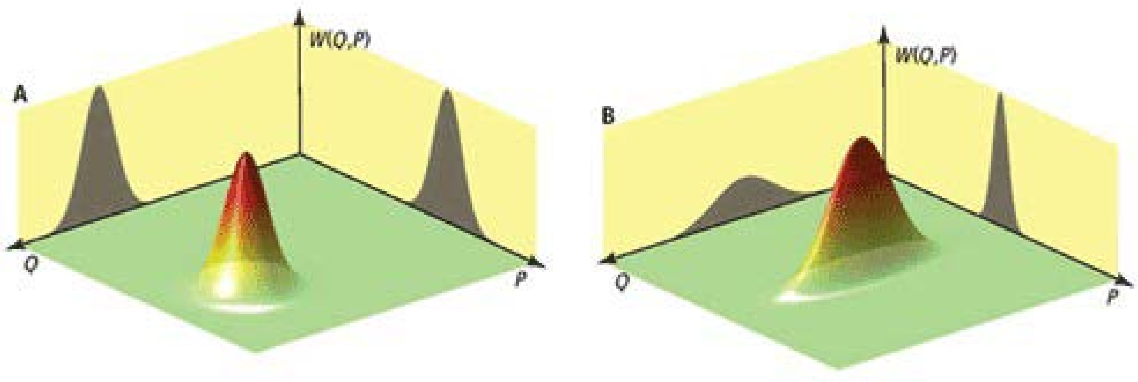
All you need is squeezing!

optimal schemes for realistic quantum metrology



R. Demkowicz-Dobrzański[1], K. Banaszek[1], J. Kołodyński[1], M. Jarzyna[1],
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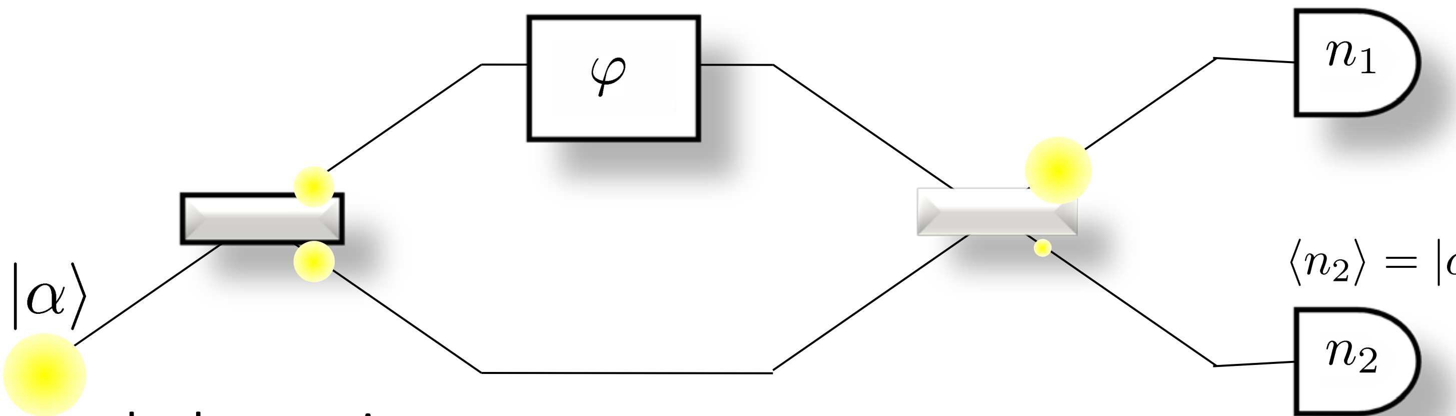
INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY
FNP
Foundation for Polish Science
UNIVERSITAS VARSOVIENSIS
chist-era
EUROPEAN UNION
EUROPEAN REGIONAL DEVELOPMENT FUND

„Classical'' interferometry

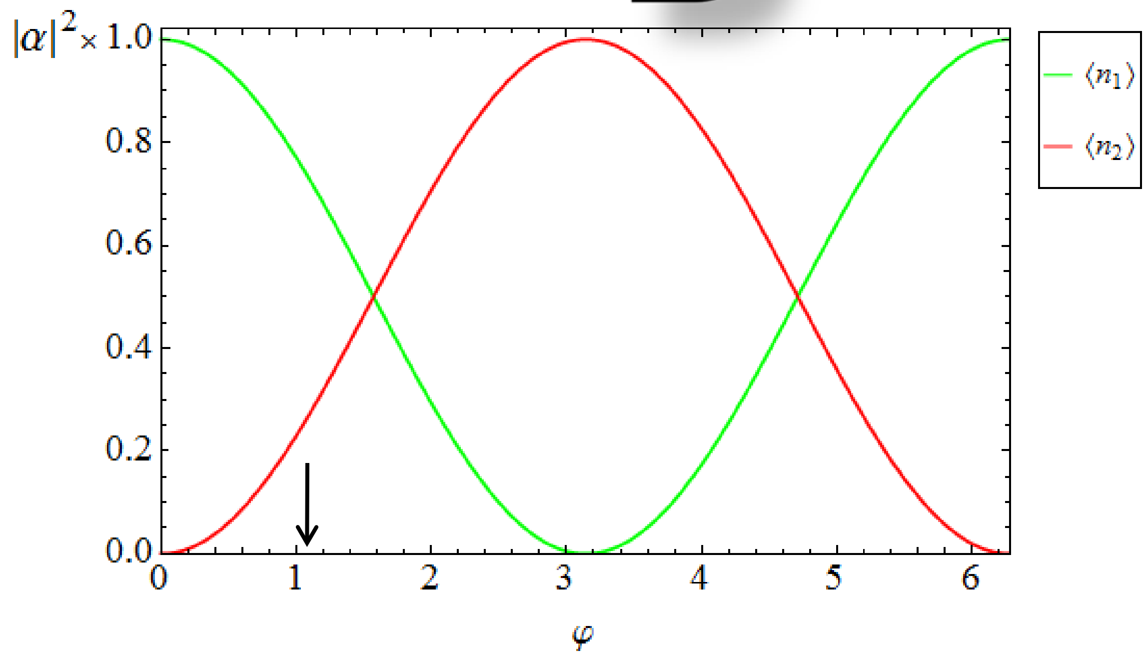
$$\langle n_1 \rangle = |\alpha|^2 (1 + \cos\varphi)/2$$

$$\langle n_2 \rangle = |\alpha|^2 (1 - \cos\varphi)/2$$



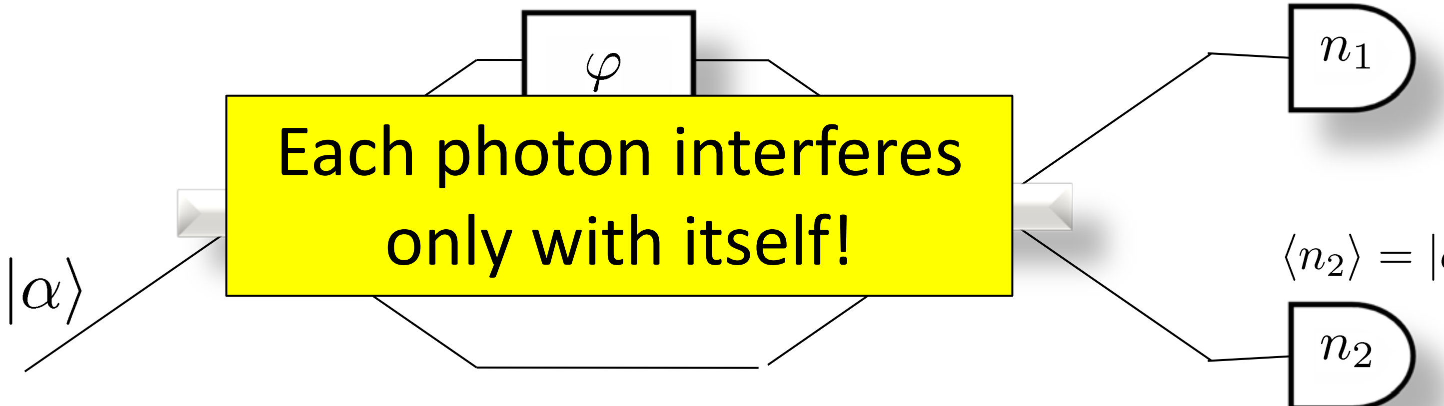
the best estimator

$$\varphi(n_1, n_2) = \arccos\left(\frac{n_1 - n_2}{|\alpha|^2}\right)$$



„Classical” interferometry

$$\langle n_1 \rangle = |\alpha|^2 (1 + \cos\varphi)/2$$



$$\langle n_2 \rangle = |\alpha|^2 (1 - \cos\varphi)/2$$

the best estimator

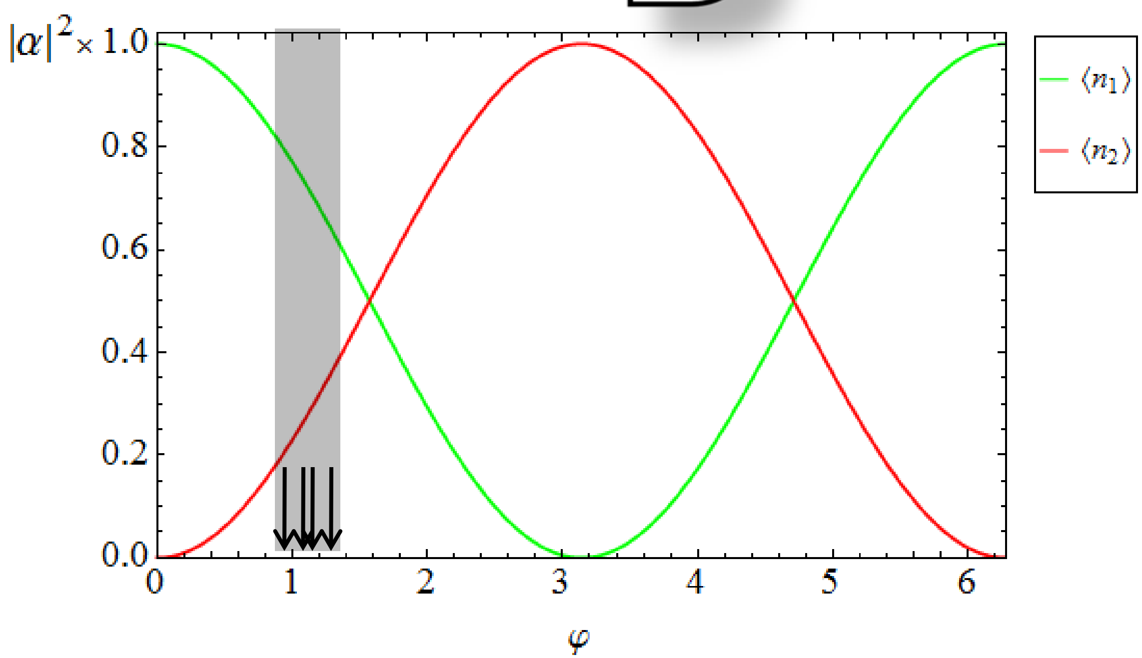
$$\varphi(n_1, n_2) = \arccos\left(\frac{n_1 - n_2}{|\alpha|^2}\right)$$

Poisson statistics

$$n_i = \langle n_i \rangle \pm \sqrt{\langle n_i \rangle}$$

$$\Delta\varphi \propto \frac{1}{|\alpha|} = \frac{1}{\sqrt{\langle n \rangle}}$$

classical (standard) scaling

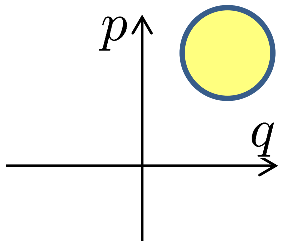


Quantum enhancement thanks to the squeezed states

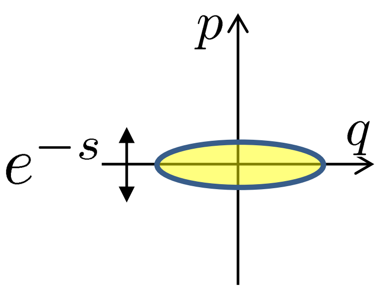
PHYSICAL REVIEW D VOLUME 23, NUMBER 8 15 APRIL 1981

Quantum-mechanical noise in an interferometer

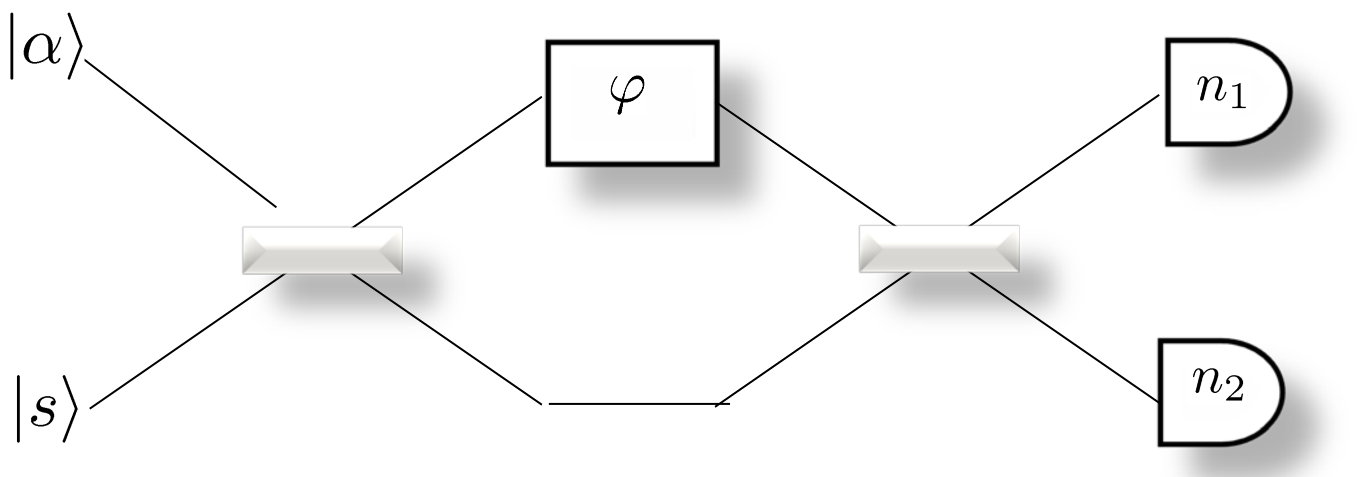
Carlton M. Caves
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
(Received 15 August 1980)



Coherent state



Squeezed vacuum

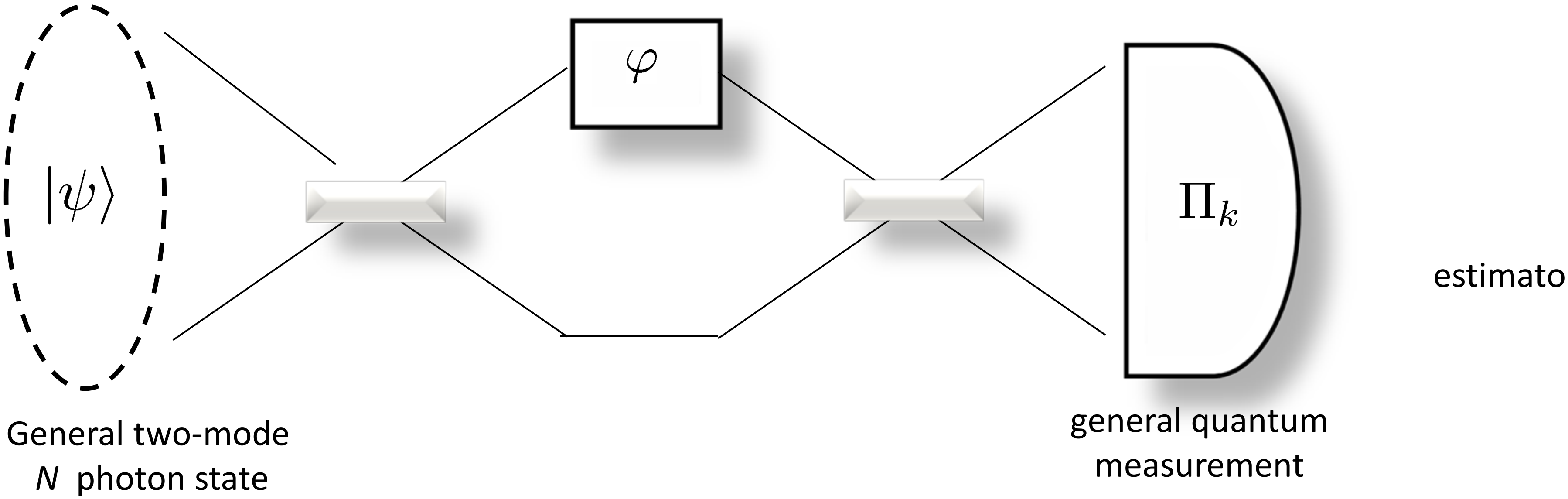


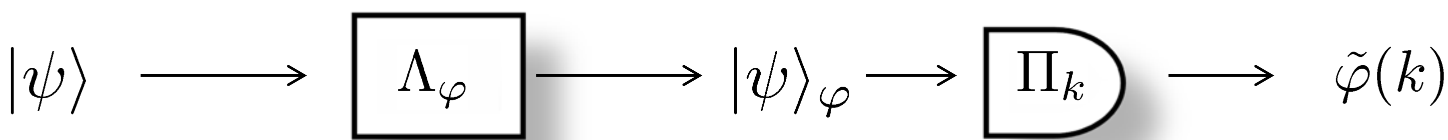
Precision enhancement thanks to subpoissonian fluctuations of n_1- n_2!

simple estimator based on n_1- n_2:

$$\Delta\varphi \propto \frac{1}{\langle n\rangle^{3/4}}$$

Optimal strategy?

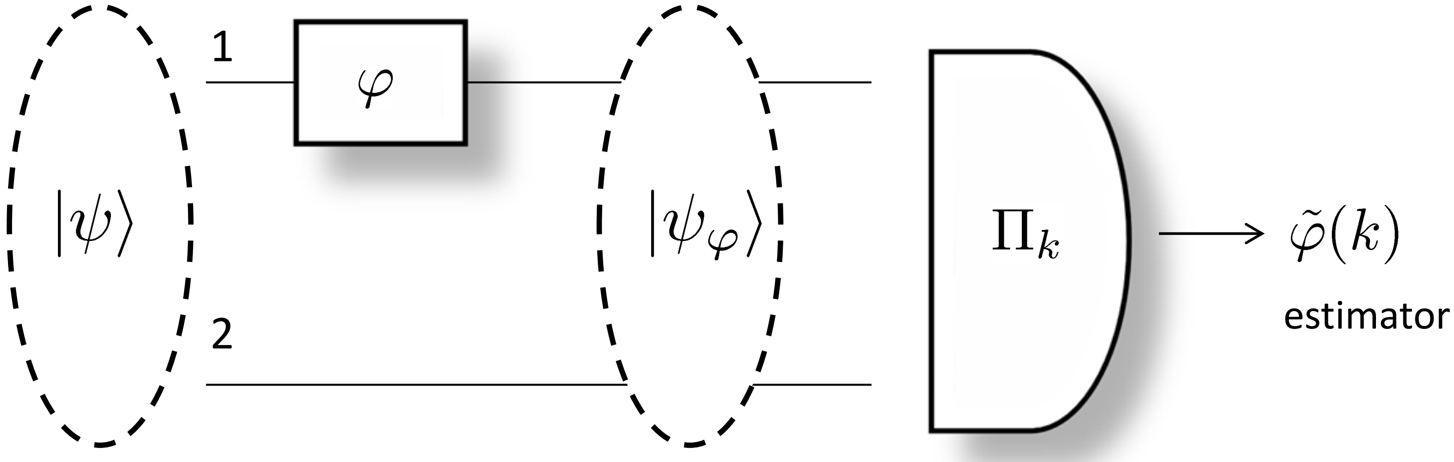




single parameter quantum channel estimation

$$\text{Minimize } \Delta\varphi = \sqrt{\langle(\tilde{\varphi} - \varphi)^2\rangle} \text{ over } |\psi\rangle, \Pi_k \text{ i } \tilde{\varphi}$$

Quantum Cramer-Rao bound and the optimal *N* photon state



$$|\psi_\varphi\rangle = e^{-\mathrm{i}\varphi\hat{n}_1}|\psi\rangle$$

$$\Delta\varphi \geq \frac{1}{\sqrt{F}} \qquad F = 4(\langle\dot{\psi}_\varphi|\dot{\psi}_\varphi\rangle - |\langle\dot{\psi}_\varphi|\psi_\varphi\rangle|)$$

$$F = 4\Delta^2 n_1 \qquad \Delta^2 n_1 = \langle\hat{n}_1^2\rangle - \langle\hat{n}_1\rangle^2$$

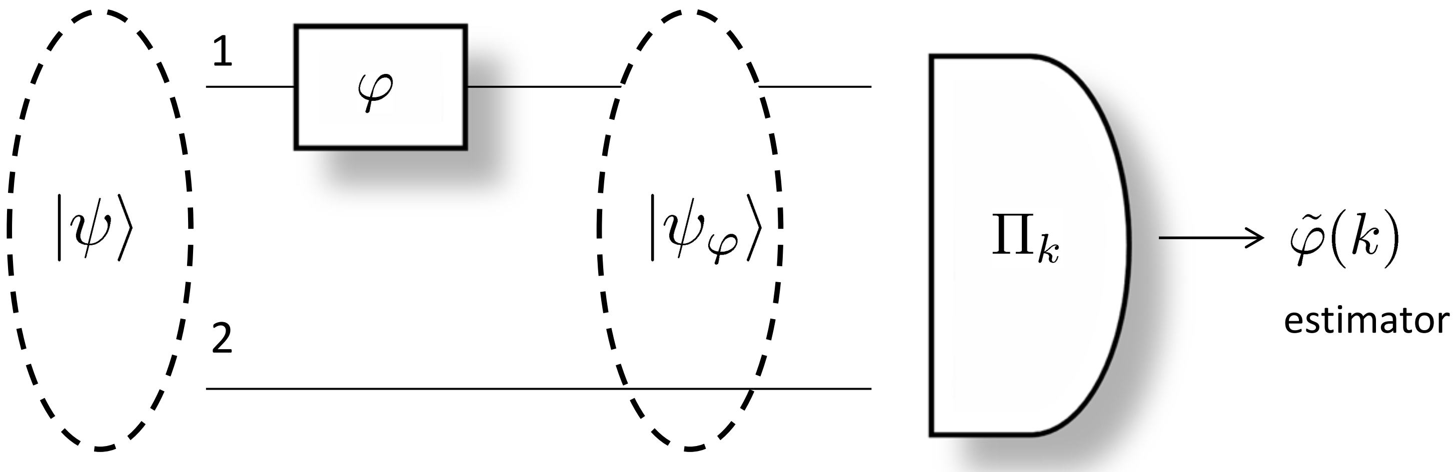
Good estimation possible only for states with high Δn_1

$$\frac{1}{\sqrt{2}}\left(|0\rangle|N\rangle + |N\rangle|0\rangle\right) \qquad F = N^2 \qquad \Delta\varphi \geq \frac{1}{N}$$

NOON states

Heisenberg limit

Other „Heisenberg limits”



In a Bayesian approach

$$p(\varphi) \approx \frac{1}{2\pi}$$

no a priori knowledge about the phase

Covariant measurements are optimal:

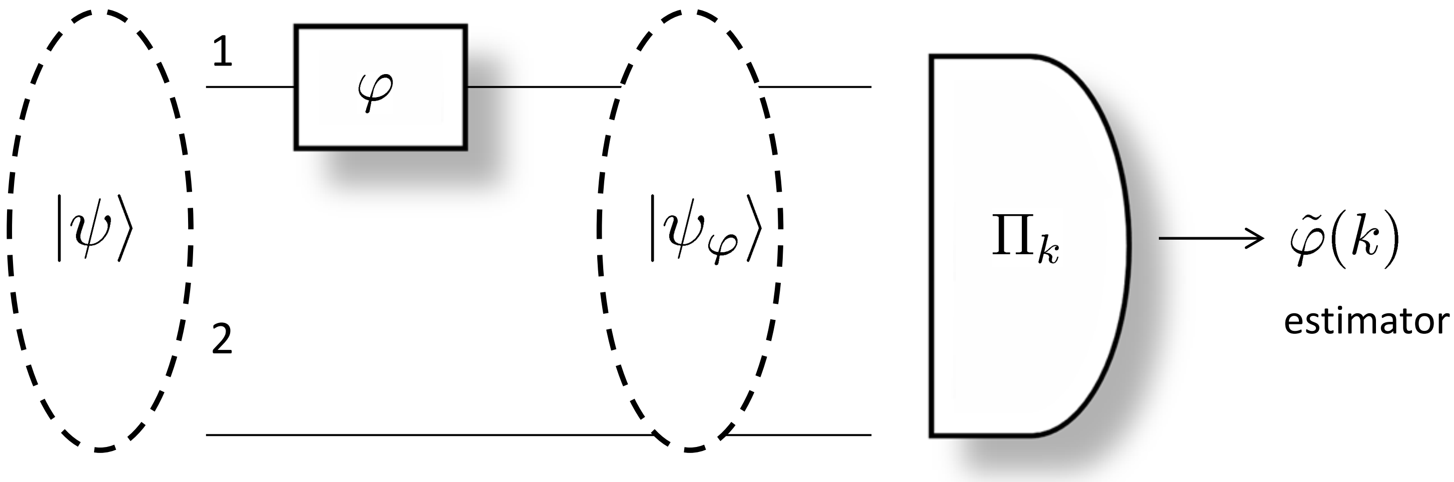
Optimal state: $|\psi\rangle = \sum_{n=0}^{N} \alpha_n |n, N-n\rangle$

$$\alpha_n = \sqrt{\frac{2}{N+2}} \sin\left[\frac{(n+1)\pi}{N+2}\right]$$

$$\Delta\tilde{\varphi} \approx \frac{\pi}{N+2}$$

D. W. Berry and H. M. Wiseman, Phys. Rev. Lett. **85**, 5098 (2000).

Other „Heisenberg limits”



For states with indefinite photon number

$$|\psi\rangle = \sum_N \sqrt{p_N} |\psi^{(N)}\rangle \qquad F(|\psi\rangle) \geq \sum_N p_N F(|\psi^{(N)}\rangle)$$

So in principle we can have $F = \sum_N p_N N^2 = \langle N^2 \rangle > (\sum p_N N)^2 = \langle N \rangle^2$

And beat the ``naive'' Heisenberg limit: $\Delta\varphi \not\geq \frac{1}{\langle N \rangle}$

H. Hoffman, Phys. Rev. A 79, 033822 (2009)

P.M. Anisimov, et al., Phys. Rev. Lett. 104, 103602 (2010)

Sub Heisenberg strategies are ineffective

V. Giovannetti, L. Maccone, Phys. Rev. Lett 108, 210404 (2012)

$$\Delta\varphi \geq \frac{1}{\sqrt{\langle N^2 \rangle}}$$

Complicated....

$$\Delta\varphi \geq \frac{1}{N}$$

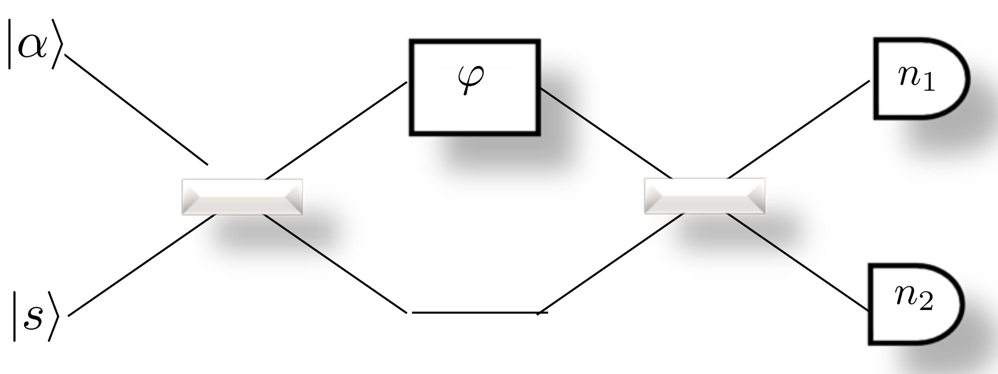
a priori knowledge?

$$\Delta\varphi \geq \frac{1}{\sqrt{\langle N^2\rangle}}$$

beating Heisenberg limit?

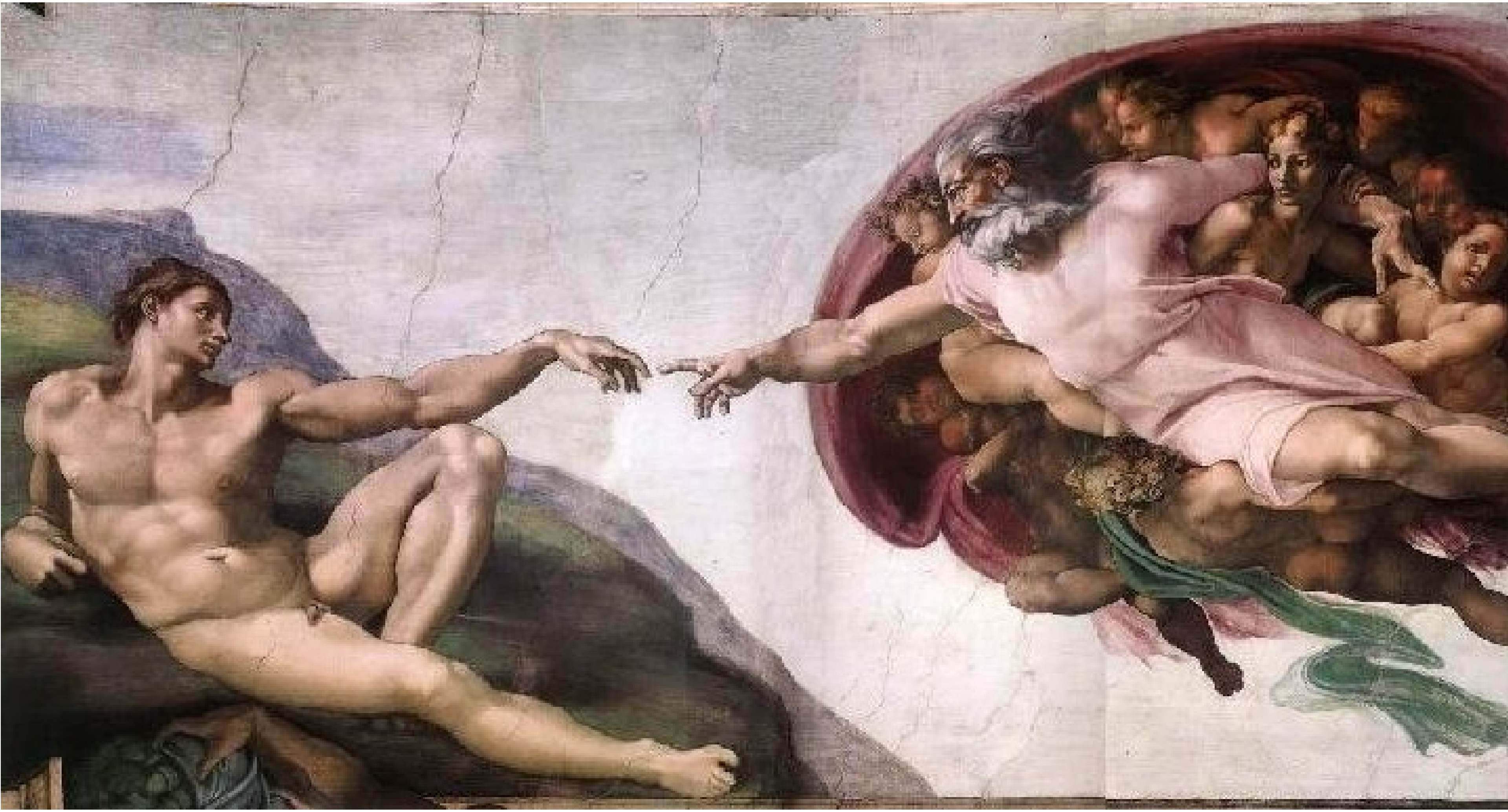
$$\Delta\varphi \geq \frac{\pi}{N+2}$$

How to saturate the bounds?



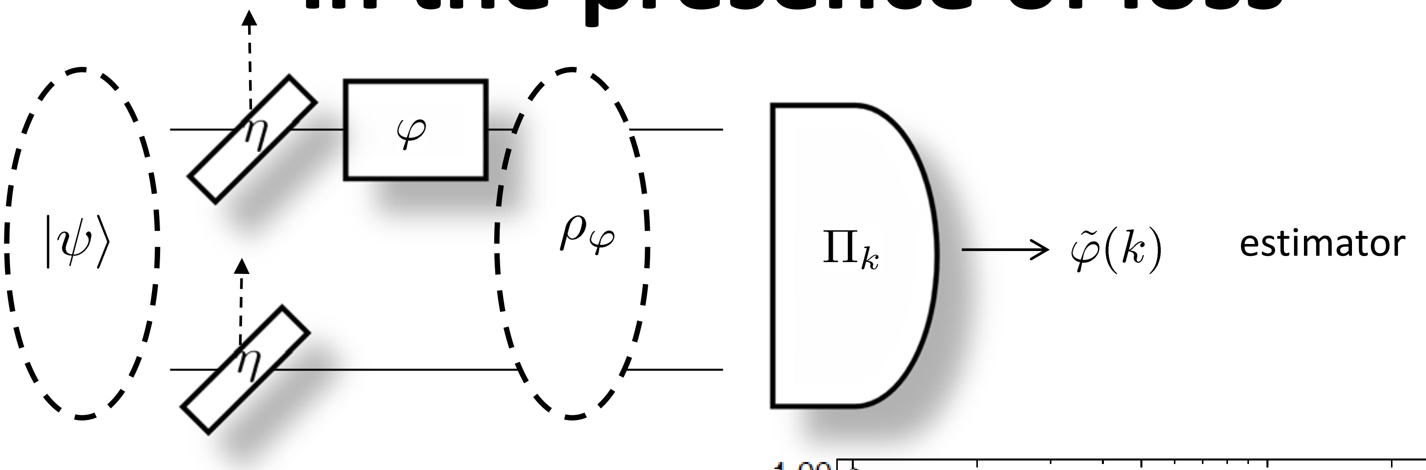
$$\Delta\varphi \propto \frac{1}{\langle N\rangle^{3/4}}$$

then God added decoherence…

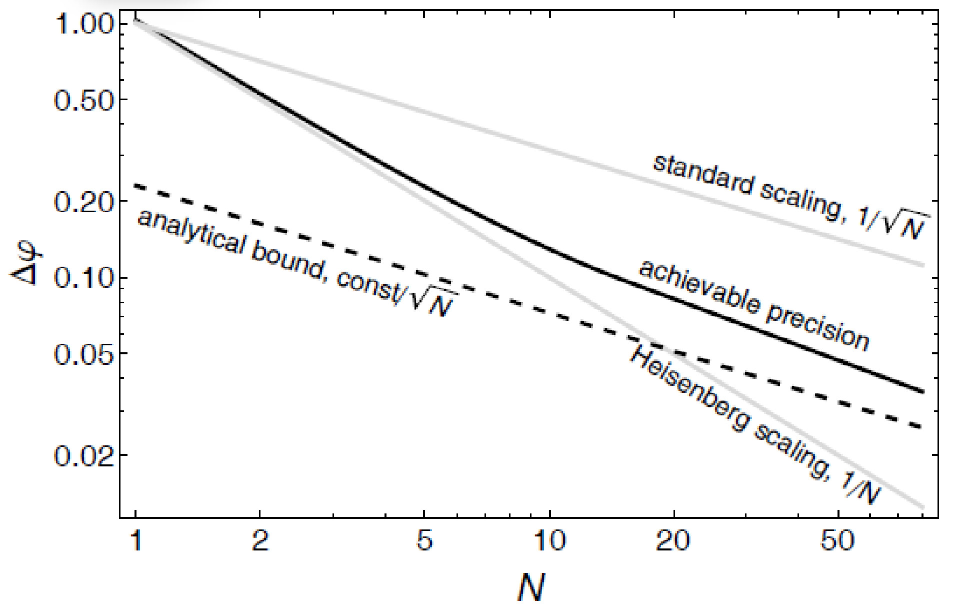


and everything became… simpler

Quantum interferometry in the presence of loss



$$\Delta\tilde{\varphi} \geq \sqrt{\frac{1-\eta}{\eta}}\frac{1}{\sqrt{N}}$$



Heisenberg scaling is lost!

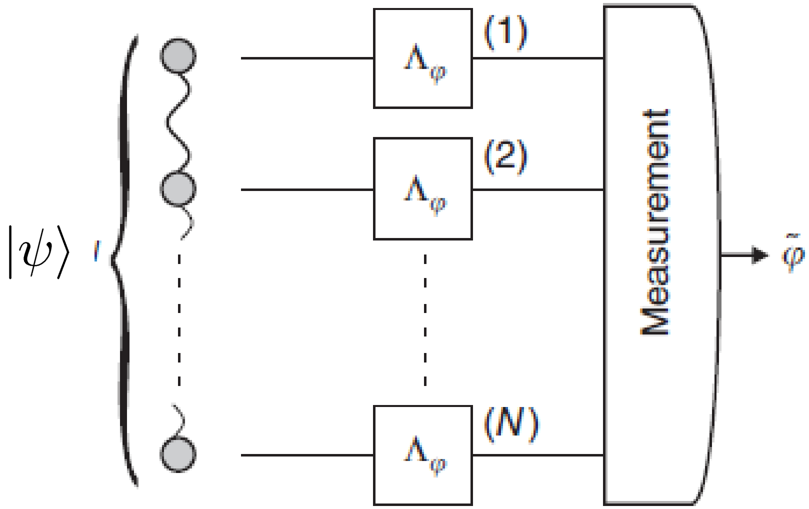
also valid if N replaced with $\langle N\rangle$

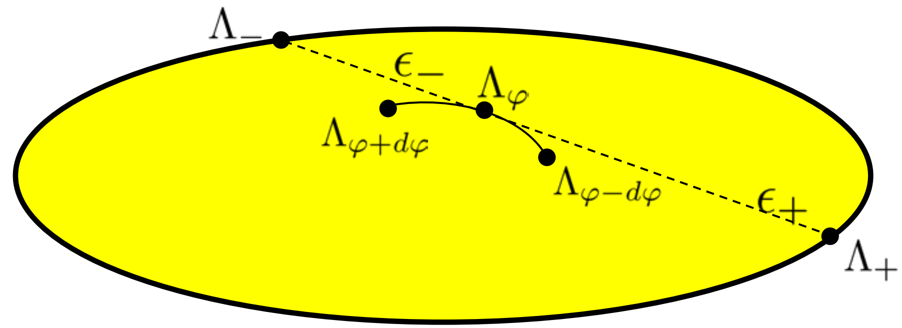
J. Kolodyński, R. Demkowicz-Dobrzański, PRA **82**,053804 (2010) – Bayesian approach

S. Knysh, V. Smelyanskiy, G. Durkin , PRA **83**, (2011) – Fisher information approach

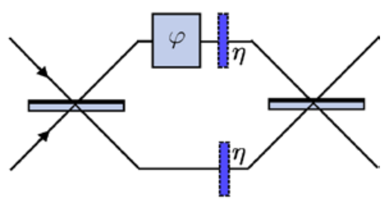
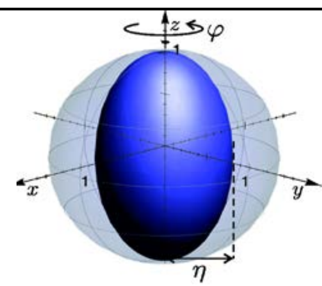
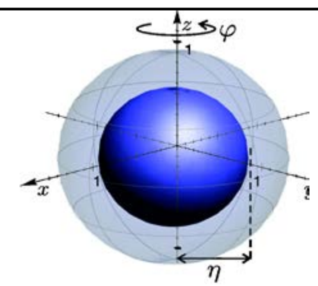
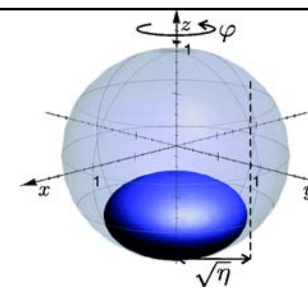
B. M. Escher, R. L. de Matos Filho, L. Davidovich, Nat. Phys. **7**, 406 (2011)

Quantum metrology with decoherence





$$\Delta\varphi \geq \sqrt{\frac{\epsilon_+ \epsilon_-}{N}} \qquad \Lambda_\pm = \Lambda_\varphi \pm \frac{d\Lambda_\varphi}{d\varphi}\epsilon_\pm$$

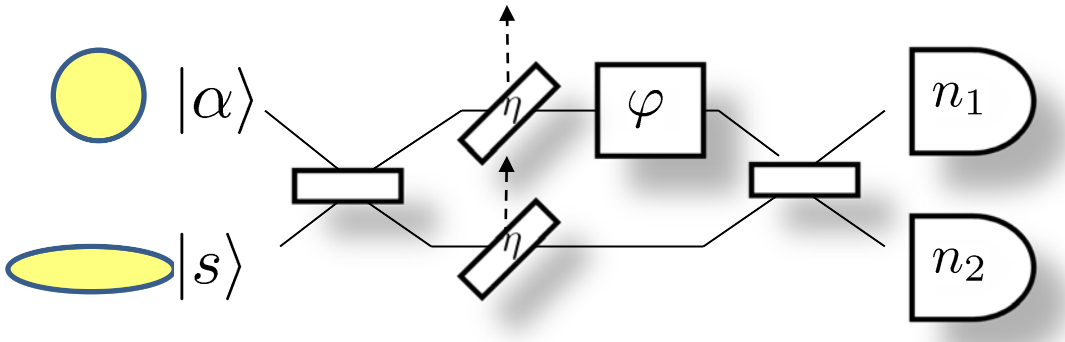
	Lossy interferometer	Dephasing	Depolarization	Spontaneous emission
$\Lambda_\varphi =$				
$\Delta\varphi \geq$	$\sqrt{\frac{1-\eta}{\eta}}\frac{1}{\sqrt{N}}$	$\frac{\sqrt{1-\eta^2}}{\eta}\frac{1}{\sqrt{N}}$	$\frac{\sqrt{(1-\eta)(1+3\eta)}}{2\eta}\frac{1}{\sqrt{N}}$	$\frac{1}{2}\sqrt{\frac{1-\eta}{\eta}}\frac{1}{\sqrt{N}}$

R. Demkowicz-Dobrzański, J. Kolodyński, M. Guta, Nat. Commun. **3**, 1063 (2012)

Saturating the fundamental bound is simple!

$$\Delta\tilde{\varphi} \geq \sqrt{\frac{1-\eta}{\eta}} \frac{1}{\sqrt{N}}$$

Fundamental bound



For strong beams:

$$\Delta\tilde{\varphi} \approx \sqrt{\frac{1-\eta+\eta e^{-2s}}{\eta|\alpha|^2}}$$

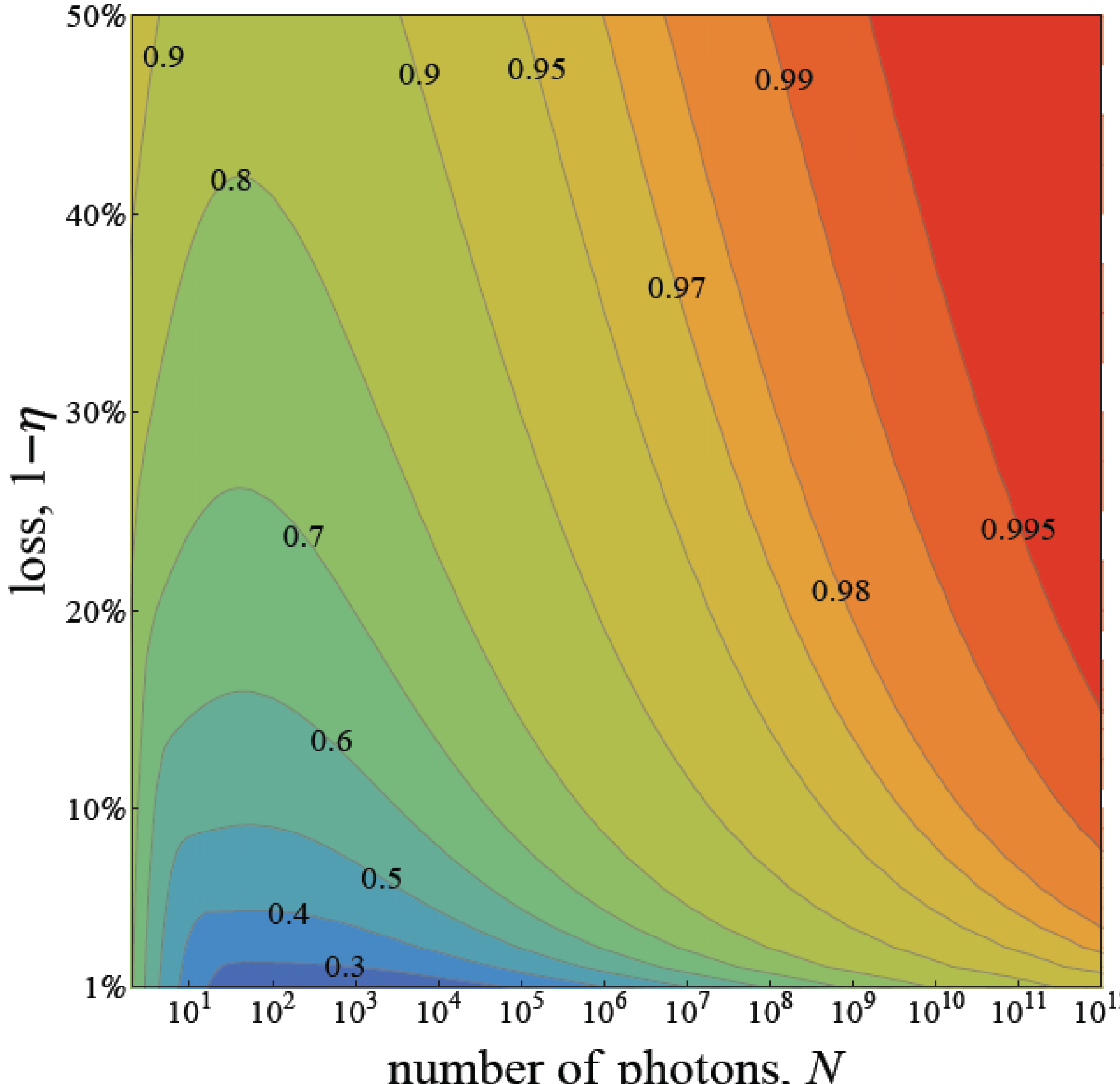
Simple estimator based on n_1 - n_2 measurement

C. Caves, Phys. Rev D **23**, 1693 (1981)

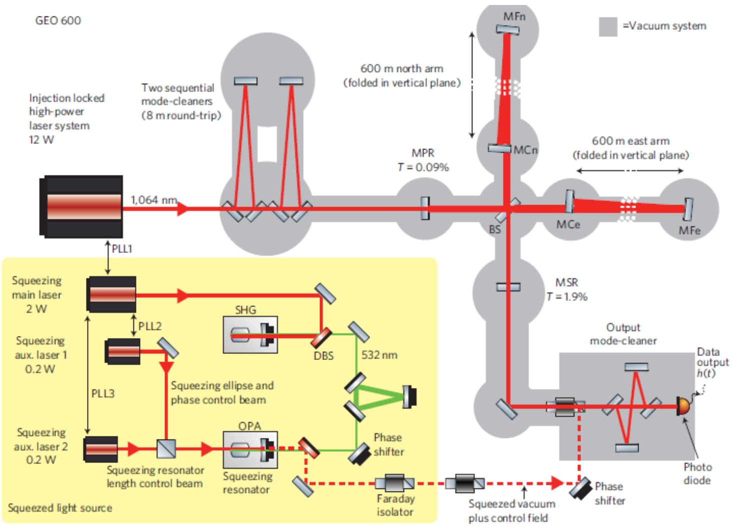
Weak squezing + simple measurement + simple estimator = optimal strategy! (thanks to decoherence)

Optimality of the squeezed vacuum+coherent state strategy

$$\frac{\Delta\varphi^{\text{optimal}}}{\Delta\varphi^{\text{squeezed}}}$$



Quantum precision enhancement in the GEO600 interferometer



Quantum precision enhancement in the GEO600 interferometer


LETTERS
PUBLISHED ONLINE: 11 SEPTEMBER 2011 | DOI: 10.1038/NPHYS2083

nature physics

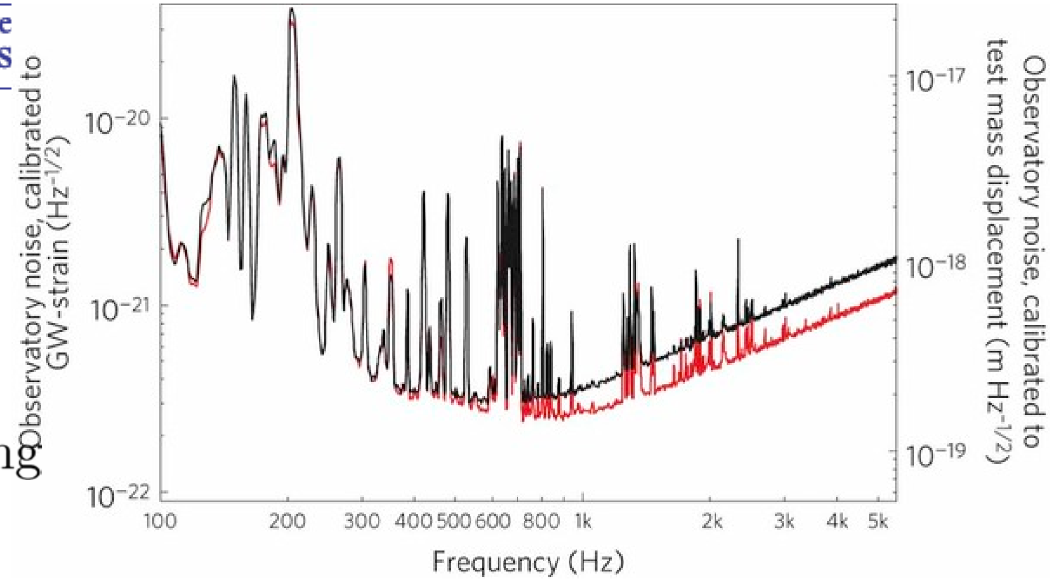
A gravitational wave observatory operating beyond the quantum shot-noise limit

The LIGO Scientific Collaboration †*

$$\frac{\Delta x_{\text{squeezed}}}{\Delta x_{\text{standard}}} \approx 0.66$$

$\eta = 0.62$

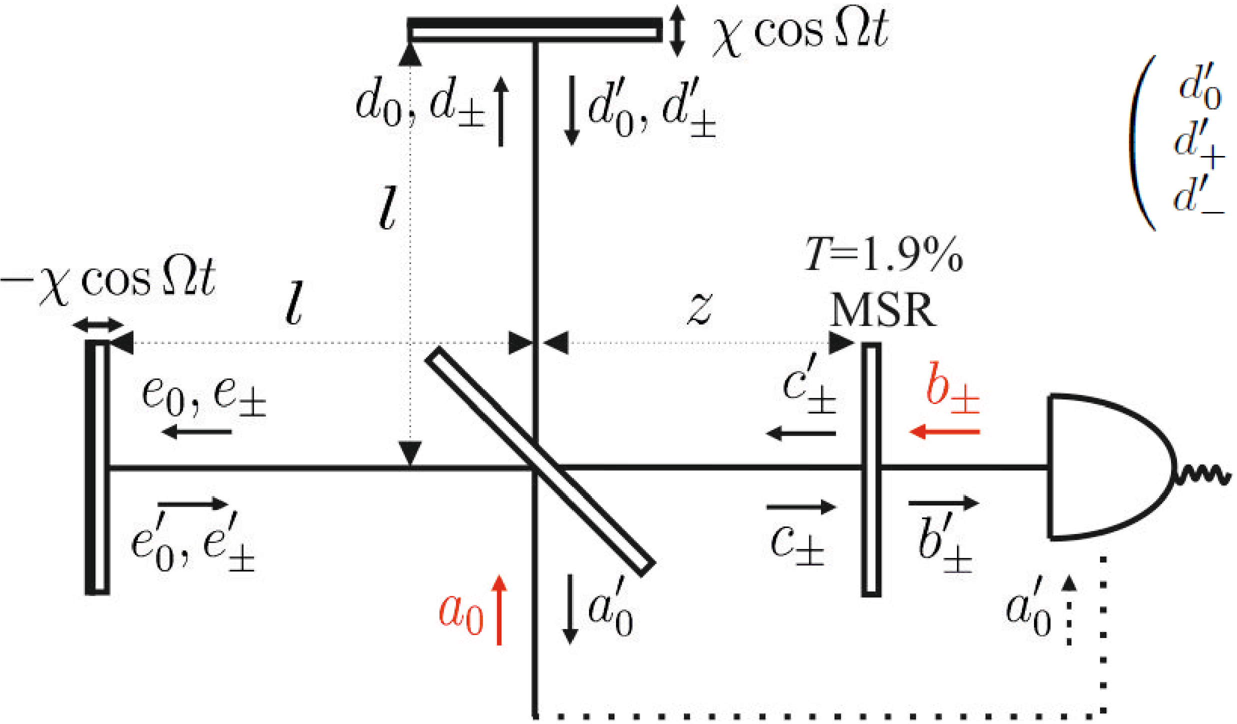
10dB squeezing



Can the precision be improved by using better quantum states and better measurements?

$$\Delta\varphi^{\text{quantum}} \geq \sqrt{\frac{1-\eta}{\eta}}\frac{1}{\sqrt{N}} \qquad \Delta\varphi^{\text{standard}} = \frac{1}{\sqrt{\eta N}} \qquad \frac{\Delta\varphi^{\text{quantum}}}{\Delta\varphi^{\text{standard}}} \geq \sqrt{1-\eta} = 0.61$$

Quantum optical model



$$\begin{pmatrix} d'_0 \\ d'_+ \\ d'_- \end{pmatrix} = \begin{pmatrix} 1 & i\epsilon & i\epsilon \\ i\epsilon & 1 & 0 \\ i\epsilon & 0 & 1 \end{pmatrix} \begin{pmatrix} d_0 \\ d_+ \\ d_- \end{pmatrix}.$$

$$\epsilon = 2\pi\chi/\lambda$$

$$\begin{pmatrix} a'_0 \\ b'_s \end{pmatrix} = \begin{pmatrix} 1 & \sqrt{2}g\epsilon \\ -\sqrt{2}g\epsilon & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ b_s \end{pmatrix}$$

$$\begin{pmatrix} a'_0 \\ b'_+ \\ b'_- \end{pmatrix} = U \begin{pmatrix} a_0 \\ b_+ \\ b_- \end{pmatrix}, \quad U = \begin{pmatrix} 1 & g_+\epsilon & g_-\epsilon \\ -g_+\epsilon & 1 & 0 \\ -g_-\epsilon & 0 & 1 \end{pmatrix} \qquad g_\pm = \sqrt{\frac{T}{2 - T - 2\sqrt{1-T}\cos[2(\omega_0 \pm \Omega)(l+z)/c]}}$$

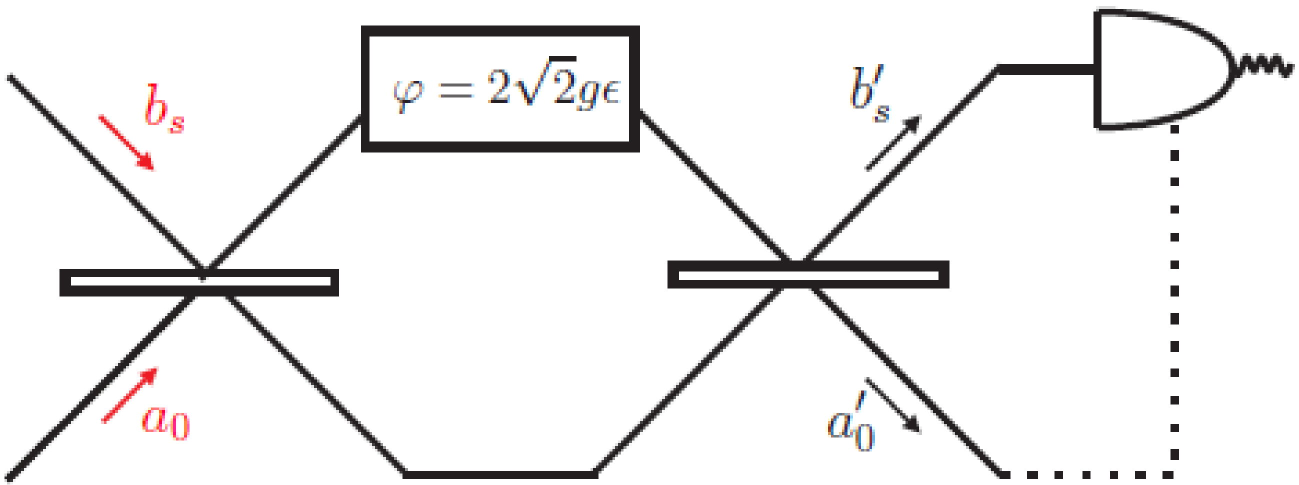
up to irrelevant phase factors and assuming no transition to higher order sidebands

If recycling cavity is tuned to the central frequency: $g_+ = g_-$

Effectively two modes: $a_0, b_s = (b_- + b_+)/\sqrt{2}$

Equivalent quantum model

$$\begin{pmatrix} a_0' \\ b_s' \end{pmatrix} = \begin{pmatrix} 1 & \sqrt{2}g\epsilon \\ -\sqrt{2}g\epsilon & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ b_s \end{pmatrix}$$

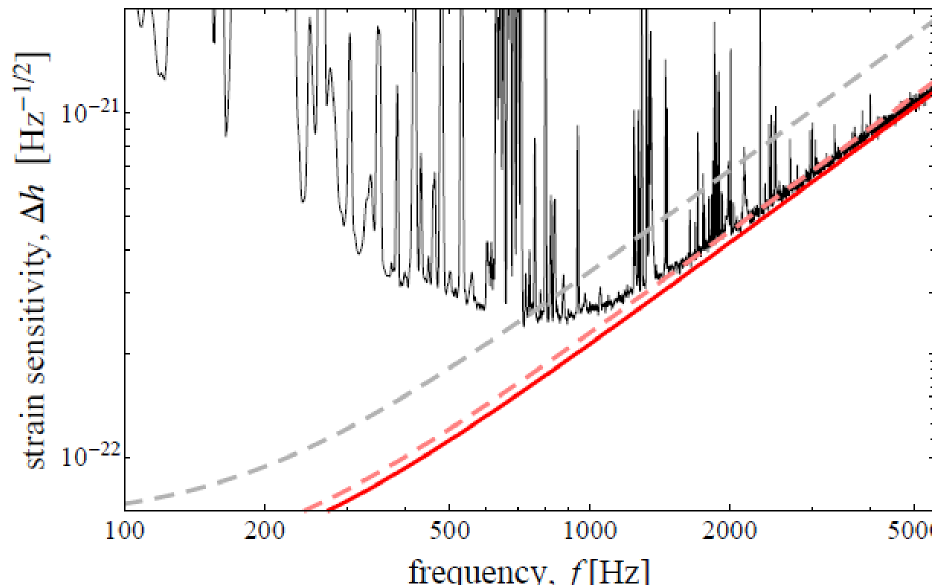


$\epsilon = 2\pi\chi/\lambda$

$$g = \sqrt{\frac{T}{2-T-2\sqrt{1-T}\cos[2\Omega l/c]}}$$

the problem reduced Mach-Zehnder interferometry!

GEO600 interferometer at the fundamental quantum bound



Gravitational wave strain sensitivity:

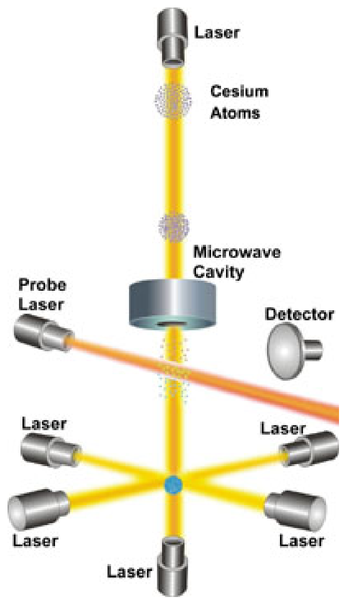
$$\Delta h = \frac{2\chi}{l}$$ ← detectable mirror oscillation amplitude

- - - coherent light
- - - +10dB squeezed
— fundamental bound

$$\Delta h(f) = \frac{1}{lg}\sqrt{\frac{\hbar c \lambda}{16P}}\sqrt{\frac{1-\eta}{\eta}} \qquad g = \sqrt{\frac{T}{2-T-2\sqrt{1-T}\cos[2\Omega l/c]}}$$

R. Demkowicz-Dobrzanski, K. Banaszek, R. Schnabel, arxiv:1302????

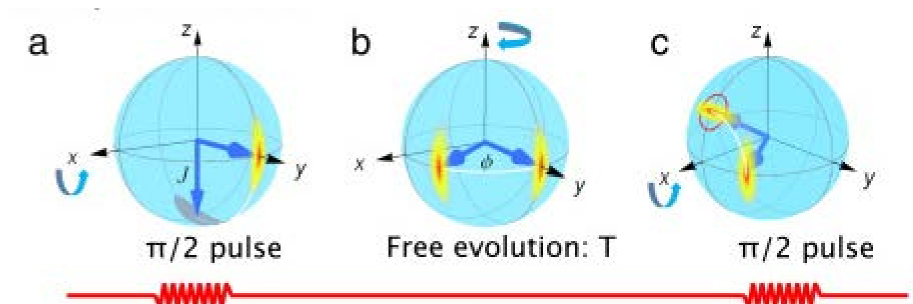
Ramsey interferometry



Atomic clocks calibration, *N* two level atoms

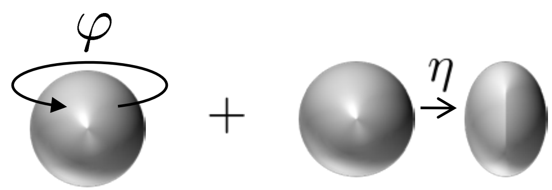
$$U = \exp[-\mathrm{i}\hat{J}_z\varphi], \quad \varphi = \omega T$$

cavity vs atom transition detuning



precision improved thanks to squeezing $|\psi\rangle = e^{s(J_+^2 - J_-^2)}|\uparrow\rangle^{\otimes N}$

Taking into account dephasing:



$$\Delta\varphi = \sqrt{\frac{\Delta^2\hat{J}_x}{\langle\hat{J}_y\rangle^2} + \frac{1-\eta^2}{\eta^2}\frac{N}{4\langle\hat{J}_y\rangle^2}}. \longrightarrow \Delta\varphi \approx \frac{\sqrt{1-\eta^2}}{\eta}\frac{1}{\sqrt{N}}$$

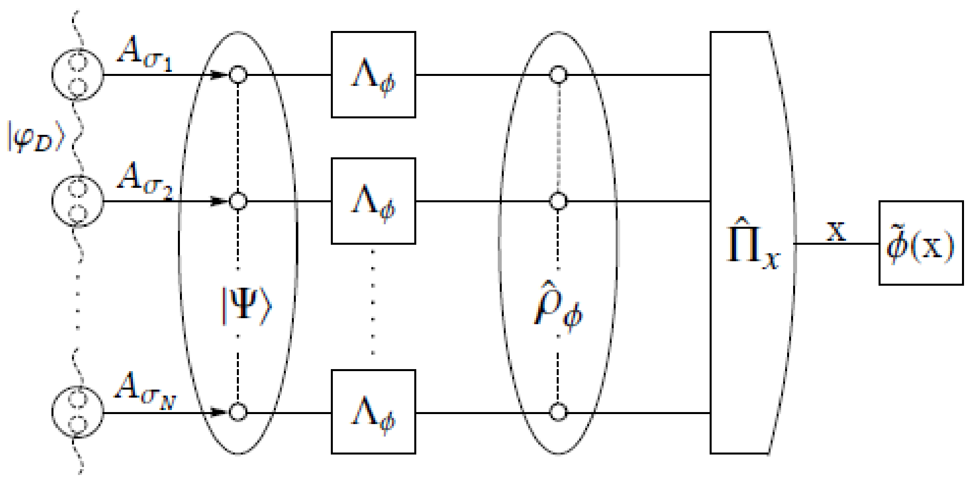
For large N using spin-squeezed states:

$$\langle J_y\rangle \approx N/2 \qquad \frac{\Delta^2 J_x}{\langle J_y\rangle^2} \ll \frac{1}{N}$$

Fundamental bound!

S. Huelga et al. Phys. Rev. Lett 79, 3865 (1997)

Matrix product states and metrology?



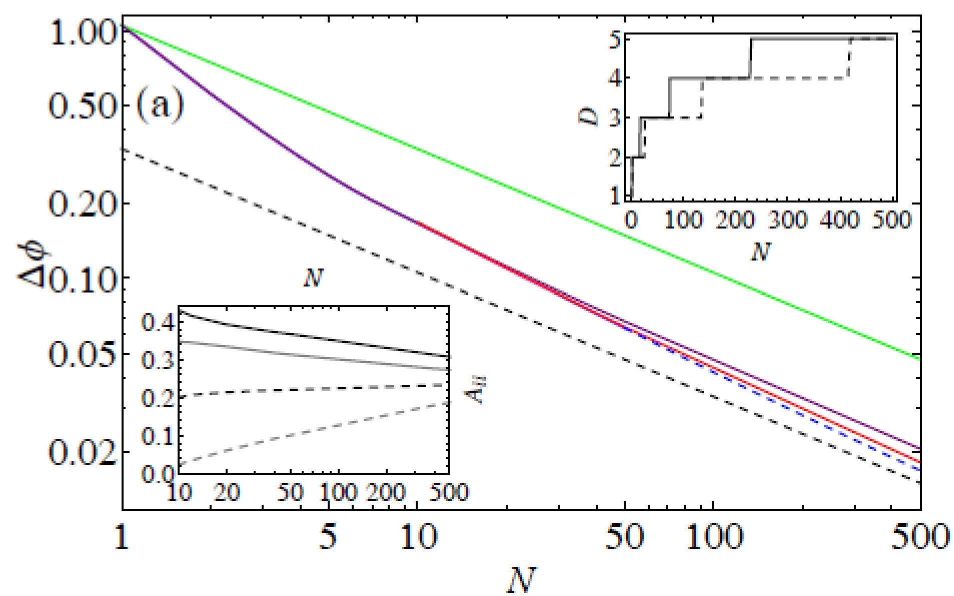
$$\Delta\varphi = \frac{c}{\sqrt{N}}$$

No need to entangle large number of probes

↑

Matrix product state with bond dimension D

Low bond dimension matrix product states are sufficient.



M.Jarzyna, R. Demkowicz-Dobrzanski, arxiv:1301.4246

Summary

- Heisenberg scaling asymptotically destroyed ☹
- Simple schemes saturate the fundamental bounds ☺
- GEO600 optimal ☺
- The same applies to atoms with loss/dephasing – spin squeezed states + Ramsey interferometry optimal ☺
- Matrix product states and metrology…. ?
- Translate results to quantum oracle algortihms with noise?

R. Demkowicz-Dobrzański, J. Kolodyński, M. Guta, Nat. Commun. **3**, 1063 (2012)
R. Demkowicz-Dobrzanski, K. Banaszek, R. Schnabel, arxiv:1302????
M.Jarzyna, R. Demkowicz-Dobrzanski, arxiv:1301.4246