The Point of E_8 In F-theory GUTs

Jonathan J. Heckman

Based on work with C. Vafa, as well as:

C. Beasley, V. Bouchard, S. Cecotti, M. Cheng, G.L. Kane, J. Marsano

S. Schäfer-Nameki, N. Saulina, J. Shao, J. Seo, A. Tavanfar

Outline

- Motivation: Bottom Up GUTs
- F-theory Ingredients
- Flavor and E_8
- ~~SUSY~~ and Cosmology

Motivation

Standard Model/MSSM $\subset$ Strings?

What is possible in string constructions?

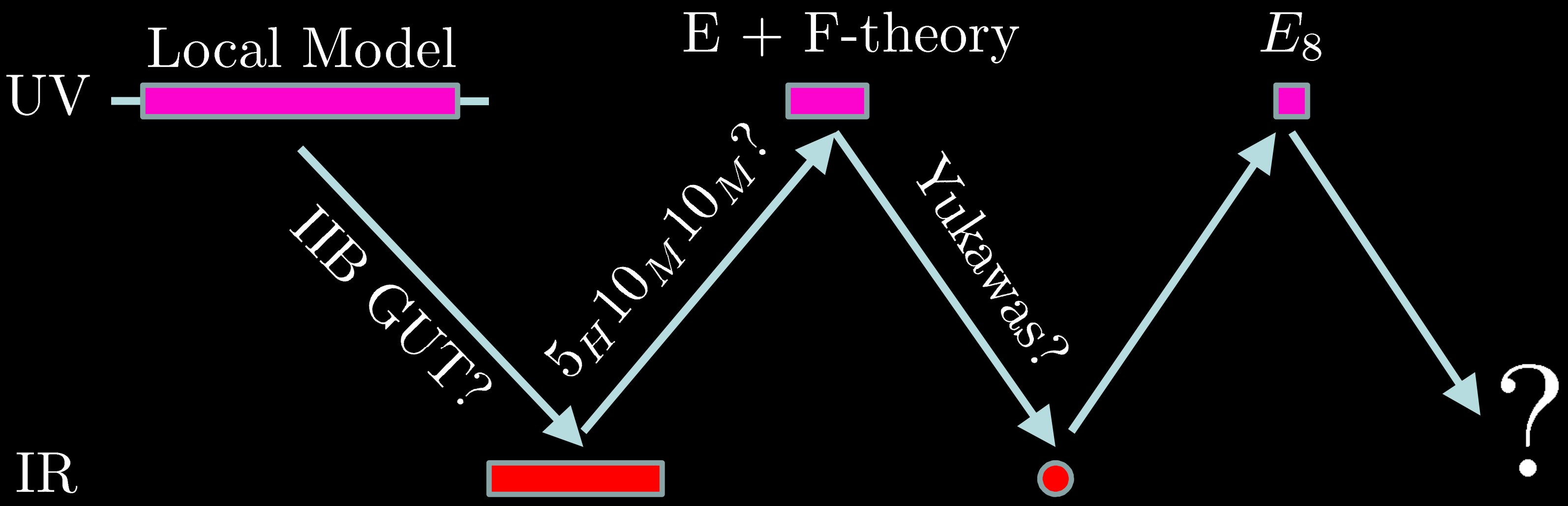
Hybrid Strategy:

Top Down: Specify All Details in UV (Global Models)

Where to look first?

Bottom Up: Decouple some of UV (Local Models)

Too flexible?



UV

Local Model

E + F-theory

E_8

IIB GUT?

$5_H 10_M 10_M$?

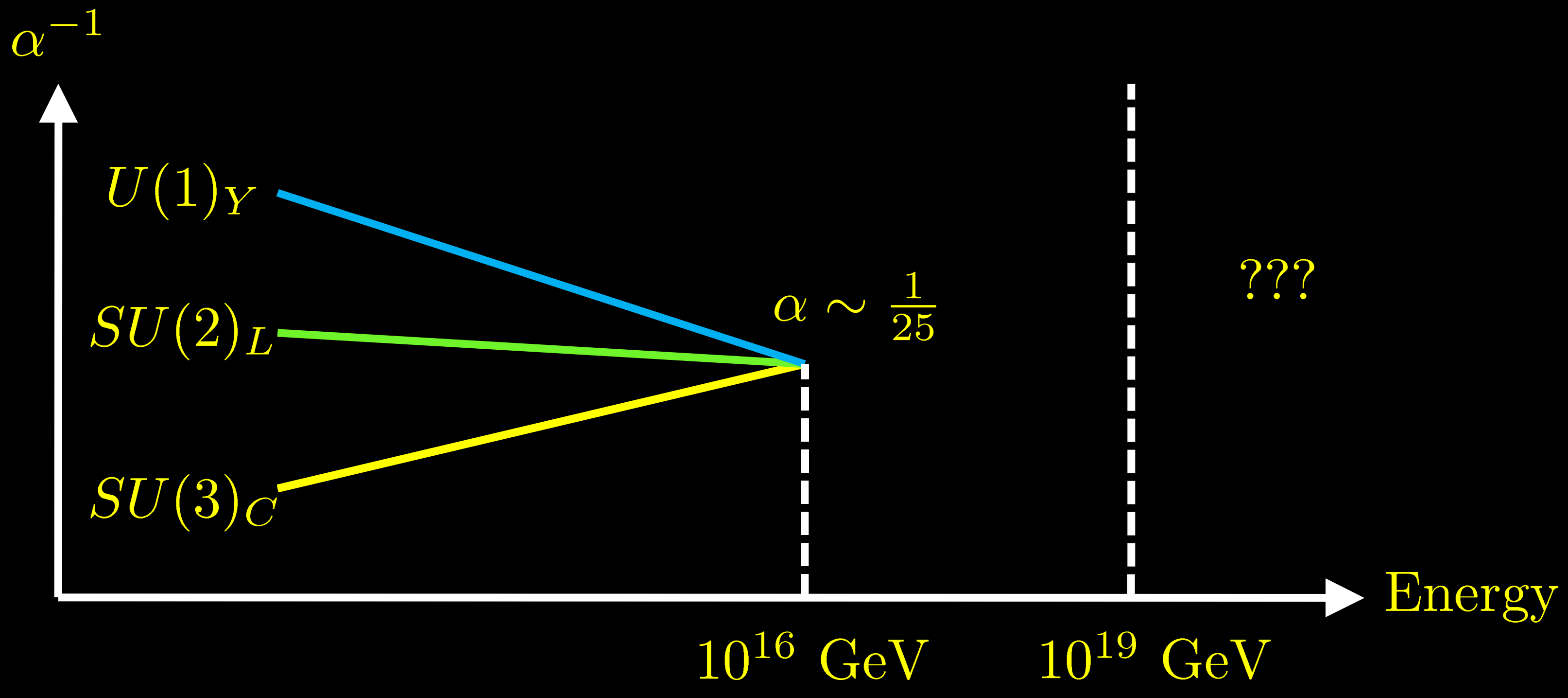
Yukawas?

?

IR

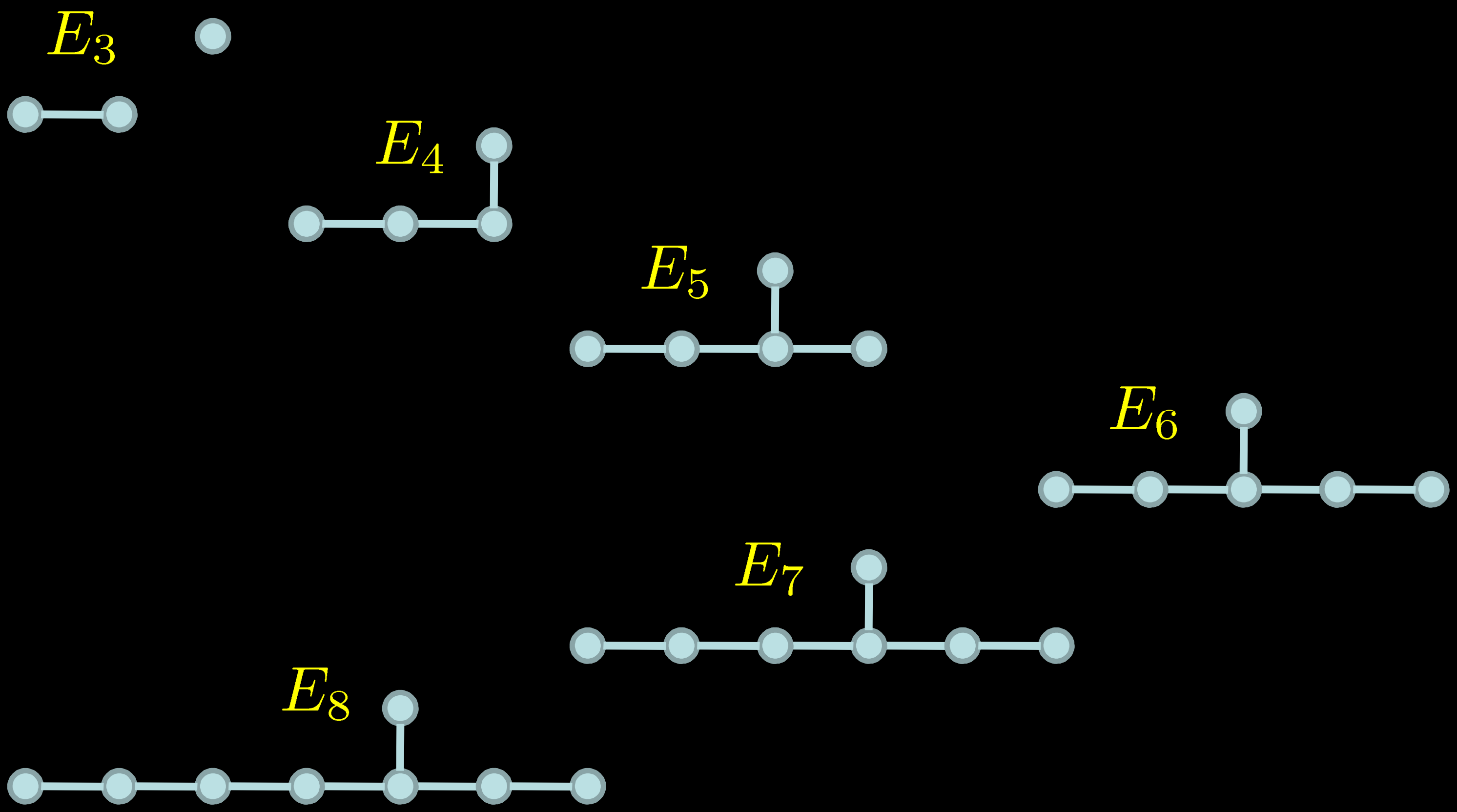
Simplifying Assumptions:

1) Low energy supersymmetry & Unification:



2) $M_{GUT}/M_{pl} \ll 1$

Assumption 1: GUTs



How much E is necessary? How much is aesthetics?

Assumption 2: $M_{GUT}/M_{pl} \ll 1$

10D Gravity: $R^{3,1} \times \mathcal{M}_6 \Rightarrow G_{Newton} \sim \frac{1}{Vol(\mathcal{M}_6)}$

Gravity decouples when $Vol(\mathcal{M}_6) \rightarrow \infty$

Gauge Theory on $R^{3,1} \times \mathcal{M}_k \subset R^{3,1} \times \mathcal{M}_6$:

$\Rightarrow g_{YM}^2 \sim \frac{1}{Vol(\mathcal{M}_k)} \Rightarrow Vol(\mathcal{M}_k) \not\rightarrow \infty$

Local Flexibility

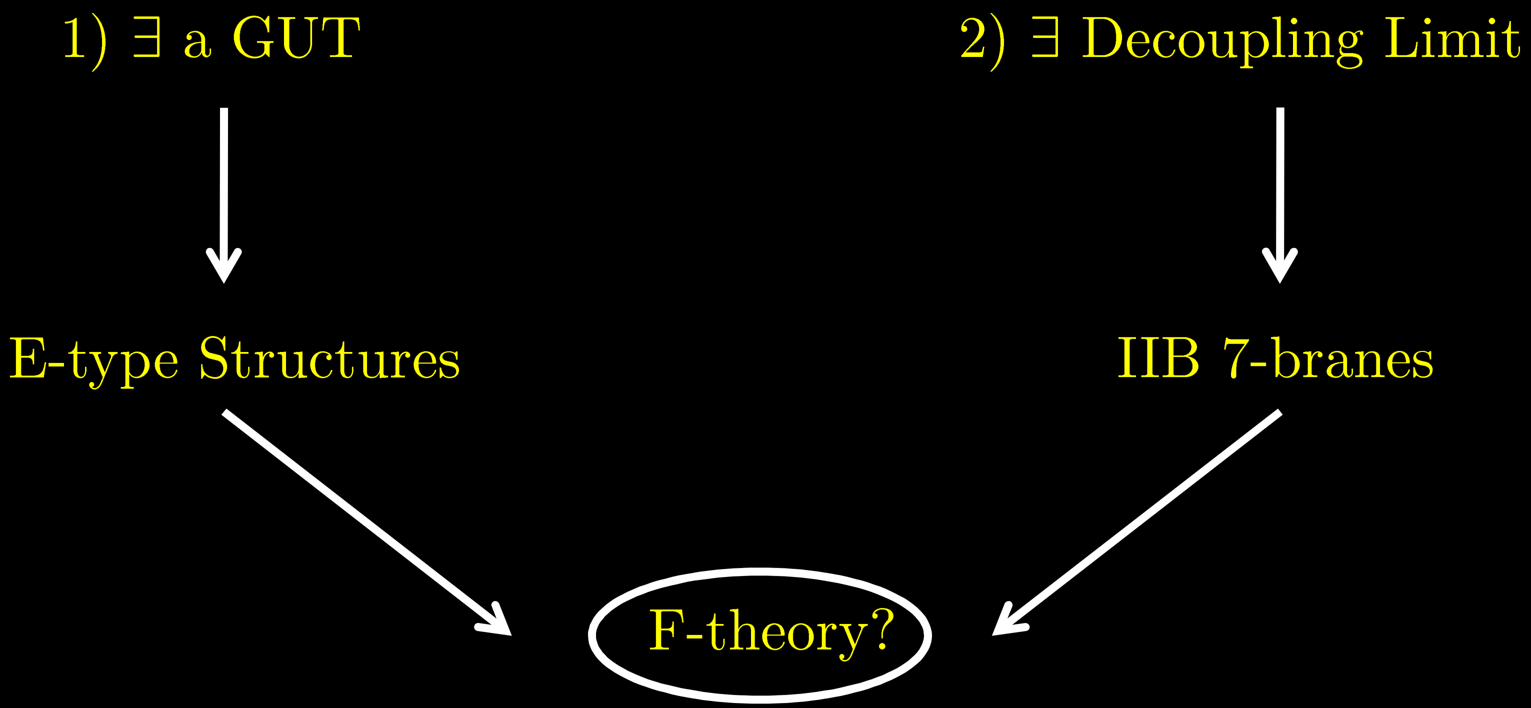
Local Model suggests GUT from p - brane, $p = 3, 4, 5, 6, 7$

$\Rightarrow$ Type II strings

E-type Structure: $g_s \rightarrow O(1)$

F-theory branes: 3-branes & 7-branes

E-type and 4d Chiral Matter $\Rightarrow$ 7-branes



1) ∃ a GUT

E-type Structures

2) ∃ Decoupling Limit

IIB 7-branes

F-theory?

Roadmap

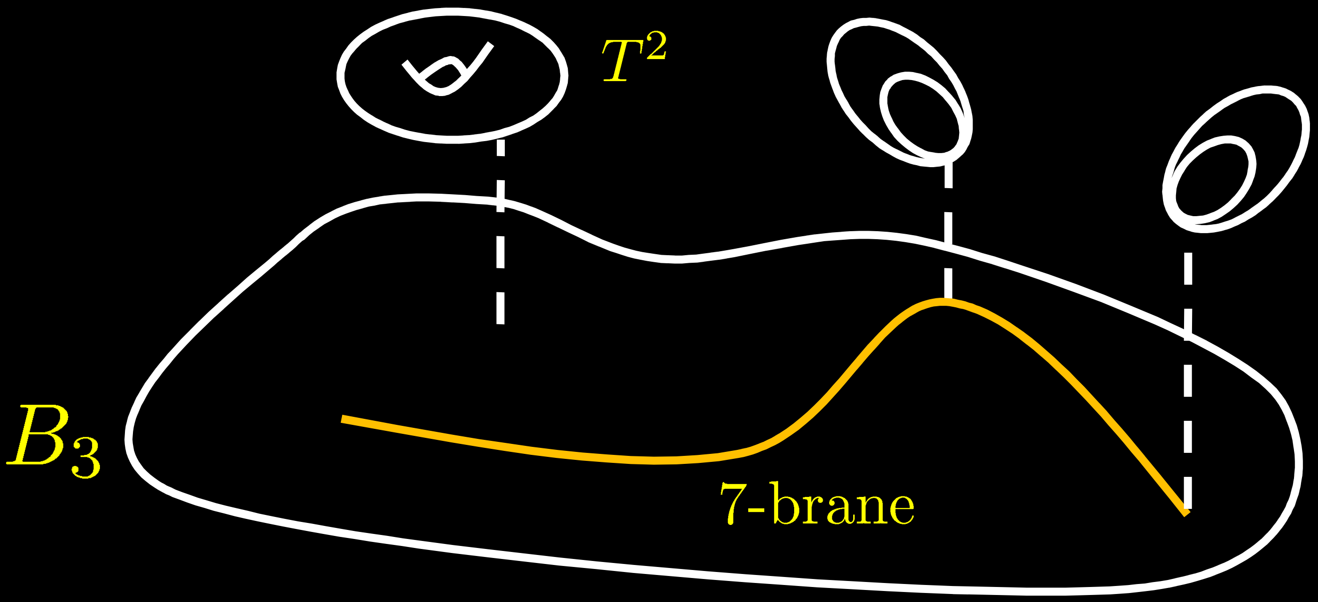
- Motivation: Bottom Up GUTs
- F-theory Ingredients

F-theory Review I

F-theory = Strongly Coupled Formulation of IIB in 12d

$\tau_{IIB} = C_0 + ie^{-\phi}$ is cplx str. of a T^2

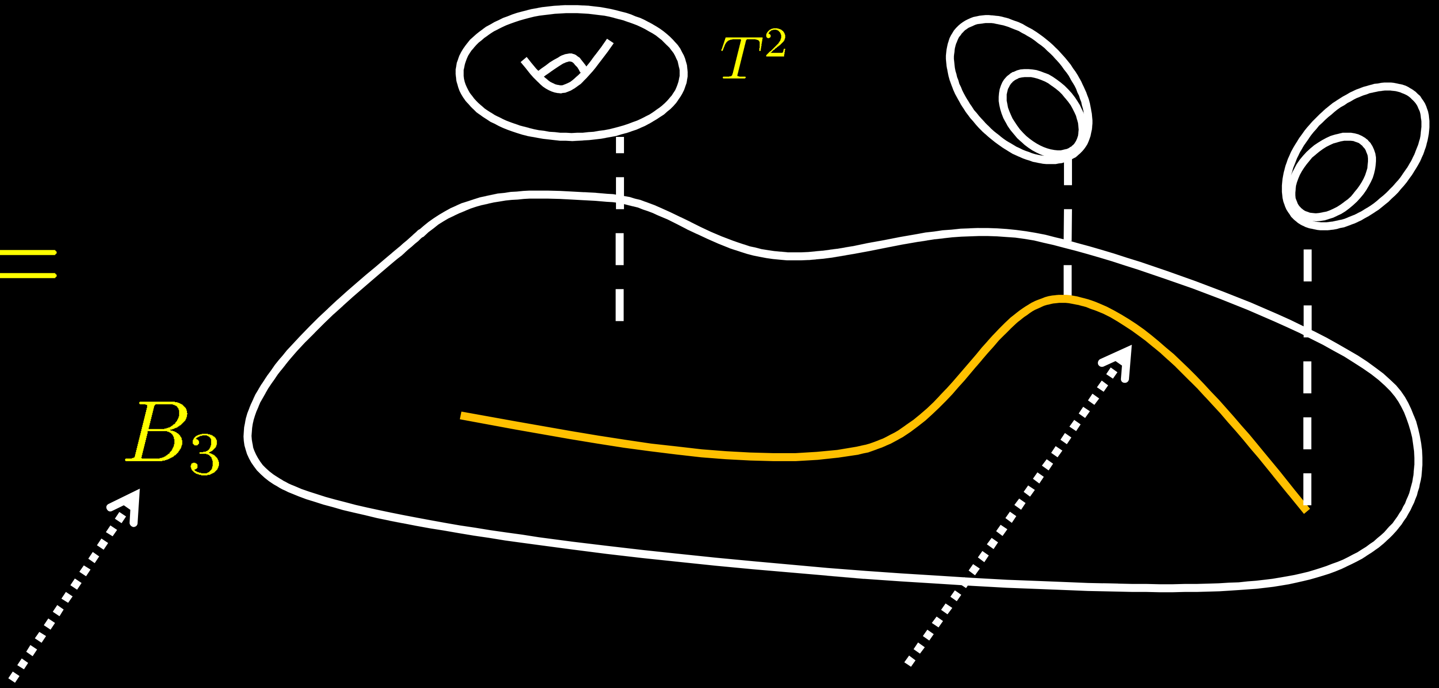
This T^2 pinches off near 7-branes:



F-theory Review II

4d $\mathcal{N} = 1 \Rightarrow$ F / $R^{3,1} \times$ Elliptic CY_4

$$CY_4 =$$



Local Model $\Rightarrow Vol(B_3) \to \infty$

$CY_4 \sim S \times C^2/\Gamma_{ADE} \Rightarrow$ 7-brane on S

Geometry ⇒ Gauge Theory

$F-th/R^{3,1}\times S\times C^2/\Gamma_{ADE}\Rightarrow 8d$ SYM w/gp G_{ADE}

Example: 8d $SU(N)$ at $z=0$ from $y^2=x^2+z^N$

$10d\Rightarrow$ Gravity (decoupled)

$8d:7\Rightarrow$ Gauge Group

$6d:7\cap 7'\Rightarrow$ Matter

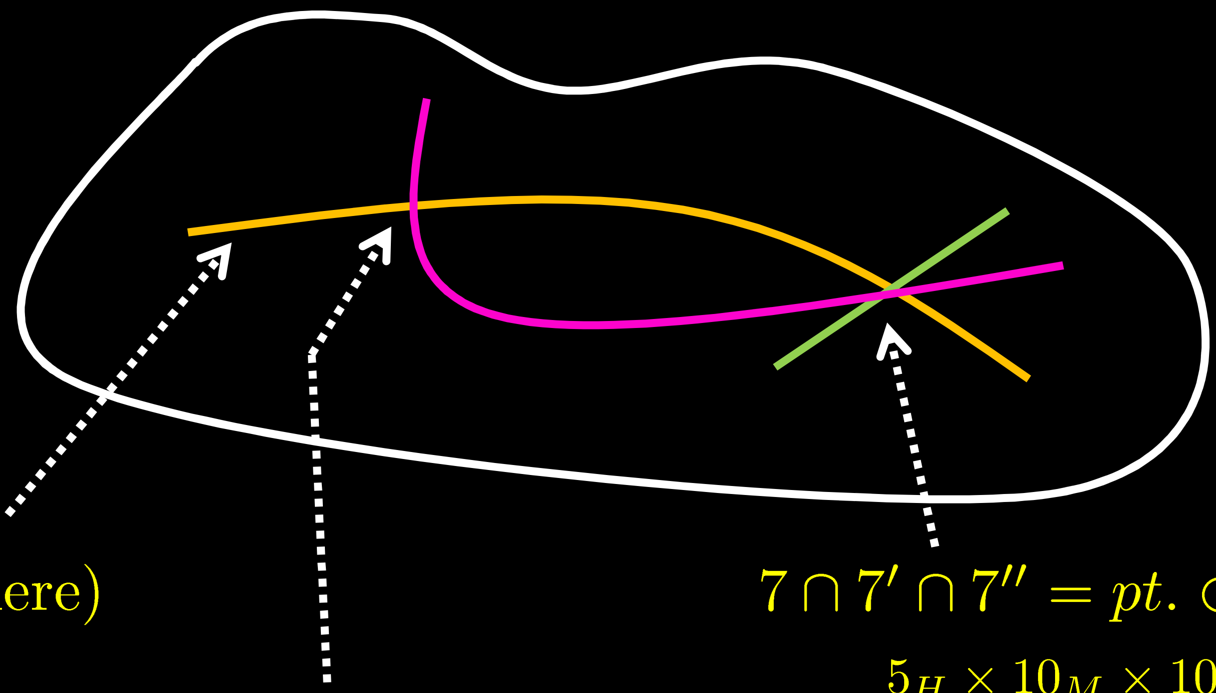
$4d:7\cap 7'\cap 7''\Rightarrow$ Yukawas

F-theory GUTs

Beasley JJH Vafa '08 (BHV I, II), Donagi Wijnholt I II '08
(see also Hayashi et al. '08 '09)

See also talks by Tatar,
Blumenhagen, Weigand

B_3:



7-brane / S
(GUT lives here)

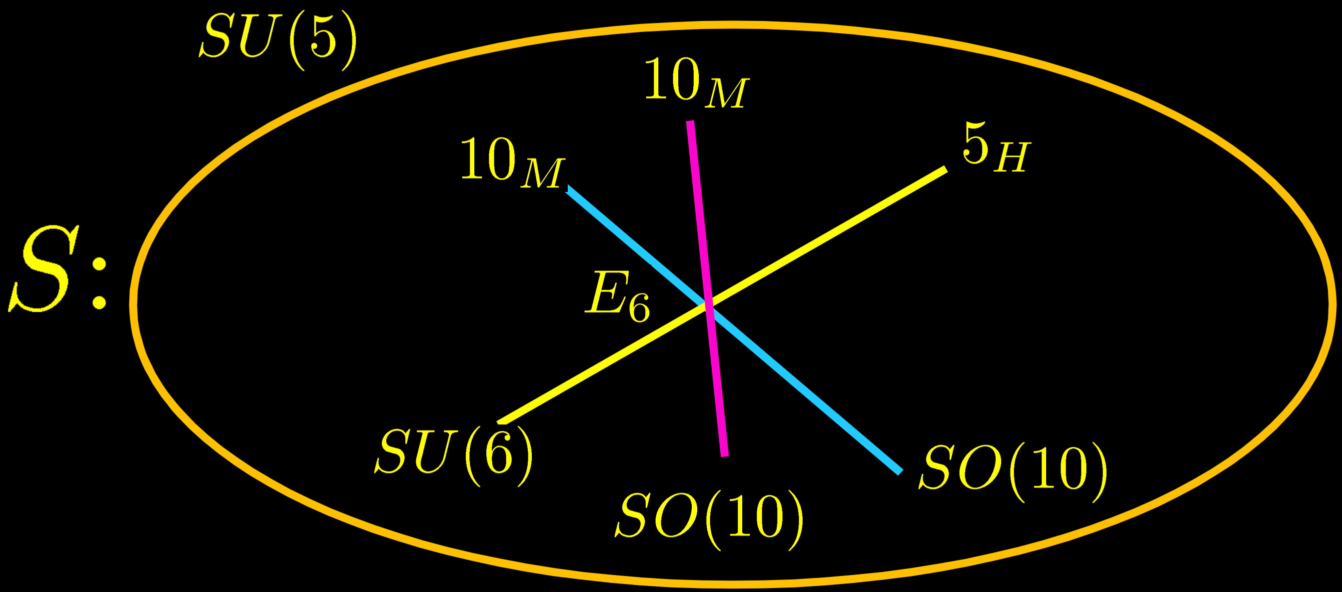
$7 \cap 7' = \text{curve} \subset S$

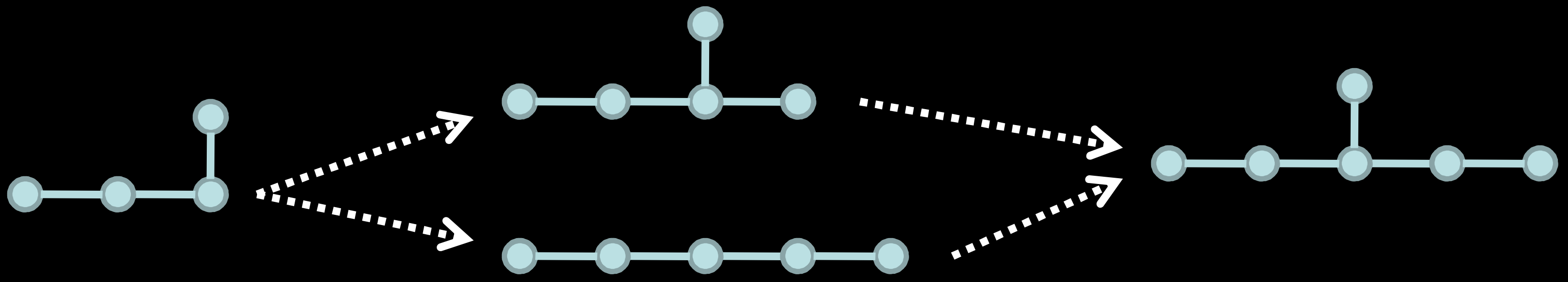
$\bar{5}, 10 \in SU(5),\ 16 \in SO(10)\ldots$

$7 \cap 7' \cap 7'' = pt. \subset S$

$5_H \times 10_M \times 10_M \ldots$

Higgsing By Geometry





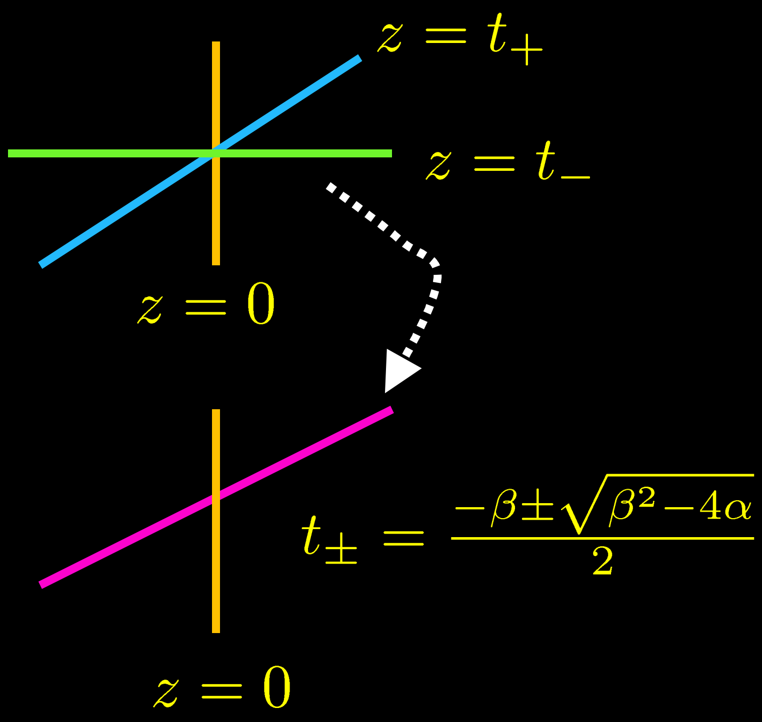
$5_H 10_M 10_M$ needs E_N

Monodromy

Location of 7-branes = roots of polynomials

$$y^2 = x^2 + z^5(z - t_+)(z - t_-)$$

$$y^2 = x^2 + z^5(z^2 + \alpha z + \beta)$$



Hayashi et al. '09

Important for two light gens from $5_H \times 10_M^{(1)} \times 10_M^{(2)}$

4d Spectrum

$$G_S \xrightarrow{\text{instanton}} \Gamma_S \times \cancel{H_S}$$

4d matter $\Longleftrightarrow$ zero modes in instanton background

S Modes: $\overline{\partial}_A \Psi = 0$

Σ Modes: $\overline{\partial}_{A+A'} \sigma = 0$

$\Rightarrow$ Index Computation

$$\int_M ch(V)Td(M)$$

Beasley JJH Vafa I '08
Donagi Wijnholt I '08

Minimal Spectrum

Beasley JJH Vafa II '08

$$G_S = SU(5) \xrightarrow{U(1)_Y \text{ flux}} SU(3) \times SU(2) \times U(1)_Y$$

No bulk exotics ⇒ unique internal flux

Higgs: $\int_{\Sigma_H} F_{U(1)_Y} \neq 0$:

$$\begin{array}{r|ccc|cc|} \cline{2-6} \bar{5}_H = & T_d & T_d & T_d & H_d & H_d \\ 5_H = & T_u & T_u & T_u & H_u & H_u \\ \cline{2-6} \multicolumn{1}{r}{} & \multicolumn{3}{c}{\text{out}} & \multicolumn{2}{c}{\text{in}} \end{array}$$

Matter: $\int_{\Sigma_M} F_{U(1)_Y} = 0$

Matter: $\int_{\Sigma_M} F_{U(1)_\perp} = 3$

$$\begin{array}{|c|} \hline 3 \times \bar{5}_M \\ 3 \times 10_M \\ \hline \multicolumn{1}{c}{\text{in}} \end{array}$$

Roadmap

- F-theory Ingredients
- Flavor and E_8

Quark Wishlist

Two Light Generations

Hierarchical CKM Matrix:

$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{bmatrix}$$

Minimal Ingredients

With Minimal Ingredients

What Yukawas Do We Get?

---

3 10_M's on Σ_{10} curve

3 $\overline{5}_M$'s on $\Sigma_{\overline{5}}$ curve

1 $5_H \times 10_M \times 10_M$ point

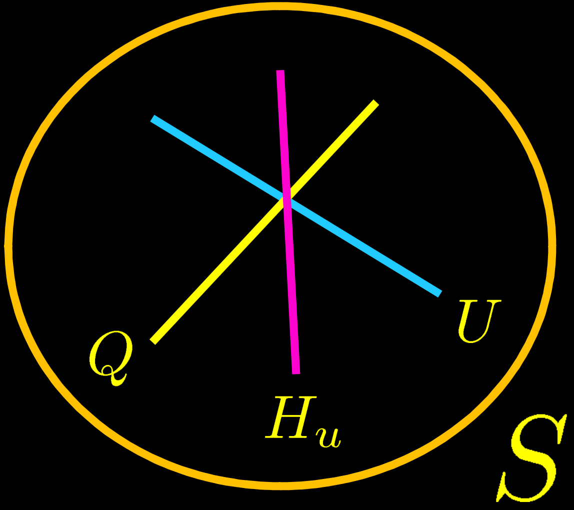
1 $\overline{5}_H \times \overline{5}_M \times 10_M$ point

$\boxed{+\text{ Flux}}$

Quark Yukawas:

$$R^{3,1}\text{:}\ \ W \supset \lambda_u^{ij} \cdot Q^i U^j H_u + \cdots$$

$$\mathcal{M}_6\text{:}\ \bar{\partial}\Psi = 0\text{:}\ \ \Psi_Q^i,\ \Psi_U^i,\ \Psi_{H_u},\ \cdots$$



$$\lambda_u^{ij} = \underbrace{\Psi_Q^i(p)\Psi_U^j(p)}\Psi_{H_u}(p) + \cdots$$

(outer product)

See Beasley JJH Vafa II ‘08
And Hayashi et al. ‘09

$$\Rightarrow$$

$$\begin{bmatrix} m_u & & \\ & m_c & \\ & & m_t \end{bmatrix} = \begin{bmatrix} 0 & & \\ & 0 & \\ & & m \end{bmatrix}$$

$$+\cdots ?$$

$$\lambda_u^{ij} = \Psi_Q^i(p)\Psi_U^j(p)\Psi_{H_u}(p) + \cdots$$

$$\lambda_d^{ij} = \Psi_Q^i(p)\Psi_D^j(p)\Psi_{H_d}(p) + \cdots$$

Two light generations

$$\lambda_u^{ij} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & \lambda_t \end{bmatrix}$$

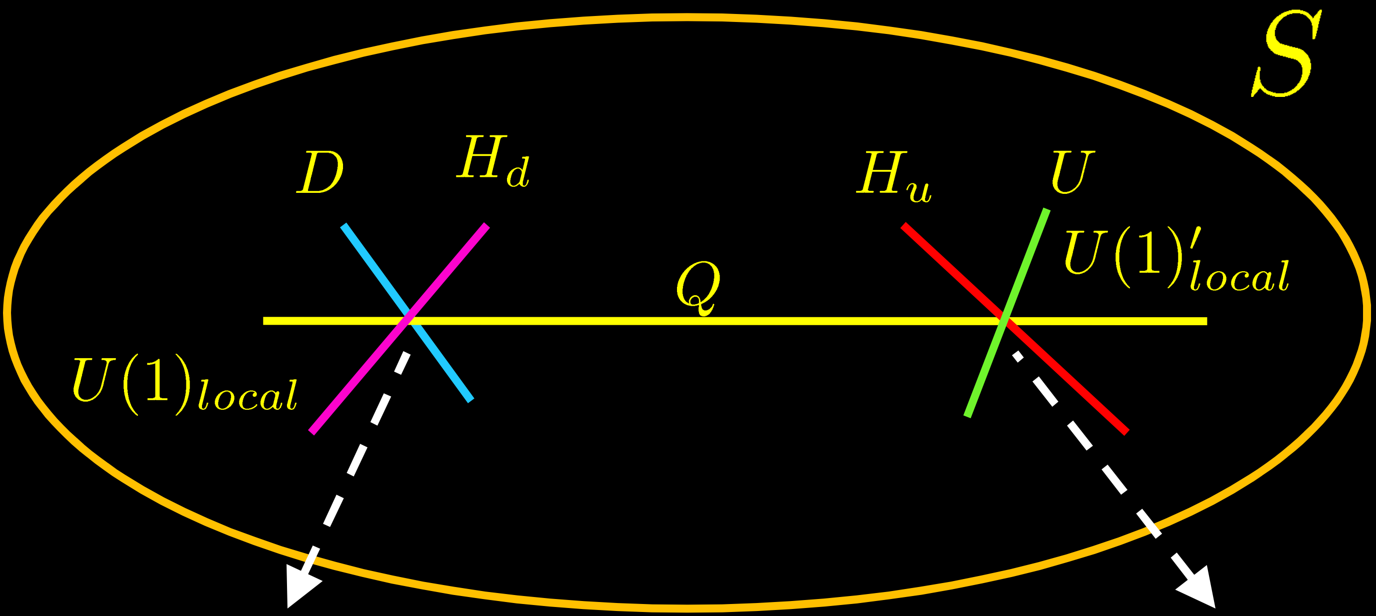
$$\lambda_d^{ij} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & \lambda_b \end{bmatrix}$$

What corrects this structure?

$$\boxed{\partial B_{(0,2)} \neq 0}$$

Cecotti, Cheng, JJH, Vafa
In Progress

$U(1)$ Selection Rules

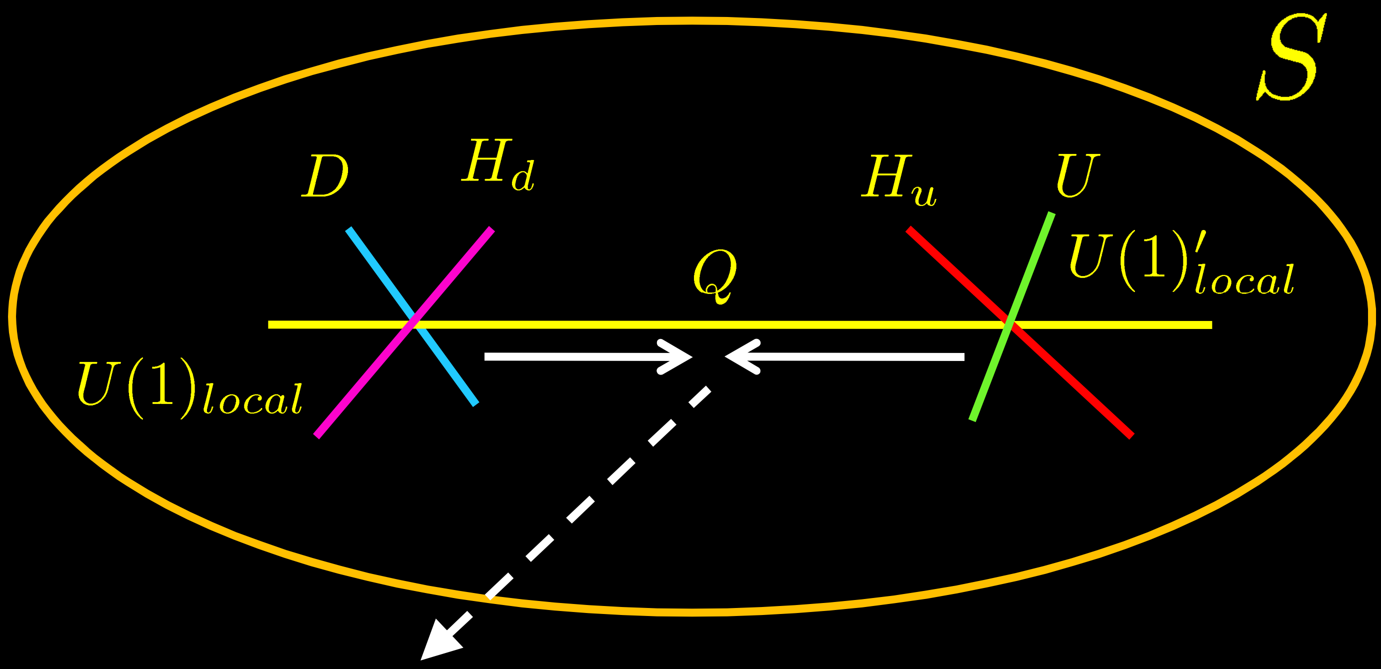


$$\lambda_d \sim \begin{bmatrix} \varepsilon_D^5 & \varepsilon_D^4 & \varepsilon_D^3 \\ \varepsilon_D^4 & \varepsilon_D^3 & \varepsilon_D^2 \\ \varepsilon_D^3 & \varepsilon_D^2 & 1 \end{bmatrix} \qquad T_L \cdot \lambda_u \cdot T_R^\dagger \sim \begin{bmatrix} \varepsilon_U^8 & \varepsilon_U^6 & \varepsilon_U^4 \\ \varepsilon_U^6 & \varepsilon_U^4 & \varepsilon_U^2 \\ \varepsilon_U^4 & \varepsilon_U^2 & 1 \end{bmatrix}$$

IF $U(1)_{local} \neq U(1)'_{local}$: No CKM Hierarchy

$$p_{down} \to p_{up}$$

$U(1)_{local} \to U(1)'_{local} \Rightarrow$ CKM Hierarchy



$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix}$$

$$\begin{tikzcd} E_6 : 5_H 10_M 10_M \arrow[dr] & \\ & E_7 \\ SO(12) : \overline{5}_H \overline{5}_M 10_M \arrow[ur] & \end{tikzcd}$$

CKM Matrix

$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix} \qquad \varepsilon^2 \sim Flux^2/M_*^4$$

$$\sim Vol(S)^{-1}/M_*^4 \sim \alpha_{GUT}$$

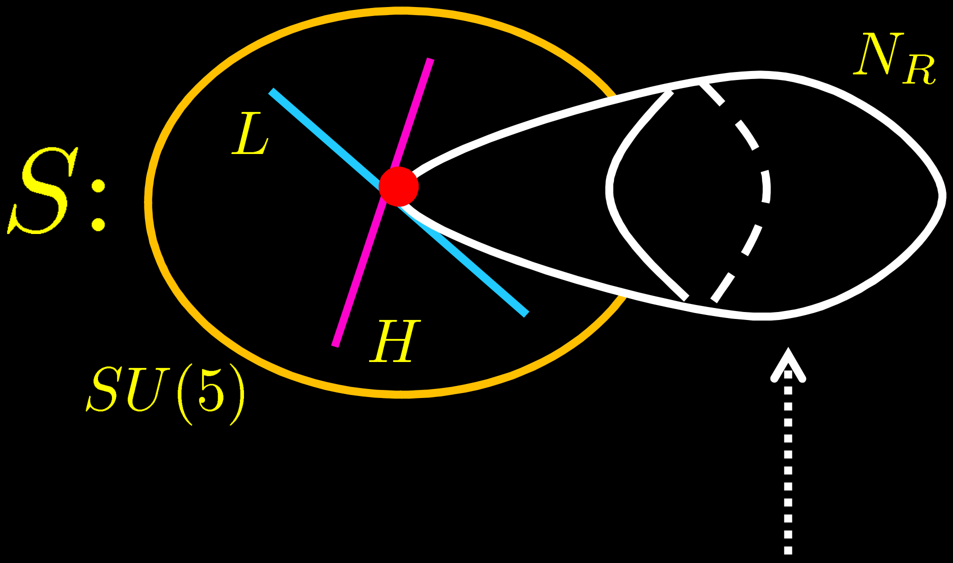
$$\Downarrow$$

$$|V_{CKM}^{F-th}| \sim \begin{bmatrix} 1 & 0.2 & 0.008 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{bmatrix}$$

$$|V_{CKM}^{obs}| \sim \begin{bmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{bmatrix}$$

Neutrinos

Beasley, JJH, Vafa ’08
Bouchard, JJH, Seo Vafa ‘09



Curve touching S

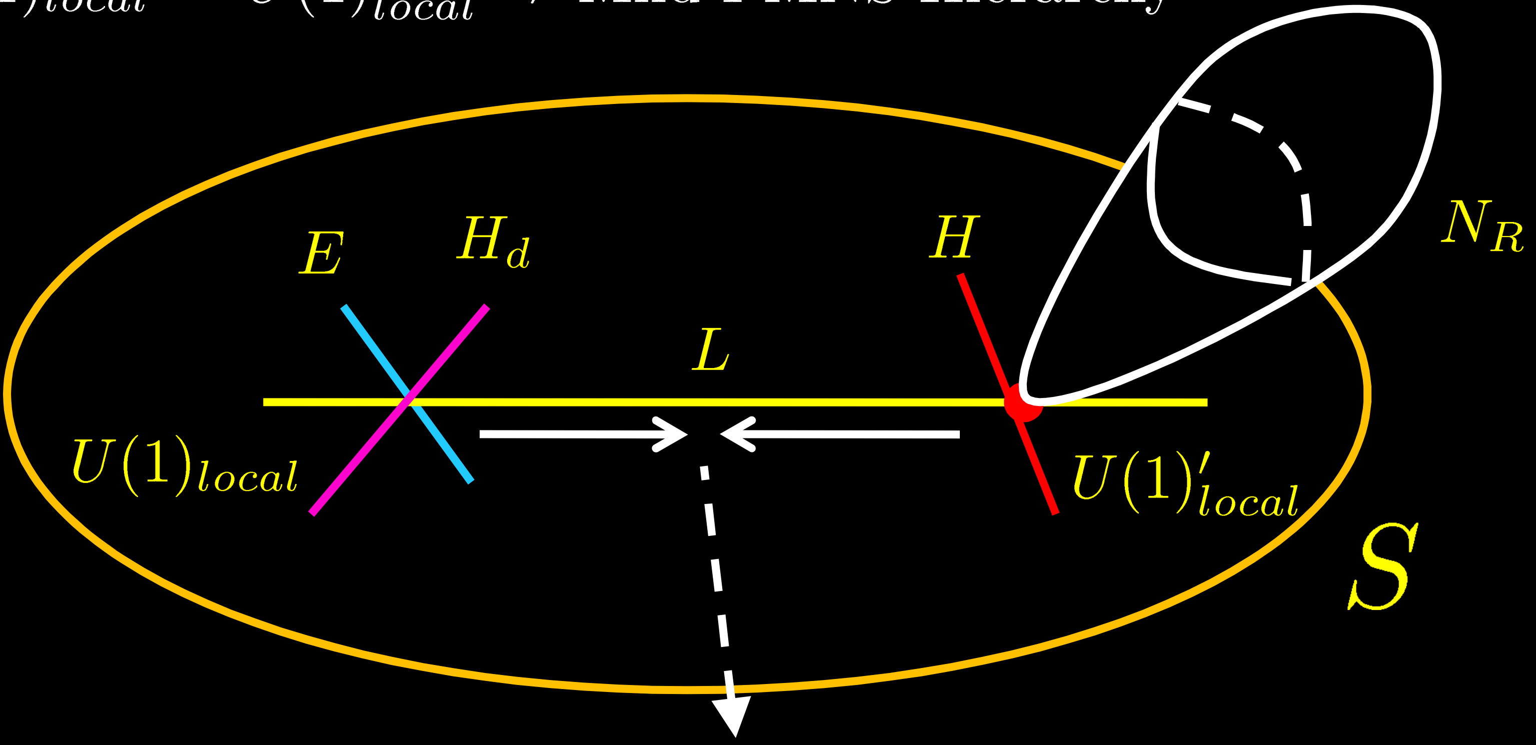
$$H = 7_{GUT} \cap 7'_{\perp}$$

$$L = 7_{GUT} \cap 7''_{\perp}$$

$$\boxed{N_R = 7'_{\perp} \cap 7''_{\perp}}$$

ν Mixing Hierarchy

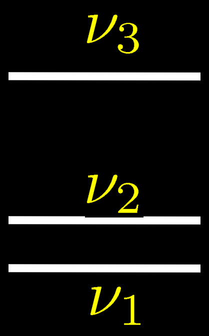
$U(1)_{local} \rightarrow U(1)'_{local} \Rightarrow$ Mild PMNS Hierarchy



$$|V_{PMNS}| \sim \begin{bmatrix} U_{e1} & \varepsilon^{1/2} & \varepsilon \\ \varepsilon^{1/2} & U_{\mu 2} & \varepsilon^{1/2} \\ \varepsilon & \varepsilon^{1/2} & U_{\mu 3} \end{bmatrix}$$

ν Masses

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \varepsilon_N^2 : \varepsilon_N : 1 \Rightarrow$$



"Normal Hierarchy"

Predict: $\frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} \sim \alpha_{GUT} \sim 0.04$

Close!

Observe: $\frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} = \frac{m_{sol}^2}{m_{atm}^2} \sim 0.03$

PMNS Matrix

Bouchard, JJH, Seo Vafa ‘09

$$\left|V_{PMNS}^{F-th}\right| \sim \begin{array}{cc} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ & \left[\begin{array}{ccc} 0.87 & 0.45 & \boxed{0.2} \\ 0.45 & 0.77 & 0.45 \\ 0.2 & 0.45 & 0.87 \end{array}\right] \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \end{array}$$

$$\left|V_{PMNS}^{obs(3\sigma)}\right| \sim \left[\begin{array}{ccc} 0.77-0.86 & 0.50-0.63 & \boxed{0.00-0.22} \\ 0.22-0.56 & 0.44-0.73 & 0.57-0.80 \\ 0.21-0.55 & 0.40-0.71 & 0.59-0.82 \end{array}\right]$$

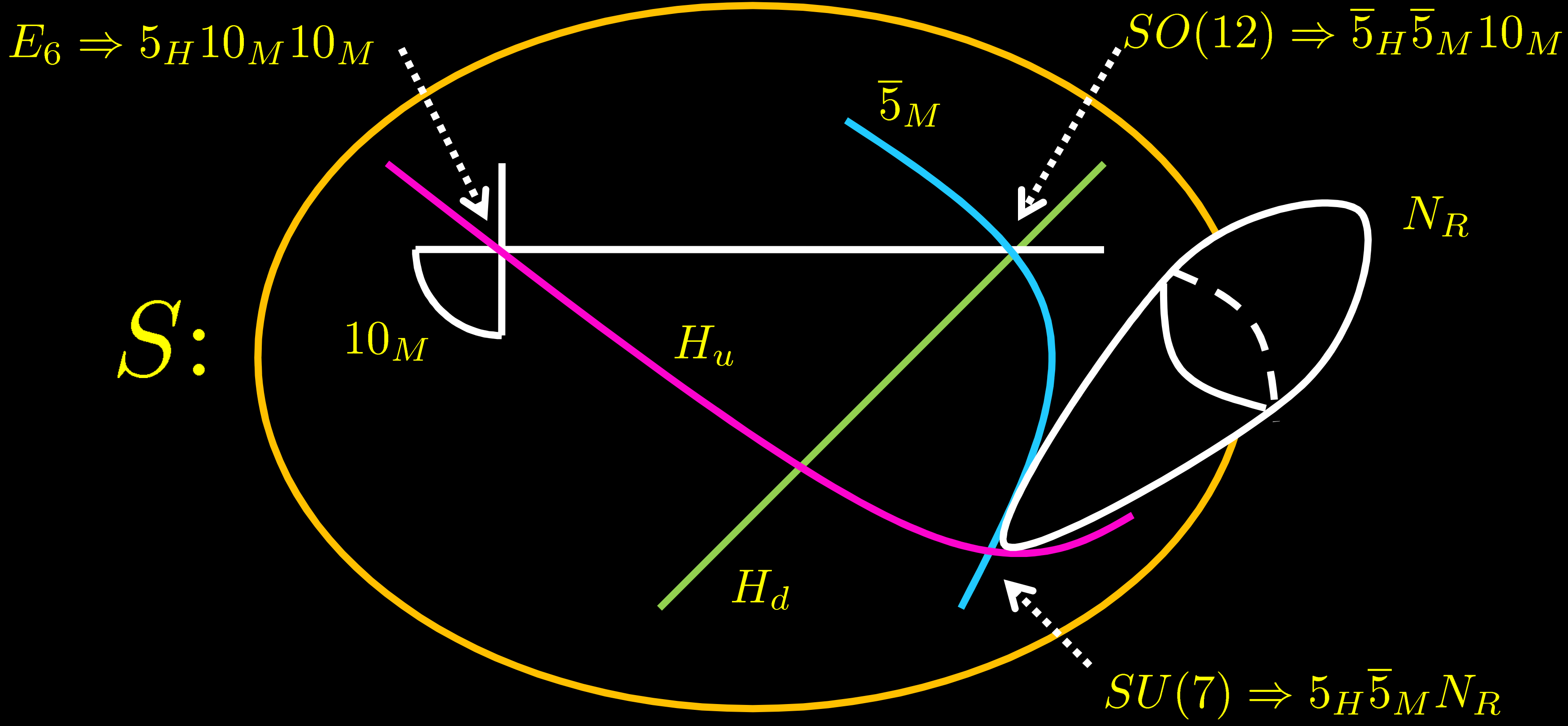
$$\boxed{\Rightarrow \text{Predict } V_{PMNS}^{1,3} \text{ close to current bound}}$$

CKM + PMNS $\Rightarrow E_8$

$$\begin{tikzcd}
 & E_8 & \\
E_6 \arrow[r, dotted, "\mathrm{CKM}"] & E_7 \arrow[u, dotted, "\mathrm{PMNS}"'] & SO(12) \arrow[l, dotted, "\mathrm{CKM}"'] \\
 & SU(5)_{GUT} \arrow[ul, dotted, "5_H 10_M 10_M"] \arrow[ur, dotted, "\overline{5}_H \overline{5}_M 10_M"'] &
\end{tikzcd}$$

Point Unification

Beasley JJH Vafa II ’08



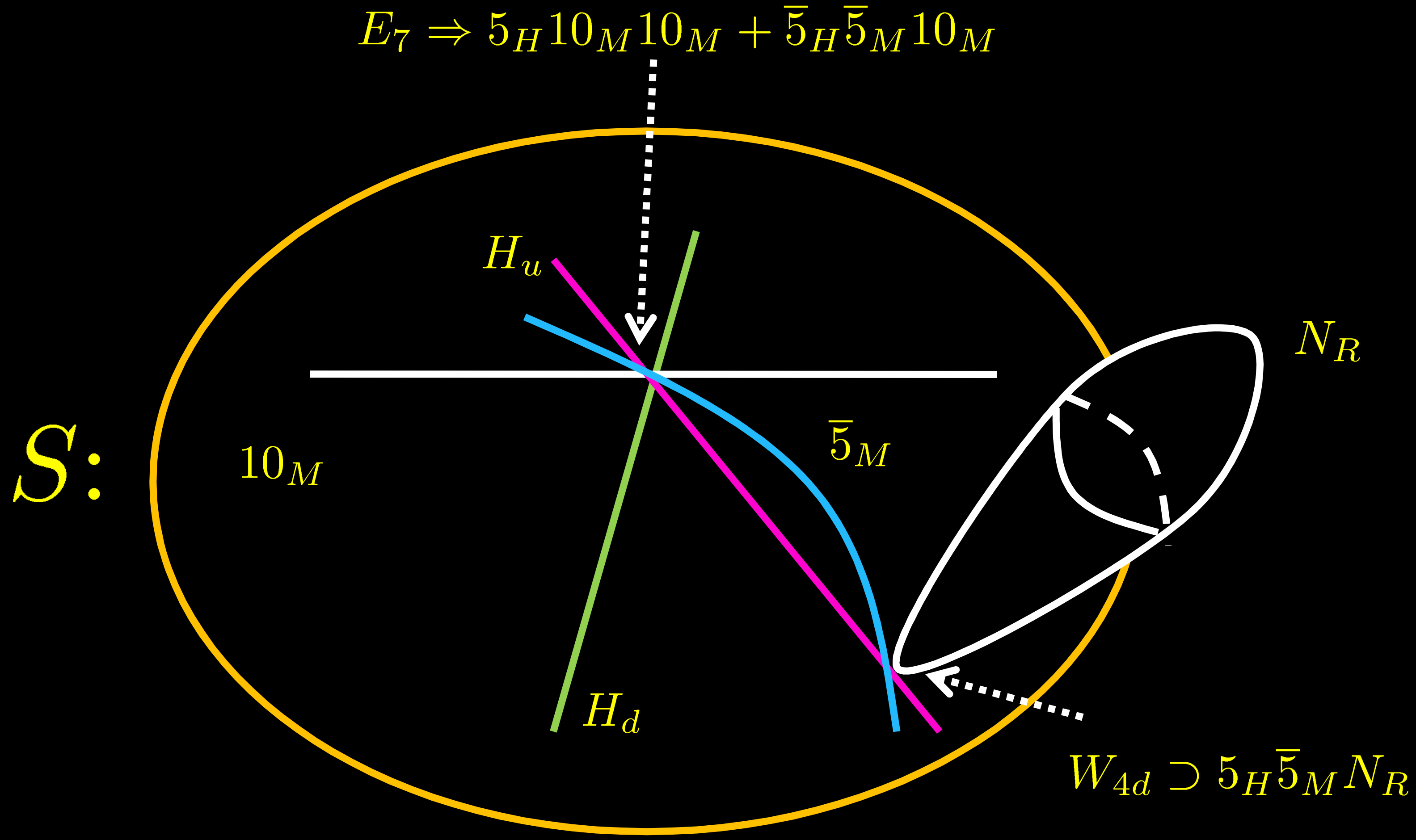
$E_6 \Rightarrow 5_H 10_M 10_M$

$SO(12) \Rightarrow \overline{5}_H \overline{5}_M 10_M$

$SU(7) \Rightarrow 5_H \overline{5}_M N_R$

S:

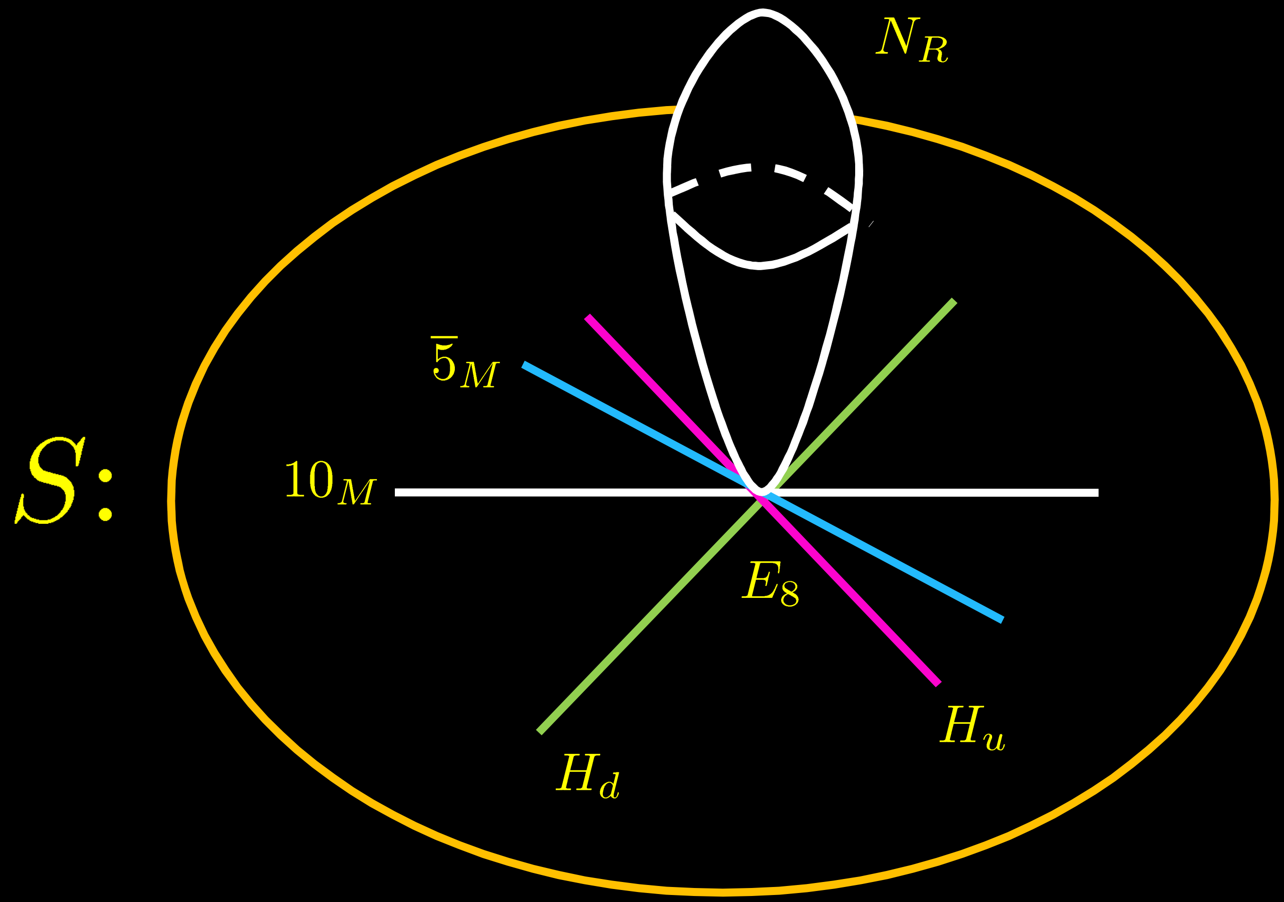
Point Unification



$E_7 \Rightarrow 5_H 10_M 10_M + \bar{5}_H \bar{5}_M 10_M$

$W_{4d} \supset 5_H \bar{5}_M N_R$

Point Unification



Extra Matter?

Roadmap

- Flavor and E_8
- ~~SUSY~~ and Cosmology

Monodromy and E_8

$E_8 \supset SU(5)_{GUT} \times SU(5)_{\perp}$

$$248 \rightarrow (5_G, 10_{\perp}) + (\overline{5}_G, \overline{10}_{\perp}) + (10_G, \overline{5}_{\perp}) + (\overline{10}_G, 5_{\perp}) + adj$$

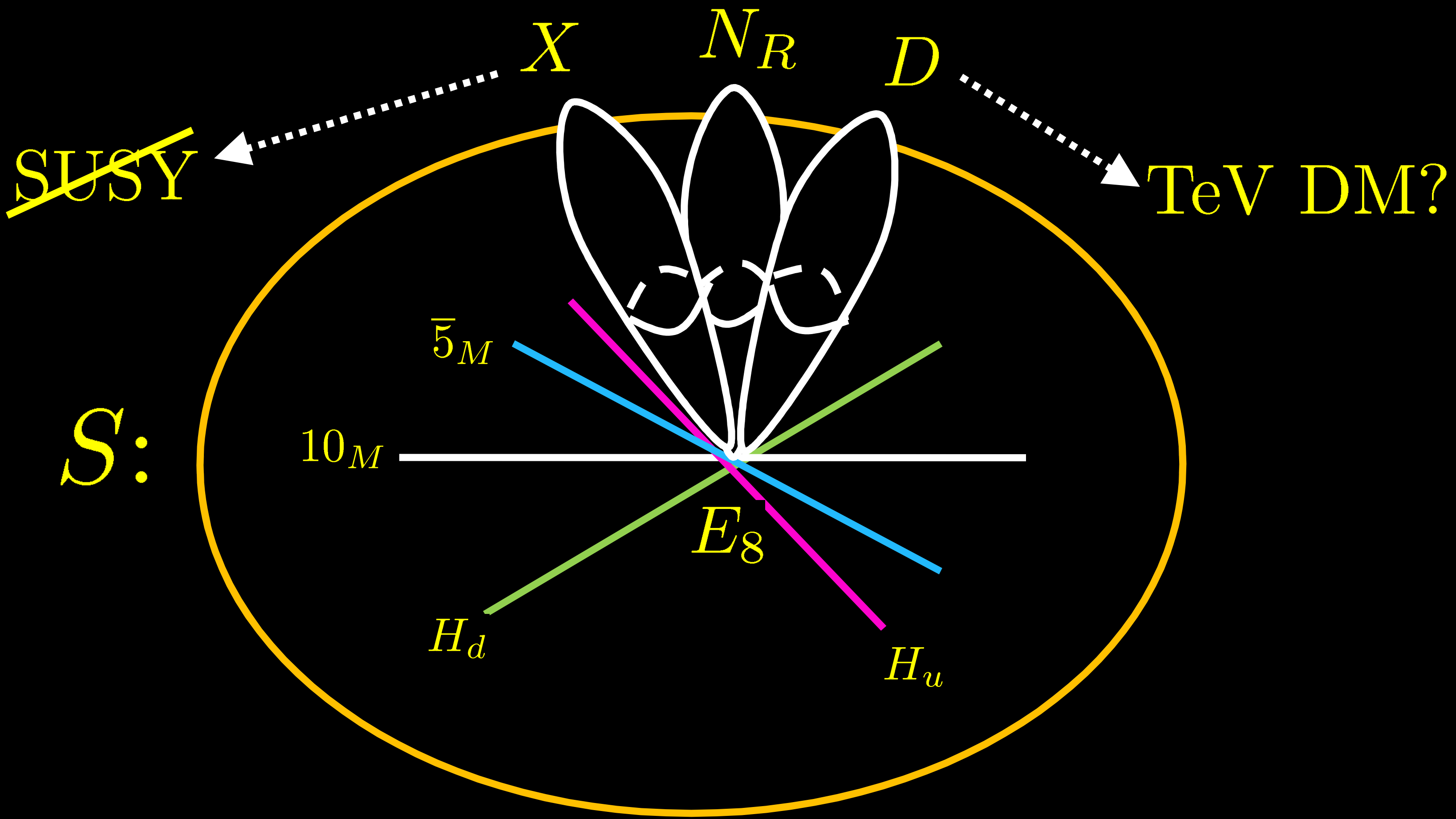
Monodromy:
$$\begin{tikzcd} 10_G^{(1)} \arrow[r, <->, dotted, bend right] \arrow[rr, <->, dotted, bend left] & 10_G^{(2)} \arrow[r, <->, dotted, bend right] & 10_G^{(3)} \end{tikzcd}$$

$G_{mono} \subset S_5$

Flavor + Monodromy $\Rightarrow$ Small List of Available Groups:

$$Z_2, Z_2 \times Z_2, Z_3, S_3, Dih_4$$

GUT Singlets



~~SUSY~~

X

N_R

D

TeV DM?

S:

Left-Overs?

$E_8 \supset SU(5)_{GUT} \times SU(5)_{\perp}$+ Monodromy $\Rightarrow$

Almost no room left except:

JJH Tavanfar Vafa '08

~~SUSY~~: $U(1)_{PQ}$, $\int d^4\theta \frac{X^\dagger H_u H_d}{\Lambda_{UV}}$ and $\int d^2\theta X Y_R Y'_{\overline{R}}$

$\langle X \rangle = x + \theta^2 F \Rightarrow \mu$-term and min. Gauge Mediation

In nearly all cases messengers in $10 \oplus \overline{10}$

Some Singlets: TeV Dark Matter?

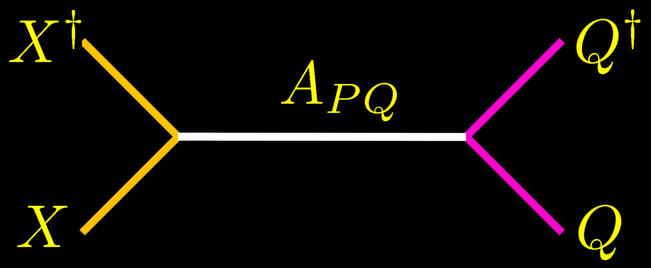
PQ Deformed GMSB

JJH, Vafa ‘08

String Theory ⇒ $U(1)_{PQ}$ gauge boson

Heavy $U(1)_{PQ}$ exchange ⇒

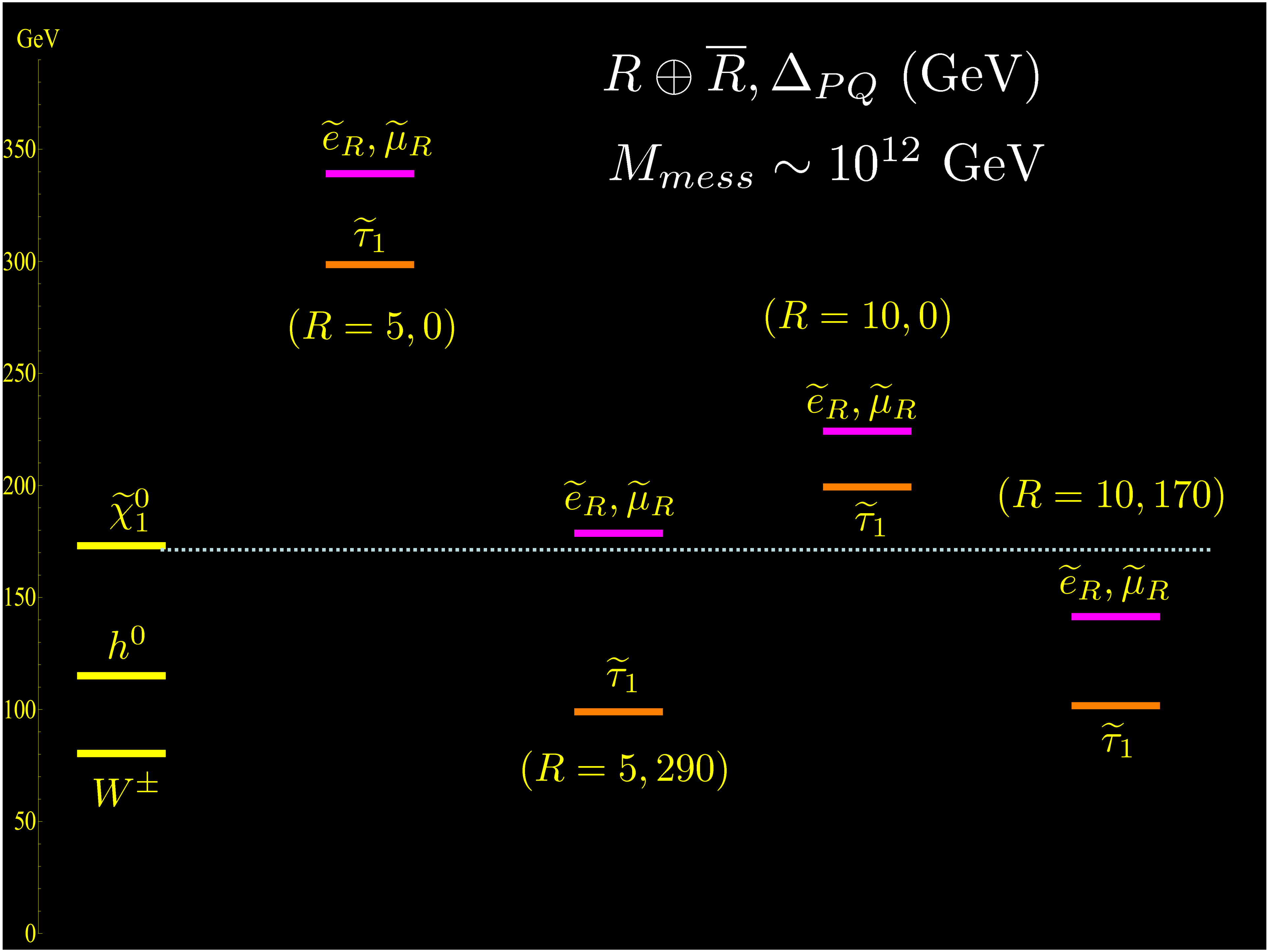
(see Arkani-Hamed, Dine, Martin ’97)



@ UV: $m^2_{soft} = m^2_{mGMSB} - q\Delta^2_{PQ}$

For most common $10 \oplus \overline{10}$ scenario

$\widetilde{\tau}_1$ is typically the quasi-stable NLSP



$R \oplus \overline{R}, \Delta_{PQ}$ (GeV)

$M_{mess} \sim 10^{12}$ GeV

LHC?

JJH, Kane, Shao Vafa '09

NLSP is either quasi-stable $\widetilde{\tau}_1$ or $\widetilde{\chi}_1^0$

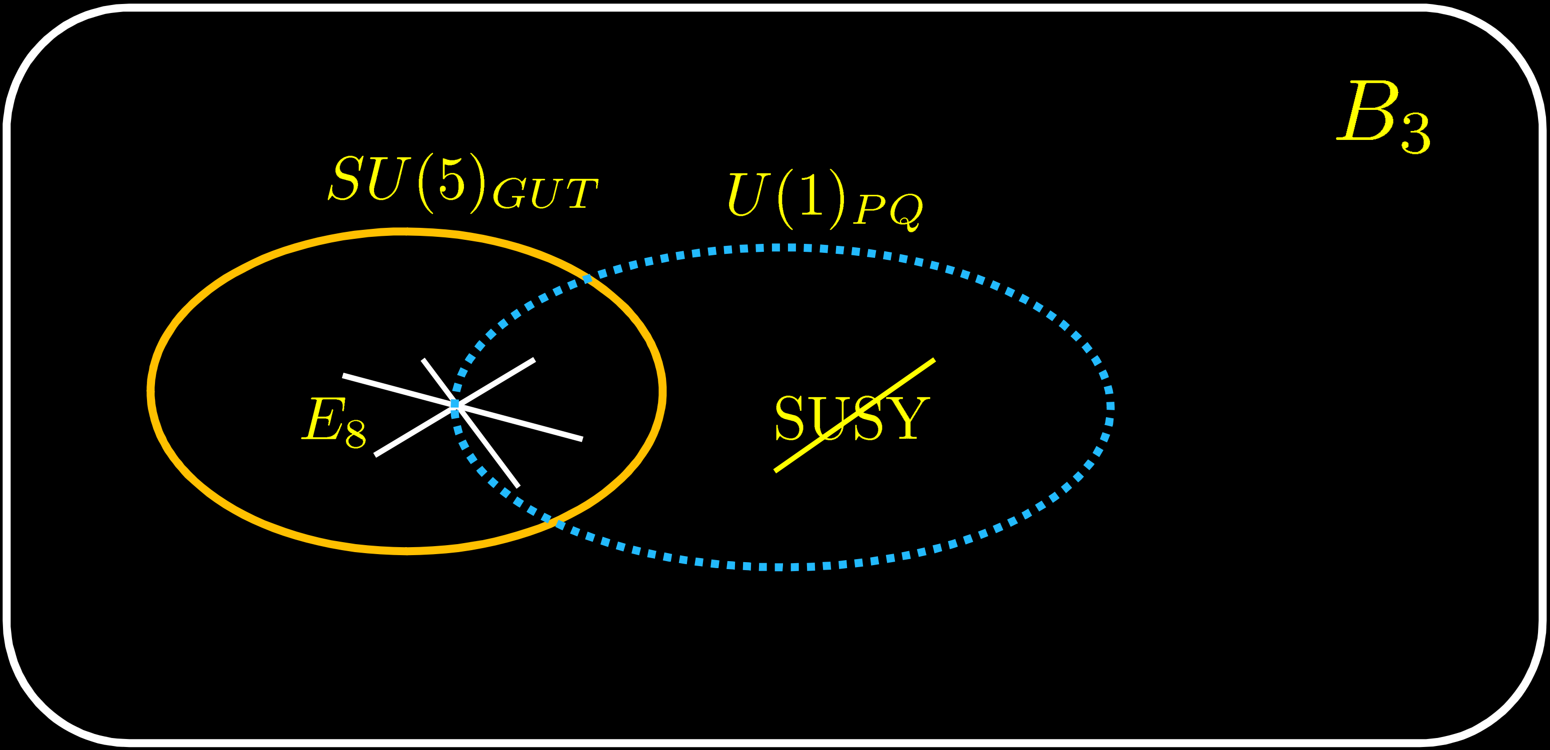
$\widetilde{\tau}_1$ NLSP: $\widetilde{\tau}_1$ track $\Rightarrow$ mass reconstruction of Δ_{PQ}

$\widetilde{\chi}_1^0$ NLSP e.g. $5 \oplus \overline{5}$ case

can STILL distinguish F-th($\Delta_{PQ}^{(1)}$) and F-th($\Delta_{PQ}^{(2)}$)

5 fb^{-1}: ± 100 GeV 50 fb^{-1}: ± 10 GeV

TeV Dark Matter?



Must be nearby PQ brane to get TeV mass JJH Tavanfar Vafa ’09

Singlets from E_8 all decay too rapidly ⇒ no candidate

Any Dark Matter?

JJH Tavanfar Vafa ’08

~~SUSY~~ $\Rightarrow$ 10 – 100 MeV LSP Gravitino

“Gravitino Problem”: Predicted $\Omega_{3/2}h^2 \gg 1$?

Oscillation and decay of saxion dilutes thermal relics

$\Omega_{3/2}^{F-th}h^2 \sim 0.1$

$\Rightarrow$ astrophysical origin for PAMELA, ATIC, FERMI, ...

JJH Tavanfar Vafa ’09

Conclusions

- Bottom Up GUTs and F-theory
- Flavor $\Rightarrow E_8$
- $10 \oplus \overline{10}$ Messengers Most Common
- $10 - 100$ MeV Gravitino = Dark Matter