4 marca 2011
17:40

4D

$\partial(12345) =$

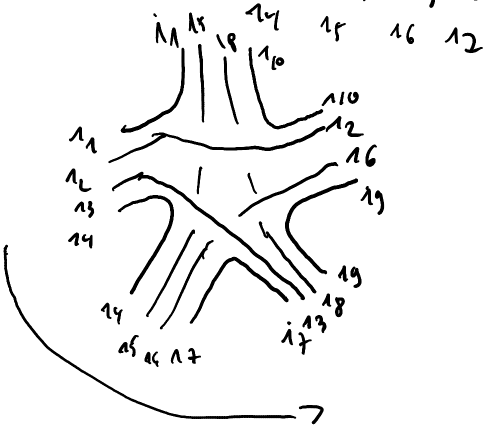
$$\begin{aligned}
&(2345) \times M_{345,\overline{245},235,\overline{234}} \quad (i_1\ i_2\ i_3\ i_4)\\
&\overline{(1345)} \times M^*_{345,\overline{145},135,\overline{134}}\\
&(1245) \times M_{245,\overline{145},125,\overline{124}}\\
&\overline{(1235)} \times M^*_{235,\overline{135},125,\overline{123}}\\
&(1234) \times M_{234,\overline{134},124,\overline{123}}
\end{aligned}$$

$3 \to 4 \to 5 \to 1 \quad M_{451,\overline{351},341,\overline{345}} \quad (i_{10}\ i_8\ i_5\ i_1)$

$5 \leftrightarrow 2 \quad 1 \leftrightarrow 4 \quad M_{512,\overline{412},452,\overline{451}} \quad (i_9\ i_6\ i_2\ i_{10})$

3F

$2 \to 1 \to 5 \quad M_{123,\overline{523},513,\overline{512}} \quad (i_7\ i_3\ i_8\ i_9)$



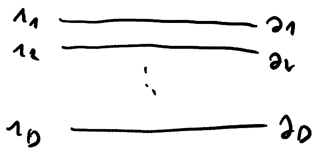
$M_{i_1,i_2,i_3,i_4} \; M_{i_4\, i_5\, i_6\, i_7}$

$M_{i_7,i_3,i_8,i_9} \; M_{i_9,i_6,i_2,i_{10}}$

M_{i_{10},i_8,i_5,i_1}

$M_{\underbrace{i_1,\dots,i_D}_{D\ D-2\text{ simplices}}} \longrightarrow D-1$ simplex

M

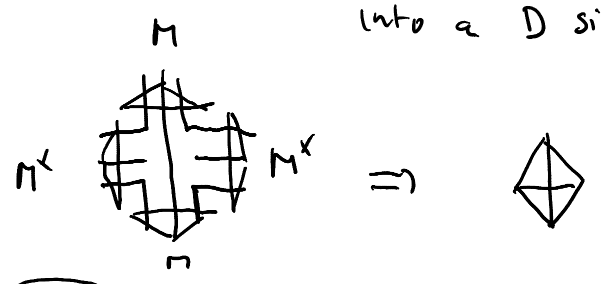


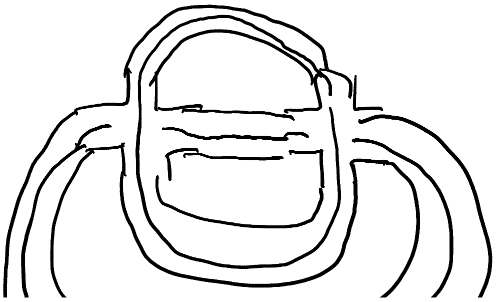
M^*

$$\sum_{\sigma\ \text{even}} \delta_{j_1, i_{\sigma(1)}} \cdots \delta_{j_D, i_{\sigma(D)}}$$

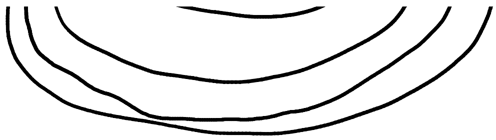
Vertex : gluing of D+1 (D) simplices into into a D simplex

D = 3





any GFT graph describes a gluing of



D simplices along the edges of the graph

Closed loops (faces) correspond to D-2 simplices (Regge)

$\rightarrow$ 2-complex which the 2-skeleton dual to a gluing of D simplices

generates objects more general that dual of triangulations of manifolds

+ M is real and invariant under all σ

$\rightarrow$ non oriented objects

+ M is real and invariant under even σ

$\rightarrow$ oriented objects with a twist in the quadratic part of the action

see di Pietri, Freidel, Krasnov, Rovelli "BC model from a BO field over homogenous space" hep-th/9907157

III. BF theory

Discretization of BF theory

$$Z = \int DA\, DB \; e^{i \int \mathrm{tr}(B \wedge F)}$$

A : Lie algebra valued 1 form

B: D-2 form, $F = dA + A^2$

$$\text{"}=\text{"} \int DA \; \delta(F)$$

D manifold $\rightarrow$ triangulation

B lives on the face of the dual of the

B lives on the faces of the dual of the triangulation

$$Z "=" \int_{SU(2)} dg_e \prod_{f\in\Gamma} \delta\Big(\overrightarrow{\prod_{e\in\partial f}} g_e^{\varepsilon_{fe}}\Big)$$

$A \to g_e \in SU(2)$ associated to edges of Γ

$$\varepsilon_{fe} = \begin{cases} +1 & \text{if the orientation of } f \text{ and } e \text{ agree} \\ -1 & \text{" disagree} \end{cases}$$

$g_{v,e}$ $\qquad g_{s(e),e} = g_e$

s, t source and target

$g_e \to g_{s(e),e}\, g^{-1}_{t(e),e}$

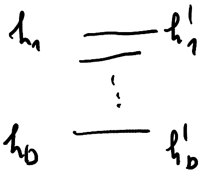
$$Z = \int_{SU(2)} dg_{v,e} \prod_{f} \delta\Big(\prod_{v\in\partial f} g^{-1}_{v,e}\, g_{v,e'}\Big)$$

e = edge entering v

e' = edge leaving v

Group field theory

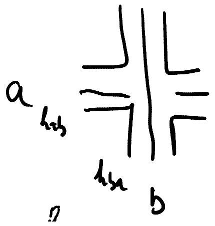
$\varphi(h_1, h_2, \dots, h_D)$ field defined on $SU(2)^D$



$$\sum_{\sigma \text{ even}} \delta(h_1, h'_{\sigma(1)}) \cdots \delta(h_D, h'_{\sigma(D)})$$

group elements are just indices of the generalized matrix model

$V(h_{ab})$



$$\prod_{1\le a<b\le D} \int dg_a\, \delta(h_{ab}\, g_a\, g_b^{-1}\, h_{ba}^{-1})$$

h_{ab}

D^2 elements

Kernel of the interaction

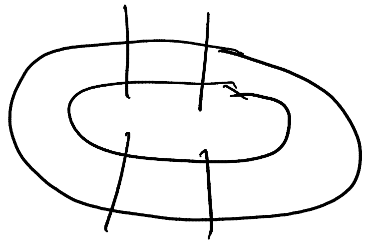
$$V(\varphi) = \underbrace{\int dh_{ab}\; V(h_{ab})\; \varphi^{D+1}(h_{ab})}_{\text{symbolical expression}}$$

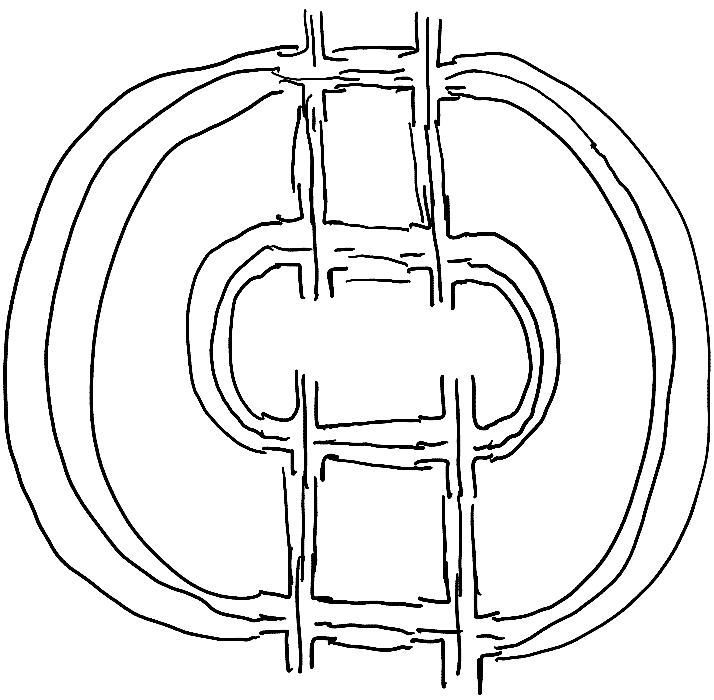
Here we used a projector $\int dg$ on the vertices but many use it on the edges

Divergence of GFT graphs

$$\underbrace{\delta(1)}_{\text{divergent}} \int dg\, \delta(h_1 g h_1'^{-1})\, \delta(h_2 g h_2'^{-1})\, \delta(h_3 g h_3'^{-1})$$

3D





$\rightarrow \delta(1)$

primitively divergent graph with 8 external legs

Two recent breakthroughs

+ $1/N$ expansion of colored tensor model
  R. Gurau 1102.5759

+ Formulation using using products
  A. Baratin, F. Girelli, D. Oriti
  "Diffeomorphisms in group field theory"
  1101.0590

IV. EPRL/FK

$h_f = \prod_{\text{vert}} h_{v,f}$ with h_f for external edges (links)

$$\mathcal{A}_\gamma = \int_{SU(2)} dh_{v,f} \; \delta(h_f) \prod_v \mathcal{A}_v(h_{fv})$$

$$\mathcal{A}_v(h_{fv}) = \int_{SL(2,\mathbb{C})} dg_n \; \prod_e K(h_e, g_{se}\, g_{te}^{-1})$$

$$K(h,g) = \sum_j \int_{SU(2)} dk \; d_j^2 \; \underbrace{\mathrm{Tr}_{V_j}(\bar{h}k)}_{SU(2)} \; \underbrace{\mathrm{Tr}_{V_{j,\gamma j}}(kg)}_{SL(2,\mathbb{C})}$$

$$V_{j,\gamma j} = \bigoplus_{j' \geq j} V_{j'} \quad \text{as an } SU(2) \text{ representation}$$

$$\mathrm{Tr}_{V_j}(\bar{h}k)\, \mathrm{Tr}_{V_{j,\gamma j}}(kg) = \qquad |j',m\rangle \; j' \geq j \text{ basis } V_{j,\gamma j}$$

$$\int dk \sum_{m,m'} \overline{\langle j,m|h|j,m'\rangle} \, \overline{\langle j,m'|k|j,m\rangle}$$

$$\sum_{j',m'',m''' \atop \cancel{j''}} \langle \underset{j'}{j',m''}|k|\underset{j'}{j'',m'''}\rangle \langle j'',m'''|g|j' m'\rangle$$

$m'' = m'$
$m''' = m$ $\quad j' = j$

$$\longrightarrow \frac{1}{d_j} \sum_{m, m'} \overline{\langle j\, m | h | j, m' \rangle} \langle j, m | g | j, m' \rangle$$

$$K(h, g) = \sum_j d_j \, \mathrm{Tr}_{V_j} (\bar{h}\, P g P)$$

$P_j \quad V_{j, \gamma r} \to V_j$

SL(2,C) rep $\qquad$ SU(2) rep

$$\delta\Big(\prod_{v \in \partial f} h_{vf}\Big) = \sum_j d_j \, \mathrm{Tr}_{V_j}\Big(\prod_{v \in \partial f} h_{v,f}\Big)$$

perform the integration over h_{vf}
using Schur's orthogonality relation

$$\mathcal{A}_f = \sum_j d_j \, \mathrm{Tr}_{V_j} \Big[\prod_{v \in \partial f} P_j \, g_{v,e}^{-1} g_{v,e'} P_j \Big]$$

with insertion of extra h_e
for the boundary of the 2-complex

GFT for the EPRL model

$\varphi(h_1, h_2, h_3, h_4) \qquad h_i \in SU(2)$

Propagator as in BF

Vertex slightly different from BF

$$\prod_{1 \leq a < b \leq 4} \int_{SL(2,C)} dg_a' \sum_j d_j \mathrm{Tr}_{V_j} \left(h_{ab} P_j g_a^{-1} g_b P_j h_{ba} \right)$$

Correlation function with N external legs equipped with fields $\varphi(h_1, \dots h_4)$ depending on $SU(2)$ group elements can be related to elements of the LQG Hilbert space

If GFT is physical, its prescription for summing Feynman graphs into correlations function must have some consequences in LQG

Ex: Schwinger-Dyson Equation

Wilsonian renormalization of GFT

renormalization :

+ regularization $\int dk \to \int_0^{\Lambda_0} dk$ UV cut off
+ renormalization $(\lambda, m) \to (\lambda(\Lambda_0), m(\Lambda_0))$

$\to$ Effective action : work with scale depend parameters (action)

$$\int [D\varphi]_{0,\Lambda_0} \varphi(k_1) \dots \varphi(k_n) e^{-S_0[\varphi]} =$$

$$\varphi = \varphi_{0,\Lambda} + \varphi_{\Lambda,\Lambda_0}$$

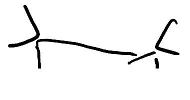
$$\int [D\varphi]_{0,\Lambda} \varphi(k_1) \dots \varphi(k_n) \underbrace{\int [D\varphi]_{\Lambda,\Lambda_0} e^{-S_0[\varphi]}}_{e^{-S_\Lambda[\varphi]}}$$

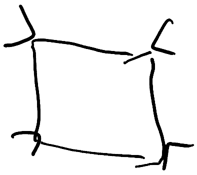
$|k| \leq \Lambda$

$\varphi_{0,\Lambda}$

effective action at scale Λ

Coarse graining of geometries

 interaction . . .



integration

over
short
distance
degrees of freedom