4 marca 2011
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ASYMPTOTICS of THE LORENTZIAN EPRL MODEL

Based on [arXiv:0907.2440] with J.W. Barrett, R.J Dowdall, F. Hellmann and R. Pereira

Outline: I 4-simplex amplitude
II Asymptotic analysis
III Asymptotic formula

I 4-simplex amplitude

σ be a 4-simplex
Corresponding amplitude A_σ determined by data associated to $\partial\sigma$

I.1 Boundary state space

Notation: • $\Pi_k : SU(2) \to \mathrm{Aut}\, V_k$, $k \in \mathbb{N}/2$
is the spin k UIR of $SU(2)$
$(V_k \cong \mathbb{C}^{2k+1})$

• Tetrahedra $t \in \partial\sigma$ labelled $a = 1, .., 5$
$\Rightarrow (ab)$, $a \neq b$, label the triangles

Given the assignments

$$k : \text{triangles} \longrightarrow \mathrm{Irrep}(SU(2))$$
$$ab \longmapsto k_{ab} \,,$$

One can associate a state space to each tetrahedron $t \in \partial\sigma$:

$$\mathcal{H}_a = \mathrm{Inv}_{SU(2)}\Big(\bigotimes_{b : b \neq a} V_{k_{ab}} \Big)$$

State space for $\partial\sigma$: $\mathcal{H}_{\partial\sigma} = \bigotimes_a \mathcal{H}_a$

Amplitude for σ : linear map

$$A_\sigma : \mathcal{H}_{\partial\sigma} \longrightarrow \mathbb{C}$$

Key step for the asymptotics :

parametrise $\mathcal{H}_a$, $a = 1, \dots, 5$, using coherent states [Livine, Speziale '07]

• Coherent state (CS) for a the direction $n \in \mathbb{R}^3$ and spin k :

unit vector $\xi \in V_k$ defined by :

$$(J \cdot n)\, \xi = i k\, \xi$$

generators of $su(2)$ $J = (J_1, J_2, J_3)$

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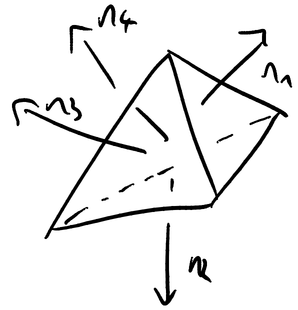
- In the coherent state basis,

$\Psi_a \in \mathcal{H}_a$ is written



$$\Psi_a(k,n) = \int_{SU(2)} dX \bigotimes_{b:b\neq a} \Pi_{k_{ab}}(X)\, \zeta_{ab}$$

State for $\partial\sigma$: $\quad \Psi(k,n) = \bigotimes_a \Psi_a(k,n)$

Data $\{k_{ab}, n_{ab}\}_{a\neq b}$ specifying $\Psi(k,n)$ up to a phase is called the boundary data (BD)

Important class of BD : geometric BD

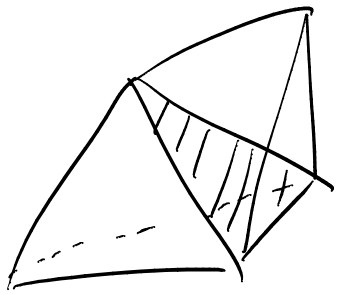
i) Non degenerate : $\forall a$, $\mathrm{Span}\{n_{ab}\}_{b\neq a} \cong \mathbb{R}^3$

$\Rightarrow$ If $\sum_{b:b\neq a} k_{ab}\, n_{ab} = 0$ (closure condition)

$\{n_{ab}\}_{b\neq a}$ specify an embedding of tetrahedron a in $\mathbb{R}^3$

$\hookrightarrow$ Each tetrahedron inherits a metric and orientation

ii) Gluing : individual tetrahedra metrics and orientations glue together consistently



For geometric boundaries : canonical choice of phase for $\Psi(k,n)$

$\exists!$ set of 10 SU(2) elements $g_{ab} = g_{ba}^{-1}$ that glue the tetrahedra

$$g_{ba} \, n_{ab} = - n_{ba}$$

Select the phase of the CS by

$$\zeta_{ba} = g_{ba} J \zeta_{ab} ,$$

where $J : V_k \to V_k$ is the SU(2) antilinear structure

Ex $k = \frac{1}{2}$ $\quad J \begin{pmatrix} z_0 \\ z_1 \end{pmatrix} = \begin{pmatrix} -\bar{z}_1 \\ \bar{z}_0 \end{pmatrix}$

$$\begin{array}{ccc} \zeta & \longrightarrow & n \\ J\downarrow & & \downarrow \text{ antipodal map} \\ J\zeta & \longrightarrow & -n \end{array}$$

I.2 Amplitude

Lorentzian EPRL model based $SL(2,\mathbb{C})_{\mathbb{R}}$

Principal series of unitary, irreducibles of $SL(2,\mathbb{C})_{\mathbb{R}}$ are labelled $\chi = (n, p)$

$$n \in \mathbb{Z}/2 \ , \quad p \in \mathbb{R}$$

Representations act in the Hilbert space V_χ of homogeneous functions on $\mathbb{C}^2$

$$f(\lambda z) = \lambda^{-1+ip+k} \bar{\lambda}^{-1+ip-k} f(z)$$

$$z = (z_0, z_1) \in \mathbb{C}^2 \ , \ \lambda \in \mathbb{C}$$

with $$\Pi_\chi(X) f(z) = f(X^t z)$$

$$\forall X \in SL(2,\mathbb{C})_{\mathbb{R}}$$

Inner product: $$(f, g) = \int_{\mathbb{CP}^1} \Omega \, \bar{f} g$$

Ω standard invariant 2-form on

$\mathbb{C}^2 - \{0\}$

Representations factorise into representations of $SU(2)$ subgroup

$$V_\alpha = \bigoplus_{j=|n|}^{\infty} V_j$$

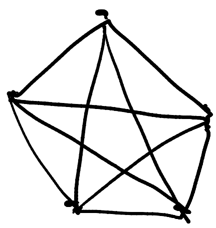
Lorentzian EPRL construction
[Engle, Pereira, Rovelli, Livine '08]

$$f_\gamma : \mathrm{Irrep}(SU(2)) \longrightarrow \mathrm{Irrep}(SL(2,\mathbb{C})_{\mathbb{R}})$$
$$k \longmapsto (n(k), \rho(k)) = (k, \gamma k)$$

$\gamma \in \mathbb{R}^+$ is a fixed parameter

This identification : embedding of the cs for the boundary into the space of $SL(2,\mathbb{C})_{\mathbb{R}}$ intertwiners

Pairing of these intertwiners :



↳ Amplitude

$$A_\sigma(k,n) = \int_{SL(2,\mathbb{C})_{\mathbb{R}}} \prod_a dX_a \, \delta(X_s) \prod_{a<b} P_{ab}$$

$$A_\sigma(k,n) = \int_{SL(2,\mathbb{C})_n} \prod_a dX_a \; \delta(X_s) \prod_{a<b} P_{ab}$$

gauge fixing

<u>Kernel</u>: $$P_{ab} = \int_{\mathbb{C}P^1} \Omega \, \langle X_a^\dagger z, X_a^\dagger z \rangle^{-1-i\rho_{ab}-n_{ab}} \langle X_a^\dagger z, \zeta_{ab} \rangle^{2n_{ab}} \times$$

$$\times \langle X_b^\dagger z, X_b^\dagger z \rangle^{-1+i\rho_{ab}-n_{ab}} \langle J \zeta_{ba}, X_b^\dagger z \rangle^{2n_{ab}}$$

where: $\langle \, , \rangle$ Hermitian inner product on $\mathbb{C}^2$

$\zeta \in \mathbb{C}^2$ are CS in the $k=\frac{1}{2}$ representation

$(n,\rho) = (k, \gamma k)$

II Asymptotic analysis

Semi-classical limit:

$$\text{Area}(ab) \sim 8\pi G \hbar \gamma k_{ab}$$

$\hbar \to 0$ keeping Area(ab) fixed

$\hookrightarrow$ All spins rescalled $k_{ab} \to \lambda k_{ab}$ with

$\lambda \to \infty$