Cosmological perturbations from multi-field inflation

David Langlois
(APC, Paris)

Introduction

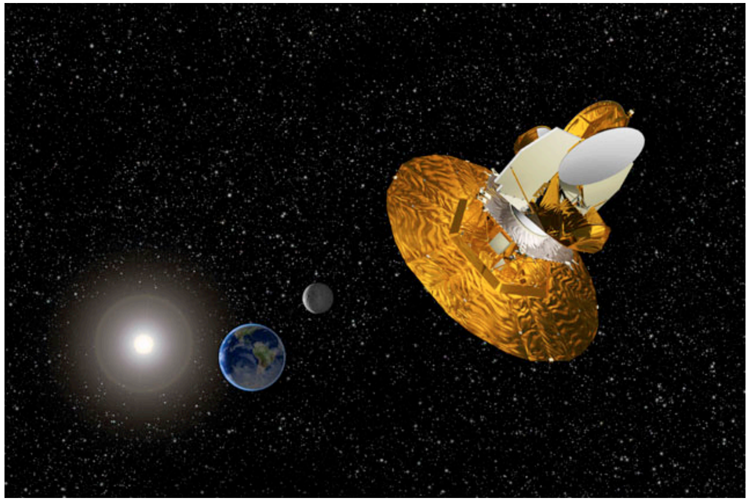
- Theoretical issues
  - Nature of the inflaton(s) ?
  - Number of fields involved for inflation and for the generation of cosmological perturbations ?
  - Dynamics of these fields ? During and after inflation…

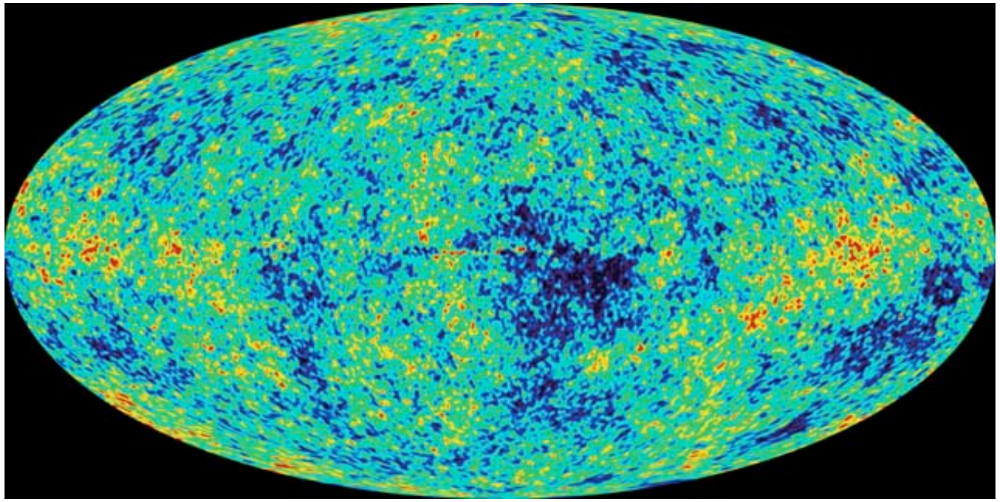
- Observational « windows »
  - Primordial gravitational waves ?
  - Entropy perturbations ?
  - Non-Gaussianities ?

- In this talk: multi-field inflation & non-Gaussianities

  Particular case: multi-field DBI inflation

Observations

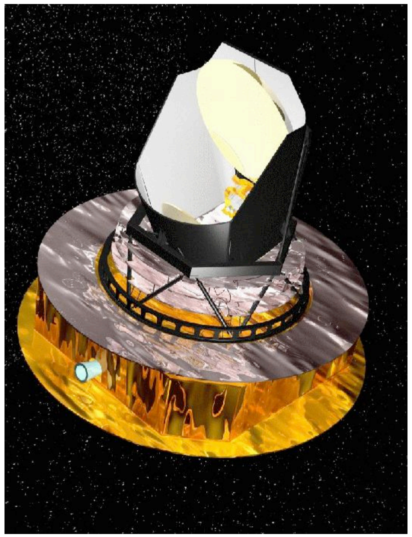


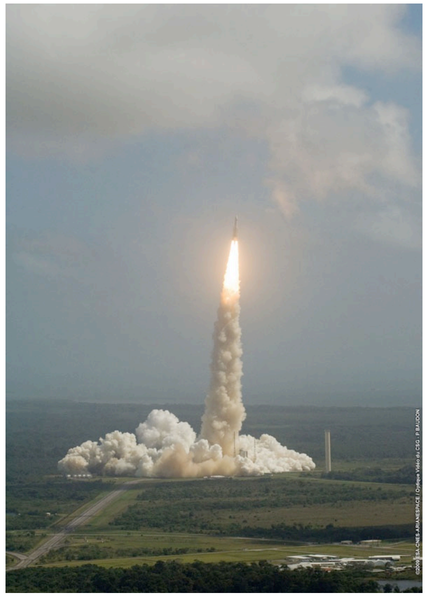


WMAP

- Power spectra of perturbations
- Non-gaussianities
- Tensor modes…

Planck





Cosmological perturbations

- Perturbed metric (longitudinal gauge, scalar perts)

$$ds^2 = -(1+2A)dt^2 + a^2(t)(1-2\psi)\delta_{ij}dx^i dx^j$$

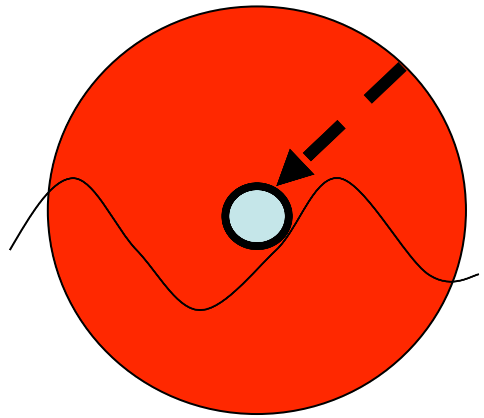
- Constant energy curvature perturbation $\zeta \equiv -\psi - \frac{H}{\dot{\rho}}\delta\rho$
- Comoving curvature perturbation $\mathcal{R} = \psi - \frac{H}{\rho + p}\delta q$
- In single field inflation: $\mathcal{R} = \psi + \frac{H}{\dot{\phi}}\delta\phi \equiv \frac{H}{\dot{\phi}}Q_\phi$

- **On large scales,** $\mathcal{R}$ **is conserved for adiabatic perturbations (such as** $\delta P/\delta\rho = \dot{P}/\dot{\rho}$ **)**

Standard single field inflation

- The vacuum quantum fluctuations of the scalar field are amplified at **Hubble crossing** $(k = aH)$

$$\mathcal{P}_{Q_\phi} = \left(\frac{H}{2\pi}\right)^2 \qquad \mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2$$



- Since $\mathcal{R}$ is conserved on large scales, it is sufficient to compute $\mathcal{P}_{\mathcal{R}}$ at Hubble crossing.

Multi-field inflation

- $\mathcal{R}$ is **not conserved** on large scales, in general.

[Starobinsky, Yokayama '95]

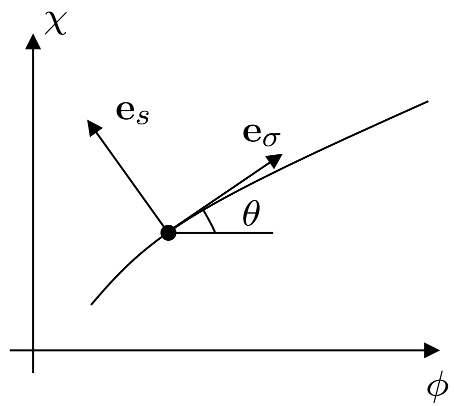
- **Transfer** from the **entropy** (or isocurvature) mode(s) into the **adiabatic** (or curvature) mode.

Gordon et al. '00
Groot Nibbelink & Van Tent '00

$$\begin{aligned} \delta\sigma &= \cos\theta\ \delta\phi + \sin\theta\ \delta\chi, \\ \delta s &= -\sin\theta\ \delta\phi + \cos\theta\ \delta\chi \end{aligned}$$

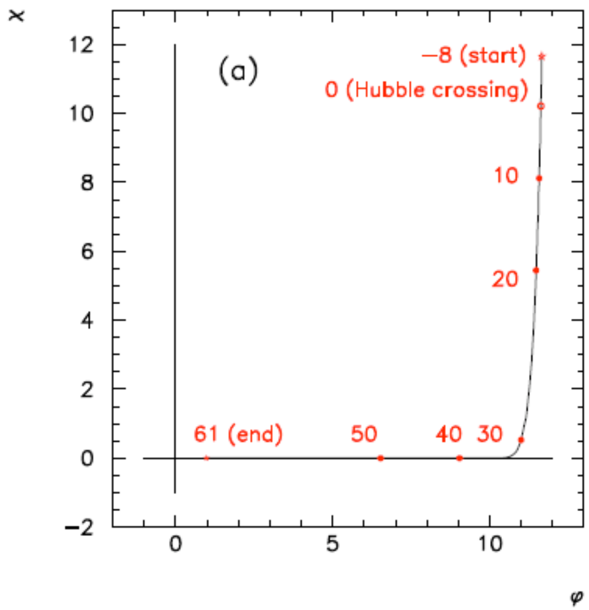
$$\mathcal{R} = \frac{H}{\dot\sigma} Q_\sigma = \psi + \frac{H}{\dot\sigma}\delta\sigma \qquad \dot\sigma = \sqrt{\dot\phi^2 + \dot\chi^2}$$

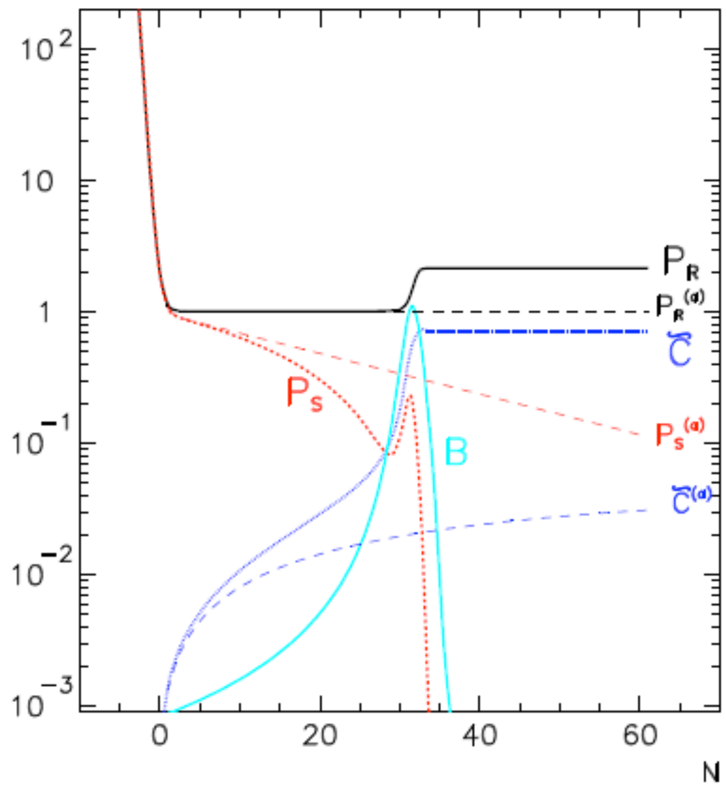
$$\dot{\mathcal{R}} = \frac{2H}{\dot\sigma}\dot\theta\delta s + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$



Double inflation

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$





$$\mathcal{S} = \frac{H}{\dot\sigma}\delta s$$

Can be solved analytically …

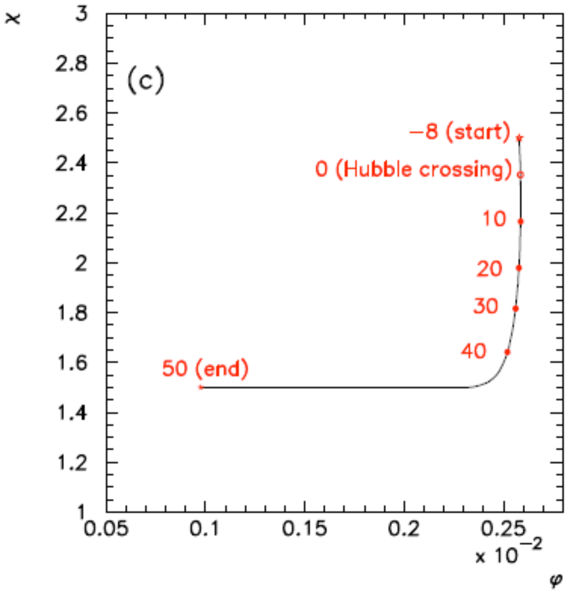
[Polarski & Starobinsky '92 '94; DL'99]

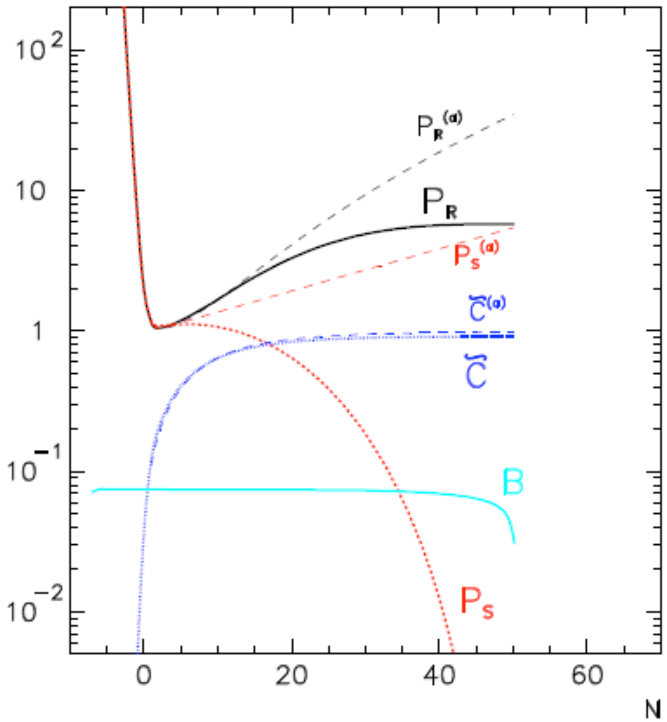
"Roulette" inflation

Conlon & Quevedo '05
Bond, Kofman, Prokushkin, Vaudevrange '06

$$L_{\text{kin}} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}e^{2b(\phi)}\partial_\mu\chi\partial^\mu\chi$$

$$b(\phi) = b_0 - \frac{1}{3}\ln\left(\frac{\phi}{M_P}\right)$$





Lalak, DL, Pokorski, Turzynski '07

General multi-field inflation

- Generalized Lagrangians

[DL, S. Renaux-Petel, D.Steer & T. Tanaka,' 08]

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + P(X^{IJ}, \phi^I)\right]$$ with $$X^{IJ} \equiv -\frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^J$$

  - particular case: $P(X, \phi^I) = X - V(\phi^I)$ with $X = G_{IJ}X^{IJ}$

  - single field k-inflation

[Armendariz-Picon, Damour, Mukhanov '99]

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + P(X, \phi^I)\right]$$ $$X \equiv -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

  - Single field DBI inflation

[Silverstein, Tong '04; Alishahiha, Silverstein, Tong'04]

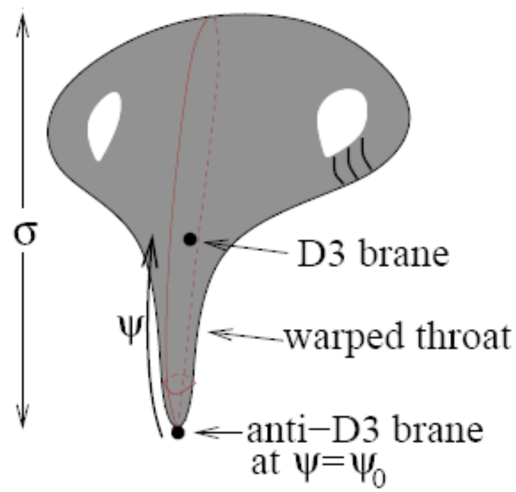
$$P(X, \phi^I) = -\frac{1}{f}\left(\sqrt{1 - 2fX} - 1\right) - V(\phi^I)$$

DBI inflation

- Brane inflation: inflaton as the position of a brane

  ⇨ effective 4D scalar

- Moving D3-brane in a higher-dimensional background



$$ds^2 = h^{-1/2}(y^K) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^K) G_{IJ}(y^K) dy^I dy^J$$

Its dynamics is governed by a Dirac-Born-Infeld action

$$L_{DBI} = -\frac{1}{f}\sqrt{-\det\left(g_{\mu\nu} + f\, G_{IJ}\,\partial_\mu\phi^I \partial_\nu\phi^J\right)}$$

DBI inflation

- One dimensional effective motion (radial motion)

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{f} \sqrt{1 + f\, \partial_\mu \phi \partial^\mu \phi} - V(\phi) \right] = \int d^4x \sqrt{-g}\, P(X, \phi)$$

- In the homogeneous case,

$$S = \int dt\, a^3 \left[-\frac{1}{f} \sqrt{1 - f\, \dot{\phi}^2} - V(\phi) \right]$$

1. Slow-roll regime: $f\, \dot{\phi}^2 \ll 1$ [KKLMMT]

2. "Relativistic" regime:

$$1 - f\, \dot{\phi}^2 \ll 1 \Rightarrow |\dot{\phi}| \simeq 1/\sqrt{f}$$

[Silverstein, Tong '04;
Alishahiha, Silverstein, Tong'04]

Multi-field DBI inflation

- Take into account the other internal coordinates [Easson, Gregory, Tasinato & Zavala '07; Huang, Shiu & Underwood '07]

  ⇨ **multi-field effective description !**

$$P(X^{IJ},\phi^K) = \tilde{P}(\tilde{X},\phi^K) = -\frac{1}{f}\sqrt{1-2f\tilde{X}} - V$$

[DL, Renaux-Petel, Steer & Tanaka, PRL '08]

$$\det\left(\delta^\mu_\nu + f\,G_{IJ}\,\partial^\mu\phi^I\partial_\nu\phi^J\right) = 1-2f\tilde{X}$$

$$\tilde{X} \equiv G_{IJ}X^{IJ} - 2fX^{[I}_I X^{J]}_J + 4f^2 X^{[I}_I X^J_J X^{K]}_K - 8f^3 X^{[I}_I X^J_J X^K_K X^{L]}_L$$

$$X^{IJ} \equiv -\frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^J, \qquad X^I_K = X^{IJ}G_{JK}$$

- Homogeneous case

$$S = \int dt\, a^3\left[-\frac{1}{f}\sqrt{1-f\,G_{IJ}\,\dot{\phi}^I\dot{\phi}^J} - V(\phi^K)\right]$$

Linear perturbations: general case

- Scalar degrees of freedom = scalar field fluctuations in the flat gauge $\phi^I = \bar{\phi}^I + Q^I$

- Their dynamics is described by the second order action

$$S_{(2)} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3 \quad \Big[\left(P_{<IJ>} + P_{<MJ>,<IK>}\dot{\phi}^M\dot{\phi}^K\right)\dot{Q}^I\dot{Q}^J$$
$$-P_{<IJ>}h^{ij}\partial_i Q^I\partial_j Q^J - \mathcal{M}_{KL}Q^K Q^L + 2\,\Omega_{KI}Q^K\dot{Q}^I\Big]$$

where $P_{<IJ>} \equiv \dfrac{\partial P}{\partial X^{(IJ)}}$, etc

[DL, Renaux-Petel,
Steer & Tanaka, PRD '08]

and the coefficients $\mathcal{M}_{IJ}$ and Ω_{IJ} depend on the background values of the fields and of the derivatives of P.

DBI case

- Lagrangian $P(X^{IJ}, \phi^K) = \tilde{P}(\tilde{X}, \phi^K) = -\frac{1}{f}\sqrt{1 - 2f\tilde{X}} - V$

- 2nd order action

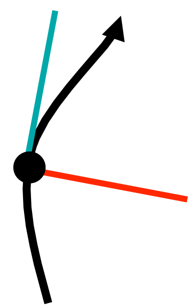
$$
\begin{aligned}
S_{(2)} &= \frac{1}{2}\int \mathrm{d}t\, d^3x\, a^3 \Big[\tilde{P}_{,\tilde{X}}\tilde{G}_{IJ}\mathcal{D}_t Q^I \mathcal{D}_t Q^J + 2\tilde{P}_{,\tilde{X}J}\dot{\phi}_I Q^J \mathcal{D}_t Q^I \\
&\quad - \frac{1}{a^2} c_s^2 \tilde{P}_{,\tilde{X}}\tilde{G}_{IJ}\partial_i Q^I \partial^i Q^J - \mathcal{M}_{IJ} Q^I Q^J\Big]
\end{aligned}
$$

with $\tilde{G}_{IJ} = G_{IJ} + \frac{\tilde{P}_{\tilde{X}\tilde{X}}}{\tilde{P}_{,\tilde{X}}}\dot{\phi}_I\dot{\phi}_J$ $\qquad c_s^2 \equiv \frac{\tilde{P}_{\tilde{X}}}{\tilde{P}_{\tilde{X}} + 2\tilde{X}P_{\tilde{X}\tilde{X}}}$ Effective speed of sound

$$\mathcal{D}_t Q^I \equiv \dot{Q}^I + \Gamma^I_{JK}\dot{\phi}^I Q^K$$

- **Adiabatic/entropy decomposition**

Kinetic terms $\tilde{P}_{,\tilde{X}}\tilde{G}_{IJ}\dot{Q}^I\dot{Q}^J \rightarrow \tilde{P}_{,\tilde{X}}\left(\frac{1}{c_s^2}\dot{Q}_\sigma^2 + \dot{Q}_s^2\right)$



Quantum fluctuations

- Canonically normalized variables $v_\sigma \equiv \frac{a}{c_s}\sqrt{\tilde{P}_X}\, Q_\sigma\,, \qquad v_s \equiv a\sqrt{\tilde{P}_X}\, Q_s$

- Equations of motion

$$v_\sigma'' + \left(k^2 c_s^2 - \frac{z''}{z}\right) v_\sigma = 0\,, \qquad v_s'' + \left(k^2 c_s^2 - \frac{\alpha''}{\alpha}\right) v_s = 0\,.$$

with $z = \frac{a\dot{\sigma}}{c_s H}\sqrt{\tilde{P}_X}, \quad \alpha = a\sqrt{\tilde{P}_X}$

- **Amplification at sound horizon !** $(k c_s \simeq aH)$

$$\mathcal{P}_{Q_\sigma} \simeq \frac{H^2}{4\pi^2 c_s \tilde{P}_X} = \frac{H^2}{4\pi^2} \qquad \mathcal{P}_{Q_s} \simeq \frac{H^2}{4\pi^2 c_s^3 \tilde{P}_X} = \frac{H^2}{4\pi^2 c_s^2}$$

- Entropy modes are enhanced $Q_s \simeq \frac{1}{c_s} Q_\sigma$

Primordial spectra

- Relating to the curvature perturbation $\mathcal{R} = \frac{H}{\dot{\sigma}} Q_\sigma$

$$\Rightarrow \quad \mathcal{P}_{\mathcal{R}_*} \simeq \frac{H^4}{8\pi^2 c_s \tilde{X} \tilde{P}_{\tilde{X}}} = \frac{H^2}{8\pi^2 \,\epsilon\, c_s}$$

[same as single-field k-inflation:
Garriga & Mukhanov '99]

- In the multi-field case, $\mathcal{R}$ can evolve on large scales

$$\mathcal{R} = \mathcal{R}_* + T_{\mathcal{RS}} \mathcal{S}_* \quad \left[\mathcal{S} = c_s \frac{H}{\dot{\sigma}} Q_s \right]$$

$$\mathcal{P}_{\mathcal{R}} = (1 + T_{\mathcal{RS}}^2) \mathcal{P}_{\mathcal{R}_*} = \frac{\mathcal{P}_{\mathcal{R}_*}}{\cos^2 \Theta}$$

- Tensor modes

$$\mathcal{P}_{\mathcal{T}} = \left(\frac{2H^2}{\pi^2} \right)_{k=aH} \quad \Rightarrow \quad r = \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = 16\,\epsilon\, c_s \cos^2 \Theta$$

$$r = 16\,\epsilon\, c_s \qquad\qquad r = 16\,\epsilon \cos^2 \Theta$$

single-field limit [Garriga & Mukhanov]

standard multi-field [Wands et al. '02]

Non-Gaussianities

- **Bispectrum**

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta^{(3)}(\sum_i \mathbf{k}_i) B_\zeta(k_1, k_2, k_3).$$

One also uses the f_{NL} parameter

$$\frac{6}{5} f_{\mathrm{NL}} \equiv \frac{\Pi_i k_i^3}{\sum_i k_i^3} \frac{B_\zeta}{4\pi^4 m_{\mathrm{P}}^2 \mathcal{P}_\zeta^2}$$

$$\left[\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = (2\pi)^3 \delta_{IJ} \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \frac{2\pi^2}{k_1^3} \mathcal{P}_\zeta(k_1) \right]$$

- **Link with inflation**

  Using the δN-formalism [Lyth & Rodriguez '05]

$$\zeta \simeq \sum_I N_{,I} \delta\varphi_*^I + \frac{1}{2} \sum_{IJ} N_{,IJ} \delta\varphi_*^I \delta\varphi_*^J$$

Non-Gaussianities

$$\begin{aligned}\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle &= \sum_{IJK} N_{,I}N_{,J}N_{,K}\langle\delta\varphi^I_{k_1}\delta\varphi^J_{k_2}\delta\varphi^K_{k_3}\rangle + \\ &\quad \frac{1}{2}\sum_{IJKL} N_{,I}N_{,J}N_{,KL}\langle\delta\varphi^I_{k_1}\delta\varphi^J_{k_2}(\delta\varphi^K\star\delta\varphi^L)_{k_3}\rangle + \text{perms},\end{aligned}$$

- If the scalar field perturbations are quasi-Gaussian, **local NG**

$$\frac{6}{5}f_{\rm NL} = \frac{N_I N_J N^{IJ}}{(N_K N^K)^2}$$

- If the scalar field three-point function is significant, like in models with non standard kinetic terms, **equilateral NG**

- **Observational constraints** $\qquad -9 < f_{NL}^{(\rm local)} < 111 \quad (95\%\ \rm CL)$

  [WMAP5: Komatsu et al '08] $\qquad -151 < f_{NL}^{(\rm equil)} < 253 \quad (95\%\ \rm CL)$

Non-Gaussianities

- In the small c_s limit, the dominant terms in the third order action are

$$S_3^{(\mathrm{main})} = \int dt d^3x \left\{ \frac{a^3}{2c_s^5 \dot\sigma} \left[\dot Q_\sigma^3 + c_s^2 \dot Q_\sigma \dot Q_s^2 \right] - \frac{a}{2c_s^3 \dot\sigma} \left[\dot Q_\sigma (\nabla Q_\sigma)^2 \right.\right.$$
$$\left.\left. + c_s^2 \dot Q_\sigma (\nabla Q_s)^2 - 2c_s^2 \dot Q_s \nabla Q_\sigma \nabla Q_s) \right] \right\}$$

- Three-point function

$$\begin{aligned} \langle \mathcal{R}(\boldsymbol{k}_1)\mathcal{R}(\boldsymbol{k}_2)\mathcal{R}(\boldsymbol{k}_3)\rangle &= (\mathcal{A}_\sigma)^3 \langle Q_\sigma(\boldsymbol{k}_1) Q_\sigma(\boldsymbol{k}_2) Q_\sigma(\boldsymbol{k}_3)\rangle \\ &\quad + \mathcal{A}_\sigma (\mathcal{A}_s)^2 (\langle Q_\sigma(\boldsymbol{k}_1) Q_s(\boldsymbol{k}_2) Q_s(\boldsymbol{k}_3)\rangle + \text{perm.}) \\ &= (\mathcal{A}_\sigma)^3 \langle Q_\sigma(\boldsymbol{k}_1) Q_\sigma(\boldsymbol{k}_2) Q_\sigma(\boldsymbol{k}_3)\rangle \left(1 + T_{\mathcal{RS}}^2\right) \end{aligned}$$

- Writing

$$\langle \mathcal{R}(\boldsymbol{k}_1)\mathcal{R}(\boldsymbol{k}_2)\mathcal{R}(\boldsymbol{k}_3)\rangle = -(2\pi)^7 \delta(\sum_i \boldsymbol{k}_i)(\frac{3}{10} f_{NL} (\mathcal{P}_\mathcal{R})^2) \frac{\sum_i k_i^3}{\prod_i k_i^3}$$

  one finds

$$f_{NL}^{(\mathrm{equil})} = -\frac{35}{108} \frac{1}{c_s^2} \frac{1}{1 + T_{\mathcal{RS}}^2} = -\frac{35}{108} \frac{1}{c_s^2} \cos^2\Theta$$

$$\text{WMAP5}: \quad -151 < f_{NL}^{(\mathrm{equil})} < 253 \quad (95\%\ \text{CL})$$

Including bulk forms

[DL, Renaux-Petel, Steer, 0902.2941]

- One can include the NS-NS and R-R bulk forms

$$S_{\rm DBI} = -T_3 \int \mathrm{d}^4x \, e^{-\Phi} \sqrt{-\det\left(\hat{\gamma}_{\mu\nu} + \hat{B}_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)}$$

$$S_{\rm WZ} = -T_3 \int_{\rm brane} \sum_{n=0,2,4} \hat{C}_n \wedge e^{\left(\hat{B}_2 + 2\pi\alpha' F_2\right)}\Big|_{4-\rm form}$$

- Variation w.r.t A_0 yields the constraint

$$2\pi\alpha' h^{1/2} A_0 - b_{IJ}\dot{\phi}^I Q^J = 0$$

⇨ all new scalar terms cancel in the 2nd and 3rd order actions !

- Vector degrees of freedom

Observational constraints

[DL, Renaux-Petel, Steer, 0902.2941]

- Spectral index

$$1 - n_{\mathcal{R}} \equiv \frac{\mathrm{d}\ln\mathcal{P}_{\mathcal{R}}}{\mathrm{d}\ln k} = -2\epsilon_* - \eta_* - s_* - \alpha_*\sin(2\Theta) - 2\beta_*\sin^2\Theta$$

$$\simeq \frac{\sqrt{3|f_{\rm NL}|}r}{4\cos^3\Theta} - \frac{\dot{f}}{Hf} + \alpha_*\sin(2\Theta) + 2\beta_*\sin^2\Theta$$

$$\left[\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{H\epsilon}, \quad s = \frac{\dot{c}_s}{Hc_s}\right]$$

- Single field UV scenario ($\dot{f} > 0$)

$$r > \frac{4}{\sqrt{3|f_{\rm NL}|}}(1 - n_{\mathcal{R}}) \gtrsim 0.1\,(1 - n_{\mathcal{R}}) \gtrsim 10^{-3}$$

[Lidsey & Huston ’07]

incompatible with the Baumann-McAllister bound (’06)

In the multi-field scenario, this constraint no longer applies !

Modulated trapping

[DL + Sorbo, arXiv:0906.1813]

- **Particle production during inflation**

[Chung, Kolb, Riotto, Tkachev '99]

e.g. coupling of the inflaton to a massive fermion field $\mathcal{L}_{\rm int} = \lambda\phi\,\bar{\psi}\psi$

Particle production occurs when $m_{\rm eff} = m - \lambda\phi = 0$, i.e. $\phi_* = \dfrac{m}{\lambda}$

- Particle occupation number

$$n_* = \frac{\lambda^{3/2}}{2\pi^3} v_*^{3/2}, \qquad v_* \equiv |\dot{\phi}_*|,$$

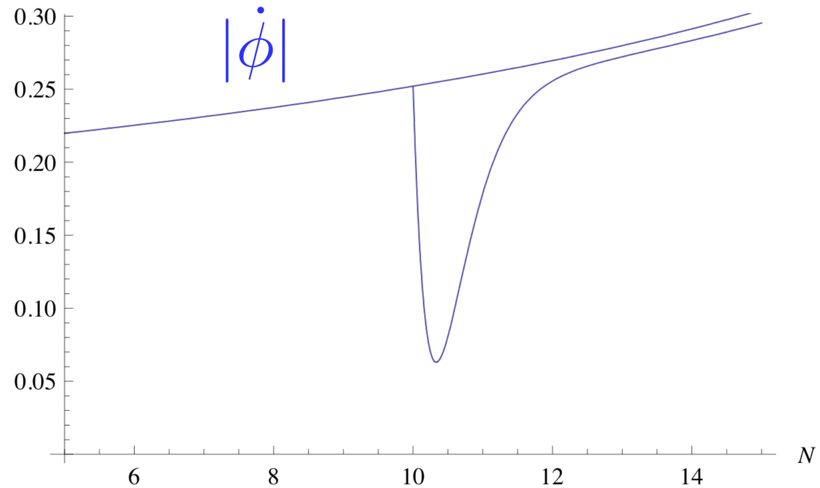
- Dilution by expansion $n(t) = n_* \left(\dfrac{a}{a_*}\right)^{-3} \Theta(t - t_*)$

Trapping

- Backreaction on the inflaton

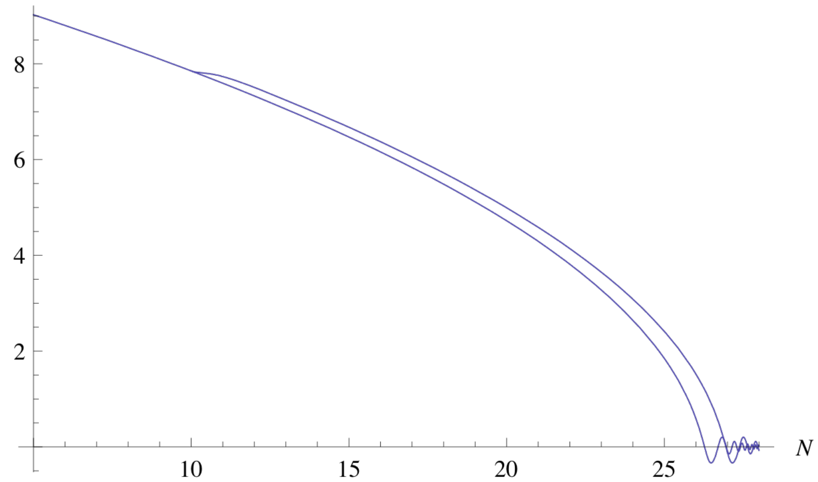
$$\ddot{\phi} + 3\,H\,\dot{\phi} + V'(\phi) = \mathcal{N}\,\lambda\,\langle\bar{\chi}\chi\rangle = \lambda\mathcal{N}n_*\left(\frac{a}{a_*}\right)^{-3}\Theta(t-t_*)$$

Slow down of the inflaton



- Time delay for the end of inflation

$$\Delta N = -H_*\,\frac{\Delta\phi}{\dot{\phi}_*} = \frac{\mathcal{N}\lambda n_*}{9H_*v_*} = \frac{\lambda^{5/2}\mathcal{N}v_*^{1/2}}{18\pi^3 H_*}$$



Modulaton

- Extra field σ

$$m_{\text{eff}} = \lambda(\sigma)\phi - m(\sigma) \quad \Rightarrow \quad \phi_* = \phi_*(\sigma)$$

- Fluctuations of σ: $\delta\sigma \sim \frac{H_k}{2\pi}$
- Number of e-folds $N = N_{\text{standard}}(\phi) + \Delta N_{\text{trapping}}(\sigma)$
- Power spectrum $\zeta = \delta N = \delta N_{\text{slow-roll}} + \Delta N_{,\sigma}\delta\sigma$

$$\mathcal{P}_\zeta = \left[\frac{1}{2\epsilon_k} + (\Delta N_{,\sigma})^2 M_P^2\right]\left(\frac{H_k}{2\pi\, M_P}\right)^2 \equiv \mathcal{P}_\zeta^{\text{inf}} + \mathcal{P}_\zeta^{\text{trapping}}$$

Perturbations

- Example

$$\lambda = \frac{\sigma}{M}, \quad m = g\,\sigma$$

- Power spectrum

$$\mathcal{P}_{\rm trap}^{1/2} = 3 \times 10^{-2}\,\beta^{3/5}\,\mathcal{N}^{2/5}\,\epsilon_*^{1/10}\,\frac{M_P}{M}\,\left(\frac{H_*}{H_k}\right)^{-1/5}\,\left(\frac{H_k}{M_P}\right)^{4/5}$$

$$\beta \equiv |\Delta\dot\phi|_{\rm max}/|\dot\phi_*|$$

- Non-Gaussianities $\zeta = \delta N = \delta N_{\text{slow-roll}} + \Delta N_{,\sigma}\delta\sigma + \frac{1}{2}\Delta N_{,\sigma\sigma}\delta\sigma^2$

  "Local type"

$$f_{\rm NL} = \frac{1}{2\,e\,\beta}\left(3 + 2\frac{\lambda\lambda''}{\lambda'^2}\right)\Xi^2$$

- Also the trispectrum…

Conclusions

Multi-field inflation generates entropy perturbations in addition to adiabatic perturbations.

⇨ Specific signatures with respect to single field inflation !

- Entropy perturbations affect the evolution of the curvature perturbation **after Hubble crossing** !
  - At the linear level: power spectrum
  - At the non-linear level: non-Gaussianities

- Depending on the models (reheating), the entropy perturbation mode can survive after inflation, and can be correlated with the adiabatic mode.

  An isocurvature contribution in the primordial perturbations can in principle be detected in cosmological observations.

⇨ **additional window on the early universe physics**

After inflation

- In some models, the isocurvature perturbations could survive after inflation

- In the radiation era:
  - Adiabatic / curvature perturbations $\mathcal{R} \simeq -\zeta$
  - Entropy / isocurvature perturbations

$$S \equiv \frac{\delta n_c}{n_c} - \frac{\delta n_\gamma}{n_\gamma} = \frac{\delta \rho_c}{\rho_c} - \frac{3}{4}\frac{\delta \rho_\gamma}{\rho_\gamma}$$

- They can be related to the perturbations during inflation:

⇨ **Correlation between adiabatic and isocurvature perts !**

DL '99

Observational constraints

- Adiabatic and isocurvature produce different peak structures in the CMB

- Sachs-Wolfe effect: $\frac{\delta T}{T} \simeq \frac{1}{5}\left(\mathcal{R}_{\rm rad} - 2\mathcal{S}_{\rm rad}\right)$ [DL '99]

$$\frac{\mathcal{P}_{\mathcal{S}}}{\mathcal{P}_{\mathcal{R}}} \equiv \frac{\alpha}{1-\alpha}$$

Impact on the CMB depends on the correlation [D.L. & Riazuelo '99]

$$\beta = \frac{\mathcal{P}_{\mathcal{S},\mathcal{R}}}{\sqrt{\mathcal{P}_{\mathcal{S}}\mathcal{P}_{\mathcal{R}}}}$$

- Present constraints [WMAP5: Komatsu et al '08]

$$\beta = 0 : \ \alpha_0 < 0.067 \quad (95\%\text{C.L.})$$

$$\beta = -1 : \ \alpha_{-1} < 0.0037 \quad (95\%\text{C.L.})$$

Observational constraints

[DL, Renaux-Petel, Steer, 0902.2941]

- Spectral index

$$1 - n_{\mathcal{R}} \equiv \frac{\mathrm{d}\ln \mathcal{P}_{\mathcal{R}}}{\mathrm{d}\ln k} = -2\epsilon_* - \eta_* - s_* - \alpha_* \sin(2\Theta) - 2\beta_* \sin^2\Theta$$

$$\simeq \frac{\sqrt{3|f_{\mathrm{NL}}|}r}{4\cos^3\Theta} - \frac{\dot{f}}{Hf} + \alpha_* \sin(2\Theta) + 2\beta_* \sin^2\Theta$$

$$\left[\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{H\epsilon}, \quad s = \frac{\dot{c}_s}{Hc_s}\right]$$

- Single field UV scenario ($\dot{f} > 0$)

$$r > \frac{4}{\sqrt{3|f_{\mathrm{NL}}|}}(1 - n_{\mathcal{R}}) \gtrsim 0.1\,(1 - n_{\mathcal{R}}) \gtrsim 10^{-3}$$

[Lidsey & Huston ’07]

incompatible with the Baumann-McAllister bound (’06)

In the multi-field scenario, this constraint no longer applies !