wsp. cylindryczne

$$\begin{cases} x^2+y^2 \le 2az \\ x^2+y^2+z^2 \le 3a^2 \end{cases}$$

$$\begin{cases} \rho^2 \le 2az \\ \rho^2+z^2 \le 3a^2. \end{cases}$$

$\operatorname{div} F = z + 2y + 2z$

$= 3z + 2y.$

$z \ge \frac{\rho^2}{2a} \ge 0$

$z^2 \le 3a^2 - \rho^2 \Rightarrow$ $\boxed{3a^2 - \rho^2 \ge 0}.$

$\Updownarrow$

$\rho \le \sqrt{3}a.$ (*)

$\frac{\rho^4}{4a^2} \le z^2 \le 3a^2 - \rho^2$

$$\rho^4 \le 4a^2(3a^2-\rho^2) = 12a^4 - 4a^2\rho^2$$

$$\rho^2 + 4a^2\rho^2 - 12a^4 \le 0$$

$$\Delta = 16a^4 + 4\cdot 12a^4 = (16+48)a^4 = 64a^4 \Rightarrow \sqrt{\Delta} = 8a^2$$

$$\rho_1^2 = \frac{-4a^2+8a^2}{2} = 2a^2 \ge 0$$

$$\rho_2^2 = \frac{-4a^2-8a^2}{2} = -6a^2 \le 0$$

$$\rho^2 \in [0, 2a^2] \qquad \rho \in [0, a\sqrt{2}] \subset [0, a\sqrt{3}]$$

Zatem:

$$\int_{\partial\Omega} \vec{F}\,\vec{n}\,dA = \int_{\Omega} \operatorname{div} F\,dV = \int_0^{2\pi} d\varphi \int_0^{a\sqrt{2}} \varrho\,d\varrho \int_{\frac{\varrho^2}{2a}}^{\sqrt{3a^2-\varrho^2}} ($$

$$(3z + 2\varrho\sin\varphi)\,dz\,)$$

$$= 2\pi \int_0^{a\sqrt{2}} \varrho\,d\varrho \cdot \frac{3}{2} z^2 \Big|_{\frac{\varrho^2}{2a}}^{\sqrt{3a^2-\varrho^2}} =$$

$$3\pi \int_0^{a\sqrt{2}} \left(3a^2 - \varrho^2 - \frac{\varrho^4}{4a^2}\right) \cdot \varrho\,d\varrho =$$

$$3\pi \left(3a^2\frac{\varrho^2}{2} - \frac{\varrho^4}{4} - \frac{\varrho^6}{24a^2}\right)\Big|_0^{a\sqrt{2}} =$$

$$3\pi\left(3a^2\cdot\frac{2a^2}{2}-\frac{24a^4}{2}-\frac{a^4\cdot 8}{24\cdot 3}\right)$$

$$=3\pi\left(3a^4-a^4-\frac{a^4}{3}\right)=3\pi\cdot\frac{5}{3}a^4=5\pi a^4$$