Bose-Einstein condensates and nonlinear waves

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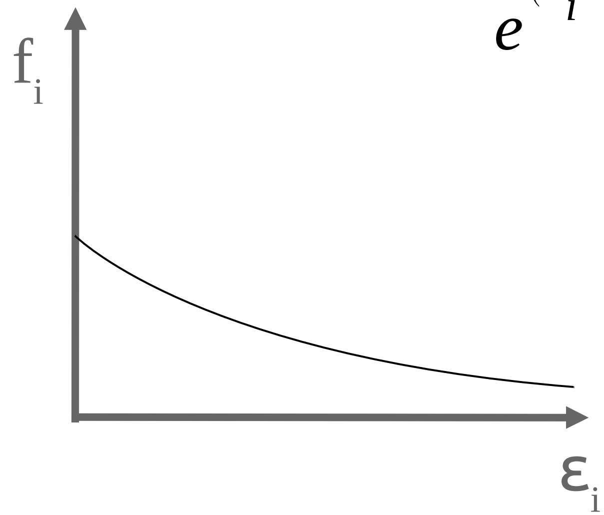
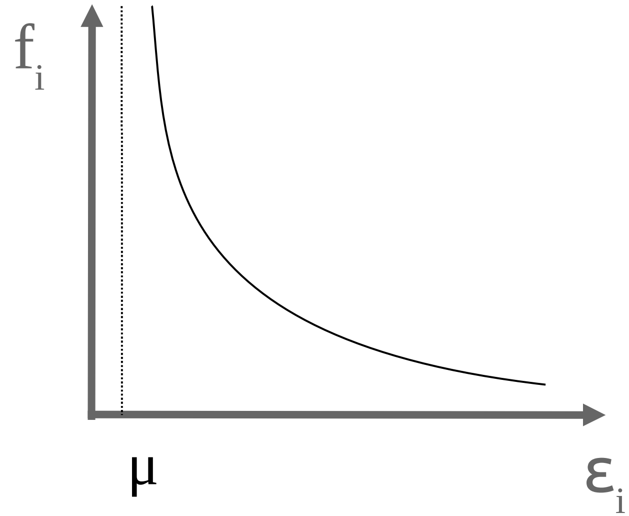
Bose-Einstein condensation

At some energy, divergence may occur

This means that the occupation of the lowest state becomes macroscopic

$$f_i = \frac{1}{e^{(\epsilon_i - \mu)/k_B T} - 1}$$





This can be achieved in two ways: decreasing T or increasing μ (number of particles)

(Berezinskii-)Kosterlitz-Thouless phase transition



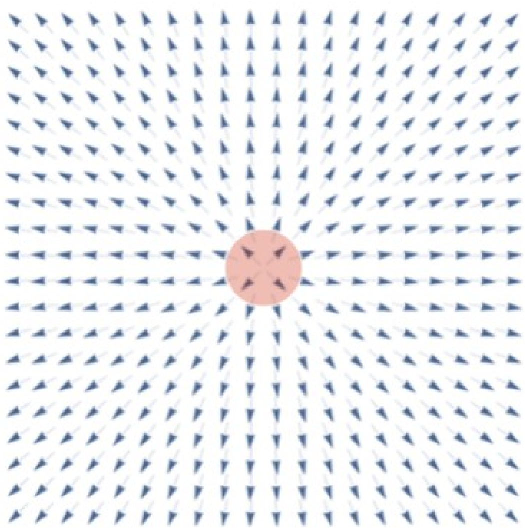
- Nobel prize in Physics 2016

- Awarded to D. J. Kosterlitz, J.M. Thouless and F. D. M.Haldane “for theoretical discoveries of topological phase transitions and topological phases of matter" (in part for the prediction of the KT transition)

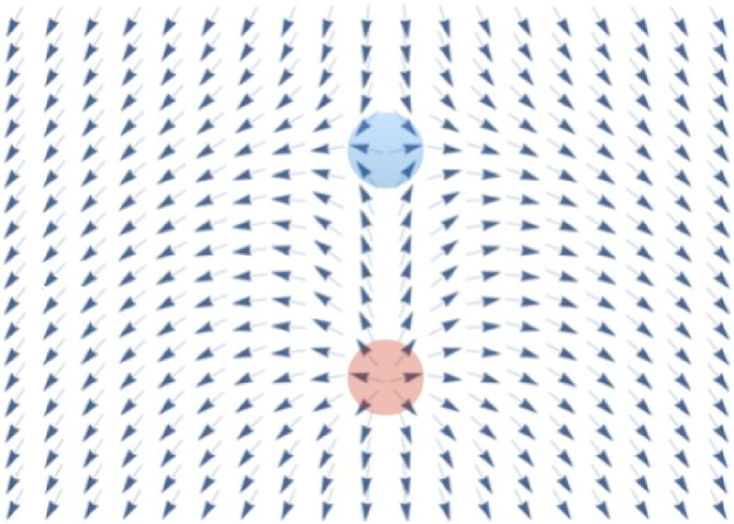
2D XY model and vortices

- A classical model of an array of spins in 2D

$$H_{XY} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$



Vortex



Vortex-Antivortex pair

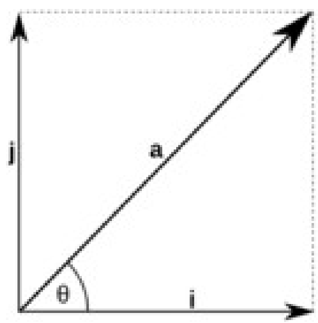
2D XY model and vortices

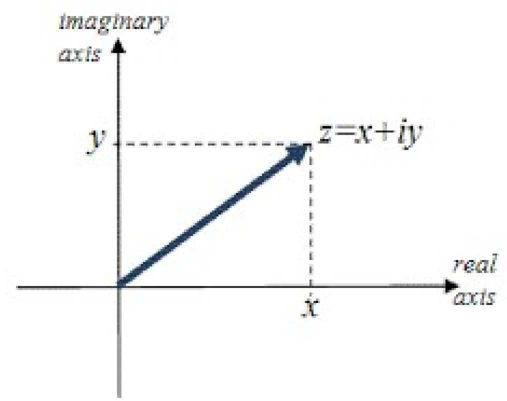
- Correspondence with complex fields

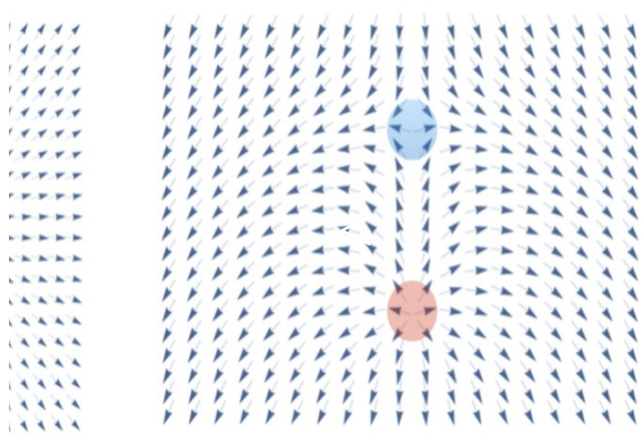
$$H_{XY} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

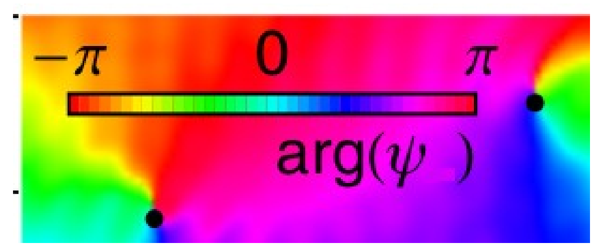
$$H_{XY} = \frac{J}{2} \int d^2r \, (\vec{\nabla}\theta(\vec{r}))^2 \, .$$

$$\psi = A\, e^{i\theta}$$



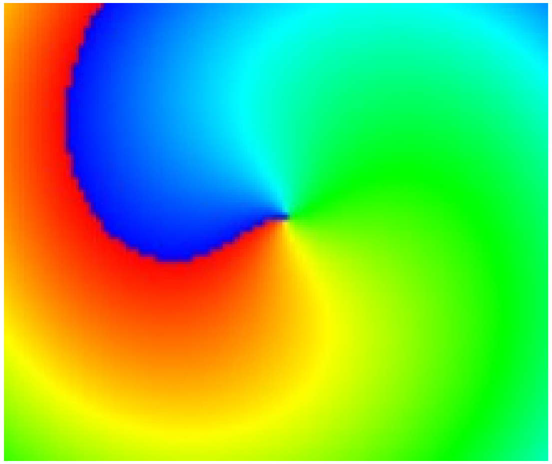


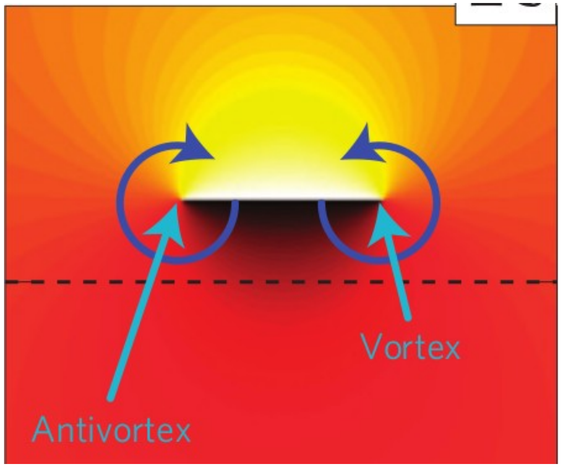




KT transition

- Phase circulation far from vortex core: Energy of a vortex diverges logarithmically with system size!





- Vortex-antivortex pairs have finite energy

- Considering free energy, even the state with free vortices can exist

$$F = E - TS = J\pi \ln\left(\frac{L}{a}\right) - Tk_B \ln\left(\frac{L^2}{a^2}\right) \qquad T_{KT} = J\pi/2k_B$$

KT transition

- The phase with bound vortex-antivortex pairs has a nonzero superfluid fraction and power-law decay of coherence

$$\langle e^{i(\theta(\vec{r})-\theta(\vec{0}))}\rangle \sim \left(\frac{a}{r}\right)^{\frac{k_B T}{2\pi J}}$$

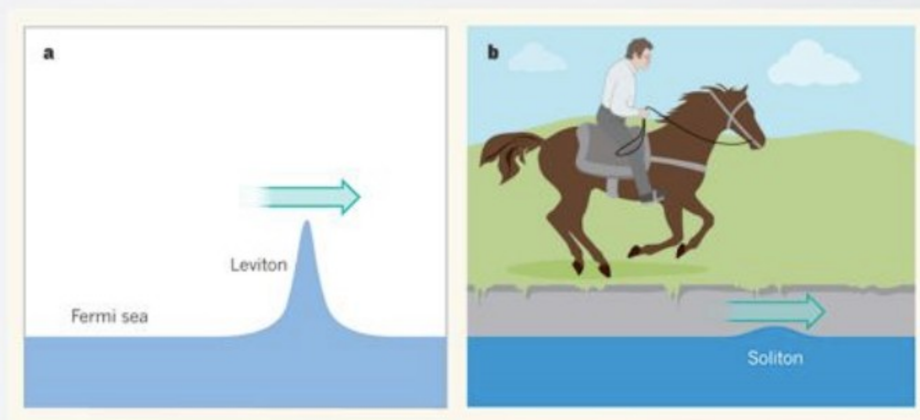
- The phase with free vortices has a standard exponential decay of coherence

Solitons
history

Scott Russell in 1834 observed a heap of water in a canal that propagated undistorted over several kilometer

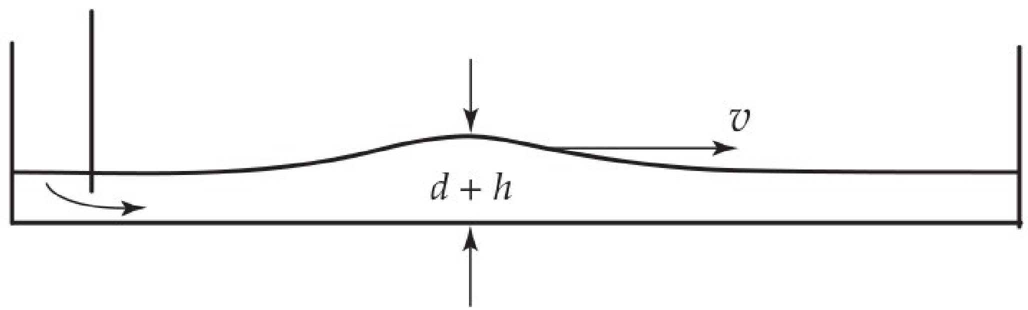
„a rounded, smooth and well-defined heap of water, which continued its curse along the channel apparently without change of form or diminution of speed. I followed it on a horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height."

after „*Nonlinear Fiber Optics*" G. P. Agraval

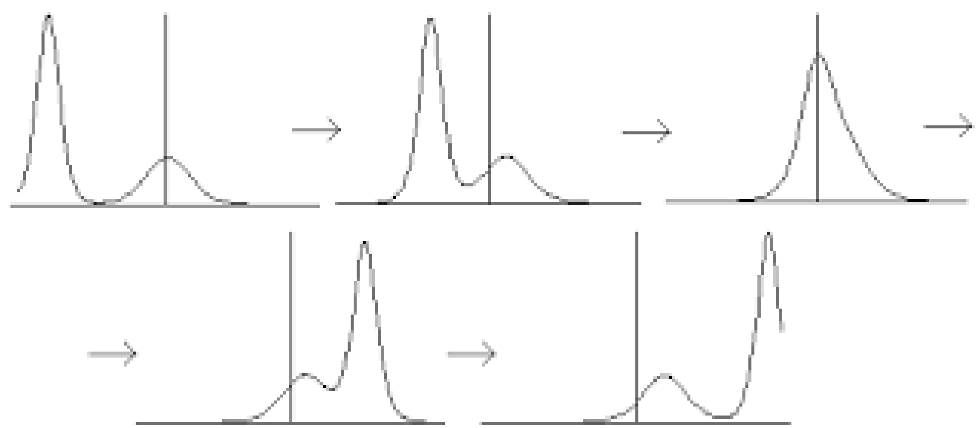


Quantum physics: Single electrons pop out of the Fermi sea
Ch. Flindt, Nature 502, 630–632 (31 October 2013)

Solitons

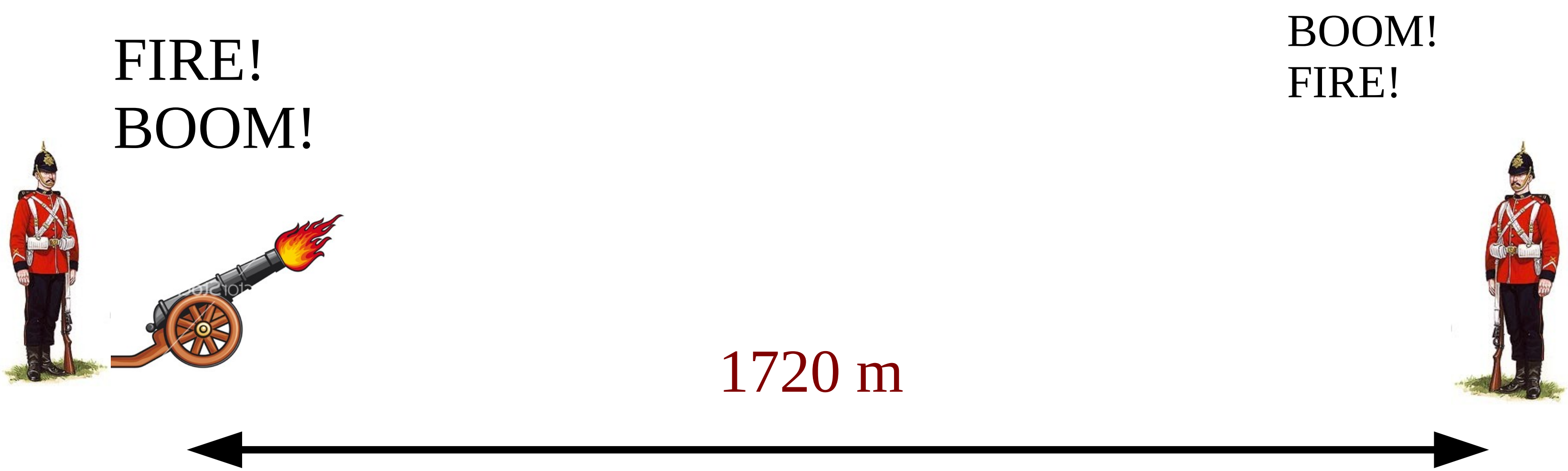


$$v = \sqrt{g(d+h)}$$



Captain Parry’s observation

- Second (unsuccesful) expedition to the North Pole, 1822
- The sound of the cannon arrived ½ second before the “fire” command



Fermi-Pasta-Ulam paradox

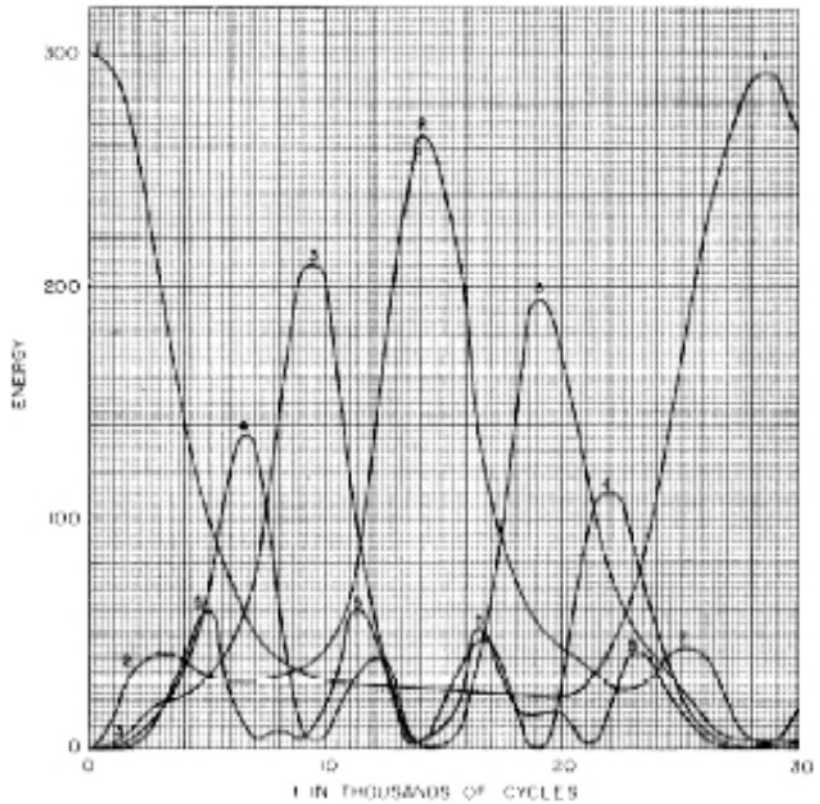
- Array of particles connected by anharmonic springs

$$H(P,Q) = \frac{1}{2}\sum_{i=1}^{N-1} P_i^2 + \frac{1}{2}\sum_{i=0}^{N-1}(Q_{i+1}-Q_i)^2 + \frac{\alpha}{3}\sum_{i=0}^{N-1}(Q_{i+1}-Q_i)^3$$

- Equivalent to the KdV equation in the continuous (many small oscillators) limit

- Inevstigated numerically on MANIAC computer in search for ergodicity and thermalization

Fermi-Pasta-Ulam paradox

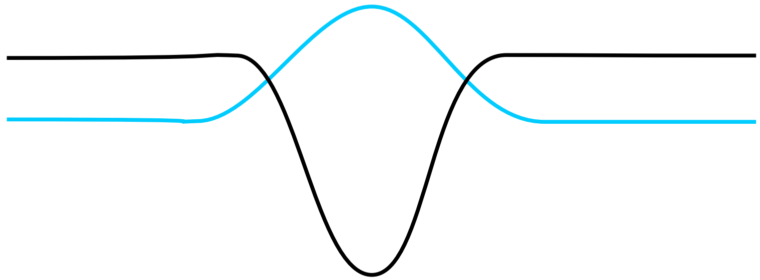


- Instead of thermalization, periodic revivals occurred
- The behaviour has been explained by Zabusky and Kruskal in terms of solitonic interference

What is NOT a soliton?

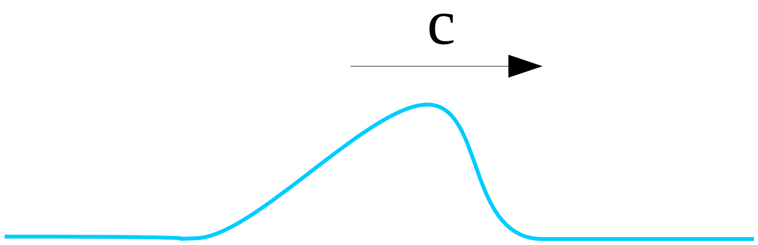
- Defect or bound state

$$i\frac{\partial\psi}{\partial t}=-\frac{\partial^2\psi}{\partial x^2}+V(x)\psi$$



- Dispersionless linear wave

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$$



Solitons are self-localized, i.e. their existence is the reason why they do not decay. In particular, they cannot exist in the low-amplitude (linear) limit.