Hubble redshift in Einstein's universe

W. Jim Jastrzebski

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It is shown that the observed features of Hubble redshift can be explained within the framework of Einstein's general relativity. The observed Hubble redshift could be attributed to thus far unnoticed mechanism of time dilation coupled to the curvature of space. Einstein's universe regains its status as a viable model predicting cosmological observations such as the (*apparent*) expansion of space and the observed acceleration of this expansion.

Derivation of Hubble constant of Einstein's universe

Consider Einstein's homogeneous universe filled with dust. Let photons move through this dust interacting with it only gravitationally. We will assume that energy conservation holds and that Newton's approximation can be applied. With these assumptions one can readily calculate energy transfer from photons to dust. To the observer at some distance from the light source this energy transfer will manifest itself as a change in wavelength, which is exactly what was observed by Hubble. The relativistic interpretation of this result allows the derivation of Hubble redshift (HR), including discovery that Hubble's constant depends on the distance between the place in deep space and the observer.

Let $E_d = E_0 - E$ be the gravitational energy acquired by the dust due to gravitational interaction between dust and photons of energy E and initial energy E_0 and let $\Lambda_E = 4\pi G\rho/c^2$, where G is Newtonian gravitational constant, ρ is density of dust, and c is speed of light (which makes, seemingly accidentally, Λ_E equal to Einstein's value of cosmological constant of Einstein's universe or R_E^{-2}, where R_E is radius of Einstein's universe). The linear density (force per unit length) of Newton's gravitational force per unit mass which is identically equal to d^2E/dr^2, where r is distance travelled by photons, can be written (using relativistic relation between mass and energy $m = E/c^2$) as $4\pi G\rho(E_0 - E_d)/c^2$ leading to equation

$$d^2E/dr^2 = \Lambda_E E \tag{1}$$

Solving the equation with initial conditions $E(r = 0) = E_0$ and $(dE/dr)(r = 0) = -E_0/R_E$ (selecting a solution that makes physical sense) one gets

$$E = E_0 exp(-r/R_E) \tag{2}$$

Since in Einstein's general relativity (EGR) there is nothing else but time dilation and the curvature of space as the media controlling gravitation, EGR interpretation of the above result is that time is running slower at a distance from (any) observer according to relation

$$d\tau/dt = exp(-r/R_E) \tag{3}$$

where τ is proper time in deep space and t is coordinate time at observer. The effect might be called *Hubble Time Dilation* (HTD) in honor of its discoverer, and as distinguished from the *gravitational time dilation* predicted by Einstein.

After differentiating the above equation at $r = 0$ we get a relation between the HTD in deep space ($\partial\tau^2/\partial t\partial r$) and the curvature of space ($1/R_E$) as

$$\partial\tau^2/\partial t\partial r + 1/R_E = 0 \tag{4}$$

It follows from equation (2) or (3) equivalently, that the redshift, produced by HTD, is equal to

$$Z = (E_0 - E)/E = exp(r/R_E) - 1 \tag{5}$$

simulating the expansion of space, with the Hubble constant of this apparent expansion at $r = 0$

$$H_0 = c/R_E \tag{6}$$

For Einstein's universe of density $\rho \approx 6 \times 10^{-27} kg/m^3$ Hubble constant $H_0 \approx 70 km/s/Mpc$.

After expanding the Hubble "constant", H(t), into Taylor series around $t = 0$ and neglecting the higher order terms, the acceleration of this apparent expansion is

$$dH/dt \approx -H_0^2/2 \tag{7}$$

which agrees within one standard deviation with 1998 observations by the Supernova Cosmology Project team [1].

Conclusions

The analysis of Hubble time dilation (HTD) as presented by eq. (3) can be carried out using Einstein's general relativity and the law of conservation of energy. The observed HTD can be attributed to the geometry of spacetime of the stationary homogenous dust universe. Such a model may be oversimplified, however, it does reproduce the essential properties of HTD, and allows one to estimate the important properties of the universe such as the average density of space and the acceleration of its apparent expansion. While the formula for Hubble redshift, equation (5), can be derived directly from equation (2) obtained using Newtonian approximation, it is equation (3) that expresses the essential transition from Newtonian approach in which space and time are distinct, to a general relativistic spacetime. Excellent agreement of the calculated acceleration of apparent expansion of the model universe with the measured value [1], lends additional support for the model.

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References

[1] Kuznetsova N, Barbary K, et al, A New Determination of the High-Redshift Type Ia Supernova Rates with the Hubble Space Telescope Advanced Camera for Surveys, 2008, The Astrophysical Journal, Volume 673, Issue 2, The American Astronomical Society, pp. 981-998, Bibliographic Code: 2008 ApJ...673..981K, http://supernova.lbl.gov/