Resolution of $T^6/Z_{6\text{-}II}$ Heterotic (MSSM) orbifolds

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Based on:
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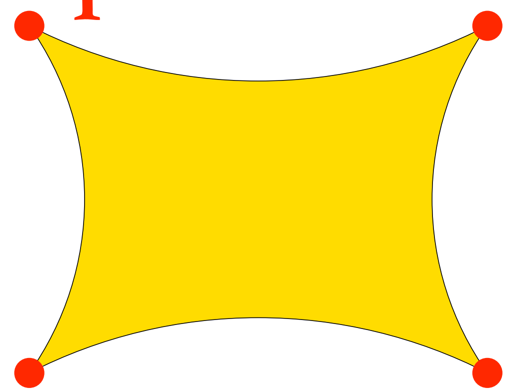
Introduction

Two main different paths to **heterotic string** phenomenology

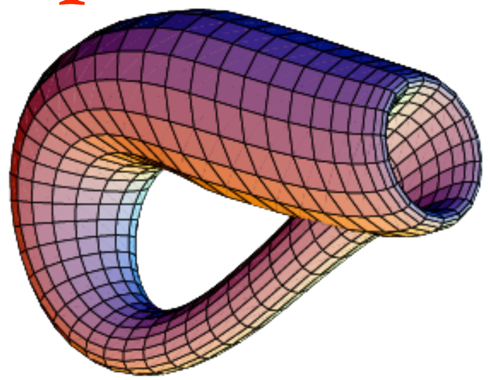
Orbifold compactifications



My task:

Reconciliate them!

Smooth manifold compactifications



Reproduce the orbifold models in a SUGRA language, i.e. as

- **compactifications of 10d SUGRA/SYM**
- **on smooth manifolds (blown-up orbifolds)**
- **in the presence of gauge fluxes.**

See also Stefan's talk tomorrow

Outline

1) Getting the smooth CY space

1.1) Resolution of orbifold singularities using toric geometry

- Local resolution of orbifold singularities
- Gluing the resolved singularities

1.2) The $\mathbf{T^6/Z_{6\text{-}II}}$ case (a source of MSSM's)

2) 10d SUGRA on the smooth CY space

2.1) Consistency conditions (flux quantization, SYM e.o.m, ...)

2.2) Matching the orbifold models

3) Matching in the $\mathbf{T^6/Z_{6\text{-}II}}$ case:

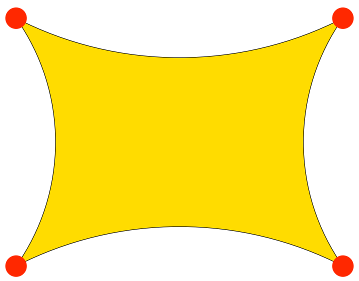
the fate of the hypercharge

4) Conclusions and outlook

1 - Getting the smooth CY space:
orbifold resolutions

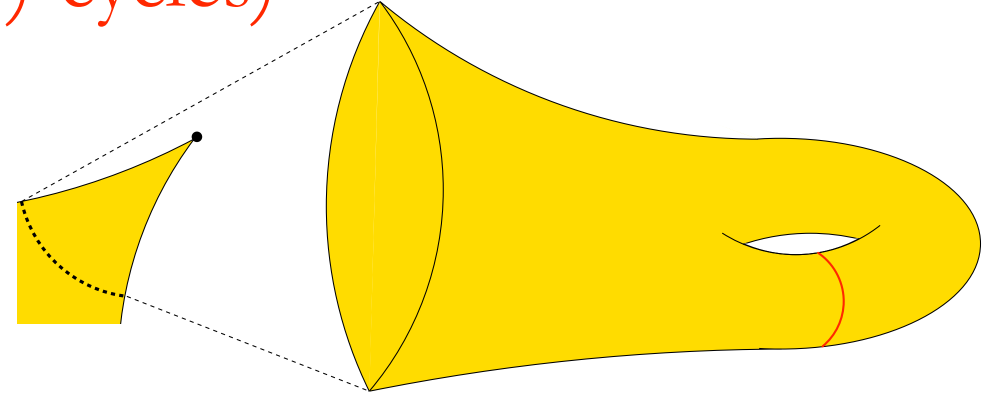
1.1 Resolution: the spirit

Ia - Given the orbifold



Ib - Cut apart each singularity and resolve it:

characterize the local geometric structure “hidden” in the singularity (localized (1,1)-cycles)



Ic - Glue together the resolved singularities:

characterize the topology of the whole CY space (non-localized cycles)

Get a smooth compact CY space
(having the original orbifold as singular limit)

Resolution of local C^n/Z_m singularities see e.g. Fulton's book

- Before resolution, the space has n divisors D_i,
- The resolution is obtained by providing
  - r new "exceptional" divisors, E_i : .
  - with n linear relations: $D_i \sim a_{ij} E_j$.
- and giving all the intersection numbers.

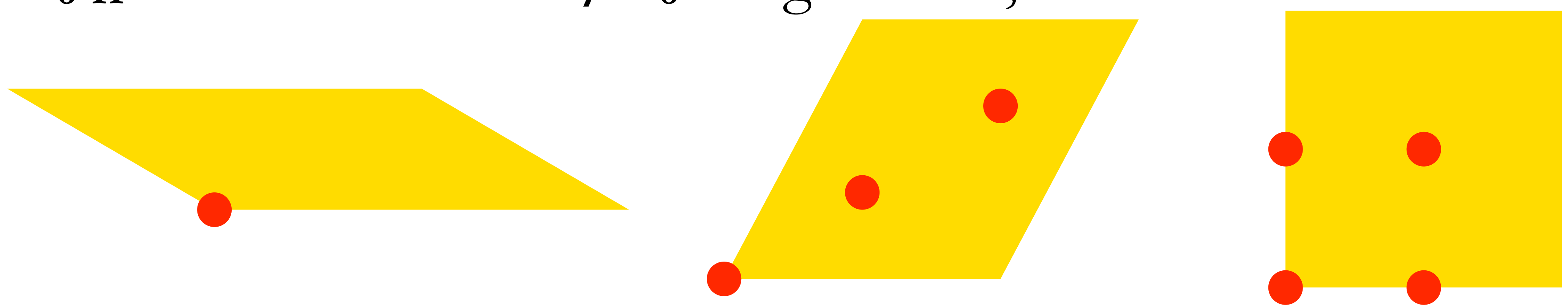
Gluing together the singularities into T^{2n}/Z_m

Lust, Reffert, Scheidegger, Stieberger

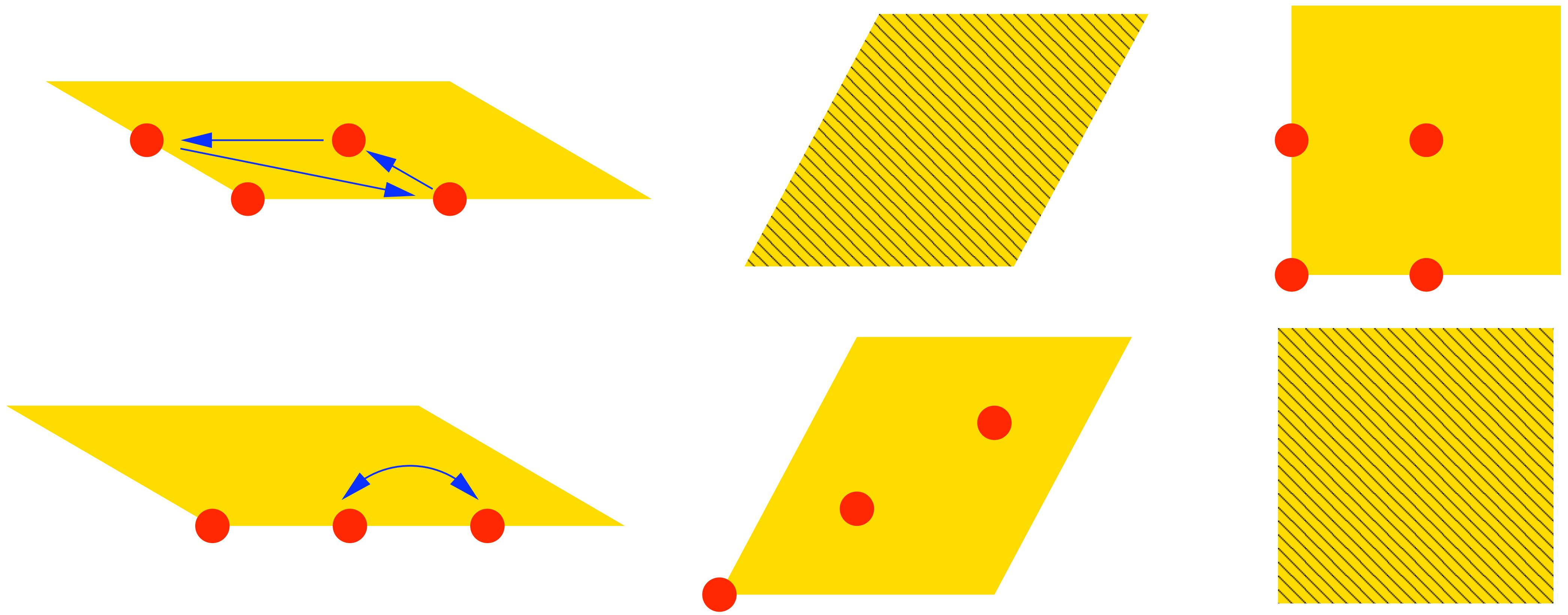
- For ach resolved singularity:
  a set $\{D_i, E_j\}$ with $D_i \sim a_{ij} E_j$ and local intersection #.
- Gluing:
  - "put together" the divisors in a single set
  - extend the linear equivalences to include all the objects
  - compute the intersections among the various divisors.
- Caveats:
  - $\mathbf{T^{2n}}$ is topologically different than $\mathbf{C^n}$
    - extra "inherited" divisors R_i .
  - Divisors may be "shared" between different singularities.

1.2 - The $T^6/Z_{6\text{-}II}$ case

- $\mathbf{T^6} = \mathbf{T^2} \times \mathbf{T^2} \times \mathbf{T^2}$, complex coordinates z_1, z_2, z_3.
- $\mathbf{Z_{6\text{-}II}}$ has $1 \times 3 \times 4$ $\mathbf{C^3/Z_6}$ singularities,



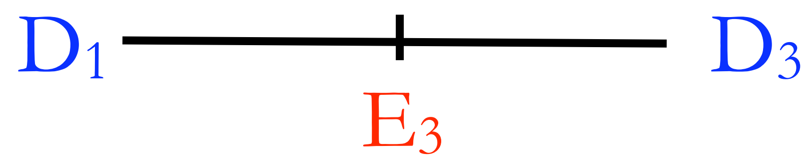
- but there are also $\mathbf{C^2/\,Z_2}$ and $\mathbf{C^2/\,Z_3}$ singularities to be resolved



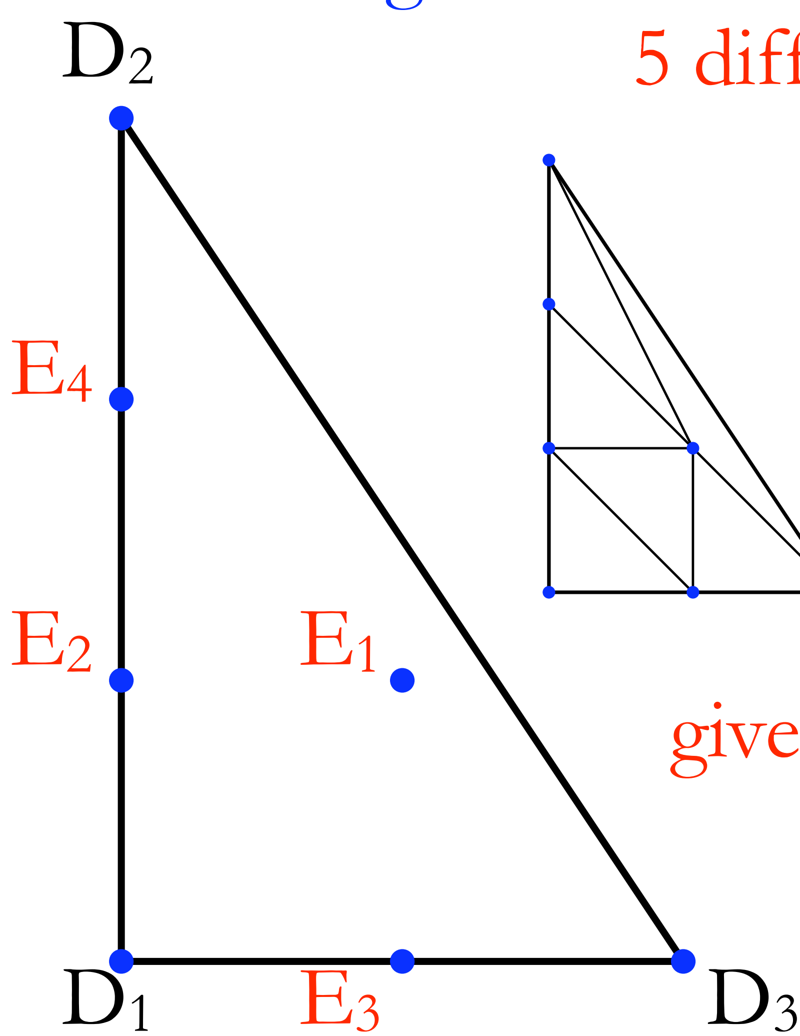
Resolve all the singularities and glue them together.

- **C^2/Z_3** singularities: 2 exceptional divisors

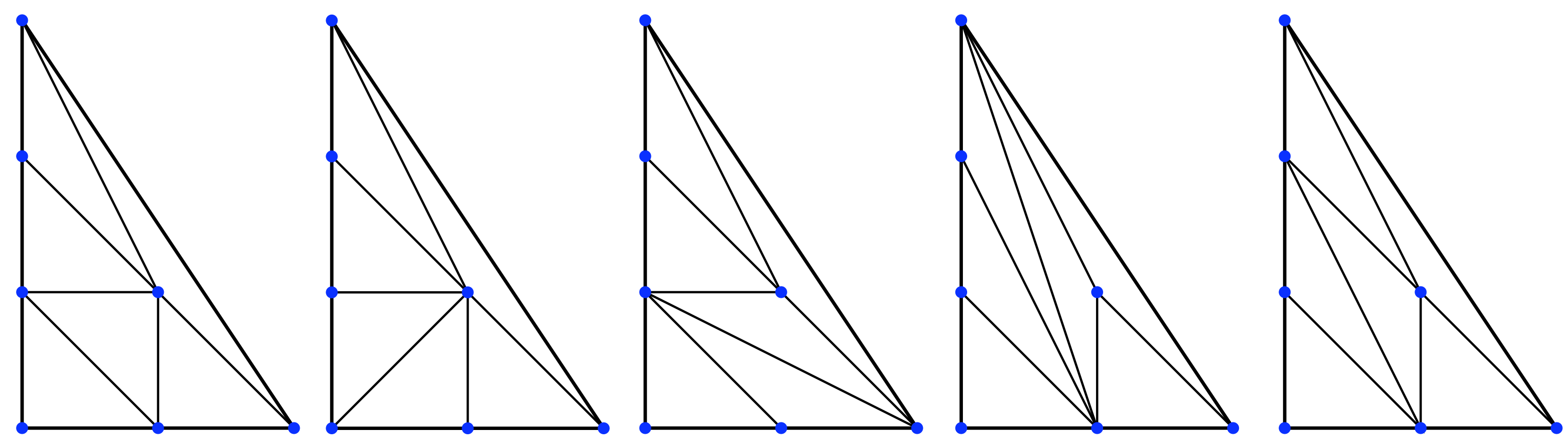
- **C^2/Z_2** singularities: 1 exceptional divisor



- **C^3/Z_6** singularities: 4 exceptional divisor



5 different possible triangulations (resolutions)



given that there are 12 such singularities, each with 5 options, we have "many" ways of resolving the **$T^6/Z_{6\text{-}II}$** orbifold.

Counting the E's:

$\mathbf{E_1}$ - localized in the 12 $\mathbf{Z_6}$ singularities, one each: 12

$\mathbf{E_3}$ - "shared" in the second torus: 4 from the $\mathbf{Z_6}$ singularities
+ $3/3 \times 4 = 4$ from the $\mathbf{Z_2}$ singularities not "inside" $\mathbf{Z_6}$

$\mathbf{E_{2/4}}$ - "shared" in the third torus: 3 from the $\mathbf{Z_6}$ singularities
+ $2/2 \times 3 = 3$ from the $\mathbf{Z_3}$ singularities not "inside" $\mathbf{Z_6}$

Counting the (1,1) forms: 32 E's + 3 R's = 35

What about the (1,2) forms?

These can be reconstructed from the "untwisted" 1-form $\mathbf{dz_i}$ and the non-orbifold-invariant exceptional divisors $\mathbf{E_i}$ ($i = 2, 3, 4$): we have exactly 10 of them (+ an extra "untwisted" 3 form): 11

Complete reconstruction of the Hodge diamond

Including the intersection number - but this depends on the triangulations we choose!

Summary:

Divisors: $R_1, R_2, R_3\,;\quad D_{1,1}\,, D_{1,2}\,, D_{1,3}\,, D_{2,\beta}\,, D_{3,\gamma}\,;$
$E_{1,\beta\gamma}\,, E_{2/4,1\beta}\,, E_{2/4,3\beta}\,, E_{3,1\gamma}, E_{3,2\gamma}$

Linear equivalences:

$$R_1 \sim 6D_{1,1} + \sum_{\beta=1}^{3}\sum_{\gamma=1}^{4} E_{1,\beta\gamma} + \sum_{\beta=1}^{3}(2E_{2,1\,\beta} + 4E_{4,1\,\beta}) + 3\sum_{\gamma=1}^{4} E_{3,1\,\gamma}\ ,$$

$$R_1 \sim 2D_{1,2} + \sum_{\gamma=1}^{4} E_{3,2\,\gamma}\ , \qquad R_1 \sim 3D_{1,3} + \sum_{\beta=1}^{3}(E_{2,3\,\beta} + 2E_{4,3\,\beta})\ ,$$

$$R_2 \sim 3D_{2,\beta} + \sum_{\gamma=1}^{4} E_{1,\beta\gamma} + \sum_{\alpha=1,3}(2E_{2,\alpha\beta} + E_{4,\alpha\beta})\ ,$$

$$R_3 \sim 2D_{3,\gamma} + \sum_{\beta=1}^{3} E_{1,\beta\gamma} + \sum_{\alpha=1,2} E_{3,\alpha\gamma}\ ,$$

Triple intersection numbers:

- triangulation independent

$$R_1R_2R_3 = 6\,,\quad R_2E_{3,1\,\gamma}^2 = -2\,,\quad R_2E_{3,2\,\gamma}^2 = -6\,,\quad R_3E_{2,1\,\beta}^2 = -2\,,$$
$$R_3E_{2,3\,\beta}^2 = -4\,,\quad R_3E_{4,1\,\beta}^2 = -2\,,\quad R_3E_{4,3\,\beta}^2 = -4\,,\quad R_3E_{2,1\,\beta}E_{4,1\,\beta} = 1$$
$$R_3E_{2,3\,\beta}E_{4,3\,\beta} = 2\,.$$

- triangulation dependent (here one specific choice)

$$E_{1,\beta\gamma}^3 = 6\,,\quad E_{2,1\,\beta}^3 = 8\,,\quad E_{3,1\,\gamma}^3 = 8\,,\quad E_{4,1\,\beta}^3 = 8\,,$$
$$E_{1,\beta\gamma}E_{2,1\,\beta}^2 = -2\,,\quad E_{1,\beta\gamma}E_{3,1\,\gamma}^2 = -2\,,\quad E_{1,\beta\gamma}E_{4,1\,\beta}^2 = -2\,,\quad E_{1,\beta\gamma}E_{2,1\,\beta}E_{4,1\,\beta} = 1$$
$$E_{2,1\,\beta}^2E_{4,1\,\beta} = -2\,.$$

Summary:

Hodge diamond

			1			
		0		0		
	0		35		0	
1		11		11		1
	0		35		0	
		0		0		
			1			

Characteristic classes

$$c(X) \;=\; \prod_{J=1}^{10} \prod_{r=1}^{32} (1 + D_J)(1 + E_r)(1 - R_1)(1 - R_2)(1 - R_3)^2$$

- first Chern class: 0
- integrated second Chern class:

$$c_2(X)E_{1,\beta\gamma} = 0\,, \quad c_2(X)E_{2,1\,\beta} = -4\,, \quad c_2(X)E_{3,1\,\gamma} = -4\,, \quad c_2(X)E_{4,1\,\beta} = -4\,,$$
$$c_2(X)E_{2,3\,\beta} = 0\,, \quad c_2(X)E_{3,2\,\gamma} = 0\,, \quad c_2(X)E_{4,3\,\beta} = 0\,,$$
$$c_2(X)R_1 = 0\,, \quad c_2(X)R_2 = 24\,, \quad c_2(X)R_3 = 24\,.$$

- Euler number: $\chi(X) = 48$

2 - 10d SUGRA on the resolved space

2.1 - Consistency conditions

1) Flux quantization: $\int_\gamma F \in \mathbf{Z}$

2) Equations of motion/SUSY:

- F must be a (1,1)-form, fulfilling the DUY condition

$\int J \wedge J \wedge F = 0$ (6d case), $\int J \wedge F = 0$ (4d case)

3) The Bianchi Identity for H must be fulfilled

$dH \sim F \wedge F - R \wedge R$ implies $\int_\gamma F \wedge F - R \wedge R = 0$

In the language of divisors:

- F can be written as $F = E_i V_i^{gI} H^I$
  - E_i the localized (1,1)-forms (flux invisible in blow-down)
  - H^I elements in the Cartan algebra of SO(32) or $E_8 \times E_8$
- Quantization: V_i^{gI} must be integers (half-integers)

- E.o.m.: conditions on the Kaehler moduli:

$$J = \sum_i \tau_i X_i \Rightarrow \int J \wedge J \wedge F = P_2(\tau_i), \quad \int J \wedge F = P_1(\tau_i)$$

quadratic (linear) polinomial equation in the moduli
(+ loop corrections, see e.g. Blumenhagen et al. '05)

- Bianchi Identity: use the splitting principle and the F definition

$$R \wedge R \sim \prod_i (1 + X_i)_{|\,\mathrm{q.o.}}, \quad F = \mathrm{E_i V_i^{gI} H^I}$$

we can write

$$\int_\gamma (F \wedge F - R \wedge R) = Y_\gamma \left(\mathrm{E_i E_j V_i^g \cdot V_j^g}\right) - Y_\gamma \prod_i (1 + X_i)_{|\,\mathrm{q.o.}} = Q_2(\mathrm{V_i}) = 0$$

a quadratic polinomial equation in the V's (for each cycle)

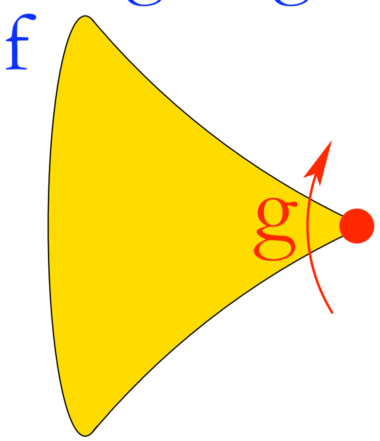
Spectrum: from the Dirac index (reduction of the 10d anomaly polynomial): all the states from the adjoint of SO(32) or $E_8 \times E_8$

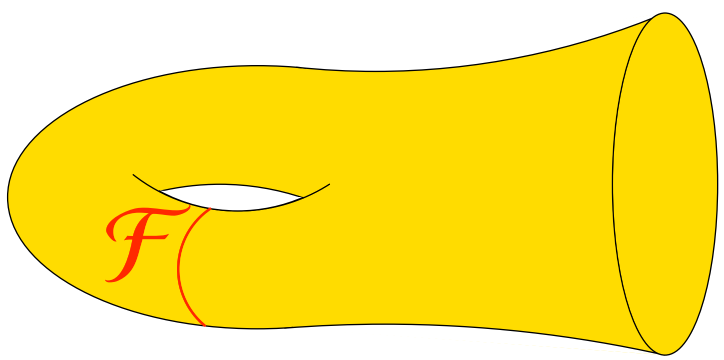
2.2 - Matching the orbifolds

Local Informations:

- on the orbifold side there are non-trivial identifications “going “round” the singularity, dictated by the embedding of the orbifold action in the gauge degrees of freedom

$$g : T^a \;\rightarrow\; e^{2\pi i H^I V_I / n}\; T^a \; e^{-2\pi i H^I V_I / n}$$





- on the bundle side the same identifications are generated by the presence of the flux (depending on how it is embedded in SO(32) or $E_8 \times E_8$)

“Simple” example: C^3/Z_3

- the resolution is obtained adding a single exceptional divisor E.
- take then $\mathcal{F} = V_I^g H^I E/3$, quantization fixes the vector to integer or half integer values, the boundary effect (and identification) is

$$\int_{D_2 D_3} \mathcal{F} = \frac{V_I^g}{3} H^I \; E D_2 D_3 = \frac{V_I^g}{3} H^I \sim \frac{V_I}{3} H^I$$

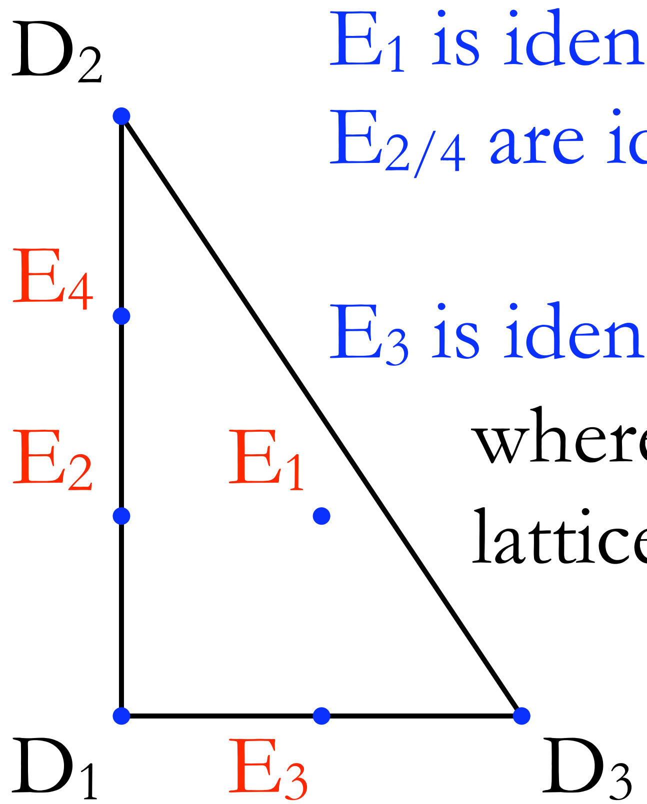
N.B. The Bianchi identity is $V^{g^2} = 12$, to be compared with the modular invariance condition $V^2 = 0 \bmod 6$!

3 - The $T^6/Z_{6\text{-}II}$ case

3.1 - Flux identification

Z_6 singularity

- Orbifold side: a shift vector V
- Resolution side: 4 shift vectors V^{g_i}, one for each line bundle E_i



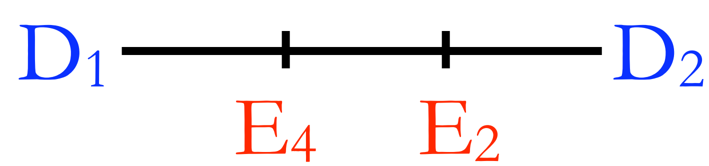
E_1 is identified with a genuine Z_6 "action": $V^{g_1} \equiv V$

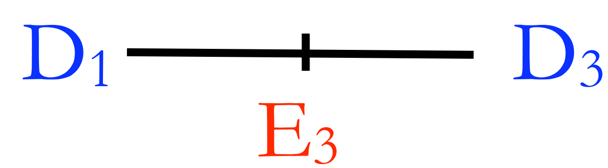
$E_{2/4}$ are identified with the C^2/Z_3 action in Z_6: $V^{g_{2/4}} \equiv \pm 2\ V$

see also T.-W. Ha, S. Groot Nibbelink, MT

E_3 is identified with the C^2/Z_2 action . . . : $V^{g_3} \equiv 3\ V$

where $A \equiv B$ means $A = B + \Lambda$, Λ being some E8×E8 lattice element





Z_3 / Z_2 singularity "out of a **Z_6** singularity"

- As for the **Z_3 / Z_2** singularity "inside a **Z_6** singularity":

$$V^{g_{2/4}} \equiv \pm 2\ V,\ V^{g_3} \equiv \pm 3\ V$$

Addition of discrete Wilson lines by just giving different shift vectors in different (resolved) singularities.

3.2 - Bianchi Identity ... we can find a solution

We have, in principle

- 32 different shift vectors, one per "exceptional" line bundle
- 35 different compact 4-cycles giving rise to 35 Bianchi Identity conditions
- 35 quadratic equations in 32 "variable" vectors, made of 16 integer entries each .....

But we also have, luckily

- not all the 35 equations are independent: we can reduce to 24
- a large reduction of the 32 shifts from the orbifold identifications (still freedom on the $E_8 \times E_8$ lattice elements Λ, but the freedom is largely reduced once we require the SM group to remain unbroken in the resolution)
- the quadratic equations can be cast in sum of squares!
- we can find a good choice of gauge fluxes and identify the "resolved version" of the MSSM models listed in th/0611095
- we did so for the so-called "Benchmark Model II" and find . . .

3.3 - MSSM's in blow-up

Gauge Symmetry

benchmark model II of 0611095 (Lebedev et al.)

- Orbifold: $SU(3)\times SU(2)\times U(1)_Y\times U(1)^4 \times SO(8)\times SU(2)\times U(1)^3$
- Blow-up: $SU(3)\times SU(2)\times U(1)_Y\times U(1)^4 \times SU(4)\times U(1)^4$

gauge symmetry breaking in the "hidden" E8

$$SO(8)\times SU(2) \rightarrow SU(4)\times U(1)$$

+ the "breaking" of some U(1)'s that are (now) anomalous

(from the orbifold perspective it's the usual fact that "blow-up" means giving a vev to some (charged) twisted fields, and this induces a Higgs mechanism)

Spectrum

- SM chiral spectrum
- SM vector-like exotics (chiral with respect to some hidden U(1))
- 2 additional singlets with non-trivial hypercharge (extra r.h. electrons)!

$\Rightarrow$ The $U(1)_Y$ is anomalous!

Focus on the anomalous hypercharge

- In the orbifold model a unified SU(5) symmetry is broken to SM "locally", in some of the singularities (while others preserve a unified group)
- In the SUGRA version this breaking is realized by U(1) fluxes, and $U(1)_Y$ is in SU(5) ⇒ $U(1)_Y$ is anomalous

Orbifold perspective

- The blow-up modes of the singularities where SU(5) is broken are all charged under the SM gauge group
  ⇒ no blow-up without breaking the SM gauge group

see also S. Groot Nibbelink, H.P. Nilles, M. T. `07

3.4 - Solving the $U(1)_Y$ "problem"

Simple way out: just do not resolve the "bad" singularities

- Orbifold perspective: $U(1)_Y$ breaking particle at zero vev → no breaking
- Resolved perspective: $U(1)_Y$ is anomalous, but the anomalous mass is zero (but out of the SUGRA perspective studied in Blumenhagen et al. '05)

More complicated, still keeping the $Z_{6\text{-}II}$ construction:

consider models where $U(1)_Y$ is not embedded "standardly" in SU(5) (see e.g. S. Raby and A. Wingerter '07)

Complicated & more interesting

consider orbifold geometries where all the fixed points preserve GUT, and SU(5) is broken "truly non-locally"

-- in progress ... see Patrick's talk

3.4 - "Delocalized" orbifold breaking: a simple $Z_2 \times Z_2'$ model

A. Hebecker and M.T. '04

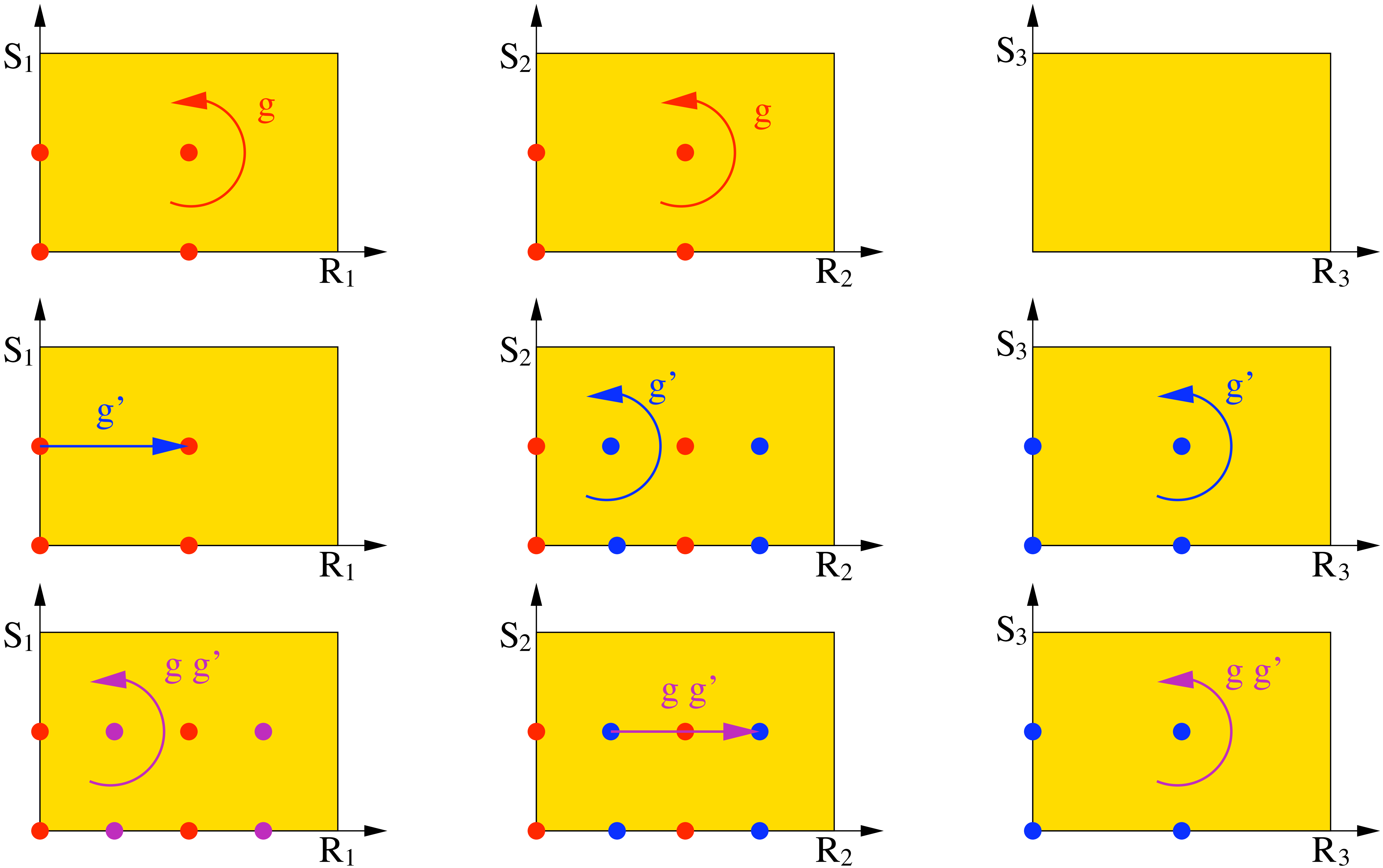
Basic idea: the orbifold actions breaking the GUT group must have free action (no fixed points)

- We need at least two different orbifold operators
  1) g', breaking the GUT group,
     acting as a translation in some internal dimension
  2) g, acting as a rotation in that direction
     otherwise too much SUSY -- N=2

- Minimal construction $Z_2 \times Z_2' = \{I, g', g, g \times g'\}$, with action on T^6

$$g': \begin{array}{l} z_1 \to z_1 + \pi R_1 \\ z_2 \to -z_2 + \pi R_2 \\ z_3 \to z_3 \end{array} \qquad g: \begin{array}{l} z_1 \to -z_1 \\ z_2 \to -z_2 \\ z_3 \to z_3 \end{array} \qquad g \times g': \begin{array}{l} z_1 \to -z_1 + \pi R_1 \\ z_2 \to z_2 + \pi R_2 \\ z_3 \to -z_3 \end{array}$$

g'	g	g×g'
free action	fixed points	free action!
GUT breaking gauge embedding	GUT preserving gauge embedding	gauge embedding fixed to be GUT breaking



S1
R1
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g'
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R2
g'
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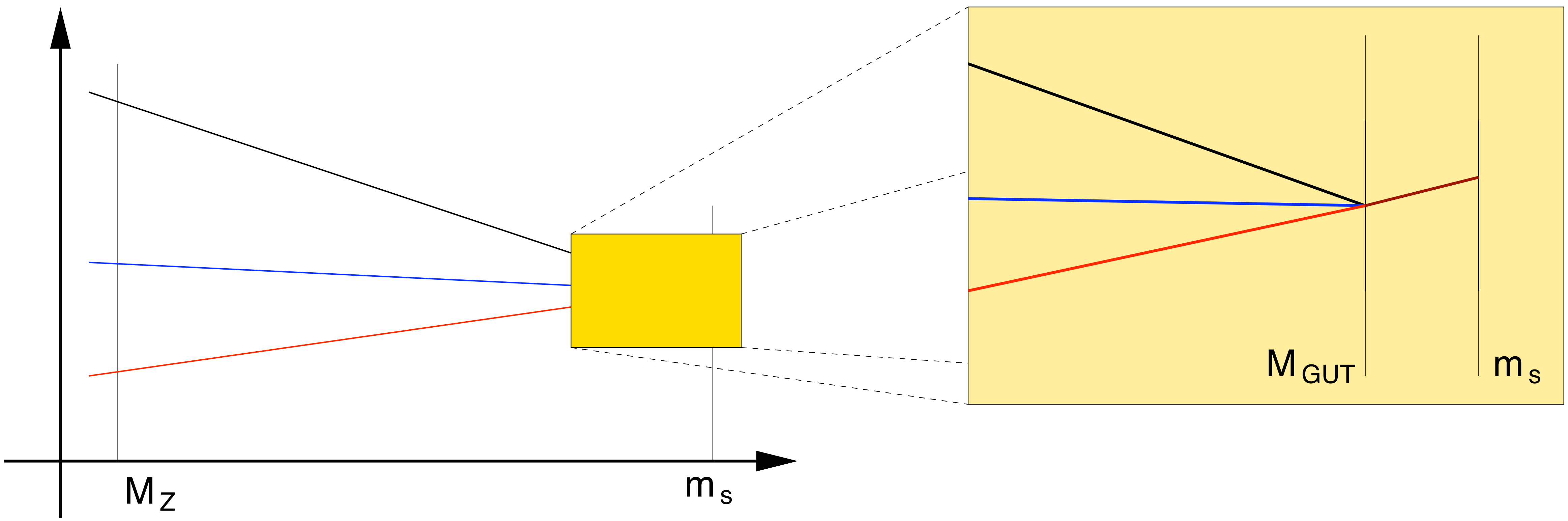
S1
R1
g g'
S2
R2
g g'
S3
R3
g g'

Byproduct: split M_{GUT} from m_s

Due to the de-localization the scale of gauge symmetry breaking is decoupled from the string scale

$$M_{GUT} \sim (R_1 R_2)^{-1/2}$$



But usual problem with "large" volumes in heterotic string

4 - Conclusions & Outlook

... see also Stefan's talk on friday

- **Reproduced the heterotic orbifold models as SUGRA construction with gauge bundles (metric resolution and toric geometry resolution).**
  - local $\mathbf{C^n/Z_m}$ and $\mathbf{C^n/Z_m \times Z_p}$ — S. Groot Nibbelink, MT, M. Walter; T.-W. Ha, S. Groot Nibbelink, MT.
  - global case $\mathbf{T^6/Z_3}$ — S. Groot Nibbelink, D. Klevers, F. Ploger, MT, P. K. S. Vaudrevenge
  - K3 with U(1) bundles — G. Honecker, MT.
  - K3 with generic bundles (and "jumping" between bundles) — S. Groot Nibbelink, F. Paccetti, MT
  - **the pheno appealing $\mathbf{T^6/Z_{6\text{-}II}}$ model** — S. Groot Nibbelink, MT, J.Held, F. Ruehle, P. K. S. Vaudrevange

- **The breaking of SU(5) to the SM group by U(1) fluxes is problematic (anomalous hypercharge)**
  - **consider different orbifolds** (e.g. A. Hebecker, MT '04) **with "de-localized" gauge symmetry breaking**

... Patrick's talk