, Nachr. Math.-Phys. Kl. **1915**, 395.
[792] Hilbert D., von Neumann J., Nordheim L., 1927, *Über die Grundlagen der Quantenmechanik*, Math. Ann. **98**, 1.
[793] Hille E., 1948, *Functional analysis and semi-groups*, American Mathematical Society, New York.
[794] Hilsum M., 1981, *Les espaces L^p d'une algèbre de von Neumann définies par la dérivée spatiale*, J. Funct. Anal. **40**, 151.
[795] Hinkelmann K., Kempthorne O., 2005, *Design and analysis of experiments*, Vol.1-2, Wiley, Hoboken (2nd ed. of Vol.1, 2008).
[796] Hirakawa M., 1992, *A generalization of φ-conditional expectation and operator valued weight*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **28**, 289.
[797] Hobson A., 1969, *A new theorem of information theory*, J. Stat. Phys. **1**, 383.
[798] Hobson A., 1971, *Concepts in statistical mechanics*, Gordon & Breach, New York.
[799] Hobson A., Cheng B.-K., 1973, *A comparison of the Shannon and Kullback information measures*, J. Stat. Phys. **7**, 301.
[800] Hölder O., 1889, *Über einen Mittelwertsatz*, Nachr. Gesellsch. Wissensch. Göttingen **38**, 38.
[801] Holdsworth D.G., Hooker C.A., 1983, *A critical survey of quantum logic*, in: Dalla Chiara M.L. (ed.), *Logic in the 20th Century*, Scientia, Milano, p.127.
[802] Holevo A.S. (Kholevo A.S.), 1972, *[Towards mathematical theory of quantum communication channels]*, Probl. Pered. Inf. **8**, 62.
[803] Holevo A.S. (Kholevo A.S.), 1973, *Nekotorye ocenki dlya kolichestva informacii peredavaemogo kvantovym kanalom svyazi*, Probl. Pered. Inf. **9**, 3.
[804] Holevo A.S. (Kholevo A.S.), 1976, *Issledovaniya obshheĭ teorii statisticheskikh resheniĭ*, Trudy Matem. Inst. V.A. Steklova

124, Nauka, Moskva (engl. transl. 1978, *Investigations in the general theory of statistical decisions*, Proc. Steklov Inst. Math. **124**, American Mathematical Society, Providence).
[805] Holevo A.S. (Kholevo A.S.), 1976, *Noncommutative analogs of the Cramer–Rao inequality in the quantum measurement theory*, in: *Proceedings of srd Japan–USSR symposium on probability theory*, LNM **550**, Springer, Berlin, p.194.
[806] Holevo A.S. (Kholevo A.S.), 1977, *Commutative superoperator of a state and its application in the noncommutative statistics*, Rep. Math. Phys. **12**, 251.
[807] Holevo A.S. (Kholevo A.S.), 1980, *Veroyatnostnye i statisticheskie aspekty kvantovoĭ teorii*, Nauka, Moskva. (engl. transl.: 1982, *Probabilistic and statistical aspects of quantum theory*, North-Holland, Amsterdam; 2nd rev. ed. 2003, IKI, Moskva/Izhevsk; 2nd rev. ed. engl. tranl. 2011, Edizioni della Normale, Pisa).
[808] Holevo A.S. (Kholevo A.S.), 1991, *Kvantovaya veroyatnost' i kvantovaya statistika*, Itogi Nauk. Tekhn. Ser. Sovr. Probl. Math. Fundam. Napravl. **83**, 5.
[809] Holevo A.S. (Kholevo A.S.), 2001, *Statistical structure of quantum theory*, LNP **67**, Springer, Berlin.
[810] Horn A., Tarski A., 1948, *Measures in boolean algebras*, Trans. Amer. Math. Soc. **64**, 467. Available at: www.ams.org/journals/tran/1948-064-03/S0002-9947-1948-0028922-8/S0002-9947-1948-0028922-8.pdf.
[811] Horuzhy S.S. (Khoruzhiĭ S.S.), 1986, *Vvedenie v algebraicheskuyu kvantovuyu teoriyu polya*, Moskva, Nauka (engl. transl. 1990, *An introduction to algebraic quantum field theory*, Kluwer, Dordrecht).
[812] Howson C., Urbach P., 1989, *Scientific reasoning: the bayesian approach*, Open Court, La Salle (2nd. ed., 1993).
[813] Huber P.J., 2002, *Discussion*, Ann. Statist. **30**, 1289.
[814] Hübner M., 1992, *Explicit computation of the Bures distance for density matrices*, Phys. Lett. A **163**, 239.
[815] Hugenholtz N.M., 1967, *On the factor type of equilibrium states in quantum statistical mechanics*, Commun. Math. Phys. **6**, 189.
[816] Huijsmans C.B., de Pagter B., 1982, *Ideal theory in f-algebras*, Trans. Amer. Math. Soc. **269**, 225.
[817] Huijsmans C.B., de Pagter B., 1984, *Subalgebras and Riesz subspaces of an f-algebra*, Proc. London Math. Soc. **48**, 161.
[818] Hurwitz A., 1897, *Ueber die Erzeugung der Invarianten durch Integration*, Nachr. Königl. Gesellsch. Wisennsch. Göttingen, Math.-phys. Kl. **1897**, 71.
[819] Husimi K., 1937, *Studies on the foundations of qunautm mechanics I*, Proc. Phys.-Math. Soc. Japan **19**, 766.
[820] Huygens C., 1657, *De ratiocinniis in ludo aleæ*, Johannis Elsevirri, Leiden. Available at: www.leidenuniv.nl/fsw/verduin/stathist/huygens/1657.pdf (engl. transl. 1714, *Libellus de ratiociniis in ludo aleæ*, Keimer, London. Available at: www.stat.ucla.edu/history/huygens.pdf).
[821] Ibragimov I.A., Has'minskiĭ R.Z., 1979, *Asimptoticheskaya teoriya ocenivaniya*, Nauka, Moskva (engl. transl. 1981, *Statistical estimation: asymptotic theory*, Springer, New York).
[822] Im G.S., 1996, *Experimental constraints on formal quantum mechanics: the emergence of Born's quantum theory of collision processes in Göttingen, 1924–1927*, Arch. Hist. Exact Sci. **50**, 73.
[823] Imparato D., 2008, *Exponential models and Fisher information*, Ph.D. thesis, Politecnico di Torino, Torino.
[824] Imparato D., Trivellato B., 2010, *Geometry of extended exponential models*, in: Gibilisco P. et al (eds.), *Algebraic and geometric methods in statistics*, Cambridge University Press, Cambridge, p.307.
[825] Ingarden R.S., 1963, *Information theory and variational principles in statistical theories*, Bull. Acad. Polon. Sci. Sér. Math. Astr. Phys. **11**, 541.
[826] Ingarden R.S., 1974, 1976, *Information theory and thermodynamics*, Inst. of Physics Preprints No. **270**, **275**, **276**, **330**, N. Copernicus Univ., Toruń.
[827] Ingarden R.S., 1981, *Information geometry in function spaces of classical and quantum finite statistical systems*, Intern. J. Engineering Sci. **19**, 1609.
[828] Ingarden R.S., Janyszek H., Kossakowski A., Kawaguchi T., 1982, *Information geometry of quantum statistical systems*, Tensor (N.S.) **37**, 105.
[829] Ingarden R.S., Kossakowski A., 1975, *On the connection of nonequilibrium information thermodynamics with non-hamiltonian quantum mechanics of open systems*, Ann. Phys. **89**, 451.
[830] Ingarden R.S., Kossakowski A., 1986, *The principle of maximum entropy and Boltzmann type kinetic equations*, Rep. Math. Phys. **24**, 177.
[831] Ingarden R.S., Kossakowski A., Ohya M., 1997, *Information dynamics and open systems – classical and quantum approach*, Kluwer, Dordrecht.
[832] Ingarden R.S., Urbanik K., 1961, *Information as a fundamental notion of statistical physics*, Bull. Acad. Polon. Sci., Sér. Sci. Math. Astronom. Phys. **9**, 313.
[833] Ingarden R.S., Urbanik K., 1962, *Information without probability*, Colloq. Math. **9**, 131.
[834] Ingarden R.S., Urbanik K., 1962, *Quantum informational thermodynamics*, Acta Phys. Polon. **21**, 281.
[835] Iochum B., 1984, *Cônes autopolaires et algèbres de Jordan*, Springer, Berlin.
[836] Ischi B., 2000, *Endomorphisms of the separated product of lattices*, Int. J. Theor. Phys. **39**, 2559.
[837] Ishwar P., Moulin P., 2005, *On the existence and characterization of the maxent distribution under general moment inequality constraints*, IEEE Trans. Inf. Theor. **51**, 3322. arXiv:cs/0506013.
[838] Ivanickaya O.S., 1979, *Lorencev bazis i gravitacionnye e'ffekty v e'ĭnshteĭnovoĭ teorii tyagoteniya*, Nauka i tekhnika, Minsk.
[839] Izumi H., 1996, *Non-commutative L^p-spaces*, Sūrikaisekikenkyūsho Kōkyūroku **956**, 24. Available at: www.kurims.kyoto-u.ac.jp/∼kyodo/kokyuroku/contents/pdf/0956-3.pdf.
[840] Izumi H., 1997, *Construction of non-commutative L^p-spaces with a complex parameter arising from modular actions*, Intern. J. Math. **8**, 1029.
[841] Izumi H., 1998, *Natural bilinear forms, natural sesquilinear forms and the associated duality on non-commutative L^p-spaces*, Intern. J. Math. **9**, 975.
[842] Izumi H., 1998, *Non-commutative L_p-spaces constructed by the complex interpolation*, Ph.D. thesis, Tōhoku Math. Publ. **9**, Tōhoku University, Sendai.
[843] Izumi H., 2000, *Non-commutative L^p-spaces*, Sūrikaisekikenkyūsho Kōkyūroku **1131**, 1. Available at: www.kurims.kyoto-u.ac.jp/∼kyodo/kokyuroku/contents/pdf/1131-1.pdf.
[844] Jacobs K., 2002, *How do two observers pool their knowledge about a quantum system?*, Quant. Inf. Proc. **1**, 73.
[845] Jadczyk A., 1988, *On Berry's phase*, unpublished preprint. Available at: arkadiusz-jadczyk.org/papers/berry.pdf.
[846] Jakšić V., Ogata Y., Pautrat Y., Pillet C.-A., 2011, *Entropic fluctuations in quantum statistical mechanics: an introduction*, arXiv:1106.3786.
[847] Jakšić V., Pillet C.-A., 2001, *On entropy production in quantum statistical mechanics*, Commun. Math. Phys. **217**, 285.
[848] Jammer M., 1974, *The philosophy of quantum mechanics: the interpretations of quantum mechanics in historical perspec-*

tive, Wiley, New York.
[849] Janyszek H., 1986, *On the geometrical structure of the generalized quantum Gibbs states*, Rep. Math. Phys. **24**, 11.
[850] Jauch J.M., 1960, *Systems of observables in quantum mechanics*, Helv. Phys. Acta **33**, 711.
[851] Jauch J.M., 1968, *Foundations of quantum mechanics*, Addison-Wesley, Reading.
[852] Jauch J.M., 1970, *What is quantum-logic?*, in: ? (ed.), *Quanta*, University of Chicago Press, Chicago.
[853] Jauch J.M., 1977, *The quantum probability calculus*, in: Leite Lopes J., Paty M. (eds.), *Quantum mechanics, a half century later*, Reidel, Dordrecht, p.39.
[854] Jauch J.M., Misra B., Gibson A.G., 1968, *On the asymptotic condition of scattering theory*, Helv. Phys. Acta **41**, 513.
[855] Jauch J.M., Piron C., 1969, *On the structure of quantal proposition systems*, Helv. Phys. Acta **42**, 842.
[856] Jaynes E.T., 1957, *Information theory and statistical mechanics*, Phys. Rev. **106**, 620.
[857] Jaynes E.T., 1957, *Information theory and statistical mechanics II*, Phys. Rev. **108**, 171.
[858] Jaynes E.T., 1963, *Information theory and statistical mechanics*, in: Ford K.W. (ed.), *Statistical physics*, Benjamin, New York.
[859] Jaynes E.T., 1965, *Gibbs vs. Boltzmann entropies*, Am. J. Phys. **33**, 391.
[860] Jaynes E.T., 1967, *Foundations of probability theory and statistical mechanics*, in: Bunge M. (ed.), *Delaware seminars in the foundations of science*, Vol.1, Springer, Berlin, p.77.
[861] Jaynes E.T., 1968, *Prior probabilities*, IEEE Trans. Sys. Sc. Cyb. **3**, 227.
[862] Jaynes E.T., 1976, *Confidence intervals vs bayesian intervals*, in: Harper W.L., Hooker C.A. (eds.), *Foundations of probability theory, statistical inference, and statistical theories of science*, Reidel, Dordrecht, p.175.
[863] Jaynes E.T., 1979, *Where do we stand on maximum entropy?*, in: Levine R.D., Tribus M. (eds.), *The maximum entropy formalism*, MIT Press, Cambridge, p.15.
[864] Jaynes E.T., 1982, *On the rationale of maximum-entropy methods*, Proc. IEEE **70**, 939.
[865] Jaynes E.T., 1985, *Generalized scattering*, in: Smith C.R., Grandy W.T. Jr. (eds.), *Maximum–entropy and bayesian methods in inverse problems*, Reidel, Dordrecht, p.377.
[866] Jaynes E.T., 1985, *Macroscopic prediction*, in: Haken H. (ed.), *Complex systems – operational approaches in neurobiology, physics and computers*, Springer, Berlin, p.254.
[867] Jaynes E.T., 1986, *Predictive statistical mechanics*, in: Moore G.T., Scully M.O. (eds.), *Frontiers of nonequilibrium statistical mechanics*, Plenum, New York, p.33.
[868] Jaynes E.T., 1990, *Probability in quantum theory*, in: Żurek W.H. (ed.), *Complexity, entropy and the physics of onformation*, Addison Wesley, Reading.
[869] Jaynes E.T., 1993, *Inferential scattering*, unpublished preprint, available at bayes.wustl.edu/etj/articles/cinfscat.pdf.
[870] Jaynes E.T., 2003, *Probability theory: the logic of science*, Cambridge University Press, Cambridge.
[871] Jaynes E.T., Scalapino D.J., 1963, *Irreversible statistical mechanics*, unpublished preprint, available at: bayes.wustl.edu/etj/articles/irreversible.stat.mac.pdf.
[872] Jeffrey R.C., 1965, *The logic of decision*, Chicago University Press, Chicago.
[873] Jeffrey R.C., 1977, *Mises redux*, in: Butts R.E., Hintikka J. (eds.), *Basic problems in methodology and linguistics*, Reidel, Dordrecht, p.213.
[874] Jeffreys H., 1939, *Theory of probability*, Oxford University Press, Oxford. (1961, 3rd rev. ed.).
[875] Jeffreys H., 1946, *An invariant form for the prior probability in estimation problems*, Proc. Roy. Soc. London A **186**, 453.
[876] Jensen U., 1995, *Review of the derivation and calculation of Rao distances with an application to portfolio theory*, in: Maddala P., Phillips G.S., Srinivasan T. (eds.), *Advances in econometrics and economics: essays in honor of C.R. Rao*, Blackwell, Cambridge, p.413.
[877] Jenčová A., 2001, *Dualistic properties of the manifold of quantum states*, in: Sollich P., Hughston L.P., Streater R.F., Coolen A.C.C. (eds.), *Disordered and complex systems (King's College, London, UK, 10-14 July 2000)*, AIP Conf. Proc. **553**, AIP, Melville, p.147.
[878] Jenčová A., 2001, *Geometry of quantum states: dual connections and divergence functions*, Rep. Math. Phys. **47**, 121. Available at: www.mat.savba.sk/∼jencova/geom.pdf.
[879] Jenčová A., 2002, *Quantum information geometry and standard purification*, J. Math. Phys. **43**, 2187.
[880] Jenčová A., 2003, *Affine connections, duality and divergences for a von Neumann algebra*, arXiv:math-ph/0311004.
[881] Jenčová A., 2003, *Flat connections and Wigner–Yanase–Dyson metrics*, Rep. Math. Phys. **52**, 331. Available at: www.mat.savba.sk/∼jencova/wyd.pdf.
[882] Jenčová A., 2004, *Generalized relative entropies as contrast functionals on density matrices*, Int. J. Theor. Phys. **43**, 1635. Available at: www.mat.savba.sk/∼jencova/ijtp.pdf.
[883] Jenčová A., 2005, *Quantum information geometry and non-commutative L_p spaces*, Inf. Dim. Anal. Quant. Prob. Relat. Top. **8**, 215. Available at: www.mat.savba.sk/∼jencova/lpspaces.pdf.
[884] Jenčová A., 2006, *A construction of a nonparametric quantum information manifold*, J. Funct. Anal. **239**, 1. arXiv:math-ph/0511065.
[885] Jenčová A., 2010, *On quantum information manifolds*, in: Gibilisco P. et al (eds.), *Algebraic and geometric methods in statistics*, Cambridge University Press, Cambridge, p.265.
[886] Jenčová A., Petz D., 2006, *Sufficiency in quantum statistical inference*, Commun. Math. Phys. **263**, 259.
[887] Jenčová A., Petz D., 2006, *Sufficiency in quantum statistical inference. A survey with examples*, Inf. Dim. Anal. Quant. Prob. Relat. Top. **9**, 331. arXiv:quant-ph/0604091.
[888] Jenčová A., Ruskai M.B., 2010, *A unified treatment of convexity of relative entropy and related trace functions, with conditions for equality*, Rev. Math. Phys. **22**, 1099. arXiv:0903.2895.
[889] Johnson R.W., 1979, *Axiomatic characterization of the directed divergences and their linear combinations*, IEEE Trans. Inf. Theory **25**, 709.
[890] Johnson R.W., Shore J.E., 1983, *Comments and correction to "Axiomatic derivation of the principle of maximum entropy and the principle of minimum cross-entropy"*, IEEE Trans. Inf. Theory **29**, 942.
[891] Johnstone P.T., 1982, *Stone spaces*, Cambridge University Press, Cambridge.
[892] Jones L.K., Byrne C.L., 1990, *General entropy criteria for inverse problems, with applications to ada compression, pattern classification and cluster analysis*, IEEE Trans. Inf. Theory **36**, 23.
[893] Jordan P., 1927, *Über eine neue Begründung der Quantenmechanik I, II*, Z. Phys. **40**, 809; **44**, 1.
[894] Jordan P., 1932, *Über eine Klasse nichtassoziativer hyperkomplexer Algebren*, Nachr. Ges. Wiss. Göttingen, Math.-Phys. Kl. **1932**, 569.
[895] Jordan P., 1933, *Über die Multiplikation quantenmechanischer Größen*, Z. Phys. **80**, 285.

[896] Jordan P., 1933, *Über Verallgemeinerungsmöglichkeiten des Formalismus der Quantenmechanik*, Nachr. Ges. Wiss. Göttingen, Math.-Phys. Kl. **1933**, 209.
[897] Jordan P., 1949, *Zur Quantum-Logik*, Arch. Math. **2**, 166.
[898] Jordan P., 1952, *Algebraische Betrachtungen zur Theorie des Wirkungsquantums und der Elementarlange*, Abhandl. Math. Sem. Univ. Hamburg **18**, 99.
[899] Jordan P., 1959, *Quantenlogik und das kommutative Gesetz*, in: Suppes P., Tarski A. (eds.), *The axiomatic method*, North-Holland, Amsterdam, p.365.
[900] Jordan P., von Neumann J., Wigner E.P., 1934, *On an algebraic generalization of the quantum mechanical formalism*, Ann. Math. **35**, 29.
[901] Jordan T.F., 1993, *Reconstructing a nonlinear dynamical framework for testing quantum mechanics*, Ann. Phys. **225**, 83.
[902] Jumarie G., 1990, *Relative information*, Springer, Berlin.
[903] Junge M., Parcet J., 2010, *Mixed-norm inequalities and operator space L_p embedding theory*, Memoirs of American Mathematical Society **203**, American Mathematical Society, Providence.
[904] Junge M., Ruan Z.-J., 2004, *Decomposable maps on non-commutative L_p-spaces*, Contemp. Math. **365**, 355.
[905] Junge M., Ruan Z.-J., Sherman D., 2005, *A classification of 2-isometries of noncommutative L_p-spaces*, Israel J. Math. **150**, 25. arXiv:math/0402181.
[906] Junge M., Ruan Z.-J., Xu Q., 2005, *Rigid $\mathcal{OL}_p$ structures of non-commutative L_p-spaces associated with hyperfinite von Neumann algebras*, Math. Scand. **96**, 63.
[907] Junge M., Sherman D., 2005, *Noncommutative L^p modules*, J. Op. Theor. **53**, 3. arXiv:math/0301044.
[908] Kadison R.V., 1951, *Isometries of operator algebras*, Ann. Math. **54**, 325.
[909] Kadison R.V., 1952, *A generalized Schwarz inequality and algebraic invariants for operator algebras*, Ann. Math. **56**, 494.
[910] Kadison R.V., 1963, *Remarks on the type of von Neumann algebras of local observables in quantum field theory*, J. Math. Phys. **4**, 1511.
[911] Kadison R.V., 1965, *Transformation of states in operator theory and dynamics*, Topology **3** (Suppl.) 2, 177.
[912] Kadison R.V., 1966, *Derivations of operator algebras*, Ann. Math. **83**, 280.
[913] Kadison R.V., Ringrose J.R., 1983, 1986, 1991, 1992, *Fundamentals of the theory of operator algebras*, Vol.1-4, Academic Press, New York.
[914] Kagan A.M., 1963, *K teorii informacionnogo kolichestva Fishera*, Dokl. Akad. Nauk SSSR **151**, 277.
[915] Kakutani S., 1940, *Weak topology, bicompact set and the principle of duality*, Proc. Imp. Acad. Tōkyō **16**, 63. Available at: euclid:pja/1195579206.
[916] Kakutani S., 1941, *Concrete representation of abstract (L)-spaces and the mean ergodic theorem*, Ann. Math. **42**, 523.
[917] Kakutani S., 1941, *Concrete representation of abstract (M)-spaces (a characterization of the space of continuous functions)*, Ann. Math. **42**, 994.
[918] Kakutani S., 1948, *On equivalence of infinite product measures*, Ann. Math. **49**, 214.
[919] Kallman R.R., 1971, *Spatially induced groups of automorphisms of certain von Neumann algebras*, Trans. Amer. Math. Soc. **156**, 505.
[920] Kalmbach G., 1983, *Orthomodular lattices*, Academic Press, New York.
[921] Kalmbach G., 1986, *Measures on Hilbert lattices*, World Scientific, Singapore.
[922] Kamber F., 1964, *Die Struktur des Aussagenkalküls in einer physikalischen Theorie*, Nachr. Akad. Wiss. Göttingen, Math.-phys. Kl. **10**, 103.
[923] Kamber F., 1965, *Zweiwertige Wahrscheinlichkeitsfunktionen auf orthokomplementären Verbanden*, Math. Ann. **158**, 158.
[924] Kamiński W., Kostecki R.P., 2012, *Quantum measurements as entropic projections*, preprint.
[925] Kampé de Fériet J., Forte B., 1967, *Information et probabilité*, Compt. Rend. Acad. Sci. Paris **265**, 110, 142, 350.
[926] Kantorovich L.V., 1935, *O poluuporyadochennykh lineĭnykh prostranstvakh i ikh primeneniyakh v teorii lineĭnykh operaciĭ*, Dokl. Akad. Nauk SSSR **4**, 11.
[927] Kantorovich L.V., 1937, *Lineare halbgeordnete Räume*, Matem. sb. N.S. **2**, 121.
[928] Kantorovich L.V., Vulikh B.Z., Pinsker A.G., 1950, *Funkcional'nyĭ analiz v poluuporyadochennykh prostranstvakh*, Gosudarstvennoe Izdatel'stvo Tekhniko-Teoreticheskoĭ Literatury, Moskva/Leningrad.
[929] Kaplansky I., 1950, *Quelques résultats sur les anneaux d'opèrateurs*, Compt. Rend. Acad. Sci. Paris **231**, 485.
[930] Kaplansky I., 1951, *Projections in Banach algebras*, Ann. Math. **53**, 235.
[931] Kaplansky I., 1951, *A theorem on rings of operators*, Pacific J. Math. **1**, 227.
[932] Kaplansky I., 1952, *Algebras of type I*, Ann. Math. **56**, 460.
[933] Kaplansky I., 1953, *Modules over operator algebras*, Amer. J. Math. **75**, 839.
[934] Kaplansky I., 1955, *Rings of operators*, Lecture notes, Mathematics Department of Chicago University, Chigago (rev. ed.: 1968, Benjamin, New York).
[935] Kappos D.A., 1948, *Ein Beitrag zur Carathéodoryschen Definition der Ortsfunktionen in Booleschen Algebren*, Math. Z. **51**, 616.
[936] Kappos D.A., 1949, *Zur mathematischen Begründung der Wahrscheinlichkeitstheorie*, Sitzungsber. Math.-Naturw. Kl. Bayer. Akad. Wiss. München **1948**, 27.
[937] Kappos D.A., 1951, *Über die Unabhängigkeit in der Wahrscheinlichkeitstheorie*, Sitzungsber. Math.-Naturw. Kl. Bayer. Akad. Wiss. München **1950**, 157.
[938] Kappos D.A., 1960, *Strukturtheorie der Wahrscheinlichkeits-Felder und -Räume*, Ergebnisse der Mathematik und ihrer Grenzgebiete. Neue Folge. Heft **24**, Springer, Berlin.
[939] Kappos D.A., 1969, *Probability algebras and stochastic spaces*, Academic Press, New York.
[940] Kappos D.A., 1976, *Measure theory on orthomodular posets and lattices*, in: Bellow A., Kölzow D. (eds.), *Measure theory. Proceedings of the conference held at Oberwolfach, 15-21 June, 1975*, LNM **541**, Springer, Berlin, p.323.
[941] Kapur J.N., Kesavan H., 1992, *Entropy optimization principles with applications*, Academic Press, New York.
[942] Karbelkar S.N., 1986, *On the axiomatic approach to the maximum entropy principle of inference*, Pramana J. Phys. **26**, 301.
[943] Kass R.E., 1980, *The riemannian structure of model spaces: a geometrical approach to inference*, Ph.D. thesis, University of Chicago, Chicago.
[944] Kass R.E., 1989, *The geometry of asymptotic inference (with comments and rejoinder)*, Statist. Sci. **4**, 188.
[945] Kass R.E., Vos P.W., 1997, *Geometrical foundations of asymptotic inference*, Wiley, New York.
[946] Kass R.E., Wasserman L., 1996, *The selection of prior distributions by formal rules*, J. Amer. Stat. Assoc. **91**, 1343. (corr.: 1998, J. Amer. Stat. Assoc. **93**, 412).

[947] Kastler D., 1976, *Equilibrium states of matter and operator algebras*, in: *Convegno sulle algebre C^* e loro applicazioni in fisica teoria (12-18 Marzo 1975, Convegno sulla teoria degli operatori indice e teoria K (13-18 Ottobre 1975), del Istituto Nazionale di Alta Matematica, Roma*, Symposia Mathematica **20**, Academic Press, New York, p.49.
[948] Kastler D., Pool J.C.T., Poulsen E.T., 1969, *Quasi-unitary algebras attached to temperature states in statistical mechanics. A comment to the work of Haag, Hugenholtz, and Winnink*, Commun. Math. Phys. **12**, 175.
[949] Kastler D., Robinson D.W., 1966, *Invariant states in statistical mechanics*, Commun. Math. Phys. **3**, 151.
[950] Katavolos A., 1976, *Isometries of non-commutative L^p-spaces*, Canad. J. Math. **28**, 1180.
[951] Katavolos A., 1981, *Are non-commutative L_p-spaces really non-commutative?*, Canad. J. Math. **33**, 1319.
[952] Katavolos A., 1982, *Non commutative L_p spaces II*, Canad. J. Math. **34**, 1208.
[953] Kato T., 1952, *On the perturbation theory of closed linear operators*, J.Math. Soc Japan **4**, 323.
[954] Kato T., 1966, *Perturbation theory for linear operators*, Springer, Berlin.
[955] Kelley J.L., 1966, *Decomposition and representation theorems in measure theory*, Math. Ann. **163**, 89.
[956] Kelley J.L., Namioka I., 1963, *Linear topological spaces*, Van Nostrand, Princeton.
[957] Kelley J.L., Vaught R.L., 1953, *The positive cone in Banach algebras*, Trans. Amer. Math. Soc. **74**, 44.
[958] Kelly G.M., Laplaza M.L., 1980, *Coherence for compact closed categories*, J. Pure Appl. Alg. **19**, 193.
[959] Kempthorne O., Folks L., 1971, *Probability, statistics and data analysis*, Iowa State University Press, ?.
[960] Kendall M.G., Stuart A., 1958, *The advanced theory of statistics*, Vol.1-2, Griffin, London.
[961] Keynes J.M., 1921, *A treatise on probability*, Macmillan, London.
[962] Kivinen J., Warmuth M.K., 1999, *Averaging expert predictions*, in: ? (ed.), *Computational learning theory: 4th european conference, EuroCOLT'99, Nordkirchen, Germany, March 29-31, 1999*, Springer, Berlin, p.153. users.soe.ucsc.edu/~manfred/pubs/C50.pdf.
[963] Kiwiel K.C., 1997, *Free-steering relaxation methods for problems with strictly convex costs and linear constraints*, Math. Oper. Res. **22**, 326.
[964] Kiwiel K.C., 1997, *Proximal minimization methods with generalized Bregman functions*, Soc. Industr. Appl. Math. J. Contr. Optim. **35**, 1142.
[965] Kiwiel K.C., 1998, *Generalized Bregman projections in convex feasibility problems*, J. Optim. Theor. Appl. **96**, 139.
[966] Kępiński A., 1972, *Rytm życia*, Wydawnictwo Literackie, Kraków.
[967] Kępiński A., 1972, *Schizofrenia*, Państwowy Zakład Wydawnictw Lekarskich, Warszawa.
[968] Kępiński A., 1977, *Lęk*, Państwowy Zakład Wydawnictw Lekarskich, Warszawa.
[969] Kochen S., Specker E.P., 1965, *The calculus of partial propositional functions*, in: Bar-Hillel Y. (ed.), *Proceedings of the 1964 International congress of logic, methodology and philosophy of science*, North-Holland, Amsterdam, p.65.
[970] Kochen S., Specker E.P., 1965, *Logical structures arising in quantum theory*, in: Addison J., Henkin L., Tarski A. (eds.), *Theory of models. Proceedings of the 1963 International symposium at Berkeley*, North-Holland, Amsterdam, p.177.
[971] Kochen S., Specker E.P., 1967, *The problem of hidden variables in quantum mechanics*, J. Math. Mech. **17**, 59.
[972] Kolmogorov A.N., 1933, *Grundbergriffe der Wahrscheinlichkeitrechnung*, Ergebnisse der Mathematik und Ihrer Grenzgebiete Bd. **2**, Springer, Berlin. (engl. transl. 1950, *Foundations of the theory of probability*, Chelsea, New York).
[973] Koopman B.O., 1931, *Hamiltonian systems and transformations in Hilbert space*, Proc. Nat. Acad. Sci. U.S.A. **17**, 315.
[974] Koopman B.O., 1936, *On distributions admitting a sufficient statistic*, Trans. Amer. Math. Soc. **39**, 399.
[975] Koppelberg S., 1989, *General theory of boolean algebras*, in: Monk J.D., Bonnet R. (eds.), *Handbook of boolean algebras*, Vol.1, North-Holland, Amsterdam, p.1.
[976] Kosaki H., 1980, *Canonical L^p-spaces associated with an arbitrary abstract von Neumann algebra*, Ph.D. thesis, University of California, Los Angeles. Available at: dmitripavlov.org/scans/kosaki-thesis.pdf.
[977] Kosaki H., 1980, *Positive cones associated with von Neumann algebra*, Math. Scand. **47**, 295.
[978] Kosaki H., 1981, *Noncommutative Lorentz spaces associated with a von Neumann algebra and applications*, Proc. Japan Acad. Ser. A **57**, 303.
[979] Kosaki H., 1981, *Positive cones and L^p-spaces associated with a von Neumann algebra*, J. Oper. Theor. **6**, 13.
[980] Kosaki H., 1982, *Interpolation theory and the Wigner–Yanase–Dyson–Lieb concavity*, Commun. Math. Phys. **87**, 315.
[981] Kosaki H., 1982, *A Radon–Nikodym theorem for natural cones associated with von Neumann algebras*, Proc. Amer. Math. Soc. **84**, 207.
[982] Kosaki H., 1982, *T-theorem for L^p-spaces associated with a von Neumann algebra*, J. Op. Theor. **7**, 267.
[983] Kosaki H., 1983, *On the Bures distance and Uhlmann transition probability of states on a von Neumann algebra*, Proc. Nat. Acad. Sci. U.S.A. **89**, 285.
[984] Kosaki H., 1984, *Applications of the complex interpolation method to a von Neumann algebra: non-commutative L^p-spaces*, J. Funct. Anal. **56**, 29.
[985] Kosaki H., 1984, *On the continuity of the map $\varphi \to |\varphi|$ from the predual of a W^*-algebra*, J. Funct. Anal. **59**, 123.
[986] Kosaki H., 1985, *Lebesgue decomposition of states on a von Neumann algebra*, Am. J. Math. **107**, 697.
[987] Kosaki H., 1986, *Relative entropy for states: a variational expression*, J. Op. Theor. **16**, 335.
[988] Koshevnik Yu.A., Levit B.Ya., 1976, *On a non-parametric analogue of the information matrix*, Theor. Prob. Appl. **21**, 738.
[989] Kossakowski A., 1969, *On quantum informational thermodynamics*, Bull. Acad. Polon. Sci. Ser. Math. Astr. Phys. **17**, 263.
[990] Kossakowski A., 1972, *On necessary and sufficient conditions for a generator of a quantum dynamical semigroup*, Bull. Acad. Polon. Sci. Ser. Math. Astr. Phys. **20**, 1021.
[991] Kossakowski A., 1972, *On quantum statistical mechanics on non-hamiltonian systems*, Rep. Math. Phys. **3**, 247.
[992] Kostecki R.P., 2010, *Quantum theory as inductive inference*, in: Mohammad-Djafari A., Bercher J.-F., Bessière P. (eds.), *Proceedings of the 30th International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, AIP Conf. Proc. **1305**, Springer, Berlin, p.24. arXiv:1009.2423.
[993] Kostecki R.P., 2011, *The general form of γ-family of quantum relative entropies*, Open Sys. Inf. Dyn. **18**, 191. arXiv:1106.2225.
[994] Kostecki R.P., 2011, *On principles of inductive inference*, in: Goyal P. et al (eds.), *Proceedings of the 31st International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering, 9-16 July 2011, Waterloo, Canada*, AIP Conf. Proc. **1443**, Springer, Berlin, p.22. arXiv:1109.3142.
[995] Kostecki R.P., 2012, *Information dynamics and new geometric foundations for quantum theory*, in: D'Ariano M. et al (eds.), *Foundations of Probability and Physics – 6, Växjö, Sweden, 14-16 June 2011*, AIP Conf. Proc. **1424**, American Institute of Physics, Melville, p.200. arXiv:1110.4492.
[996] Koszul J.L., 1961, *Domaines bornés homogènes et orbites de groupes de transformations affines*, Bull. Soc. Math. France **89**, 515.

[997] Kraft C.H., 1955, *Some conditions for consistency and uniform consistency of statistical procedures*, Univ. Calif. Publ. Statist. **1**, 125.
[998] Krasnosel'skiĭ M.A., Rutickiĭ Ya.B., 1958, *Vypuklye funkcii i prostranstva Orlicha*, Gosudarstvennoe izdatel'stvo fiziko-matematicheskoĭ literatury, Moskva (engl. trans. 1961, *Convex functions and Orlicz spaces*, Nordhoff, Groningen).
[999] Kraus F., 1936, *Über konvexe Matrixfunktionen*, Math. Z. **41**, 18.
[1000] Kraus K., 1971, *General state changes in quantum theory*, Ann. Phys. **64**, 331.
[1001] Kraus K., 1983, *States, effects and operations: fundamental notions of quantum theory*, Springer, Berlin.
[1002] Kreĭn M.G, Kreĭn S.G., 1940, *On an inner characteristic of the set of all continuous functions defined on a bicompact Hausdorff space*, Dokl. Akad. Nauk SSSR **27**, 427.
[1003] Kreĭn M.G, Kreĭn S.G., 1943, *Sur l'espace de fonctions continues définies sur un bicompact Hausdorff et ses sous espaces semiordonnés*, Matem. sb. **13**, 1.
[1004] Kreĭn M.G., Milman D., 1940, *On extreme points of regularly convex sets*, Stud. Math. **9**, 133.
[1005] Krieger W., 1976, *On ergodic flows and the isomorphism of factors*, Math. Ann. **223**, 19.
[1006] Kriegl A., Michor P.W., 1997, *The convenient setting for global analysis*, Mathematical Surveys and Monographs **53**, American Mathematical Society, Providence. Available at: www.mat.univie.ac.at/∼michor/apbookh-ams.pdf.
[1007] Kristensen P., Mejlbo L., Thue Poulsen E., 1965, *Tempered distributions in infinitely many dimensions. I. Canonical field operators*, Commun. Math. Phys. **1**, 175.
[1008] Kronfli N.S., 1970, *States on generalised logics*, Int. J. Theor. Phys. **3**, 191.
[1009] Kruskal M., 1960, *Maximal extension of Schwarzschild manifold*, Phys. Rev. **119**, 1743.
[1010] Kruszyński P., 1984, *Extensions of Gleason theorem*, in: Accardi L., Frigerio A., Gorini V. (eds.), *Quantum probability and applications to the quantum theory of irreversible processes*, LNM **1055**, Springer, Berlin, p.210.
[1011] Kubo F., Andō T., 1980, *Means of positive linear operators*, Math. Ann. **246**, 205.
[1012] Kubo R., 1957, *Statistical mechanical theory of irreversible processes. I. General theory and simple applications to magnetic and conduction problems*, J. Phys. Soc. Japan **12**, 570.
[1013] Kulhavý R., 1996, *Recursive nonlinear estimation: a geometric approach*, Springer, Berlin.
[1014] Kullback S., 1959, *Information theory and statistics*, Wiley, New York (2nd ed. 1968).
[1015] Kullback S., Khairat M.A., 1966, *A note on minimum discrimination information*, Ann. Math. Statist. **37**, 279.
[1016] Kullback S., Leibler R.A., 1951, *On information and sufficiency*, Ann. Math. Statist. **22**, 79. euclid:aoms/1177729694.
[1017] Kumagai W., 2011, *A characterization of extended monotone metrics*, Lin. Alg. Appl. **434**, 224.
[1018] Kuna M., Czachor M., Leble S.B., 1999, *Nonlinear von Neumann-type equations: Darboux invariance and spectra*, Phys. Lett. A **225**, 42. arXiv:quant-ph/9810023.
[1019] Kunze R., 1958, *L_p Fourier transform on locally compact unimodular groups*, Trans. Amer. Math. Soc. **89**, 519.
[1020] Kunze W., 1990, *Noncommutative Orlicz spaces and generalized Arens algebras*, Math. Nachr. **147**, 123.
[1021] Kurdila A.J., Zabarankin M., 2005, *Convex functional analysis*, Birkhäuser, Basel.
[1022] Kurose T., 1990, *Dual connections and affine geometry*, Math. Z. **203**, 115.
[1023] Kurose T., 1993, *Dual connections and projective geometry*, Hiroshima Math. J. **23**, 327.
[1024] Kusraev A.G., Kutateladze S.S., 1999, *Boolean valued analysis*, Kluwer, Dordrecht.
[1025] Labuschagne L.E., 2012, *A crossed product approach to Orlicz spaces*, arXiv:1205.2192.
[1026] Labuschagne L.E., Majewski W.A., 2008, *Quantum L_p and Orlicz spaces*, in: García J.C., Quezada R., Sontz S.B.(eds.), *Quantum probability and related topics. Proceedings of the 28th Conference (CIMAT-Guanajuato, Mexico, 2-8 September 2007)*, World Scientific, Singapore, p.176. arXiv:0902.4327.
[1027] Labuschagne L.E., Majewski W.A., 2009, *Maps on noncommutative Orlicz spaces*, arXiv:0902.3078.
[1028] Lacey H.E., 1974, *The isometric theory of classical Banach spaces*, Springer, Berlin.
[1029] Landau L.D., 1927, *Das Dämpfungsproblem in der Wellenmechanik*, Z. Phys. **45**, 430.
[1030] Landé A., 1965, *New foundations of quantum mechanics*, Cambridge University Press, Cambridge.
[1031] Landsman N.P., 1991, *Algebraic theory of superselection sectors and the measurement problem in quantum mechanics*, Int. J. Theor. Phys. A **30**, 5349.
[1032] Landsman N.P., 2008, *Macroscopic observables and the Born rule I. Long run frequencies*, Rev. Math. Phys. **20**, 1173. arXiv:0804.4849.
[1033] Landstad M.B., 1979, *Duality theory for covariant systems*, Trans. Amer. Math. Soc. **248**, 223.
[1034] Lanford O.E. III, 1970, *The KMS sates of quantum spin system*, in: Michel L., Ruelle D. (eds.), *Systèmes à un nombre infinies de degrès de liberté*, Centre National de la Recherche Scientifique, Paris, p.146.
[1035] Lanford O.E. III, Robinson D.W., 1968, *Statistical mechanics of quantum spin systems III*, Commun. Math. Phys. **9**, 327.
[1036] Laurant F., Oheix M., Raoult J.P., 1969, *Tests d'hypothèses*, Ann. Inst. Henri Poincaré B **5**, 385.
[1037] Lauritzen S.L., 1987, *Conjugate connections in statistical theory*, in: Dodson C.T.J. (ed.), *Geometrization of statistical theory*, Univ. Lancaster Dept. Math. Publ., Lancaster, p.33.
[1038] Lauritzen S.L., 1987, *Statistical manifolds*, in: Gupta S.S. (ed.), *Differential geometry in statistical inference*, Institute of Mathematical Statistics, Hayward, p.163.
[1039] Lawvere F.W., 1962, *The category of probabilistic mappings with applications to stochastic processes, statistics, and pattern recognition*, unpublished preprint. Available at: www.fuw.edu.pl/∼kostecki/scans/lawvere1962.pdf.
[1040] Lax P., 1968, *Integrals of nonlinear equations of evolution and solitary waves*, Comm. Pure Appl. Math. **21**, 467. Available at: archive.org/download/integralsofnonli00laxp/integralsofnonli00laxp.pdf.
[1041] Le Besnerais G., 1993, *Méthode du maximum d'entropie sur la moyenne, critères de reconstruction d'image, et synthèse d'ouverture en radio-astronomie*, Ph.D. thesis, Université de Paris-Sud, Orsay.
[1042] Le Besnerais G., Bercher J.-F., Demoment G., 1999, *A new look at entropy for solving linear inverse problems*, IEEE Trans. Inf. Theor. **45**, 1565.
[1043] Le Cam L., 1960, *Locally asymptotically normal families of distributions*, Univ. California Publ. Stat. **3**, 37.
[1044] Le Cam L., 1964, *Sufficiency and approximate sufficiency*, Ann. Math. Statist. **35**, 1419.
[1045] Le Cam L., 1969, *Théorie asymptotique de la décision statistique*, Les Presses de l'Université Montréal, Montréal.
[1046] Le Cam L., 1986, *Asymptotic methods in statistical decision theory*, Springer, Berlin.
[1047] Le Cam L., Yang G.L., 1990, *Asymptotics in statistics: some basic concepts*, Springer, Berlin (2nd ed. 2000).
[1048] Lebesgue H., 1901, *Sur une généralisation de l'intégrale définie*, Compt. Rend. Hebd. Séanc. l'Acad. Sci. **132**, 1025.
[1049] Lebesgue H., 1902, *Intégrale, longeur, aire*, Ann. Matem. Pura Appl. **7**, 231.
[1050] Lebesgue H., 1904, *Leçons sur l'intégration et la recherche des fonctions primitives*, Gauthier–Villars, Paris (2nd. ed. 1924).
[1051] Lebesgue H., 1910, *Sur l'intégration des fonctions discontinues*, Ann. Sci. École Norm. Sup. 3^e ser. **27**, 361. num-

dam:ASENS_1910_3_27__361_0.
[1052] Leble S.B., Czachor M., 1998, *Darboux-integrable nonlinear Liouville-von Neumann equation*, Phys. Rev. E **58**, 7091. arXiv:quant-ph/9804052.
[1053] Leinert M., 1986, *On integration with respect to a trace*, in: Nagel R. et. al (eds.), *Aspects of positivity in functional analysis*, North-Holland, Amsterdam, p.231.
[1054] Leinert M., 1991, *Integration with respect to a weight*, Intern. J. Math. **2**, 177.
[1055] Leinert M., 1992, *Integration with respect to a trace*, Arch. Math. **59**, 475.
[1056] Lem S., 1974, *Profesor A. Dońda*, in: Jęczmyk L. (ed.), *Kroki w nieznane 5*, Iskry, Warszawa, p.63.
[1057] Léonard L., 2001, *Minimization of energy functionals applied to some inverse problems*, Appl. Math. Optim. **44**, 273.
[1058] Léonard L., 2001, *Minimizers of energy functionals*, Acta Math. Hungar. **93**, 281.
[1059] Léonard L., 2003, *Minimizers of energy functionals under not very integrable constraints*, J. Conv. Anal. **10**, 63.
[1060] Léonard L., 2008, *Minimization of entropy functionals*, J. Math. Anal. Appl. **346**, 186.
[1061] Léonard L., 2010, *Convex minimization problems with weak constraint qualifications*, J. Conv. Anal. **17**, 321.
[1062] Léonard L., 2010, *Entropic projections and dominating points*, ESAIM Prob. Statist. **14**, 343.
[1063] Lesniewski A., Ruskai M.B., 1999, *Monotone riemannian metrics and relative entropy on non-commutative probability space*, J. Math. Phys. **40**, 5702. arXiv:math-ph/9808016.
[1064] Levi-Civita T., 1917, *Nozione di parallelismo in una varietà qualunque e consequente specificazione geometrica della curvatura riemanniana*, Rend. Circ. Mat. Palermo **42**, 173.
[1065] Lewin J., Lewin M., 1975, *A reformulation of the Radon-Nikodym theorem*, Proc. Amer. Math. Soc. **47**, 393.
[1066] Lewis A.S., 1995, *Consistency of moment systems*, Can. J. Math. **47**, 995.
[1067] Lie S., 1890, *Theorie der Transformationsgruppen*, Zweiter Abschnitt, Teubner, Leipzig.
[1068] Lieb E.H., 1973, *Convex trace functions and the Wigner–Yanase–Dyson conjecture*, Adv. Math. **11**, 267.
[1069] Lieb E.H., Ruskai M.B., 1973, *Proof of the strong subadditivity of quantum mechanical entropy*, J. Math. Phys. **14**, 1938.
[1070] Liese F., 1977, *On the existence of f-projections*, in: Csiszár I., Elias P. (eds.), *Topics in information theory*, North-Holland, Amsterdam, p.431.
[1071] Liese F., Vajda I., 1987, *Convex statistical distances*, Teubner–Texte zur Mathematik **95**, Teubner, Leipzig.
[1072] Liese F., Vajda I., 2006, *On divergences and informations in statistics and information theory*, IEEE Trans. Inf. Theor. **52**, 4394.
[1073] Liese F., Vajda I., 2008, *f-divergences: sufficiency, deficiency and testing of hypotheses*, in: Barnett N.S., Dragomir S.S. (eds.), *Advances in inequalities from probability theory and statistics*, Nova Publishers, New York.
[1074] Lindblad G., 1974, *Expectations and entropy inequalities for finite quantum systems*, Commun. Math. Phys. **39**, 111.
[1075] Lindblad G., 1975, *Completely positive maps and entropy inequalities*, Commun. Math. Phys. **40**, 147.
[1076] Lindblad G., 1976, *Dissipative operators and cohomology of operator algebras*, Lett. Math. Phys. **1**, 219.
[1077] Lindblad G., 1976, *On the generators of quantum dynamical semigroups*, Commun. Math. Phys. **48**, 119.
[1078] Lindelöf E., 1894, *Sur l'application de la méthode des apporximation successives aux équations differentielles ordinaires du premier orde*, Compt. Rend. Hebd. Séanc. Acad. Sci. **114**, 454.
[1079] Lindenstrauss J., Tzafriri L., 1977, 1979, *Classical Banach spaces*, Vol.1-2, Springer, Berlin.
[1080] Linhard J., Nielsen V., 1971, *Studies in statistical dynamics*, Kong. Danske Vid. Selkab. Mat.-fys. Medd. **38**, 1.
[1081] Linton F.E.J., 1970, *Applied functorial semantics, I*, Ann. Mat. **86**, 1.
[1082] Lipschitz R., 1869, *Untesuchungen in Betreff der ganzen homogenen Functionen von n Differentialen*, J. reine angew. Math. **70**, 71.
[1083] Littaye-Petit M., Piednoir J.L., Van Cutsem B., 1969, *Exhausivité*, Ann. Inst. Henri Poincaré B. **5**, ?.
[1084] Lodkin A.A., 1974, *Vsyakaya mera na proektorakh W^*-algebry prodolzhaetsya do sostoyaniya*, Funkc. Anal. Prilozh. **8**, 54 (engl. transl. 1974, *Every measure on the projectors of a W^*-algebra can be extended to a state*, Funct. Anal. Appl. **8**, 318; erratum: 1975, Funct. Anal. Appl. **9**, 271).
[1085] Loebl R.I., 1974, *Injective von Neumann algebras*, Proc. Nat. Acad. Sci. U.S.A. **44**, 46.
[1086] Łomnicki A., 1923, *Nouveaux fondements du calcul des probabilités*, Fund. Math. **4**, 34.
[1087] Longo R., 1979, *Notes on algebraic invariants for non-commutative dynamical systems*, Commun. Math. Phys. **69**, 195.
[1088] Loomis L.H., 1947, *On the representation of σ-complete boolean algebras*, Bull. Amer. Math. Soc. **53**, 757.
[1089] Loomis L.H., 1955, *The lattice-theoretic background of the dimension theory of operator algebras*, Memoirs of the American Mathematical Society **18**, American Mathematical Society, Providence.
[1090] Loubenets E.R., 2001, *Quantum stochastic approach to the description of quantum measurements*, J. Phys. A **34**, 7639.
[1091] Lovrić M., Min-Oo M., Ruh E.A., 2000, *Multivariate normal distributions parametrized as a riemannian symmetric space*, J. Multivar. Anal. **74**, 36.
[1092] Lowdenslager D.B., 1957, *On postulates for general quantum mechanics*, Proc. Nat. Acad. Sci. U.S.A. **8**, 88.
[1093] Löwner K., 1934, *Über monotone Matrixfunctionen*, Math. Z. **38**, 177.
[1094] Lüders G., 1951, *Über die Zustandsänderung durch den Messprozess*, Ann. Phys. Leipzig **8**, 322. (engl. transl.: 2004, *Concerning the state-change due to the measurement process*, arXiv:quant-ph/0403007).
[1095] Ludwig G., 1964, *Versuch einer axiomatischen Grundlegung der Quantenmechanik und allgemeinerer physikalischer Theorien*, Z. Phys. **181**, 233.
[1096] Ludwig G., 1967, *Attempt of an axiomatic foundation of quantum mechanics and more general theories. II*, Commun. Math. Phys. **4**, 331.
[1097] Ludwig G., 1968, *Attempt of an axiomatic foundation of quantum mechanics and more general theories. III*, Commun. Math. Phys. **9**, 1.
[1098] Ludwig G., 1970, *Deutung des Begriffes physikalische Theorie und axiomatische Grundlegung der Hilbertraumstruktur der Quantenmechanik durch Hauptsätze des Messens*, Springer, Berlin.
[1099] Ludwig G., 1972, *The measuring proces and an axiomatic foundation of quantum mechanics*, in: D'Espagnat B. (ed.), *Foundations of quantum mechanics*, Academic Press, New York, p.287.
[1100] Ludwig G., 1983, 1985, *Foundations of quantum mechanics*, Vol.1-2, Springer, Berlin.
[1101] Ludwig G., 1985, 1987, *An axiomatic basis for quantum mechanics*, Vol.1-2, Springer, Berlin.
[1102] Ludwig G., Neumann H., 1981, *Connections between different approaches to the foundations of quantum mechanics*, in: Neumann H. (ed.), *Interpretations and foundations of quantum theory*, Bibliographisches Institut Wissenschaftsverlag, Mannheim, p.133.
[1103] Łukasiewicz J., 1913, *Die Logischen Grundlagen der Wahrscheinlichkeitsrechnung*, Polska Akademia Umiejętności, Kraków (engl. transl. 1970, *Logical foundations of probability theory*, in: Łukasiewicz J., *Selected works*, Państwowe Wydawnictwo

Naukowe, Warszawa, p.16).
[1104] Łukasiewicz J., 1920, *O logice trójwartościowej*, Ruch Filozof. **5**, 170 (engl. transl. 1970, *On three-valued logic*, in: Łukasiewicz J., *Selected works*, Państwowe Wydawnictwo Naukowe, Warszawa, p.87).
[1105] Łukasiewicz J., 1930, *Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls*, Compt. Rend. Séanc. Soc. Sci. Lett. Varsovie **23**, 51 (engl. transl. 1967, *Philosophical remarks on many-valued systems of propositional logics*, in: McCall S., *Polish logic 1920-1939*, Clarendon, Oxford, p.40).
[1106] Lurie J., 2011, *von Neumann algebras*, Lecture notes, available at: www.math.harvard.edu/∼lurie/261y.html.
[1107] Luschgy H., Mussmann D., 1985, *Equivalent properties and completion of statistical experiments*, Sankhyā A **47**, 174.
[1108] Luxemburg W.A.J., 1955, *Banach function spaces*, Ph.D. thesis, Delft.
[1109] Luxemburg W.A.J., Zaanen A.C., 1971, *Riesz spaces*, Vol.1, North-Holland, Amsterdam.
[1110] Lyubovin V.D., 1954, *Poluuporyadochennye kol'ca samospryazhennykh operatorov*, Ph.D. thesis, Leningradskiĭ Gosudarstvennyĭ Pedagogicheskiĭ Institut, Leningrad.
[1111] Lyubovin V.D., 1956, *O K-prostranstve ogranichennykh samospryazhennykh operatorov*, Uch. zap. leningr. pedagog. univ. **125**, 119.
[1112] Mackey G.W., 1945, *On infinite dimensional spaces*, Trans. Amer. Math. Soc. **57**, 155.
[1113] Mackey G.W., 1957, *Quantum mechanics and Hilbert space*, Amer. Math. Month. **64**, 45.
[1114] Mackey G.W., 1963, *Mathematical foundations of quantum mechanics*, Benjamin, New York.
[1115] MacLane S., 1963, *Natural associativity and commutativity*, Rice Univ. Stud. **49**, 28.
[1116] MacLane S., 1971, *Categories for the working mathematician*, Springer, Berlin (2nd ed., 1998).
[1117] MacNeille H.M., 1937, *Partially ordered sets*, Trans. Amer. Math. Soc. **42**, 416.
[1118] Maeda S., 1955, *Dimension functions on certain general lattices*, J. Sci. Hiroshima Univ. A **19**, 211.
[1119] Maeda S., 1990, *Probability measures on projections in von Neumann algebras*, Rev. Math. Phys. **1**, 235.
[1120] Magyar Z., Sebestyén Z., 1985, *On the definition of C^*-algebras II*, Can. J. Math. **37**, 664.
[1121] Mahalanobis P.C., 1936, *On the generalized distance in statistics*, Proc. Nat. Inst. Sci. India **2**, 49.
[1122] Maharam D., 1942, *On homogeneus measure algebras*, Proc. Amer. Math. Soc. **28**, 108.
[1123] Maharam D., 1942, *On measure in abstract sets*, Trans. Amer. Math. Soc. **51**, 413.
[1124] Majewski A.W., 1990, *On completely positive nonlinear dynamical semigroups*, J. Phys. A **23**, L359.
[1125] Majewski A.W., Zegarliński B., 1995, *Quantum stochastic dynamics I: spin systems on a lattice*, Math. Phys. Electr. J. **1**, 1.
[1126] Majewski A.W., Zegarliński B., 1996, *On quantum L_p-space technique*, Acta Phys. Polon. B **27**, 653.
[1127] Majewski A.W., Zegarliński B., 1996, *On quantum stochastic dynamics*, Markov Proc. Rel. Fields **2**, 87.
[1128] Majewski A.W., Zegarliński B., 1996, *On quantum stochastic dynamics and non-comutative L_p-spaces*, Lett. Math. Phys. **36**, 337.
[1129] Majewski A.W., Zegarliński B., 1996, *Quantum stochastic dynamics II*, Rev. Math. Phys. **8**, 689.
[1130] Maligranda L., 1989, *Orlicz spaces and interpolation*, University of Campinas, Campinas.
[1131] Manoff S., 2000, *Auto-parallel equation as Euler-Lagrange's equation over spaces with affine connections and metrics*, arXiv:gr-qc/0010048.
[1132] Marchand J.-P., 1981, *Statistical inference in quantum mechanics*, in: Gustafson K.E. et al (eds.), *Quantum mechanics in mathematics, chemistry and physics*, Plenum, New York, p.73.
[1133] Marchand J.-P., 1983, *Statistical inference by minimal Bures distance*, in: van der Merwe (ed.), *Old and new questions in physics, cosmology, philosophy, and theoretical biology. Essays in honor of Wolfgang Yourgreau*, Plenum, New York, p.275.
[1134] Marchand J.-P., 1983, *Statistical inference in non-commutative probability*, Rend. Sem. Math. Fis. Milano **52**, 551.
[1135] Marchand J.-P., Wyss W., 1977, *Statistical inference and entropy*, J. Stat. Phys. **16**, 349.
[1136] Markov A.A., 1906, *Raspostranenye zakona bol'shikh chisel na velichiny, zavisyashhie drug ot druga*, Izv. Fiz.-mat. Obshh. Kazan. Univ. **15**, 135. (Engl. transl. *The extension of the law of large numbers onto quantities depending on each other*, in: Sheynin O.B. (ed.), 2004, *Probability and statistics. Russian papers*, NG Verlag, Berlin, p.143).
[1137] Marsden J.E., Ratiu T.S., 1994, *Introduction to mechanics and symmetry*, Springer, Berlin.
[1138] Martignon L., 1980, *Banach f-algebras and Banach lattice algebras with unit*, Bol. Soc. Bras. Mat. **11**, 11.
[1139] Martin F., Petit J.-L., Petit-Littaye M., 1971, *Comparison des expériences*, Ann. Inst. Henri Poincaré Sèr. B **7**, 145.
[1140] Martin F., Vaguelsy M.D., 1969, *Propriétés asymptotiques du modèle statistique*, Ann. Inst. Henri Poincaré Sèr. B **5**, 357.
[1141] Martin P.C., Schwinger J., 1959, *Theory of many particle systems: I*, Phys. Rev. **115**, 1342.
[1142] Masuda T., 1983, *L_p-spaces for von Neumann algebra with reference to a faithful normal semifinite weight*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **19**, 673. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=19&iss=2&rank=10.
[1143] Masuda T., 1984, *Groupoid dynamical systems and crossed product, I–the case of C^*-systems*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **20**, 959.
[1144] Masuda T., 1984, *Groupoid dynamical systems and crossed product, I–the case of W^*-systems*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **20**, 929.
[1145] Masuda T., 1984, *A note on a theorem of A. Connes on Radon-Nikodym cocycles*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **20**, 131.
[1146] Masuda T., 1985, *Groupoid dynamical systems and crossed product*, in: Araki H. et al (eds.), *Operator algebras and their connections with topology and erodic theory. Proceedings of the OATE Conference held in Busteni, Romania, Aug. 29 – Sept. 9, 1983*, LNM **1132**, Springer, Berlin, p.350.
[1147] Matsuzuoe H., 1998, *On a realization of conformally projectively flat statistical manifolds and the divergences*, Hokkaido Math. J. **27**, 409.
[1148] Matumoto T., 1993, *Any statistical manifold has a contrast functions (on the C^3-functions taking the minimum at the diagonal of the product manifold)*, Hiroshima Math. J. **23**, 327. euclid:hmj/1206128255.
[1149] Matusita K., 1951, *On the theory of statistical decision functions*, Ann. Inst. Statist. Math. **3**, 17.
[1150] Matusita K., 1955, *Decision rules based on the distance, for problems of fit, two samples and estimation*, Ann. Math. Statist. **26**, 613.
[1151] Matusita K., 1964, *Distance and decision rules*, Ann. Inst. Statist. Math. **16**, 305.
[1152] Maurin K., 1969, *Generalized eigenfunction expansion and unitary representations of topological groups*, Państwowe Wydawnictwo Naukowe, Warszawa).
[1153] McCarthy C.A., 1967, c_p, Israel J. Math. **5**, 249.
[1154] McDuff D., 1969, *A countable infinity of II_1 factors*, Ann. Math. **90**, 361.

[1155] McDuff D., 1969, *Uncountably many II_1 factors*, Ann. Math. **90**, 372.
[1156] McKinsey J.C.C., Suppes P., 1954, *Review of La structure des théories physiques by Paulette Destouches-Février*, J. Symb. Log. **19**, 52.
[1157] McShane E.J., 1944, *Integration*, Princeton University Press, Princeton.
[1158] McShane E.J., 1949, *Remark concerning integration*, Proc. Nat. Acad. Sci. U.S.A. **35**, 46.
[1159] McShane E.J., 1953, *Order-preserving maps and integration processes*, Princeton University Press, Princeton.
[1160] McShane E.J., 1962, *Families of measures and representations of algebras of operators*, Trans. Amer. Math. Soc. **102**, 328.
[1161] Meyer-Nieberg P., 1991, *Banach lattices*, Springer, Berlin.
[1162] Meyer P.-A., 1993, *Quantum probability for probabilists*, Springer, Berlin (1995, 2nd corr. ed.).
[1163] Michor P.W., Petz D., Andai A., 2000, *On the curvature of a certain riemannian space of matrices*, Inf. Dim. Anal. Quant. Prob. Relat. Top. **3**, 199.
[1164] Mielnik B., 1969, *Theory of filters*, Commun. Math. Phys. **15**, 1.
[1165] Mielnik B., 1974, *Generalized quantum mechanics*, Commun. Math. Phys. **37**, 221.
[1166] Mielnik B., 1976, *Quantum logic: is it necessarily orthocomplemented?*, in: Flato M. et al (eds.), *Quantum mechanics, determinism, causality, and particles. An international collection of contributions in honor of Louis de Broglie on the occasion of the jubilee of His celebrated thesis*, Reidel, Dordrecht, p.117.
[1167] Mielnik B., 2000, *Is nonlinear quantum mechanics possible?*, in: Doebner H.-D. (ed.), *Quantum theory and symmetries: proceedings of the international symposium: Goslar, Germany, 18-22 July 1999*, World Scientific, Singapore, p.124.
[1168] Minkowski H., 1909, *Raum und Zeit*, Phys. Z. **10**, 104.
[1169] Mitchell W.C., 1967, *Statistical mechanics of thermally driven systems*, Ph.D. thesis, Washington University, St.Louis.
[1170] Mittelmeyer G., Wolff M., 1974, *Über den Absolutbetrag auf Komplexen Vektorbänden*, Math. Z. **131**, 87.
[1171] Mączyński M.J., 1967, *A remark on Mackey's axiom system for quantum mechanics*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astr. Phys. **15**, 583.
[1172] Mączyński M.J., 1972, *Hilbert space formalism of quantum mechanics without the Hilbert space axiom*, Rep. Math. Phys. **3**, 209.
[1173] Mączyński M.J., 1973, *The field of real numbers in axiomatic quantum mechanics*, J. Math. Phys. **14**, 1469.
[1174] Mączyński M.J., 1973, *On a functional representation of the lattice of projections on a Hilbert space*, Studia Math. **47**, 253.
[1175] Mączyński M.J., 1973, *Orthogonality postulae in axiomatic quantum mechanics*, Int. J. Theor. Phys. **8**, 353.
[1176] Mączyński M.J., 1974, *Functional properties of quantum logics*, Int. J. Theor. Phys. **11**, 149.
[1177] Mączyński M.J., 1981, *Commutativity and generalised transition probability in quantum logic*, in: Beltrametti E.G., van Fraasen B.C. (eds.), *Current issues in quantum logic*, Plenum Press, New York, p.355.
[1178] Moore E.H., 1909, *On a form of general analysis, with applications to linear differential and integral equations*, in: Castelnuovo G. (ed.), *Atti del IV Congresso Internazionale dei Matematici (Roma, 6-11 Aprile 1908)*, Accademia Nazionale dei Lincei, Roma, vol.2, p.98.
[1179] Moore E.H., 1915, *Definition of limit in general integral analysis*, Proc. Natl. Acad. Sci. USA **1**, 628.
[1180] Mordukhovich B.S., 2006, *Variational analysis and generalized differentiation*, Vol.1-2, Springer, Berlin.
[1181] Moreau J.-J., 1963, *Fonctionelles sous-différentiables*, Compt. Rend. Acad. Sci. Paris **257**, 4117.
[1182] Morgan S.L., Winship C., 2007, *Counterfactuals and causal inference: methods and principles for social research*, Cambridge University Press, Cambridge.
[1183] Mori H., 1956, *A quantum-statistical theory of transport processes*, J. Phys. Soc. Japan **11**, 1029.
[1184] Morimoto T., 1963, *Markov processes and the H-theorem*, J. Phys. Soc. Jap. **12**, 328.
[1185] Morozova E.A., Chencov N.N., 1982, *Struktura semeĭstv stacionarnykh raspredeleniĭ dlya nekomutitativnoĭ cepi Markova*, in: *Matematicheskiye modeli statisticheskoĭ fiziki*, Izdatel'stvo Tyumenskogo Gosudarstvennogo Universiteta, Tyumen', p.31.
[1186] Morozova E.A., Chencov N.N., 1985, *Ekvivarianty v kategorii markovskikh otovrazheniĭ*, Usp. mat. nauk **40**, 189.
[1187] Morozova E.A., Chencov N.N., 1986, *Markov maps in noncommutative probability theory and mathematical statistics*, in: Prokhorov J.V. et al (eds.), *Probability theory and mathematical statistics. Proceedings of the fourth Vilnius conference*, Vol.2, VNU Sci. Press, Utrecht, p.287.
[1188] Morozova E.A., Chencov N.N., 1986, *Markovskie invarianty par veroyatnostnykh zakonov*, Preprint **1986-121**, Institut Prikladnoĭ Matematiki AN SSSR.
[1189] Morozova E.A., Chencov N.N., 1988, *Invariantnye lineĭnye svyaznosti na sovokupnostyakh raspredeleniĭ veroyatnosteĭ*, Preprint **1988-167**, Institut Prikladnoĭ Matematiki AN SSSR.
[1190] Morozova E.A., Chencov N.N., 1989, *Markovskaya invaryantnaya geometriya na mnogoobraziyakh sostoyaniĭ*, Itogi Nauk. i Tekh. Ser. Sovrem. Probl. Mat. Nov. Dostizh. **36**, 69 (engl. transl. 1991, *Markov invariant geometry on state manifolds*, J. Soviet Math. **56**, 2648).
[1191] Morozova E.A., Chencov N.N., 1991, *Estestvennaya geometriya semeĭstv veroyatnostnykh zakonov*, Itogi Nauk. i Tekhn. Ser. Sovr. Probl. Mat. Fundam. Napravlen. **83**, 133.
[1192] Morse M., Transue W., 1950, *Functionals F bilinear over the product $A \times B$ of two pseudo-normed vector spaces*, Ann. Math. **51**, 576.
[1193] Morse N., Sacksteder R., 1966, *Statistical isomorphism*, Ann. Math. Statist. **37**, 203.
[1194] Mosonyi M., Petz D., 2004, *Structure of sufficient coarse grainings*, Lett. Math. Phys. **68**, 19.
[1195] Moy S.-T.C., 1954, *Characterisations of conditional expectation as a transformation on function spaces*, Pacific J. Math. **4**, 47. euclid:pjm/1103044954.
[1196] Moyal J.E., 1949, *Quantum mechanics as a statistical theory*, Proc. Cambridge Phil. Soc. **45**, 99.
[1197] Muller F.A., 1997, *The equivalence myth of quantum mechanics–part I, II*, Stud. Hist. Phil. Mod. Phys. **28**, 35, 219.
[1198] Muller F.A., 1999, *The equivalence myth of quantum mechanics (addendum)*, Stud. Hist. Phil. Mod. Phys. **30**, 543.
[1199] Mundici D., 1986, *Interpretation of AF C^*-algebras in Łukasiewicz sentential calculus*, J. Funct. Anal. **65**, 15.
[1200] Mundici D., 1989, *The C^*-algebras of three-valued logic*, in: Bonotto C. et al (eds.), *Logic colloquium '88*, North-Holland, Amsterdam, p.61.
[1201] Mundici D., 1996, *Uncertainty measures in MV algebras and states on AF C^*-algebras*, J. Funct. Anal. **65**, 15.
[1202] Murata N., Takenouchi T., Kanamori T., Eguchi S., 2004, *Information geometry of U-boost and Bregman divergence*, Naural Comput. **26**, 1651.
[1203] Muratov M.A., 1978, *[Noncommutative Orlicz spaces]*, Dokl. Akad. Nauk UzSSR **1978:6**, 11.
[1204] Muratov M.A., 1979, *[The Luxemburg norm in an Orlicz space of measurable operators]*, Dokl. Akad. Nauk UzSSR **1979:1**, 5.

[1205] Murphy G.J., 1990, *C^*-algebras and operator theory*, Academic Press, San Diego.
[1206] Murray F.J., von Neumann J., 1936, *On rings of operators*, Ann. Math. **37**, 116.
[1207] Murray F.J., von Neumann J., 1937, *On rings of operators II*, Trans. Am. Math. Soc. **41**, 208.
[1208] Murray F.J., von Neumann J., 1943, *On rings of operators IV*, Ann. Math. **44**, 716.
[1209] Murray M.K., Rice J.W., 1993, *Differential geometry and statistics*, Chapman and Hall, London.
[1210] Musielak J., 1983, *Orlicz spaces and modular spaces*, LNM **1034**, Springer, Berlin.
[1211] Mussmann D., 1972, *Vergleich von Experimenten im schwach dominierten Fall*, Z. Wahrschein. Geb. **24**, 295.
[1212] Mussmann D., 1979, *Sufficiency and f-divergences*, Studia Sci. Math. Hungar. **14**, 37.
[1213] Nagaoka H., 1982, *[Foundations of statistical geometry and applications to robust estimation]*, M.Sc. thesis, Tōkyō daigaku, Tōkyō.
[1214] Nagaoka H., 1987, *?*, in: *Proceedings of 10th Symposium on Information Theory and Its Applications (SITA), Enoshima, Kanagawa, Japan, November 19-21, 1987*, p.241 (engl. transl. 2005, *On Fisher information of quantum statistical models*, in: Hayashi M. (ed.), *Asymptotic theory of quantum statistical inference*, World Scientific, Singapore).
[1215] Nagaoka H., 1989, *A new approach to Cramér–Rao bounds for quantum state estimation*, IEICE Technical Report IT **89-42**, 9.
[1216] Nagaoka H., 1994, *Differential geometrical aspects of quantum state estimation and relative entropy*, Technical report METR **94-14**, University of Tōkyō, Tōkyō.
[1217] Nagaoka H., 1995, *Differential geometrical aspects of quantum state estimation end relative entropy*, in: Belavkin V.P., Hirota O., Hudson R.L. (eds.), *Proceedings of an International Workshop on Quantum Communications and Measurement, July 11-16, 1994, in Nottingham, England*, Plenum, New York, p.449.
[1218] Nagaoka H., Amari S.-i., 1982, *Differential geometry of smooth families of probability distributions*, Technical report METR **82-7**, University of Tōkyō, Tōkyō.
[1219] Nagel E., 1945, *Review of Hans Reichenbach's philosophic foundations of quantum mechanics*, J. Phil. **42**, 437.
[1220] Nagumo M., 1936, *Einige analytische Untersuchungen in linearen metrischen Ringen*, Japan J. Math. **13**, 61.
[1221] Nakagami Y., 1975, *Duality for crossed products of von Neumann algebras by locally compact groups*, Bull. Amer. Math. Soc. **81**, 1106.
[1222] Nakagami Y., 1977, *Dual action on a von Neumann algebra and Takesaki's duality for a locally compact group*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **12**, 727.
[1223] Nakagami Y., Takesaki M., 1979, *Duality for crossed products of von Neumann algebras*, LNM **731**, Springer, Berlin.
[1224] Nakamura M., Turumaru T., 1954, *Expectations in an operator algebra*, Tōhoku Math. J. **6**, 174. euclid:tmj/1178245178.
[1225] Nakamura M., Umegaki H., 1961, *A note on the entropy on operator algebras*, Proc. Jap. Acad. **37**, 149.
[1226] Nakamura M., Umegaki H., 1962, *On von Neumann theory of measurements in quantum statistics*, Math. Jap. **7**, 151.
[1227] Nakano H., 1941, *Über System aller stetigen Funktionen auf einem topologischen Raum*, Proc. Imp. Acad. Tōkyō **17**, 308. Available at: euclid:pja/1195578669.
[1228] Nakano H., 1950, *Modulared semi-ordered linear spaces*, Maruzen, Tōkyō.
[1229] Namsrai K., 1986, *Nonlocal quantum field theory and stochastic quantum mechanics*, Reidel, Dordrecht.
[1230] Naĭmark M.A., 1940, *Spekral'nye funkcii simmetricheskogo operatora*, Izv. Akad. Nauk. SSSR, Ser. Mat. **4**, 277.
[1231] Naĭmark M.A., 1956, *Normirovannye kol'ca*, Gostekhizdat, Moskva. (2nd. rev. ed., 1968; engl. transl. 1972, *Normed rings*, Noordhoff, Groningen.).
[1232] Navaza J., 1985, *On the maximum entropy estimate of electron density function*, Acta Crystallogr. A **41**, 232.
[1233] Navaza J., 1986, *The use of non-local constraints in maximum-entropy electron density reconstruction*, Acta Crystallogr. A **42**, 212.
[1234] Negrepontis J.W., 1971, *Duality in analysis from the point of view of triples*, J. Alg. **19**, 228.
[1235] Neirotti J.P., Raggio G.A., 1993, *Maximizing relative entropy at given energies*, Lett. Math. Phys. **29**, 33.
[1236] Nelson E., 1967, *Dynamical theories of brownian motion*, Princeton University Press, Princeton.
[1237] Nelson E., 1974, *Notes on non-commutative integration*, J. Funct. Anal. **15**, 103.
[1238] Nelson E., 1986, *Quantum fluctuations*, Princeton University Press, Princeton.
[1239] Neveu J., 1964, *Bases mathématiques du calcul des probabilités*, Masson et Cie, Paris (engl. transl.: 1965, *Mathematical foundations of the calculus of probability*, Holden–Day, San Francisco).
[1240] Neyman J., 1935, *Sur un teorema concernente le considette statistiche suficienti*, Giorn. Ist. Ital. Att. **6**, 320.
[1241] Nikodým O.M., 1930, *Sur uner généralisation des intégrales de M.J.Radon*, Fund. Math. **15**, 131. Available at: matwbn.icm.edu.pl/ksiazki/fm/fm15/fm15114.pdf.
[1242] Nikodým O.M., 1931, *Contribution à la théorie des fonctionnelles linéaires en connection avec la théorie de la mesure des ensembles abstraits*, Mathematica, Cluj **5**, 130; Bull Soc. Ştiinţ. Cluj **6**, 79.
[1243] Nomizu K., Sasaki T., 1994, *Affine differential geometry*, Cambridge University Press, Cambridge.
[1244] Nomizu K., Simon U., 1992, *Notes on conjugate connections*, in: Dillen F., Verstraelen L. (eds.), *Geometry and topology of submanifolds IV*, World Scientific, Singapore, p.152.
[1245] Norden A.P., 1945, *O parakh soprazhennykh parallel'nykh pereneseniĭ v n-mernykh prostranstvakh*, Dokl. Akad. Nauk SSSR **49**, 9.
[1246] Norden A.P., 1948, *O vnutrennikh geometriyakh poverkhnosteiĭ proektivnogo prostranstva*, Trudy semin. vekt. tenzorn. anal. **6**, 125.
[1247] Norden A.P., 1949, *O vnutrennikh geometriyakh poverkhnosteiĭ proektivnogo prostranstva*, Trudy semin. vekt. tenzorn. anal. **7**, 31.
[1248] Norden A.P., 1959, *O vnutrenneĭ geometrii 2-go roda na giperploskosti affinnogo prostranstva*, in: Shirokov N.A., Shirokov A.N., *Affinnaya differencial'naya geometriya*, Fizmatgiz, Moskva.
[1249] Norden A.P., 1976, *Prostranstva affinoĭ svyaznosti*, (2nd. rev. ed.), Nauka, Moskva.
[1250] Ochs W., Bayer W., 1973, *Quantum states with maximum information entropy II, II*, Z. Naturf. **28a**, 1571.
[1251] Ochs W., Spohn H., 1978, *A characterization of the Segal entropy*, Rep. Math. Phys. **14**, 75.
[1252] Odzijewicz A., 2011, *Hamiltonian and quantum mechanics*, Geom. Topol. Monogr. **17**, 385.
[1253] Odzijewicz A., Ratiu T.S., 2003, *Banach Lie–Poisson spaces and reduction*, Commun. Math. Phys. **243**, 1. arXiv:math/0210207.
[1254] Odzijewicz A., Ratiu T.S., 2004, *Extensions of Banach–Lie–Poisson spaces*, J. Funct. Anal. **217**, 103. arXiv:math/0310312.
[1255] Odzijewicz A., Ratiu T.S., 2008, *Induced and coinduced Banach Lie–Poisson spaces and integrability*, J. Funct. Anal. **255**, 1225.
[1256] Odzijewicz A., Sliżewska A., 2011, *Groupoids and inverse semigroups associated to W^*-algebras*, arXiv:1110.6305.

[1257] Ogasawara T., Yoshinaga K., 1955, *Extension of ♮-application to unbounded operators*, J. Sci. Hiroshima Univ. **19**, 273. Available at: dmitripavlov.org/scans/ogasawara-yoshinaga-1.pdf.
[1258] Ogasawara T., Yoshinaga K., 1955, *A non-commutative theory of integration for operators*, J. Sci. Hiroshima Univ. **18**, 311. Available at: dmitripavlov.org/scans/ogasawara-yoshinaga-0.pdf.
[1259] Ohya M., Petz D., 1993, *Quantum entropy and its use*, Springer, Berlin (2nd. ed., 2004).
[1260] Ojima I., 2004, *Temperature as order parameter of broken scale invariance*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **40**, 731. arXiv:math-ph/0311025.
[1261] Olesen D., Pedersen G.K., 1978, *Applications of the Connes spectrum to C^*-dynamical systems*, J. Funct. Anal. **30**, 179.
[1262] Olmsted J.H.M., 1942, *Lebesgue theory on a boolean algebra*, Trans. Amer. Math. Soc. **51**, 164.
[1263] Ono T., 1959, *Note on a B^*-algebra*, J. Math. Soc. Japan **11**, 140.
[1264] Ore Ø., 1935, *On the foundations of abstract algebra. I*, Ann. Math. **36**, 406.
[1265] Orlicz W., 1932, *Über eine gewisse Klasse von Räumen vom Typus B*, Bull. Acad. Polon. Sci. Sér. A **1932**:8/9, 207.
[1266] Orlicz W., 1936, *Über Räume (L^M)*, Bull. Int. Acad. Polon. Sci. Lett. A **1936**, 93.
[1267] Ovchinnikov V.I., 1970, *Simmetrichnye prostranstva izmerimykh operatorov*, Dokl. Akad. Nauk SSSR **191**, 769 (engl. transl. 1970, *Symmetric spaces of measurable operators*, Sov. Math. Dokl. **11**, 448).
[1268] Ozawa M., 1984, *Quantum measuring processes of continuous observables*, J. Math. Phys. **25**, 79.
[1269] Ozawa M., 1985, *Conditional probability and a posteriori states in quantum mechanics*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **21**, 279.
[1270] Padmanabhan A.R., 1967, *Convergence in measure and related results in finite rings of operators*, Trans. Amer. Math. Soc. **128**, 359 (err. 1968, **132**, 563).
[1271] Padmanabhan A.R., 1979, *Probabilistic aspects of von Neumann algebras*, J. Funct. Anal **31**, 139.
[1272] Palge V., Konrad T., 2008, *A remark on Fuchs' bayesian interpretation of quantum mechanics*, Stud. Hist. Phil. Mod. Phys. **39**, 273.
[1273] Palmer T.W., 1994, 2001, *Banach algebras an the general theory of ∗-algebras*, Vol.1-2, Cambridge University Press, Cambridge.
[1274] Pardo L., 2006, *Statistical inference based on divergence measures*, Chapman & Hall, London.
[1275] Pardo M.C., Vajda I., 1997, *About distances of discrete distributions satisfying the data processing theorem of information theory*, IEEE Trans. Inf. Theor. **43**, 1288.
[1276] Park J.L., Margenau H., 1968, *Simultaneous measurability in quantum theory*, Int. J. Theor. Phys. **1**, 211.
[1277] Parthasarthy K.R., 1992, *An introduction to quantum stochastic calculus*, Birkhäuser, Basel.
[1278] Paszkiewicz A., 1985, *Measures on projections in W^*-factors*, J. Funct. Anal. **62**, 87.
[1279] Pauli W., 1926, *A letter to P. Jordan (12th April)*, in: van der Waerden B.L., 1973, *From matrix mechanic and wave mechanics to unfied quantum mechanics*, in: Mehra J. (ed.), *The physicist's conception of nature*, Reidel, Dordrecht, p.276 (reprinted in: 1997, Not. Amer. Math. Soc. **44**, 323).
[1280] Pauli W., 1927, *Über Gasentartung und Paramagnetismus*, Z. Phys. **41**, 81.
[1281] Pavičić M., 1992, *Bibliography on quantum logics and related structures*, Int. J. Theor. Phys. **31**, 373.
[1282] Pavičić M., 2009, *Is quantum logic a logic?*, in: Engesser K., Gabbay D.M., Lehmann D. (eds.), *Handbook of quantum logic and quantum structures: quantum logic*, Elsevier, Amsterdam, p.23.
[1283] Pavlov D., 2010, *Posts at mathoverflow.net*, Available at: mathoverflow.net/questions/49426.
[1284] Pavlov D., 2011, *A decomposition theorem for noncommutative L_p-spaces and a new symmetric monoidal bicategory of von Neumann algebras*, Ph.D. thesis, University of California, Berkeley.
[1285] Pearl J., 2000, *Causality: models, reasoning and inference*, Cambridge University Press, Cambridge (2nd ed. 2009).
[1286] Pearl J., 2009, *Causal inference in statistics: an overview*, Statist. Surv. **3**, 96. Available at: ftp.cs.ucla.edu/pub/stat_ser/r350.pdf.
[1287] Pearson K., 1900, *On the criterion that a given system of deviations from the probable in the case of correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling*, Phil. Mag. **50**, 157.
[1288] Pechukas P., 1994, *Reduced dynamics need not be completely positive*, Phys. Rev. Lett. **73**, 1060.
[1289] Pechukas P., 1995, *Pechukas replies*, Phys. Rev. Lett. **75**, 3021.
[1290] Pedersen G.K., 1966, *Measure theory for C^*-algebras*, Math. Scand. **19**, 131.
[1291] Pedersen G.K., 1968, *Measure theory for C^*-algebras II*, Math. Scand. **22**, 63.
[1292] Pedersen G.K., 1969, *Measure theory for C^*-algebras III*, Math. Scand. **25**, 71.
[1293] Pedersen G.K., 1969, *Measure theory for C^*-algebras IV*, Math. Scand. **25**, 121.
[1294] Pedersen G.K., 1972, *C^*-integrals: an approach to non-commutative measure theory*, Ph.D. thesis, University of Copenhagen, Copenhagen.
[1295] Pedersen G.K., 1979, *C^*-algebras and their automorphism groups*, Academic Press, London.
[1296] Pedersen G.K., 1986, *SAW^*-algebras and corona C^*-algebras, contributions to non-commutative topology*, J. Op. Theor. **15**, 15.
[1297] Pedersen G.K., Takesaki M., 1973, *The Radon–Nikodym theorem for von Neumann algebras*, Acta Math. **130**, 53. Available at: www.kryakin.com/files/Acta_Mat_%282_55%29/acta150_107/130/130_02.pdf.
[1298] Peirce C.S., 1880, *On the algebra of logic*, Amer. J. Math. **3**, 15.
[1299] Peirce C.S., 1884, *On the algebra of logic*, Amer. J. Math. **7**, 180.
[1300] Perdizet F., 1970, *Élements positifs relatifs à une algèbre hilbertienne à gauche*, Comp. Rend. Acad. Sci. Paris **270**, 322.
[1301] Perdizet F., 1971, *Élements positifs relatifs à une algèbre hilbertienne à gauche*, Comp. Math. **23**, 25.
[1302] Peres A., 1990, *Neumark's theorem and quantum inseparability*, Found. Phys. **20**, 1441.
[1303] Pérez A., 1967, *Information-theoretic risk estimates in statistical decision*, Kybernetika **3**, 1. Available at: www.kybernetika.cz/content/1967/1/1/paper.pdf.
[1304] Petz D., 1985, *Properties of quantum entropy*, in: Accardi L., von Waldenfels W. (eds.), *Quantum probability and applications II*, LNM **1136**, Springer, Berlin, p.428.
[1305] Petz D., 1985, *Quasi-entropies for states of a von Neumann algebra*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **21**, 787. Available at: www.math.bme.hu/∼petz/pdf/26quasi.pdf.
[1306] Petz D., 1986, *Properties of the relative entropy of states of von Neumann algebra*, Acta Math. Hungar. **47**, 65.
[1307] Petz D., 1986, *Quasi-entropies for finite quantum systems*, Rep. Math. Phys. **23**, 57. Available at: www.renyi.hu/∼petz/pdf/52.pdf.
[1308] Petz D., 1986, *Sufficient subalgebras and the relative entropy of states of a von Neumann algebra*, Commun. Math. Phys. **105**, 123.

[1309] Petz D., 1988, *A variational expression for the relative entropy*, Comm. Math. Phys. **114**, 345.
[1310] Petz D., 1991, *On certain properties of the relative entropy of states of operator algebras*, Math. Z. **206**, 351.
[1311] Petz D., 1992, *Characterization of the relative entropy of states of matrix algebras*, Acta Math. Hung. **59**, 449. Available at: www.renyi.hu/∼petz/pdf/52.pdf.
[1312] Petz D., 1992, *Entropy in quantum probability I*, in: Accardi L. (ed.), *Quantum probability and related topics VII*, World Scientific, Singapore, p.275.
[1313] Petz D., 1994, *Geometry of canonical correlation on the state space of a quantum system*, J. Math. Phys. **35**, 780. Available at: www.renyi.hu/∼petz/pdf/61.pdf.
[1314] Petz D., 1994, *On entropy functionals of states of operator algebras*, Acta Math. Hungar. **64**, 333. Available at: www.renyi.hu/∼petz/pdf/59.pdf.
[1315] Petz D., 1996, *Monotone metrics on matrix spaces*, Lin. Alg. Appl. **224**, 81. Available at: www.renyi.hu/∼petz/pdf/68mon.pdf.
[1316] Petz D., 2007, *Bregman divergence as relative operator entropy*, Acta Math. Hungar. **116**, 127. Available at: www.renyi.hu/∼petz/pdf/112bregman.pdf.
[1317] Petz D., 2010, *From f-divergence to quantum quasi-entropies and their use*, Entropy **12**, 304. Available at: www.mdpi.com/1099-4300/12/3/304/pdf.
[1318] Petz D., Hasegawa H., 1996, *On the riemannian metric of α-entropies of density matrices*, Lett. Math. Phys. **38**, 221. Available at: www.renyi.hu/∼petz/pdf/71.pdf.
[1319] Petz D., Jenčová A., 2006, *Sufficiency in quantum statistical inference*, Commun. Math. Phys. **263**, 259.
[1320] Petz D., Raggio G.A., Verbeure A., 1989, *Asymptotics of Varadhan-type and the Gibbs variational principles*, Commun. Math. Phys. **121**, 271.
[1321] Petz D., Sudár C., 1996, *Geometries of quantum states*, J. Math. Phys. **37**, 2662.
[1322] Petz D., Sudár C., 1999, *Extending the Fisher metric to density matrices*, in: Barndorff-Nielsen O.E., Vedel Jensen E.B. (eds.), *Geometry in present day science*, World Scientific, Singapore, p.21.
[1323] Petz D., Toth G., 1993, *The Bogoliubov inner product on quantum statistics*, Lett. Math. Phys. **27**, 205.
[1324] Petz D., Zemánek J., 1988, *Characterizations of the trace*, Lin. Alg. Appl. **111**, 43.
[1325] Pfanzagl J., 1973, *Asymptotic expansions related to minimum contrast estimators*, Ann. Statist. **1**, 993.
[1326] Pfanzagl J., 1981, *The second order optimality of tests and estimators for minimum contrast functionals. I*, Prob. Math. Statist. **2**, 55.
[1327] Pfanzagl J., 1982, *Contribution to general asymptotic statistical theory*, LNS **13**, Springer, Berlin.
[1328] Pfeffer W.F., 1977, *Integrals and measures*, Marcel Dekker, New York.
[1329] Phan V.T., 1999, *L^p spaces for C^*-algebras*, Ph.D. thesis, University of Łódź, Łódź.
[1330] Phelps R.R., 1989, *Convex functions, monotone operators and differentiablity*, LNM **1364**, Springer, Berlin (1993, 2nd rev. ed.).
[1331] Picard C.E., 1890, *Memoire sur la théorie des équations aux dérivées partielles et la méthode des approximations successives*, J. Math. Pures Appl. **6**, 145.
[1332] Picard D.B., 1992, *Statistical morphisms and related invariance properties*, Ann. Inst. Statist. Math. **44**, 45.
[1333] Pillet C.-A., 2006, *Quantum dynamical systems*, in: Attal S., Joye A., Pillet C.-A. (eds.), *Open quantum systems I: the hamiltonian approach*, LNM **1880**, Springer, Berlin, p.107. Available at: hal.archives-ouvertes.fr/docs/00/12/88/67/PDF/qds.pdf.
[1334] Pinsker M.S., 1960, *Informaciya i informacionnaya ustoĭchivost' sluchaĭnykh velichin i processov*, Izdatel'stvo Akademii nauk SSSR, Moskva (engl. transl. 1964, *Information and information stability of random variables and processes*, Holden-Day, San Francisco).
[1335] Piron C., 1964, *Axiomatique quantique*, Helv. Phys. Acta **37**, 439.
[1336] Piron C., 1976, *Foundations of quantum physics*, Benjamin, New York.
[1337] Piron C., 1990, *Méchanique quantique. Bases et applications*, Presses polytechniques et univesitaites romandes, Lausanne (2nd ed., 1998).
[1338] Pisier G., Xu Q., 2003, *Noncommutative L^p spaces*, in: *Handbook of the geometry of Banach spaces*, vol.2, North-Holland, Amsterdam, p.1459. Available at: dmitripavlov.org/scans/pisier-xu.pdf.
[1339] Pistone G., 2001, *New ideas in nonparametric estimation*, in: Sollich P., Hughston L.P., Streater R.F., Coolen A.C.C. (eds.), *Disordered and complex systems (King's College, London, UK, 10-14 July 2000)*, AIP Conf. Proc. **553**, AIP, Melville, p.159.
[1340] Pistone G., 2010, *Algebraic varietes vs. differentiable manifolds in statistical models*, in: in: Gibilisco P. et al (eds.), *Algebraic and geometric methods in statistics*, Cambridge University Press, Cambridge, p.341.
[1341] Pistone G., Rogantin M.P., 1999, *The exponential statistical manifold, mean parameters, orthogonality and space transformations*, Bernoulli **5**, 721.
[1342] Pistone G., Sempi C., 1995, *An infinite-dimensional geometric structure on the space of all the probability measures equivalent to a given one*, Ann. Statist. **23**, 1543.
[1343] Pitcher T.S., 1957, *Sets of measure not admitting necessary and sufficient statistics on subfields*, Ann. Math. Statist. **28**, 267.
[1344] Pitcher T.S., 1965, *A more general property than domination for sets of probability measures*, Pacific J. Math. **15**, 597.
[1345] Pitman E.J.G., 1936, *Sufficient statistics and intrinsic accuracy*, Proc. Cambridge Phil. Soc. **32**, 567.
[1346] Pitowsky I., 2006, *Quantum mechanics as a theory of probability*, in: Demopoulos W., Pitowsky I. (eds.), *Physical theory and its interpretation: essays in honor of Jeffrey Bub*, Springer, Berlin, p.213. arXiv:quant-ph/0510095.
[1347] Plancherel M., 1910, *Contribution à l'étude de la répresentation d'une fonction arbitraire par des intégrales définies*, Rend. Circ. Mat. Palermo **30**, 289.
[1348] Plymen R.J., 1968, *C^*-algebras and Mackey's axioms*, Commun. Math. Phys. **8**, 132.
[1349] Plymen R.J., 1968, *A modification of Piron's axioms*, Helv. Phys. Acta **41**, 69.
[1350] Poincaré H., 1902, *La science et l'hypothese*, Flammarion, Paris.
[1351] Poincaré H., 1905, *La valeur de la science*, Flammarion, Paris.
[1352] Poincaré H., 1905, *Sur la dynamique de l'électron*, Compt. Rend. Acad. Sci. **140**, 1504.
[1353] Poincaré H., 1906, *Sur la dynamique de l'électron*, Rend. Circ. Mat. Palermo **21**, 129.
[1354] Pontryagin L.S., 1934, *The theory of topological commutative groups*, Ann. Math. **35**, 361.
[1355] Popa S., 1986, *Correspondences*, INCREST preprint. Available at: www.math.ucla.edu/∼popa/popa-correspondences.pdf.
[1356] Powers R.T., 1967, *Representations of uniformly hyperfinite algebras and their associated von Neumann rings*, Ann. Math. **86**, 138.

[1357] Powers R.T., Sakai S., 1975, *Unbounded derivations in operator algebras*, J. Funct. Anal. **19**, 81.
[1358] Pratt J.W., 1961, *Review of 'Testing statistical hypothesis' (Lehman, 1959)*, J. Am. Stat. Assoc. **56**, 163.
[1359] Prugovečki E., 1967, *On a theory of measurement of incompatible observables in quantum mechanics*, Canad. J. Phys. **45**, 2173.
[1360] Prugovečki E., 1973, *A postulational framework for theories of simultaneous measurement of several observables*, Found. Phys. **3**, 3.
[1361] Prugovečki E., 1984, *Stochastic quantum mechanics and quantum spacetime*, Reidel, Dordrecht.
[1362] Prugovečki E., 1985, *A fibre bundle formulation of quantum geometry*, Nuovo Cim. A **89**, 105.
[1363] Prugovečki E., 1987, *A Poincaré gauge-invariant formulation of general-relativistic quantum geometry*, Nuovo Cim. A **97**, 597.
[1364] Prugovečki E., 1991, *Geometro-stochastic locality in quantum spacetime and quantum diffusions*, Found. Phys. **21**, 93.
[1365] Prugovečki E., 1992, *Quantum geometry: a framework for quantum general relativity*, Kluwer, Dordrecht.
[1366] Prugovečki E., 1994, *On the general covariance and strong equivalence principles in quantum general relativity*, Found. Phys. **24**, 989.
[1367] Prugovečki E., 1995, *Principles of quantum general relativity*, World Scientific, Singapore.
[1368] Prugovečki E., 1996, *On locality in quantum general relativity and quantum gravity*, Found. Phys. **26**, 1645. arXiv:gr-qc/9603044.
[1369] Prugovečki E., 1996, *On quantum-geometric connections and propagators in curved spacetime*, Class. Quant. Grav. **13**, 1007. arXiv:gr-qc/9510007.
[1370] Pták P., Pulmannová S., 1989, *Kvantové logiky*, Veda, Bratislava (engl. transl. 1991, *Orthomodular structures as quantum logics*, Kluwer, Dordrecht).
[1371] Pták P., Pulmannová S., 2007, *Quantum logics as underlying structures of generalized probability theory*, in: Engesser K., Gabbay D.M., Lehmann D. (eds.), *Handbook of quantum logic and quantum structures: quantum structures*, Elsevier, Amsterdam, p.147.
[1372] Pukánszky L., 1954, *The theorem of Radon–Nikodym in operator-rings*, Acta Sci. Math. Szeged **15**, 149.
[1373] Pukánszky L., 1956, *Some examples of factors*, Publ. Math. Debrecen **4**, 135.
[1374] Pulmannová S., 1978, *Joint distributions of observables on quanutm logics*, Int. J. Theor. Phys. **17**, 665.
[1375] Pulmannová S., 1978, *A remark on the comparison of Mackey and Segal models*, Math. Slov. **28**, 297.
[1376] Pulmannová S., 1986, *Functional properties of transition amplitude spaces*, Rep. Math. Phys. **24**, 81.
[1377] Pulmannová S., 1986, *Transition probability spaces*, J. Math. Phys. **27**, 1781.
[1378] Pulmannová S., Gudder S.P., 1987, *Geometric properties of transition amplitude spaces*, J. Math. Phys. **28**, 2393.
[1379] Pusz W., Woronowicz S.L., 1978, *Form convex functions and the WYDL and other inequalities*, Lett. Math. Phys. **2**, 505.
[1380] Pusz W., Woronowicz S.L., 1978, *Passive states and KMS states for general quantum systems*, Commun. Math. Phys. **58**, 273.
[1381] Putnam C.R., 1967, *Commutation properties of Hilbert space operators*, Springer, Berlin.
[1382] Pykacz J., 1994, *Fuzzy quantum logics and infinity-valued Łukasiewicz logic*, Int. J. Theor. Phys. **33**, 1403.
[1383] Pykacz J., 1995, *Many-valued logics in foundations of quantum mechanics*, in: Herfel W.E. et al (eds.), *Theories and models in scientific processes*, Poznań Stud. Phil. Sci. Hum. **44**, Radopi, Amsterdam, p.401.
[1384] Pykacz J., 1995, *Quantum logic as partial infinite-valued Łukasiewicz logic*, Int. J. Theor. Phys. **34**, 1697.
[1385] Pykacz J., 2000, *Łukasiewicz operations in fuzzy set and many-valued representations of quantum logics*, Found. Phys. **30**, 1503.
[1386] Pykacz J., 2003, *The many valued logic of Jan Łukasiewicz in the foundations of quantum mechanics*, Wydawnictwo Uniwersytetu Gdańskiego, Gdańsk.
[1387] Pykacz J., 2010, *Unification of two approaches to quantum logic: every Birkhoff–von Neumann quantum logic is a partial infinite-valued Łukasiewicz logic*, Studia Logica **95**, 5.
[1388] Radon J., 1913, *Theorie und Anwendungen der absolut additiven Mengenfunktionen*, Sitzungsber. Akad. Wiss. Wien, Math.-Naturwiss. Klasse, Abt. IIa **122**, 1296.
[1389] Raggio G.A., 1982, *Comparison of Uhlmann's transition probability with the one induced by the natural positive cone of von Neumann algebras in standard form*, Lett. Math. Phys. **6**, 233.
[1390] Raggio G.A., 1984, *Generalized transition probabilities and applications*, in: Accardi L. et al (eds.), *Quantum probability and appplications to quantum theory of irreversible processes*, Springer, Berlin, p.327.
[1391] Raggio G.A., Werner R.F., 1989, *Quantum statistical mechanics of general mean field systems*, Helv. Phys. Acta **62**, 980.
[1392] Raggio G.A., Werner R.F., 1990, *Minimizing the relative entropy in a face*, Lett. Math. Phys. **19**, 7.
[1393] Raggio G.A., Werner R.F., 1991, *The Gibbs variational principle for inhomogeneus mean-field systems*, Helv. Phys. Acta **64**, 633.
[1394] Randall C.H., 1966, *A mathematical foundation for empirical science – with special reference to quantum theory. Part 1, a calculus of experimental propositions*, Tech. Rep. KAPL-**3147**, Knolls Atomic Power Lab., Schenectady.
[1395] Randall C.H., Foulis D.J., 1983, *Properties and operational propositions in quantum mechanics*, Found. Phys. **13**, 843.
[1396] Randrianantonina N., 2008, *Embeddings of non-commutative L^p-spaces into preduals of finite von Neumann algebras*, Israel J. Math. **163**, 1.
[1397] Rao C.R., 1945, *Information and accuracy attainable in the estimation of statistical parameters*, Bull. Calcutta Math. Soc. **37**, 81.
[1398] Rao C.R., 1947, *The problem of classification and distance between two populations*, Nature **159**, 30.
[1399] Rao C.R., 1982, *Diversity and dissimilarity coefficients: a unified approach*, J. Theor. Popul. Biol. **21**, 24.
[1400] Rao M.M., Ren Z.D., 1991, *Theory of Orlicz spaces*, Dekker, New York.
[1401] Rao M.M., Ren Z.D., 2002, *Applications of Orlicz spaces*, Dekker, New York.
[1402] Ratiu T.S., 2011, *Coadjoint orbits and the beginnings of a geometric representation theory*, in: Neeb K.-H., Pianzolla A. (eds.), *Development and trends in infinite-dimensional Lie theory*, Springer, Berlin, p.417.
[1403] Raĭkov D.A., 1946, *K teorii normirovannykh kolec s involyucieĭ*, Dokl. Akad. Nauk SSSR **54**, 391.
[1404] Raynaud Y., Xu Q., 2001, *On the structure of subspaces of non-commutative L_p-spaces*, Compt. Rend. Acad. Sci. Paris **333**, 213.
[1405] Raynaud Y., Xu Q., 2003, *On subspaces of noncommutative L_p-spaces*, J. Funct. Anal. **203**, 149. arXiv:math/0307169.
[1406] Read T.R.C., Cressie N., 1988, *Goodness-of-fit statistics for discrete multivariate data*, Springer, Berlin.
[1407] Rédei M., 1992, *When can non-commutative statistical inference be bayesian?*, Int. Stud. Phil. Sci. **6**, 129.
[1408] Rédei M., 1998, *Quantum logic in algebraic approach*, Kluwer, Dordrecht.

[1409] Rédei M., 1999, *"Unsolved problems in mathematics": J. von Neumann's address to the International Congress of Mathematicians, Amsterdam, September 2–9, 1954*, Math. Int. **21**, 7. Available at: phil.elte.hu/∼redei/cikkek/intel.pdf.
[1410] Rédei M., 2001, *Facets of quantum logic*, Stud. Hist. Phil. Mod. Phys. **32**, 101.
[1411] Rédei M., 2001, *John von Neumann's concept of quantum logic and quantum probability*, Rédei M., Stoeltzner M. (eds.), *John von Neumann and the foundations of quantum physics*, Kluwer, Dordrecht, p.153.
[1412] Rédei M., 2004, *Operator algebras and quantum logic*, in: Weingartner P. (ed.), *Alternative logics: do sciences need them?*, Springer, Berlin, p.349.
[1413] Rédei M., 2009, *The Birkhoff–von Neumann concept of quantum logic*, in: Engesser K., Gabbay D.M., Lehmann D. (eds.), *Handbook of quantum logic and quantum structures: quantum logic*, Elsevier, Amsterdam, p.1.
[1414] Rédei M., Summers S.J., 2007, *Quantum probability theory*, Stud. Hist. Phil. Mod. Phys. **38**, 390. arXiv:quant-ph/0601158.
[1415] Reeh H., 1988, *A remark concerning canonical commutation relations*, J. Math. Phys. **29**, 1535.
[1416] Reichenbach H., 1944, *Philosophic foundations of quantum mechanics*, University of California Press, Berkeley.
[1417] Reichenbach H., 1946, *Reply to Ernest Nagel's criticism of my views on quantum mechanics*, J. Phil. **43**, 239.
[1418] Reichenbach H., 1948, *The principle of anomaly in quantum mechanics*, Dialectica **7-8**, 337.
[1419] Reichenbach H., 1951, *Über die erkenntnistheoretische Problemlage und den Gebrauch einer dreiwertigen Logik in der Quantenmechanik*, Z. Naturf. **6a**, 569.
[1420] Reichenbach H., 1952, *Les fondements logiques de la méchanique des quanta*, Ann. Inst. Henri Poincaré **13**, 109.
[1421] Reichenbach H., 1954, *Les fondements logiques de la théorie des quanta*, in: ? (ed.), *Applications scientifiques de la logique mathématique*, ?, Paris, p.103.
[1422] Reid M., Williamson R., 2009, *Generalised Pinsker inequalities*, in: Dasgupta S., Klivans A. (eds.), *Annual conference on computational learning theory (COLT 2009*, Asociation for Computing Machinery, Montreal, p.125. arXiv:0906.1244.
[1423] Rellich F., 1937, *Störungstheorie der Spektralzerlegung I, II*, Math. Ann. **113**, 600, 677.
[1424] Rellich F., 1939, *Störungstheorie der Spektralzerlegung III*, Math. Ann. **116**, 555.
[1425] Rellich F., 1940, *Störungstheorie der Spektralzerlegung IV*, Math. Ann. **117**, 356.
[1426] Rellich F., 1942, *Störungstheorie der Spektralzerlegung V*, Math. Ann. **118**, 462.
[1427] Rellich F., 1946, *Der Eindeutigkeitssatz für die Lösungen der quantenmechanischen Vertauschungsrelationen*, Nachr. Akad. Wiss. Göttingen, Math. Phys., Kl. II A **11**, 107.
[1428] Rényi A., 1960, *Az információelmélet néhány alapvető kérdése*, MTA III. Oszt. Közl. **10**, 251 (engl. transl. 1976, *Some fundamental questions of information theory*, in: Rényi A., *Selected papers of Alfred Rényi*, Vol.2, Akadémiai Kiadó, Budapest, p.526).
[1429] Rényi A., 1961, *On measures of entropy and information*, in: Neyman J. (ed.), *Proceedings of the fourth Berkeley symposium on mathematical statistics and probability*, Vol.1, University of California Press, Berkeley, p.547. euclid:bsmsp/1200512181.
[1430] Rényi A., 1962, *Wahrscheinlichkeitsrechnung: mit einem Anhang über Informationstheorie*, Deutscher Verlag der Wissenschaften, Berlin (engl. transl. 1970, *Probability theory*, North-Holland, Amsterdam).
[1431] Resmerita E., 2004, *On total convexity, Bregman projections and stability in Banach spaces*, J. Conv. Anal. **11**, 1.
[1432] Ricci-Curbastro G., 1887, *Sulla derivazione covariante ad una forma quadratica differenziale*, Rend. Acc. Lincei **3**, 15.
[1433] Ricci-Curbastro G., 1888, *Delle derivazioni covarianti e controvarianti e del loro uso nell'analisi applicata*, Studi Ed. Univ. Padova **3**, 3.
[1434] Ricci-Curbastro G., Levi-Civita T., 1901, *Méthodes de calcul différentiel absolu et leurs applications*, Math. Ann. **54**, 125.
[1435] Rickart C.E., 1946, *Banach algebras with an adjoint operation*, Ann. Math. **47**, 528.
[1436] Rickart C.E., 1960, *Banach algebras*, Van Nostrand, Princeton.
[1437] Ridder J., 1941, *Mass- und Integrationstheorie in Strukturen*, Acta. Math. **73**, 131.
[1438] Ridder J., 1946, *Mass- und Integrationstheorie in Strukturen II*, Neder. Akad. Wetensch. **49**, 167.
[1439] Rieffel M., van Daele A., 1977, *A bounded operator approach to Tomita–Takesaki theory*, Pacific J. Math. **69**, 187.
[1440] Riemann B., 1854, *Über die Hypothesen, welche der Geometrie zu Grunde liegen*, Habilitationsschrift, Göttingen, publ. in: 1876, Abhandl. Königl. Gess. Wiss. Göttingen **13**, 87. Available at: www.emis.de/classics/Riemann/Geom.pdf (engl. transl. 1873, *On the hypotheses which lie at the bases of geometry*, Nature **8**, 183:14, 184:36).
[1441] Riemann B., 1861, *Commentatio mathematica, qua respondere tentatur quaestioni ab Ill^{ma} Academia Parisiensi propositiae:*, in: 1876, *Gesammelte mathematische Werke und wissenschaftlicher Nachlass*, Teubner, Leipzig, p.391. Available at: www.maths.tcd.ie/pub/HistMath/People/Riemann/Paris/Paris.pdf (engl. transl. 1990, *A mathematical treatise in which an attempt is made to answer the question prposed by the most illustrious Academy of Paris:*, Hist. Math. **17**, 240).
[1442] Riesz F., 1907, *Sur les systèmes orthogonaux de fonctions*, Compt. Rend. Acad. Sci. Paris **144**, 615.
[1443] Riesz F., 1909, *Sur les opérations fonctionelles linéaires*, Compt. Rend. Acad. Sci. Paris **149**, 974.
[1444] Riesz F., 1910, *Untesuchungen über Systeme integrierbarer Funktionen*, Math. Ann. **69**, 449.
[1445] Riesz F., 1912, *Sur quelques points de la théorie des fonctions sommables*, Compt. Rend. Acad. Sci. Paris **154**, 641.
[1446] Riesz F., 1917, *Lineáris függvényegyenletekről*, Matematikai és Természettudományi Értesítő **35**, 544 (germ. transl. 1918, *Über lineare Funktionalgleichungen*, Acta Math. **41**, 71; engl. transl. available at: www.math.technion.ac.il/hat/fpapers/rie1ineng.pdf).
[1447] Riesz F., 1920, *Sur intégrale de Lebesgue*, Acta Math. **42**, 191.
[1448] Riesz F., 1930, *Sur la décomposition des opérations fonctionelles linéaires*, in: Zanichelli N. (ed.), *Atti Congr. Intern. Mat. Bologna 3-10 Settembre 1928 (VI)*, Vol.3, Società Tipografica già Compositori, Bologna, p.143. Available at: www.mathunion.org/ICM/ICM1928.3/Main/icm1928.3.0143.0148.ocr.pdf.
[1449] Riesz F., 1930, *Über die linearen Transformationen des komplexen Hilbertschen Raumes*, Acta Litt. Sci. Szeged **5**, 23.
[1450] Riesz F., 1937, *A lineáris operációk általános elméletének néhány alapvető fogalomalkotásáról*, Matematikai és Természettudományi Értesítő **56**, 1.
[1451] Riesz F., 1940, *Sur quelques notions fondamentales dans la théorie générale des opérations linéaires*, Ann. Math. **41**, 174.
[1452] Riesz F., Szőkefalvi-Nagy B., 1952, *Leçons d'analyse functionelle*, Akadémiai Kiadó, Budapest (engl. transl. 1955, *Functional analysis*, Ungar, New York).
[1453] Riečan B., 1964, *Sur une extension continue des certaines functions monotones*, Spisy Přir. Fak. UJEP Brně **459**, 481.
[1454] Riečan B., 1965, *O nepreryvnom prodolzhenii monotonnykh funkcionalov nekatorogo tipa*, Mat.-fyz. čas. **15**, 116.
[1455] Riečan B., Neubrunn T., 1997, *Integral, measures and ordering*, Kluwer, Dordrecht.
[1456] Roberts J.E., 1966, *The Dirac bra and ket formalism*, J. Math. Phys. **7**, 1097.
[1457] Roberts J.E., 1966, *Rigged Hilbert spaces in quantum mechanics*, Commun. Math. Phys. **3**, 98.
[1458] Roberts J.E., 1990, *Lectures on algebraic quantum field theory*, in: Kastler D. (ed.), *The algebraic theory of superselection sectors*, World Scientific, Singapore, p.1.

[1459] Roberts J.E., Roepstorff G., 1969, *Some basic concepts of algebraic quantum theory*, Commun. Math. Phys. **11**, 321.
[1460] Robertson B., 1966, *Equations of motion in nonequilibrium statistical mechanics*, Phys. Rev. **144**, 151.
[1461] Robinson D.W., 1968, *Statistical mechanics of quantum spin systems II*, Commun. Math. Phys. **7**, 337.
[1462] Rockafellar R.T., 1968, *Integrals which are convex functionals*, Pacif. J. Math. **24**, 525.
[1463] Rockafellar R.T., 1970, *Conjugate convex functions in optimal control and the calculus of variations*, J. Math. Anal. Appl. **32**, 174.
[1464] Rockafellar R.T., 1970, *Convex analysis*, Princeton University Press, Princeton. Available at: www.math.washington.edu/∼rtr/papers/rtr-ConvexAnalysis.djvu.
[1465] Rockafellar R.T., 1971, *Convex integral functionals and duality*, in: Zarantonello E. (ed.), *Contribution to nonlinear functional analysis*, Academic Press, New York, p.215.
[1466] Rockafellar R.T., 1971, *Integrals which are convex functionals II*, Pacif. J. Math. **39**, 439.
[1467] Rockafellar R.T., 1974, *Conjugate duality and optimization*, Regional Conferences Series in Applied Mathematics **16**, Society for Industrial and Applied Mathematics, Philadelphia.
[1468] Rockafellar R.T., Wets R.J.-B., 1998, *Variational analysis*, Springer, Berlin (2006, 2nd ed.).
[1469] Rodríguez C.C., 1991, *Entropic priors*, in: Grandy W.T. Jr., Schick L.H. (eds.), *Maximum entropy and bayesian methods*, Kluwer, Dordrecht.
[1470] Rodríguez C.C., 1996, *Bayesian robustness: a new look from geometry*, in: Heidbreder G.R. (ed.), *Maximum entropy and bayesian methods*, Kluwer, Dordrecht.
[1471] Rodríguez C.C., 1999, *Are we cruising a hypothesis space?*, in: von der Linde W. et al. (eds.), *Maximum entropy and bayesian methods*, Kluwer, Dordrecht. arXiv:physics/9808009.
[1472] Rodríguez C.C., 2003, *A geometric theory of ignorance*, preprint, available at: omega.albany.edu:8008/ignorance/ignorance03.pdf.
[1473] Roepstorff G., 1976, *Correlation inequalities in quantum statistical mechanics and their application in the Kondo problem*, Commun. Math. Phys. **46**, 253. euclid:cmp/1103899639.
[1474] Rogers L.J., 1888, *An extension of a certain theorem in inequalities*, Messeng. Math. **17**, 145.
[1475] Rokhlin V.A., 1949, *Ob osnovnykh ponyatiyakh teorii mery*, Matem. Sb. **25**, 107 (engl. tranl. 1952, *On the fundamental ideas of measure theory*, Amer. Math. Soc. Transl. **71**, American Mathematical Society, Providence.
[1476] Romier G., 1969, *Décision statistique*, Ann. Inst. Henri Poincaré B **5**, ?.
[1477] Romier G., 1969, *Modél d'expérimentation statistique*, Ann. Inst. Henri Poincaré B **5**, 275.
[1478] Rota G.-C., 2001, *Twelve problems in probability no one likes to bring up*, Crapo H., Senato D. (eds.), *Algebraic combinatorics and computer science: a tribute to Gian-Carlo Rota*, Springer Italia, Milano, p.57.
[1479] Ruelle D., 1966, *States of physical systems*, Commun. Math. Phys. **3**, 133.
[1480] Ruelle D., 1969, *Statistical mechanics: rigorous results*, Benjamin, New York.
[1481] Ruelle D., 1970, *Integral representation of states on a C^*-algebra*, J. Funct. Anal. **6**, 116.
[1482] Ruelle D., 1970, *Symmetry breakdown in statistical mechanics*, in: Kastler D. (ed.), *Cargèse Lectures in Physics*, Vol.4, Gordon & Breach, New York, p.169.
[1483] Rüschendorf L., 1984, *On the minimum discrimination information theorem*, Statist. Dec. Suppl. **1**, 263.
[1484] Rüschendorf L., 1987, *Projections of probability measures*, Statistics **18**, 123.
[1485] Ruskai M.B., 1973, *A generalisation of entropy using traces on von Neumann algebras*, Ann. Inst. Henri Poincaré A **19**, 357.
[1486] Ruskai M.B., 1988, *Extremal properties of relative entropy in quantum statistical mechanics*, Rep. Math. Phys. **26**, 143.
[1487] Ruskai M.B., 1994, *Beyond strong subadditivity? Improved bounds on the contraction of generalized relative entropy*, Rev. Math. Phys. **6**, 1147.
[1488] Russo B., 1968, *Isometries of L^p-spaces associated with finite von Neumann algebras*, Bull. Amer. Math. Soc. **74**, 228.
[1489] Rüttiman G.T., 1980, *Non-commutative measure theory*, Habilitationsschrift, Universitat Bern.
[1490] Rüttiman G.T., 1989, *The approximate Jordan–Hahn decomposition*, Canad. J. Math. **41**, 1124.
[1491] Sá N.B., 1997, *Geodesics and autoparallels from a variational principle?*, arXiv:gr-qc/9712013.
[1492] Sacksteder R., 1967, *A note on statistical equivalence*, Ann. Math. Statist. **38**, 787.
[1493] Sacksteder R., 1968, *On products of experiments*, Z. Wahrschein. Gebiete **10**, 203.
[1494] Saitō S., 1969, *On the algebra of measurable operators for a general AW^*-algebra*, Tōhoku Math. J. **21**, 249.
[1495] Saitō S., 1970, *A non-commutative integration theory for a semi-finite AW^*-algebra and a problem of Feldman*, Proc. Japan Acad. **46**, 463.
[1496] Sakai S., 1956, *A characterization of W^*-algebras*, Pacific J. Math. **6**, 763.
[1497] Sakai S., 1958, *On linear functionals of W^*-algebras*, Proc. Japan Acad. **34**, 571.
[1498] Sakai S., 1960, *On a conjecture of Kaplansky*, Tōhoku Math. J. **12**, 31.
[1499] Sakai S., 1962, *The theory of W^*-algebras*, Lecture notes, Yale University, New Heaven.
[1500] Sakai S., 1964, *On the reduction theory of von Neumann*, Bull. Amer. Math. Soc. **70**, 393. euclid:bams/1183526019.
[1501] Sakai S., 1965, *On central decomposition for positive functionals on C^*-algebras*, Trans. Amer. Math. Soc. **118**, 406.
[1502] Sakai S., 1965, *A Radon–Nikodym theorem in W^*-algebras*, Bull. Amer. Math. Soc. **71**, 149.
[1503] Sakai S., 1966, *Derivations of W^*-algebras*, Ann. Math. **83**, 273.
[1504] Sakai S., 1968, *Asymptotically abelian II_1 factor*, Publ. Res. Inst. Math. Sci., Kyōto Univ. **4**, 299.
[1505] Sakai S., 1970, *An uncountable family of non-hyperfinite type III factors*, in: Wilde C.O. (ed.), *Functional analysis*, Academic Press, New York, p.65.
[1506] Sakai S., 1970, *An uncountable number of II_1 and II_∞ factors*, J. Funct. Anal. **5**, 236.
[1507] Sakai S., 1971, *C^*-algebras and W^*-algebras*, Springer, Berlin.
[1508] Sakai S., 1991, *Operator algebras in dynamical system: the theory of unbounded derivations in C^*-algebras*, Cambridge University Press, Cambridge.
[1509] Saks S., 1933, *Théorie de l'intégrale*, Monografje Matematyczne **2**, Garasiński–Fundusz Kultury Narodowej, Warszawa. (2nd rev. ed., 1937, *Theory of the integral*, Stechert, New York).
[1510] Sanov I.N., 1957, *O veroyatnosti bol'shikh otkloneniĭ sluchaĭnykh velichin*, Matem. Sbornik **42**, 11.
[1511] Sato Y., Sugawa K., Kawaguchi M., 1979, *The geometrical structure of the parametric space of two-dimensional normal distribution*, Rep. Math. Phys. **16**, 111.
[1512] Sauvageot J., 1974, *Sur le type de produit croisé d'une algèbre de von Neumann par un groupe localement compact d'automorphismes*, Compt. Rend. Acad. Sci. Paris **278**, 941.
[1513] Sauvageot J., 1977, *Sur le type de produit croisé d'une algèbre de von Neumann par un groupe localement compact*

d'automorphismes, Bull. Soc. Math. France **105**, 349.
[1514] Savage L.J., 1954, *The foundations of statistics*, Dover, New York. (1972, 2nd rev. ed.).
[1515] Savvidou K., 1999, *The action operator in continuous time histories*, J. Math. Phys. **40**, 5657. arXiv:gr-qc/9811078.
[1516] Savvidou K., 1999, *Continuous time in consistent histories*, Ph.D. thesis, Imperial College, London. arXiv:gr-qc/9912076.
[1517] Scalapino D.J., 1961, *Irreversible statistical mechanics and the principle of maximum entropy*, Ph.D. thesis, Stanford University, Stanford.
[1518] Schack R., Brun T.A., Caves C.M., 2001, *Quantum Bayes rule*, Phys. Rev. A **64**, 014305. arXiv:quant-ph/0008113.
[1519] Schaefer H.H., 1974, *Banach lattices and positive operators*, Springer, Berlin.
[1520] Schatten R., 1946, *The cross-space of linear transformations*, Ann. of Math. **47**, 73.
[1521] Schatten R., 1950, *A theory of cross spaces*, Princeton University Press, Princeton.
[1522] Schatten R., 1960, *Norm ideals of completely continuous operators*, Springer, Berlin.
[1523] Schatten R., von Neumann J., 1946, *The cross-space of linear transformations II*, Ann. of Math. **47**, 608.
[1524] Schatten R., von Neumann J., 1947, *The cross-space of linear transformations III*, Ann. of Math. **49**, 557.
[1525] Schauder J., 1930, *Über lineare, vollstetige Funktionaloperationen*, Studia Math. **2**, 183.
[1526] Schmidt E., 1907, *Zur Theorie der linearen und nicht linearen Integralgleichungen Zweite Abhandlung: Auflösung der allgemeinen linearen Integralgleichung*, Math. Ann. **64**, 161.
[1527] Schmidt E., 1907, *Zur Theorie der linearen und nichtlinearen Integralgleichungen I. Teil: Entwicklung willkürlicher Funktionen nach Systemen vorgeschriebener*, Math. Ann. **63**, 433.
[1528] Schmidt E., 1907, *Zur Theorie der linearen und nichtlinearen Integralgleichungen. III. Teil. Über die Auflösung der nichtlinearen Integralgleischung und die Verzweigung ihrer Lösungen*, Math. Ann. **64**, 370.
[1529] Schmidt E., 1908, *Über die Anflösung linearer Gleichungen mit endlich vielen Unbekannten*, Rend. Circ. Mat. Palermo **25**, 53.
[1530] Schmidt W., Baumann K., 1956, *Quantentheorie der Felder als Distributionstheorie*, Nuovo Cim. **4**, 860.
[1531] Schmüdgen K., 1983, *On the Heisenberg commutation relations, II*, Publ. Res. Inst. Math. Sci., Kyōto Univ. **19**, 601.
[1532] Schouten J.A., Haantjes J., 1934, *Generelle Feldtheorie VIII. Autogeodätische Linien und Weltlinien*, Z. Phys. **89**, 357.
[1533] Schröder E., 1890, 1891, 1895, *Vorlesungen über die Algebra der Logik*, Vol.1-3, Teubner, Leipzig.
[1534] Schrödinger E., 1926, *Quantisierung als Eigenwertproblem I, II, II, IV*, Ann. Phys, **384**, 361, 489; **385**, 437; **386**, 109.
[1535] Schrödinger E., 1926, *Über das Verhältnis der Heisenberg-Born-Jordanchen Quantenmechanik zu der meinem*, Ann. Phys. **384**, 734.
[1536] Schrödinger E., 1935, *Die gegenwärtige Situation in der Quantenmechanik*, Naturwiss. **23**, 807, 824, 844. (engl. transl.: 1983, *The present situation in quantum mechanics*, in: Wheeler J.A., Żurek W.H. (eds.), *Quantum theory and measurement*, Princeton University Press, Princeton, p.152).
[1537] Schrödinger E., 1935, *Discussion of probability relations between separated systems*, Proc. Cambridge Phil. Soc. **31**, 555.
[1538] Schrödinger E., 1936, *Probability relations between separated systems*, Proc. Cambridge Phil. Soc. **32**, 446.
[1539] Schroer B., Wiesbrock H.-W., 2000, *Modular constructions of quantum field theories with interactions*, Rev. Math. Phys. **12**, 301. arXiv:hep-th/981225.
[1540] Schur I., 1909, *Über die charakterischen Wurzein einer linearen Subsitution mit einer Anwendung auf die Theorie der Integralgleichungen*, Math. Ann. **66**, 488.
[1541] Schwartz J., 1963, *Non-isomorphism of a pair of factors of type III*, Comm. Pure Appl. Math. **16**, 111.
[1542] Schwartz J., 1963, *Two finite, non-hyperfinite, non-isomorphic factors*, Comm. Pure Appl. Math. **16**, 19.
[1543] Schwartz J., 1967, *W^*-algebras*, Gordon & Breach, New York.
[1544] Schwartz L.-M., 1945, *Généralisation de la notion de fonction, de dérivation, de transforation de Fourier et applications mathématiques et physiques*, Ann. Univ. Grenoble **21**, 57.
[1545] Schwartz L.-M., 1950, 1951, *Théorie des distributions*, Vol.1-2, Actualités Scientifiques et Industrielles **1091**, **1122**, Hermann, Paris.
[1546] Schwartz L.-M., 1973, *Radon measures on arbitrary spaces and cylindrical measures*, ?, Oxford.
[1547] Schwarzschild K., 1916, *Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie*, Sitzungsber. Königl. Preuss. Akad. Wiss. **1**, 189.
[1548] Schwinger J., 1960, *Unitary operator bases*, Proc. Nat. Acad. Sci. U.S.A. **46**, 570.
[1549] Schwinger J., 1969, *Particles and sources*, Gordon & Breach, New York.
[1550] Schwinger J., 1970, *Particles, souces, and fields*, Vol.1-2, Addison-Wesley, Reading.
[1551] Segal I.E., 1947, *Irreducible representations of operator algebras*, Bull. Amer. Math. Soc. **61**, 69.
[1552] Segal I.E., 1947, *Postulates for general quantum mechanics*, Ann. Math. **48**, 930.
[1553] Segal I.E., 1949, *A kind of abstract integration pertinent to locally compact groups*, Bull. Amer. Math. Soc. **55**, 46.
[1554] Segal I.E., 1949, *Two-sided ideals in operator algebras*, Ann. Math. **50**, 856.
[1555] Segal I.E., 1951, *Decompositions of operator algebras I and II*, Memoirs of American Mathematical Society **9**, American Mathematical Society, New York.
[1556] Segal I.E., 1951, *Equivalences of measure spaces*, Amer. J. Math. **73**, 275.
[1557] Segal I.E., 1953, *A non-commutative extension of abstract integration*, Ann. Math. **57**, 401, 595.
[1558] Segal I.E., 1954, *Abstract probability spaces and a theorem of Kolmogoroff*, Am. J. Math. **76**, 721.
[1559] Segal I.E., 1959, *Caractérisation mathématique des observables en théorie quantique des champs et ses conséquences pour la structure des particules libres*, in: Deheuvels R., Michel L. (eds.), *Les problèmes mathématiques de la théorie quantique des champs. Lille 3-8 Juin, 1957*, Centr. Natl. Rech. Sci., Paris, p.57.
[1560] Segal I.E., 1960, *A note on the concept of entropy*, J. Math. Mech. **9**, 623. Available at: www.iumj.indiana.edu/IUMJ/FTDLOAD/1960/9/59036/pdf.
[1561] Segal I.E., 1963, *Mathematical problems in relativistic physics, with an appendix on group representations in Hilbert space by George W. Mackey*, American Mathematical Society, Providence.
[1562] Segal I.E., 1965, *Algebraic integration theory*, Bull. Amer. Math. Soc. **71**, 571.
[1563] Segal I.E., 1994, *C^*-algebras and quantization*, in: Doran R.S. (ed.), *C^*-algebras: 1945–1993: a fifty year celebration: AMS special session commemorating the first fifty years of C^*-algebra theory, January 13-14, 1993, San Antonio, Texas*, American Mathematical Society, Providence.
[1564] Segal I.E., 1996, *Noncommutative geometry, by Alain Connes*, Bull. Amer. Math. Soc. **33**, 459.
[1565] Seilinger P., 2007, *Dagger compact closed categories and completely positive maps*, in: Seilinger P. (ed.), *Proceedings of the 3rd international workshop on quantum programming languages (QPL 2005)*, Electr. Notes Theor. Comp. Sci. **170**, 139.
[1566] Semadeni Z., 1971, *Banach spaces of continuous functions*, Monografie Matematyczne **55**, Państwowe Wydawnictwo

Naukowe, Warszawa.
[1567] Sen R.N., 1944, *On parallelism in riemannian space I*, Bull. Calcutta Math. Soc. **36**, 102.
[1568] Sen R.N., 1945, *On parallelism in riemannian space II*, Bull. Calcutta Math. Soc. **37**, 153.
[1569] Sen R.N., 1946, *On parallelism in riemannian space III*, Bull. Calcutta Math. Soc. **38**, 161.
[1570] Sewell G.L., 1977, *KMS conditions and local thermodynamical stability of quantum lattice systems. II*, Commun. Math. Phys. **55**, 53.
[1571] Sewell G.L., 1986, *Quantum theory of collective phenomena*, Oxford University Press, Oxford.
[1572] Sewell G.L., 2002, *Quantum mechanics and its emergent macrophysics*, Princeton University Press, Princeton.
[1573] Shafer G., Vovk V., 2005, *The origins and legacy of Kolmogorov's Grundbegriffe*, preprint. Available at: www.probabilityandfinance.com/articles/04.pdf.
[1574] Shannon C., 1948, *A mathematical theory of communication*, Bell Syst. Tech. J. **27**, 379, 623.
[1575] Shannon C., Weaver W., 1949, *The mathematical theory of communication*, University of Illinois Press, Urbana.
[1576] She C.Y., Heffner H., 1966, *Simultaneous measurements of noncommuting observables*, Phys. Rev. **152**, 1103.
[1577] Sherman D.E., 2001, *The application of modular algebras to relative tensor products and noncommutative L^p modules*, Ph.D. thesis, University of California, Los Angeles.
[1578] Sherman D.E., 2003, *Relative tensor products for modules over von Neumann algebras*, in: Jarosz K. (ed.), *Function spaces: fourth conference on function spaces, May 14-19, 2002, Southern Illinois University at Edwardsville*, Contemp. Math. **328**, American Mathematical Society, Providence, p.275. arXiv:math/0301061.
[1579] Sherman D.E., 2005, *Noncommutative L^p structure encodes exactly Jordan structure*, J. Funct. Anal. **221**, 150. arXiv:math/0309365.
[1580] Sherman D.E., 2006, *A new proof of the noncommutative Banach-Stone theorem*, in: Bożejko M., Młotkowski W., Wysoczański J. (eds.), *Quantum probability*, Banach Center Publications **73**, Instytut Matematyki Polskiej Akademii Nauk, Warszawa, p.363. arXiv:math/0409488.
[1581] Sherman D.E., 2006, *On the structure of isometries between noncommutative L_p spaces*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **42**, 45. arXiv:math/0311336.
[1582] Sherman S., 1951, *Non-negative observables are squares*, Proc. Nat. Acad. Sci. U.S.A. **2**, 31.
[1583] Sherman S., 1951, *Order in operator algebras*, Amer. J. Math. **73**, 227.
[1584] Sherman S., 1956, *On Segal's postulates for general quantum mechanics*, Ann. Math. **64**, 593.
[1585] Sherstnëv A.N., 1974, *K obshheĭ teorii sostoyaniĭ na algebrakh fon Neĭmana*, Funkts. Anal. Prilozh. **8**, 89 (engl. transl. 1974, *States on von Neumann algebras*, Funct. Anal. Appl. **8**, 272).
[1586] Sherstnëv A.N., 1977, *Kazhdyĭ gladkiĭ ves yavlyaetsya ℓ-vesom*, Izv. Vyssh. Uchebn. Zaved. Matem. **183**, 88 (engl. transl. 1977, *Every smooth weight is an ℓ-weight*, Sov. Math. **21**:8, 66).
[1587] Sherstnëv A.N., 1982, *K obshheĭ teorii mery i integrala v algebrakh Neĭmana*, Izv. Vyssh. Uchebn. Zaved. Matem. **243**, 20 (engl. transl. 1982, *On the general theory of the measure and integral in von Neumann algebras*, Sov. Math. **26**:8, 21).
[1588] Sherstnëv A.N., 2008, *Metody bilineĭnykh form v nekommutativnoĭ teorii mery i integrala*, Fizmatlit, Moskva.
[1589] Shewell J.R., 1959, *On the formation of quantum-mechanical operators*, Amer. J. Phys. **27**, 16.
[1590] Shilov G.E., Gurevich B.L., 1964, *Integral, mera i proizvodnaya*, Nauka, Moskva (2nd rev. ed. 1967; engl. transl. 1966, *Integral, measure and derivative: a unified approach*, Prentice–Hall, Englewood Cliffs; 3rd ed. 2012, Dover, New York).
[1591] Shima H., 1976, *On certain locally flat homogeneus manifolds of solvable Lie groups*, Osaka J. Math. **13**, 213.
[1592] Shima H., 1977, *Symmetric spaces with invariant locally hessian structures*, J. Math. Soc. Japan **29**, 581.
[1593] Shima H., 1986, *Vanishing theorems for compact hessian manifolds*, Ann. Inst. Fourier Grenoble **36**, 183.
[1594] Shima H., 2007, *The geometry of hessian structures*, World Scientific, Singapore.
[1595] Shima H., Yagi K., 1997, *Geometry of hessian manifolds*, Diff. Geom. Appl. **7**, 277.
[1596] Shiryayev A.N., Koshevnik Yu.A., 1989, *Review of: Asymptotic methods in statistical decision theory by Lucien Le Cam*, Bull. Amer. Math. Soc. **20**, 280.
[1597] Shiryayev A.N., Spokoiny V.G., 2000, *Statistical experiments and decisions: asymptotic theory*, World Scientific, Singapore.
[1598] Shmul'yan V.L., 1940, *Über lineare topologische Räume*, Mat. Sbornik **7**, 425.
[1599] Shore J.E., Johnson R.W., 1980, *Axiomatic derivation of the principle of maximum entropy and the principle of minimum cross-entropy*, IEEE Trans. Inf. Theory **26**, 26.
[1600] Shore J.E., Johnson R.W., 1981, *Properties of cross-entropy minimization*, IEEE Trans. Inf. Theory **27**, 472.
[1601] Siebert E., 1979, *Pairwise sufficiency*, Z. Warschein. Geb. **46**, 237.
[1602] Sierpiński W., 1927, *Sur les ensembles boreliens abstraits*, Ann. Soc. Polon. Math. **6**, 50.
[1603] Sierpiński W., 1928, *Un thèoréme général sur les familles des ensembles*, Fund. Math. **12**, 206.
[1604] Sikorski R., 1948, *On the representation of boolean algebras as fields of sets*, Fund. Math. **35**, 247.
[1605] Sikorski R., 1960, *Boolean algebras*, Springer, Berlin (3rd corr. ed. 1969).
[1606] Simon B., 1976, *Quantum dynamics: from automorphism to hamiltonian*, in: Lieb E.H., Simon B., Wightman A.S. (eds.), *Studies in mathematical physics. Essays in honor of Valentine Barmann*, Princeton University Press, Princeton, p.327.
[1607] Simon B., 1979, *Trace ideals and their applications*, Cambridge University Press, Cambridge (2nd.ed. 2005, American Mathematical Society, Providence).
[1608] Simons S., 1998, *Minimax and monotonicity*, LNM **1693**, Springer, Berlin.
[1609] Singh R., Warmuth M.K., Raj B., Lamere P., 2003, *Classification with free energy at raised temperatures*, in: ? (ed.), *Proceedings of Eurospeech 2003*, ?, ?, p.1773. Available at: users.soe.ucsc.edu/∼manfred/pubs/C66.pdf.
[1610] Skau C.F., 1975, *Orthogonal measures on the state space of a C^*-algebra*, in: Williamson J.H. (ed.), *Algebras in analysis*, Academic Press, New York, p.272.
[1611] Skau C.F., 1980, *Geometric aspects of the Tomita–Takesaki theory I*, Math. Scan. **47**, 311.
[1612] Skovgaard L.T., 1984, *A riemannian geometry of the multivariate normal model*, Scand. J. Stat. **11**, 211.
[1613] Slater J., 1927, *Radiation and absorption in Schrödinger's theory*, Proc. Nat. Acad. Sci. U.S.A. **13**, 7.
[1614] Solèr M.P., 1995, *Characterization of Hilbert spaces by orthomodular spaces*, Commun. Alg. **23**, 219.
[1615] Sołtan P.M., 2007, *C^*-algebras, group actions and crossed products*, Lecture notes, University of Warsaw, Warsaw. Available at: www.fuw.edu.pl/∼psoltan/prace/C*-algebras,%20group%20actions%20and%20crossed%20products%20(lecture%20notes).pdf.
[1616] Specker E.P., 1960, *Die Logik nicht gleichzeitig entscheidbarer Aussagen*, Dialectica **14**, 239 (engl. transl. 1975, *The logic of propositions which are not simultaneously decidable*, in: Hooker C.A. (ed.), *The logico-algebraic approach to quantum mechanics*, Vol.1, Reidel, Dordrecht, p.135).
[1617] Spengler O., 1918, 1923, *Der Untergang des Abendlandes. Umriße einer Morphologie der Weltgeschichte*, Vol.1-2,

Braumüller, Wien. (2nd rev. ed. of Vol.1: 1922, Beck, München; engl. transl.: 1926, *The decline of the west*, Knopf, New York).
[1618] Spirtes P., Glymour C., Scheines R., 1993, *Causation, prediction and search*, Springer, Berlin (2nd ed. 2000, MIT Press, Cambridge).
[1619] Srinivas M.D., 1975, *Foundations of a quantum probability theory*, J. Math. Phys. **16**, 1672.
[1620] Srinivas M.D., 1979, *Successive observations in relativistic quantum theory*, J. Math. Phys. **20**, 1593.
[1621] Srinivas M.D., Wolf E., 1975, *Some non-classical features of phase space representations of quantum mechanics*, Phys. Rev. D **11**, 1477.
[1622] Steinhaus H., 1919, *Additive und stetige Funktionaloperationen*, Math. Z. **5**, 186.
[1623] Steinhaus H., 1923, *Les probabilités denomerables et leur rapport à théorie de la mesure*, Fund. Math. **4**, 286.
[1624] Steinmann O., 1968, *Particle localization in field theory*, Commun. Math. Phys. **7**, 112.
[1625] Stieltjes T.-J., 1894, *Recherches sur les fractions continues*, Ann. Fac. Sci. Tolouse **8**, 1. numdam:AFST_1894_1_8_4_J1_0.
[1626] Stieltjes T.-J., 1895, *Recherches sur les fractions continues [suite et fin]*, Ann. Fac. Sci. Tolouse **9**, 5. numdam:AFST_1895_1_9_1_A5_0.
[1627] Stinespring W.F., 1955, *Positive functions on C^*-algebras*, Proc. Amer. Math. Soc. **6**, 211.
[1628] Stinespring W.F., 1959, *Integration theorems for gages and duality for unimodular groups*, Trans. Amer. Math. Soc. **90**, 15.
[1629] Stirzaker D., 1994, *Probability vicit expectation*, in: Kelly F.P. (ed.), *Probability, statistics and optimization. A tribute to Peter Whittle*, Wiley, Chisterter.
[1630] Stolz P., 1969, *Attempt of an axiomatic foundation of quantum mechanics and more general theories. V*, Commun. Math. Phys. **11**, 303.
[1631] Stolz P., 1971, *Attempt of an axiomatic foundation of quantum mechanics and more general theories. VI*, Commun. Math. Phys. **23**, 117.
[1632] Stone M.H., 1930, *Linear transformations in Hilbert space, III: operational methods and group theory*, Proc. Natl. Acad. Sci. U.S.A. **16**, 172.
[1633] Stone M.H., 1932, *Linear transformations in Hilbert space and their applications to analysis*, Amer. Math. Soc. Colloq. Publ. **15**, American Mathematical Society, Providence.
[1634] Stone M.H., 1932, *On one-parameter unitary groups in Hilbert space*, Ann. Math. **33**, 643.
[1635] Stone M.H., 1936, *The theory of representations of boolean algebras*, Trans. Amer. Math. Soc. **40**, 37.
[1636] Stone M.H., 1937, *Algebraic characterization of special boolean rings*, Fund. Math. **29**, 223. Available at: matwbn.icm.edu.pl/ksiazki/fm/fm29/fm29127.pdf.
[1637] Stone M.H., 1937, *Applications of the theory of boolean rings in topology*, Trans. Amer. Math. Soc. **41**, 375.
[1638] Stone M.H., 1940, *A general theory of spectra I*, Proc. Nat. Acad. Sci. U.S.A. **26**, 280.
[1639] Stone M.H., 1948, *Notes on integration I, II, III*, Proc. Natl. Acad. Sci. U.S.A. **34**, 336, 447, 483.
[1640] Stone M.H., 1949, *Boundedness properties in function lattices*, Canad. J. Math. **1**, 176.
[1641] Stone M.H., 1949, *Notes on integration IV*, Proc. Natl. Acad. Sci. U.S.A. **35**, 50.
[1642] Strasser H., 1985, *Mathematical theory of statistics: statistical experiments and asymptotic decision theory*, de Gruyter, Berlin.
[1643] Ştrătilă S., 1981, *Modular theory in operator algebras*, Edituria Academiei/Abacus Press, Bucureşti/Tunbridge Wells.
[1644] Ştrătilă S., Voiculescu D., Zsidó L., 1976, *On crossed products I*, Revue. Roum. Math. Pures Appl. **21**, 1411.
[1645] Ştrătilă S., Voiculescu D., Zsidó L., 1977, *On crossed products II*, Revue. Roum. Math. Pures Appl. **22**, 83.
[1646] Ştrătilă S., Zsidó L., 1975, *Lecţii de algebre von Neumann*, Edituria Academiei, Bucureşti (rev. upd. engl. transl. 1979, *Lectures on von Neumann algebras* Edituria Academiei/Abacus Press, Bucureşti/Tunbridge Wells).
[1647] Ştrătilă S., Zsidó L., 1977, 1979, *Operator algebras*, INCREST, Bucureşti (2nd ed. 1995, Tipografia Univestităţii din Timişoara, Timişoara; 3rd ed. 2009, Theta Foundation, Bucureşti).
[1648] Strauss M., 1936, *Zur Begründung der statistischen Transformationstheorie der Quantenphysik*, Sitzungber. Preuss. Akad. Wiss., Phys.-math. Kl. **1936**, 382 (engl. transl. 1972, *The logic of complementarity and the foundation of quantum theory*, in: Strauss M., *Modern physics and its philosophy: selected papers in the logic, history, and philosophy of science*, Reidel, Dordrecht, p.186).
[1649] Strauss M., 1937, *Mathematics as logical syntax: a method to formalize the language of a physical theory*, Erkenntnis **7**, 147.
[1650] Strauss M., 1938, *Mathematische und logische Beiträge zur quantenmechanischen Komplementaritätstheorie*, Ph.D. thesis, Praha.
[1651] Strauss M., 1967, *Grundlagen der modernen Physik – Teil III: h-Theorie (Quantenmechanik)*, in: Ley H., Löther R. (eds.), *Mikrokosmos, Makrokosmos: philosophisch-teoretische Probleme der Naturwissenschaft, Technik und Medizin*, Vol.2, Akademie, Berlin, p.55 (engl. transl. 1975, *Foundations of quantum mechanics*, in: Hooker C.A. (ed.), *The logico-algebraic approach to quantum mechanics*, Vol.1, Reidel, Dordrecht, p.351).
[1652] Strauss M., 1973, *Logics for quantum mechanics*, Found. Phys. **3**, 265.
[1653] Strauss M., 1973, *Two concepts of probability in physics*, in: Suppes P. et al (eds.), *Proceedings of the Fourth International Congress for Logic, Methodology and Philosophy of Science, Bucharest, 1971*, Vol.4, North-Holland, Amsterdam, p.603.
[1654] Streater R.F., 1996, *Information geometry and reduced quantum description*, Rep. Math. Phys. **38**, 419.
[1655] Streater R.F., 1997, *Statistical dynamics and information geometry*, in: Borglignon J.-P., Nencka H. (eds.), *Geometry and nature (Madeira, 1995)*, Contemporary Mathematics **203**, American Mathematical Society, Providence, p.117.
[1656] Streater R.F., 2000, *The analytic quantum information manifold*, in: Gesztesy F., Paycha S., Holden H. (eds.), *Stochastic processes, physics and geometry: new interplays. II: A volume in honor of Sergio Albeverio (Leipzig, 1999)*, Canadian Math. Soc. Conf. Proc. **29**, American Mathematical Society, Providence, p.603. arXiv:math-ph/9910036.
[1657] Streater R.F., 2000, *Classical and quantum info-manifolds*, Sūrikaisekikenkyūsho Kōkyūroku **1196**, 32. arXiv:math-ph/0002050.
[1658] Streater R.F., 2000, *Classical and quantum probability*, J. Math. Phys. **41**, 3556.
[1659] Streater R.F., 2000, *The information manifold for relatively bounded potentials*, in: Vladimirov V.S., Mishhenko E.F. (eds.), *Problems of the modern mathematical physics. Collection of papers dedicated to the 90th anniversary of academician Nikolai Nikolaevich Bogolyubov*, Tr. Mat. Inst. Steklova **228**, Nauka, Moskva, p.217. arXiv:math-ph/9910035.
[1660] Streater R.F., 2004, *Duality in quantum information geometry*, Open Sys. Inf. Dyn. **11**, 71. arXiv:math-ph/0308037.
[1661] Streater R.F., 2004, *Quantum Orlicz spaces in information geometry*, Open Sys. Inf. Dyn. **11**, 350. arXiv:math-ph/0407046.

[1662] Streater R.F., 2007, *Lost causes in and beyond physics*, Springer, Berlin.
[1663] Streater R.F., 2008, *The set of density operators modelled on a quantum Orlicz space*, in: Belavkin V.P., Guţă M. (eds.), *Quantum stochastics and information: statistics, filtering and control*, World Scientific, Singapore, p.99.
[1664] Streater R.F., 2009, *Statistical dynamics. A stochastic approach to nonequilibrium thermodynamics*, 2nd rev. ed., Imperial College Press, London.
[1665] Streater R.F., 2010, *The Banach manifold of quantum states*, in: Gibilisco P. et al (eds.), *Algebraic and geometric methods in statistics*, Cambridge University Press, Cambridge, p.257.
[1666] Streater R.F., 2010, *Introduction to non-parametric estimation*, in: Gibilisco P. et al (eds.), *Algebraic and geometric methods in statistics*, Cambridge University Press, Cambridge, p.241.
[1667] Streit L., 1969, *A generalisation of Haag's theorem*, Nuovo Cimento A **62**, 673.
[1668] Strocchi F., 1985, *Elements of quantum mechanics of infinite systems*, World Scientific, Singapore.
[1669] Stroock D.W., 1994, *A concise introduction to the theory of integration*, Birkhäuser, Basel (3rd ed. 1999).
[1670] Study E., 1905, *Kürzeste Wege in komplexen Gebiet*, Math. Ann. **60**, 321.
[1671] Stummer W., Vajda I., 2010, *On divergences of finite measures and their applicability in statistics and information theory*, Statistics **44**, 169.
[1672] Sudár C., 1996, *Radial extension of monotone riemannian metrics on density matrices*, Publ. Math. Debrecen **49**, 243.
[1673] Sukochev F.A., 1996, *Non-isomorphism of L_p-spaces associated with finite and infinite von Neumann algebras*, Proc. Amer. Math. Soc. **124**, 1517.
[1674] Sunder V.S., 1987, *An invitation to von Neumann algebras*, Springer, Berlin.
[1675] Suppes P., 1966, *L'argument probabiliste pur une logique nonclassique de la méchanique quantique*, Synthese **16**, 74.
[1676] Suppes P., 1966, *The probabilistic argument for non-classical logic of quantum mechanics*, Phil. Sci. **33**, 14.
[1677] Sýkora S., 1974, *Quantum theory and the bayesian inference problems*, J. Stat. Phys. **11**, 17.
[1678] Szabó L.E., 2001, *Critical reflections on quantum probability theory*, in: Rédei M., Stölzner M. (eds.), *John von Neumann and the foundations of quantum physics*, Kluwer, Dordrecht, p.201.
[1679] Szekeres G., 1960, *On the singularities of a Riemannian manifold*, Publ. Math. Debrecen **7**, 285.
[1680] Szőkefalvi-Nagy B., 1942, *Spektraldarstellung linearer Transformationen des Hilbertschen Raumes*, Springer, Berlin.
[1681] Szpilrajn E., 1934, *Remarques sur les fonctions complètment additives d'ensemble et sur les ensembles jouissant de la proprieté de Baire*, Fund. Math. **22**, 303. Available at: matwbn.icm.edu.pl/ksiazki/fm/fm22/fm22129.pdf.
[1682] Takai H., 1974, *Dualité dans les produits croisés de C^*-algèbres*, Compt. Rend. Acad. Sci. Paris **278**, 1041.
[1683] Takai H., 1975, *On a duality for crossed products of C^*-algebras*, J. Funct. Anal. **19**, 25.
[1684] Takesaki M., 1970, *The theory of operator algebras*, Lecture notes 1969-1970, University of California, Los Angeles (unpublished).
[1685] Takesaki M., 1970, *Tomita's theory of modular Hilbert algebras and its applications*, LNM **128**, Springer, Berlin.
[1686] Takesaki M., 1971, *One parameter automorphism groups and states of operator algebras*, in: Proceedings of International Congress of Mathematicians (Nice, 1970), Vol.2, Gauthier–Villars, Paris, p.427.
[1687] Takesaki M., 1972, *Conditional expectations in von Neumann algebras*, J. Funct. Anal. **9**, 306.
[1688] Takesaki M., 1973, *Duality for crossed products and the structure of von Neumann algebras of type III*, Acta Math. **131**, 249.
[1689] Takesaki M., 1973, *States and automorphisms of operator algebras, standard representations and the Kubo–Martin–Schwinger boundary condition*, in: Lennard A. (ed.), *Statistical mechanics and mathematical problems*, LNP **20**, p.205.
[1690] Takesaki M., 1973, *The structure of a von Neumann algebra with a homogeneous periodic state*, Acta Math. **131**, 79.
[1691] Takesaki M., 1978, *Introduction to the flow of weights on factors of type III*, in: Dell'Antonio G., Doplicher S., Jona-Lasinio G. (eds.), *Mathematical problems in theoretical physics: international conference held in Rome, June 6-15, 1977*, LNP **80**, p.134.
[1692] Takesaki M., 1979, 2003, *Theory of operator algebras, Vol.1-3*, Springer, Berlin.
[1693] Takesaki M., 1983, *Structure of factors and automorphism groups*, CBMS **51**, American Mathematical Society, Providence.
[1694] Takesaki M., 2003, *Sayōsokan e no iriguchi*, Sūgaku **55**, 89. jstage:sugaku1947/55/1/55_1_89 (engl. transl. 2008, *Entrance to operator algebras*, Amer. Math. Soc. Transl. **223**, 1).
[1695] Takiyama R., 1974, *[On geometrical structures of parameter spaces of one-dimensional distributions]*, Trans. Inst. Elect. Comm. Eng. Japan A **57**, 67.
[1696] Tam P.K., 1979, *Isometries of L^p-spaces associated with semifinite von Neumann algebras*, Trans. Amer. Math. Soc. **254**, 339.
[1697] Taneja I., Kumar P., 2004, *Relative information of type s, Csiszár f-divergence, and information inequalities*, Inf. Sci. **166**, 131.
[1698] Teboulle M., 1992, *Entropic proximal mappings with applications to nonlinear programming*, Math. Oper. Res. **17**, 670.
[1699] Teboulle M., Vajda I., 1993, *Convergence of best ϕ-entropy estimates*, IEEE Trans. Inf. Theor. **39**, 297.
[1700] Teller P., 1983, *The projection postulate as a fortuitous approximation*, Phil. Sci. **50**, 413.
[1701] Terp M., 1981, *L^p-spaces associated with von Neumann algebras*, Københavns Univ. Math. Inst. Rapp. No. **3a+3b**, Matematisk Institut, Københavns Universitet, København. Available at: www.fuw.edu.pl/~kostecki/scans/terp1981.pdf.
[1702] Terp M., 1982, *Interpolation spaces between von Neumann algebra and its predual*, J. Oper. Theor. **8**, 327. Available at: www.theta.ro/jot/archive/1982-008-002/1982-008-002-008.pdf.
[1703] Terras A., 1979, *Harmonic analysis on symmetric spaces and applications*, Springer, Berlin.
[1704] Thirring W.E., 1979, 1980, *Lehrbuch der mathematischen Physik*, Bd.3-4, Springer, Wien (engl. transl. 1981, 1983, *A course in mathematical physics*, Vol.3-4, Springer, Wien).
[1705] Thompson C.W., 1978, *Dissipations on von Neumann algebras*, Commun. Math. Phys. **62**, 71.
[1706] Tikhonov O.E., 1982, *Prostranstva tipa L_p otnositel'no vesa na algebre Neĭmana*, Izv. Vyssh. Uchebn. Zaved. Matem. **243**, 73 (engl. transl. 1982, *Spaces of L_p type with respect to a weight on a von Neumann algebra*, Sov. Math. **29**:8, 93).
[1707] Timpson C.G., 2008, *Quantum bayesianism: a study*, Stud. Hist. Phil. Mod. Phys. **39**, 579.
[1708] Toeplitz O., 1918, *Das algebraische Analogen zu einem Satze von Fejér*, Math Z. **2**, 187.
[1709] Tomamichel M., Colbeck R., Renner R., 2009, *A fully quantum asymptotic equipartition property*, IEEE Trans. Inf. Theor. **55**, 5840. arXiv:0811.1221.
[1710] Tomita M., 1953, *On rings of operators in non-separable Hilbert spaces*, Mem. Fac. Sci., Kyūsyū Univ., Ser. A **7**, 129. jstage:kyushumfs/7/2/7_2_129.
[1711] Tomita M., 1956, *Harmonic analysis on locally compact groups*, Math. J. Okayama Univ. **5**, 133.
[1712] Tomita M., 1959, *Spectral theory of operator algebras I*, Math. J. Okayama Univ. **9**, 63.

[1713] Tomita M., 1960, *Spectral theory of operator algebras II*, Math. J. Okayama Univ. **10**, 19.
[1714] Tomita M., 1967, *Quasi-standard von Neumann algebras*, mimeographed notes, Kyushu University (unpublished).
[1715] Tomita M., 1967, *Standard forms of von Neumann algebras*, in: *The Fifth Functional Analysis Symposium of Mathematical Society of Japan*, Mathematical Institute, Tōhoku University, Sendai, p.101 (in Japanese).
[1716] Tomiyama J., 1957, *On the projection of norm one in W^*-algebras*, Proc. Japan Acad. **33**, 608.
[1717] Tomiyama J., 1958, *On the projection of norm one in W^*-algebras II*, Tōhoku Math. J. **10**, 204.
[1718] Tomiyama J., 1959, *On the projection of norm one in W^*-algebras III*, Tōhoku Math. J. **11**, 125.
[1719] Tomiyama J., 1987, *Invitation to C^*-algebras and topological dynamics*, World Scientific, Singapore.
[1720] Topping D.M., 1971, *Lectures on von Neumann algebras*, Van Nostrand, New York.
[1721] Topsoe F., 2001, *Bounds for entropy and divergence for distributions over a two element set*, J. Ineq. Pure Appl. Math. **2**, 25.
[1722] Torgersen E.N., 1991, *Comparison of statistical experiments*, Cambridge University Press, Cambridge.
[1723] Tornebohm H., 1957, *On two logical systems proposed in the philosophy of quantum mechanics*, Theoria **23**, 84.
[1724] Toussaint G.T., 1975, *Sharper lower bounds for discrimination information in terms of variation*, IEEE Trans. Inf. Theor. **21**, 99.
[1725] Tribus M., 1961, *Thermostatics and thermodynamics*, Van Nostrand, New York.
[1726] Tribus M., 1969, *Rational descriptions, decisions and designs*, Pergamon, New York.
[1727] Tribus M., Rossi R., 1973, *On the Kullback information measure as a basis for information theory: comments on a proposal by Hobson and Chang*, J. Stat. Phys. **9**, 331.
[1728] Tropp J.A., 2011, *From joint convexity of quantum relative entropy to a concavity theorem of Lieb*, arXiv:1101.1070.
[1729] Trunov N.V., 1979, *O nekomutativnom analoge prostranstva L_p*, Izv. Vyssh. Uchebn. Zaved. Matem. **210**, 69 (engl. transl. 1979, *A noncommutative analogue of the space L_p*, Sov. Math. **23**:11, 71).
[1730] Trunov N.V., 1980, *K teorii integrirovaniya v algebrakh Neĭmana otnositel'no vesa*, Ph.D. thesis, Kazanskiĭ universitet, Kazan.
[1731] Trunov N.V., 1981, *Prostranstva L_p, associirovannye s vesom na polukonechnoĭ algebre Neĭmana*, Konstr. Teor. Funkts. Funkts. Anal. **3**, 88.
[1732] Trunov N.V., Sherstnëv A.N., 1978, *K obshheĭ teorii integrirovaniya v algebrakh operatorov otnositel'no vesa, I*, Izv. Vyssh. Uchebn. Zaved. Matem. **194**, 79 (engl. transl. 1978, *On the general theory of integration with respect to a weight in algebras of operators. I*, Sov. Math. **22**:8, 65).
[1733] Trunov N.V., Sherstnëv A.N., 1978, *K obshheĭ teorii integrirovaniya v algebrakh operatorov otnositel'no vesa, II*, Izv. Vyssh. Uchebn. Zaved. Matem. **199**, 88 (engl. transl. 1978, *On the general theory of integration with respect to a weight in algebras of operators. II*, Sov. Math. **22**:12, 62).
[1734] Trunov N.V., Sherstnëv A.N., 1985, *Vvedenie v teoriyu nekommutativnogo integrirovaniya*, Itogi Nauk. Tekhn. Ser. Sovr. Probl. Math. Nov. Dostizh. **27**, 167 (engl. transl. 1987, *Introduction to the theory of noncommutative integration*, J. Sov. Math. **37**:6, 1504).
[1735] Tsallis C., 1988, *Possible generalization of Boltzmann–Gibbs statistics*, J. Stat. Phys. **52**, 479.
[1736] Tuckerman M.E., Liu Y., Ciccotti G., Martyna G.J., 2001, *Non-hamiltonian molecular dynamics: generalizing hamiltonian phase space principles to non-hamiltonian systems*, J. Chem. Phys. **115**, 1678.
[1737] Tuckerman M.E., Mundy C.J., Klein M.L., 1997, *Toward a statistical thermodynamics of steady states*, Phys. Rev. Lett. **78**, 2042.
[1738] Tuckerman M.E., Mundy C.J., Klein M.L., 1998, *Reply to comment on "Toward a statistical thermodynamics of steady states"*, Phys. Rev. Lett. **80**, 4105.
[1739] Tuckerman M.E., Mundy C.J., Martyna G.J., 1999, *On the classical statistical mechanics of non-hamiltonian systems*, Europhys. Lett. **45**, 149.
[1740] Tumpach A.B., 2009, *On the classification of infinite-dimensional irreducible hermitian-symmetric affine coadjoint orbits*, Forum Math. **21**, 375.
[1741] Uffink J.B.M., 1995, *Can the maximum entropy principle be explained as a consistency requirement?*, Stud. Hist. Phil. Mod. Phys. B **26**, 223.
[1742] Uffink J.B.M., 1996, *The constraint rule of the maximum entropy principle*, Stud. Hist. Phil. Mod. Phys. **27**, 47. Available at: www.phys.uu.nl/igg/jos/mep2def/mep2def.pdf.
[1743] Uhlmann A., 1976, *The "transition probability" in the state space of a *-algebra*, Rep. Math. Phys. **9**, 273. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh76a.pdf.
[1744] Uhlmann A., 1977, *Relative entropy and the Wigner–Yanase–Dyson–Lieb concavity in an interpolation theory*, Commun. Math. Phys. **54**, 21. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh77a.pdf.
[1745] Uhlmann A., 1985, *The transition probability for states of $*$-algebras*, Ann. Phys. **42**, 524. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh85a.pdf.
[1746] Uhlmann A., 1986, *Parallel transport and "quantum holonomy" along density operators*, Rep. Math. Phys. **24**, 229. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh86a.pdf.
[1747] Uhlmann A., 1987, *Parallel transport and holonomy along density operators*, in: Doebner H.D., Hennig J.D. (eds), *Differential geometric methods in theoretical physics*, World Scientific, Singapore, p.246. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh87a.pdf.
[1748] Uhlmann A., 1992, *An energy dispersion estimate*, Phys. Lett A **161**, 329. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh92a.pdf.
[1749] Uhlmann A., 1992, *The metric of Bures and the geometric phase*, in: Gielerak R., Lukierski J., Popowicz Z. (eds.), *Quantum groups and related topics, Proceedings of the First Max Born Symposium, Wrocław 1991*, Kluwer, Dordrecht, p.267. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh92b.pdf.
[1750] Uhlmann A., 1993, *Density operators as an arena for differential geometry*, Rep. Math. Phys. **33**, 253. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh93e.pdf.
[1751] Uhlmann A., 1995, *Geometric phases and related structures*, Rep. Math. Phys. **36**, 461. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/Uh95a.pdf.
[1752] Uhlmann A., Crell B., 2009, *Geometry of state spaces*, in: Buchleitner A. et al. (eds.), *Entanglement and decoherence*, Lecture Notes in Physics **768**, Springer, Berlin, p.1. Available at: www.physik.uni-leipzig.de/∼uhlmann/PDF/UC07.pdf.
[1753] Ulam S., 1930, *Zur Masstheorie in der allgemeinen Mengenlehre*, Fund. Math. **16**, 140.
[1754] Umegaki H., 1954, *Conditional expectations in an operator algebra*, Tōhoku Math J. **6**, 177.
[1755] Umegaki H., 1956, *Conditional expectations in an operator algebra II*, Tōhoku Math. J. **8**, 86.

[1756] Umegaki H., 1959, *Conditional expectations in an operator algebra III*, Kōdai Math. Sem. Rep. **11**, 51.
[1757] Umegaki H., 1961, *On information in operator algebras*, Proc. Jap. Acad. **37**, 459. euclid:pja/1195523632.
[1758] Umegaki H., 1962, *Conditional expectations in an operator algebra IV (entropy and information)*, Kōdai Math. Sem. Rep. **14**, 59.
[1759] Urbanik K., 1961, *Joint probability distributions of observables in quantum mechanics*, Studia Math. **21**, 117.
[1760] Urbanik K., 1985, *Joint distributions and compatibility of observables*, Demonstr. Math. **18**, 31.
[1761] Ustinov N.V., Czachor M., Kuna M., Leble S.B., 2001, *Darboux integration of* $i\frac{d\rho}{dt} = [H, f(\rho)]$, Phys. Lett. A **279**, 333. arXiv:quant-ph/0005030.
[1762] Vaisman I., 1994, *Lectures on the geometry of Poisson manifolds*, Birkhäuser, Basel.
[1763] Vajda I., 1970, *Note on discrimination information and variation*, IEEE Trans. Inf. Theor. **16**, 99.
[1764] Vajda I., 1972, *On the f-divergence and singularity of probability measures*, Period. Math. Hung. **2**, 223.
[1765] Vajda I., 1973, *χ^α-divergence and generalized Fisher's information*, in: Kozesnik J. (ed.), *Transactions of the sixth Prague conference on information theory, statistical decision functions and random processes*, Academia, Praha, p.873.
[1766] Vajda I., 1982, *Teória informácie a štatistického rozhodovania*, Alfa, Bratislava. (engl. transl.: 1989, *Theory of statistical inference and information*, Kluwer, Dordrecht).
[1767] Valente G., 2007, *Is there a stability problem for bayesian noncommutative probabilities?*, Stud. Hist. Phil. Mod. Phys. **38**, 832.
[1768] van Daele A., 1974, *A new approach to the Tomita–Takesaki theory of generalized Hilbert algebras*, J. Funct. Anal. **15**, 378.
[1769] van Daele A., 1976, *A Radon–Nikodym for weights on von Neumann algebras*, Pacific J. Math. **61**, 527.
[1770] van Daele A., 1978, *Continuous crossed products and type III von Neumann algebras*, London Mathematical Society LNS **31**, Cambridge University Press, Cambridge.
[1771] van Dantzig D., 1931, *Studiën over topologische Algebra*, Paris, Amsterdam.
[1772] van der Vaart A., 1998, *Asymptotic statistics*, Cambridge University Press, Cambridge.
[1773] van der Vaart A., 2002, *The statistical work of Lucien Le Cam*, Ann. Statist. **30**, 631.
[1774] van der Waerden B.L., 1932, *Die gruppentheoretische Methode in der Quantenmechanik*, Springer, Berlin (engl. transl. rev. ed. 1974, *Group theory and quantum mechanics*, Springer, Berlin).
[1775] van Fraasen B.C., 1974, *The labirynth of quantum logics*, in: Cohen R., Wartofsky M. (eds.), *Boston studies in the philosophy of science*, Vol.13, Reidel, Dordrecht, p.?.
[1776] van Hove L., 1951, *Sur le probléme des relations entre les transformations unitaires de la méchanique quantique et les transformations canoniques de la méchanique classique*, Bull. Class. Sci. Acad. Roy. Belgique **37**, 610.
[1777] van Hove L., 1951, *Sur l'opérateur hamiltonien de deux champs quantifiés en interaction*, Bull. Class. Sci. Acad. Roy. Belgique **37**, 1055.
[1778] van Hove L., 1952, *Les difficultés de divergences pour un modelle particulier de champ quantifié*, Physica **18**, 145.
[1779] van Hove L., 1959, *Quelques méthodes et problèmes de la théorie quantique des champs et interaction*, in: Deheuvels R., Michel L. (eds.), *Les problèmes mathématiques de la théorie quantique des champs. Lille 3-8 Juin, 1957*, Centr. Natl. Rech. Sci., Paris, p.39.
[1780] van Kampen E., 1935, *Locally bicompact abelian groups and their character groups*, Ann. Math. **36**, 448.
[1781] van Lambalgen M., 1987, *Von Mises' notion of random sequence reconsidered*, J. Symb. Log. **52**, 725.
[1782] van Neerven J.M.A.M., 1997, *The norm of a complex Banach lattice*, Positivity **1**, 381.
[1783] Varadarajan V.S., 1961, *Mery na topologicheskikh prostranstvakh*, Matem. Sb. **55**, 35 (engl. transl. 1965, *Measures on topological spaces*, Amer. Math. Soc. Transl. **48**, 161).
[1784] Varadarajan V.S., 1962, *Probability in physics and a theorem on simultaneous observability*, Commun. Pure Appl. Math. **115**, 189.
[1785] Varadarajan V.S., 1968, *The geometry of quantum theory*, Vol.1-2, Van Nostrand, New York (2nd ed. 1985).
[1786] Varadarajan V.S., 1977, *Review of: Foundations of quantum physics, by C. Piron*, Bull. Amer. Math. Soc. **83**, 226.
[1787] Vedernikov V.I., 1965, *Simmetricheskie prostranstva i sopryazhennye svyaznosti*, Uchen. zap. kazansk. univ. **125**, 7.
[1788] Vetterlein T., 2011, *Partial quantum logics revisited*, Int. J. Gen. Sys. **40**, 23.
[1789] Vinberg E'.B., 1963, *Teoriya odnorodnykh vypuklykh konusov*, Trudy mosk. mat. obshh **12**, 303 (engl. transl. 1963, *The theory of convex homogeneous cones*, Trans. Moscow Math. Soc. **12**, 340).
[1790] Voiculescu D.V., 2000, *Lectures on free probability*, LNM **1738**, Springer, Berlin.
[1791] Voiculescu D.V., Dykema K.J., Nica A., 1992, *Free random variables*, American Mathematical Society, Providence.
[1792] Volle M., Hiriart-Urruty J.-B., 2012, *A characterization of essentially strictly convex functions on reflexive Banach spaces*, Nonlin. Anal. **75**, 1617.
[1793] Volčič A., 1970, *Sul teorema di Radon–Nikodym nel caso non σ-finito*, Rend. Inst. mat. Univ. Trieste **2**, 42. Available at: rendiconti.dmi.units.it/volumi/02/04.pdf.
[1794] von Neumann J., 1927, *Mathematische Begrüdung der Quantenmechanik*, Nachr. Ges. Wiss. Göttingen, Math.-phys. Kl. **1927**, 1.
[1795] von Neumann J., 1927, *Thermodynamik quantenmechanischer Gesamtheiten*, Nachr. Ges. Wiss. Göttingen, Math.-phys. Kl. **1927**, 273.
[1796] von Neumann J., 1927, *Wahrscheinlichkeitstheoretischer Aufbau der Quantenmechanik*, Nachr. Ges. Wiss. Göttingen, Math.-phys. Kl. **1927**, 245.
[1797] von Neumann J., 1929, *Allgemeine Eigenwerttheorie Hermitescher Funktionaloperatoren*, Math. Ann. **102**, 49.
[1798] von Neumann J., 1930, *Zur algebra der Funktionaloperatoren und Theorie der normalen Operatoren*, Math. Ann. **102**, 370.
[1799] von Neumann J., 1931, *Die Eindeutigkeit der Schrödingerchen Operatoren*, Math. Ann. **104**, 570.
[1800] von Neumann J., 1932, *Mathematische Grundlagen der Quantenmechanik*, Springer, Berlin. (engl. transl. 1955, *Mathematical foundations of quantum mechanics*, Princeton University Press, Princeton).
[1801] von Neumann J., 1932, *Über adjungierte Funktionaloperatoren*, Ann. Math. **33**, 294.
[1802] von Neumann J., 1932, *Über einen Satz von Herrn M. H. Stone*, Ann. Math. **33**, 567.
[1803] von Neumann J., 1932, *Zur Operatorenmethode in der Klassischen Mechanik*, Ann. Math. **33**, 587.
[1804] von Neumann J., 1932, *Zusätze zur Arbeit „Zur Operatorenmethode...”*, Ann. Math. **33**, 789.
[1805] von Neumann J., 1934, *Zur Haarschen Mass in topologischen Gruppen*, Comp. Math. **1**, 106.
[1806] von Neumann J., 1936, *On a certain topology for rings of operators*, Ann. Math. **37**, 111.
[1807] von Neumann J., 1936, *On an algebraic generalisation of the quantum mechanical formalism (part I)*, Matem. sb. N.S. **1**, 415.
[1808] von Neumann J., 1936, *The uniqueness of Haar's measure*, Matem. sb. N.S. **1**, 721.

[1809] von Neumann J., 1937, *Quantum logic (strict- and probability logics)*, unpublished manuscript, John von Neumann Archive, Library of Congress, Washington (reviewed by Taub A.H., 1961, in: von Neumann J., 1961, *Collected Works*, Pergamon, London, p.195).
[1810] von Neumann J., 1937, *Some matrix-inequalities and metrization of matrix-spaces*, Tomsk Univ. Rev. **1**, 286.
[1811] von Neumann J., 1939, *On infinite direct product*, Comp. Math. **6**, 1. numdam:CM_1939__6__1_0.
[1812] von Neumann J., 1943, *On rings of operators III*, Ann. Math. **41**, 94.
[1813] von Neumann J., 1949, *On rings of operators. Reduction theory*, Ann. Math. **50**, 401.
[1814] von Neumann J., 1981, *Continuous geometries with a transition probability*, American Mathematical Society, Providence.
[1815] Vulikh B.Z., 1957, *Chastichnoe uporyadochenie kolec ogranichennykh samosopryazhennykh operatorov*, Vestnik Leningr. Univ. **12**, 13.
[1816] Vulikh B.Z., 1957, *Primenenie teorii poluuporyadochennykh prostranstv k issledovaniyu samospryazhennykh operatorov v gil'bertovom prostranstve*, Usp. Mat. Nauk **12**, 169.
[1817] Vulikh B.Z., 1961, *Vvedenie v teoriyu poluuporyadochennykh prostranstv*, Gosudarstvennoe Izdatel'stvo Fiziko-Matematicheskoĭ Literatury, Moskva (engl. transl., 1967, *Introduction to the theory of partially ordered spaces*, Woolters–Noordhoff, Groningen).
[1818] Wada J., 1957, *Stonian spaces and the second conjugate spaces of AM spaces*, Osaka Math. J. **9**, 195. euclid:ojm/1200689162.
[1819] Wald A., 1939, *Contributions to the theory of statistical estimation and testing hypothesis*, Math. Statist. **10**, 299.
[1820] Wald A., 1947, *Sequential analysis*, Wiley, New York.
[1821] Wald A., 1950, *Statistical decision functions*, Wiley, New York.
[1822] Warmuth M.K., 2005, *A Bayes rule for density matrices*, in: Weiss Y., Schölkopf B., Platt J. (eds.), *Advances in neural information processing systems **18** (NIPS 05)*, MIT Press, p.1457.
[1823] Watanabe K., 1992, *On isometries between noncommutative L^p-spaces associated with arbitrary von Neumann algebras*, J. Op. Theor. **28**, 267.
[1824] Watanabe K., 1995, *Finite measures on preduals and non-commutative L^p-isometries*, J. Op. Theor. **33**, 371.
[1825] Watanabe K., 1996, *An application of orthoisomorphisms to non-commutative L^p-isometries*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **32**, 493.
[1826] Watanabe K., 1999, *Problems on isometries of non-commutative L^p-spaces*, Contemp. Math. **232**, 349.
[1827] Wecken F., 1940, *Abstrakte Integrale und fastperiodische Funktionen*, Math. Z. **45**, 377.
[1828] Wegge-Olsen N.E., 1993, *K-theory and C^*-algebras*, Oxford University Press, Oxford.
[1829] Wehrl A., 1978, *General properties of entropy*, Rev. Mod. Phys. **50**, 221.
[1830] Weil A., 1940, *Calcul des probabilités, méthode axiomatique, intégration*, Revue Sci. (Rev. Rose Illus.) **78**, 201.
[1831] Weil A., 1940, *L'intégration dans les groupes topologiques et ses applications*, Actualités Scientifiques et Industrielles **869**, Hermann, Paris.
[1832] Weinstein A., 1983, *The local structure of Poisson manifolds*, J. Diff. Geom. **18**, 523.
[1833] Weinstein A., 1998, *Poisson geometry*, Diff. Geom. Appl. **9**, 213.
[1834] Weir A.J., 1974, *General integration and measure*, Cambridge University Press, Cambridge.
[1835] Wessels L., 1980, *What was Born's statistical interpretation?*, in: *Proceedings of the biennal meeting of the Philosophy of Science Association*, Vol.2, The University of Chicago Press, Chicago, p.187.
[1836] Weyl H.K.H., 1918, *Raum–Zeit–Materie*, Springer, Berlin.
[1837] Weyl H.K.H., 1918, *Reine Infinitesimalgeometrie*, Math. Z. **2**, 384.
[1838] Weyl H.K.H., 1927, *Quantenniechanik und Gruppentheorie*, Z. Phys. **46**, 1.
[1839] Weyl H.K.H., 1928, *Gruppentheorie und Quantenmechanik*, Hirzel, Leipzig. (engl. transl.: 1931, *The theory of groups and quantum mechanics*, Dutton, New York).
[1840] Weyl H.K.H., 1929, *Elektron und Gravitation I*, Z. Phys. **56**, 330.
[1841] Whitehead A.N., 1898, *A treatise on universal algebra, with applications*, Cambridge University Press, Cambridge.
[1842] Whittle P., 1970, *Probability*, Penguin, Harmondsworth.
[1843] Whittle P., 2000, *Probability via expectation*, (4th ed.), Springer, Berlin.
[1844] Wichmann E.H., 1963, *Density matrices arising from incomplete measurements*, J. Math. Phys. **4**, 884.
[1845] Wick G.C., 1954, *Properties of Bethe–Salpeter wave functions*, Phys. Rev. **96**, 1124.
[1846] Wick G.C., Wightman A.S., Wigner E.P., 1952, *Intrisic parity of elementary particles*, Phys. Rev. **88**, 101.
[1847] Wick G.C., Wightman A.S., Wigner E.P., 1970, *Superselection rule for charge*, Phys. Rev. D **1**, 3267.
[1848] Wielandt H., 1949, *Über die Unbeschränkheit der Operatoren der Quantenmechanik*, Math. Ann. **121**, 21.
[1849] Wiener N., 1920, *The mean of a functional of arbitrary elements*, Ann. Math. **22**, 66.
[1850] Wiener N., 1923, *Differential space*, J. Math. Phys. **2**, 131.
[1851] Wiesbrock H.-W., 1997, *Symmetries and modular intersections of von Neumann algebras*, Lett. Math. Phys. **39**, 203.
[1852] Wightman A.S., 1956, *Quantum field theory in terms of vacuum expectation values*, Phys. Rev. **101**, 860.
[1853] Wightman A.S., 1957, *Quelques problèmes mathématiques de la théorie quantique relativiste*, Lecture notes, Faculté des Sciences, Université de Paris, Paris (reprinted in: Deheuvels R., Michel L. (eds.), 1959, *Les problèmes mathématiques de la théorie quantique des champs. Lille 3-8 Juin, 1957*, Centr. Natl. Rech. Sci., Paris, p.1).
[1854] Wightman A.S., 1959, *Relativistic invariance and quantum mechanics*, Nuovo Cim. Suppl. **14**, 81.
[1855] Wightman A.S., 1964, *La théorie quantique locale et la théorie quantique des champs*, Ann. Inst. Henri Poincaré A **1**, 403.
[1856] Wightman A.S., 1995, *Superselection rules; old and new*, Nuovo Cim. B **110**, 751.
[1857] Wightman A.S., Gårding L., 1964, *Fields as operator-valued distributions in relativistic quantum theory*, Ark. Fys. **28**, 129.
[1858] Wightman A.S., Schweber S.S., 1955, *Configuration space methods in relativistic quantum field theory, I*, Phys. Rev. **98**, 812.
[1859] Wigner E.P., 1931, *Gruppentheorie und ihre Anwendung in der Theorie der Atomspektren*, Vieweg, Braunschweig (engl. transl. 1959, *Group theory and its application to the quantum mechanics of atomic spectra*, Academic Press, New York).
[1860] Wigner E.P., 1932, *On the quantum correction for thermodynamic equilibrium*, Phys. Rev. **40**, 749.
[1861] Wigner E.P., 1939, *On unitary representations of the inhomogeneus Lorentz group*, Ann. Math. **40**, 149.
[1862] Wigner E.P., 1962, *Theorie der quantenmechanischen Messung*, in: Brüche E. (ed.), *Physikertagung, Wien: Hauptvorträge von der gemeinsamen Jahrestagung 1961 des Verbandes Deutscher Physikalischer Gesellschaften und der Österreichischen Physikalischen Gesellschaft*, Physik Verlag, Mosbach/Baden, p.1.
[1863] Wigner E.P., 1971, *Quantum mechanical distribution functions revisited*, Yourgrau W., van der Merwe A. (eds.), *Perspectives on quantum theory*, MIT Press, Cambridge, p.25.
[1864] Wigner E.P., 1973, *Epistemological perspective on quantum theory*, in: Hooker C.A. (ed.), *Contemporary research in the*

foundations and philosophy of quantum theory: proceedings of a conference held at the University of Western Ontario, London, Canada, Reidel, Dordrecht, p.369.
[1865] Wigner E.P., Yanase M.M., 1963, *Information contents of distributions*, Proc. Nat. Acad. Sci. U.S.A. **49**, 910.
[1866] Wilce A., 1992, *Tensor products in generalized measure theory*, Int. J. Theor. Phys. **31**, 1915.
[1867] Wilce A., 2000, *Test spaces and orthoalgebras*, in: Coecke B., Moore D., Wilce A. (eds.), *Current research in operational quantum logic: algebras, categories, languages*, Kluwer, Dordrecht, p.81.
[1868] Wils W., 1968, *Désintégration centrale des formes positives sur les C^*-algèbres*, Compt. Rend. Acad. Sci. Paris **267**, 810.
[1869] Wils W., 1969, *Désintégration centrale dans une partie convexe compacte d'un espace localement convexe*, Compt. Rend. Acad. Sci. Paris **269**, 702.
[1870] Wils W., 1971, *The ideal center of partially ordered vector spaces*, Acta Math. **127**, 41.
[1871] Winnink M., 1968, *An application of C^*-algebras to quantum statistical mechanics of systems in equilibrium*, Ph.D. thesis, Rijks–Universiteit, Gröningen.
[1872] Winnink M., 1970, *Algebraic aspects of the Kubo–Martin–Schwinger condition*, in: Kastler D. (ed.), *Cargèse Lectures in Physics*, Vol.4, Gordon & Breach, New York, p.235.
[1873] Wintner A., 1947, *The unboundedness of quantum-mechanical matrices*, Phys. Rev. **71**, 738.
[1874] Wiśniewski-Snerg A., 1990, *Jednolita teoria czasoprzestrzeni*, Pusty Obłok, Warszawa.
[1875] Woods E.J., 1973, *The classification of factors is not smooth*, Canad. J. Math. **25**, 96.
[1876] Woronowicz S.L., 1972, *On the purification of factor states*, Commun. Math. Phys. **28**, 221.
[1877] Woronowicz S.L., 1979, *Operator systems and their application to the Tomita–Takesaki theory*, LNS **52**, Aarhus Universitet, Matematisk Institut, Aarhus. Reprinted in: J. Op. Theor. **2**, 169. Available at: www.theta.ro/jot/archive/1979-002-002/1979-002-002-003.pdf.
[1878] Woronowicz S.L., 1991, *Unbounded elements affiliated with C^*-algebras and noncompact quantum groups*, Commun. Math. Phys. **136**, 399. euclid:cmp/1104202358.
[1879] Wozniak F., 2007, *Conditions and constraints in the teaching of statistics: the scale of levels of determination*, ?.
[1880] Wright R., 1978, *Spin manuals*, in: Marlow A.R. (ed.), *Mathematical foundations of quantum theory*, Academic Press, New York, p.?.
[1881] Wúmén Huìkāi, 1228, *Chán zong wúménguān*, (engl. transl. 2007, *The gateless checkpoint of the zen lineage*, available at: home.pon.net/wildrose/gateless.htm).
[1882] Xu Q., 1991, *Analytic functions with values in lattices and symmetric spaces of measurable operators*, Math. Proc. Cambridge Phil. Soc. **109**, 541.
[1883] Yamada S., Morimoto H., 1992, *Sufficiency*, in: Ghosh M., Pathak P.K. (eds.), *Current issues in statistical inference: essays in honor of D. Basu*, Institute of Mathematical Statistics, Hayward, p.86. euclid:lnms/1215458840.
[1884] Yamagami S., 1992, *Algebraic aspects in modular theory*, Publ. Res. Inst. Math. Sci. Kyōto Univ. **28**, 1075. Available at: www.ems-ph.org/journals/show_pdf.php?issn=0034-5318&vol=28&iss=6&rank=8.
[1885] Yamagami S., 1994, *Modular theory for bimodules*, J. Funct. Anal. **125**, 327.
[1886] Yamagami S., 2008, *Geometric mean of states and transition amplitudes*, Lett. Math. Phys. **84**, 123. arXiv:0801.0858.
[1887] Yamagami S., 2010, *Geometry of quasi-free states of CCR algebras*, Int. J. Math. **21**, 875. arXiv:0801.1739.
[1888] Yeadon F.J., 1971, *A new proof of the existence of a trace in a finite von Neumann algebra*, Bull. Amer. Math. Soc. **77**, 257.
[1889] Yeadon F.J., 1973, *Convergence of measurable operators*, Math. Proc. Cambridge Phil. Soc. **74**, 257.
[1890] Yeadon F.J., 1975, *Non-commutative L_p-spaces*, Math. Proc. Cambridge Phil. Soc. **77**, 91.
[1891] Yeadon F.J., 1981, *Isometries of non-commutative L^p spaces*, Math. Proc. Cambridge Phil. Soc. **90**, 41.
[1892] Yeadon F.J., 1984, *Finitely additive measures on projections infinite W^*-algebras*, Bull. London Math. Soc. **16**, 145.
[1893] Yngvason J., 2004, *The role of type III factors in quantum field theory*, arXiv:math-ph/0411058.
[1894] Yoshida K., 1948, *On the differentiability and the representation of one-parameter semigroups of linear operators*, J. Math. Soc. Japan **1**, 15.
[1895] Yoshida K., Fukamiya M., 1941, *On regularly convex sets*, Proc. Imp. Acad. Tōkyō **17**, 49.
[1896] Yoshizawa S., Tanabe K., 1999, *Dual differential geometry associated with the Kullback–Leibler information on the gaussian distributions and its 2-parameter deformations*, SUT J. Math. **1**, 113.
[1897] Yoshizawa T., 1971, *A geometrical interpretation of location and scale parameters*, Memorandom TYH-**3**, Statistics Department, Harvard University, Cambridge.
[1898] Yoshizawa T., 1971, *A geometry of parameter space and its statistical interpretation*, Memorandom TYH-**2**, Statistics Department, Harvard University, Cambridge.
[1899] Younce M.B., 1987, *Random variables on non-boolean structures*, Ph.D. thesis, University of Massachusetts.
[1900] Young W.H., 1911, *On a new method in the theory of integration*, Proc. London Math. Soc. **9**, 15.
[1901] Young W.H., 1913, *On the new theory of integration*, Proc. Royal Soc. London Ser. A. **88**, 170.
[1902] Young W.H., 1914, *On integration with respect to a function of bounded variation*, Proc. London Math. Soc. **13**, 109.
[1903] Yuen H.P.H., Lax M., 1973, *Multiple-parameter quantum estimation and measurement of nonselfadjoint observables*, IEEE Trans. Inf. Theor. **19**, 740.
[1904] Zaanen A.C., 1952, *Integral transformations and their resolvents in Orlicz and Lebesgue spaces*, Compositio Math. **10**, 56.
[1905] Zaanen A.C., 1961, *The Radon–Nikodym theorem I, II*, Nederl. Akad. Wetensch. Proc. Ser. A **64**, 157.
[1906] Zaanen A.C., 1983, *Riesz spaces II*, North-Holland, Amsterdam.
[1907] Zawirski Z., 1931, *Logika trójwartościowa Jana Łukasiewicza. O logice L.E.J. Brouwera. Próby stosowania logiki wielowartościowej do współczesnego przyrodoznawstwa*, Sprawozd. Poznańsk. Tow. Przyj. Nauk **2-4/1931**, 1.
[1908] Zawirski Z., 1932, *Les logiques nouvelles et le champ de leur application*, Rev. Métaphys. Morale **39**, 503.
[1909] Zawirski Z., 1936, *Über die Anwendung der mehwertigen Logik in der empirischen Wissenschaft*, Erkenntnis **61**, 430.
[1910] Zeller-Meier G., 1969, *Deux autres facteurs de type II_1*, Invest. Math. **7**, 235.
[1911] Zhang J., 2004, *Divergence function, duality, and convex analysis*, Neural Comp. **16**, 159. Available at: www.lsa.umich.edu/psych/junz/Neural%20Comp%202004.pdf.
[1912] Zhang J., 2004, *Dual scaling of comparision and reference stimuli in multi-dimensional psychological space*, J. Math. Psych. **48**, 409.
[1913] Zhang J., 2006, *A note on curvature of α-connections of a statistical manifold*, Inst. Statist. Math. DOI:**10.1007/s10463-006-0105-1**.
[1914] Zhang J., Matsuzoe H., 2009, *Dualistic riemannian manifold structure induced from convex functions*, in: *Advances in applied mathematics and global optimization in honor of Gilbert Strang*, Advances in Mechanics and Mathematics **17**,

Springer, Berlin, p.1. Available at: www.lsa.umich.edu/psych/junz/Zhang%20and%20Matsuzoe%20hessian.pdf.
[1915] Zhu H., 1998, *Generalized Lebesgue spaces and application to statistics*, Santa Fe Inst. Tech. Rep. **98-06-44**, Santa Fe. Available at: www.santafe.edu/media/workingpapers/98-06-044.ps.
[1916] Zhu H., 1998, *Unique extension of metric and affine connections to finite measures on infinite spaces*, preprint.
[1917] Zhu H., Rohwer R., 1995, *Information geometric measurements of generalisation*, Tech. Rep. Neural Comp. Res. Group **4350**, Aston. Available at: eprints.aston.ac.uk/507/1/NCRG_95_005.pdf.
[1918] Zhu H., Rohwer R., 1997, *Measurements of generalisation based on information geometry*, in: Ellacott S.W. et al. (eds.), *Mathematics of neural networks: models, algorithms and applications*, Kluwer, Dordrecht, p.394. Available at: eprints.aston.ac.uk/514/1/NCRG_95_012.pdf.
[1919] Zhu H., Rohwer R., 1998, *Information geometry, bayesian inference, ideal estimates and error decomposition*, Santa Fe Inst. Tech. Rep. **98-06-45**, Santa Fe. Available at: omega.albany.edu:8008/ignorance/zhu98.pdf.
[1920] Zierler N., 1961, *Axioms for non-relativistic quantum mechanics*, Pacific J. Math. **11**, 1151.
[1921] Zolotarëv A.A., 1982, *Prostranstva L^p otnositel'no sostoyaniya na algebre Neĭmana i interpolyaciya*, Izv. Vyssh. Uchebn. Zaved. Matem. **243**, 36. (engl. transl. 1982, *L^p space with respect to a state on a von Neumann algebra, and interpolation*, Sov. Math. **26**:8, 41).
[1922] Zolotarëv A.A., 1985, *K interpolyacionnoĭ teorii prostranstv L_p otnositel'no sostoyaniya na algebre Neĭmana*, Izv. Vyssh. Uchebn. Zaved. Matem. **272**, 59. (engl. transl. 1985, *On the interpolation theory of L_p spaces with respect to a state on von Neumann algebra*, Sov. Math. **29**:1, 78).
[1923] Zsidó L., 1973, *[Topological decomposition of W^*-algebras I, II]*, Studii Cercet. Matem. **26**, 859, 1037.
[1924] Zsidó L., 1979, *Invariance and dual weights*, in: *Algèbres d'opérateurs et leurs applications en physique mathématique (Proc. Colloques Internationaux, Marseille 20-24 juin 1977)*, Colloques Internationaux C.N.R.S. **274**, Éditions du C.N.R.S., Paris, p.529.
[1925] Zălinescu C., 2002, *Convex analysis in general vector spaces*, World Scientific, Singapore.
[1926] Zumbrägel J., 2004, *Modular-Theorie und die Konstuktion nicht-komutativer L_p-Räume nach Haagerup*, Diplomarbeit, Universität Oldenburg, Oldenburg. Available at: shannoninstitute.ucd.ie/∼jzumbr/files/Diplomarbeit.pdf.

Index

$(\mathbf{g}, \nabla_1, \nabla_2)$-projection, 254
(n, m)-tensor bundle, 247
(n, m)-tensor fields, 146
$*$-algebra, 164
$*$-antihomomorphism, 165
$*$-antiisomorphism, 165
$*$-automorphism, 165
$*$-homomorphism, 164
$*$-isomorphism, 165
AW^*-algebra, 166
C^*-algebra, 164
C^*-covariant system, 192
C^*-crossed product, 197
C^*-dynamical system, 192
W^*-algebra, 167
W^*-covariant system, 192
W^*-dynamical system, 192
X-open, 235
γ-embedding, 78
γ-representation, 89, 92
$\mathbb{K}$-linear, 166
$\mathbf{A}$-propagation, 143
$\mathfrak{f}$-connections, 103
$\mho(\mathcal{X})$-measurable, 233
∇-affine, 249, 253
∇-autoparallel, 253
∇-convex, 254
∇-flat, 253
∇-geodesic, 248
ω-stable, 214
τ-measurable, 209
τ-topology, 210
ϑ-connections, 250
ϑ-hessian manifold, 251
n-form field, 147
n-positive, 59
$\tilde{\mu}$-almost every, 233
$\tilde{\mu}$-integrable, 233
$\tilde{\mu}$-integral, 233
$\tilde{\mu}$-null, 233
$\tilde{\mu}$-simple, 233
Ştrătilă–Zsidó, 166

abelian, 189
absolute value, 165
absolutely continuous, 235
abstract L_p space, 227
act standardly, 180
adequate, 66
adjoint, 177
adjoint map, 135
admissible codomain, 73
admissible domain, 73
affiliated, 205
affine, 96
affine connection, 248
affine geometry, 248
affine manifold, 248
Alber functional, 68
Alber projection, 70
algebraic hamiltonian vector field, 139
Amari conjecture, 95
Amari–Cressie–Read γ-distance, 80
Amari–Nagaoka geometries, 95
anti-self-adjoint, 165, 178
antilinear $*$-homomorphism, 165
antilinear $*$-isomorphism, 165
antirepresentation, 170
approximately finite dimensional, 191
Araki distance, 80
Araki–Masuda spaces, 218
archimedean, 227

Bóna equation, 138
Bóna–Odzijewicz–Ratiu equation, 137
Banach, 227
Banach $*$-algebras, 164
Banach algebra, 164
Banach dual, 166
Banach Lie algebra, 135
Banach Poisson manifold, 136
Banach–Lie–Poisson space, 136
band, 228
barycenter, 173
basically disconnected, 238
Bhattacharyya distance, 81
bimodule, 187
Bogolyubov–Kubo–Mori metric, 99
boolean, 226, 227
boolean algebra, 227
boolean homomorphism, 227
boolean isomorphism, 228
bounded, 60
bounded above, 226
bounded below, 226
Bregman functional, 67
Bregman functional projection, 69
Bregman pre-distance, 72
bregmanian renormalisation, 132

canonical $\mathcal{N}$-$\mathcal{N}$ bimodule, 187
canonical core, 203
canonical embedding, 167
canonical Hilbert space, 186
canonical integral, 222, 238
canonical liouvillean, 195

canonical relative modular operator, 188
canonical representation, 186
canonical right $\mathcal{N}$ module, 187
canonical standard form, 186
canonical trace, 220
Cantoni–Uhlmann transition probability, 149
cartesian, 249
ccb-algebra, 228
center, 172
central, 173, 189
central support, 169
centrally faithful, 171
Chencov–Amari γ-connections, 95
Christoffel symbols, 249
clopen, 238
closable, 177
closed, 65, 177
closure, 177
coadjoint map, 135
coarse graining, 59
Codazzi structure, 252
cofinite, 66
commutant, 171
compact, 211, 238
compactly inner regular, 238
complete, 166
complete morphism, 232
completely positive, 59
complex Riesz lattice, 227
conditional expectation, 213
cone, 180
configuration parameters, 48
conjugation, 175
Connes cocycle, 184
Connes spatial quotient, 183
Connes' cocycle, 182
Connes' spatial quotient, 188
Connes–Hilsum spaces, 218
constraint mapping, 113
continuous, 189
contraction coefficient, 132
control parameter, 109
converge in τ, 210
convex, 65
convex hull, 173
correspondence, 187
cotangent bundle, 247
cotangent space, 247
countably additive, 228, 233
countably additive algebra, 232
countably additive complete, 226, 227
countably additive ideal, 232
countably additive probability measure, 33
countably additively orthocomplete, 32
countably finite, 172
covariance equation, 192, 198
covariance matrix, 107
covariant derivative, 248
covariant representation, 196
cozero, 238
crossed product, 198
Csiszár–Morimoto $\mathfrak{f}$-distance, 61
cumulant function, 105
curve, 247
cyclic, 170
Daniell lattice, 234
Daniell system, 235
Daniell–Stone extension, 235
Daniell–Stone integral, 234
Dcb-algebra, 228
Dedekind–MacNeille complete, 226, 227
densely defined, 177
density operator, 58
dequantisation, 155
Dereziński–Jakšić–Pillet perturbation, 141
derivation, 193
dimension function, 188
discrete, 189
distance, 60
distributive, 226, 227
dominated, 168
downwards directed, 226
dual action, 198
dual coordinate system, 72
dual pair, 65
dual povm-instrument, 110
dual representation, 72
dual weight, 214
dualisable, 73
dualiser, 73
dually flat coordinates, 251
dually flat manifold, 251
Dye–Segal density, 209

effective domain, 65
Eguchi equations, 250
Eguchi geometry, 250
Eguchi manifold, 250
Eguchi system, 250
emergent, 156
entropic effect measure, 114
enveloping von Neumann algebra, 172
epigraph, 65
essentially Fréchet differentiable, 67
essentially Gâteaux differentiable, 67
essentially strictly convex, 67
estimable, 48
evaluation, 228
evolutional transition probability, 149
expansional, 141
exponential, 96
exponential coordinates, 105
exponential family, 105
exponential map, 249
extended liouvillean instrument, 145
extended liouvillean model, 145
extended liouvillean operator, 145
extended positive cone, 211
extremal point, 167
extremally disconnected, 238

f-algebra, 227
face, 172
factor, 172
factorial, 173
faithful, 167, 168, 170, 212–214
Fenchel dual, 65
Fenchel subdifferential, 65
Fenchel subgradients, 65
finite, 58, 168, 189, 228

finite coarse graining, 58
finite curve, 247
Fisher matrix, 94
Fisher–Rao–Jeffreys metric, 94
fixed point, 133, 165, 199
flat, 248, 253
flow of weights, 200
Fréchet, 60
Fréchet derivative, 66
Fréchet differentiable, 66
Fréchet–Legendre, 67

Gâteaux derivative, 66
Gâteaux differentiable, 66
gauge, 141
gauge geodesic propagation, 143
gauge geodesic propagation model, 143
Gel'fand ideal, 170
Gel'fand spectrum, 240
Gel'fand–Naĭmark–Segal bundle, 139
Gel'fand–Naĭmark–Segal representation, 171
generalised Alber functional, 71
generalised Bregman distance, 74
generalised Bregman functional, 73
generalised Bregman functional projection, 75
generalised Bregman projection, 76
generalised cosine equation, 69, 75
generalised Legendre transformation, 73
generalised orthogonal decomposition, 75
generalised pythagorean equation, 70, 76, 96, 254
generalised statistical Bregman distance, 76
generalised transition probability, 149
generated, 166, 173
generator, 193
Gibbs–Helmholtz equation, 106
Gibbs–Shannon entropy, 105
global charge, 147
global gauge, 141
global source strength, 144
grade, 221
greatest element, 226
greatest lower bound, 226
group homomorphism, 191

Haagerup–Terp spaces, 217
Hamilton equation, 137
Hamilton function, 135, 136
hamiltonian flow, 137
hamiltonian vector field, 135, 136
Hasegawa γ-distance, 80
Hausdorff, 238
Helstrom–Uhlmann metric, 101
hereditary, 166
hessian, 252
hessian manifold, 252
hessian structure, 252
Hilbert–Schmidt, 211
Hilbert–Schmidt space, 211
Hurwitz measure, 197
hyperstonean, 238

ideal entropic projection, 113
identity correspondence, 187
increasing, 212
indicator function, 70
infimum, 226
infinite, 189
information distance, 60
information inequality, 107
inner $*$-automorphisms, 190
integrable, 201, 214
invertible, 165
irreducible, 172
isometric, 228
isometric isomorphism, 165
isometry, 165

Jordan $*$-homomorphism, 165
Jordan $*$-isomorphism, 165

Kakutani–Hellinger distance, 62, 81
Kosaki–Petz $\mathfrak{f}$-distance, 63
Kubo transform, 99
Kubo–Martin–Schwinger condition, 176

lattice, 226
lattice homomorphism, 227
Lauritzen manifolds, 251
least element, 226
least upper bound, 226
Lebesgue measure, 197
left Haar measure, 197
left support, 169
Legendre, 67
Legendre transformation, 72
length of a finite curve, 247
length of a vector, 247
Levi-Civita, 248
Lie algebra, 135
Lie bracket, 135
Liese–Vajda γ-distances, 76
lineal, 182
linear, 167
local gauge, 141
local gauge liouvillean, 142
local gauge liouvillean model, 142
local gauge model, 141
local gauge quantum field models, 147
local gauge quantum fields, 147
local gauge structure, 141
local source liouvillean, 144
local source liouvillean model, 144
local source model, 144
local source operator, 144
local source strength, 144
local source term, 144
localisable, 228, 233
locally bounded, 66
locally compact, 192, 238
locally finite, 238

Maharam, 228, 233
majorises, 167, 168
Markov instrument, 110
Markov map, 59
markovian renormalisation semi-group, 133
Massieu–Planck functional, 105
maximal central, 173
Mazur map, 216
mcb-algebra, 228

meager, 238
measurable space, 232
measure, 228, 233
measure algebra, 228
measure preserving, 230
measure space, 233
metric tensor, 248
metric–affine geometry, 248
metric-compatible, 248
metrical, 60
MI-space, 227
mixed, 167
mixture coordinates, 105
modular, 179
modular algebra, 224
modular automorphisms, 179
modular conjugation, 178
modular function, 197
modular hamiltonian, 179
modular operator, 178
modular spectrum, 190
monotone, 95, 97
Morozova–Chencov function, 97
Morozova–Chencov–Petz metrics, 97
morphismo, 166
mutually dual foliations, 129

natural cone, 180
natural trace, 215
noncommutative flow of weights, 221
nondegenerate, 170
nonparametric quantum manifold, 94
nonparametric statistical manifold, 91
nonsingular, 208
nonzero, 170, 189
Norden–Sen dual, 249
Norden–Sen geometry, 249
Norden–Sen manifold, 249
norm continuous, 166
norm preserving, 165, 228
norm topology, 166, 192
normal, 165, 167, 168, 170, 190, 213, 214, 238
normal extension, 172
normal model, 108
normalised, 97, 167, 168
normalised quantum expectations, 58
normalised quantum model, 57
normalised state, 168
normalised states, 58
nowhere dense, 238
nuclear, 211

open, 239
operator concave, 63
operator convex, 63
operator monotone decreasing, 63
operator monotone increasing, 63
operator valued weight, 213
opposite algebra, 165
optimal decision, 112
optimal entropic projection, 112
optimal estimate, 112
order closed, 226
order continuous, 227
order preserving, 227
order unit, 227
order unit norm, 227
Orlicz space, 90
orthocomplementation, 32
orthocomplemented, 32
orthogonal, 75, 76, 150, 173, 174
Orthogonality, 32
orthogonality condition, 75
orthomodular, 32
ortochronous Poincaré group, 142

parallel transport, 248
parametric probabilistic manifold, 89
parametric quantum manifold, 92
parametric statistical manifold, 89
partial isometry, 165
partially ordered set, 226
partition function, 105
Pearson–Kagan χ^2-distance, 61
Pedersen–Takesaki density, 208
Petz function, 97
Plancherel formula, 199
Poincaré–Bloch–Wick rotation, 156
Poisson compatibility condition, 139
Poisson manifold, 135
Poisson map, 136
Poisson structure, 135
polar decomposition, 170, 178, 181
Pontryagin dual group, 198
Pontryagin duality map, 198
poset, 226
positive, 59, 165, 167, 178, 228
povm effects, 110
povm-instrument, 110
predual, 167
premeasurable space, 232
primary, 173
prior, 112
probabilistic expectation, 56
probabilistic model, 56
probabilistic state, 56
probability, 57
probability density, 56
probability manifold, 88
probability simplex, 58
projection, 165
projection band, 228
proper, 65, 227
properly infinite, 189
pseudosupported, 173
pure, 167
purely infinite, 189

quadrilateral equation, 69, 75
quantum γ-distance, 79
quantum γ-entropic instrument, 113
quantum n-form field, 147
quantum Bregman distance, 76
quantum Bregman functional, 76
quantum coarse graining, 60
quantum distance, 60
quantum entropic instrument, 113
quantum expectations, 57
quantum exponential family, 107
quantum field extended liouvillean model, 146

quantum field model, 146
quantum fields, 146
quantum information manifold, 88
quantum information model, 57
quantum information states, 57
quantum instrument, 109
quantum manifold, 88
quantum Markov map, 60
quantum model, 57
quantum operation, 109
quantum Schwarzschild space-time, 160
quantum space-time, 155
quantum state, 168
quantum states, 57
quasi-open, 239
quasi-stonean, 238

Radon, 238
Radon–Nikodým quotient, 236
Raggio transition probability, 149
rare, 238
reduced, 169
reducible, 172
registration parameter, 109
regularity conditions, 88
relative entropy, 60
relative free energy, 85
relative modular conjugation, 181
relative modular hamiltonian, 182
relative modular operator, 181
relative prior, 112
representation, 140, 170, 191
Ricci curvature scalar, 249
Rickart, 166, 238
Riemann–Christoffel curvature tensor, 248
riemannian distance, 247
riemannian geometry, 247
riemannian manifold, 247
riemannian metric, 247
riemannian quantum manifold, 94
riemannian statistical manifold, 94
Riesz, 226
Riesz dual, 227
Riesz homomorphism, 227
Riesz isomorphism, 228
Riesz–Schauder, 211
right $\mathcal{N}$ module, 187
right annihilator, 166
right Gâteaux derivative, 66
right Haar measure, 197
right support, 169

scalar potentials, 252
Schwarz, 63
Schwarz–Kadison inequality, 63
self-adjoint, 165, 167, 178
self-polar, 180
semi-direct product, 142
semi-finite, 168, 189, 212, 213, 228, 233
semi-spectral measure, 41
separable, 172
separating, 170
sequentially order continuous, 227
skew information, 102
skewness tensor, 250
source term, 131
space-time, 155
spectral measure, 38
spectral principle, 39
square root, 165
standard, 195
standard Bregman distance, 74
standard cone, 180
standard conjugation, 180
standard core, 202
standard form, 180
standard liouvillean, 195
standard representation, 180
standard unitary equivalence, 181
standard unitary transition, 182
standard vector representative, 180
state, 168
states, 57
statistical Bregman distance, 76
statistical density, 56
statistical distance, 60
statistical manifold, 88
statistical model, 56
statistical state, 56
stochastic matrix, 58
Stone, 238
Stone lattice, 234
Stone representation map, 228
Stone spectrum, 228
stonean, 238
strictly convex, 65
strictly positive, 228
strong topology, 192
strong-$\star$ topology, 168
strong-$\star\star$ topology, 168
strongly continuous, 176, 192
strongly disjoint, 173
subprojection property, 115
support, 169, 178
supported, 173
supremum, 226
symmetric, 60, 97, 98, 248

tangent bundle, 247
time dependent n-point correlation functions, 148
time independent n-point correlation function, 148
topological, 164, 192
torsion tensor, 248
torsion-free, 248
total variation distance, 62
totally convex, 70
totally disconnected, 238
trace, 168
trace class, 211
tracial, 167
transition amplitude, 149
transition distance, 154
transition probability, 149
triangle equality, 115
triangle inequality, 60
two sided annihilator, 166
type I, 189
type I_∞, 189
type I_n, 189
type II, 189

type II_1, 189
type II_∞, 189
type III, 189
type III_0, 190
type III_1, 190
type III_λ, 190

Umegaki distance, 80
unit preserving, 228
unital, 165
unitarily equivalent, 170
unitarily inequivalent, 170
unitary, 165
unitary implementation, 192
upwards directed, 226

von Neumann algebra, 172
von Neumann entropy, 106
von Neumann equation, 138
von Neumann–Schatten p-class, 211

Wald–Good–Kullback–Leibler distance, 77
weak-$\star$ topology, 167, 192
weakly-$\star$ continuous, 176, 192
weight, 168
Wigner–Yanase metric, 100
Wigner–Yanase–Dyson skew information, 101
Wigner–Yanase–Dyson–Hasegawa metric, 101

Young function, 90
Young–Fenchel inequality, 65
Yuen–Lax metric, 100

zero-dimensional, 238
zone consistent, 71