Quantum interferometry with and without reference beam

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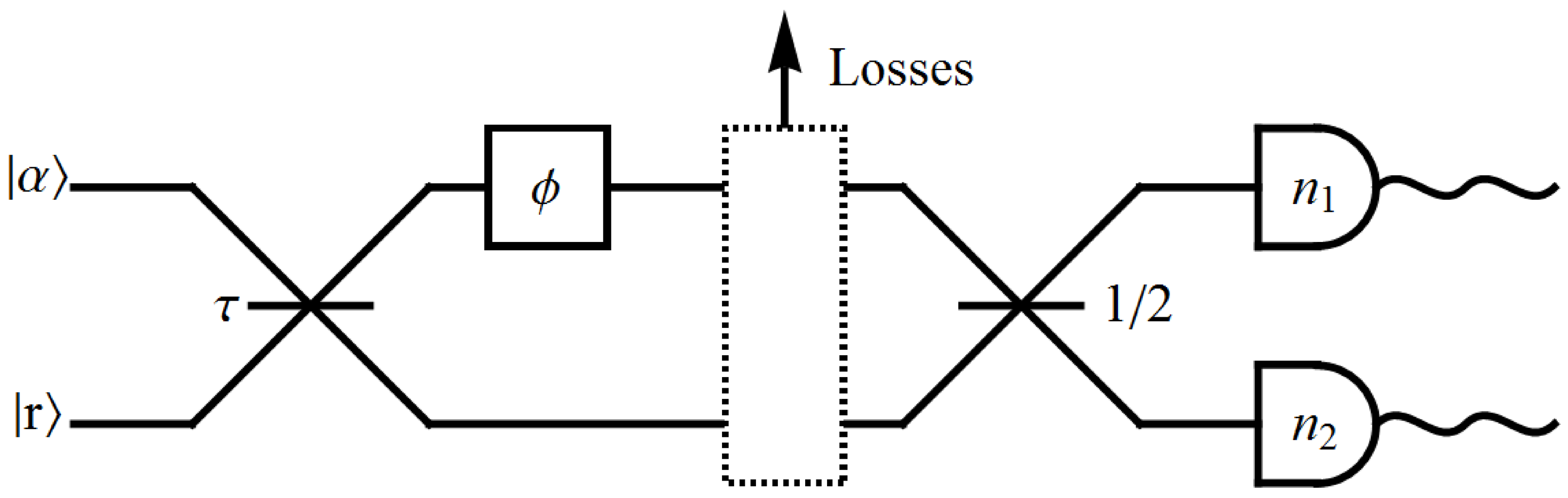
Introduction

Precision of phase estimation with Mach-Zehnder interferometer was considered in four different cases. It was shown that in general it is a two-parameter problem and to obtain valid precision additional reference beam should be considered as well. As a result, full Fischer information matrix should be used, however it rarely occurs in current literature.

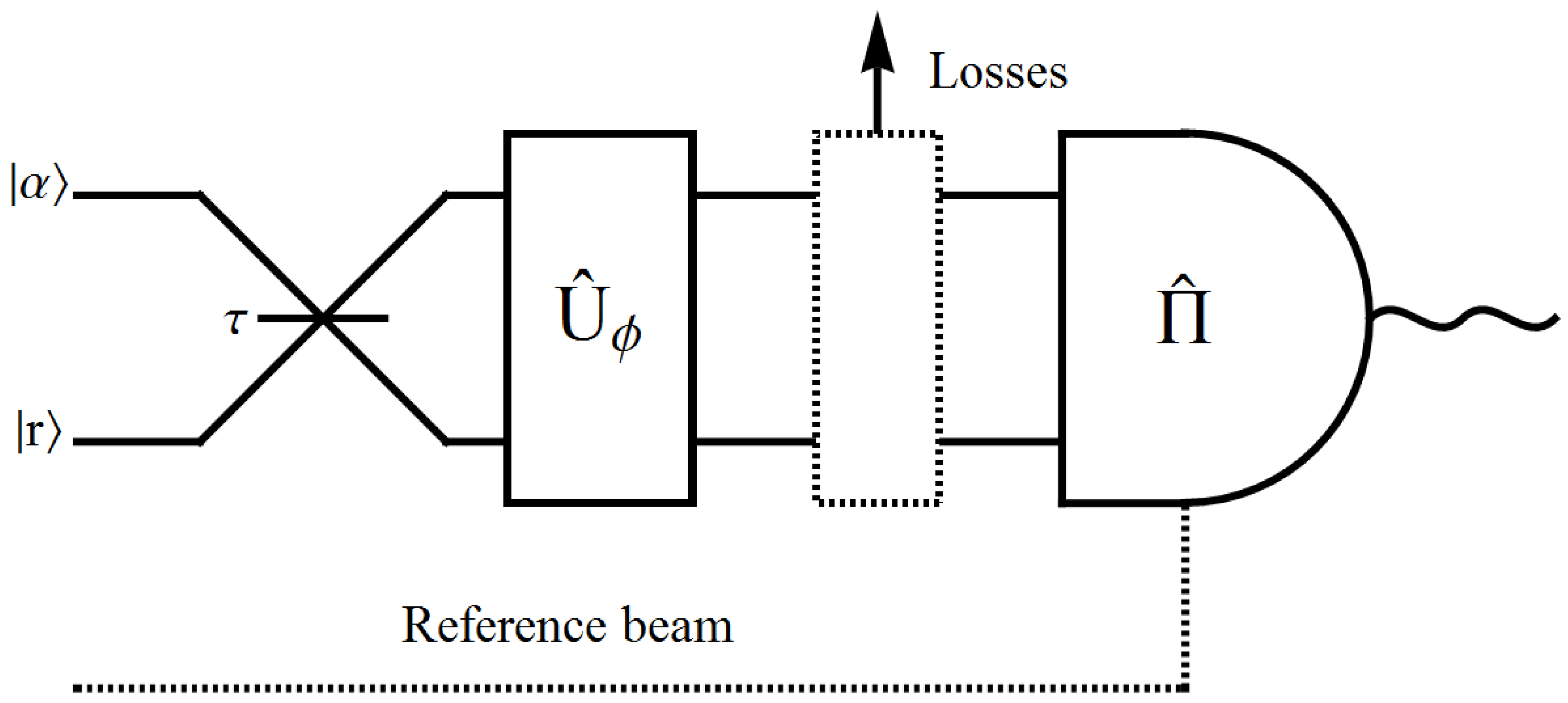
Schemes for interferometry

Standard scheme for quantum interferometry



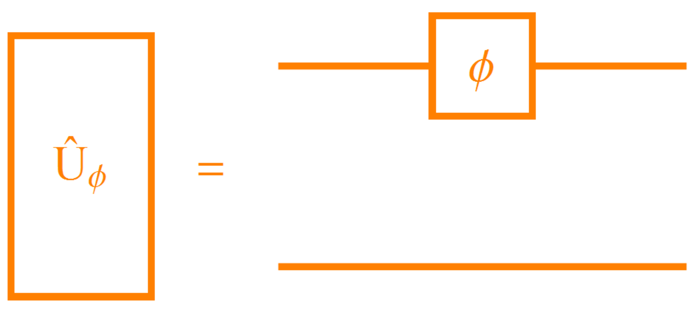
$$F_{cl}=\sum_{n_1,n_2}\frac{1}{p(n_1,n_2|\phi)}\left(\frac{dp(n_1,n_2|\phi)}{d\phi}\right)^2,\quad \Delta\phi_{cl}\geq\frac{1}{\sqrt{F_{cl}}}$$

General scheme with arbitrary measurement and additional reference beam



Three cases of phase delay

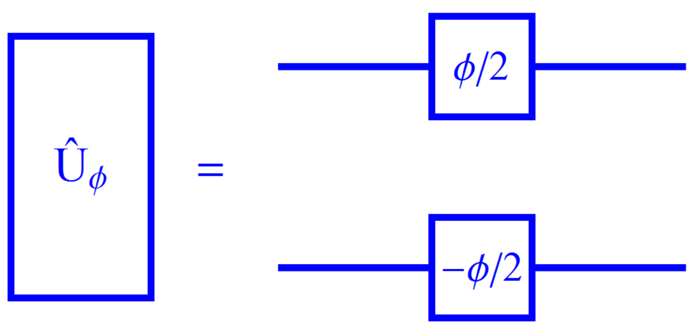
1.



Phase delay in one arm

$$F_1=\mathrm{Tr}\left(\hat\rho\hat\Lambda^2\right),\quad \Delta\phi_1=\frac{1}{\sqrt{F_1}}$$

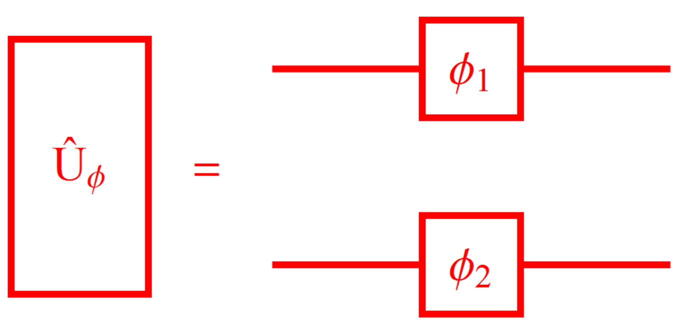
2.



Opposite phase delays in both arms

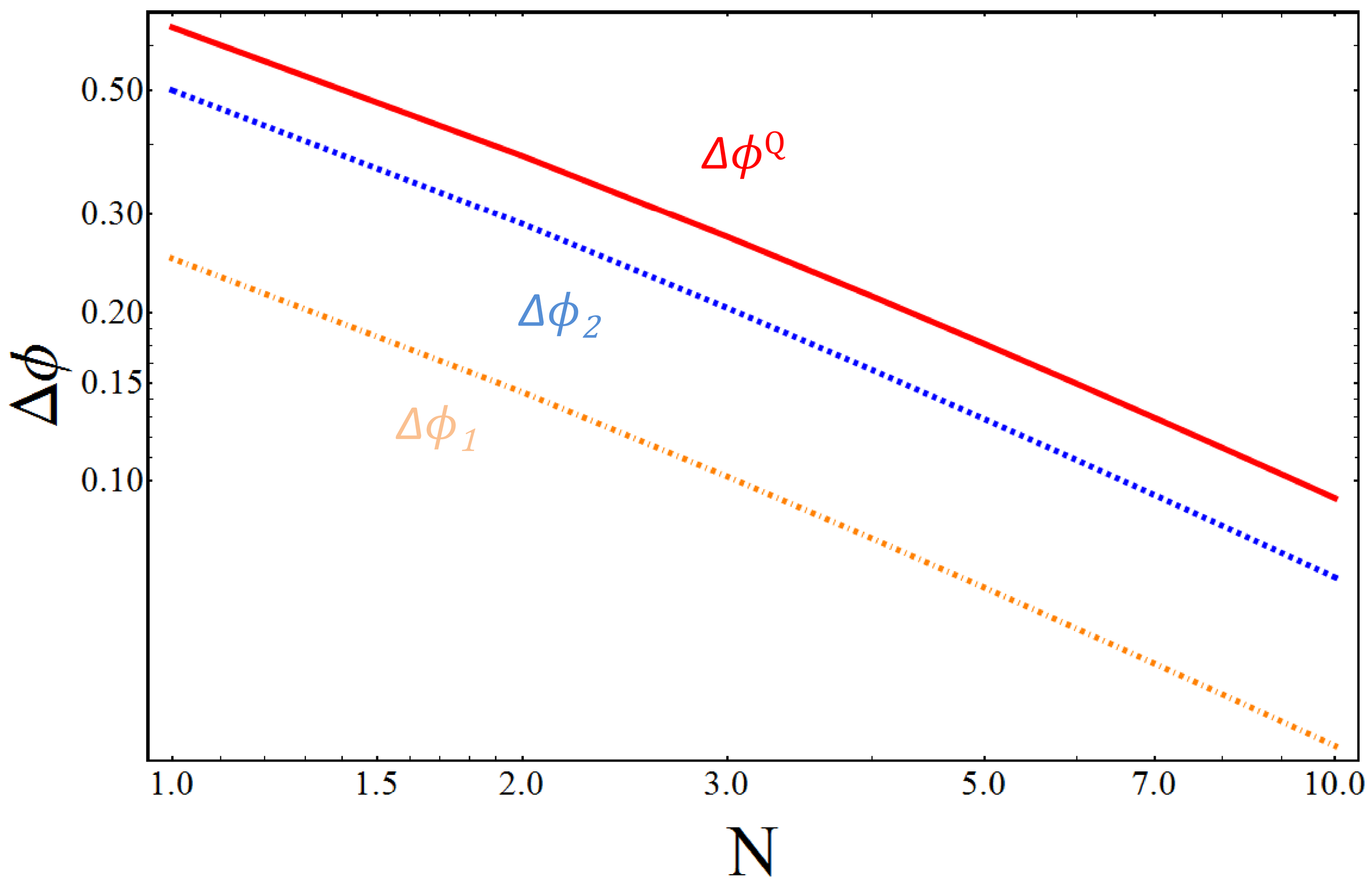
$$F_2=\mathrm{Tr}\left(\hat\rho\hat\Lambda^2\right),\quad \Delta\phi_2\geq\frac{1}{\sqrt{F_2}}$$

3.

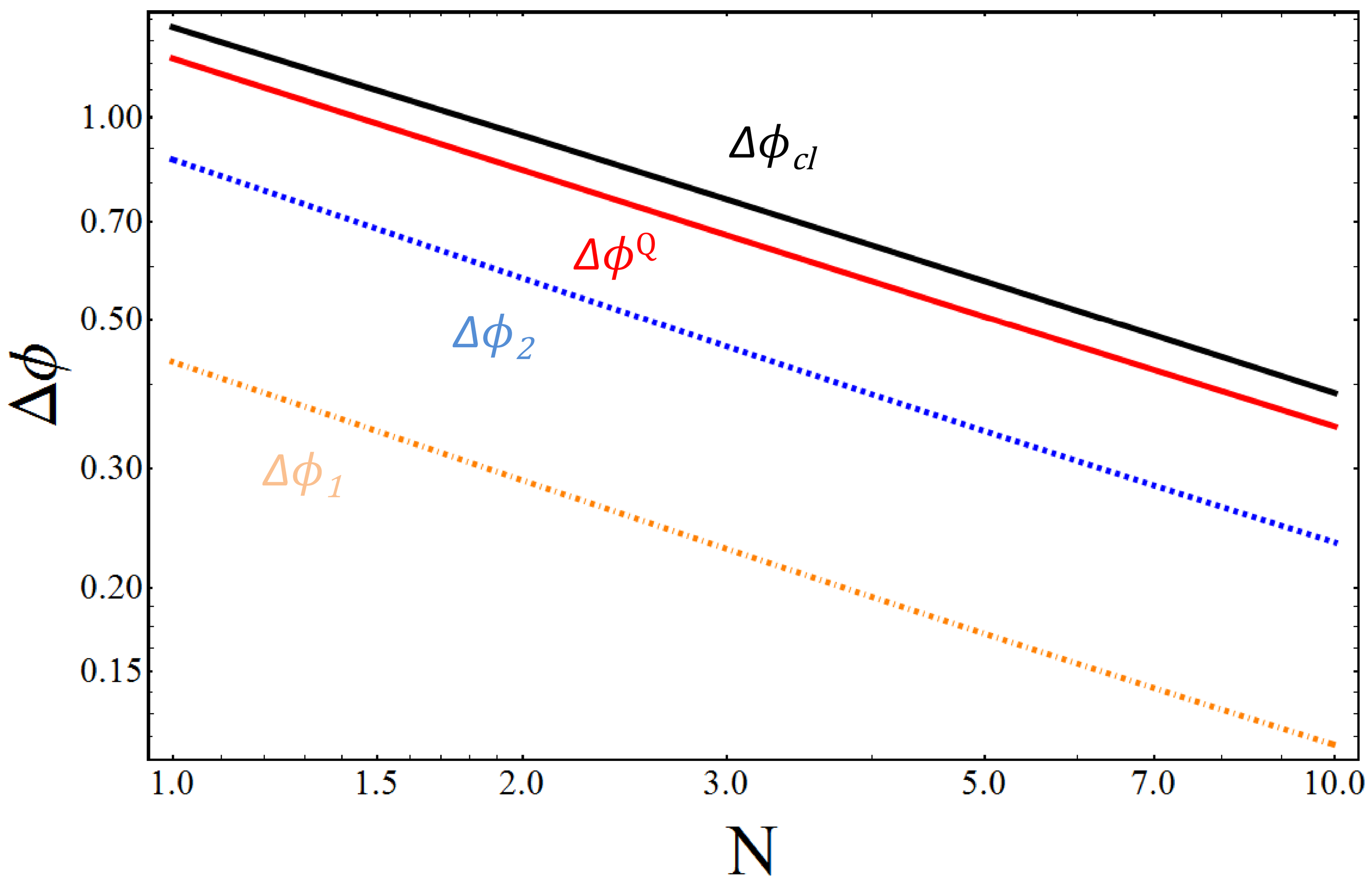


Different phase delays in both arms

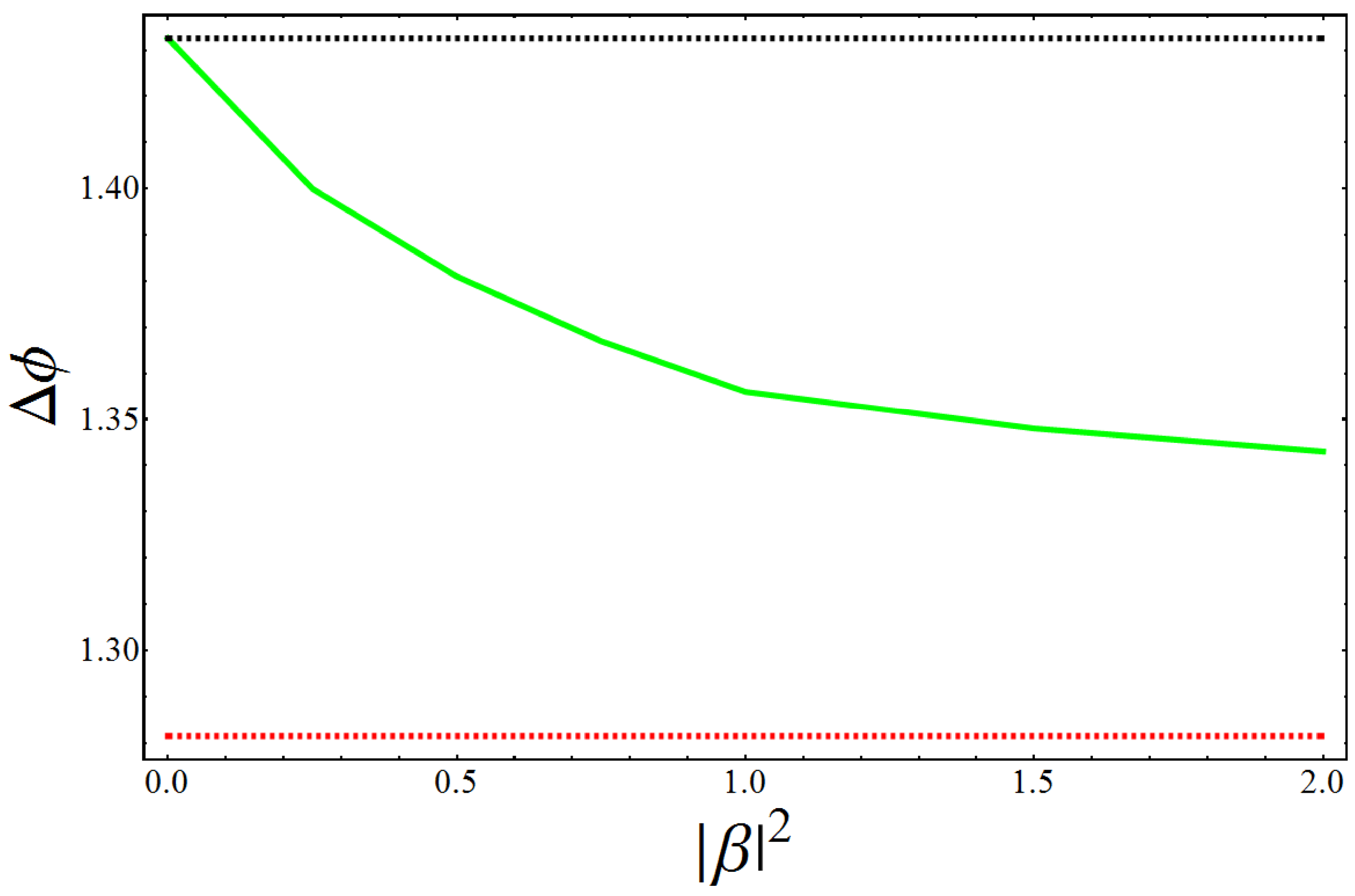
$$\mathcal{F}=\begin{pmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{pmatrix},\quad F_{ij}=\frac{1}{2}\mathrm{Tr}\left(\hat\rho\hat\Lambda_i\hat\Lambda_j+\hat\rho\hat\Lambda_j\hat\Lambda_i\right),\quad \Delta\phi^Q_i\geq\left(\mathcal{F}^{-1}\right)_{ii}$$



Best precision for optimal τ and without losses as a function of total photon number. In such case $\Delta\phi_Q=\Delta\phi_{cl}$. For $\Delta\phi_1$, $\Delta\phi_2$, $\tau=0$, for $\Delta\phi_Q$, $\tau=0.5$.



Best precision for optimal τ and with losses as a function of total photon number. In such case $\Delta\phi_Q\neq\Delta\phi_{cl}$. For $\Delta\phi_1$, $\Delta\phi_2$, $\tau=0$, for $\Delta\phi_Q$, $\Delta\phi_{cl}$, $\tau=0.5$.



Best precision as a function of average photon number in the reference beam (green). Black dashed line indicates case with $|\beta|^2=0$, and red dashed line with $|\beta|^2=1$.

All three cases give different precisions only in the presence of reference beam. In other case state changes from $|\alpha\rangle|r\rangle$ to mixed state

$$\hat\rho=\int\frac{d\xi}{2\pi}\hat U_\xi|\alpha\rangle\langle\alpha|\otimes|r\rangle\langle r|U_\xi^+$$

which is averaged over phase, and one gets $\Delta\phi_1=\Delta\phi_2=\Delta\phi^Q=\Delta\phi_{cl}$.

Summary

In general, whole Fischer information matrix should be taken into consideration to obtain valid precision of phase estimation. If not, different ways of placing phase delays result in different precisions. On the other hand, when the reference beam is absent, all cases give the same precision, which is equal to reference-beam case only when there are no losses. In the last situation, it can be obtained that the more photons reference beam contain, the better precision is achieved.

Acknowledgments

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