Polaritons and polariton condensation: A theoretist's point of view

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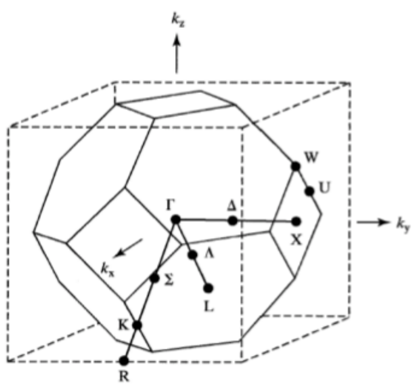
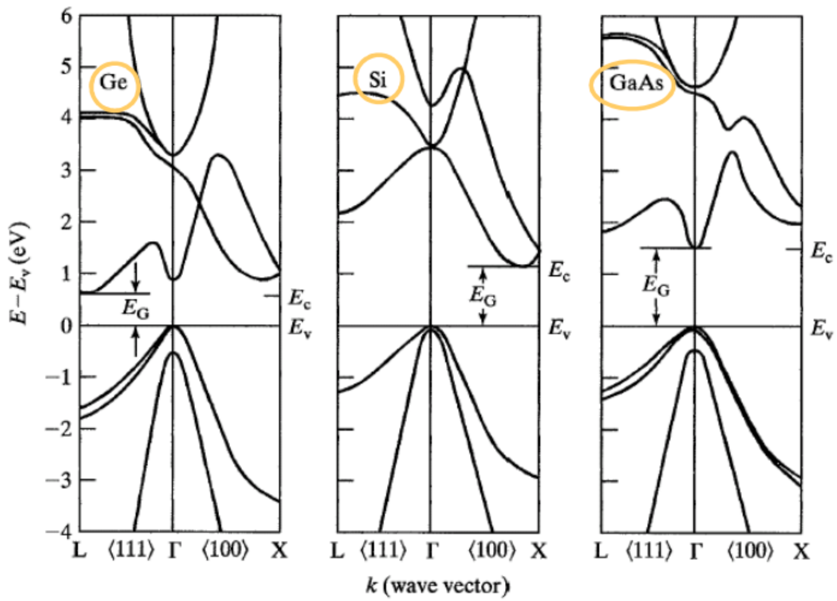
Band structure of semiconductors

Schrödinger equation for an electron in a crystal lattice

$$\frac{-h^2}{2\mathrm{m}_0}\nabla^2\Psi_{\vec{k},n}+V\left(\vec{r}\right)\Psi_{\vec{k},n}=E_{\vec{k},n}\Psi_{\vec{k},n}$$

Solutions result in a band structure, valence and conduction bands





Conduction bands (electrons)

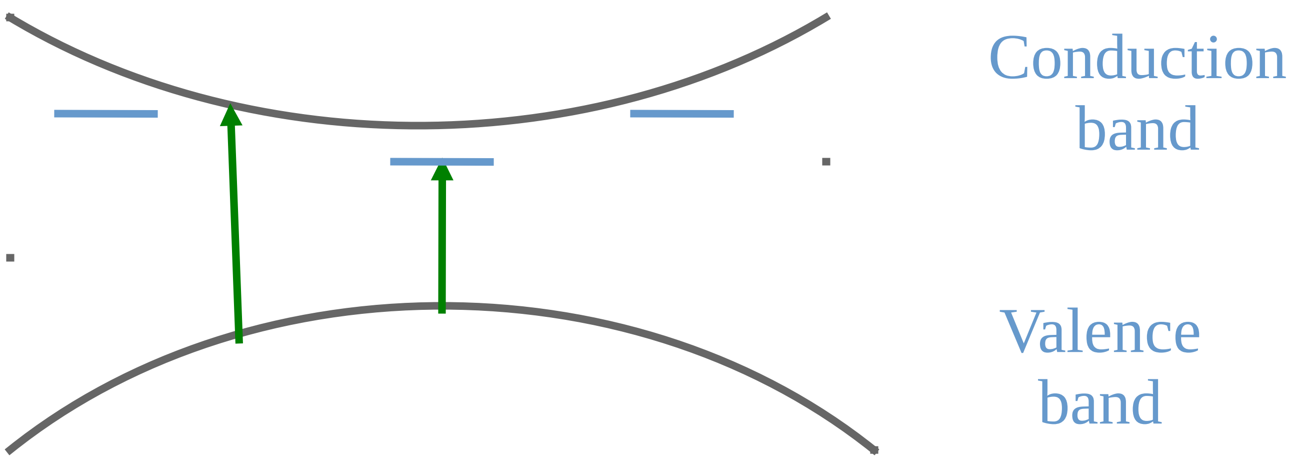
Valence bands (holes)

Excitons

Interaction between electrons and holes

$$\sum\left(\frac{-h^2}{2m_0}\nabla_i^2\Psi+V\left(\vec{r}_i\right)\Psi\right)+\sum V\left(\vec{r}_i-\vec{r}_j\right)\Psi=E\Psi\left(r_1,\ldots,r_N\right)$$

results in bound states of electrons and holes – excitons



Exciton energy is lower than E_g by the interaction energy

Excitons

- The second-quantized Hamiltonian

$$H = \int \Psi(\mathbf{r})^\dagger \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) \right) \Psi(\mathbf{r})\, d\mathbf{r}$$
$$+ \frac{1}{2} \int \int \Psi(\mathbf{r})^\dagger \Psi(\mathbf{r}')^\dagger \frac{e^2}{|\mathbf{r} - \mathbf{r}'|} \Psi(\mathbf{r}')\Psi(\mathbf{r})\, d\mathbf{r}\, d\mathbf{r}' ,$$

where $\Psi(\mathbf{r})^\dagger$ is a creation operator of an electron at position r

$$\Psi^\dagger(\mathbf{r}) = \sum_{i\in\{c,v\}} \sum_{\mathbf{k}} \varphi_{i,\mathbf{k}}(\mathbf{r}) e^{+}_{i,\mathbf{k}} \qquad \varphi_{i,\mathbf{k}}(\mathbf{r}) \propto e^{i\mathbf{k}\cdot\mathbf{r}} u_{i,\mathbf{k}}(\mathbf{r})$$

- The creation of a hole is annihilation of an electron in valence band

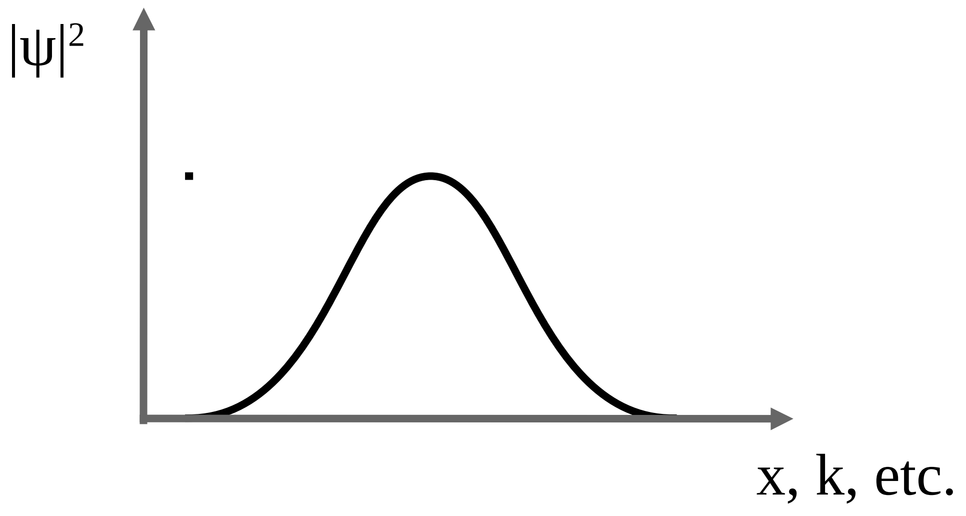
$$h_{\mathbf{k}} = e^{\dagger}_{v,-\mathbf{k}}$$

Quantum superposition

- In quantum mechanics, addition of states means that the particle is in a superposition of these states, eg. the Schrödinger cat state

  $|\psi\rangle = \alpha|\text{electron here}\rangle + \beta|\text{electron there}\rangle + \ldots$

- The probability of finding the particle in a given state is $|\psi|^2$

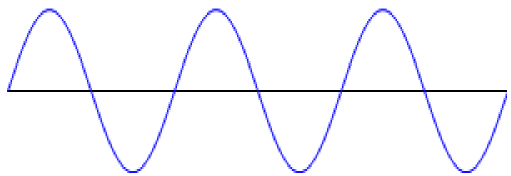


Real and Fourier space – Heisenberg uncertainty priciple

Real space

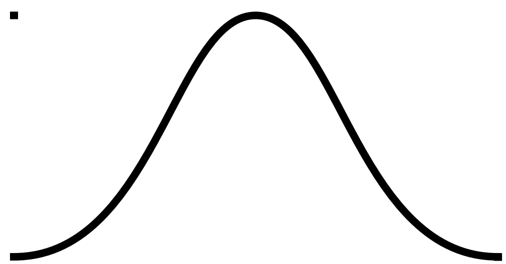
Fourier space

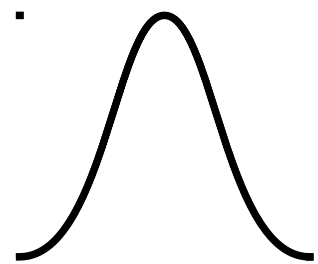
$$e^{i k_0 x}$$



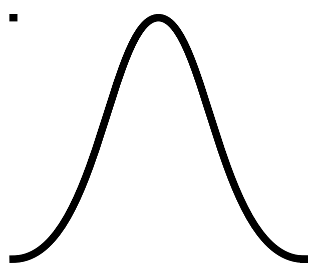
$$\delta\left(k-k_0\right)$$

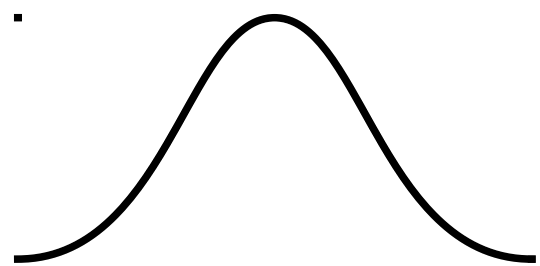
$$e^{\frac{1}{2}\left(\frac{x}{W}\right)^2}$$





$$e^{\frac{1}{2}(kW)^2}$$





Excitons

The exciton operator is a "wave packet" made of electrons and holes

$$X_\nu(\mathbf{k}) \equiv \sum_{\mathbf{p}} \varphi_\nu(\mathbf{p}) h_{\mathbf{k}/2-\mathbf{p}} e_{\mathbf{k}/2+\mathbf{p}}$$

How to understand this?

|G> - Ground state (all electrons in the valence band)

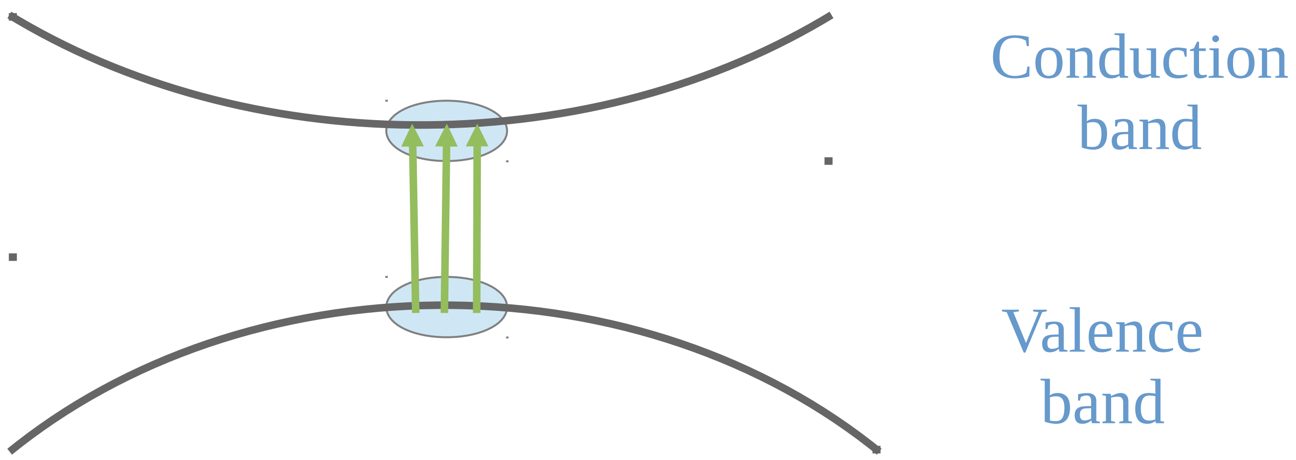
$X^+(0)$ |G> = α|electron in state k1, hole in state -k1> +
+β|electron in state k2, hole in state -k2> + …

The φ(p) function is such that electron and hole are close to each other in real space

Excitons

$X^+(0)$ |G> = α|electron in state k1, hole in state -k1> +
+β|electron in state k2, hole in state -k2> + ...

$$X_\nu(\mathbf{k}) \equiv \sum_{\mathbf{p}} \varphi_\nu(\mathbf{p}) h_{\mathbf{k}/2-\mathbf{p}} e_{\mathbf{k}/2+\mathbf{p}}$$



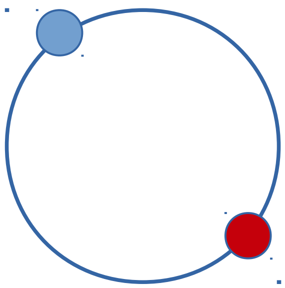
A more correct picture!

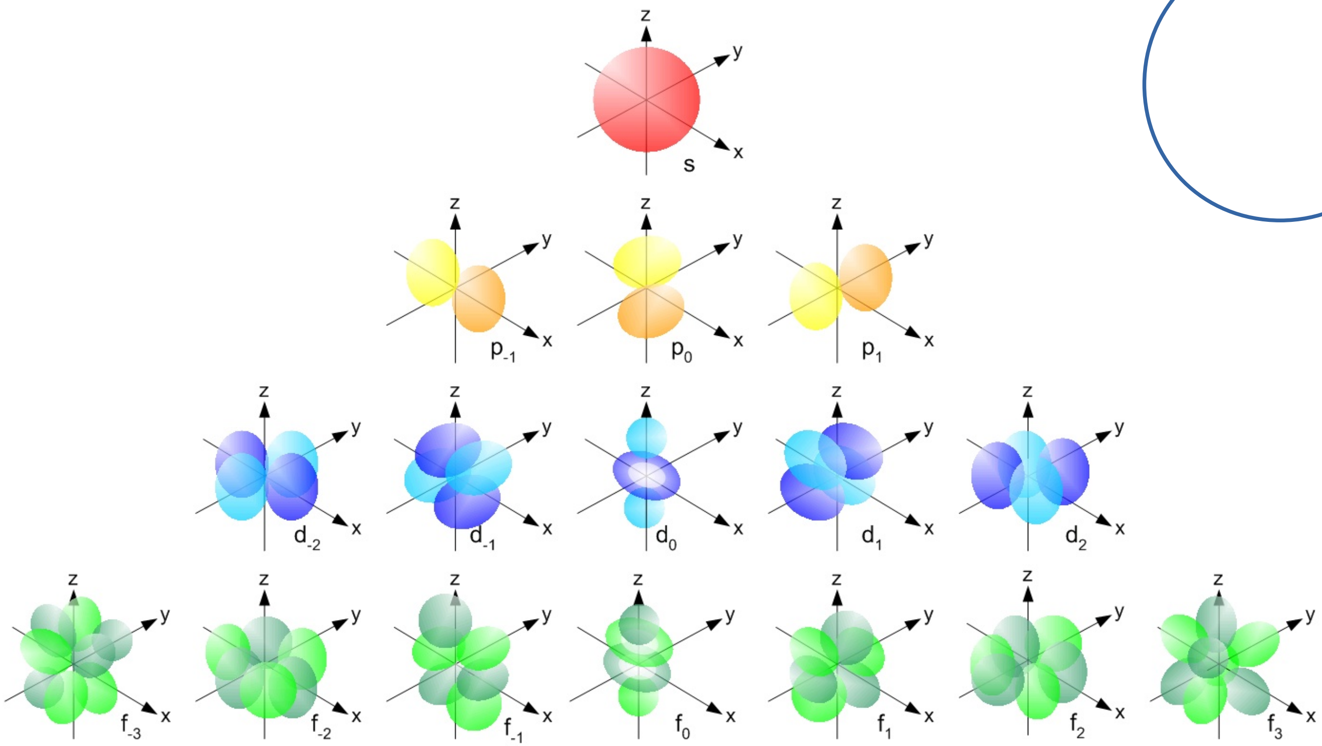
$$h_{\mathbf{k}} = e^\dagger_{v,-\mathbf{k}}$$

Exciton state

The relative electron-hole motion is analogous to the hydrogen atom

$$\Psi_{rel}\left(\vec{r}_e, \vec{r}_h\right) \approx \Psi_{nlm}\left(\vec{r}_e - \vec{r}_h\right)$$

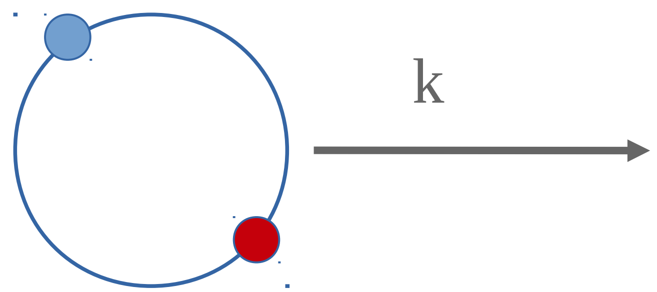




Exciton state

The motion of exciton as a whole

$$\Psi\left(\vec{r}_e, \vec{r}_h\right) \approx \Psi_{rel}\left(\vec{r}_e - \vec{r}_h\right) \exp\left(i \vec{k} \vec{R}\right), \quad \vec{R} = \frac{\vec{r}_e m_e + \vec{r}_h m_h}{m_e + m_h}$$



Are excitons bosons or fermions?

An electron and a hole both have half-integer spins, an exciton has integer spin

The commutation relation of true bosons

$$[X_\nu(\mathbf{k}), X_\mu^\dagger(\mathbf{q})] = \delta_{\nu,\mu}\delta_{\mathbf{k},\mathbf{q}}$$

For excitons we have

$$[X_\nu(\mathbf{k}), X_\mu^\dagger(\mathbf{q})] = \delta_{\nu,\mu}\delta_{\mathbf{k},\mathbf{q}} - \sum_{\mathbf{p}} |\varphi_{1s}(\mathbf{p})|^2 (c_{\mathbf{k}}^\dagger c_{\mathbf{q}} + h_{-\mathbf{k}}^\dagger h_{-\mathbf{q}})$$

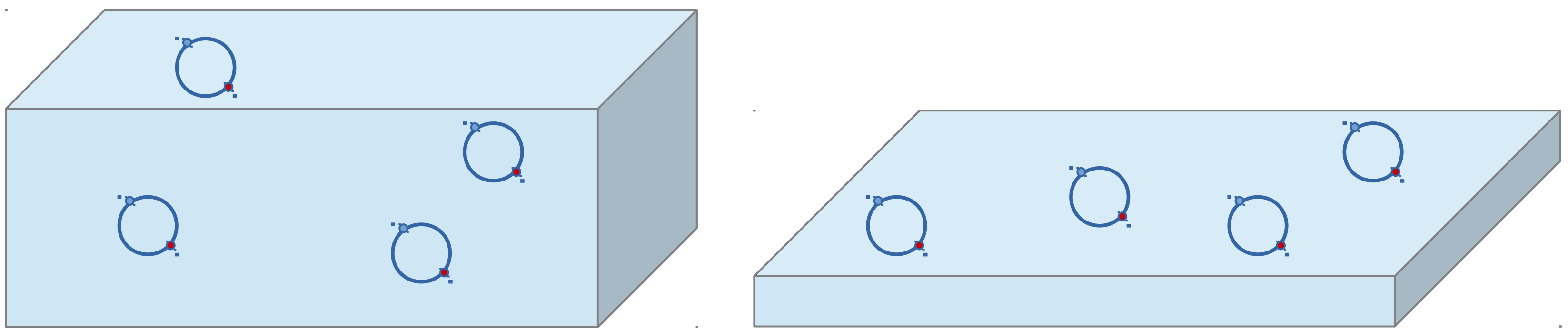
Excitons are well approximated by bosons in the low density limit

$$Na_0^2 \ll 1$$

Are excitons bosons or fermions?

The exciton is in a superposition of being all over the place!
But the electron and hole are bound together

$$\Psi\left(\vec{r}_e,\vec{r}_h\right)\approx\Psi_{rel}\left(\vec{r}_e-\vec{r}_h\right)\exp\left(i\vec{k}\vec{R}\right) \qquad Na_0^2\ll 1$$

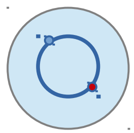


This is true for bulk (3D) and quantum well (2D) excitons.

They behave as bosons, because we can put many of them in the same state.

Are excitons bosons or fermions?

In small quantum dots (0D), the excitons behave as fermions:
we cannot put more than one in the same state



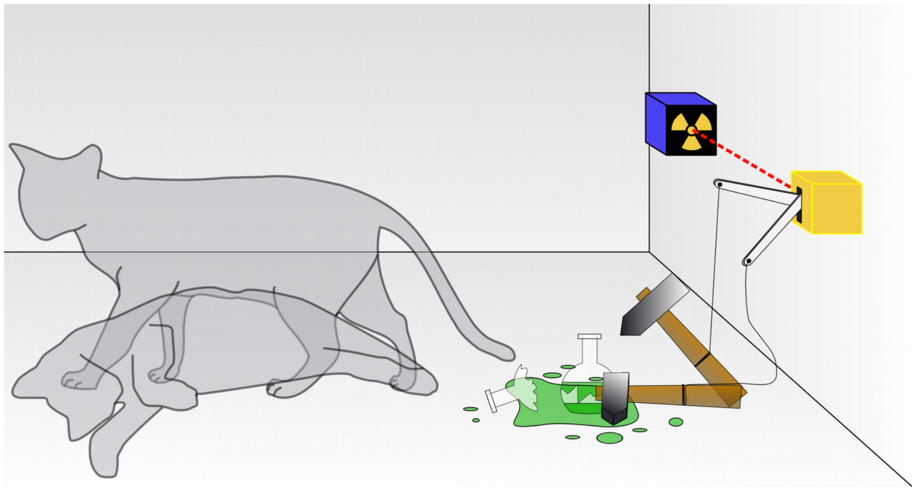
$$Na_0^2 \approx 1$$

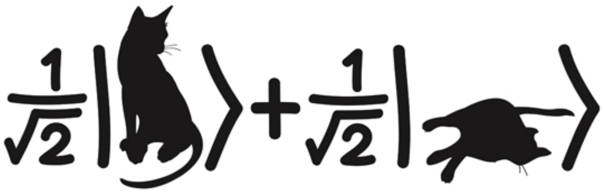
In this limit, we „see” the fermionic nature of the constituents

Polaritons

The excitons are no longer correct quantum states if the strong coupling of excitons to photons is introduced.

Polariton is a superposition of an exciton and a photon (a Shrödinger cat)





|polariton> = α |exciton> +β |photon>

Exciton-Polaritons

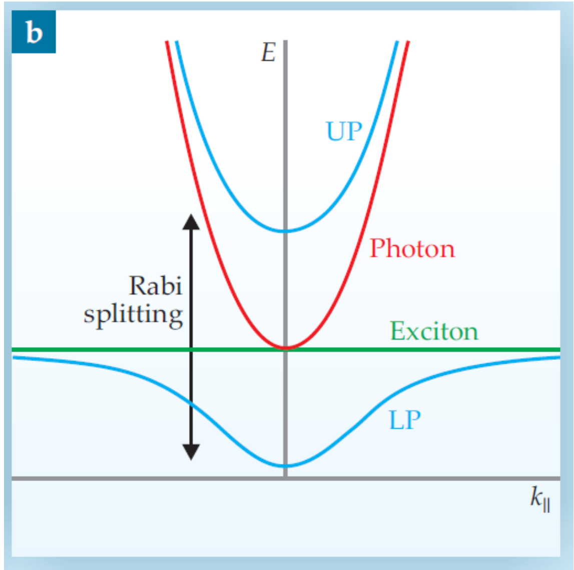
Exciton: a_k, photon: $b_{k,}$ strong coupling: Ω_k

$$\hat{H}_k = E_{ex} a^+ a + E_{ph} b^+ b + \frac{\Omega}{2} \left(a^+ b + a b^+ \right)$$

As a and b are bosonic operators, we can reduce the problem to a single 2x2 matrix

Eigenstates consist of lower and upper polaritons, mixed states of excitons and photons

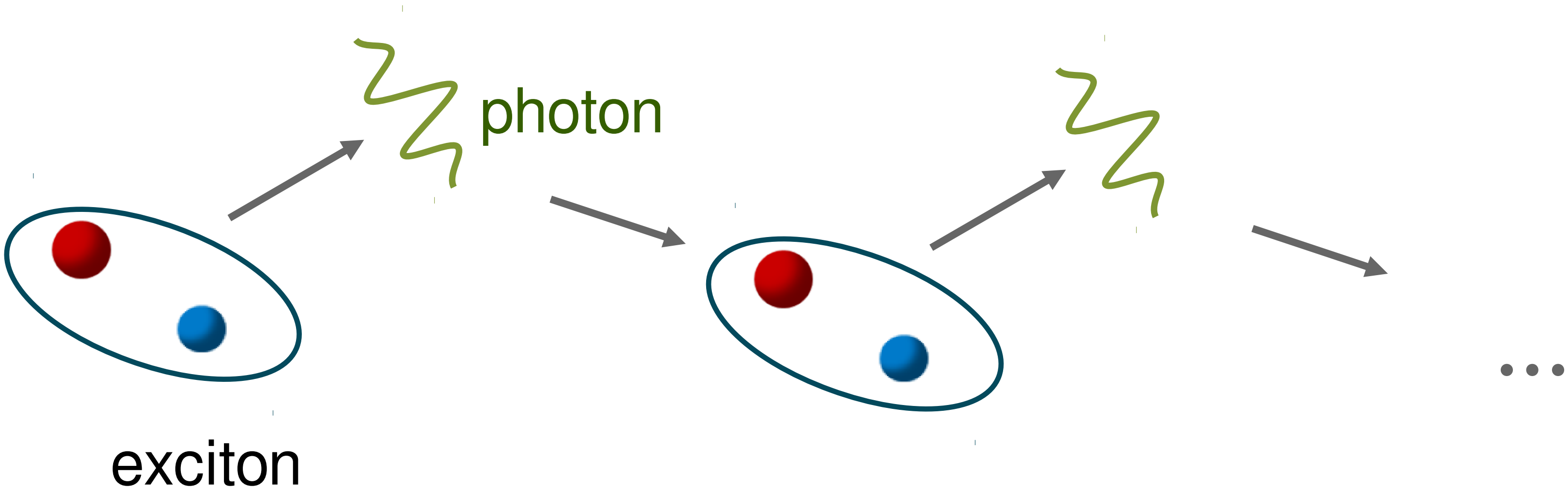
$$\hat{H}_k = \begin{pmatrix} E_{ex} & \frac{\Omega}{2} \\ \frac{\Omega}{2} & E_{ph} \end{pmatrix}$$

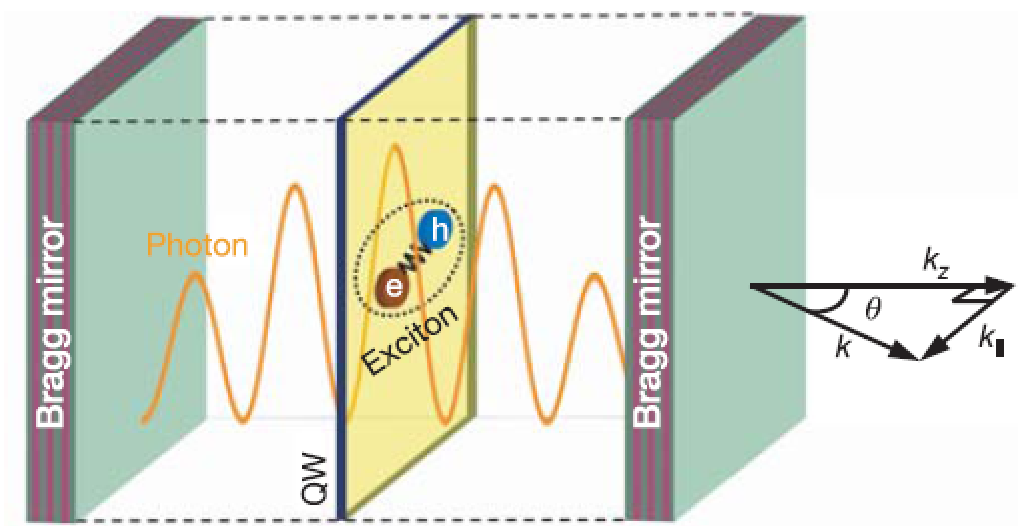


|polariton> = α |1,0> +β |0,1>

Polaritons

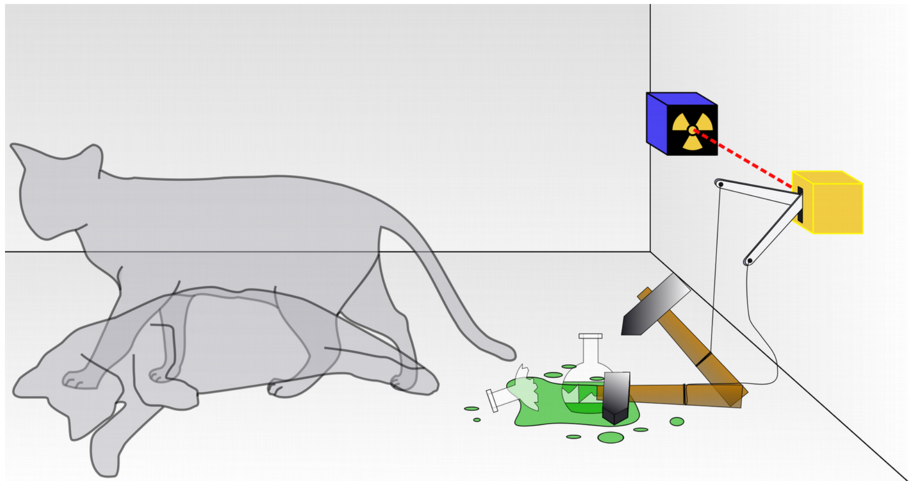
„Classical picture”: exciton periodically transforms into photon (with a Rabi frequency)

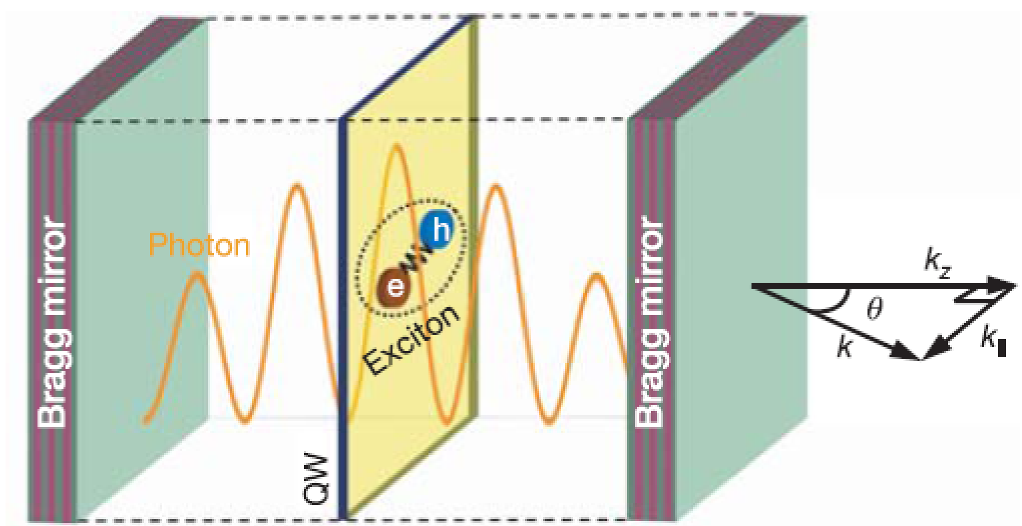




Polaritons

„Quantum picture”: there is a superposition of the two possibilities





Do polaritons exist without pumping?

Polaritons are correct quantum eigenstates even in the vaccum

The vacuum fluctuations provide virtual polaritons even without pumping!

$$\hat{H} = \sum_{\mathbf{k}} E_X(\mathbf{k}, B)\hat{a}^{\dagger}_{\mathbf{k}}\hat{a}_{\mathbf{k}} + E_C(\mathbf{k})\hat{c}^{\dagger}_{\mathbf{k}}\hat{c}_{\mathbf{k}} + \frac{1}{2}\Omega(B, \mathbf{k})(\hat{a}^{\dagger}_{\mathbf{k}}\hat{c}_{\mathbf{k}} + \hat{c}^{\dagger}_{\mathbf{k}}\hat{a}_{\mathbf{k}})$$

In the basis of states |1,0> and |0,1>

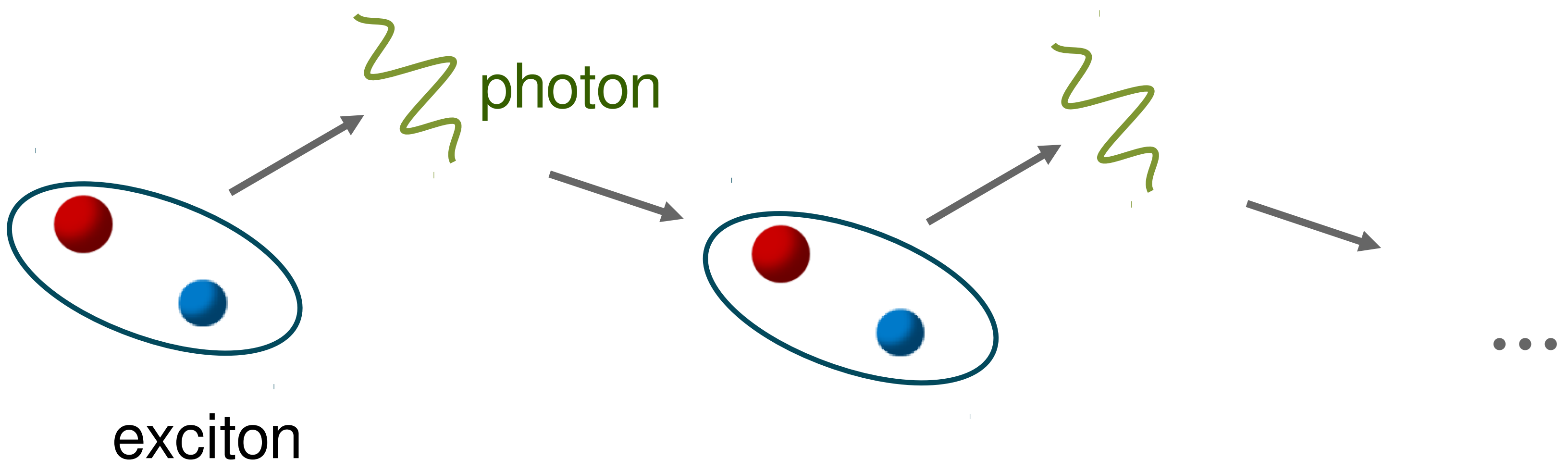
$$\hat{H} = \begin{pmatrix} E_X & \Omega/2 \\ \Omega/2 & E_C \end{pmatrix}$$

|polariton> = α |1,0> +β |0,1>

Polaritons: weak and strong coupling

Strong coupling means that the decoherence rate is lower than the conversion rate (Rabi oscillations)

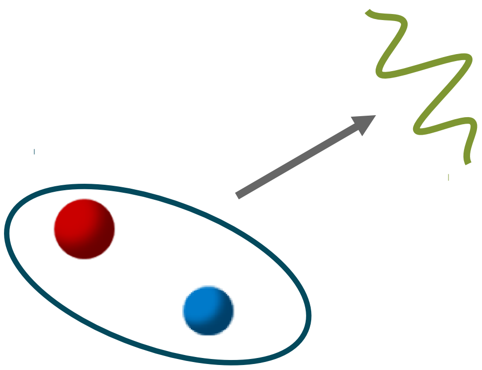
Possible sources of decoherence: finite lifetime, interactions with the crystal lattice, interctions between polaritons etc.



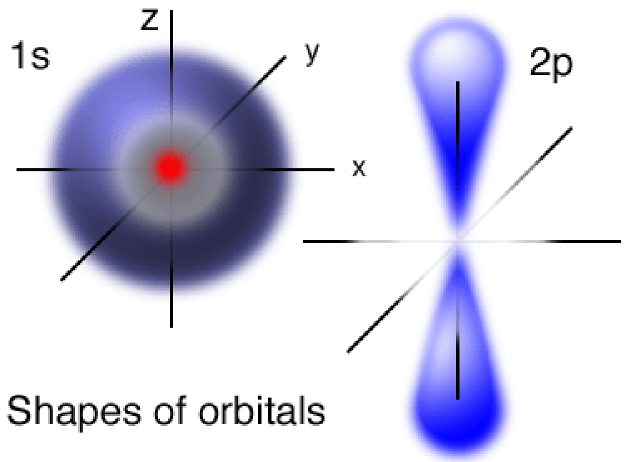
Polariton coupling strength

Coupling strength is proportional to the probability of conversion of a photon to an exciton and vice versa

This can occur only when electron and hole are "in the same lattice cell" so that they can annihilate or be created



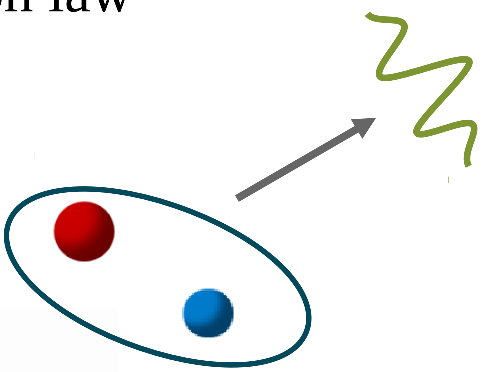
Selection rules: only *s* states are active

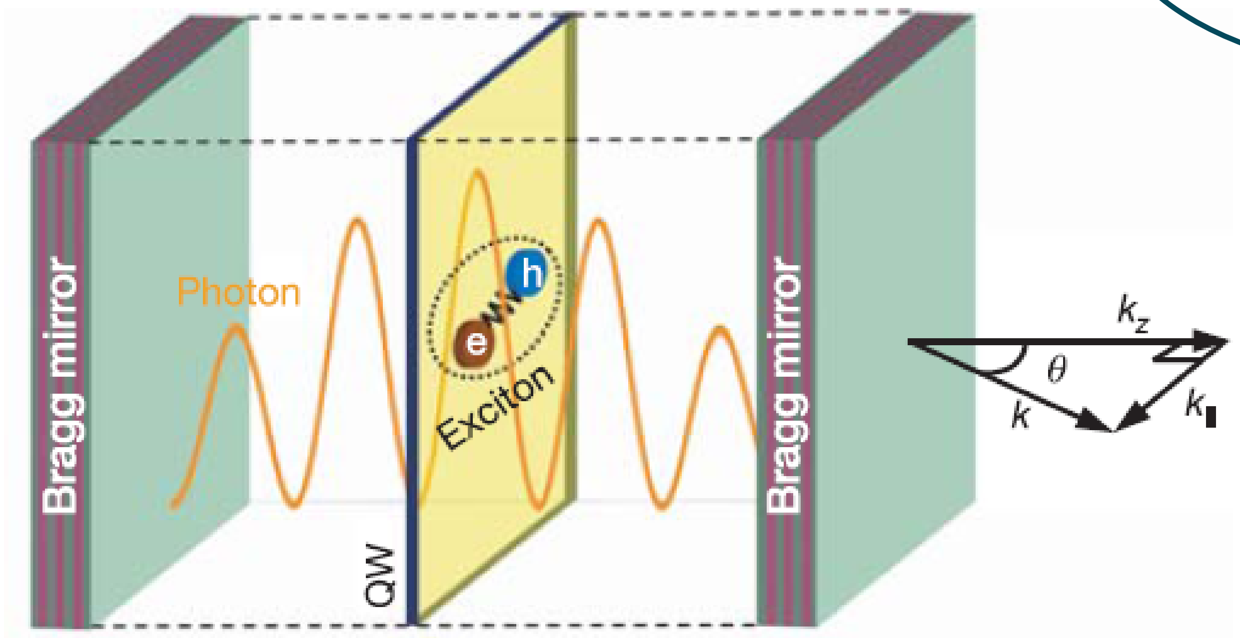


$$\Psi_{rel}\left(\vec{r}_e, \vec{r}_h\right) \approx \Psi_{nlm}\left(\vec{r}_e - \vec{r}_h\right)$$

Polariton momentum

Excitons and photons share the same momentum parallel to the quantum well plane due to the conservation law





Bosons, fermions and...

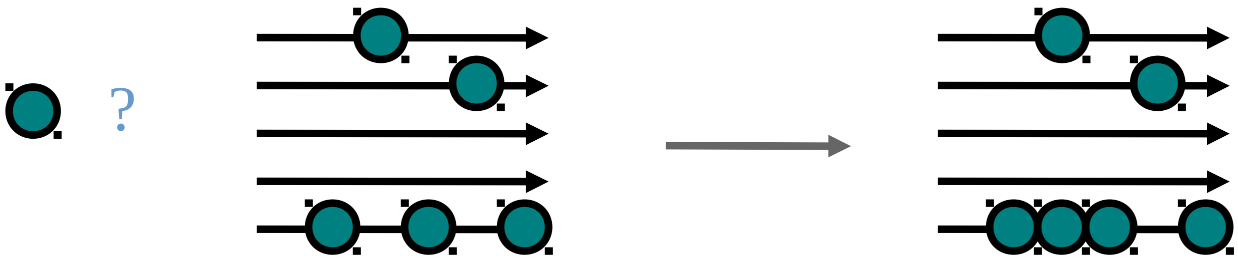
Bosons

Classical particles

Fermions

Bosonic enhancement

Bosonic particles prefer to be where the others are



This is the most probable choice!

Thermal equilibrium distributions

In thermal equilibirum, the probability of finding the **system** in a certain state A is proportional to (Boltzmann distribution)

$$p_A \sim e^{-\frac{E_A - \mu N}{k_B T}}$$

The probability of finding a **particle** in the state a is

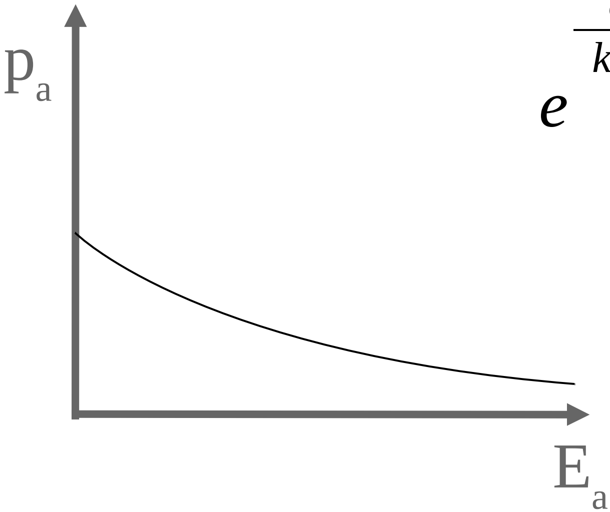
$$p_A \sim \frac{1}{e^{\frac{E_a - \mu}{k_B T}} - 1} \qquad p_A \sim e^{\frac{E_a - \mu}{k_B T}} \qquad p_A \sim \frac{1}{e^{\frac{E_a - \mu}{k_B T}} + 1}$$

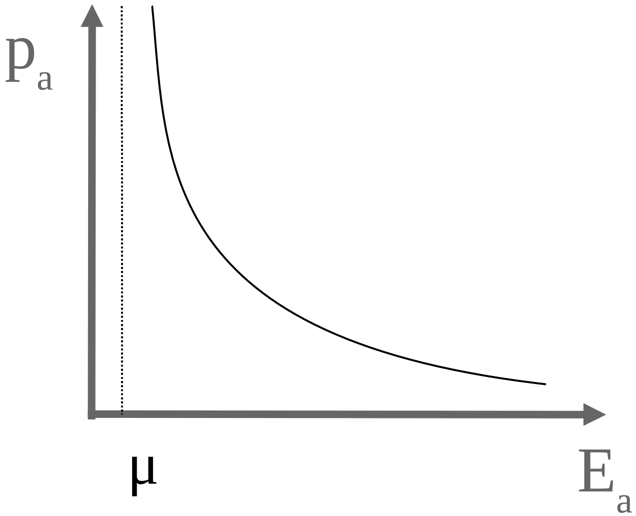
Bosons Classical particles Fermions

Bose-Einstein condensation

At some point, divergence may occur.
This means that the occupation of the lowest state becomes macroscopic

$$p_A \sim \frac{1}{e^{\frac{E_a - \mu}{k_B T}} - 1}$$

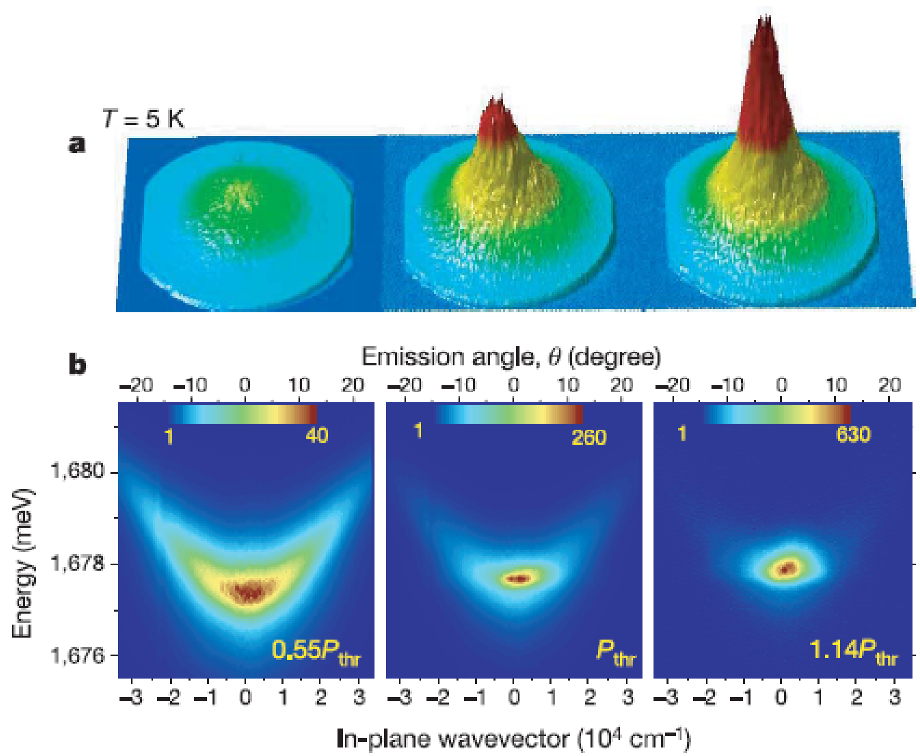


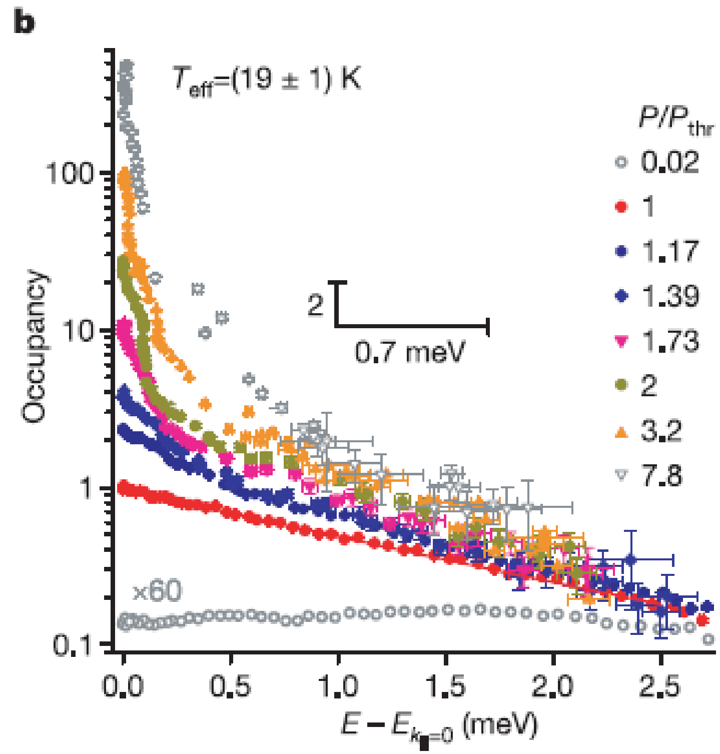


This can be achieved in two ways: decreasing T or increasing μ (number of particles)

Bose-Einstein condensation

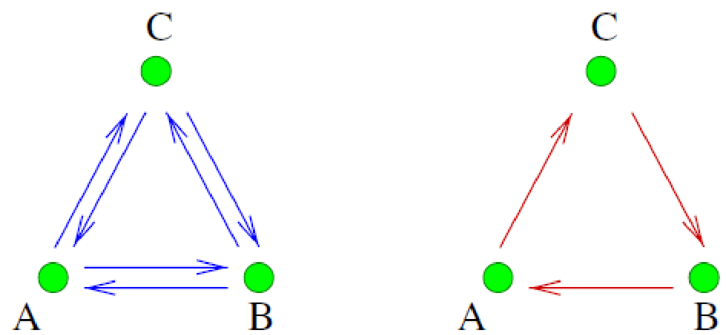
At some point, divergence may occur.
This means that the occupation of the lowest state becomes macroscopic





Above threshold pumping power, the ground state becomes massively occupied. The thermal cloud is found to be at 19K. (J. Kasprzak et al., Nature 2006)

Equilibrium and out of equilibrium



Equilibrium Out-of-equilibrium

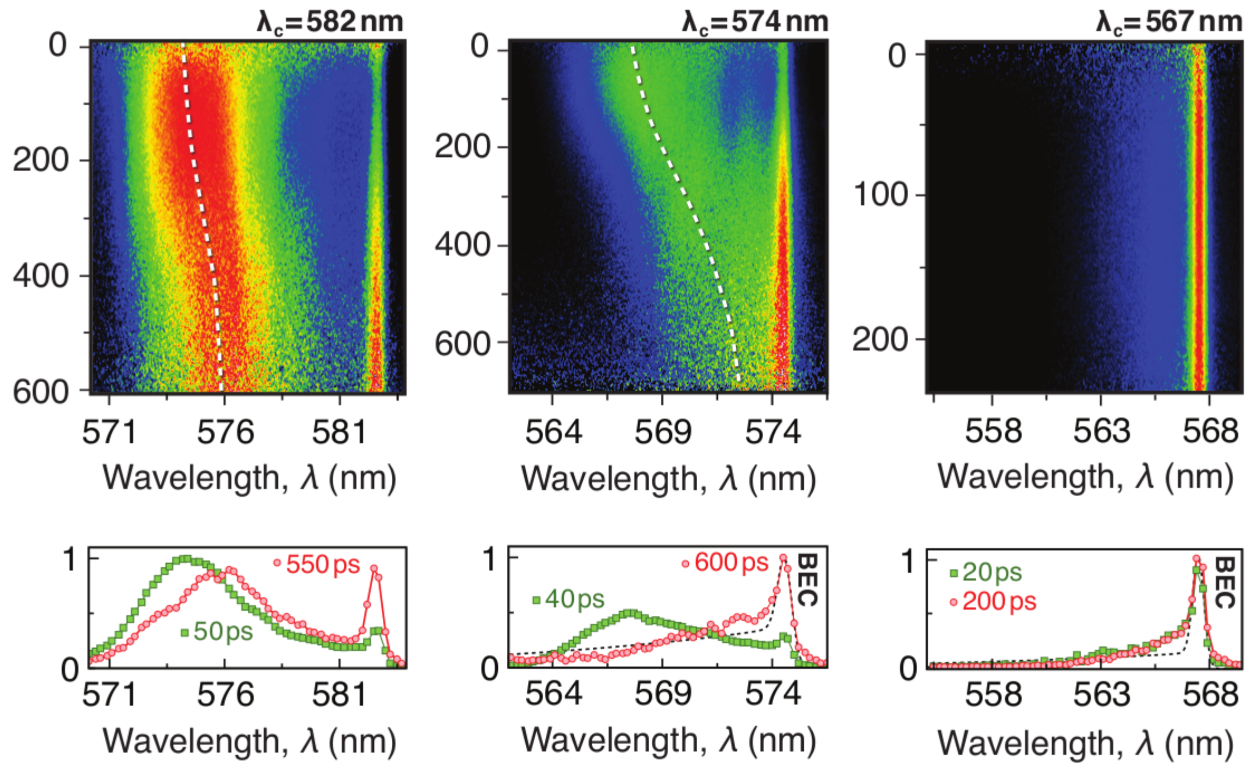
- Close to equilibrium: tools of statistical mechanics (eg. Boltzmann distribution) can be applied

- Far from equilibrium: New description required

  Lasers, Percolations, growing surfaces, traffic jams, social networks...

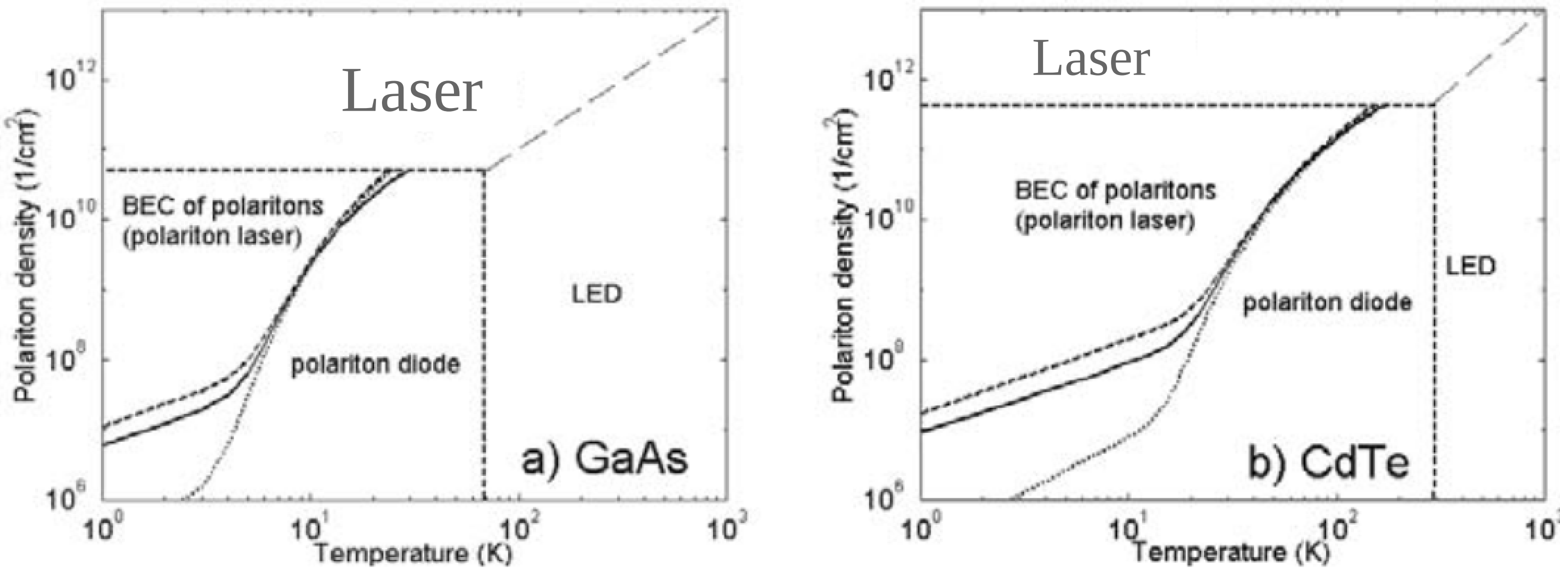
Nonequilibrium condensation

Bosonic enhancement works even away from equilibirum
Transition from equilibrium to nonequilibrium condensation

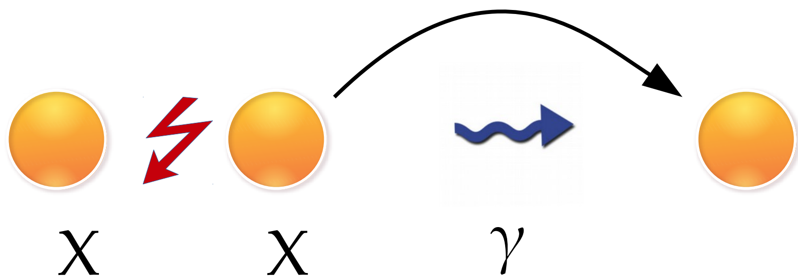


M. Weitz et al., PRA 92, 011602(R) (2015)

Phase diagram



Properties of exciton-polariton condensates



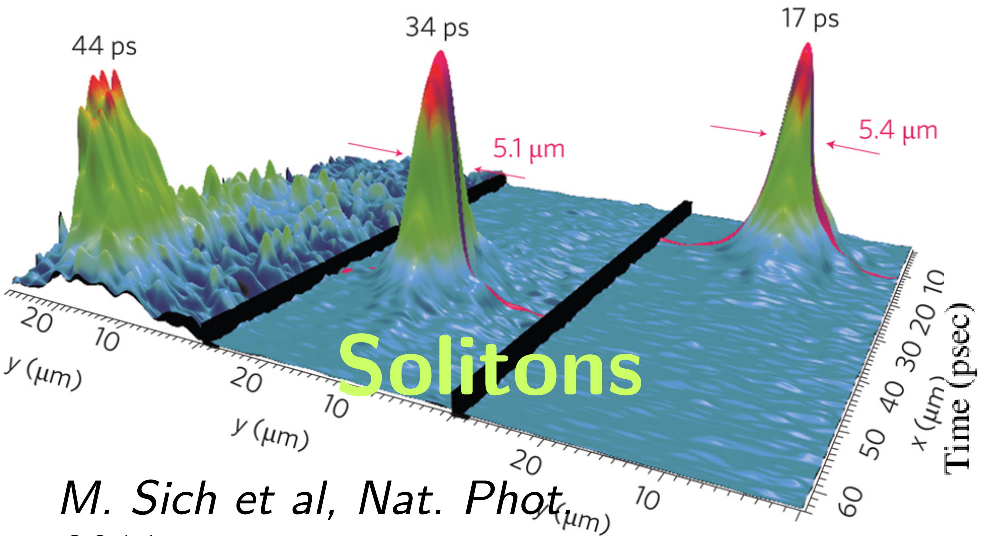
- Excellent transport properties and extremely low effective mass thanks to the photonic component

- Very strong interparticle interactions thanks to the exciton component – world record of (ultrafast) optical nonlinearity

- Short lifetime (1-200 ps) is an issue, but also interesting for fundamental research of nonequilibrium systems

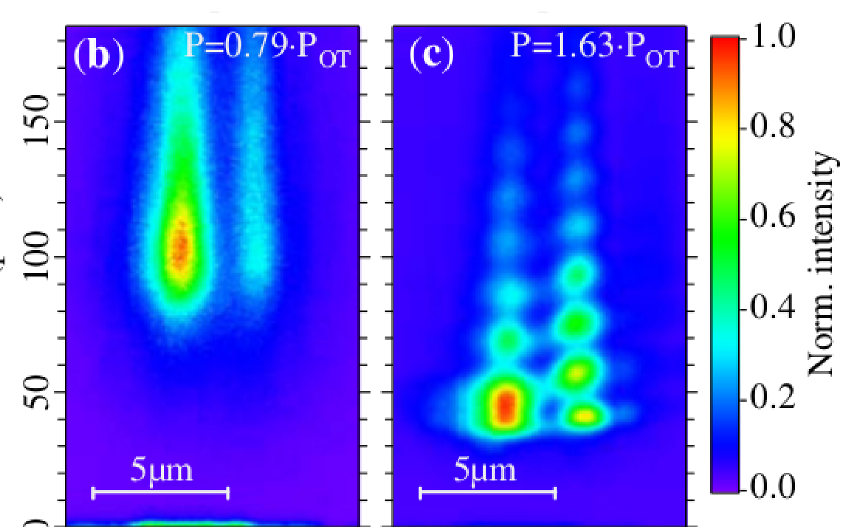
Exciton-Polariton condensate physics

Solitons

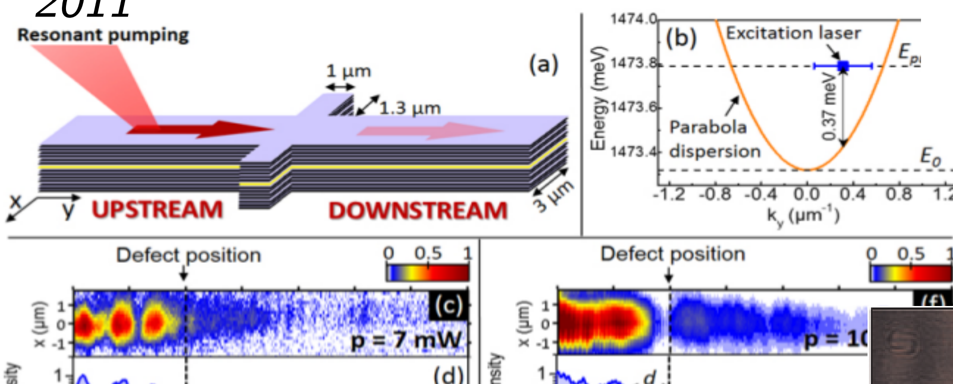


M. Sich et al, Nat. Phot 2011

Josephson junctions



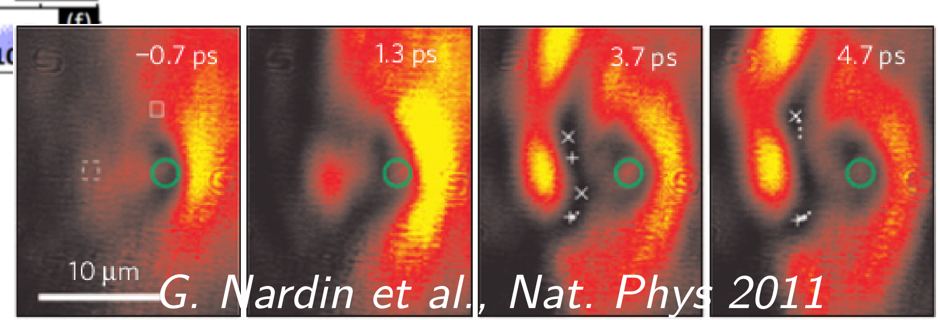
K.G. Lagoudakis et al., PRL 2010



Black hole physics

H.S. Nguyen et al., PRL 2015

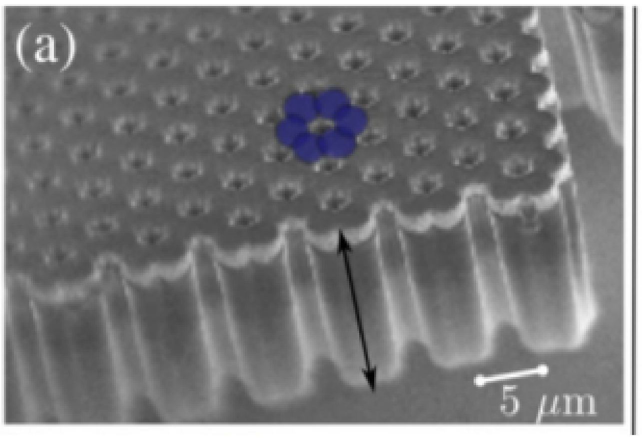
Vortices



G. Nardin et al., Nat. Phys 2011

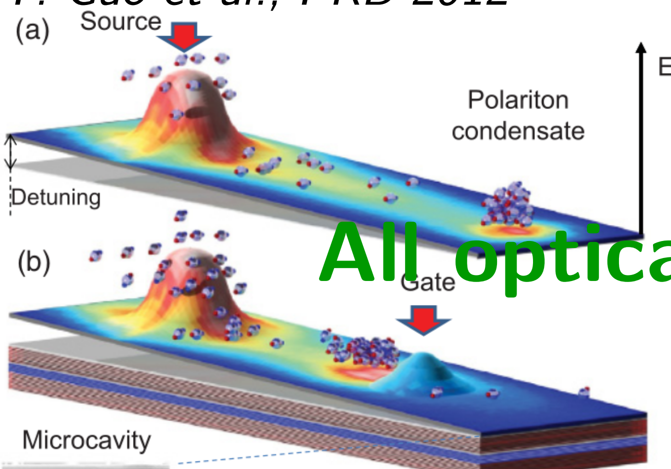
Exciton-Polariton condensate physics

Aritificial materials

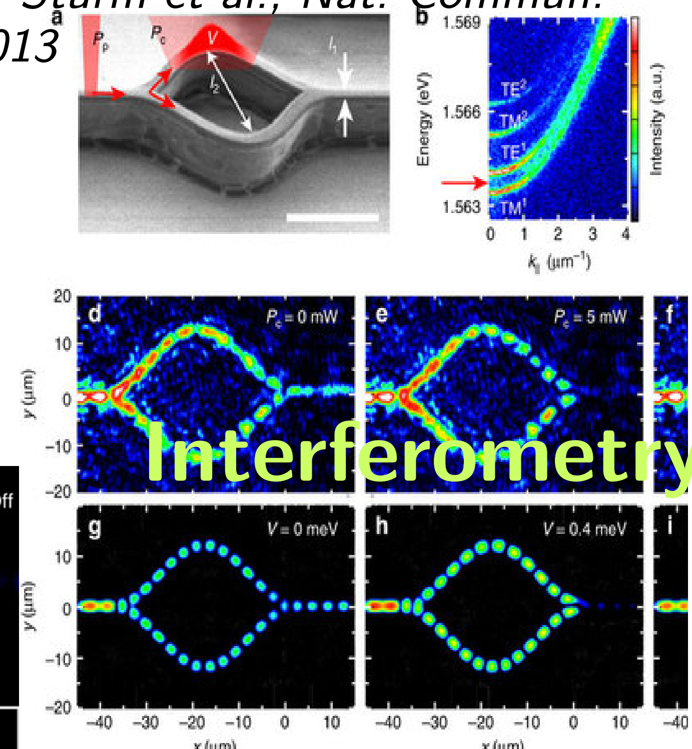


T. Jacqmin et al., PRL 2014

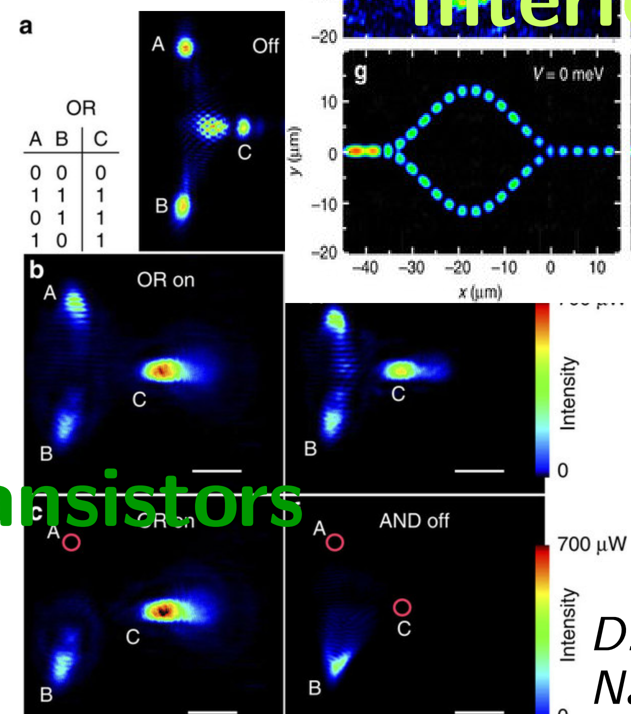
T. Gao et al., PRB 2012



C. Sturm et al., Nat. Commun. 2013



Interferometry

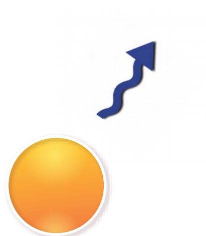


All optical transistors

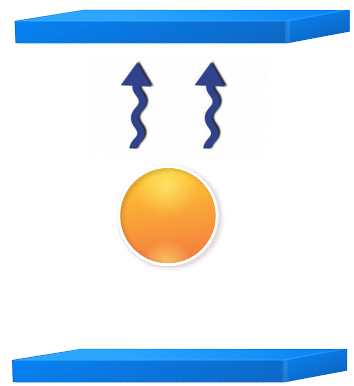
D. Ballarini et al., Nat. Commun. 201

Advantages

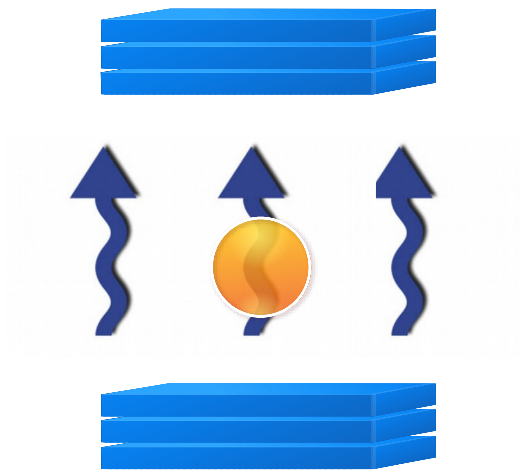
- Very short (picosecond) emission lifetime



Quantum dot



Purcell effect

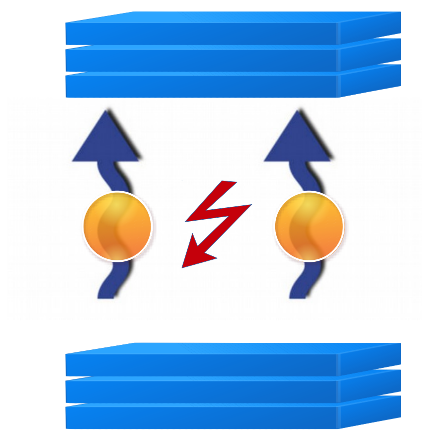


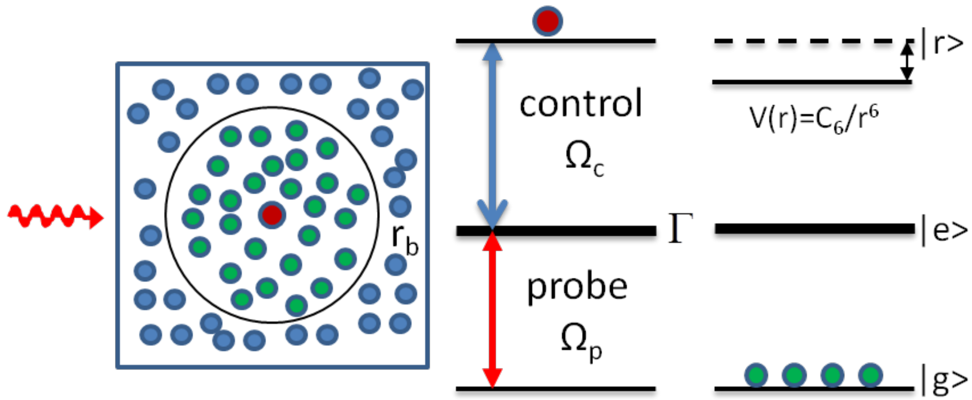
Strong coupling polariton

Advantages

- Strong interactions mediated by the matter component

"Realization of strongly interacting photonic systems is one of the holy grails of quantum optics"





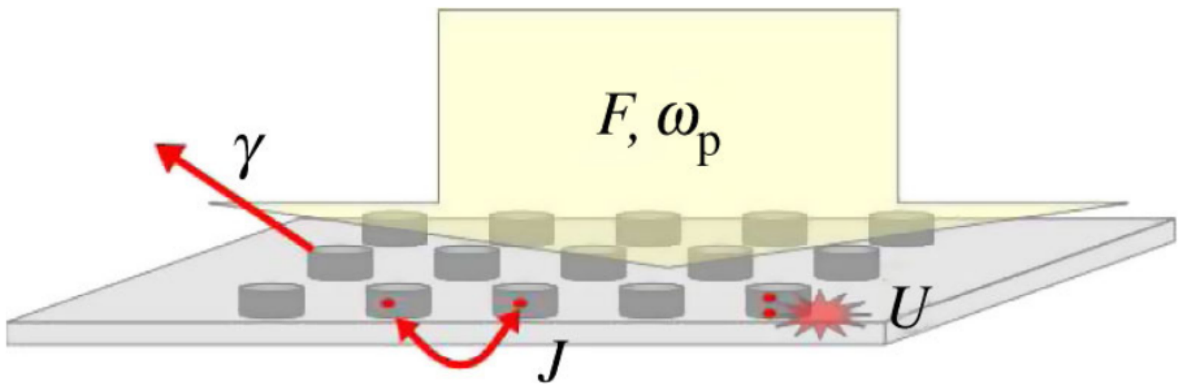
http://nqo.sdu.dk/wordpress/research/rydberg-eit/rydberg-eit-overview/

Polariton blockade

Rydberg blockade

Advantages

- Possibility of quantum simulations in an open system



$$H = -\frac{J}{z}\sum_{\langle i,j\rangle} b_i^\dagger b_j - \sum_i^N \Delta\omega b_i^\dagger b_i + \frac{U}{2} b_i^\dagger b_i^\dagger b_i b_i$$
$$+ F b_i^\dagger + F^* b_i,$$

$$i\partial_t \rho = [H, \rho] + \frac{i\gamma}{2}\sum_i^N 2 b_i \rho b_i^\dagger - b_i^\dagger b_i \rho - \rho b_i^\dagger b_i,$$

A. Le Boite et al., PRL 110, 233601 (2013)