Singularities of controllability of simplest dynamic inequalities

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A simplest dynamical inequality in Euclidean plane Oxy is defined by a smooth vector field $v = (a, b)$ of a flow acting on a controlled object and by a smooth function f characterizing the own resources of the object at a point of the plane. To be more precise, a velocity $(\dot{x}, \dot{y})$ of the object is *admissible* at a point (x, y) if

$$(\dot{x} - a(x, y))^2 + (\dot{y} - b(x, y))^2 \leq f(x, y).$$

An *admissible motion* of a dynamical inequality is, by definition, an absolutely continuous map from a time interval to the plane such that the respective velocity is admissible at all points of differentiability of this map. A point A in the phase space is *attainable from a point* B if there exists an admissible motion that moves the point B to A in finite time. The union of all points which are attainable from some point is called the *positive* of this point. The *negative* orbit of a point is defined analogously.

Structural stability of dynamic inequality is defined in classical manner like for vector fields [1-4], only here we consider positive and negative orbits of points instead their trajectories. So it is the same as the one given in [5] for control systems.

We show that a generic dynamic inequality on the plane which are completely controllable at the infinity is structurally stable. That implies structural stability of the nonlocal transitivity zones of such an inequality, that is an open domain in the phase space which coincides with the intersection of the positive and negative orbits of any its point.

For such an inequality the boundary of any its nonlocal transitivity zone is a smooth curve with singularities. To be more precise, the germ of this zone at any point of its boundary is diffeomorphic to the germ at zero of one of the five sets $y > 0$, $y > |x|$, $y < |x|$, $y < x^2h(x)$, $y > x^2h(x)$, where the graph of the function $y = x^2h(x)$ near zero is the closure of the union of two singular phase curves of the equation $dy^2 = (y - ax^2)dx^2$, $0 < a < 1/16$.

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REFERENCES

[1] D. V. Anosov, S. Kh. Aranson, I. U. Bronshtein, and V. Z. Grines - Smooth Dynamical Systems // Itogi Nauki Tekh., Ser.: Sovr. Probl. Mat., Fund. Napr. 1, 151–242 (1985); Engl. transl. in Dynamical Systems I (Springer, Berlin, 1988), Encycl. Math. Sci. 1, pp. 149–233.

[2] V. I. Arnold - Additional Chapters of the Theory of Ordinary Differential Equations //Nauka, Moscow, 1978 [in Russian].

[3] R. Garcia, C. Gutierrez, and J. Sotomayor - Structural Stability of Asymptotic Lines on Surfaces Immersed in R3,// Bull. Sci. Math. 123, 599–622, (1999).

[4] J. Palis, Jr. and W. de Melo - Geometric Theory of Dynamical Systems //Springer, New York, 1982; URSS, Moscow, 1998.

[5] A.A. Davydov - Structural Stability of Control Systems on Orientable Surfaces //Mat. Sb. 182 (1), 3–35 (1991) [Math. USSR, Sb. 72, 1–28 (1992)].

[6] A.A. Davydov - Local Controllability of Typical Dynamical Inequalities on Surfaces //Tr. Mat. Inst. im. V.A. Steklova, Ross. Akad. Nauk 209, 84–123 (1995) [Proc. Steklov Inst. Math. 209, 73–106 (1995)].

[7] Yu.A. Grishina, A.A. Davydov - Structural stability of simplest dynamical inequalities //Tr. Mat. Inst. im. V.A. Steklova, Ross. Akad. Nauk 256, 89–101 (2007) [Proc. Steklov Inst. Math. 256, 80-91 (2007)].