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H-points and Denting Points in Orlicz Spaces*

Abstract. H-points and denting points of the unit sphere in Orlicz spaces over nonatomic and purely atomic (counting) measure spaces are characterized. Some corollaries concerning the relevance of H-property and G-property in connection with MLUR-property in any Orlicz space are given.

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1. Preliminaries. For a Banach space X, we denote by $S(X)$ and $B(X)$ the unit sphere and unit ball of X, respectively. A point $x_0 \in S(X)$ is called

a) an extreme point if for every $x, y \in S(X)$ the equality $2x_0 = x + y$ implies $x = y$;

b) a strong extreme point if for any sequences $(x_n), (y_n) \subset X$ such that $\|x_n\| \to 1$, $\|y_n\| \to 1$ as $n \to \infty$ and $2x_0 = x_n + y_n$ $(n = 1, 2, \ldots)$, we have $\|x_n - y_n\| \to 0$ as $n \to \infty$;

c) an H-point if for any sequence $(x_n) \subset X$ such that $\|x_n\| \to 1$ as $n \to \infty$, the weak convergence of (x_n) to x_0 (write $x_n \xrightarrow{w} x_0$) implies that $\|x_n - x_0\| \to 0$ as $n \to \infty$;

d) a denting point if for every $\varepsilon > 0$ $x_0 \notin \overline{\text{conv}}\{B(X) \setminus (x_0 + \varepsilon B(X))\}$.

Characterizations of extreme points and strong extreme points in Orlicz spaces were obtained in [1], [2], [3], [4] and [9]. In this note we will characterize H-points and denting points of the unit sphere in Orlicz spaces over nonatomic finite and purely atomic measure space. The reader who is interested in a discussion of the relevance of denting points in connection with the Radon-Nikodym property (RNP) is referred to the monographs [2] and [7].

Let $\mathbb{R} = (-\infty, \infty)$ be the set of all real numbers, $\mathbb{N}$ the set of all natural numbers and m the set of all sequences. Further, let (G, Σ, μ) be a measure

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space with a non-negative, finite, atomless and complete measure defined on a σ-algebra Σ. We denote by $\mathcal{M}$ the set of all μ-equivalence classes of real-valued measurable functions defined on G.

A convex even function $M : \mathbb{R} \to [0,\infty)$ is called an $\mathcal{N}$-function iff $M(0) = 0$, $M \not\equiv 0$, $\frac{M(u)}{u} \to \infty$ as $u \to \infty$ and $\frac{M(u)}{u} \to 0$ as $u \to 0$.

For every $\mathcal{N}$-function M we define a complementary function $N : \mathbb{R} \to [0,\infty)$ by the formula $N(v) = \max_{u\geqslant 0}[u|v| - M(u)]$ for every $v \in \mathbb{R}$. The function N is also an $\mathcal{N}$-function.

We write $M \in \overline{\Delta}_2$ ($M \in \Delta_2$), whenever M satisfies the Δ_2-condition for large u (for small u) (cf. [11], p. 23). A real number u is said to be a point of strict convexity of M if for any $u_1, u_2 \in \mathbb{R}$, $u_1 \neq u_2$, the equality $u_1 + u_2 = 2u$ implies $M(u) < \frac{1}{2}(M(u_1) + M(u_2))$. Let S_M be the set of all points of strict convexity of M. We denote

$$S_M^+ = \{u \in S_M : \exists_{\varepsilon>0} M \text{ is a linear function on } [|u|, |u|+\varepsilon]\},$$
$$S_M^- = \{u \in S_M : \exists_{\varepsilon>0} M \text{ is a linear function on } [|u|-\varepsilon, |u|]\}$$

and $S_M^0 = S_M \setminus (S_M^+ \cup S_M^-)$.

Functionals

$$\varrho_M(x) = \sum_{i=1}^{\infty} M(x_i) \quad \text{for } x \in m$$

and

$$\overline{\varrho}_M(x) = \int_G M(x(t))\, d\mu \quad \text{for } x \in \mathcal{M}$$

are modulars on m and $\mathcal{M}$ respectively (cf. [14]). The space

$$l_M = \{x \in m : \varrho_M(kx) < \infty \text{ for some } k > 0\}$$

equipped with so called Luxemburg norm

$$\|x\|_{(M)} = \inf\{a > 0 : \varrho_M(a^{-1}x) \leqslant 1\}$$

or with the equivalent Orlicz norm (in Amemiya sense)

$$\|x\|_M = \inf_{k>0} \frac{1}{k}(1 + \varrho_M(kx))$$

is said to be an Orlicz sequence space. A subspace of finite elements $h_M \subset l_M$ is defined as the set of all $x \in m$ such that $\varrho_M(kx) < \infty$ for any $k > 0$. This subspace is equipped with the norm induced from l_M. To simplify denotations we put $l_M = (l_M, \|\cdot\|_M)$, $l_{(M)} = (l_M, \|\cdot\|_{(M)})$, $h_M = (h_M, \|\cdot\|_M)$ and $h_{(M)} = (h_M, \|\cdot\|_{(M)})$. Orlicz function spaces L_M and $L_{(M)}$ equipped with the norms $\|\cdot\|_M$ and $\|\cdot\|_{(M)}$, respectively and the space of finite elements E_M and $E_{(M)}$ are defined analogously (cf. [11]).

2. Results.

THEOREM 1. *Let $x_0 \in S(L_M)$. x_0 is an H-point if $M \in \overline{\Delta}_2$ and x_0 is an extreme point.*

Proof of necessity. Suppose $x_0 \in S(L_M)$ is an H-point. In virtue of the fact $\varrho_M(x_0) \leqslant \|x_0\|_M = 1$ there exists a number $C > 0$ such that the set $G_0 = \{t \in G : |x_0(t)| \leqslant C\}$ is of positive measure. Assume $M \notin \overline{\Delta}_2$. Then a monotonically increasing sequence of numbers u_n ($n = 1, 2, \ldots$), which tends to infinity, can be found such that $M(u_1) > \frac{1}{\mu(G_0)}$ and

$$M\left(\left(1 + \frac{1}{n}\right) u_n\right) > 2^n M(u_n) \quad (n = 1, 2, \ldots).$$

Take $G_n \subset G_0$ with

$$\mu(G_n) = \frac{1}{2^n M(u_n)} \quad (n = 1, 2, \ldots).$$

Define

$$x_n = x_n' + x_n'' \quad (n = 1, 2, \ldots),$$

where

$$x_n' = x_0 \chi_{G\setminus G_n} + \frac{1}{k_0} u_n \chi_{G_n}, \quad x_n'' = x_0 \chi_{G_n} \quad (n = 1, 2, \ldots)$$

and k_0 is a positive number such that $\|x_0\|_M = \frac{1}{k_0}(1 + \overline{\varrho}_M(k_0 x_0))$ (cf. [20] Th. 1.27 p. 46). Obviously,

$$\|x_n - x_n'\|_M = \|x_n''\|_M \leqslant C\|\chi_{G_n}\|_M \to 0 \quad \text{as } n \to \infty.$$

By the following inequalities

$$\|x_n'\|_M \geqslant \|x_0 \chi_{G\setminus G_n}\|_M \quad \text{and} \quad 1 = \|x_0\|_M \leqslant \|x_0\chi_{G\setminus G_n}\|_M + \|x_0 \chi_{G_n}\|_M$$

for $n = 1, 2, \ldots$, we can conclude that

$$\liminf_{n\to\infty} \|x_n'\|_M \geqslant 1.$$

But, in view of Theorem 10.5 from [11]

$$\|x_n'\|_M \leqslant \frac{1}{k_0}(1 + \overline{\varrho}_M(k_0 x_0 \chi_{G\setminus G_n})) + \frac{1}{k_0} M(u_n)\mu(G_n) \leqslant \|x_0\|_M + 2^{-n}\frac{1}{k_0},$$

so

$$\limsup_{n\to\infty} \|x_n'\|_M \leqslant 1.$$

Thus

$$\lim_{n\to\infty} \|x_n'\|_M = 1.$$

Therefore, taking into account the definition of the sequence (x_n), it is easy to notice that $\|x_n\|_M \to 1$ as $n \to \infty$. Now we will prove that $x_n \xrightarrow{w} x_0$. Every functional $f \in (L_M)^*$ is of the following form (see [1] or [15])

$$f = \Psi_y + \Phi,$$

where $y \in L_N$ and

$$\Psi_y(x) = \int_G x(t)y(t)\,d\mu \quad (\text{for every } x \in L_M),$$

and Φ denotes a singular functional, i.e. $\Phi(s) = 0$ for $x \in E_M$. Notice that $x_n - x_0 = \frac{1}{k_0} u_n \chi_{G_n} \in E_M$. Let $f \in (L_M)^*$ and let $d > 0$ be a number such that $\overline{\varrho}_N(dy) < \infty$. Using Young's inequality, we get

$$\begin{aligned} |f(x_n - x_0)| &\leqslant \Big|\int_G (x_n(t) - x_0(t))y(t)\,d\mu\Big| + |\Phi(x_n - x_0)| \\ &= \Big|\int_{G_n} \frac{u_n}{k_0} y(t)\,d\mu\Big| \leqslant \frac{1}{k_0}\Big(M(u_n)\mu(G_n) + \int_{G_n} N(dy(t))\,d\mu\Big) \to 0 \\ &\qquad \text{as } n \to \infty \end{aligned}$$

for any $f \in (L_M)^*$. Thus $x_n \xrightarrow{w} x_0$.

On the other hand, for any m and $n \geqslant m$ we have

$$\begin{aligned} \overline{\varrho}_M\left(\left(1 + \frac{1}{m}\right)k_0(x_n - x_0)\right) &= M\left(\left(1 + \frac{1}{m}\right)u_n\right)\mu(G_n) \\ &\geqslant M\left(\left(1 + \frac{1}{n}\right)u_n\right)\mu(G_n) > 2^n M(u_m)\frac{1}{2^n M(u_n)} = 1. \end{aligned}$$

Hence

$$\|x_n - x_0\|_M \geqslant \|x_n - x_0\|_{(M)} \geqslant \frac{1}{k_0}\left(1 + \frac{1}{m}\right)^{-1} \quad (n \geqslant m),$$

and, in virtue of the fact that m is arbitrary,

$$\liminf_{n \to \infty} \|x_n - x_0\|_M \geqslant \frac{1}{k_0}.$$

But this contradicts the fact that x_0 is a H-point. Thus, $M \in \overline{\Delta}_2$.

Now assume that the H-point x_0 is not an extreme point. Then $\mu(\{t \in G : k_0 x_0(t) \in \mathbb{R} \setminus S_M\}) > 0$ (cf. [1], [3] or [9]). Consequently, there exists at least one interval (a, b) on which $M(u) = cu + d$ and $\mu(\{t \in G : k_0 x_0(t) \in (a, b)\}) > 0$. Choose $\delta > 0$ such that the measure of the set $\tilde{E} = \{t \in G : k_0 x_0(t) \in [a + \delta, b - \delta]\}$ is positive.

Repeating the same argumentation as in the proof of Lemma 4 from [5], two sequences of subsets (E'_n) and (E''_n) can be found such that $E'_n \cap E''_n = \emptyset$,

$E'_n \cup E''_n = \tilde{E}$, $\mu(E'_n) = \mu(E''_n)$ $(n = 1, 2, \ldots)$ and for any $y \in L_N$, we have

$$\lim_{n \to \infty} \Big(\int_{E'_n} y(t)\,d\mu - \int_{E''_n} y(t)\,d\mu\Big) = 0.$$

Define

$$x_n(t) = x_0(t)\chi_{G \setminus \tilde{E}}(t) + \left(x_0(t) + \frac{\delta}{k_0}\right)\chi_{E'_n}(t) + \left(x_0(t) - \frac{\delta}{k_0}\right)\chi_{E''_n}(t),$$

$$x'_n(t) = x_0(t)\chi_{G \setminus \tilde{E}}(t) + \left(x_0(t) - \frac{\delta}{k_0}\right)\chi_{E'_n}(t) + \left(x_0(t) + \frac{\delta}{k_0}\right)\chi_{E''_n}(t)$$

$(n = 1, 2, \ldots)$. For each $n \in N$, we have

$$\begin{aligned} \|x_n\|_M &\leqslant \frac{1}{k_0}(1 + \overline{\varrho}_M(k_0 x_n)) \\ &= \frac{1}{k_0}\Big(1 + \int_{G \setminus \tilde{E}} M(k_0 x_0(t))\,d\mu + \int_{E'_n} M(k_0 x_0(t) + \delta)\,d\mu \\ &\qquad + \int_{E''_n} M(k_0 x_0(t) - \delta)\,d\mu\Big) \\ &= \frac{1}{k_0}\Big(1 + \int_{G \setminus \tilde{E}} M(k_0 x_0(t))\,d\mu + \int_{E'_n} [c(k_0 x_0(t) + \delta) + d]\,d\mu \\ &\qquad + \int_{E''_n} [c(k_0 x_0(t) - \delta) + d]d\mu\Big) \\ &= \frac{1}{k_0}\Big(1 + \int_{G \setminus \tilde{E}} M(k_0 x_0(t))\,d\mu + \int_{\tilde{E}} (c k_0 x_0(t) + d)\,d\mu\Big) \\ &= \frac{1}{k_0}(1 + \overline{\varrho}_M(k_0 x_0)) = \|x_0\|_M = 1. \end{aligned}$$

Similarly, $\|x'_n\|_M \leqslant 1$. Moreover,

$$2 = \|2x_0\|_M = \|x_n + x'_n\|_M \leqslant \|x_n\|_M + \|x'_n\|_M \leqslant 2.$$

Therefore,

$$\|x_n\|_M = 1 \quad (n = 1, 2, \ldots).$$

By the previous part of the proof $M \in \overline{\Delta}_2$, so $(L_M)^* = L_{(N)}$. Then to every $f \in (L_M)^*$ there corresponds in one-to-one fashion a function $y \in L_{(N)}$ and we have

$$f(x_n - x_0) = \int_G (x_n(t) - x_0(t))y(t)\,d\mu = \frac{2\delta}{k_0}\Big(\int_{E'_n} y(t)\,d\mu - \int_{E''_n} y(t)\,d\mu\Big) \to 0,$$

i.e. $x_n \xrightarrow{w} x_0$. But
$$\|x_n - x_0\|_M = \frac{2\delta}{k_0}\|\chi_{\bar{E}}\|_M > 0,$$
so x_0 cannot be a H-point. This contradiction completes the proof of necessity.

P r o o f o f s u f f i c i e n c y. Suppose that x_0 is an extreme point and $M \in \overline{\Delta}_2$. Let (x_n) be a sequence of functions such that $x_n \in L_M$ $(n = 1, 2, \ldots)$, $\|x_n\|_M \to 1$ as $n \to \infty$ and $x_n \xrightarrow{w} x_0$. Without loss of generality, we can assume that for every $n \in \mathrm{N}$ $\|x_n\|_M = 1$. Let (k_n) be a sequence of positive numbers such that
$$\|x_n\|_M = \frac{1}{k_n}(1 + \overline{\varrho}_M(k_n x_n)) \quad (n = 0, 1, \ldots).$$
First we will prove the following statements:

(1) $$\overline{k} = \sup_{n\in\mathrm{N}} k_n < \infty;$$

(2) $$\lim_{e\to\infty} \sup_{n\in\mathrm{N}} \mu(\{t \in G : |k_n x_n(t)| > e\}) = 0;$$

(3) $$\lim_{\mu(D)\to\infty} \sup_{n\in\mathrm{N}} \overline{\varrho}_M(k_n x_n \chi_D) = 0.$$

Suppose that $\sup_{n\in\mathrm{N}} k_n = \infty$. Then there exists a subsequence (k_{n_i}) such that $\lim_{i\to\infty} k_{n_i} = \infty$. Taking into account that
$$\lim_{u\to\infty}\frac{M(u)}{u} = \infty \quad \text{and} \quad 1 = \|x_{n_i}\|_M > \frac{1}{k_{n_i}}\overline{\varrho}_M(k_{n_i} x_{n_i}),$$
we can conclude that the subsequence (x_{n_i}) is convergent to 0 in measure ($x_{n_i} \xrightarrow{\mu} 0$). Hence, by Theorem 14.6 from [11], (x_{n_i}) is E_N-weakly convergent to 0 ($x_{n_i} \xrightarrow{E_N} 0$), so $x_{n_i} \xrightarrow{w} 0$. This contradicts to the assumption $x_n \xrightarrow{w} x_0 \neq 0$. Thus (1) is true.

Further, denoting
$$G_n^e = \{t \in G : |k_n x_n(t)| > e\},$$
we have
$$1 > \frac{1}{k_n}\overline{\varrho}_M(k_n x_n) \geqslant \frac{1}{k_n}\int_{G_n^e} M(k_n x_n(t))\, d\mu \geqslant \overline{k}^{-1} M(e)\mu(G_n^e).$$
Hence
$$\mu(G_n^e) < \frac{\overline{k}}{M(e)} \quad (n = 1, 2, \ldots)$$
and we obtain (2) in an obvious manner.

Now, suppose that (3) is false. Then there exist a $\delta > 0$ and sets $D_n \subset G$ $(n = 1, 2, \ldots)$ such that $\mu(D_n) < 2^{-n}$ and $\overline{\varrho}_M(k_n x_n \chi_{D_n}) \geqslant \delta$. Fix a positive integer m so large that for every $E \subset G$ with $\mu(E) > \mu(G) - 2^{-m}$ we have
$$\|x_0\chi_E\|_M \geqslant \|x_0\|_M - \frac{\delta}{2\overline{k}} = 1 - \frac{\delta}{2\overline{k}}.$$
In particular, putting $E = G \setminus \bigcup_{n=m+1}^{\infty} D_n$, we obtain $\|x_0\chi_E\|_M > 1 - \frac{\delta}{2\overline{k}}$. Therefore, for $n > m$, we get
$$1 = \|x_n\|_M = \frac{1}{k_n}[1 + \overline{\varrho}_M(k_n x_n \chi_E) + \overline{\varrho}_M(k_n x_n \chi_{\bigcup_{n=m+1}^{\infty} D_n})]$$
$$\geqslant \|x_n\chi_E\|_M + \frac{1}{\overline{k}}\overline{\varrho}_M(k_n x_n \chi_{D_n}) \geqslant \|x_n\chi_E\|_M + \frac{\delta}{\overline{k}},$$
and so, by the weak convergence of $(x_n\chi_E)$ to $x_0\chi_E$,
$$1 \geqslant \underline{\lim}_{n\to\infty}\|x_n\chi_E\|_M + \frac{\delta}{\overline{k}} \geqslant \|x_0\chi_E\|_M + \frac{\delta}{\overline{k}} \geqslant 1 + \frac{\delta}{2\overline{k}}$$
(cf. e.g. [23], Th. 1 ii), p. 120). This contradiction proves (3).

Denote $G^0 = \{t \in G : k_0 x_0(t) \in S_M^0\}$, $G^+ = \{t \in G : k_0 x_0(t) \in S_M^+\}$ and $G^- = \{t \in G : k_0 x_0(t) \in S_M^-\}$. Since x_0 is an extreme point, $k_0 x_0(t) \in S_M$ for almost every $t \in G$ (cf. e.g. [3], th. 6). Hence $\mu(G) = \mu(G^0 \cup G^+ \cup G^-)$.

To prove $\|x_n - x_0\|_M \to 0$ as $n \to \infty$, by [22], it is enough to show

(4) $$x_n - x_0 \xrightarrow{\mu} 0 \quad \text{on } G = G^0 \cup (G^+ \setminus G^-) \cup (G^- \setminus G^+) \cup (G^+ \cap G^-).$$

The proof of (4) requires four steps.

I. We will show that

(5) $$k_n x_n - k_0 x_0 \xrightarrow{\mu} 0 \quad \text{on } G_0.$$

Suppose (5) does not hold. Then there exist positive real numbers ε and σ such that
$$\mu(\{t \in G^0 : |k_n x_n(t) - k_0 x_0(t)| \geqslant \varepsilon\}) > \sigma \quad (n = 1, 2, \ldots)$$
Fix $e > 0$ satisfying $\mu(\{t \in G : |k_n x_n(t)| > e\}) > \frac{\sigma}{3}$ $(n = 0, 1, 2, \ldots)$. Denoting for $n = 1, 2, \ldots$
$$F_n = \{t \in G^0 : |k_n x_n(t) - k_0 x_0(t)| \geqslant \varepsilon,\ |k_n x_n(t)| \leqslant e,\ |k_0 x_0(t)| \leqslant e\},$$
it is easy to verify that $\mu(F_n) > \frac{\sigma}{3}$ $(n = 1, 2, \ldots)$. Since $k_0 x_0(t) \in S_M^0$,
$$0 < \frac{1}{1+k} \leqslant \frac{k_0}{k_0 + k_n} \quad \text{and} \quad \frac{k_n}{k_0 + k_n} \leqslant \frac{\overline{k}}{1 + \overline{k}} < 1,$$
there exists a $\delta \in (0, 1)$ such that
$$M\left(\frac{k_0 k_n}{k_0 + k_n}(x_0(t) + x_n(t))\right)$$
$$\leqslant (1 - \delta)\left[\frac{k_n}{k_0 + k_n}M(k_0 x_0(t)) + \frac{k_0}{k_0 + k_n}M(k_n x_n(t))\right]$$

for $t \in F_n$ $(n = 1, 2, \ldots)$. Hence, by the inequality $\max\{|k_0x_0(t)|, |k_nx_n(t)|\} \geqslant \frac{\varepsilon}{2}$ for $t \in F_n$, we have

$$\begin{aligned}
2 - \|x_0 - x_n\|_M &\geqslant \frac{1}{k_0}(1 + \overline{\varrho}_M(k_0x_0)) + \frac{1}{k_n}(1 + \overline{\varrho}_M(k_nx_n)) \\
&\quad - \frac{k_0 + k_n}{k_0k_n}\left[1 + \overline{\varrho}_M\left(\frac{k_0k_n}{k_0 + k_n}(x_0 + x_n)\right)\right] \\
&\geqslant \frac{k_0 + k_n}{k_0k_n}\left[\frac{k_n}{k_0 + k_n}\overline{\varrho}_M(k_0x_0)\right. \\
&\quad \left. + \frac{k_0}{k_0 + k_n}\overline{\varrho}_M(k_nx_n) - \overline{\varrho}_M\left(\frac{k_0k_n}{k_0 + k_n}(x_0 + x_n)\right)\right] \\
&\geqslant \frac{k_0 + k_n}{k_0k_n}\int_{F_n}\left[\frac{k_n}{k_0 + k_n}M(k_0x_0(t))\right. \\
&\quad \left. + \frac{k_0}{k_0 + k_n}M(k_nx_n(t)) - M\left(\frac{k_0k_n}{k_0 + k_n}(x_0(t) + x_n(t))\right)\right] d\mu \\
&\geqslant \frac{k_0 + k_n}{k_0k_n}\delta\int_{F_n}\left[\frac{k_n}{k_0 + k_n}M(k_0x_0(t)) + \frac{k_0}{k_0 + k_n}M(k_nx_n(t))\right] d\mu \\
&\geqslant \frac{\delta}{k}\int_{F_n} M\left(\frac{\varepsilon}{2}\right) d\mu \geqslant \frac{\delta}{k}M\left(\frac{\varepsilon}{2}\right)\frac{\sigma}{3}
\end{aligned}$$

and so $\|x_0 + x_n\|_M \not\to 2$. On the other hand $x_n - x_0 \xrightarrow{w} 0$ implies $\|x_0 + x_n\|_M \to 2$ as $n \to \infty$. This contradiction finishes the proof of I.

II. We will prove two following facts:

$$\lim_{n\to\infty} k_n = k_0, \tag{6}$$

$$x_n \xrightarrow{\mu} x_0 \quad \text{on } G^0. \tag{7}$$

Observe first that $x_n - x_0 \xrightarrow{E_N(G^0)} 0$, where $E_N(G^0) = \{y\chi_{G^0} : y \in E_N\}$. Moreover, by the step I and Theorem 14.6 from [11] $k_nx_n - k_0x_0 \xrightarrow{E_N(G^0)} 0$. Hence

$$(k_n - k_0)x_0 = (k_nx_n - k_0x_0) - k_n(x_n - x_0) \xrightarrow{E_N(G^0)} 0.$$

If $\mu(\{t \in G^0 : x_0(t) = 0\}) < \mu(G^0)$, then (6) is satisfied in an obvious manner and (7) is an immediate consequence of (5) and (6).

If $\mu(\{t \in G^0 : x_0(t) = 0\}) = \mu(G^0)$, then $x_n \xrightarrow{\mu} x_0 = 0$ on G^0 by (5), i.e. (7) is satisfied. Now, we have to prove (6) in this case. Obviously, the set $S_M^+ \cup S_M^-$ is at most countable. We may assume that there exists a sequence $(r_i) \subset S_M^+ \cup S_M^-$ such that $G_i = \{t \in G : k_0x_0(t) = r_i\}$, $\mu(G_i) > 0$ $(i = 1, 2, \ldots)$ and $\mu(G \setminus G^0) = \mu(\bigcup_{i=1}^\infty G_i)$. Since $x_n \to 0$ on G^0, $\overline{\varrho}_M(k_nx_n\chi_{G^0}) \to 0$ as $n \to \infty$ by (3). Hence

$$\frac{1}{k_0}\Big(1 + \sum_{i=1}^\infty M(r_i)\mu(G_i)\Big) = \|x_0\|_M = 1$$

and

$$\frac{1}{k_n}\Big(1 + \sum_{i=1}^\infty \overline{\varrho}_M(k_nx_n\chi_{G_i})\Big) \to 1 \quad \text{as } n \to \infty.$$

By (1) the sequence (k_n) is bounded. Without loss of generality we can assume that $\lim_{n\to\infty} k_n = k_0'$. Since $M \in \overline{\Delta}_2$, for any $\varepsilon > 0$ a natural number i_0 can be found that

$$\sum_{i=i_0+1}^\infty \frac{1}{k_0}M\left(\frac{k_0'}{k_0}r_i\right)\mu(G_i < \varepsilon.$$

Moreover

$$\lim_{n\to\infty}\int_{G_i} x_n(t)\, d\mu = \int_{G_i} x_0(t)\, d\mu = \frac{r_i}{k_0}\mu(G_i) \quad (i = 1, 2, \ldots).$$

Thus

$$\begin{aligned}
1 = \|x_0\|_M &\leqslant \frac{1}{k_0'}(1 + \overline{\varrho}_M(k_0'x_0)) = \frac{1}{k_0'}\left[1 + \sum_{i=1}^\infty M\Big(\frac{k_0'}{k_0}r_i\Big)\mu(G_i)\right] \\
&\leqslant \frac{1}{k_0'}\left[1 + \sum_{i=1}^{i_0} M\Big(\frac{k_0'}{k_0}r_i\Big)\mu(G_i)\right] + \varepsilon \\
&\leqslant \frac{1}{k_n}\left[1 + \sum_{i=1}^{i_0} M\Big(k_n\frac{1}{\mu(G_i)}\int_{G_i} x_n(t)\, d\mu\Big)\mu(G_i)\right] + 2\varepsilon \\
&\leqslant \frac{1}{k_n}\left[1 + \sum_{i=1}^{i_0}\int_{G_i} M(k_nx_n(t))\, d\mu\right] + 2\varepsilon \\
&\leqslant \frac{1}{k_n}\left[1 + \sum_{i=1}^{i_0}\overline{\varrho}_M(k_nx_n\chi_{G_i})\right] + 2\varepsilon \leqslant 1 + 3\varepsilon,
\end{aligned}$$

for sufficiently large n. Hence $\frac{1}{k_0}(1 + \overline{\varrho}_M(k_0'x_0)) = 1$, because ε is arbitrary. Thus $k_0 = k_0'$. This completes the proof of (6).

III. We will show here that

$$x_n \xrightarrow{\mu} x_0 \quad \text{on } (G^- \setminus G^+) \cup (G^+ \setminus G^-). \tag{8}$$

Suppose $S_M^- \setminus S_M^+ = \{r_1, r_2, \ldots\}$ and denote $G_i = \{t \in G : k_0x_0(t) = r_i\}$

$(i=1,2,\ldots)$. To prove (8) first we will show

$$\int_{G_i(x_n\geqslant x_0)} (x_n(t)-x_0(t))\,d\mu\to 0 \quad\text{as } n\to\infty \quad (i=1,2,\ldots), \tag{9}$$

where $G_i(x_n\geqslant x_0)=\{t\in G_i: x_n(t)\geqslant x_0(t)\}$.

To verify (9), suppose, to the contrary, that there are $j\in\mathrm{N}$ and $\delta>0$ such that

$$\int_{G_j(x_n\geqslant x_0)} (x_n(t)-x_0(t))\,d\mu\geqslant\delta \quad (n=1,2,\ldots).$$

Since, by (2) and (3),

$$\int_{G_j(x_n\geqslant x_0,x_n>e)} (x_n(t)-x_0(t))\,d\mu\leqslant \int_{G(x_n>e)}|x_n(t)|\,d\mu$$

$$\leqslant\int_{G(x_n>e)} M(k_nx_n(t))\,d\mu\to 0 \quad\text{as } e\to\infty,$$

a number $e>0$ can be chosen such that

$$\int_{G_j(e\geqslant x_n\geqslant x_0)} (x_n(t)-x_0(t))\,d\mu\geqslant\frac{\delta}{2} \quad (n=1,2,\ldots),$$

where sets $G_j(x_n\geqslant x_0, x_n>e)$, $G(x_n>e)$, $G_j(e\geqslant x_n\geqslant x_0)$ are defined analogously as $G_i(x_n\geqslant x_0)$. Consequently, there exist positive real numbers ε' and σ' such that

$$\mu(\{t\in G_j: e\geqslant x_n(t), x_n(t)-x_0(t)\geqslant\varepsilon'\})\geqslant\sigma' \quad (n=1,2,\ldots).$$

Hence, by the convergence of the sequence (k_n) to k_0, a natural number n_0 can be found such that $\mu(F_n)\geqslant\frac{\sigma'}{2}$ for $n\geqslant n_0$, where $F_n=\{t\in G_j: ek_0\geqslant k_nx_n(t), k_nx_n(t)-k_0x_0(t)\geqslant\varepsilon'\}$. Observe that $k_nx_n(t)$ and $k_0x_0(t)$ belong to the set S_M for $t\in F_n$ and $n\geqslant n_0$. Hence there exists $\eta'\in(0,1)$ such that

$$M\left(\frac{k_0k_n}{k_0+k_n}(x_n(t)+x_0(t))\right)$$
$$\leqslant(1-\eta')\left(\frac{k_n}{k_0+k_n}M(k_0x_0(t))+\frac{k_0}{k_0+k_n}M(k_nx_n(t))\right)$$

for $t\in F_n$ and $n\geqslant n_0$. Now, repeating the argumentation from the proof of the step I, we conclude that $\|x_0+x_n\|_M\not\to 2$. This contradiction finishes the proof of (9).

Since

$$\int_{G_i}(x_n(t)-x_0(t))\,d\mu\to 0 \quad\text{as } n\to\infty \quad (i=1,2,\ldots),$$

it follows, by (9), that

$$\int_{G_i(x_n<x_0)} (x_0(t)-x_n(t))\,d\mu\to 0 \quad\text{as } n\to\infty \quad (i=1,2,\ldots).$$

Hence, we conclude

$$\int_{G_i}|x_n(t)-x_0(t)|\,d\mu\to 0 \quad\text{as } n\to\infty \quad (i=1,2,\ldots).$$

Consequently, $x_n\xrightarrow{\mu}x_0$ on G_i $(i=1,2,\ldots)$. Since $\lim_{i_0\to\infty}\mu(\bigcup_{i=i_0+1}^{\infty}G_i)=0$, we may deduce that $x_n\xrightarrow{\mu}x_0$ on whole $(G^-\setminus G^+)$. In a similar manner, we can obtain that $x_n\to x_0$ on $(G^+\setminus G^-)$. Thus (8) is proved.

IV. Finally, we will prove

$$x_n\xrightarrow{\mu}x_0 \quad\text{on } G^+\cap G^-. \tag{10}$$

We have

$$|\overline{\varrho}_M(k_0x_n)-\overline{\varrho}_M(k_0x_0)|\leqslant|\overline{\varrho}_M(k_0x_n)-\overline{\varrho}_M(k_nx_n)|+|\overline{\varrho}_M(k_nx_n)-\overline{\varrho}_M(k_0x_0)|.$$

The right hand side of this inequality tends to 0 as $n\to\infty$ because $k_n\to k_0$ as $n\to\infty$ and $M\in\overline{\Delta}_2$. Thus

$$\overline{\varrho}_M(k_0x_n)\to\overline{\varrho}_M(k_0x_0) \quad\text{as } n\to\infty. \tag{11}$$

On the other hand, the previous part of the proof implies that $x_n\to x_0$ on $G\setminus(G^+\cap G^-)$. Hence

$$\overline{\varrho}_M(k_0x_n\chi_{G\setminus(G^+\cap G^-)})\to\overline{\varrho}_M(k_0x_0\chi_{G\setminus(G^+\cap G^-)}) \quad\text{as } n\to\infty$$

and so, by (11)

$$\overline{\varrho}_M(k_0x_n\chi_{G^+\cap G^-})\to\overline{\varrho}_M(k_0x_0\chi_{G^+\cap G^-}) \quad\text{as } n\to\infty.$$

Therefore, denoting $S_M^+\cap S_M^-=\{s_1,s_2,\ldots\}$ and $D_i=\{t\in G: k_0x_0(t)=s_i\}$ $(i=1,2,\ldots)$, we have

$$\sum_{i=1}^{\infty}\int_{D_i}M(k_0x_n(t))\,d\mu\to\sum_{i=1}^{\infty}\int_{D_i}M(k_0x_0(t))\,d\mu=\sum_{i=1}^{\infty}M(s_i)\mu(D_i)$$
$$\text{as } n\to\infty,$$

i.e.

$$\sum_{i=1}^{\infty}\int_{D_i(x_n\geqslant x_0)}[M(k_0x_n(t))-M(k_0x_0(t))]\,d\mu$$
$$-\int_{D_i(x_n<x_0)}[M(k_0x_0(t))-M(k_0x_n(t))]\,d\mu\to 0 \quad\text{as } n\to\infty. \tag{12}$$

Suppose $[s_i', s_i]$ and $[s_i, s_i'']$ are two intervals on which the function M is linear, i.e.

$$M(u) = \begin{cases} A_i'u + B_i' & \text{for } u \in [s_i', s_i] \\ A_i''u + B_i'' & \text{for } u \in [s_i, s_i''] \end{cases}$$

Obviously, $A_i' > A_i''$ $(i = 1, 2, \ldots)$.

Hereinafter, we will show that

$$(13)\quad \sum_{i=1}^{\infty} \int_{D_i(x_n \geqslant x_0)} [M(k_0 x_n(t)) - (A_i'' k_0 x_n(t) + B_i'')]\, d\mu \to 0 \quad \text{as } n \to \infty.$$

To this end, fix $\varepsilon > 0$. Since $\mu(\bigcup_{i=j+1}^{\infty} D_i) \to 0$ as $j \to \infty$, by (3) there exists $i_0 \in \mathbb{N}$ such that

$$(14)\quad \Big| \sum_{i=i_0+1}^{\infty} \int_{D_i(x_n \geqslant x_0)} [M(k_0 x_n(t)) - (A_i'' k_0 x_n(t) + B_i'')]\, d\mu \Big| < \varepsilon \qquad (n = 1, 2, \ldots).$$

Further, for $1 \leqslant i \leqslant i_0$ we have

$$(15)\quad \int_{D_i(\frac{s_i+\delta}{k_0} \geqslant x_n \geqslant x_0)} [M(k_0 x_n(t)) - (A_i'' k_0 x_n(t) + B_i'')]\, d\mu$$
$$= \int_{D_i(x_i''+\delta \geqslant k_0 x_n \geqslant s_i'')} M(k_0 x_n(t)) - (A_i'' x_n(t) + B_i'')]\, d\mu$$
$$\leqslant (M(s_i'' + \delta) - M(s_i''))\mu(G) \leqslant \frac{\varepsilon}{i_0}$$

$(i = 1, 2, \ldots)$ for sufficiently small $\delta > 0$.

Notice that $\lim_{n\to\infty} \mu(\{t \in D_i : k_n x_n(t) s_i'' + \delta\}) = 0$ (otherwise, repeating the argumentation from I, we obtain that $\|x_0 - x_n\|_M \not\to 2$, i.e. a contradiction). Therefore, by (3), we get

$$(16)\quad \Big| \int_{D_i(\frac{s_i+\delta}{k_0} \geqslant x_n \geqslant x_0)} M(k_0 x_n(t))\, d\mu - \int_{D_i(x_n \geqslant x_0)} M(k_0 x_n(t))\, d\mu \Big| < \varepsilon \qquad (i = 1, 2, \ldots, i_0;\ n \geqslant n_0)$$

and

$$(17)\quad \Big| \int_{D_i(\frac{s_i+\delta}{k_0} \geqslant x_n \geqslant x_0)} (A_i'' k_0 x_n(t) + B_i'')\, d\mu - \int_{D_i(x_n \geqslant x_0)} (A_i'' k_0 x_n(t) + B_i'')\, d\mu \Big| < \frac{\varepsilon}{i_0} \quad (i = 1, 2, \ldots, i_0;\ n \geqslant n_0)$$

Combining (15), (16) and (17), we have

$$\Big| \int_{D_i(x_n \geqslant x_0)} [M(k_0 x_n(t)) - (A_i'' k_0 x_n(t) + B_i'')]\, d\mu \Big| < \frac{3\varepsilon}{i_0} \qquad (i = 1, 2, \ldots, i_0;\ n \geqslant n_0).$$

Consequently,

$$(18)\quad \Big| \sum_{i=1}^{i_0} \int_{D_i(x_n \geqslant x_0)} [M(k_0 x_n(t)) - (A_i'' k_0 x_n(t) + B_i'')]\, d\mu \Big| < 3\varepsilon \quad (n \geqslant n_0).$$

Taking into account (18) and (14), we conclude (13). Similarly, we may obtain

$$(19)\quad \sum_{i=1}^{\infty} \int_{D_i(x_n < x_0)} [M(k_0 x_n(t)) - (A_i' k_0 x_n(t) + B_i')]\, d\mu \to 0 \quad \text{as } n \to \infty.$$

From (19), (13) and (12), it follows that

$$(20)\quad \sum_{i=1}^{\infty} [A_i'' \int_{D_i(x_n \geqslant x_0)} (x_n(t) - x_0(t))\, d\mu - A_i' \int_{D_i(x_n < x_0)} (x_0(t) - x_n(t))\, d\mu] \to 0 \quad \text{as } n \to \infty.$$

Since $x_n \xrightarrow{w} x_0$, it is easy to notice that

$$\lim_{n\to\infty} \int_{D_i(x_n \geqslant x_0)} (x_n(t) - x_0(t))\, d\mu = \lim_{n\to\infty} \int_{D_i(x_n < x_0)} (x_0(t) - x_n(t))\, d\mu = \theta_i \geqslant 0$$

for every $i \in \mathbb{N}$. Obviously, by (20), θ_i $(i = 1, 2, \ldots)$ cannot be positive because $A_i'' > A_i'$ $(i = 1, 2, \ldots)$. Therefore

$$\int_{D_i} |x_n(t) - x_0(t)|\, d\mu \to 0 \quad \text{as } n \to \infty \quad (i = 1, 2, \ldots),$$

i.e. $x_n \xrightarrow{\mu} x_0$ on D_i $(i = 1, 2, \ldots)$. Hence, noticing that $\mu(\bigcup_{i=i_0+1}^{\infty} D_i) \to 0$ as $i_0 \to \infty$, we have $x_n - x_0 \xrightarrow{\mu} 0$ on $\bigcup_{i=1}^{\infty} D_i = G^+ \cap G^-$. This finishes the proof of (10).

Combining (7), (8) and (10), we obtain immediately that $x_n - x_0 \xrightarrow{\mu} 0$ on whole G. Thus the proof of the theorem is complete.

THEOREM 2. *Let $x_0 \in S(L_{(M)})$. x_0 is an H-point iff $M \in \overline{\Delta}_2$ and x_0 is an extreme point.*

The proof of Theorem 2 is similar to the proof of Theorem 1, so it is omitted here.

THEOREM 3. *Let $x_0 \in S(l_{(M)})$. x_0 is an H-point iff $M \in \overline{\Delta}_2$.*

P r o o f o f s u f f i c i e n c y. It is obvious by [21].

P r o o f o f n e c e s s i t y. Suppose that $x^0 = (x_1^0, x_2^0, \ldots) \in S(l_{(M)})$ is an H-point. Select a subsequence $(t_1, t_2, \ldots)$ of the sequence x_0 such that $(t_1, t_2, \ldots) \in h_M$. Denote by $(s_1, s_2, \ldots)$ the remaining part of sequence x_0. Write for convenience the sequence x_0 in the following form

$$x_0 = (t_1, t_2, \ldots;\ s_1, s_2, \ldots).$$

Assume $M \notin \overline{\Delta}_2$. Then there exists a sequence $u_n \downarrow 0$ such that $M(u_n) < \frac{1}{2^{n+1}}$ and

$$M\left(\left(1+\frac{1}{n}\right)u_n\right) > 2^{n+1}M(u_n) \quad (n = 1, 2, \ldots).$$

Choose a positive integer m_n satisfying

$$\frac{1}{2^{n+1}} \leqslant m_n M(u_n) < \frac{1}{2^n} \quad (n = 1, 2, \ldots).$$

Define

$$x_n = (t_1, \ldots, t_n, t_{n+1} + u_n, \ldots, t_{n+m_n} + u_n, t_{n+m_n+1}, \ldots;\ s_1, s_2, \ldots)$$

$(n = 1, 2, \ldots)$. Obviously, the element x_n $(n = 1, 2, \ldots)$ can be written in the form $x_n = x_n' + x_n''$, where

$$x_n' = (t_1, \ldots, t_n, u_n, t_{n+m_n+1}, \ldots;\ s_1, s_2, \ldots),$$
$$x_n'' = (0, \ldots, 0, t_{n+1}, \ldots, t_{n+m_n}, 0, \ldots; 0, 0, \ldots) \quad (n = 1, 2, \ldots).$$

Since $(t_1, t_2, \ldots) \in h_{(M)}$, we conclude that $\|x_n''\|_{(M)} \to 0$ as $n \to \infty$. Hence $\|x_n - x_n'\|_{(M)} \to 0$ as $n \to \infty$. Moreover,

$$\|x_n'\|_{(M)} \geqslant \|(t_1, \ldots, t_n, 0, \ldots, 0, t_{n+m_n+1}, \ldots;\ s_1, s_2, \ldots)\|_{(M)} = \|x_0 - x_n''\|_{(M)},$$

so $\liminf_{n\to\infty} \|x_n'\|_{(M)} \geqslant \|x_0\|_{(M)} = 1$. On the other hand

$$\varrho_M(x_n') \leqslant \varrho_M(x_0) + m_n M(u_n) \leqslant 1 + 2^{-n},$$

i.e. $\|x_n'\|_{(M)} \leqslant 1 + 2^{-n}$. Hence $\limsup_{n\to\infty} \|x_n'\|_{(M)} \leqslant 1$. Therefore, $\lim_{n\to\infty} \|x_n'\|_{(M)} = 1$. Now, it is easy to notice that $\lim_{n\to\infty} \|x_n\|_{(M)} = 1$.

Every functional $f \in (l_{(M)})^*$ can be written in the form $f = \Psi_y + \Phi$, where Ψ_y is a regular functional on $h_{(M)}$ generated by $y \in l_N$ and Φ is a singular functional. Let a be a positive real number such that $\varrho_N(ay) < \infty$. Notice that $x_n - x_0 \in h_{(M)}$. Then

$$|f(x_n - x_0)| = \Big|\sum_{i=n+1}^{n+m_n} u_n y_i\Big| \leqslant \frac{1}{a}\Big[m_n M(u_n) + \sum_{i=n+1}^{\infty} N(ay_i)\Big] \to 0 \quad \text{as } n \to \infty,$$

i.e. $x_n - x_0 \xrightarrow{w} 0$. But for any positive integer m and $n > m$

$$\varrho_M\left(\left(1+\frac{1}{m}\right)(x_n - x_0)\right) = m_n M\left(\left(1+\frac{1}{m}\right)u_n\right) \geqslant m_n M\left(\left(1+\frac{1}{n}\right)u_n\right) \geqslant 1.$$

Hence

$$\|x_n - x_0\|_{(M)} > \left(1+\frac{1}{m}\right)^{-1} \quad \text{for each } m \in \mathbb{N} \text{ and } n > m.$$

Consequently,

$$\liminf_{n\to\infty} \|x_n - x_0\|_{(M)} \geqslant 1.$$

Thus x_0 cannot be any H-point. This contradiction completes the proof of Theorem 3.

THEOREM 4. *Let $x_0 \in S(l_M)$. x_0 is an H-point iff $M \in \Delta_2$.*

The proof of Theorem 4 is analogous to the proof of theorem 3, so we will omit it.

3. Corollaries. Bor-Luh Lin, Pei-Kee Lin and S.L. Troyanski proved (cf. Th. (iii) [13]) that element x in a bounded closed convex set K of a Banach space is a denting point of K iff x is a H-point of K and x is an extreme point of K. Combining this result with our results and with results concerning the characterization of strong extreme points in Orlicz spaces, given in [6], we obtain the following

COROLLARY 1. *Suppose $x_0 \in S(L_M)$ or $x_0 \in S(L_{(M)})$. TFAE:*
(a) *x_0 is a denting point.*
(b) *x_0 is an H-point.*
(c) *x_0 is a strong extreme point.*
(d) *x_0 is an extreme point and $M \in \overline{\Delta}_2$.*

COROLLARY 2. *Suppose $x_0 \in S(l_M)$ or $x_0 \in S(l_{(M)})$. TFAE:*
(a) *x_0 is a denting point.*
(b) *x_0 is a strong extreme point.*
(c) *x_0 is an extreme point and $M \in \Delta_2$.*

A Banach space X is said to posses Property (G) (Property (H)), provided every point of $S(X)$ is denting point (H-point).

A Banach space X is said to be midpoint locally uniformly rotund (MLUR), if for any $\varepsilon \in (0, 2)$ and $x \in S(X)$, there is $\delta > 0$ such that $y, z \in S(x)$ and $\|y - z\| \geqslant \varepsilon$ implies $\|x - \frac{1}{2}(y + z)\| \geqslant \delta$.

It is well known that a Banach space X is (MLUR) iff every point of $S(X)$ is a strong extreme point (see for example [16]). Hence and from the above corollaries, we can deduce

Corollary 3. *For the spaces L_M or $L_{(M)}$ we have*

$$(G) \Leftrightarrow (H) \Leftrightarrow \text{(MLUR)}.$$

Corollary 4. *For te spaces l_M or $l_{(M)}$ we have*

$$(G) \Leftrightarrow \text{(MLUR)}.$$

Corollary 3 improves essentially Theorem 2 presented in [17] by Tingfu Wang.

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