LECTURE 4

Polariton BEC

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pok. 3.64

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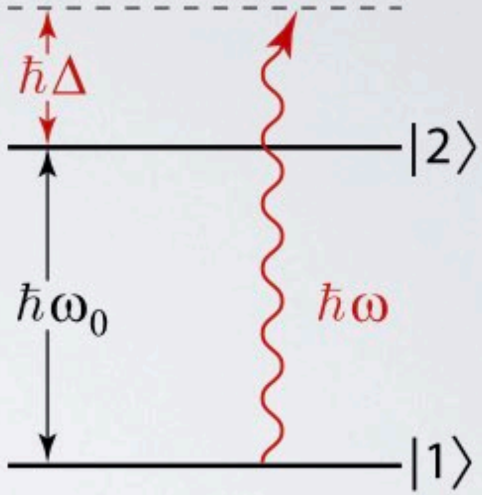
Light-matter interaction

two-levels in an external field

- $|1\rangle$ and $|2\rangle$ form an orthonormal basis for the system *i.e.* $\langle i|j\rangle = \delta_{ij}$ for $i, j = 1, 2$;
- Photon frequency : $\omega = \omega_0 + \Delta$;
- Detuning : Δ ;
- Resonant frequency : ω_0 .



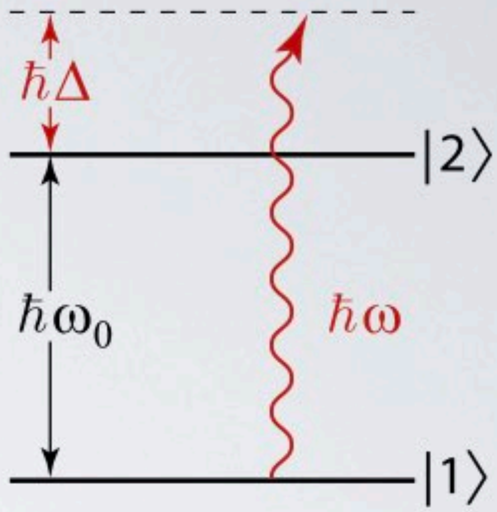
The bare hamiltonian:

$$H_0 = \hbar \begin{pmatrix} 0 & 0 \\ 0 & \omega_0 \end{pmatrix}$$

$$\hat{H}_0|1\rangle = 0\,|1\rangle,$$
$$\hat{H}_0|2\rangle = \hbar\,\omega_0|2\rangle$$

w.g. R. Anderson in *Two-level system: Rabi oscillations.*

Light-matter interaction
two-levels in an external field



$$\psi = \begin{pmatrix} \langle 1|\psi\rangle \\ \langle 2|\psi\rangle \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Any two-level quantum state can be expressed as $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$, where c_1 and c_2 are complex state amplitudes and $|c_1|^2 + |c_2|^2 = 1$. Such a state can be represented by a two-component vector;

The probability of finding the system in state $|i\rangle$ is $|\langle i|\psi\rangle|^2 = |c_i|^2$, (for $i = 1, 2$).

w.g. R. Anderson in *Two-level system: Rabi oscillations.*

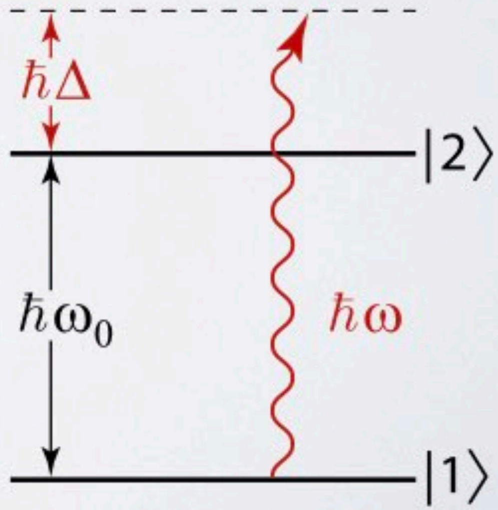
Light-matter interaction

two-levels in an external field

What about the driving field? What does it do? It induces a dipole (electric or magnetic) moment between the states $|1\rangle$ and $|2\rangle$. The electromagnetic field interacts with this dipole, resulting in an oscillatory perturbation. This perturbation is represented by the operator:

$$H_{int} = \hbar \begin{pmatrix} 0 & \Omega \cos(\omega t) \\ \Omega^* \cos(\omega t) & 0 \end{pmatrix}$$

the Rabi frequency $\Omega = \mathcal{E}\mu/\hbar$



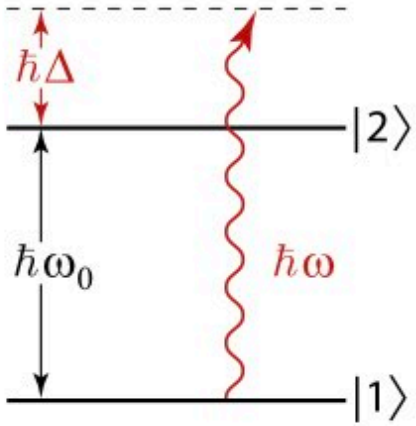
w.g. R. Anderson in *Two-level system: Rabi oscillations.*

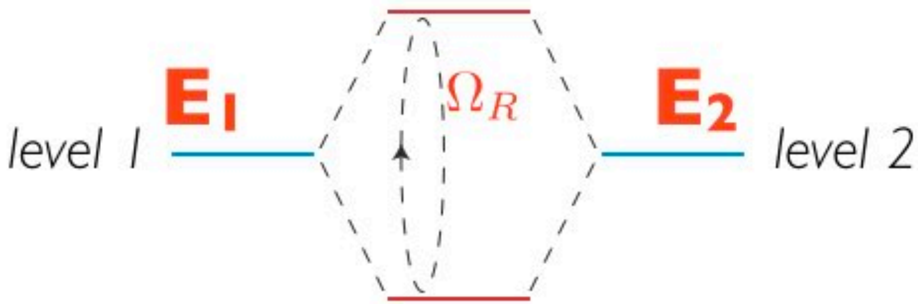
Light-matter interaction

two-levels in an external field

$$H_0 = \hbar \begin{pmatrix} 0 & 0 \\ 0 & \omega_0 \end{pmatrix}$$

$$H_{int} = \hbar \begin{pmatrix} 0 & \Omega \cos(\omega t) \\ \Omega^* \cos(\omega t) & 0 \end{pmatrix}$$





$$\psi(x,t) = c_1(t)\phi_1(x) + c_2(t)\phi_2(x)$$

The time dependent coefficients satisfy the Schrödinger equation in matrix form

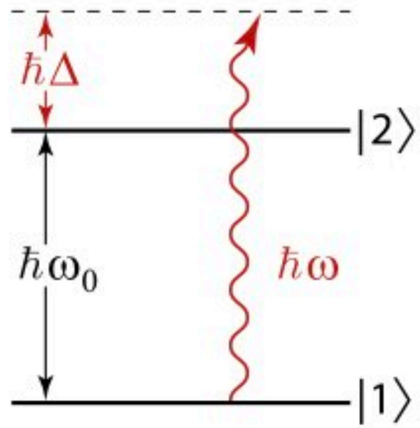
$$i\hbar \frac{d}{dt} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} .$$

Light-matter interaction

two-levels in an external field

$$H_0 = \hbar \begin{pmatrix} 0 & 0 \\ 0 & \omega_0 \end{pmatrix}$$

$$H_{int} = \hbar \begin{pmatrix} 0 & \Omega \cos(\omega t) \\ \Omega^* \cos(\omega t) & 0 \end{pmatrix}$$

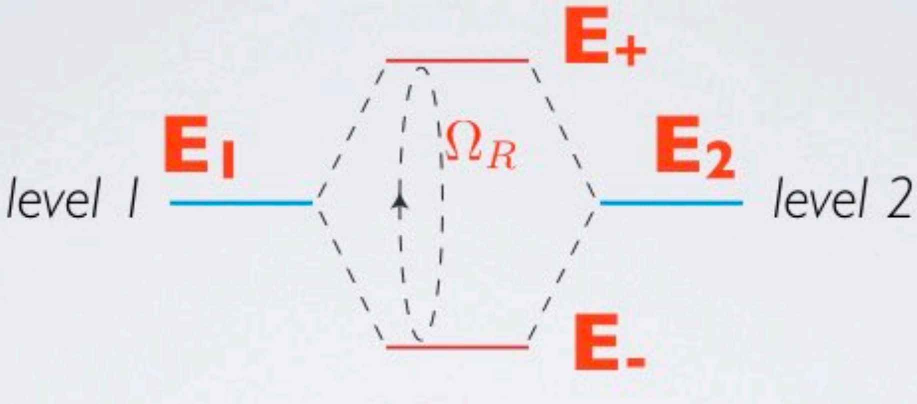


$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = E \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\begin{vmatrix} H_{11} - E & H_{12} \\ H_{21} & H_{22} - E \end{vmatrix} = 0\,.$$

Light-matter interaction

two-levels in an external field

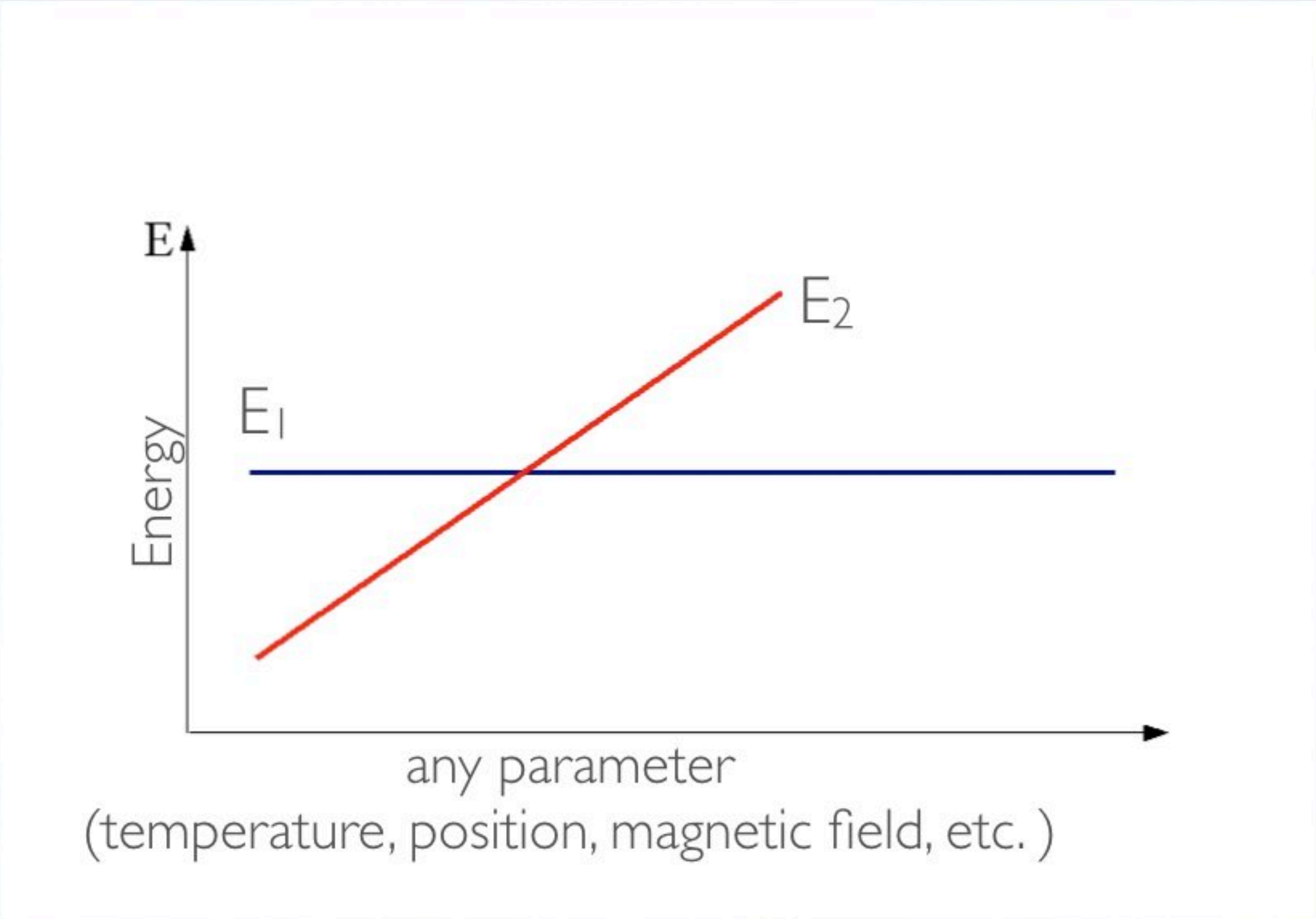


$$E_- = \frac{H_{11}+H_{22}}{2} - \sqrt{\left(\frac{H_{22}-H_{11}}{2}\right)^2 + |H_{12}|^2}$$

$$E_+ = \frac{H_{11}+H_{22}}{2} + \sqrt{\left(\frac{H_{22}-H_{11}}{2}\right)^2 + |H_{12}|^2}$$

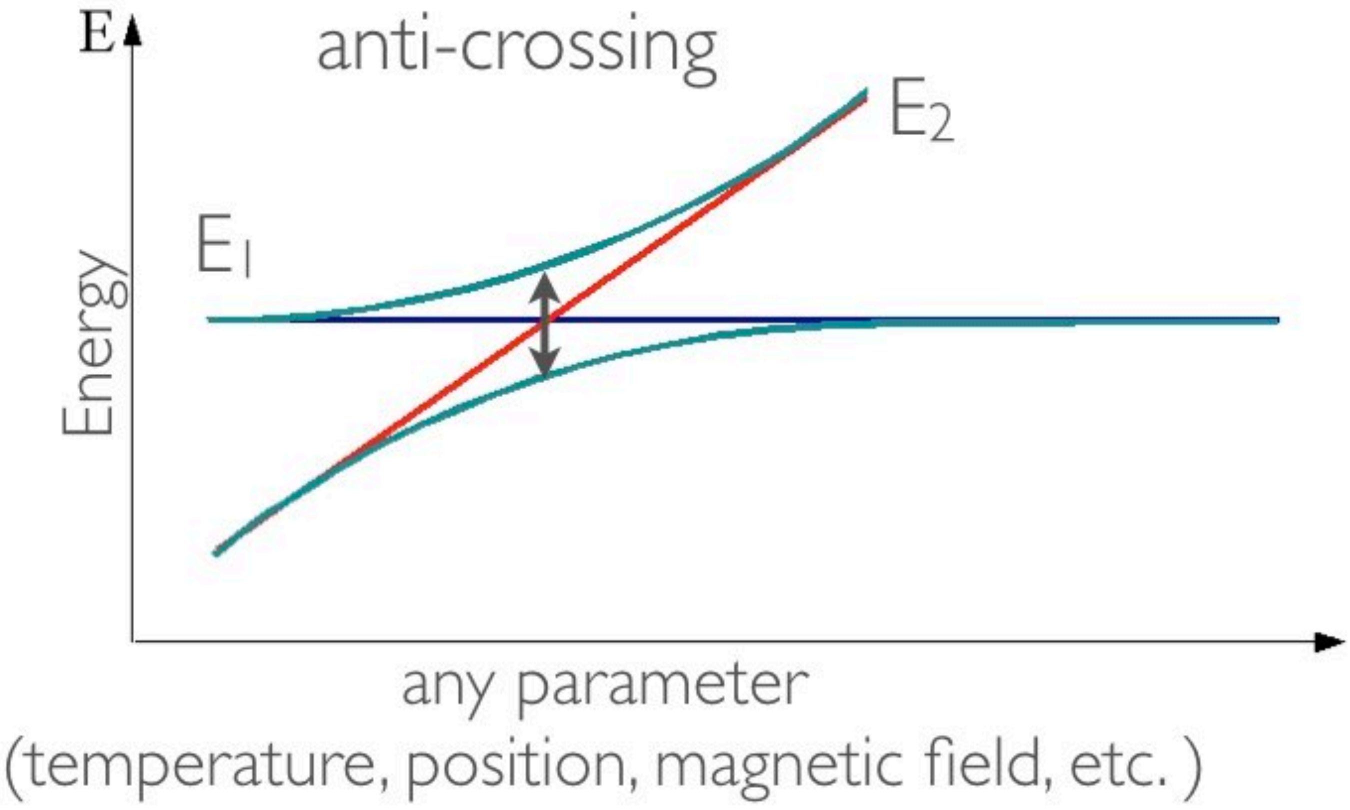
Light-matter interaction

two-level interaction



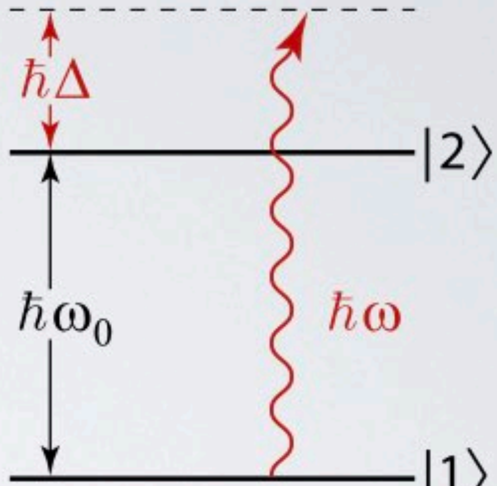
Light-matter interaction

two-level interaction



Light-matter interaction
two-levels in an external field

$$|c_1(t)|^2 = \frac{\Omega^2}{\Omega_R^2}\sin^2\left(\frac{\Omega_R t}{2}\right),$$
$$|c_2(t)|^2 = \frac{\Delta^2}{\Omega_R^2} + \frac{\Omega^2}{\Omega_R^2}\cos^2\left(\frac{\Omega_R t}{2}\right),$$
$$\Omega_R^2 \equiv \Omega^2 + \Delta^2.$$

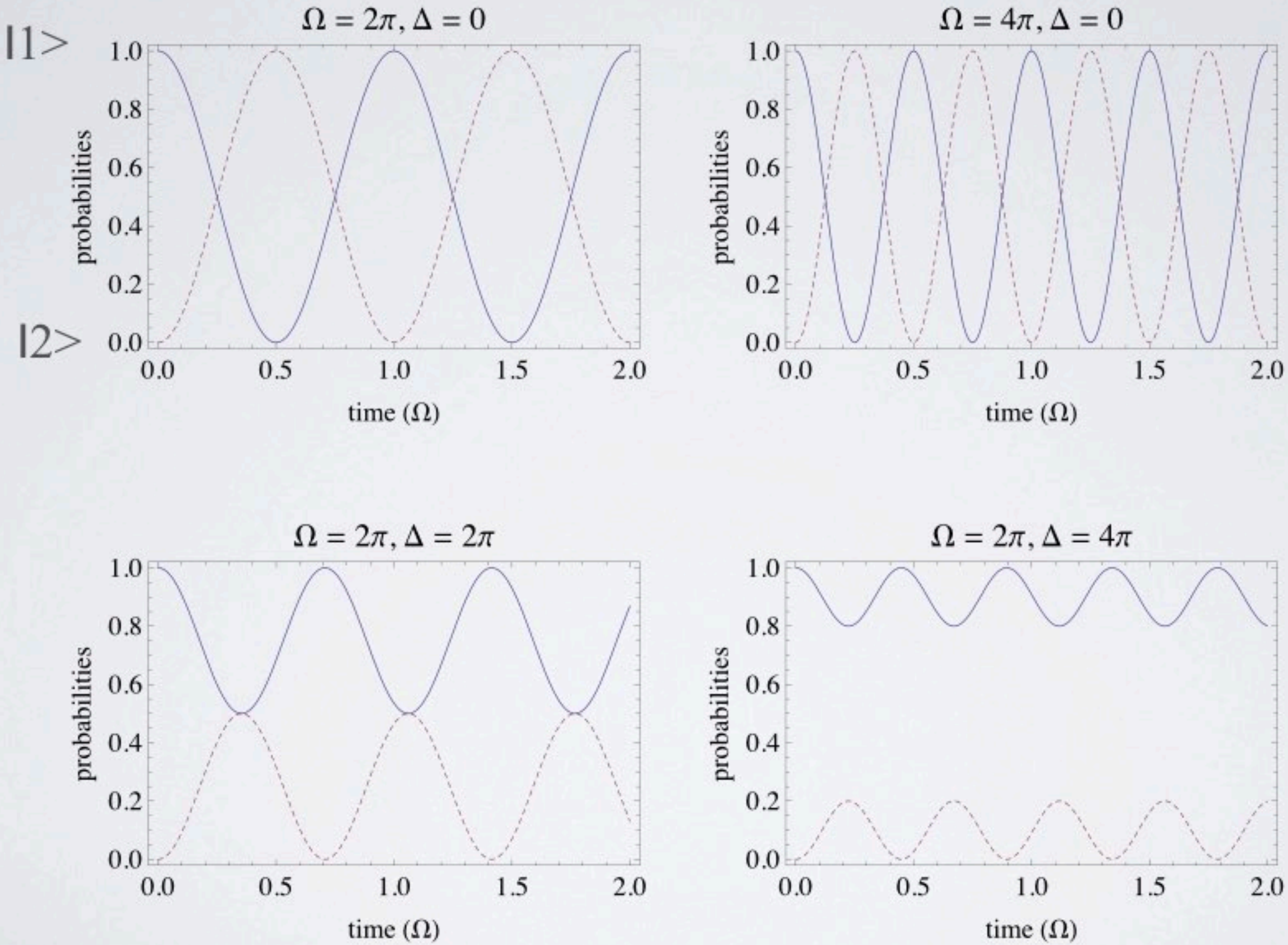


This means that the probabilities to be in state $|1\rangle$ or $|2\rangle$ oscillate with the frequency Ω_R defined above, the *total Rabi frequency*. From this result, it is clear that states $|1\rangle$ and $|2\rangle$ are no longer stationary states of the system. It is remarkable that the dynamic behaviour of the system is governed (at this point) by only two parameters. These parameters are the coupling strength Ω (proportional to the electromagnetic field strength) and the detuning Δ (how far the field is away from resonance).

w.g. R. Anderson in *Two-level system: Rabi oscillations.*

Light-matter interaction

two-levels in an external field



w.g. R. Anderson in *Two-level system: Rabi oscillations.*

Exciton polaritons

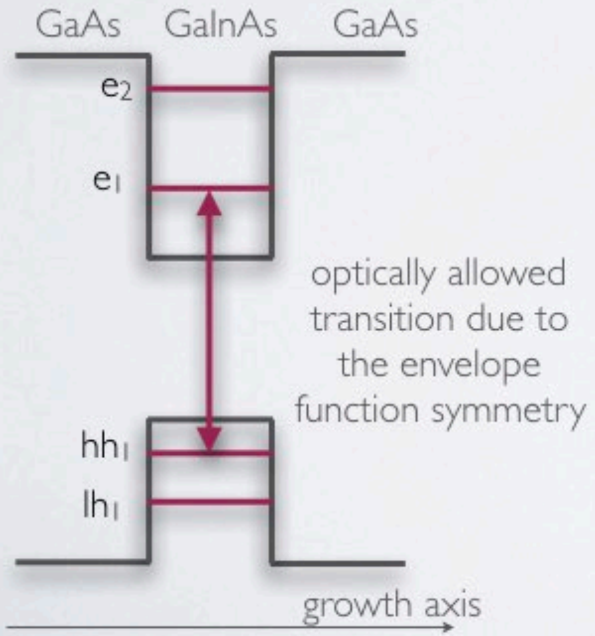
experimental realization of light - matter interaction

POLARITON = EXCITON + PHOTON

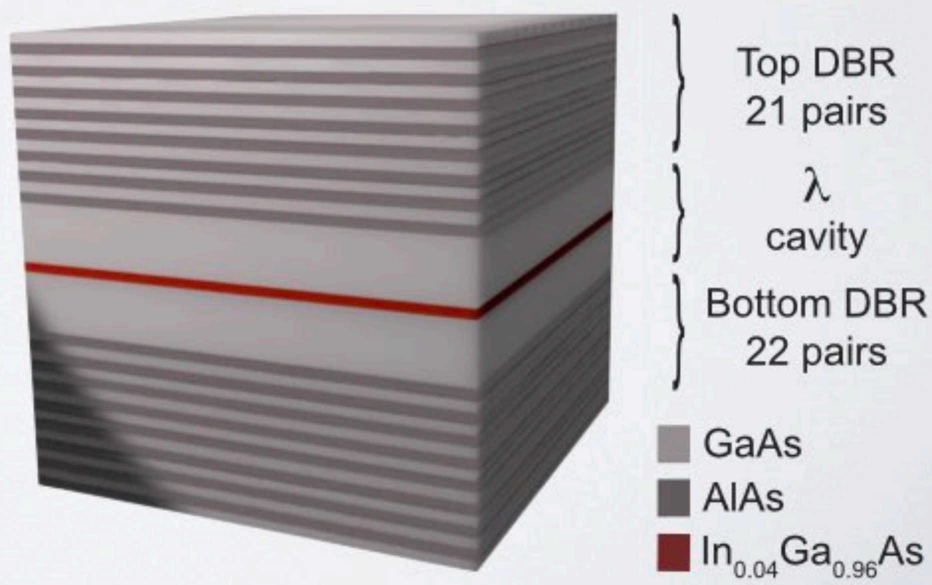
matter component — light component

in the strong coupling limit

quantum well



semiconductor microcavity



Exciton polaritons
short summary

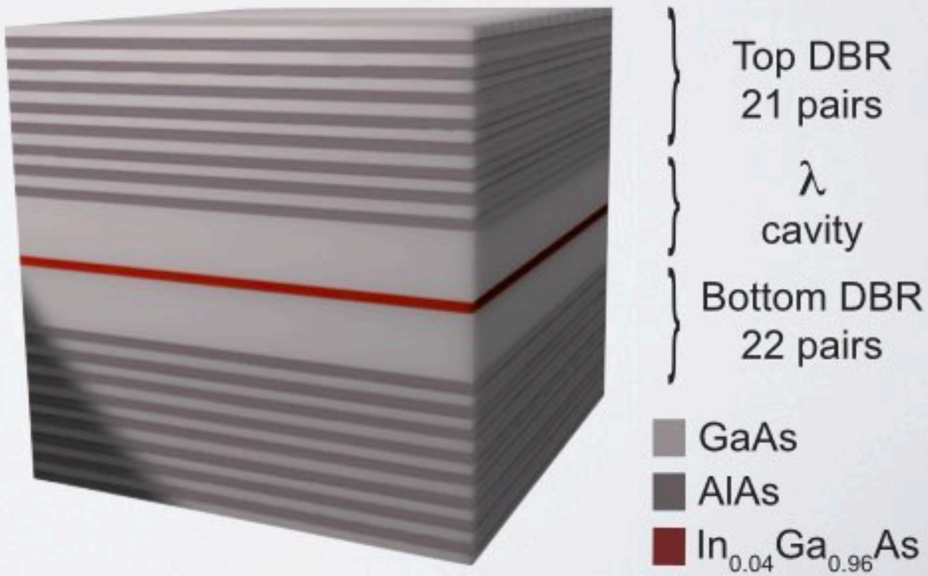
POLARITON = EXCITON + PHOTON

POLARITON ADVANTAGES

- Very light: mass $10^{-4}m_e$ - $10^{-5}m_e$
- Interact via excitonic component
- (Composite) bosons

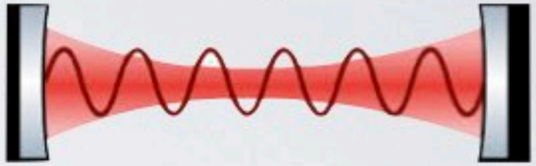
IMAGING

- Optical access to wave function : amplitude and phase



Exciton polaritons
photonic confinement

Basic laws to describe the problem:



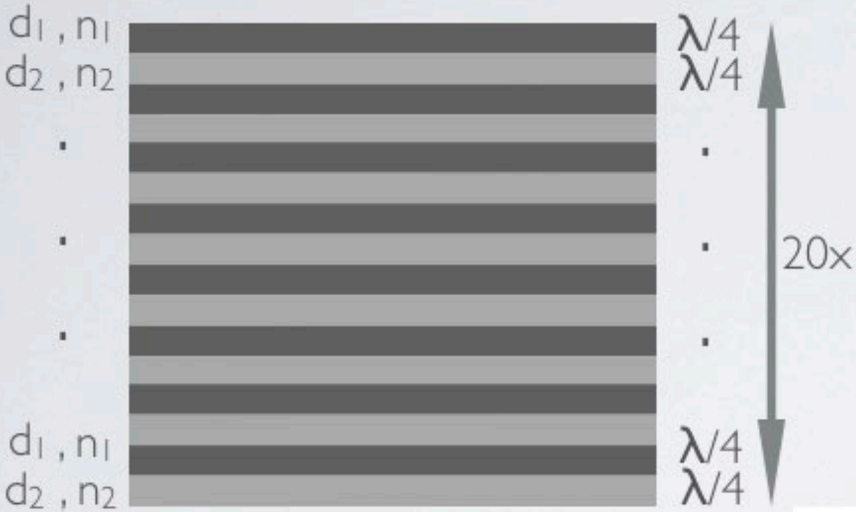
- ☑ wave equation
- ☑ Maxwell's laws
- ☑ continuity equations at the material borders
- ☑ Snell's law (law od refraction)
- ☑ Fresnel equations (amplitudes of reflected and refracted beams)

- let's find an ideal mirror -

Exciton polaritons

photonic confinement

- BRAGG mirror -

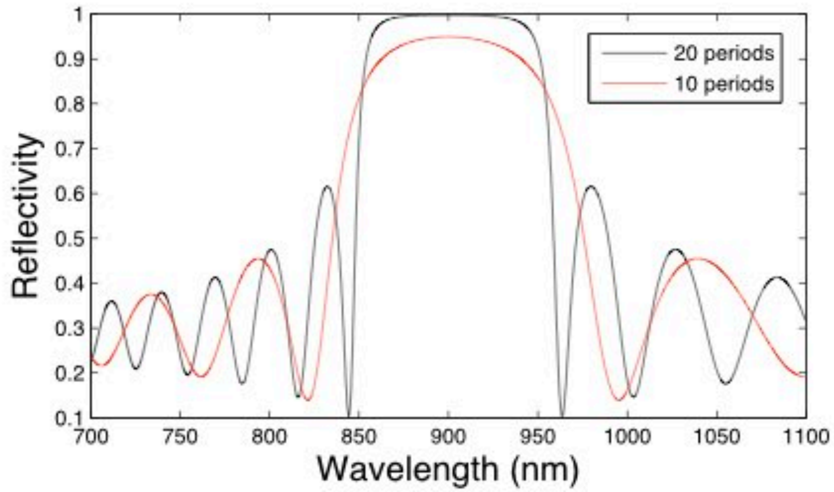


In order to achieve high reflectivity for a given wavelength.

a constructive interference is predicted for

$$n_1 d_1 = n_2 d_2 = \frac{\lambda}{4}$$

single DBR reflectivity spectrum



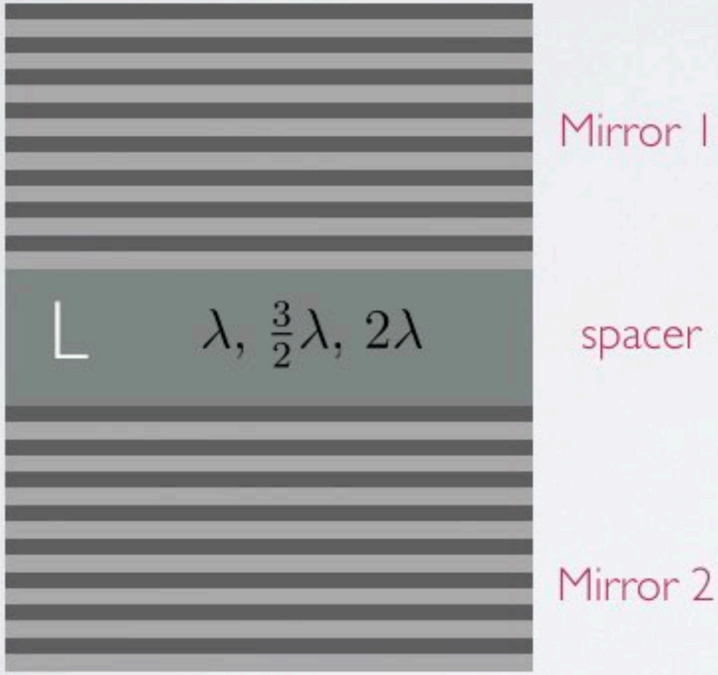
reflectivity spectrum from multiple layers of alternating materials with varying refractive index

Exciton polaritons

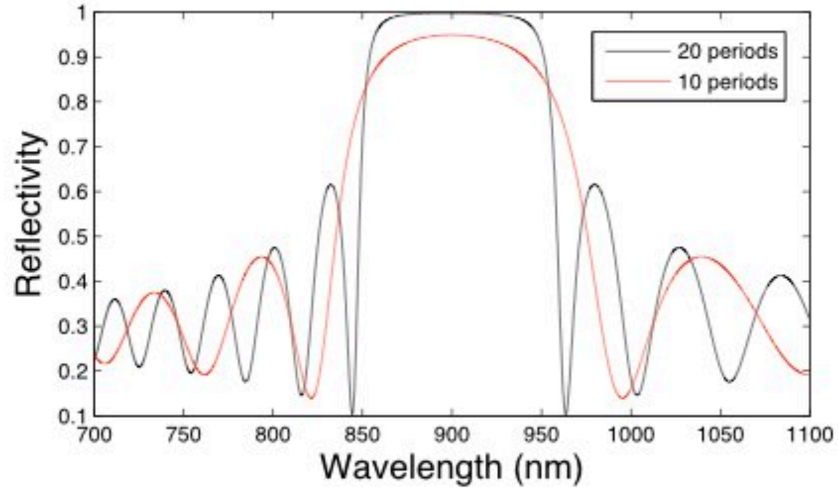
photonic confinement

- two BRAGG mirrors & cavity -

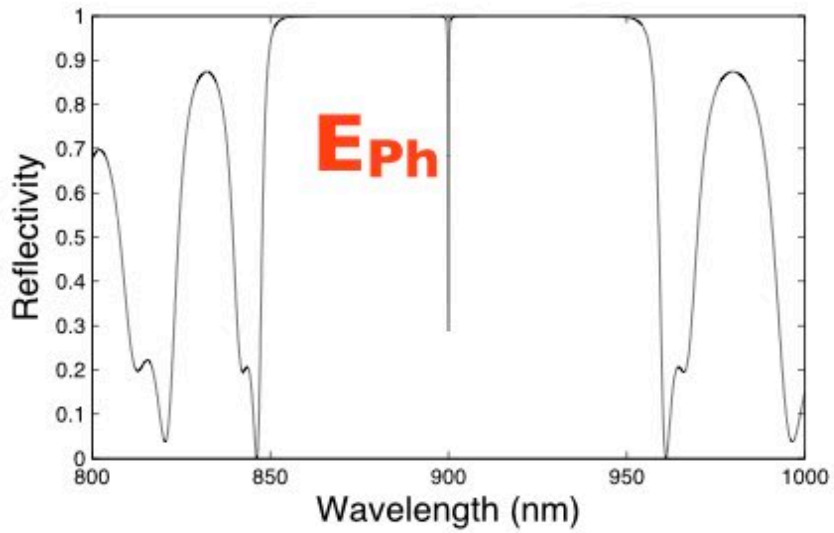
periodicity disorder implies the creation of new levels in energy gap



single DBR reflectivity spectrum



microcavity reflectivity spectrum

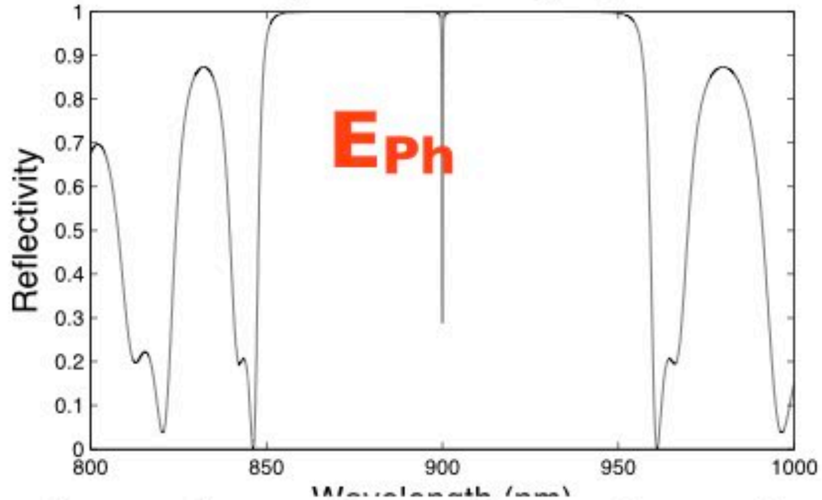


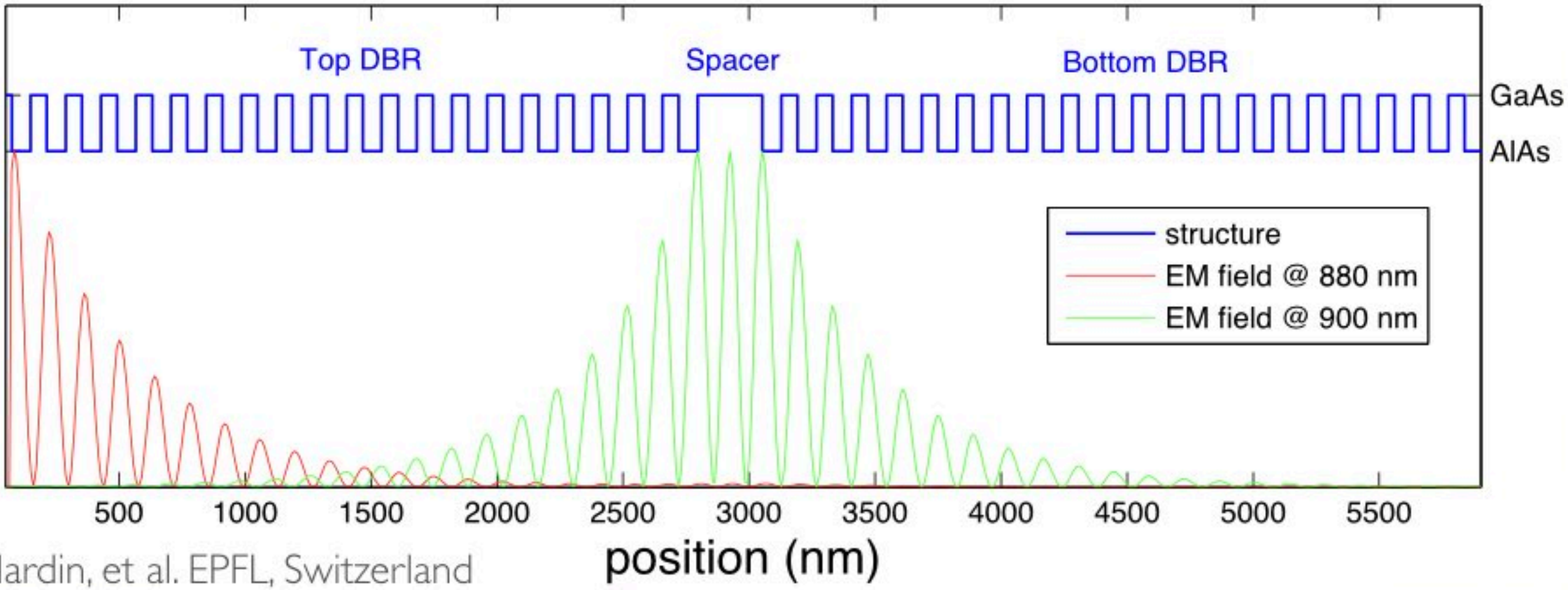
Exciton polaritons

photonic confinement

- photon cavity mode -

microcavity reflectivity spectrum





G. Nardin, et al. EPFL, Switzerland

Exciton polaritons
photonic confinement

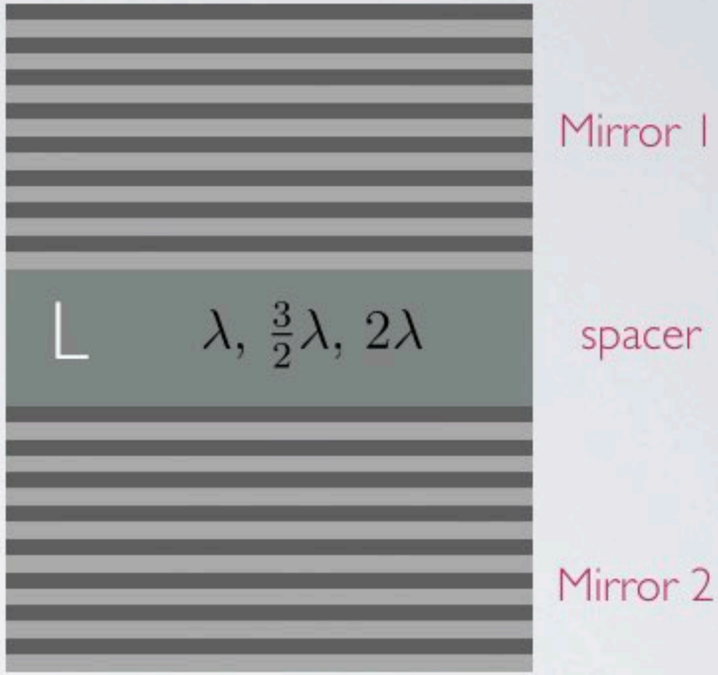
Transmission at resonant frequency

$$T = \frac{(1 - R_1)(1 - R_2)}{[1 - \sqrt{R_1 R_2}]^2 + 4\sqrt{R_1 R_2}\sin^2(\phi/2)}$$

Cavity quality factor

$$Q \doteq \frac{\lambda_c}{\Delta\lambda_c} \simeq \frac{\pi(R_1 R_2)^{1/4}}{1 - (R_1 R_2)^{1/2}}$$

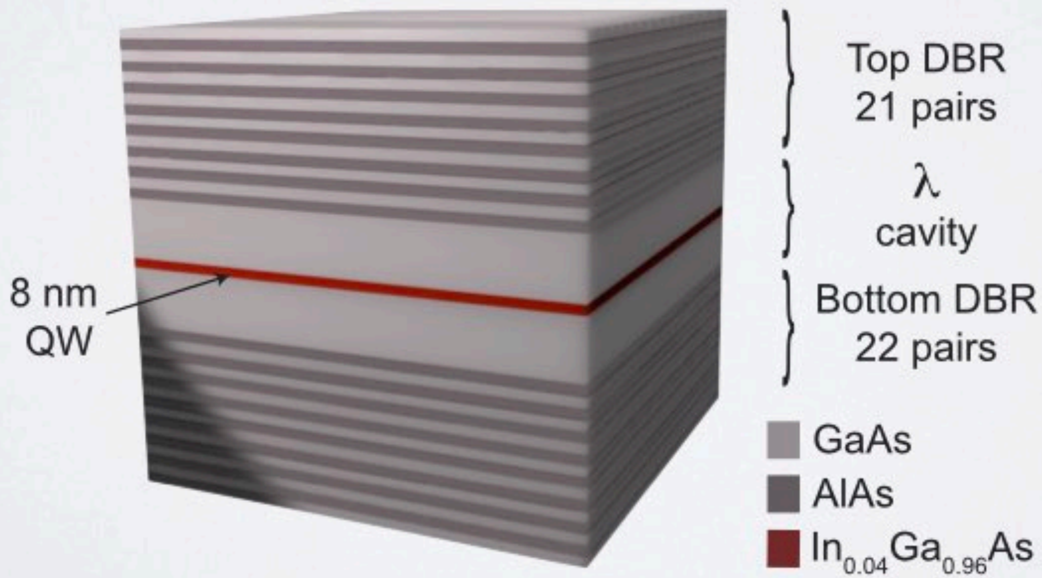
width of the resonance



Exciton polaritons

complete system

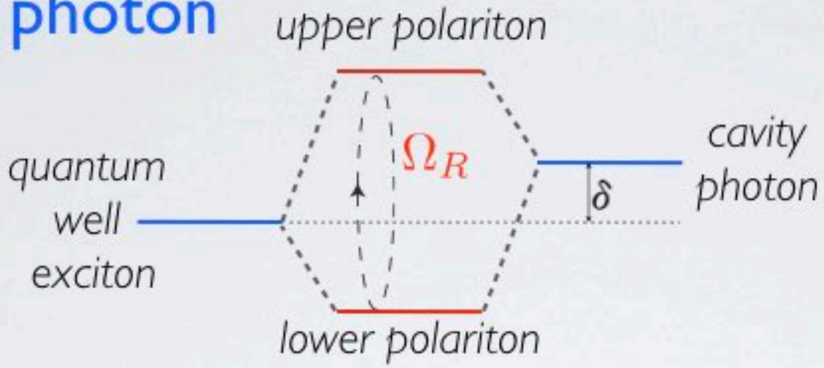
SEMICONDUCTOR MICROCAVITY



Exciton polaritons

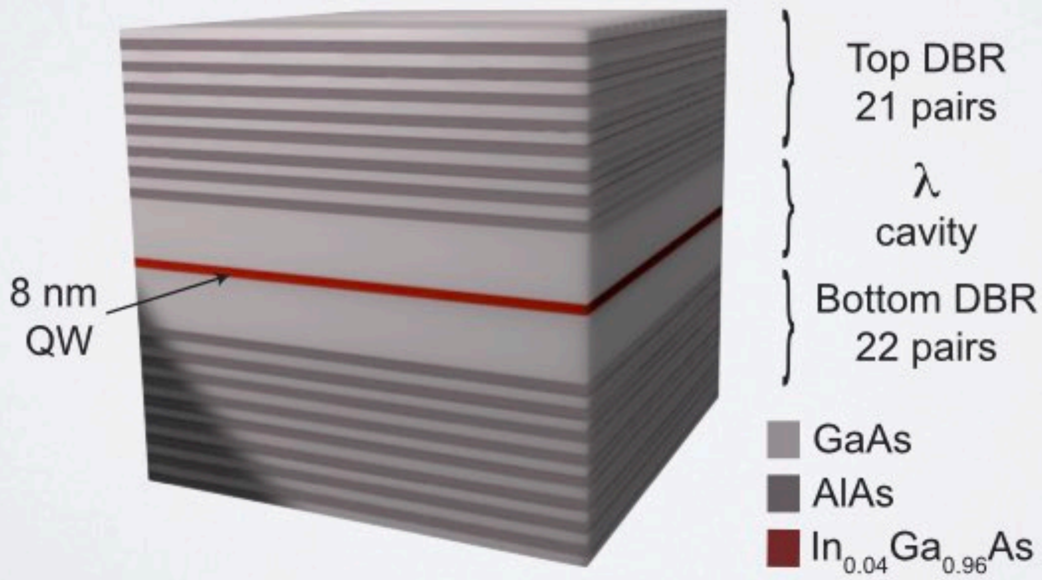
vacuum field Rabi splitting determines exciton polariton modes

Cavity exciton polaritons :
exciton + photon



STRONG interaction between the e-m wave with the QW excitation: photon + exciton

SEMICONDUCTOR MICROCAVITY



Exciton polaritons

vacuum field Rabi splitting determines exciton polariton modes

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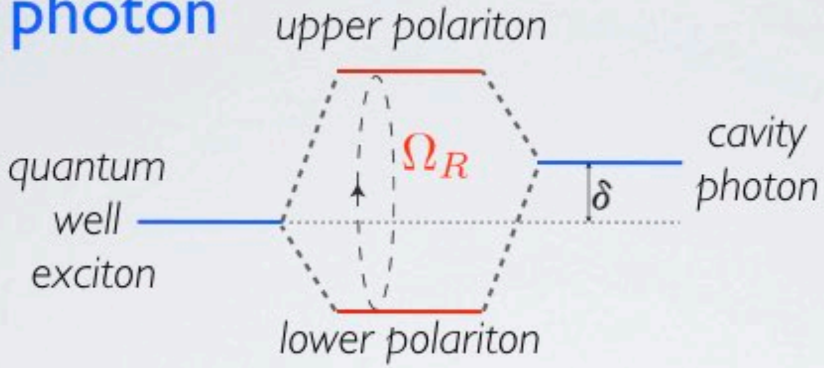
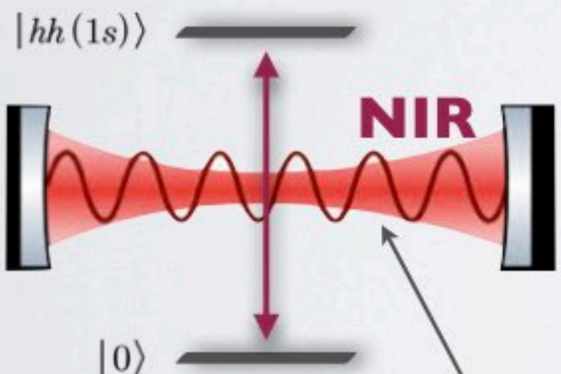


STRONG interaction between the e-m wave with the QW excitation: photon + exciton

Hamiltonian matrix form

exciton - photon coupling energy

$$H = \begin{pmatrix} E_C(k_\parallel) & \frac{\hbar\Omega}{2} \\ \frac{\hbar\Omega}{2} & E_X(k_\parallel) \end{pmatrix}$$



Rabi splitting

$$\Omega \propto \sqrt{\frac{f_{osc} N_{QW}}{L_c}}$$

$\hat{H}_{pol}$ do not depend on number of photons in a cavity

oscillations at the **vacuum Rabi** frequency !

n photons in a cavity !

true even for n = 0 photons

Exciton polaritons
coupling strength

Rabi energy

$$\Omega \propto \sqrt{\frac{f_{osc} N_{QW}}{L_c}}$$

Exciton wave function (1S):

$$\Psi_{ex}(r, z_e, z_h) = U_e(z_e) U_h(z_h) f_r(r)$$

Radial part:

$$f_r(r) = \frac{\sqrt{2/\pi}}{a} \exp\left(-r/a\right)$$ a – exciton radius

Oscillator strength for 1s exciton :

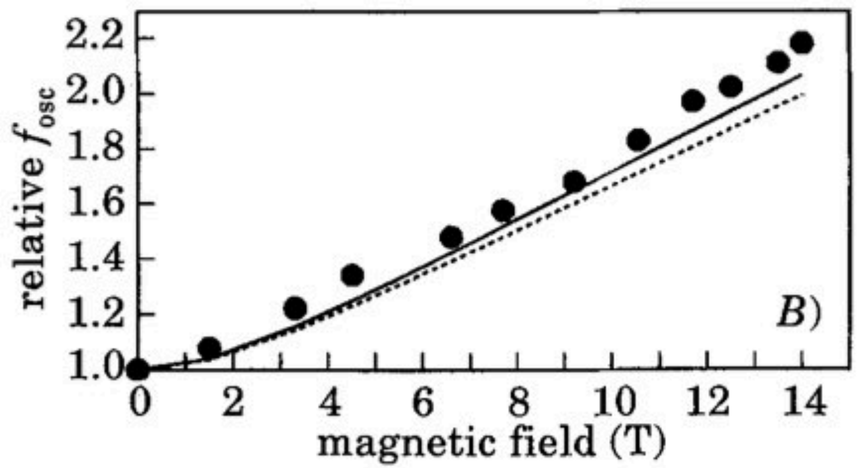
$$f \sim \left| \int U_h(z) U_e(z)\, dz \right|^2 \; |f_r(0)|^2$$

$$f_r(0) = \sqrt{2/\pi}\, \frac{1}{a}$$

IL NUOVO CIMENTO Vol. 17 D, N. 11-12 Novembre-Dicembre 1995

Vacuum Rabi Splitting in Semiconductor Microcavities with Applied Electric and Magnetic Fields (*).

D. M. Whittaker (1), T. A. Fisher (1), A. M. Afshar (1), M. S. Skolnick (1)
P. Kinsler (1), J. S. Roberts (2), G. Hill (2) and M. A. Pate (2)



Exciton polaritons

dispersion - energy-wave vector dependence

Exciton dispersion in quantum well

$$E_X(k) = E_g - E_b + \frac{\hbar^2 k^2}{2m_X}$$

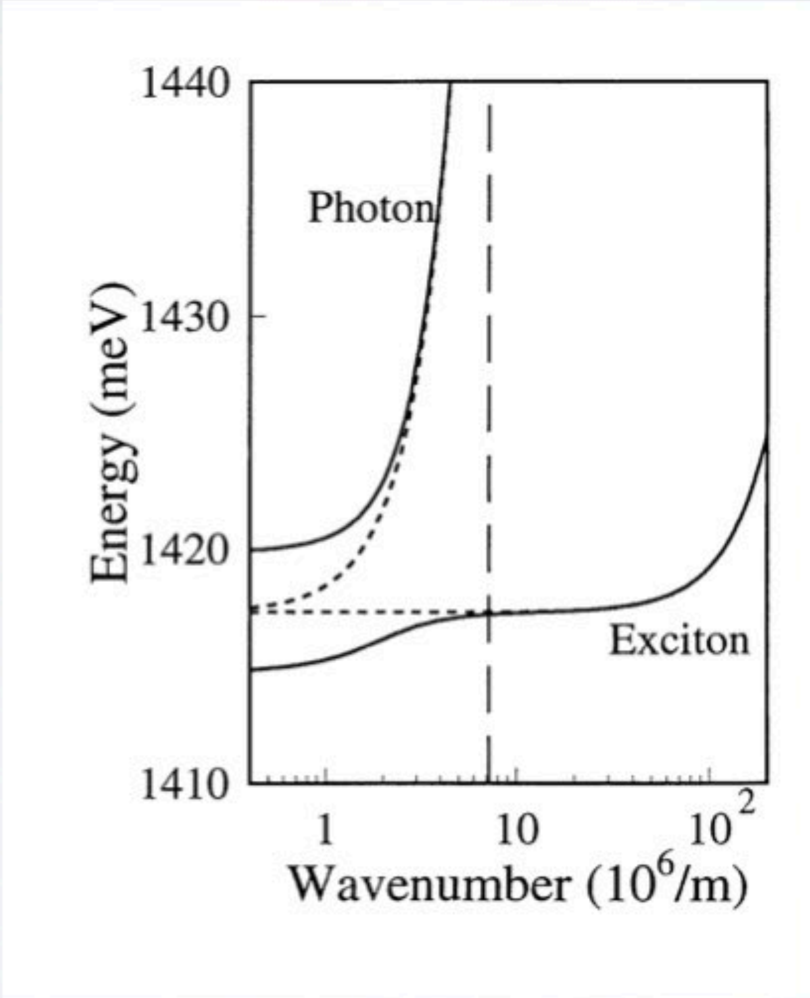


image after: M. S. Skolnick et al.
Semicond. Sci. Technol. 13, 645 (1998)

Exciton polaritons

dispersion - energy-wave vector dependence

Exciton dispersion in quantum well

$$E_X(k) = E_g - E_b + \frac{\hbar^2 k^2}{2m_X}$$

photon dispersion in quantum well

photon with wavevector

$$\vec{k} = \left(k_x, k_y, k_z\right)$$

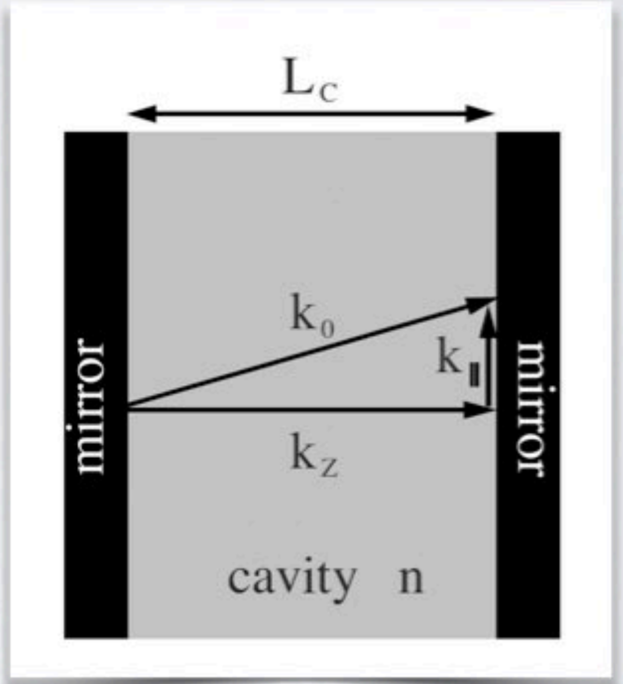
in a microcavity has a quantised z coordinate (in the growth direction of the microcavity of thickness L_C):

$$k_z = \frac{2\pi}{L_C}$$

thus has an energy:

$$E\left(\vec{k}\right) = \frac{\hbar c}{n}\left|\vec{k}\right| = \frac{\hbar c}{n}\sqrt{\left(\frac{2\pi}{L_C}\right)^2 + k_{II}^2}$$

a Fabry-Perot resonator



Schematic vision of the microcavity.

It consists of two mirrors separated by a spacer having a width Lc.

Exciton polaritons

dispersion - energy-wave vector dependence

Exciton dispersion in quantum well

$$E_X(k) = E_g - E_b + \frac{\hbar^2 k^2}{2m_X}$$

photon dispersion in quantum well

$$E(\vec{k}) = \frac{\hbar c}{n}|\vec{k}| = \frac{\hbar c}{n}\sqrt{\left(\frac{2\pi}{L_C}\right)^2 + k_{II}^2}$$

because we are interested in small wave-vectors k_{II} we can make the following approximation:

$$\sqrt{\varepsilon^2 + a^2} \approx a + \frac{\varepsilon^2}{2a}$$

and derive the energy in a form:

$$E(\vec{k}) \approx \frac{\hbar c}{n}\left[\frac{2\pi}{L_C} + \frac{k_{II}^2 L_C}{4\pi}\right] = E_0 + \frac{\hbar^2 k_{II}^2}{2m_{ph}^*}$$

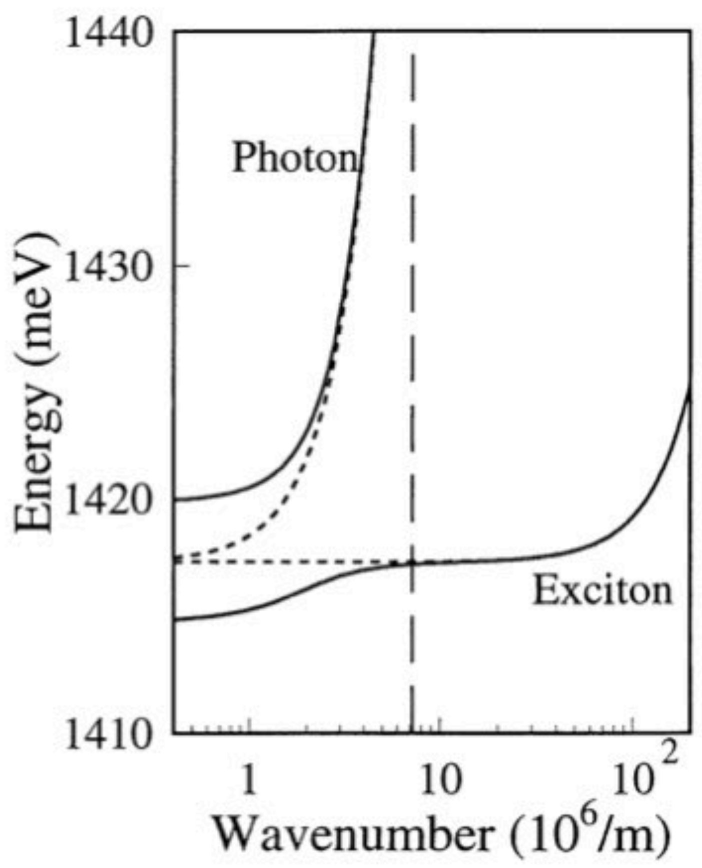


image after: M. S. Skolnick et al. Semicond. Sci. Technol. 13, 645 (1998)

Exciton polaritons

dispersion - energy-wave vector dependence

Exciton dispersion in quantum well

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photon dispersion in quantum well

$$E(\vec{k}) \approx \frac{\hbar c}{n}\left[\frac{2\pi}{L_C} + \frac{k_{II}^2 L_C}{4\pi}\right] = E_0 + \frac{\hbar^2 k_{II}^2}{2m_{ph}^*}$$

conclusions:

in a cavity photon gain an effective mass:

$$m_C^* = \frac{\hbar k_z n}{c} = \frac{h n^2}{c\lambda_0}$$

~10^{-36} kg, i.e. 6 orders of magnitude lower than the free electron mass

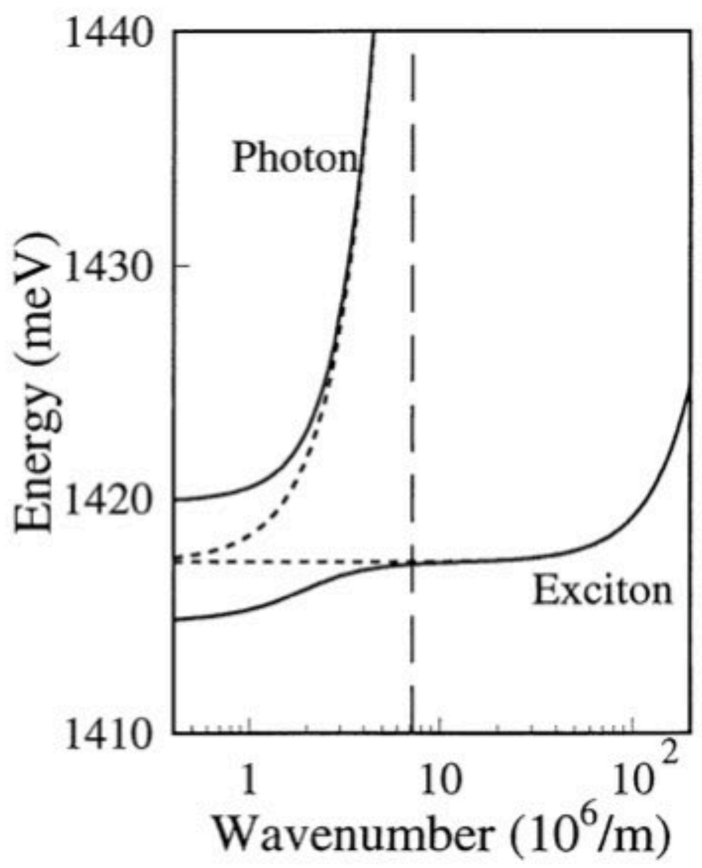
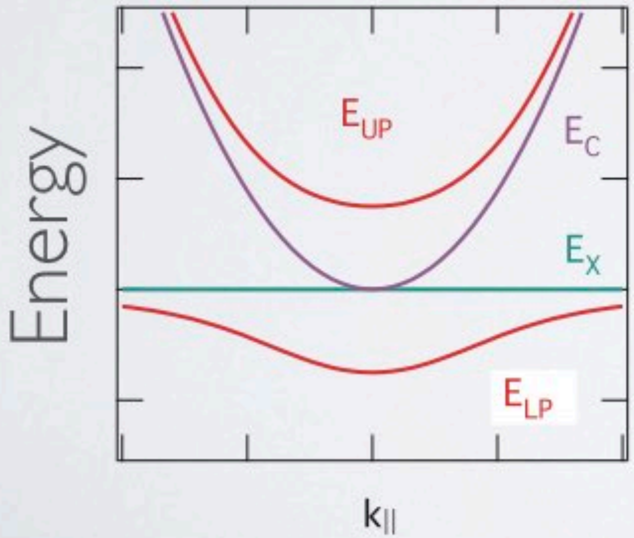


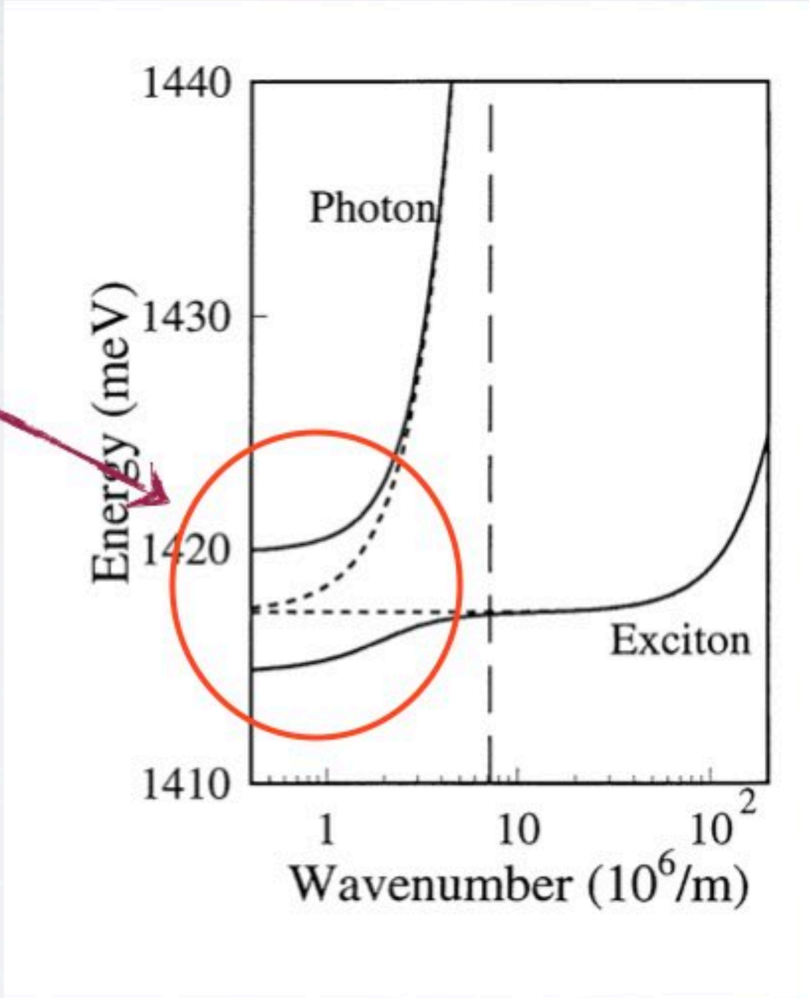
image after: M. S. Skolnick et al. Semicond. Sci. Technol. 13, 645 (1998)

Exciton polaritons

dispersion - energy-wave vector dependence

experimental access





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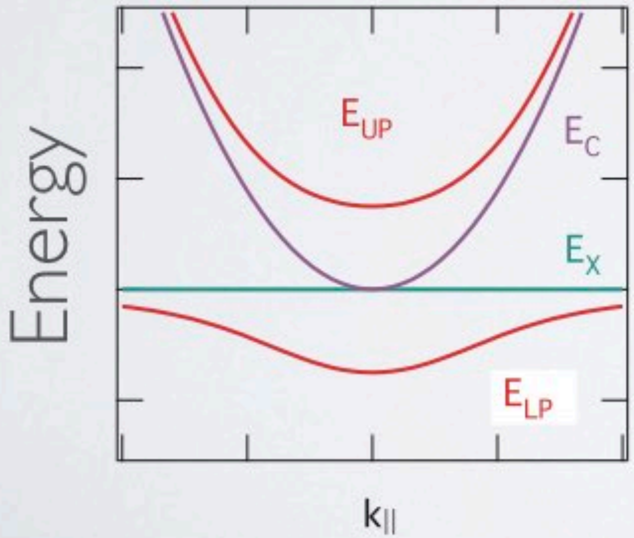
Exciton polaritons
dispersion - energy-wave vector dependence



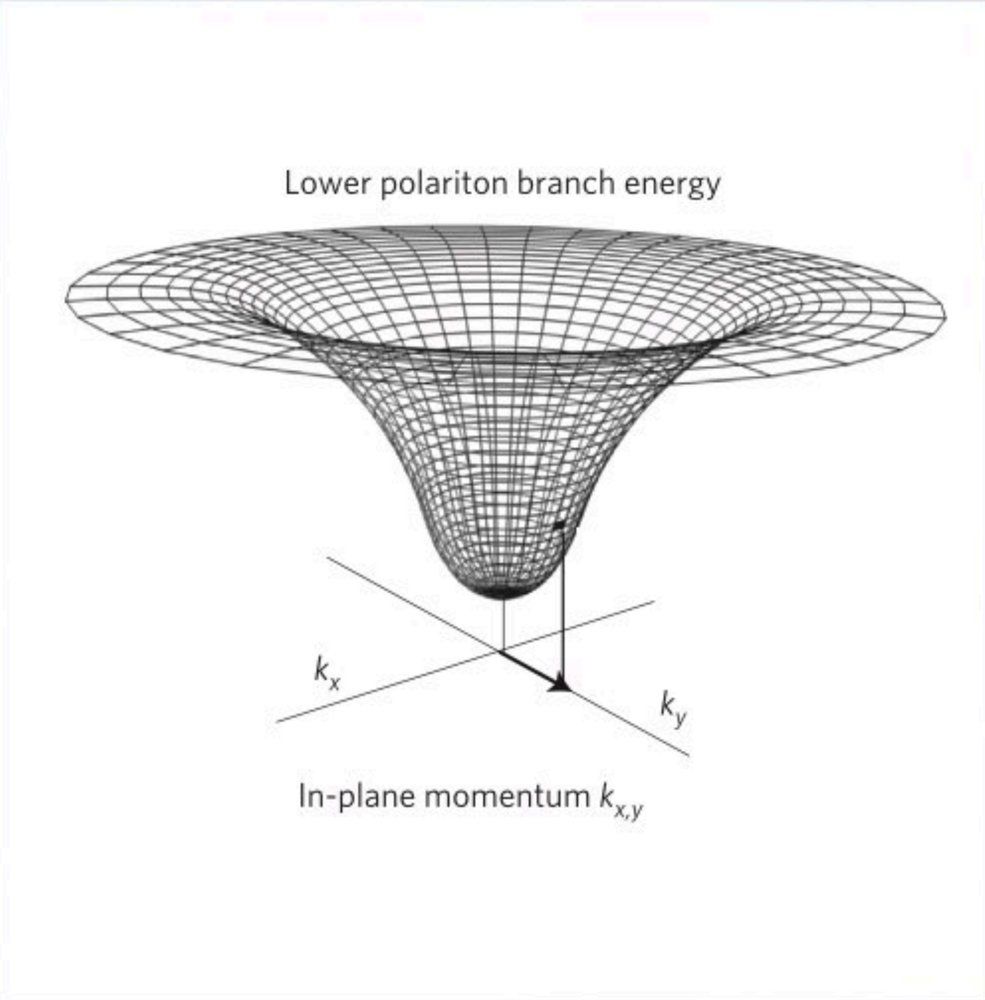
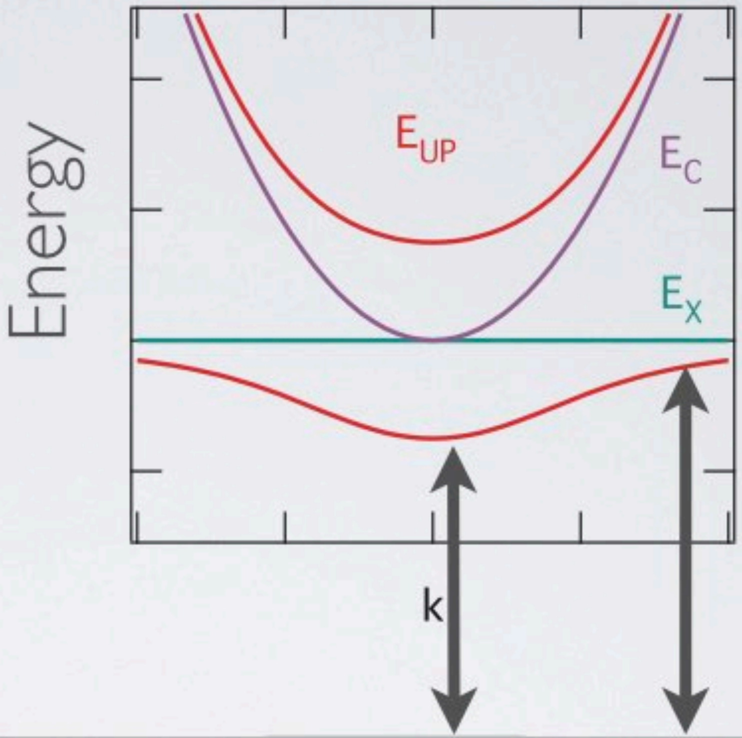


image after: D. Sanvitto et. al.

Exciton polaritons

complex properties due to
the dispersion shape



lifetime (2 orders of magnitude)	**short (1ps)**	**long (100ps)**
effective mass	**small**	**large**
relaxation along LP branch	**slow**	**rapid**
polariton DOS (4 orders of magnitude)	**small**	**large**

BEC OF POLARITONS - BEC OF ATOMS

CHARACTERISTICS:

- Solid state system
- Disordered environment
- Long range spatial coherence
- Non equilibrium
- Steady state is characterized by incoming and outgoing flow of particles
- Emitted light is linearly related to polaritons

	atoms	**polaritons**
m	Rb: $10^4 m_e$	$10^{-4} m_e$
T	10^{-7}K	>100K
N	$10^{14}/cm^3$	$<10^{11}/cm^2$
t	∞	1 ps

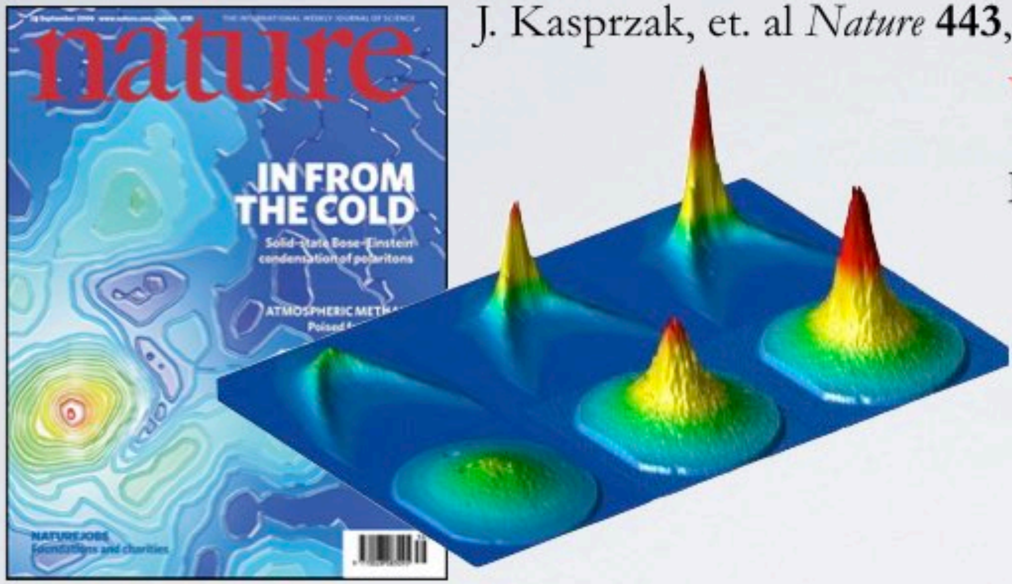
FUNDAMENTAL DIFFERENCES:

- Condensation in a disordered medium
- Interacting Bose gas
- **Out of equilibrium**
- **Non-isolated system**

Sources: J. Kasprzak *et al* Bose–Einstein condensation of exciton polaritons. Nature 443: 409-414, (2006)
V. Savona *et al.* Optical Properties of Microcavity Polaritons. *Phase Transitions* **68** 169-279 (1999) and
V. Savona *et al.* Quantum-Well Excitons in Semiconductor Microcavities - Unified Treatment of Weak and Strong-Coupling Regimes. *Sol. St. Com.* **93** 733-739 (1995)

CONDENSATE in solid-state ? yes!!!

J. Kasprzak, et. al *Nature* **443**, 409 (2006)

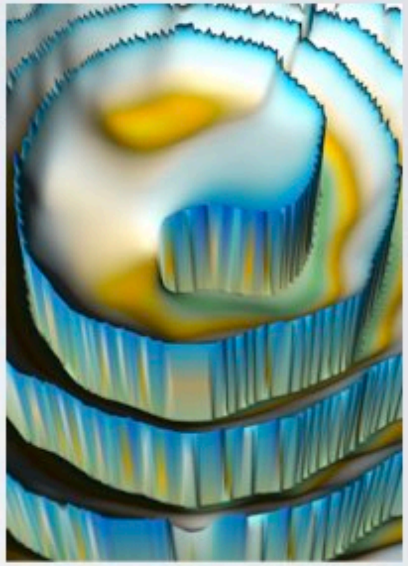


VORTEX, HALF-VORTEX !

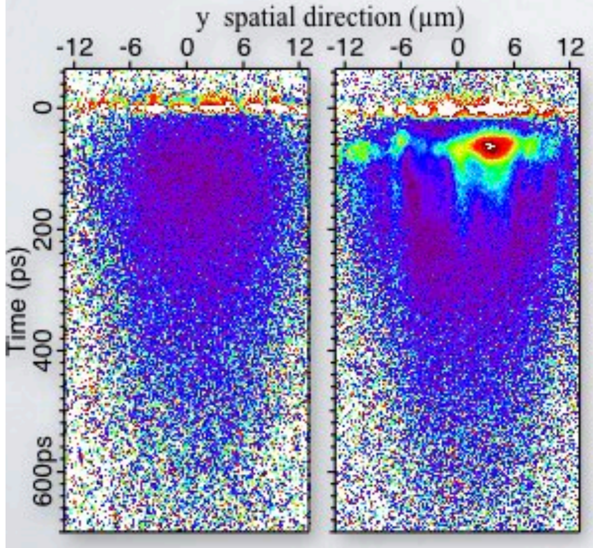
K.G.Lagoudakis, et al. *Nature Phys.* **4**, 706 (2008)

K.G.Lagoudakis, et al. *Science* **326**, 974 (2009)

phase rotation of 2π around a vortex core

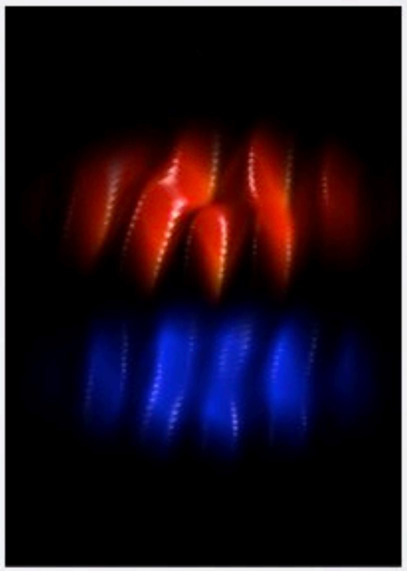


LONG RANGE ORDER



build up of quantum phase coherence

G. Nardin, B. Pietka et al. *PRL* 103, 256402 (2009)



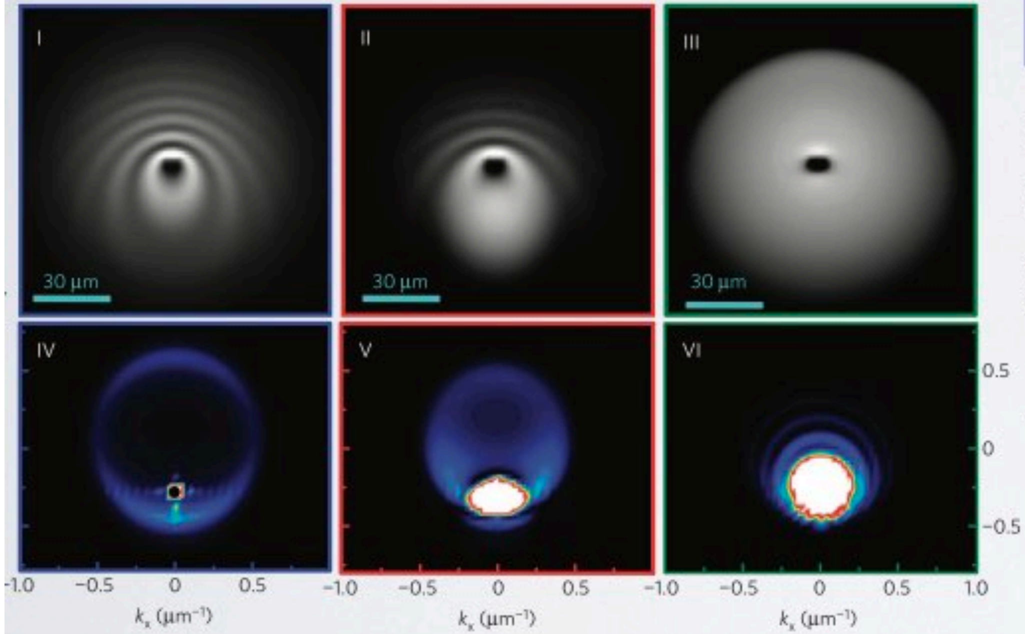
interference of polariton condensate for two polarization of light:
$\sigma+$ *: fork-like dislocation*
$\sigma-$ *: no dislocation*

SUPERFLUIDITY OF EXCITON POLARITONS

nature physics
LETTERS
PUBLISHED ONLINE: 20 SEPTEMBER 2009 | DOI: 10.1038/NPHYS1364

Superfluidity of polaritons in semiconductor microcavities

Alberto Amo[1]★, Jérôme Lefrère[1], Simon Pigeon[2], Claire Adrados[1], Cristiano Ciuti[2], Iacopo Carusotto[3], Romuald Houdré[4], Elisabeth Giacobino[1] and Alberto Bramati[1]★

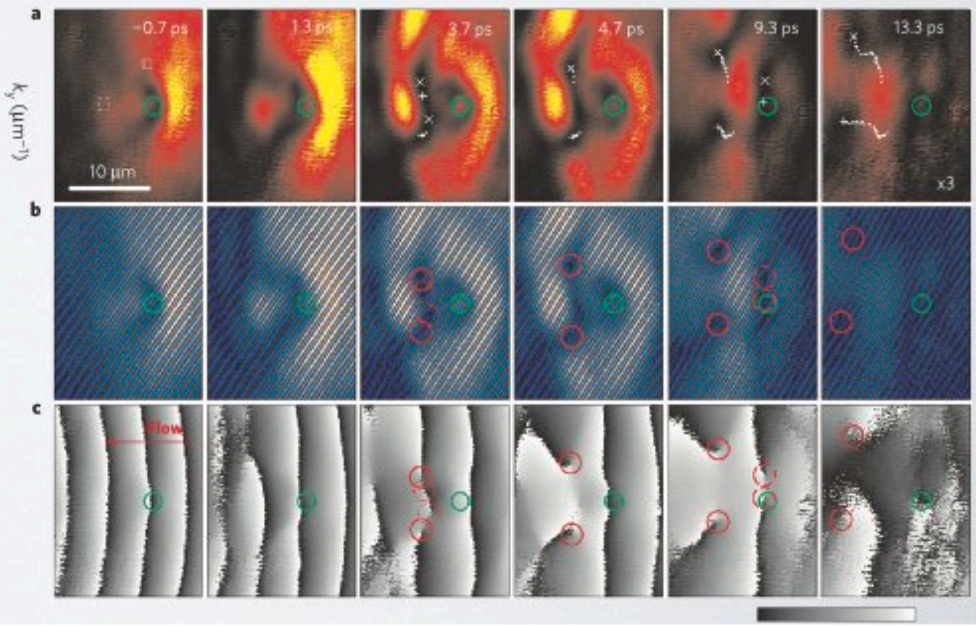


quantum turbulences

nature physics
ARTICLES
PUBLISHED ONLINE: 3 APRIL 2011 | DOI: 10.1038/NPHYS1959

Hydrodynamic nucleation of quantized vortex pairs in a polariton quantum fluid

Gaël Nardin★, Gabriele Grosso, Yoan Léger, Barbara Piętka[†], François Morier-Genoud and Benoît Deveaud-Plédran



Movie on exciton-polaritons form
Sheffield Univ.

https://www.youtube.com/watch?v=sWmvZ0IGrsU

Please have a look at the lectures: Physics@FOM Veldhoven

2016, Peter Littlewood - Polariton condensation

https://www.youtube.com/watch?v=Ff1GsWSahps

https://www.youtube.com/watch?v=pQoMVsGaZgQ

Synchronized metronomes:
https://youtu.be/fmjY3UIgcwA