LECTURE 10

solitons

dr hab. Barbara Piętka

barbara.pietka@fuw.edu.pl 3.64



Institute of Experimental Physics Faculty of Physics Warsaw University



Solitons definition

Self reinforcing solitary wave packet, that maintains its shape while it propagates at a constant velocity.

Come as a solution of a nonlinear equations in dispersive media (where the speed of the wave vary with frequency).

solitary solution - there exists a single solution to the propagation equation

Main properties:

- permanent in form
- localized within a region
- can interact with other solitons and emerge from the collisions unchanged (phase shift allowed)

Dispersion and non-linearity are necessary to produce permanent and localized wave forms.

Shallow water wave generation

University of Tasmania - solitons

Solitons history

Scott Russell in 1834 observed a heap of water in a canal that propagated undistorted over several kilometer

"a rounded, smooth and well-defined heap of water, which continued its curse along the channel apparently without change of form or diminution of speed. I followed it on a horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height."



after "Nonlinear Fiber Optics" G. P. Agraval



Quantum physics: Single electrons pop out of the Fermi sea Ch. Flindt, Nature 502, 630–632 (31 October 2013)

Solitons history

Fascinated by what he had observed, Russell constructed a 30-foot wave tank in his back yard and carried out experiments. He made the following observations:

- These "solitary" waves are stable, and can travel over very large distances without changing their shape, neither decreasing in amplitude nor breaking as waves in water often do.
- The speed of the wave depends on the height of the wave.
- These waves don't obey superposition. When a taller (faster) wave overtakes a shorter (slower) wave, they don't combine and add together. Instead they appear to swap places with the faster wave appearing to jump through the slower one.

after "*Nonlinear Fiber Optics*" G. P. Agraval http://www.acs.psu.edu/drussell/Demos/Solitons/solitons.html

Solitons two water solitons

Two solitons traveling in the same direction

- the speed depends on the height of the wave
- the taller wave is faster than the shorter wave
- the taller wave overtakes and passes the smaller wave

Collision between two solitons traveling in the same direction

- linear waves interfere with each other by adding their amplitudes (superposition) - grey
- solitary waves instead of interacting through interference and simple addition, collide in a nonlinear and complicated manner - black
- the double soliton solution is not the simple sum of the two individual solitons

Solitons



Korteweg-de Vries equation



What is NOT a soliton?

• Defect or bound state

$$i\frac{\partial \psi}{\partial t} = -\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi$$

• Dispersionless linear wave

$$\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$



Solitons are self-localized, i.e. their existence is the reason why they do not decay. In particular, they cannot exist in the lowamplitude (linear) limit.

Solitons two water solitons

Collision between two solitons traveling in the same direction

After they "collide" they keep moving each with their own speeds but they are not in the relative locations as expected, if they simply passed through each other. The contour plot shows the traces of the two pulses as they collide. The paths of the two pulses appear to jump and change places rather than just pass through each other.



• In result, solitons are immune to collisions, except for a phase and trajectory shifts

http://www.acs.psu.edu/drussell/Demos/Solitons/solitons.html

Solitons classical examples Atmospheric solitons

Tidal wave

Morning Glory cloud

000 km long - 2 km height 10 - 20 m/s

http://www.scisnack.com/2016/10/13/surfing-atmosphericwaves-the-morning-glory-phenomenon/ This soliton packet is triggered by inflowing Atlantic water accelerated by its passage through the narrow Strait and across the sill at the entrance to the Strait. At the interface between the fresher, lighter Atlantic water and more saline, denser Mediterranean water, internal wave sets are generated, usually at a depth of about 60 to 80 meters.

http://www.lpi.usra.edu/publications/slidesets/ oceans/oceanviews/oceanviews_index.shtml

Solitons vortex ring as a particular example of a soliton







The Vortex Ring, Close Up, in Slow Motion: https://www.youtube.com/watch?v=Sj9irzI-Pzw

DIY: Box Plastic bottle

https://www.youtube.com/watch?v=ijsytrR9WiE https://www.youtube.com/watch?v=NU6j_w5r-TY

Solitons vortex half ring as a particular example of a soliton



Solitons other examples

Pulse of light traveling in glass

- pulse consist of several different frequencies.
- glass is dispersive (Group Velocity Dispersion) different frequencies will travel at different speeds and the shape of the pulse will change over time
- non-linear Kerr effect the refractive index of a material at given frequency depends on the light's amplitude or strength
 in special cases the Kerr effect can exactly cancel the
 - dispersion effect and the shape will not change over a time

In optical fibers: balance between GVD and Kerr nonlinearity: 1974 Bell Labs - solitons could be generated in optical fibers 1980' - soliton pulses transmitted over 4 000 km (Raman effect phenomenon) 1990' Bell Labs : 2.5 gigabits per second over more than 14 000 km (erbium optical fiber amplifiers 1998- data transmission of 1 terabit per second

since 2001 - practical use of solitons in optical fibers

GVD

Solitons solution of the equation

An equation and its solutions developed by Koreweg and de Vries in 1895 (KdV) that is most commonly used to describe the waves that have become known as solitons.

The equation of motion for these waves can be written in dimensionless form as

$$\frac{\delta^2 a}{\delta \xi^2} + i \frac{\delta a}{\delta \zeta} + N^2 |a|^2 a = 0$$

similar to non-linear Schrodinger equation.

- if $N \ll I$ then we can neglect the nonlinear part of the equation. It will just diffract without any nonlinear behavior.
- if $N \gg 1$ then the nonlinear effect will be more evident than diffraction
- if $N \approx 1$ then the two effects balance each other

The most general solution of:

$$a(\xi,\zeta) = \operatorname{sech}(\xi)e^{i\zeta/2}$$

Solitons details

Dispersion and non-linearity are necessary to produce permanent and localized wave forms.

Both are present in the Gross-Pitaevskii equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) + U_0|\psi(\mathbf{r})|^2\psi(\mathbf{r}) = \mu\psi(\mathbf{r})$$

dependence of the velocity of an excitation on the local density and on the wavenumber

Bogoliubov dispersion relation for exciton polaritons:

$$\varepsilon_B(k) = \sqrt{\varepsilon(k)(\varepsilon(k) + 2U_0n)} + U_0n$$

• non-linear interactions U_0

Solitons details

Qualitatively:

- localized disturbance with amplitude Δn
- extends over a distance L

velocity of sound within the disturbance is different from the bulk medium due to nonlinear effects by the amount of



 ξ healing length - describes the distance over which the wave function tends to its bulk value when subjected to a localized perturbation

$$\xi \approx \sqrt{\frac{\hbar^2}{2mU_0n}}$$

kinetic energy equals interaction strength

Solitons in atomic condensates

potential edge.

Х

0



(1999)

Absorption images of BEC's with kink-wise struc-FIG. 2. tures propagating in the direction of the long condensate axis, for different evolution times in the magnetic trap, t_{ev} . $(\Delta \phi \sim \pi, N \approx 1.5 \times 10^5, \text{ and } t_{\text{TOF}} = 4 \text{ ms}).$

Solitons in atomic condensates



Fig. 1. (A) Writing a phase step onto the condensate. A far-detuned uniform light pulse projects a mask (a razor blade) onto the condensate. Because of the light shift, this imprints a phase distribution that is proportional to the light intensity distribution. A lens (not shown) is used to image the razor blade onto the condensate. The mask in **(B)** writes a phase stripe onto the condensate. The mask in **(C)** imprints an azimuthally varying phase pattern that can be used to create vortices.

J. Denschlag et al., *Generating Solitons by Phase Engineering of a Bose-Einstein Condensate*, Science 287, 97 (2000).



Fig. 3. Experimental (**A** to **E**) and theoretical (**F** to **J**) images of the integrated BEC density for various times after we imprinted a phase step of $\sim 1.5\pi$ on the top half of the condensate with a 1-µs pulse. The measured number of atoms in the condensate was 1.7 (± 0.3) × 10⁶, and this value was used in the calculations. A positive density disturbance moved rapidly in the +x direction, and a dark soliton moved oppositely at significantly less than the speed of sound. Because the imaging pulse (27) is destructive, each image shows a different BEC. The width of each frame is 70 µm.

Solitons in atomic condensates

Formation of Dispersive Shock Waves by Merging and Splitting Bose-Einstein Condensates J. J. Chang, P. Engels, and M. A. Hoefer, Phys. Rev. Lett. 101, 170404 (2008)

uniform soliton trains containing more than ten solitons or the formation of a high-density bulge as well as dispersive shock waves are observed experimentally within merged BECs. Our numerical simulations indicate the formation of many vortex rings. In the case of splitting a BEC, the transition from sound-wave formation to dispersive shock-wave formation is studied by use of increasingly stronger splitting barriers.



FIG. 1. Left column: experimental antitrapped expansion images of a BEC collision at t = (a) 2 ms, (b) 7 ms, (c) 12 ms, (d) 22 ms, and (e) 57 ms (including the 2 ms antitrapped expansion time). Middle column: numerical simulations at t = (f) 2 ms, (g) 7 ms, (h) 12 ms, (i) 22 ms, and (j) 57 ms. The antitrapped expansion was not simulated; therefore, the vertical scale of (f)–(j) is about $\frac{3}{57}$ the vertical scale of (a)–(e), see also [20]. Right column: (k)–(m) simulations showing zoomed-in density slices of the BEC by the plane z = 0 after (k) 5 ms, (l) 6.25 ms, and (m) 7.5 ms. (n) Integrated cross section of (d). (o) Typical uniform soliton train observed for lower atom numbers (experimental image; parameters see text).

Solitons dark and bright

Classification from the point of view of particle density (or light intensity in the case of optical solitons):

dark solitons - depression in density

black solitons - minimum density is zero

grey solitons - minimum density is non-zero

bright solitons with the density maximum

Solitons Time dependent Gross-Pitaevskii equation: dark $i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) + U_0 |\psi(\mathbf{r},t)|^2 \psi(\mathbf{r},t)$ has one dimensional dark soliton solution for repulsive interactions $U_0 > 0$ Taking the solution that depends on the spatial coordinate and time $\psi(x,t) = f(x-ut)e^{-i\mu t/\hbar}$ and inserting into GP, we obtain $-\frac{\hbar^2}{2m}f'' + U_0|f|^2f = -i\hbar uf' + \mu f$ where $f' = \frac{\delta f}{\delta(x - ut)}$ $\psi = \sqrt{n_0} \left| \frac{i u}{s} + \sqrt{\left(1 - \frac{u^2}{s^2}\right) \tanh\left(\frac{x - ut}{\sqrt{2}\xi_u}\right)} \right| e^{-i\mu t/\hbar}$ sound velocity of the uniform condensate $\ s = (n_0 U_0/m)^{1/2}$ $\mu = U_0 n_0$ chemical potential

Solitons bright $i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) + U_0 |\psi(\mathbf{r},t)|^2 \psi(\mathbf{r},t)$ has also one dimensional bright soliton solution for attractive interactions $U_0 < 0$

Few remarks:

- The properties of bright solitons differ from those of dark solitons in that their height and width are unrelated to their propagation velocity.
- The velocity of a dark soliton cannot exceed the sound velocity.
- There is no corresponding restriction on the velocity of bright solitons.
- For atoms in an optical lattice it is possible to observe bright solitons even in condensates with repulsive interactions. The effective mass of an atom in an optical lattice becomes negative near the Brillouin zone boundary. A change in sign of the mass will have the same effect as a change in sign of interactions in allowing for the existence of bright soliton solutions.

Soliton trains in condensate with tuned interactions bright solitons in quasi ID trap

Attraction in atomic condensate makes it unstable for collapse (it can be stabilized by confinement)

Feshbach resonance allows to tune the interactions from repulsive to attractive





Figure 1 Feshbach resonance. Calculation of the scattering length versus magnetic field for atoms in the (1, 1) state of ⁷Li using the coupled channels method¹⁸. The field axis has been scaled here by a factor of 0.91, to agree with the measured resonance position of 725 G shown in Fig. 2. The scattering length is given in units of the Bohr radius, a_0 .

Soliton trains in condensate with tuned interactions bright solitons in quasi ID trap

K. E. Strecker et al., *Formation and propagation of matter-wave soliton trains*, Nature 417, 150 (2002) Solitons may be formed when a nonlinear interactions produces a self-focusing of the wave packet that compensates for dispersion.



Figure 3 Comparison of the propagation of repulsive condensates with atomic solitons. The images are obtained using destructive absorption imaging, with a probe laser detuned 27 MHz from resonance. The magnetic field is reduced to the desired value before switching off the end caps (see text). The times given are the intervals between turning off the end caps and probing (the end caps are on for the t = 0 images). The axial dimension of each image frame corresponds to 1.28 mm at the plane of the atoms. The amplitude of

oscillation is $\sim 370 \,\mu$ m and the period is 310 ms. The a > 0 data correspond to 630 G, for which $a \approx 10a_0$, and the initial condensate number is $\sim 3 \times 10^5$. The a < 0 data correspond to 547 G, for which $a \approx -3a_0$. The largest soliton signals correspond to $\sim 5,000$ atoms per soliton, although significant image distortion limits the precision of number measurement. The spatial resolution of $\sim 10 \,\mu$ m is significantly greater than the expected transverse dimension $I_{\rm f} \approx 1.5 \,\mu$ m.

Soliton trains in condensate with tuned interactions solitons in quasi ID trap

K. E. Strecker et al., *Formation and propagation of matter-wave soliton trains*, Nature 417, 150 (2002) Solitons with a phase difference of π will repel, while those that have the same phase will attract.



Figure 4 Repulsive interactions between solitons. The three images show a soliton train near the two turning points and near the centre of oscillation. The spacing between solitons is compressed at the turning points, and spread out at the centre of the oscillation. A simple model based on strong, short-range, repulsive forces between nearest-neighbour solitons indicates that the separation between solitons oscillates at approximately twice the trap frequency, in agreement with observations. The number of

solitons varies from image to image because of shot to shot experimental variations, and because of a very slow loss of soliton signal with time. As the axial length of a soliton is expected to vary as 1/N (ref. 11), solitons with small numbers of atoms produce particularly weak absorption signals, scaling as N^2 . Trains with missing solitons are frequently observed, but it is not clear whether this is because of a slow loss of atoms, or because of sudden loss of an individual soliton.

Important characteristics of polariton superfluid







A SOUND VELOCITY CAN BE ATTRIBUTED TO THE POLARITON FLUID

$$c_{\rm s} = \sqrt{\hbar g |\psi_{\rm c}|^2}/m$$

Polariton superfluid setup

A.Amo, et al. Nature 457, 291 (2009)







PUBLISHED ONLINE: 20 SEPTEMBER 2009 | DOI: 10.1038/NPHYS1364

A.Amo, et al. Nature 457, 291 (2009)

Superfluidity of polaritons in semic microcavities

Alberto Amo¹*, Jérôme Lefrère¹, Simon Pigeon², Claire Adrados¹, Cristiano Ciuti², Iacopo Carusotto³, Romuald Houdré⁴, Elisabeth Giacobino¹ and Alberto Bramati¹*

sound speed



TO UNDERSTAND POLARITON SUPERFLUIDITY



Theory: Carusotto & Ciuti, PRL **93**, 166401 (2004) Theory + experiments: Amo *et al,* Nature Phys. **5**, 805 (2009)

DIFFERENT REGIMES vs FLUID SPEED



Carusotto *et al*, PRL **97**, 260403 (2006)

DIFFERENT REGIMES vs FLUID SPEED



Solitons in polariton condensates dark solitons

Particular solutions of the Gross-Pitaevskii equation describing boson condensates subject to repulsive interactions.

Characteristics:

- notch in the density profile shape is not subject to dispersion thanks to the non-linearity arising from particle interactions
- phase jump up to $\,\pi\,$ when crossing them

Quantum hydrodynamic solitons behind an obstacle

Dark solitons nucleate hydrodynamical due to the gradient of flow speeds occurring around the potential barrier, which result in density variations on the order of the hearestedt A. November 2015; see full text.

Once the soliton is formed, the repulsive inter particle interactions stabilize its shape as it propagates.

A. Amo et al., *Polariton Superfluids Reveal Quantum Hydrodynamic Solitons*, Science 332, 1167 (2011)

Characteristics:

notch in the density profile

phase jump up to π



Quantum hydrodynamic solitons



Fig. 1. (**A**) Real-space emission showing a soliton doublet nucleated in the wake of a photonic defect located at the origin. (**B**) Horizontal profiles at different downflow distances from the defect Δy . Arrows indicate the soliton position. (**C**) Interference between the emitted intensity and a constant-phase reference beam, showing phase jumps along the solitons (dashed lines). The curved shaped of the fringes and the decreasing interfringe distance arise from the geometry of the reference beam. (**D**) Soliton depth (black circles) and phase jump obtained from (C) (filled triangles; see fig. S4), showing a strong correlation. Open triangles: soliton depth obtained from the measured phase jump and Eq. 1.

Quantum hydrodynamic solitons behind an obstacle

 $\begin{array}{l} \text{superfluid} \\ \nu_{flow} = 0.25 c_s \end{array}$

onset of drag force $\nu_{flow} \sim 0.4 c_s$

solitons $\nu_{flow} = 0.6c_s$



Fig. 3. (**A** to **C**) Real-space images of the polariton gas flowing downward at different excitation densities in the presence of a double defect (total width: 15 µm). The gas is injected above the red line (*25*). At high density (A) (117 mW), the fluid is subsonic ($v_{flow} = 0.25\overline{c}_s$) and flows in a superfluid fashion around the defect. At lower densities (B) (36 mW; $v_{flow} = 0.4\overline{c}_s$), a turbulent pattern appears in the wake of the defect, eventually giving rise to the formation of two oblique dark solitons (C) ($v_{flow} = 0.6\overline{c}_s$; 27 mW). (**D** to **F**) Interferograms corresponding to (A) to (C), respectively. (**G**) to (**I**) show the corresponding degree of first-order coherence [$g^{(1)}$, see (*25*)]. Saturated values of $g^{(1)}$ are due to the uncertainty in the measurements.

Half-solitons - unique feature of exciton polaritons spinor character of the condensate

- Excitons with total angular momentum $\,J\,$ couple to light
- Bright excitons

excitons with $J_z=-1$ couple to $\,\sigma^-$ photons spin down polaritons (

- excitons with $J_z=+1$ couple to $\,\sigma^+$ photons $\,$ spin up polaritons
- Dark excitons, $J=\pm 2$, do not couple to light

spin polarized condensates

 n_{\perp}

condensate with two spin components



Half-solitons - unique feature of exciton polaritons spinor character of the condensate

Quantum vortices characteristics

5	Full Vortices	Half Vortices
Phase shift:	2π	π
Polarization rotation:	0	π
Density @ core:	Minimum	Minimum in σ^+ Maximum in σ^-
Quantum numbers:	<i>m</i> =1,2,	$(k, m) = (\pm \frac{1}{2}, \pm \frac{1}{2})$
Observation:	 Superfluids Condensates Superconductors Exciton-polaritons 	High T _c superconductors Exciton-polaritons

K. Lagoudakis et al., Nat. Phys. 4, 706 (2008)

Half-solitons - unique feature of exciton polaritons spinor character of the condensate R. Hivet et al., *Half-solitons in a*





Half-solitons - unique feature of exciton polaritons spinor character of the condensate



Half-solitons in a polariton quantum fluid behave 724 (2012 like magnetic monopoles, Nature Phys. 8, R. Hivet et al.,

Half-solitons - unique feature of exciton polaritons spinor character of the condensate





R. Hivet et al., Half-solitons in a polariton quantum fluid behave like magnetic monopoles, Nature Phys. 8, 724 (2012)

 $\pi/2$

Bright solitons transition from positive to negative mass



Figure 1 Experimental configuration and observation of soliton propagation. a, Dispersion (energy-momentum) diagram of the lower-branch polaritons and schematic representation of the soliton spectrum and excitation scheme. **b**, Schematic of soliton excitation in the microcavity structure. The c.w. pump with incident in-plane momentum parallel to the *x*-direction is focused into a large spot. The pulsed writing beam, also incident along *x*, is focused into a small spot and triggers soliton formation. **c**, Bistable polariton density as a function of pump momentum. Arrow indicates the state of the system created by the pump before incidence of the writing beam. **d**-**g**, Streak camera measurements of the soliton trajectories along the *x*-direction excited under different conditions. **d**,**e**, Components of the soliton in transverse electric (TE) and transverse magnetic (TM) polarizations, respectively. This soliton is excited with a 7-μm-wide writing beam with in-plane momentum the same as that of the pump. **f**, As in **d**, but the writing beam has half the momentum of the pump. **g**, As in **f**, but the writing beam width is 14 μm. (wb, writing beam.)

Bright solitons transition from positive to negative mass



Figure 3 | Two-dimensional soliton images. Two-dimensional streak camera measurements of a soliton travelling across the microcavity plane. Experimental conditions are as in Fig. 2a–c. Cross-sections taken along the *y*-direction indicate a soliton FWHM of 5 μm.