LECTURE 6

Quantum vortices
Exciton-polariton superfluidity

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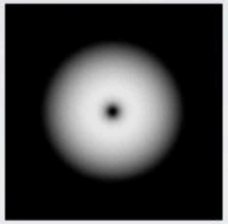
Quantum vortices
what quantity should be observed

EXPERIMENTAL QUANTITIES

- Zero particle density at the vortex core (no particles in the centre)



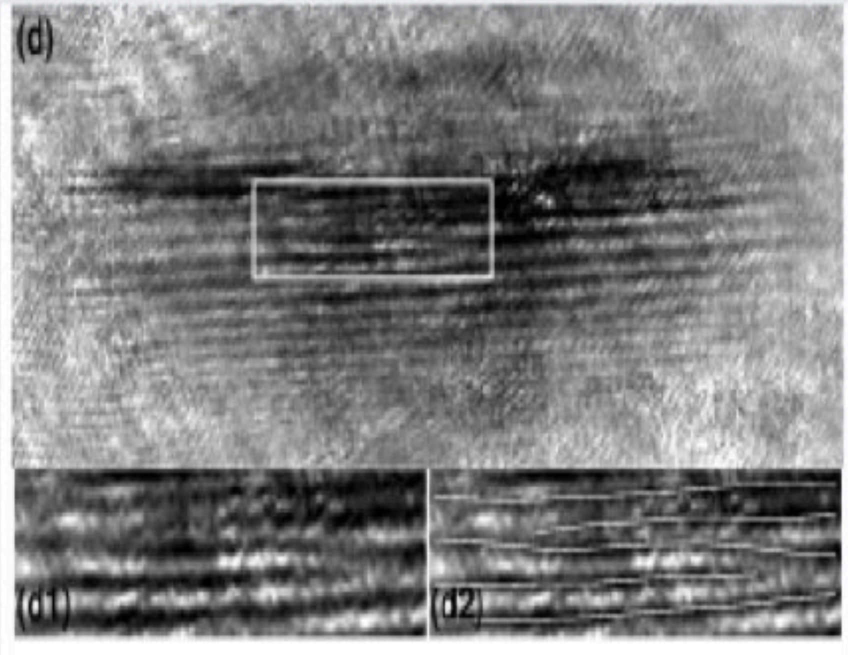
- Phase: multiplicity of 2π around a vortex core

EXPERIMENTAL METHOD TO DETECT THE PHASE

Source: E. L. Bolda *et al.* Detection of Vorticity in Bose-Einstein Condensed Gases by Matter-Wave Interference *Phys.Rev.Lett.* **81**, 5477 (1998).

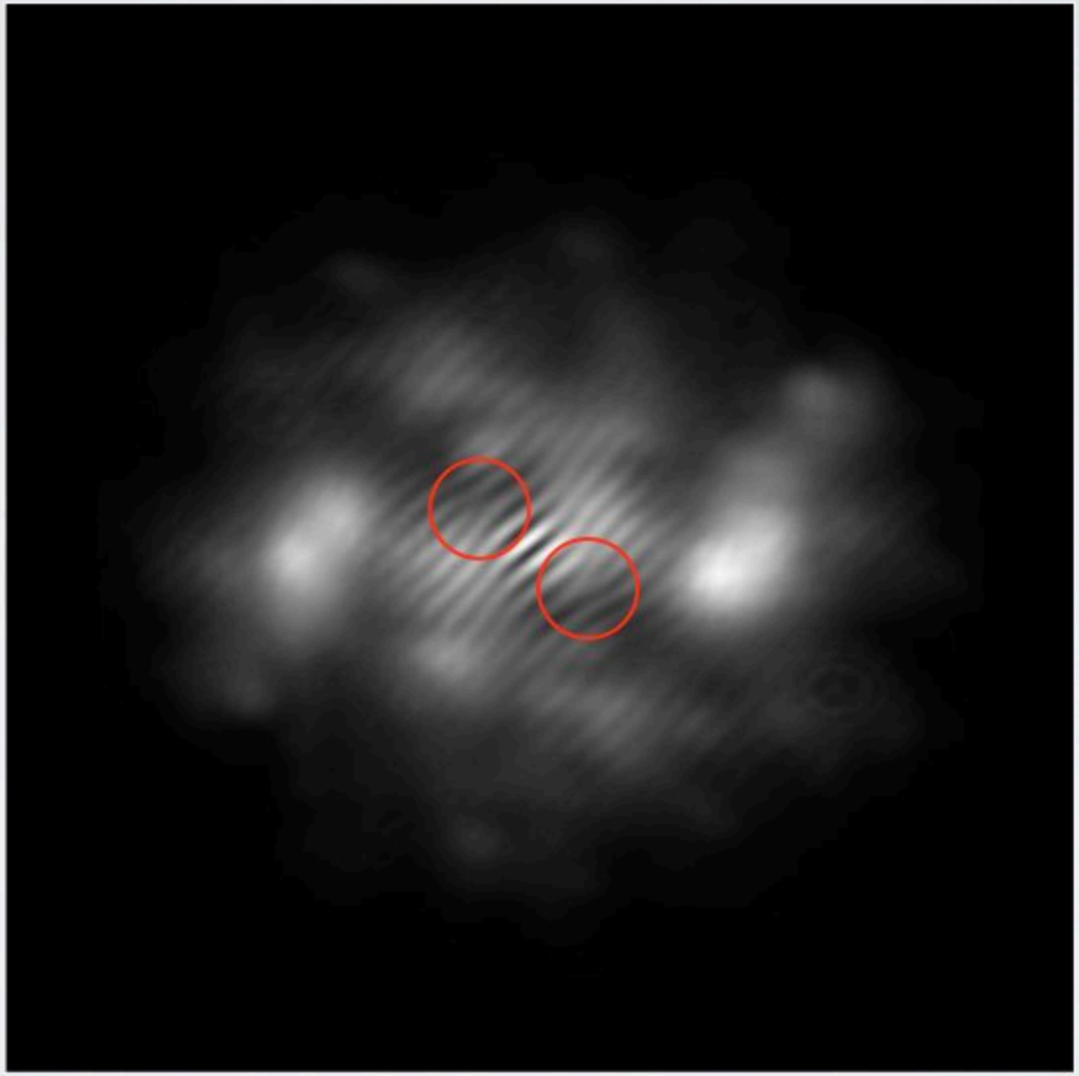
Quantum vortices
interferogram

PHASE DETECTION IN ATOMIC BEC



Observation of vortices in BEC
Inouye *et al*, PRL **87**, 080402 (2001)

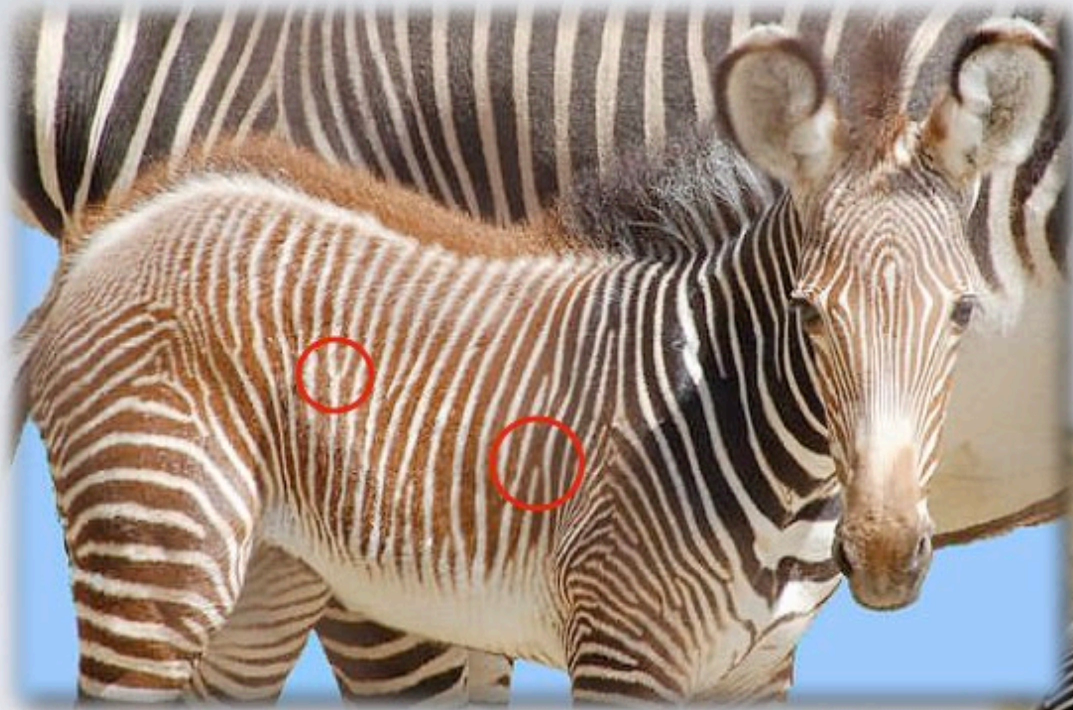
Quantum vortices
interferogram



images: K. Lagoudakis, EPFL

Are there other types of quantum vortices ?

INSPIRED BY NATURE



natural macroscopic
quantum vortices

Quantum vortices
interferogram

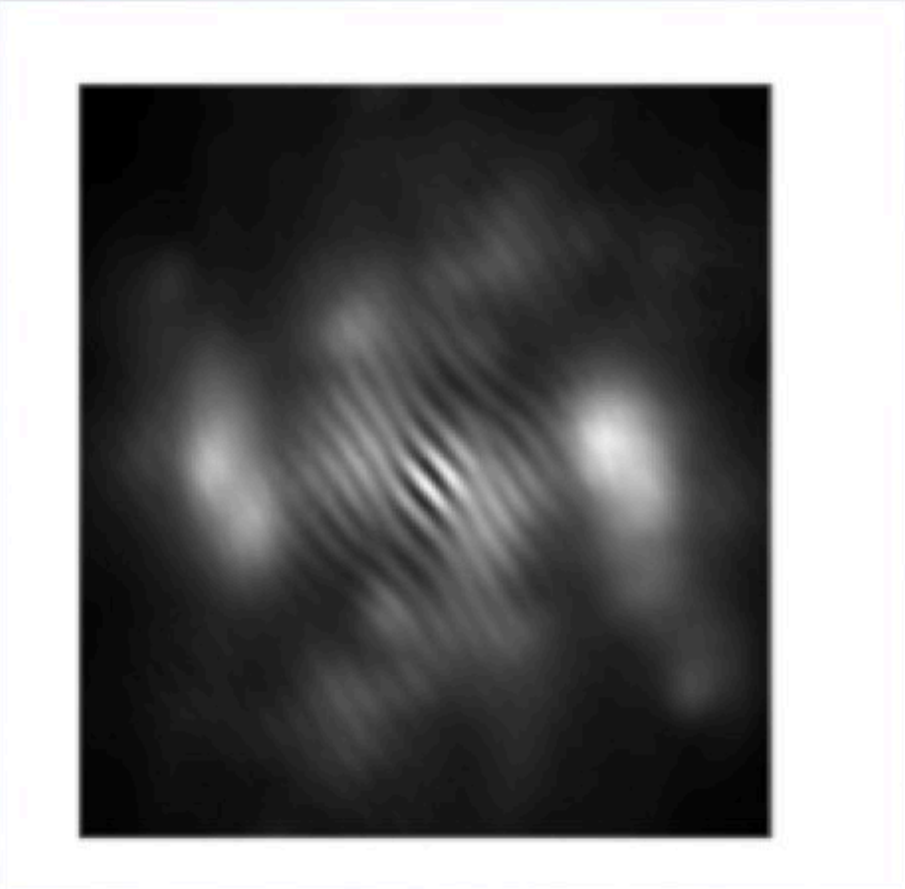
Few more words about interferogram

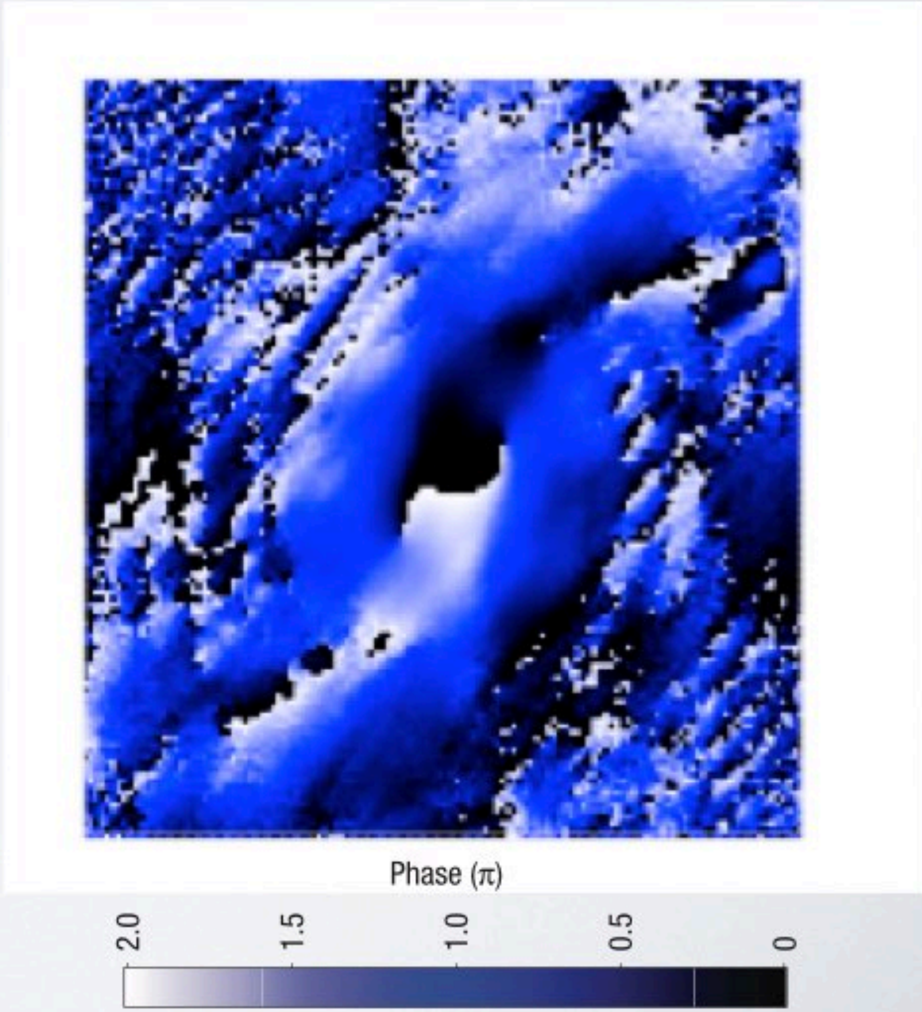
1. measure interferogram
2. calculate FFT (fast Fourier transform)
3. distinguish amplitude and phase

Quantum vortices
interferogram

Extraction of phase from interferogram

- The application of fringes is a key feature for the extraction of the phase from an interferogram
- The fork-like dislocation indeed gives a 2π winding of the phase





K. Lagoudakis et al., Nat. Phys. 4, 706 (2008)

Quantum vortices
interferogram

Extraction of phase from interferogram

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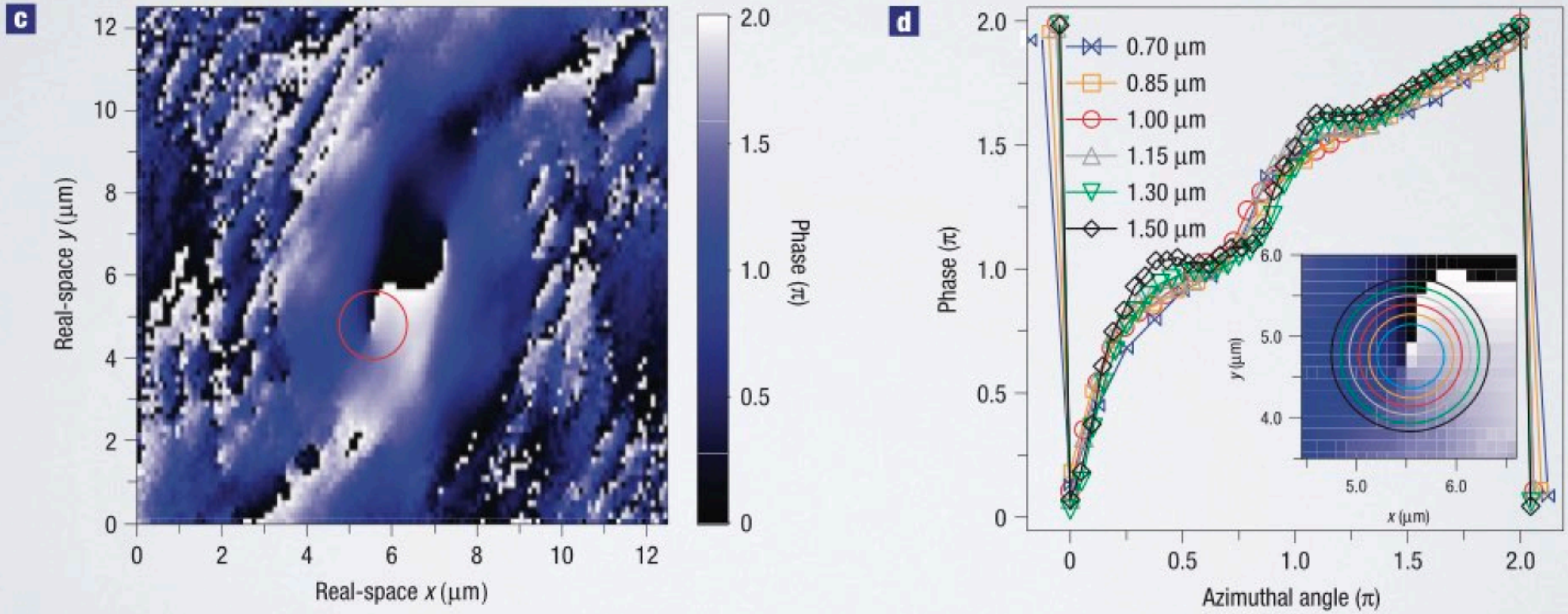
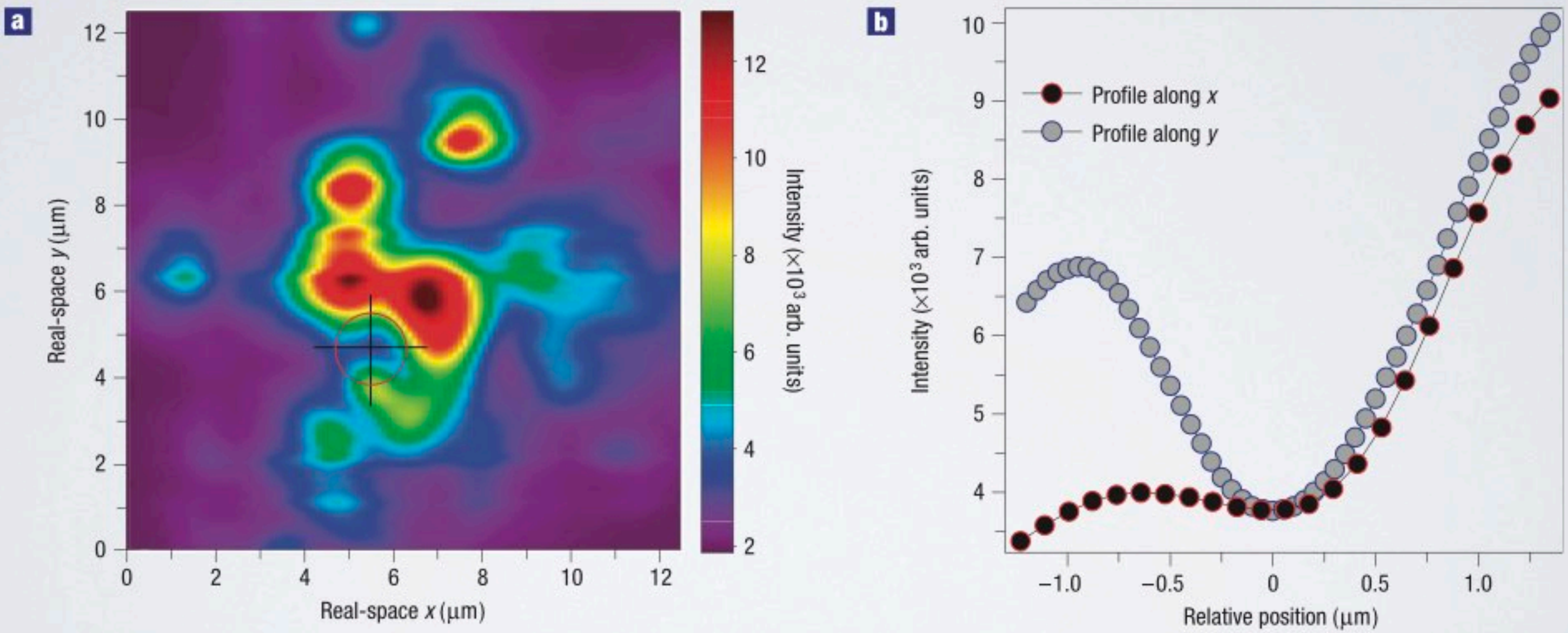


K. Lagoudakis et al., Nat. Phys. 4, 706 (2008)

Quantum vortices
interferogram

Extraction of amplitude from interferogram

- The minimum density is indeed found at the vortex core



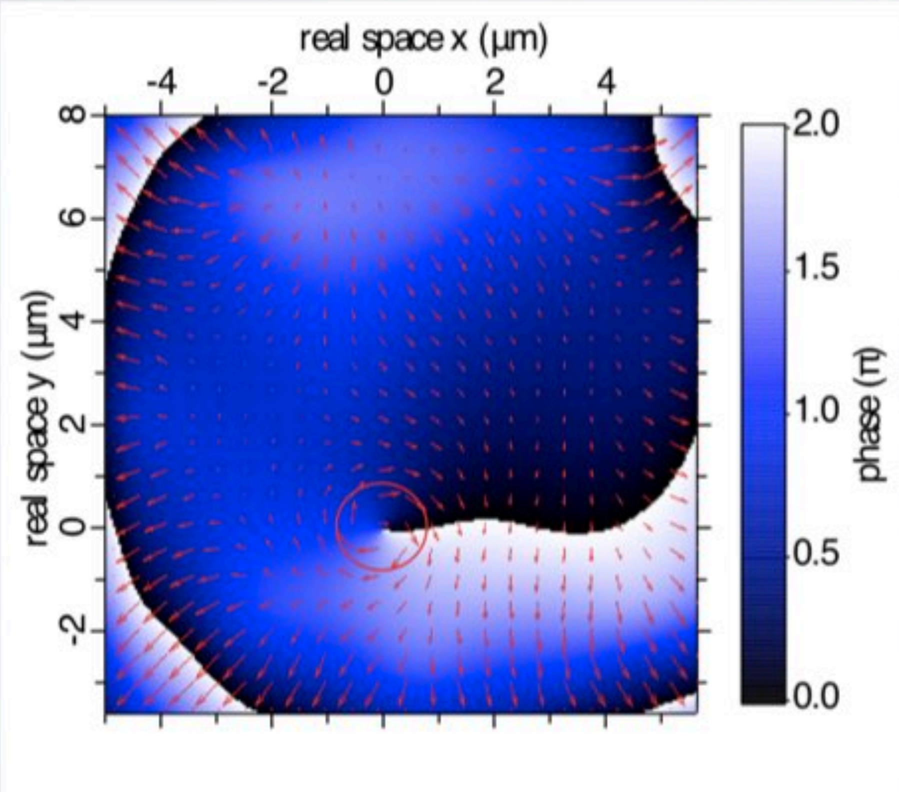
K. Lagoudakis et al., Nat. Phys. 4, 706 (2008)

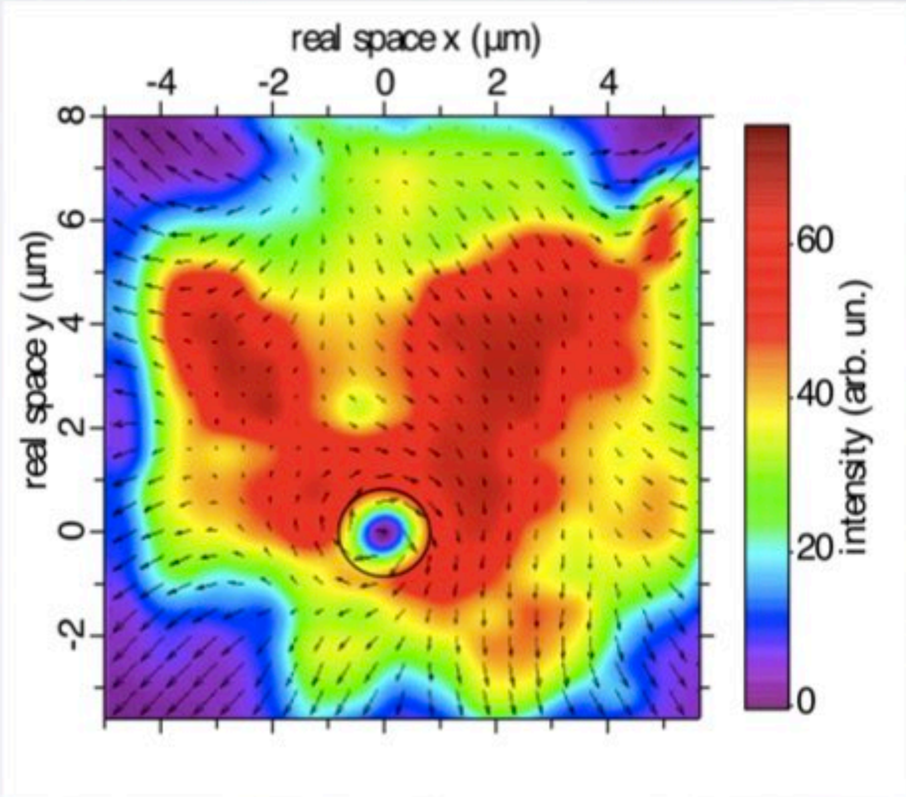
Quantum vortices

theory behind the vortex appearance

Michel Wouters et al.

- For given disorder potentials vortices appear as solutions of the system
- Phase winding and zero density are reproduced





K. Lagoudakis et al., Nat. Phys. 4, 706 (2008)

Short summary - puzzle

What is it ?

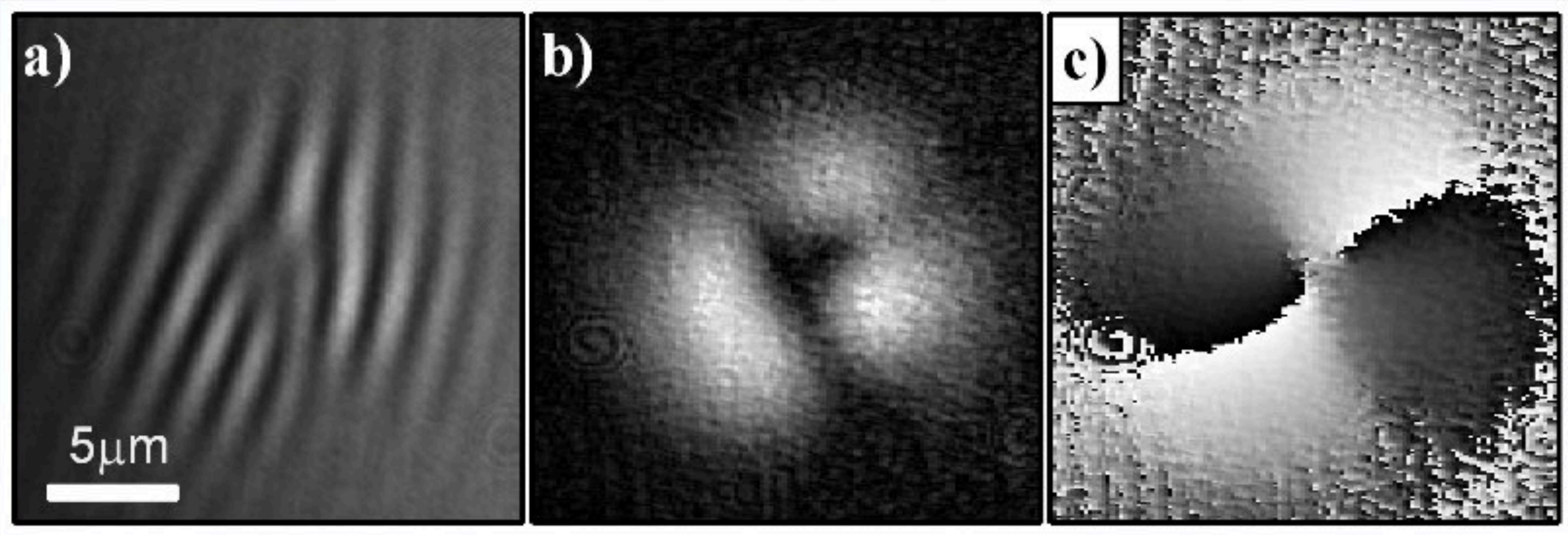
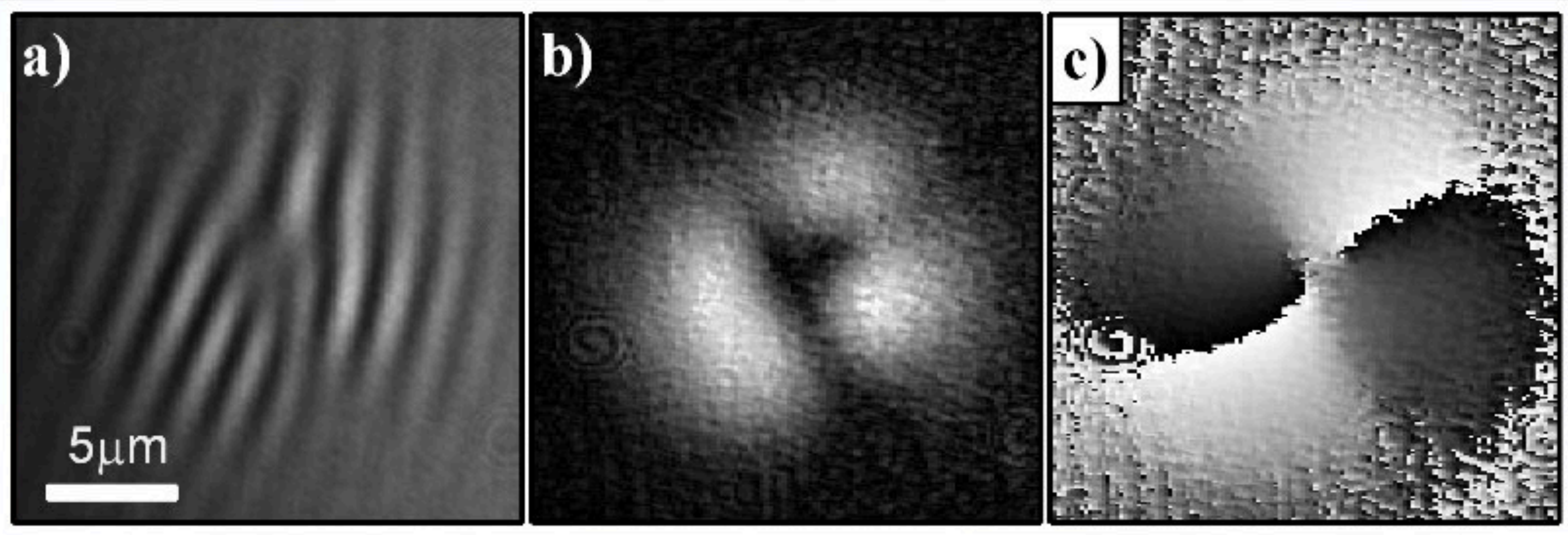


image: G. Nardin, EPFL

Vortex with topological charge $\ell = 2$

interferogram amplitude phase



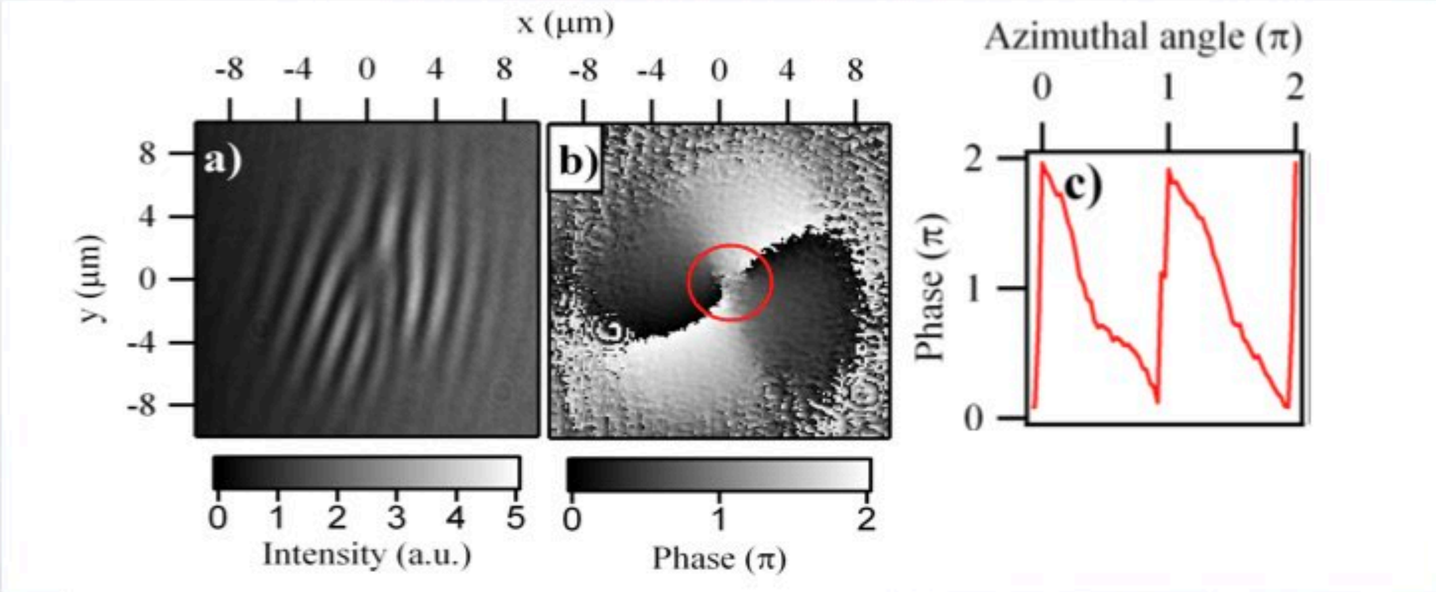


image: G. Nardin, EPFL

Quantum vortices
& link to superfluidity

Vortices!!

- The two characteristic features (zero density and quantised phase) are verified without ambiguity
- Very strong indication of superfluidity

Can superfluidity be claimed in polariton BEC?

Control over creation and annihilation of vortices would be the demonstration of superfluidity

- Need to find a way to create vortices

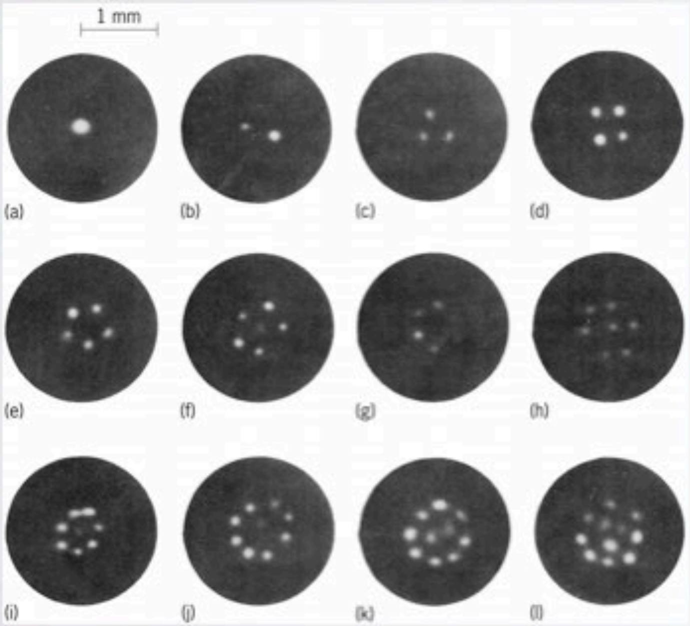
Quantum vortices in atomic BEC

lattice of vortices

rotating asymmetric traps and/or rotating potential ⟶ vortex lattice

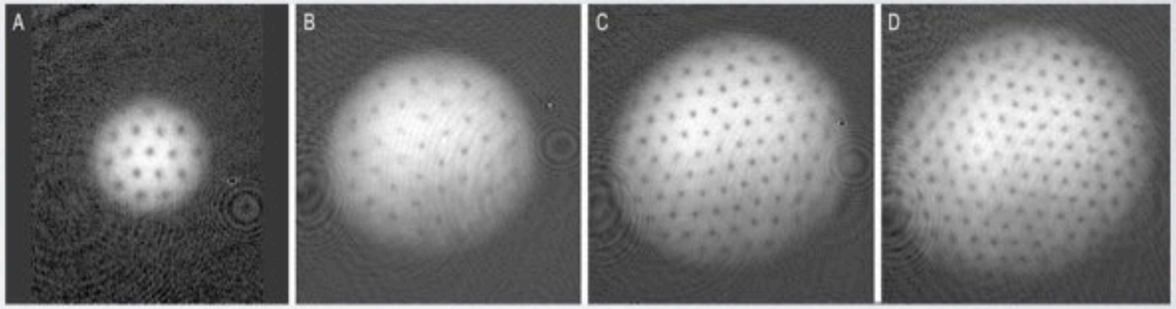
- ^{4}He below the λ point

1950s Hall and Vinen



E. J. Yarmchuk, PRL 43, 1979

- atomic BECs

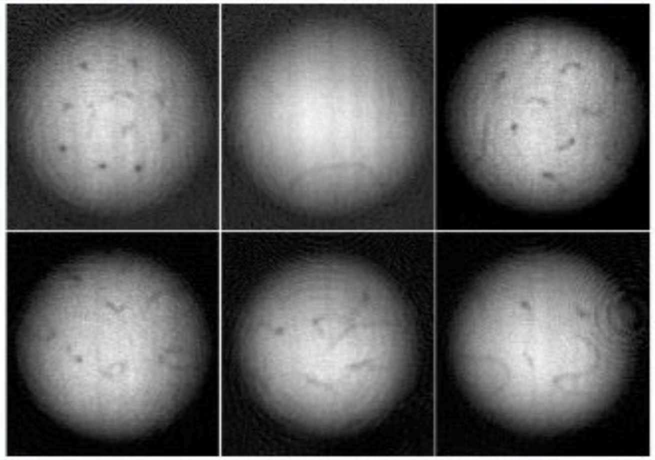


Vortex Lattices in Bose-Einstein Condensates

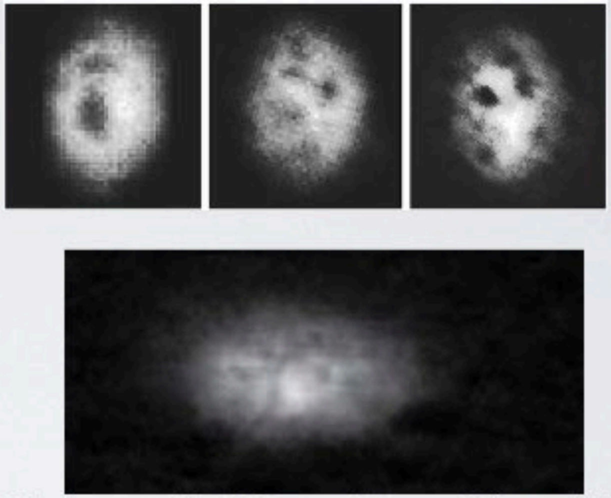
Abo-Shaeer et al., Science 292, 476 (2001)

Quantum vortices in atomic BEC

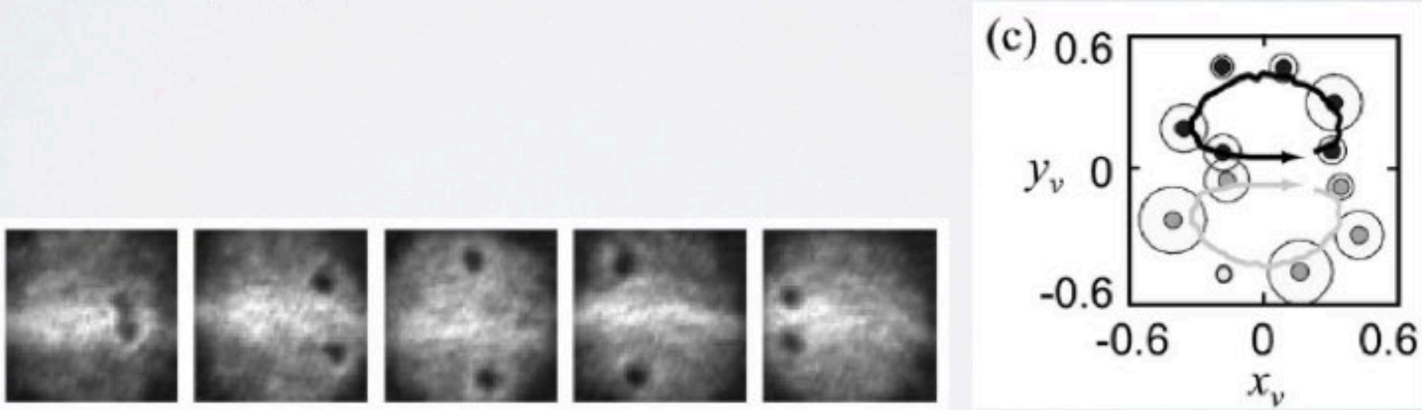
lattice of vortices - turbulent vortices - vortex dipoles



Vortex nucleation in a stirred BEC
Raman *et al*, PRL **87**, 210402 (2001)



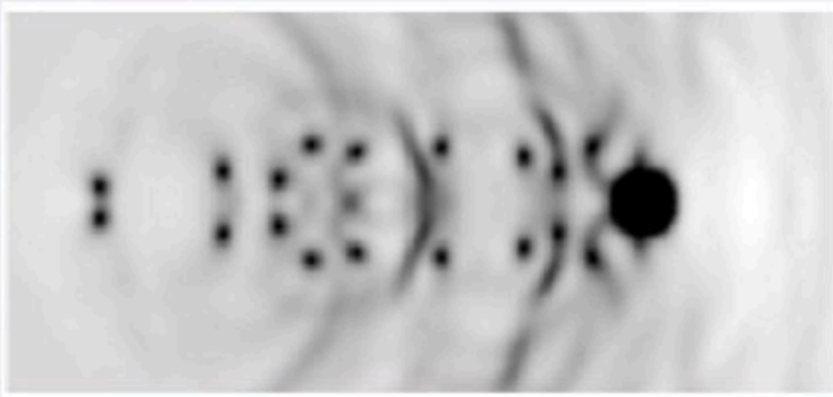
E.A.L. Henn et al., J Low Temp Phys 158, 435 (2010)



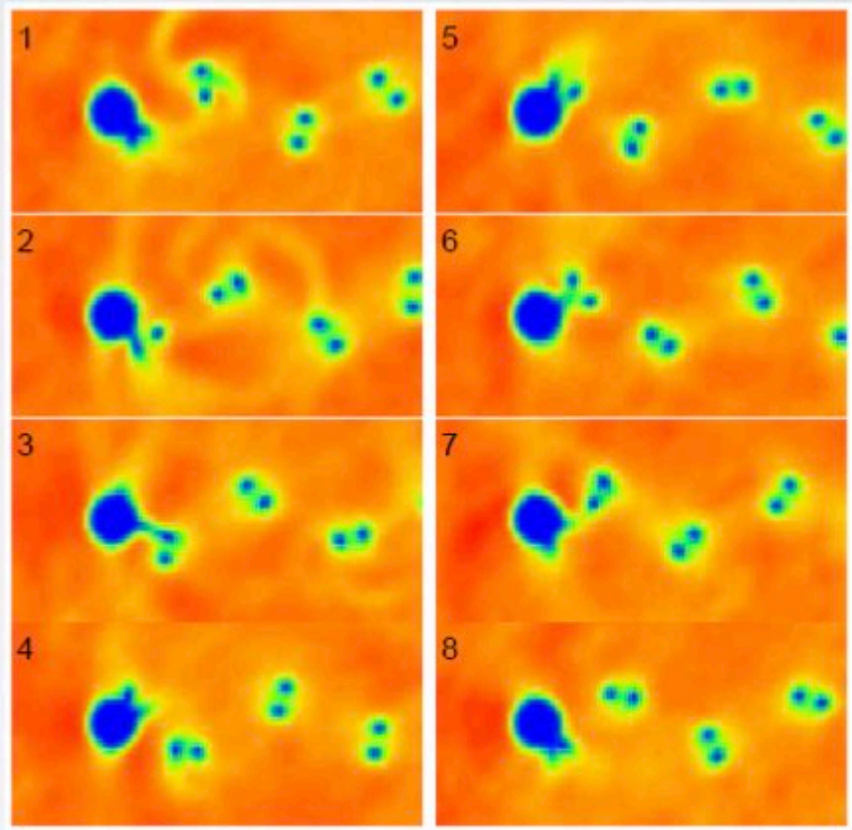
Observation of vortex dipoles in BEC, Neely *et al*, PRL **104**, 160401 (2010)

Quantum turbulences

great interest in the physics community

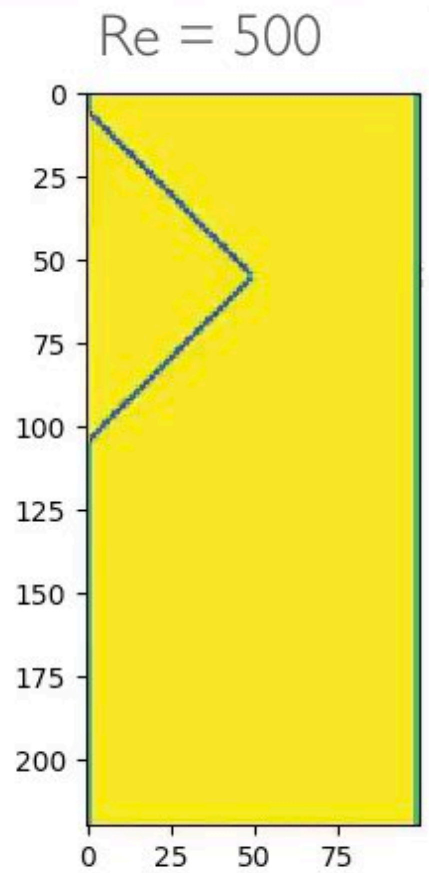


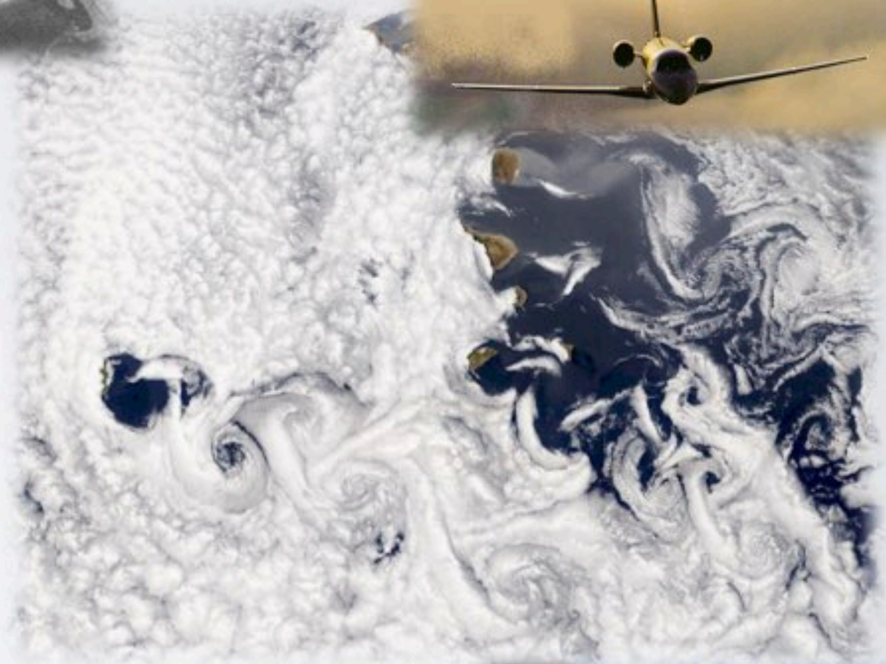
Vortex shedding and drag in a BEC
Pojawianie się wirów i siły nośnej w kondensacie Bosego - Einsteina
Winiecki *et al*, J. Phys. B **33**, 4069 (2000)



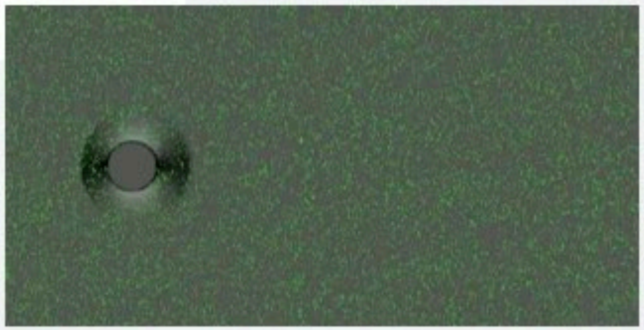
Bénard- Von Kármán street in a BEC
Ścieżka wirów von Karmana w kondensacie Bosego - Einsteina
Sasaki *et al*, PRL **104**, 150404 (2010)

VORTEX STREETS IN CLASSICAL FLUIDS





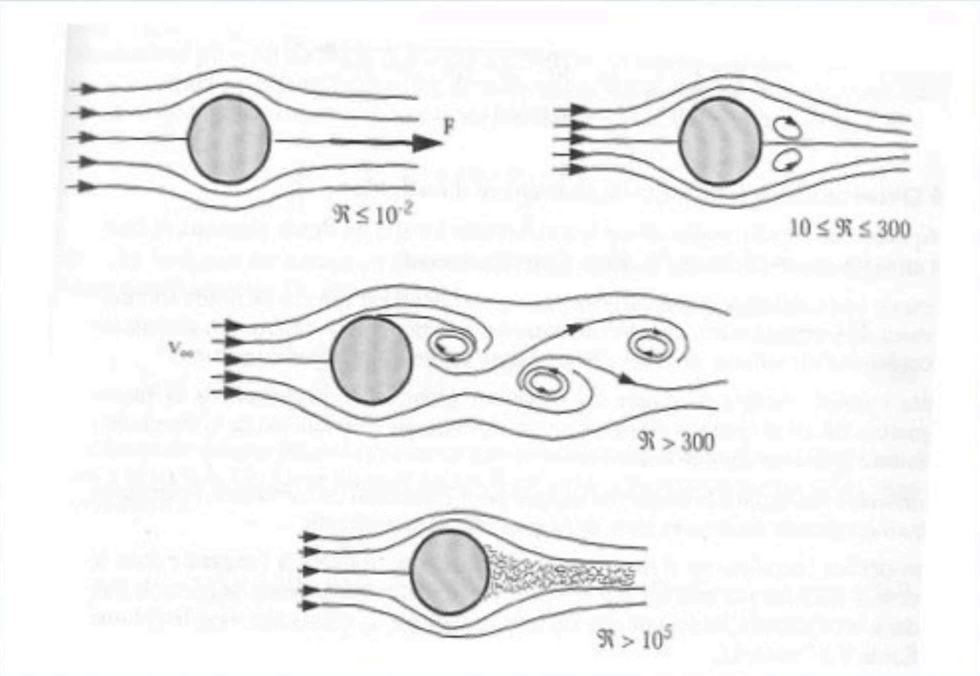
Simulation: Piotr Stawicki



VON KARMAN VORTEX STREET

image source:http://www.youtube.com/watch?v=qpDKRrS9aqE

VORTICES IN CLASSICAL FLUIDS



R_e

UNSEPARATED FLOW

A PAIR OF VORTICES FIXED IN A WAKE

BENARD-VON KARMAN VORTEX STREET - LAMINAR VORTEX STREET

TURBULENT FLOW

$$R_e = \frac{Fluid\_Velocity \cdot Obstacle\_diameter}{Kinematic\_viscosity} = \frac{predkosc \cdot promien}{lepkosc}$$

Reynolds number sets the border between the laminar flow and the different regimes of turbulent flow

Quantum vortices
& link to superfluidity

Quantum vortices have two characteristic features:

- zero density in the vortex core
- quantised phase $\Delta\phi = 2\pi l$

Very strong indication of superfluidity

Superfluidity in polariton BEC:

- Fluid exhibiting zero viscosity
- Creation and annihilation of vortices would be the demonstration of superfluidity

Superfluidity

general concepts

THE MOST SPECTACULAR EXAMPLES

- ^{4}He below λ point
- atomic BECs
- superfluid polaritons

- superfluid

! ZERO VISCOSITY!

$$\cancel{R_e} = \frac{Fluid\_Velocity \cdot Obstacle\_diameter}{\cancel{Kinematic\_viscosity}}$$ ~~(LEPKOŚĆ)~~

- consequence of a BEC state
- well defined phase in the wavefunction

$$\psi(\mathbf{r}) = \sqrt{n_0(\mathbf{r})}e^{i\theta(\mathbf{r})}$$

„Giant matter wave"

Superflow arrises whenever the condensate phase, $\theta(\mathbf{r})$ varies in space.

Superfluidity
general concepts

THE MOST SPECTACULAR EXAMPLES

- ^{4}He below λ point
- atomic BECs
- superfluid polaritons

! ZERO VISCOSITY!

- superfluid

$$\cancel{R_e} = \frac{Fluid\_Velocity \cdot Obstacle\_diameter}{\cancel{Kinematic\_viscosity \ (LEPKOŚĆ)}}$$

- DETERMINED FOR SUBSONIC SPEEDS

$$v < c_s$$

- for $v > c_s$ czerenkov waves - shock waves

⟶ Landau superfluid criterion

Superfluidity
current of particles

Condensate wave function

$$\psi(\mathbf{r}) = \sqrt{n_0(\mathbf{r})}e^{i\theta(\mathbf{r})}$$

current density defines the number of particles flowing per unit area per second:

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{2mi}\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right]$$

using $\psi(\mathbf{r})$ and the product rule for differentiation:

$$\nabla\psi(\boldsymbol{r}) = e^{i\theta}\nabla\sqrt{n_0} + i\sqrt{n_0}\,e^{i\theta}\nabla\theta$$
$$\nabla\psi^*(\boldsymbol{r}) = e^{-i\theta}\nabla\sqrt{n_0} - i\sqrt{n_0}\,e^{-i\theta}\nabla\theta$$

multiplying by $\psi^*(\mathbf{r})$ and $\psi(\mathbf{r})$ respectively:

$$\psi^*(\boldsymbol{r})\nabla\psi(\boldsymbol{r}) = \sqrt{n_0}\,e^{-i\theta}e^{i\theta}\nabla\sqrt{n_0} + \sqrt{n_0}\,e^{-i\theta}i\sqrt{n_0}\,e^{i\theta}\nabla\theta = \sqrt{n_0}\nabla\sqrt{n_0} + in_0\nabla\theta$$
$$\psi(\boldsymbol{r})\nabla\psi^*(\boldsymbol{r}) = \sqrt{n_0}\,e^{i\theta}e^{-i\theta}\nabla\sqrt{n_0} - \sqrt{n_0}\,e^{i\theta}i\sqrt{n_0}\,e^{-i\theta}\nabla\theta = \sqrt{n_0}\nabla\sqrt{n_0} - in_0\nabla\theta$$

Superfluidity

current of particles

$$\boldsymbol{j}(\boldsymbol{r}) = \frac{\hbar}{2mi}(\sqrt{n_0}\nabla\sqrt{n_0} + in_0\nabla\theta - \sqrt{n_0}\nabla\sqrt{n_0} + in_0\nabla\theta)$$

$$\boldsymbol{j}(\boldsymbol{r}) = \frac{\hbar}{2mi}2in_0\nabla\theta$$

$$\boldsymbol{j}(\boldsymbol{r}) = \frac{\hbar}{m}n_0\nabla\theta$$

since the net current of particles equals a density times a velocity:

$$\boldsymbol{j}(\boldsymbol{r}) = n_0\mathbf{v}$$

superfluid velocity:

$$\mathbf{v} = \frac{\hbar}{m}\nabla\theta$$

Superfluidity

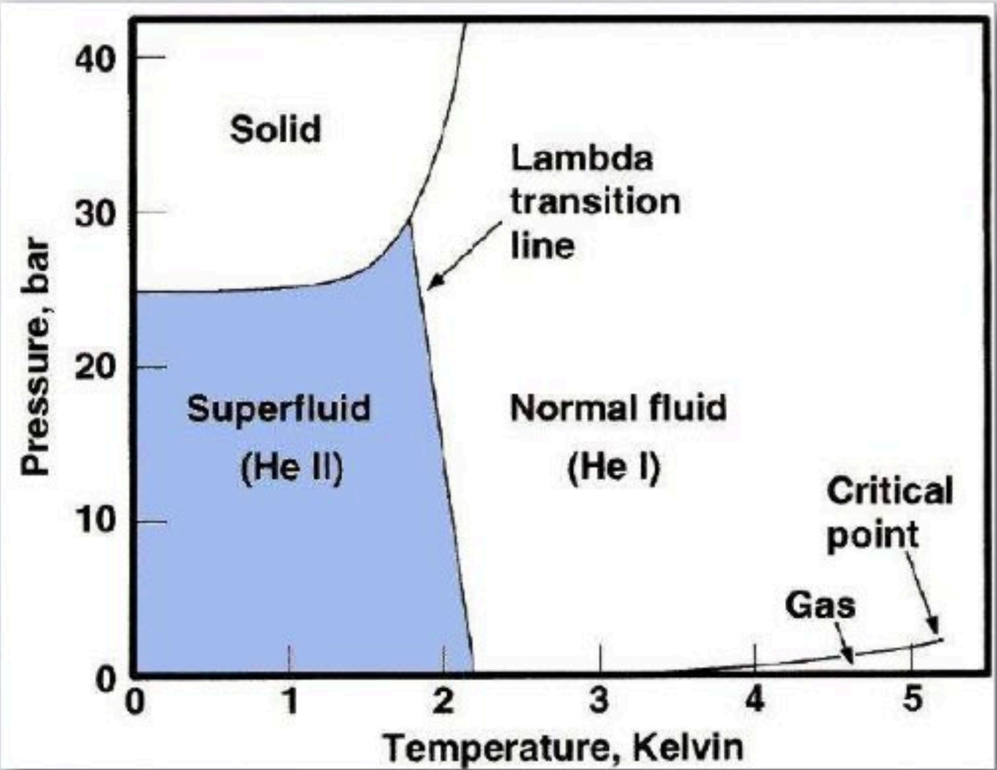
history begins with liquid helium

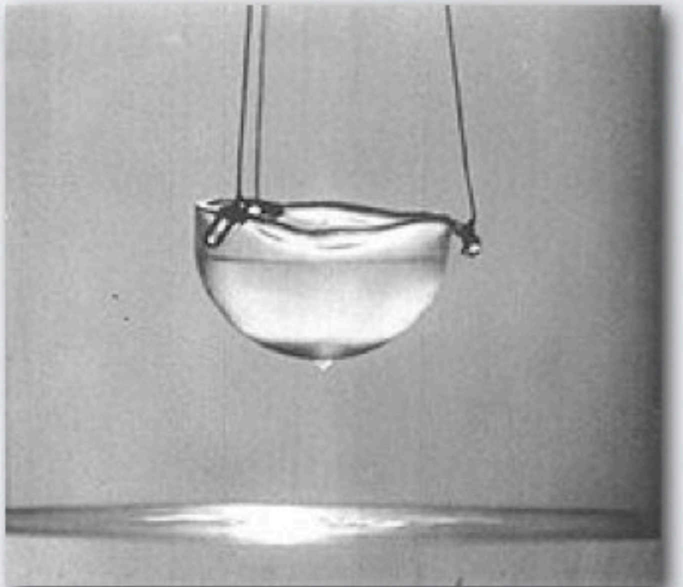
P. Kapitza, Nature 141, 74 (1938)

J. F. Allen, A. D. Misener, Nature 141, 75 (1938)

Nobel Prize, 1978

Liquid ^{4}He could flow through narrow capillaries without any resistance due to fluid viscosity





tip: search youtube for films with this phase transitions

Superfluidity
two-fluid model

Andronikashvilli experiment: stack of a circular disc immersed in a fluid can be made to undergo torsional oscillations that depends on the torsional stiffness and inertia of the disc.

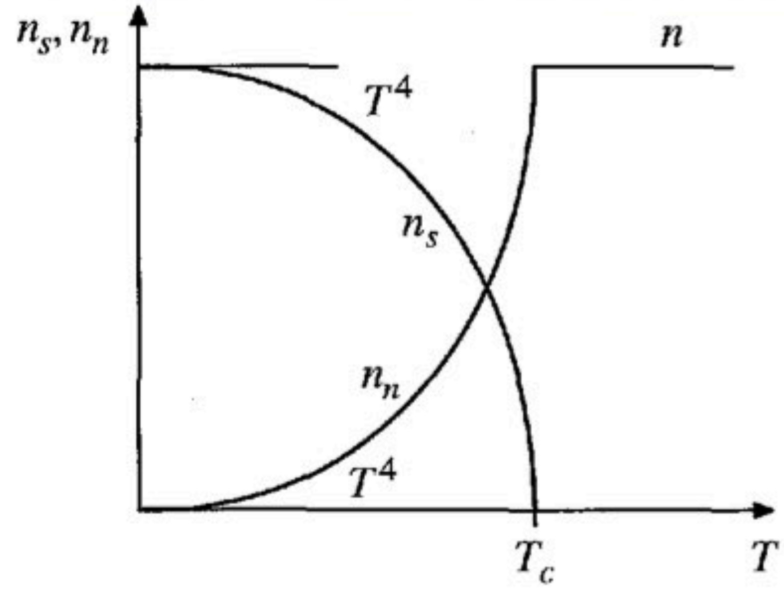
In a viscous fluid a disk will be dragged which will contribute to the moment of inertia of the disc

Total particle number:

$$n = n_s + n_n$$

superfluid component flow with zero viscosity

normal component act as a viscous fluid



Temperature dependence of the superfluid and normal components of liquid He II, as measured, for example, from in the torsional oscillation disk stack experiment.

Superfluid density is defined as mass density of the superfluid part of the fluid: $\rho_s = mn_s$

Current density:

$$\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n = n_s \mathbf{v}_s + n_n \mathbf{v}_n$$

Superfluidity

comment on condensate density and superfluid density

Condensate density n_0

defined from one-particle density matrix or from the delta function peak in the momentum distribution

Superfluid density n_s

defined from two-fluid model of superflow

$$\mathbf{j}_s = n_s \mathbf{v}_s = n_s \hbar (\nabla \theta)/m$$

Condensate density is the property of the ground state, the superfluid density is a property of a superflow.

At $T = 0$ all particles participate in superflow $n_s = n$

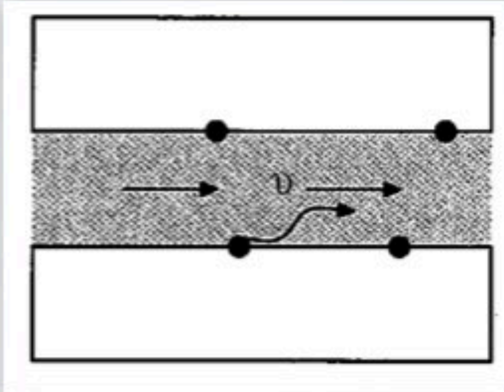
In a noninteracting Bose gas $n = n_0 = n_s$

Further reading: J. F. Annett „Superconductivity, superfluids and condensates"

Superfluidity
quasiparticle excitations

Why should the superfluid flow without friction ?

imagine a fluid flowing
in a narrow tube



Normal fluid
friction and viscosity due to scattering with hard walls
momentum is transferred from the fluid to the walls
particles can become excited in the fluid

A single particle with **i**nitial momentum and energy can be scattered elastically to the **f**inal state only if

$$\epsilon_f - \epsilon_i = \mathbf{v} \cdot (\mathbf{p}_f - \mathbf{p}_i)$$

If particle is initially in the condensate with $\mathbf{p} = 0$

Created **elementary excitation** is only if:

$$\epsilon(\mathbf{p}) = \mathbf{v} \cdot \mathbf{p}$$

Further reading: J. F. Annett „Superconductivity, superfluids and condensates"

Superfluidity
quasiparticle excitations

Created **elementary excitation** is only if:

$$\epsilon(\mathbf{p}) = \mathbf{v} \cdot \mathbf{p}$$

In a normal fluid the energy of an excited particle: $\epsilon(\mathbf{p}) = \frac{p^2}{2m}$

rough wall can always impart momentum to the fluid, leading to viscous friction.

Superfluid occurs if no scattering can be present and equation

$$\epsilon(\mathbf{p}) = \mathbf{v} \cdot \mathbf{p}$$

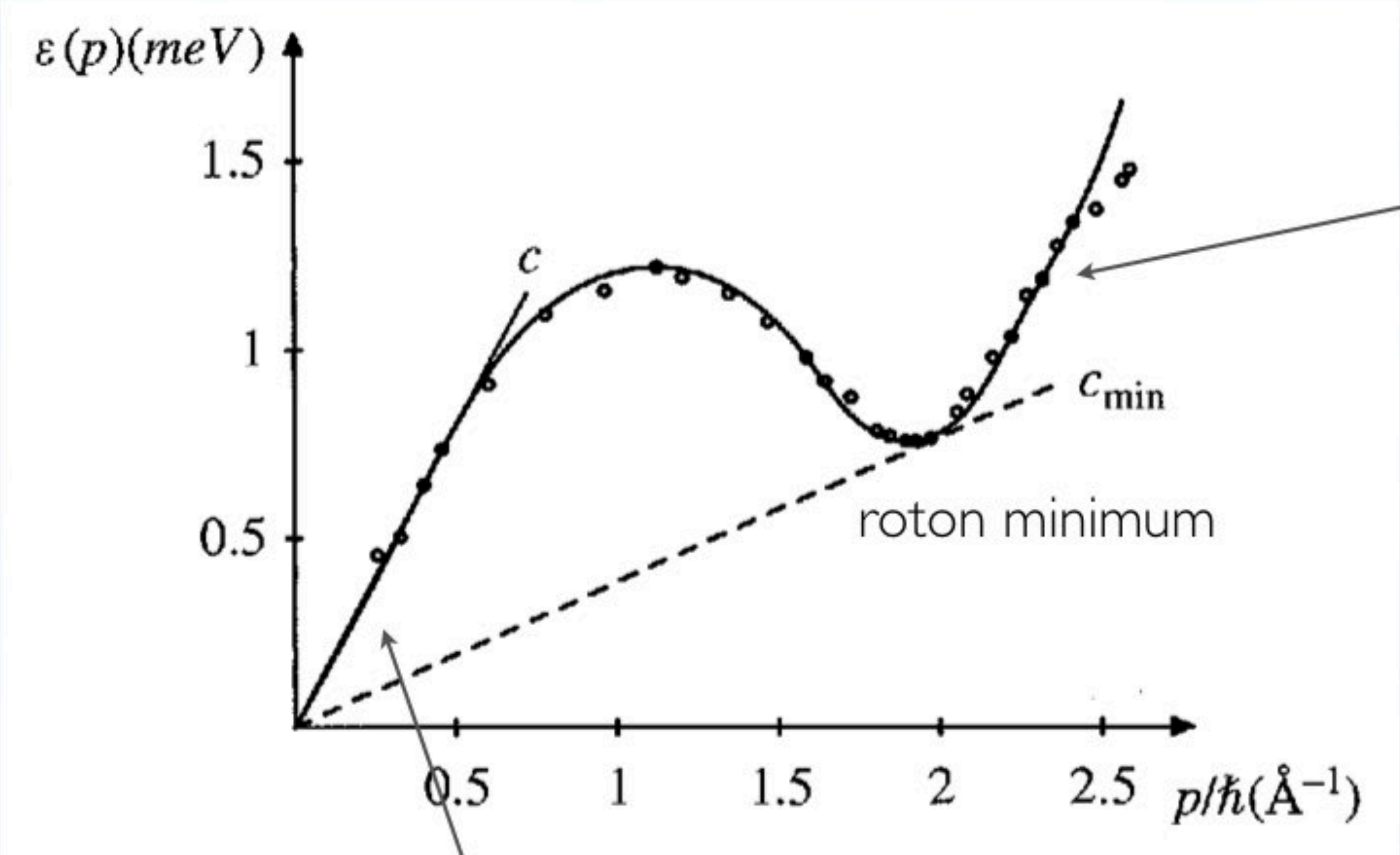
is **not** satisfied.

Landau: *the energy spectrum of particle excitations must be very different in a superfluid compared to normal fluid*

Further reading: J. F. Annett „Superconductivity, superfluids and condensates"

Superfluidity
quasiparticle excitations

Elementary excitations of the system



roton minimum

ballistic scattering:

$$\epsilon(\mathbf{p}) = \frac{p^2}{2m^*}$$

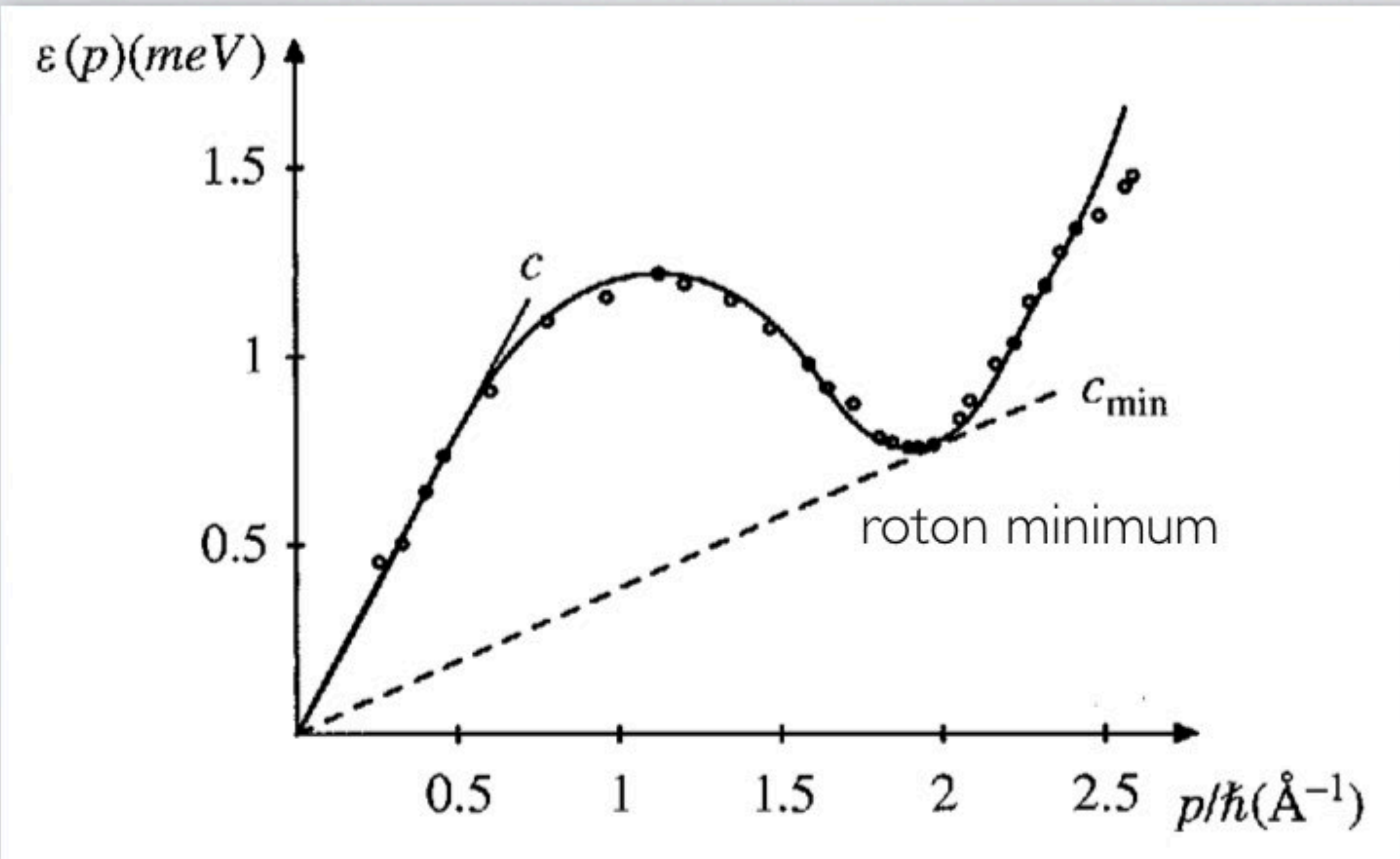
linear:

$$\epsilon(\mathbf{p}) = c|\mathbf{p}|$$

Further reading: J. F. Annett „Superconductivity, superfluids and condensates"

Superfluidity
quasiparticle excitations

Elementary excitations of the system



$$\epsilon(\mathbf{p}) = \mathbf{v} \cdot \mathbf{p}$$

this equation can not be satisfied if the superfluid velocity is small !

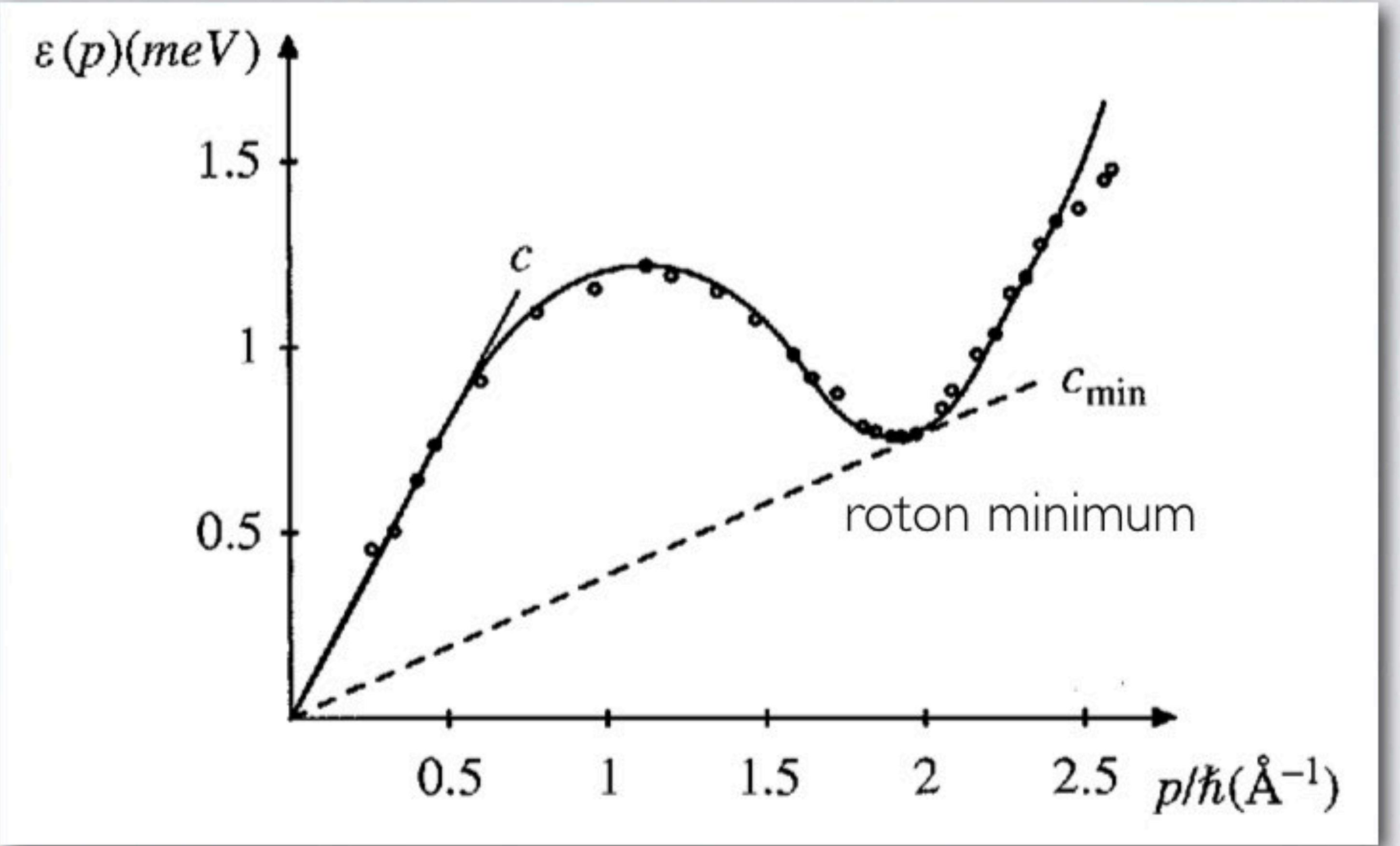
$$\epsilon(\mathbf{p}) > c_{min}|\mathbf{p}|$$

$$|\mathbf{v}_s| < c_{min}$$

Further reading: J. F. Annett „Superconductivity, superfluids and condensates"

Superfluidity
quasiparticle excitations

Elementary excitations of the system



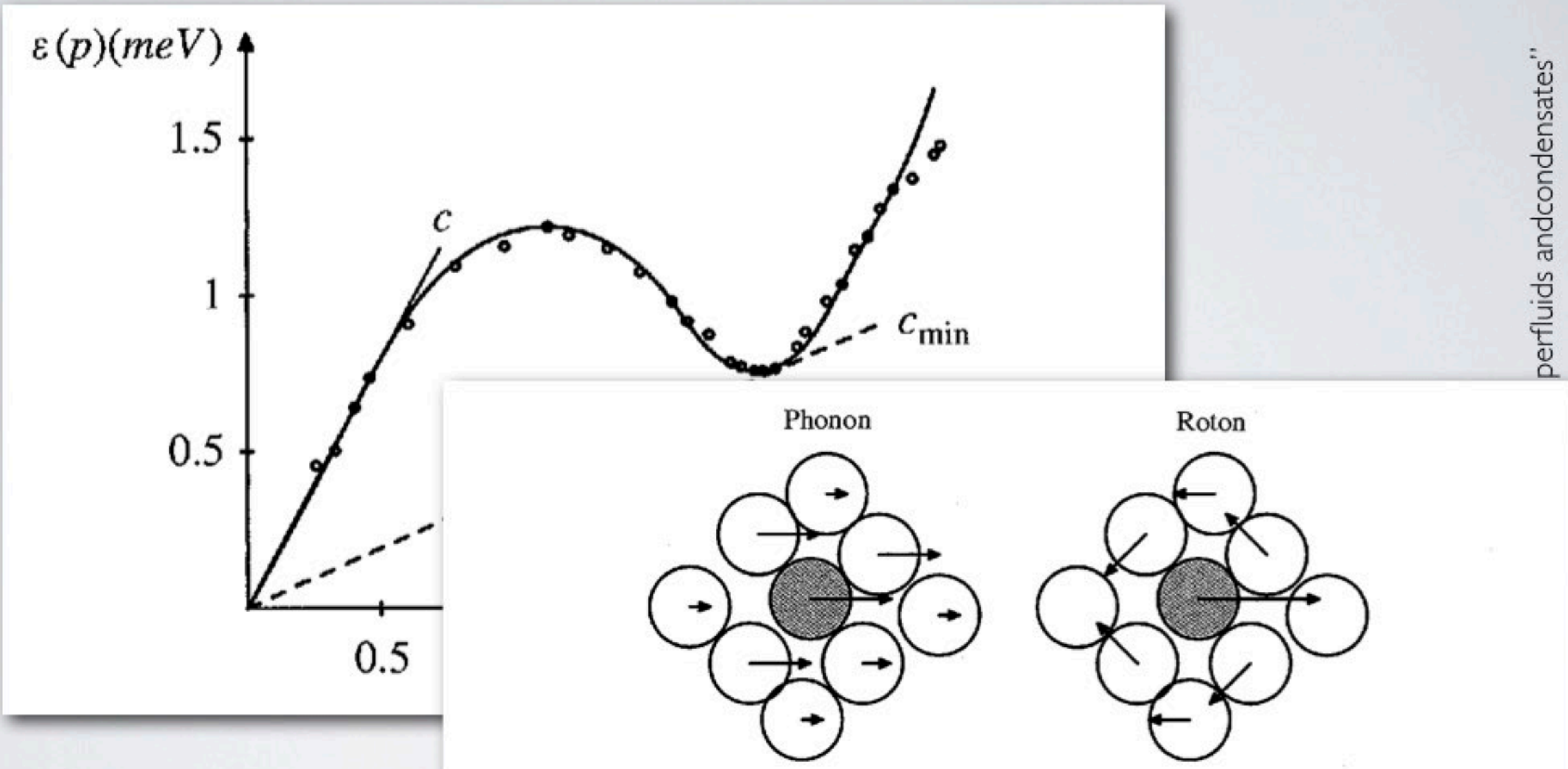
Superfluid flows without dissipation due to scattering of quasiparticles if the flow velocity is less than the critical velocity

$$|\mathbf{v}_s| < c_{min}$$

Further reading: J. F. Annett „Superconductivity, superfluids and condensates"

Superfluidity
quasiparticle excitations

Elementary excitations of the system



Physical interpretation of the phonon and roton parts of the quasiparticle spectrum. The phonon motion corresponds to de Broglie wavelengths, $p = h/\lambda$ greater than a single atomic size, and leads to coupled motion of groups of atoms moving together, rather like a phonon in a solid. The roton corresponds to de Broglie wavelengths of order the interparticle separation. It corresponds to a central particle moving forward, while the close packed neighbors must move out of the way in a circular motion. Feynman notes that this combination of linear and circular motion has some nice similarity to a moving smoke-ring.

perfluids andcondensates"

Important characteristics of polariton superfluid

- DETERMINED FOR SUBSONIC SPEEDS

$$v < c_s$$

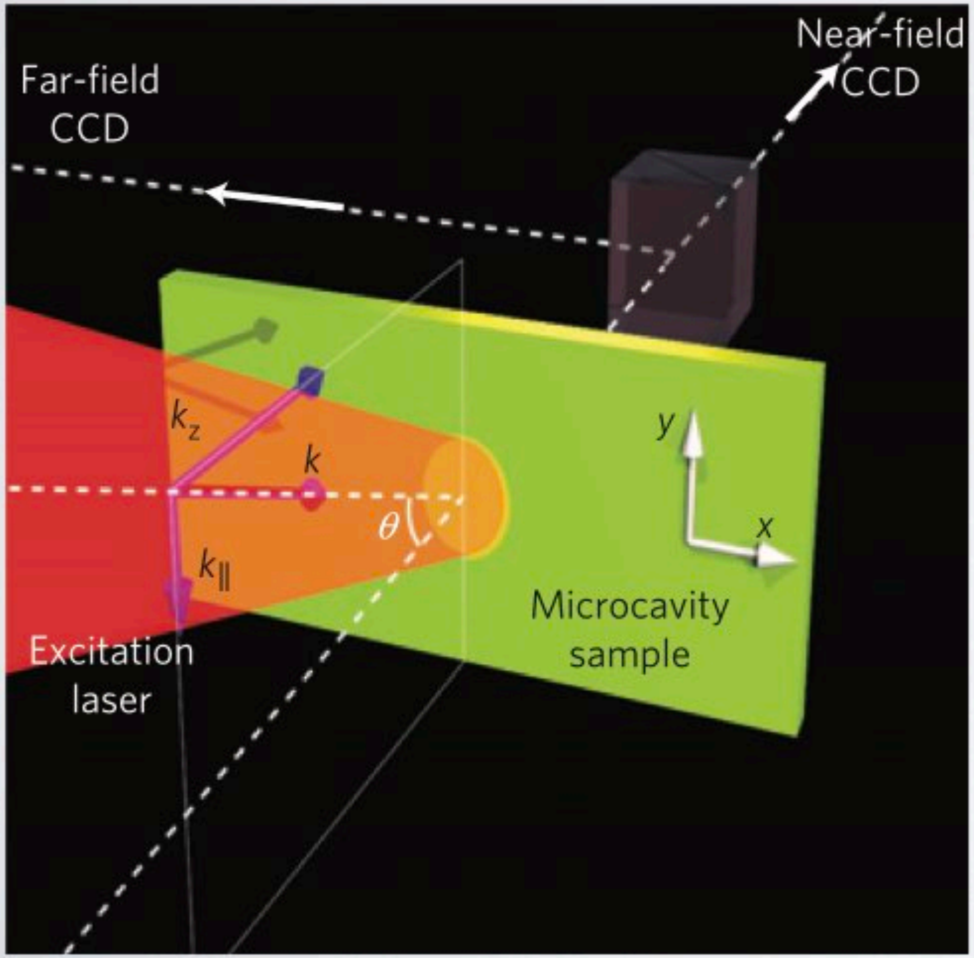
- for $v > c_s$ czerenkov waves - shock waves

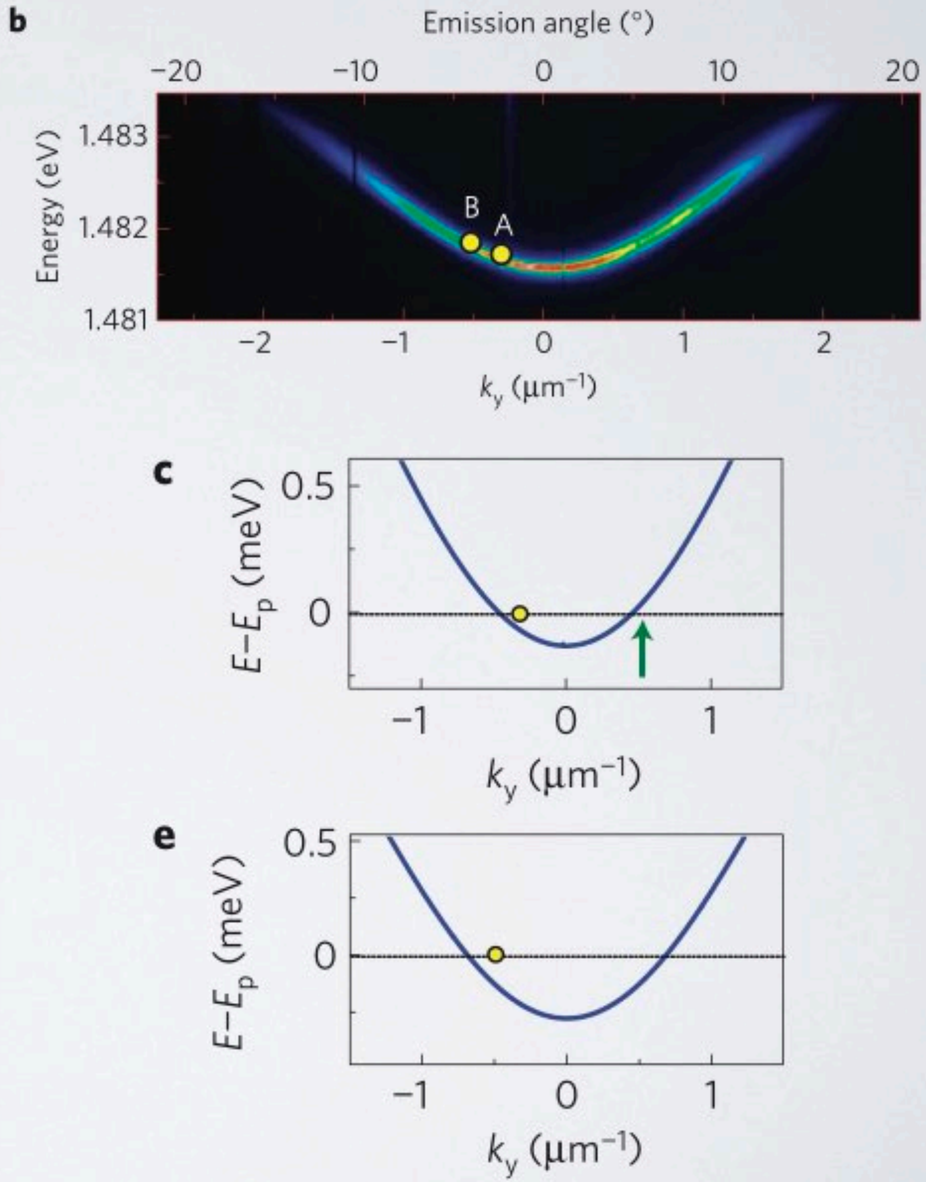
- A SOUND VELOCITY CAN BE ATTRIBUTED TO THE POLARITON FLUID

$$c_{\mathrm{s}} = \sqrt{\hbar g |\psi_{\mathrm{c}}|^2 / m}$$

First experimental realization
setup

A.Amo, et al. *Nature* **457**, 291 (2009)



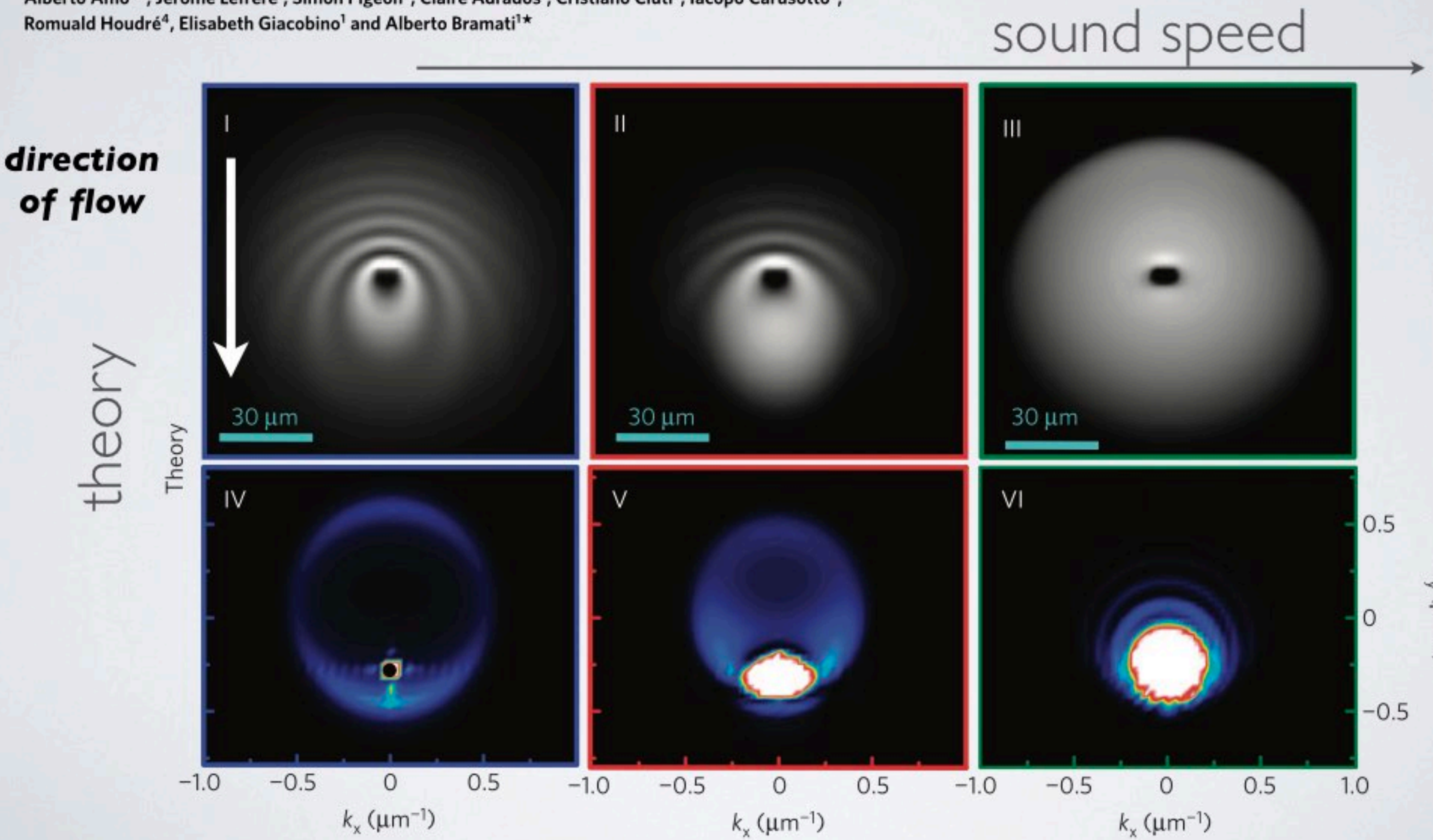


nature physics | LETTERS
PUBLISHED ONLINE: 20 SEPTEMBER 2009 | DOI: 10.1038/NPHYS1364

A.Amo, et al. *Nature* **457**, 291 (2009)

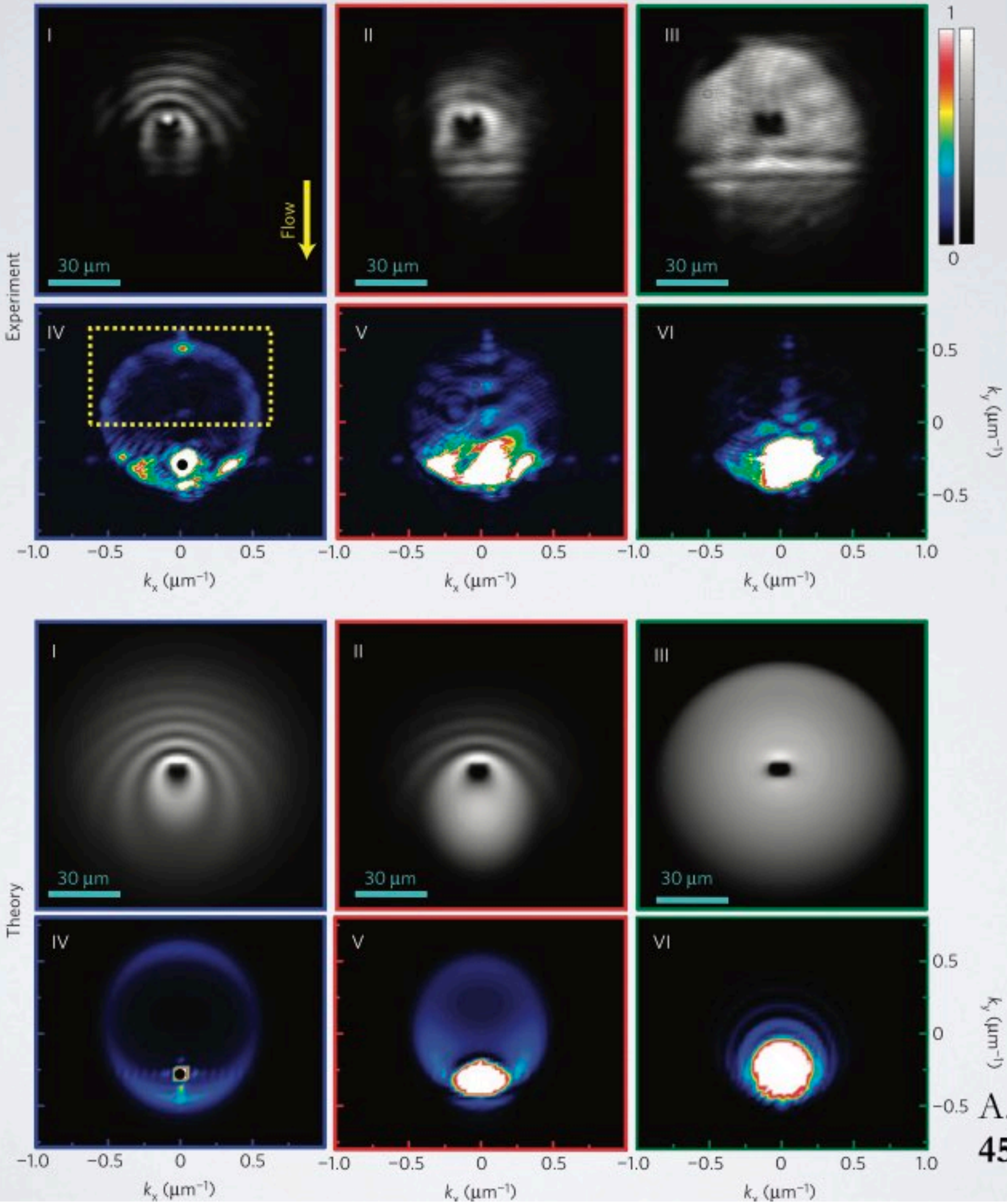
Superfluidity of polaritons in semiconductor microcavities

Alberto Amo[1]*, Jérôme Lefrère[1], Simon Pigeon[2], Claire Adrados[1], Cristiano Ciuti[2], Iacopo Carusotto[3], Romuald Houdré[4], Elisabeth Giacobino[1] and Alberto Bramati[1]*



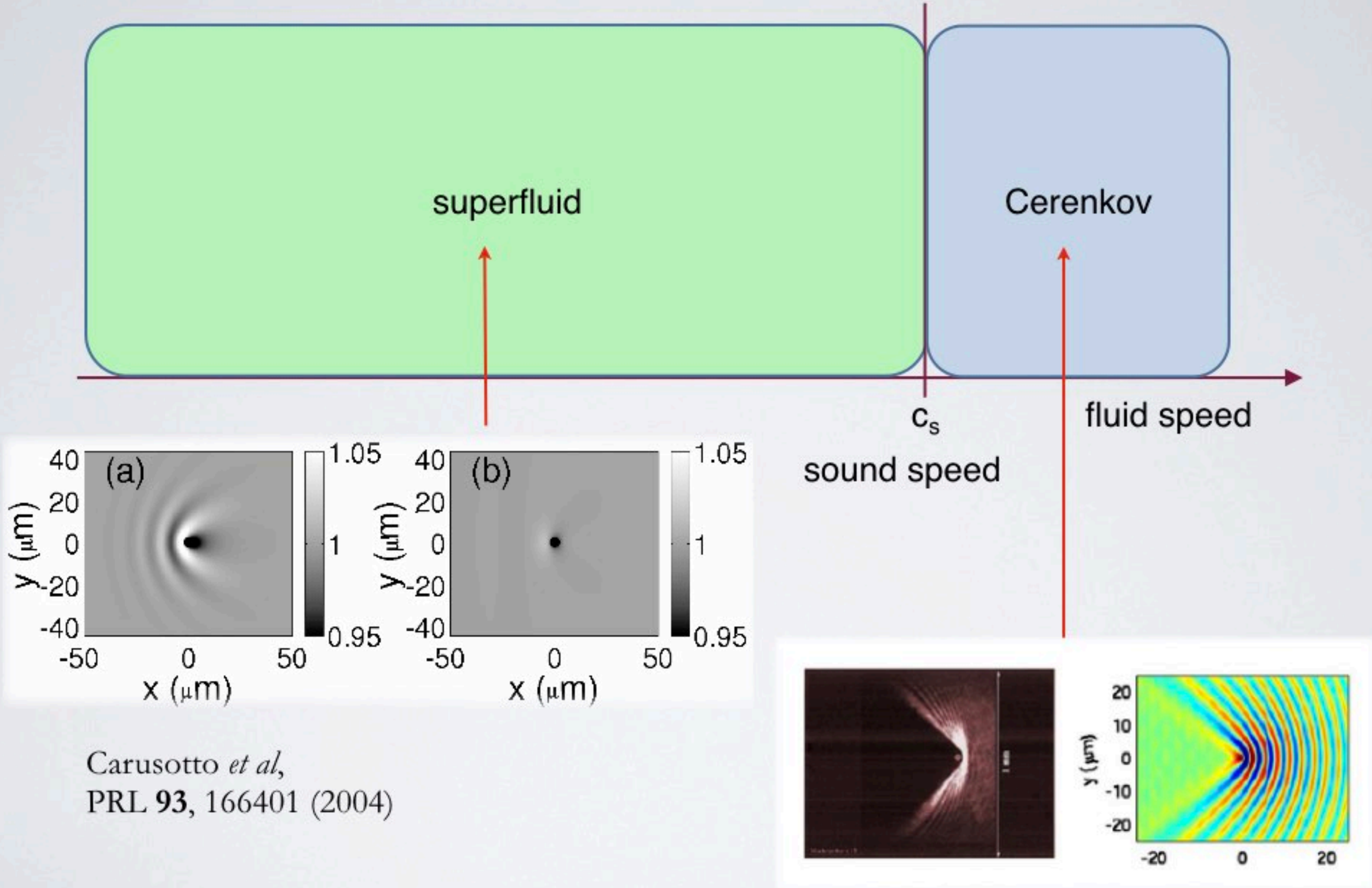
experiment

theory



A.Amo, et al. *Nature* **457**, 291 (2009)

DIFFERENT REGIMES vs FLUID SPEED



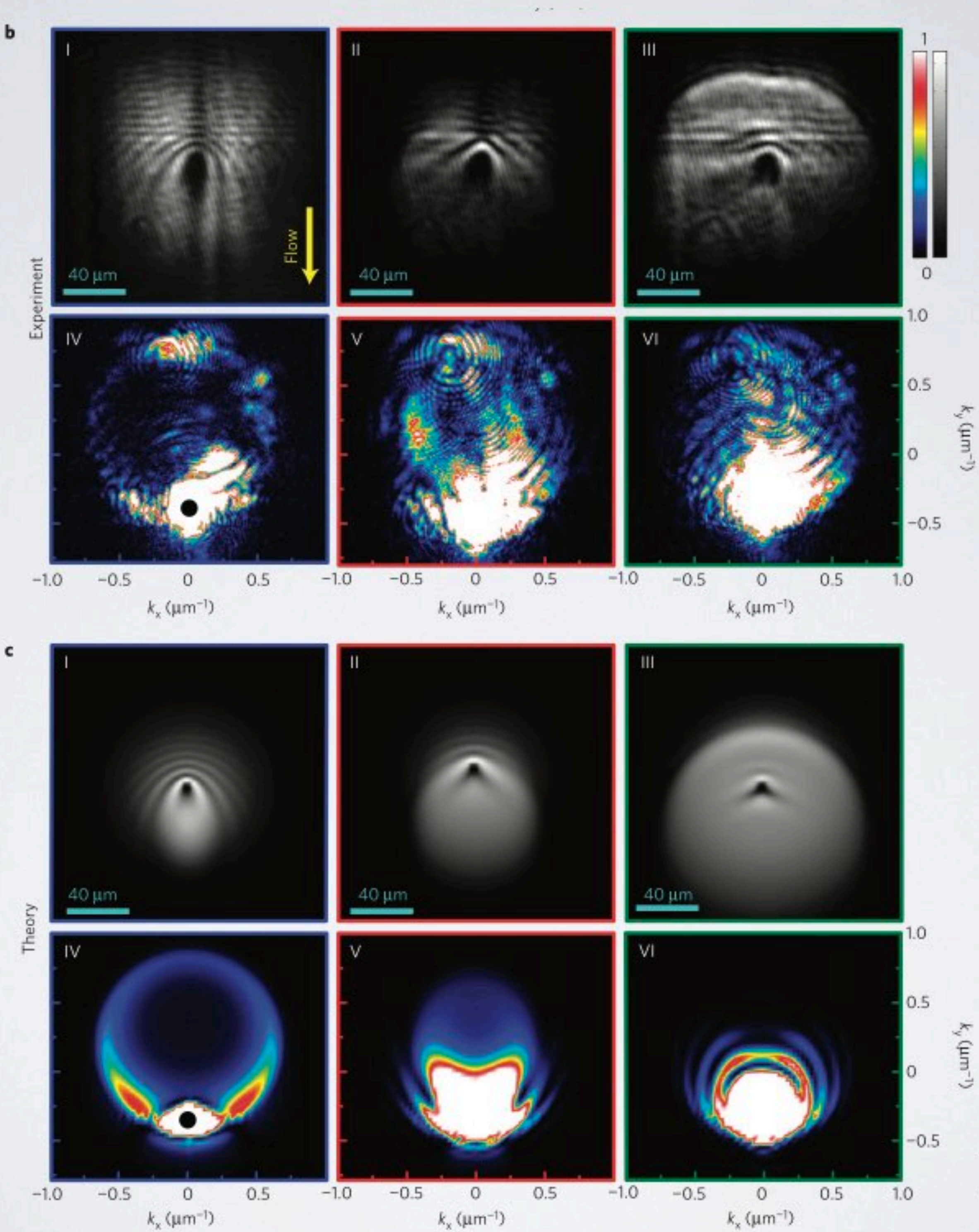
Carusotto *et al*,
PRL **93**, 166401 (2004)

Carusotto *et al*,
PRL **97**, 260403 (2006)

Shock wave

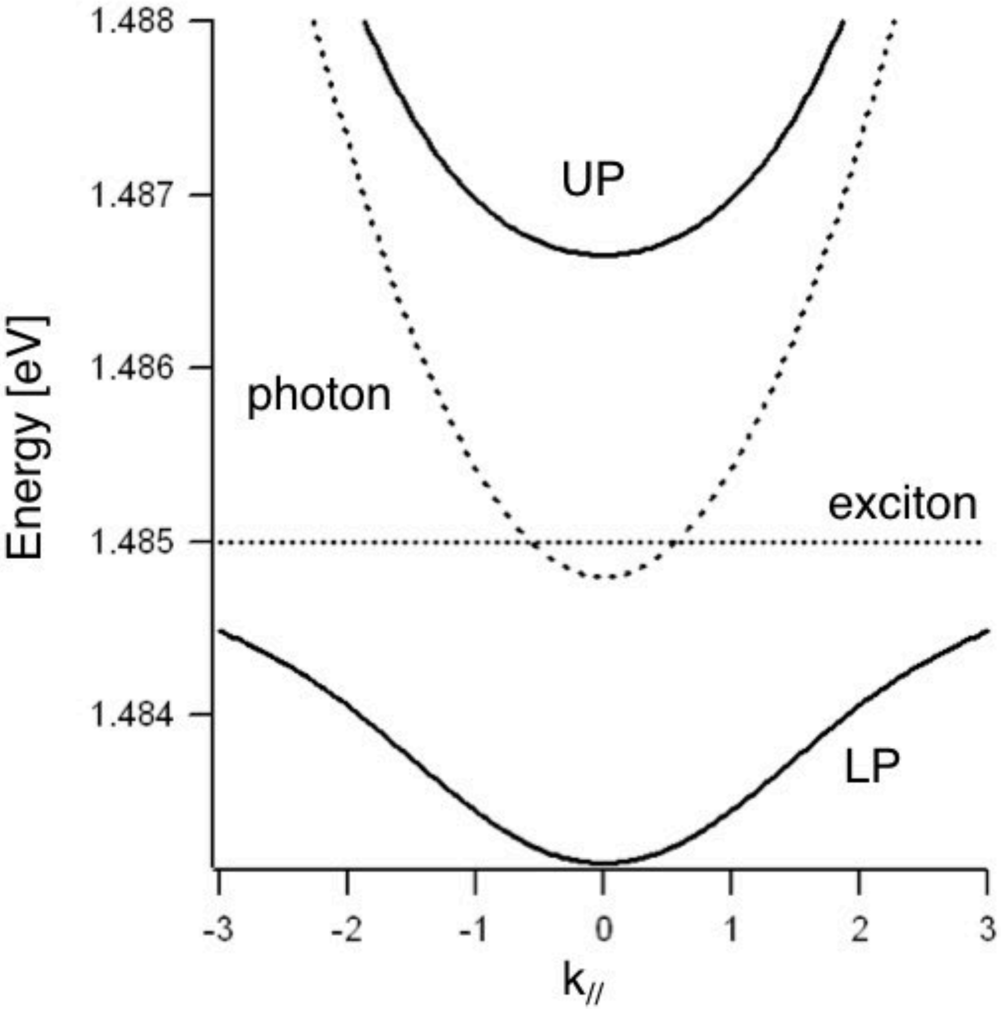
experiment

theory

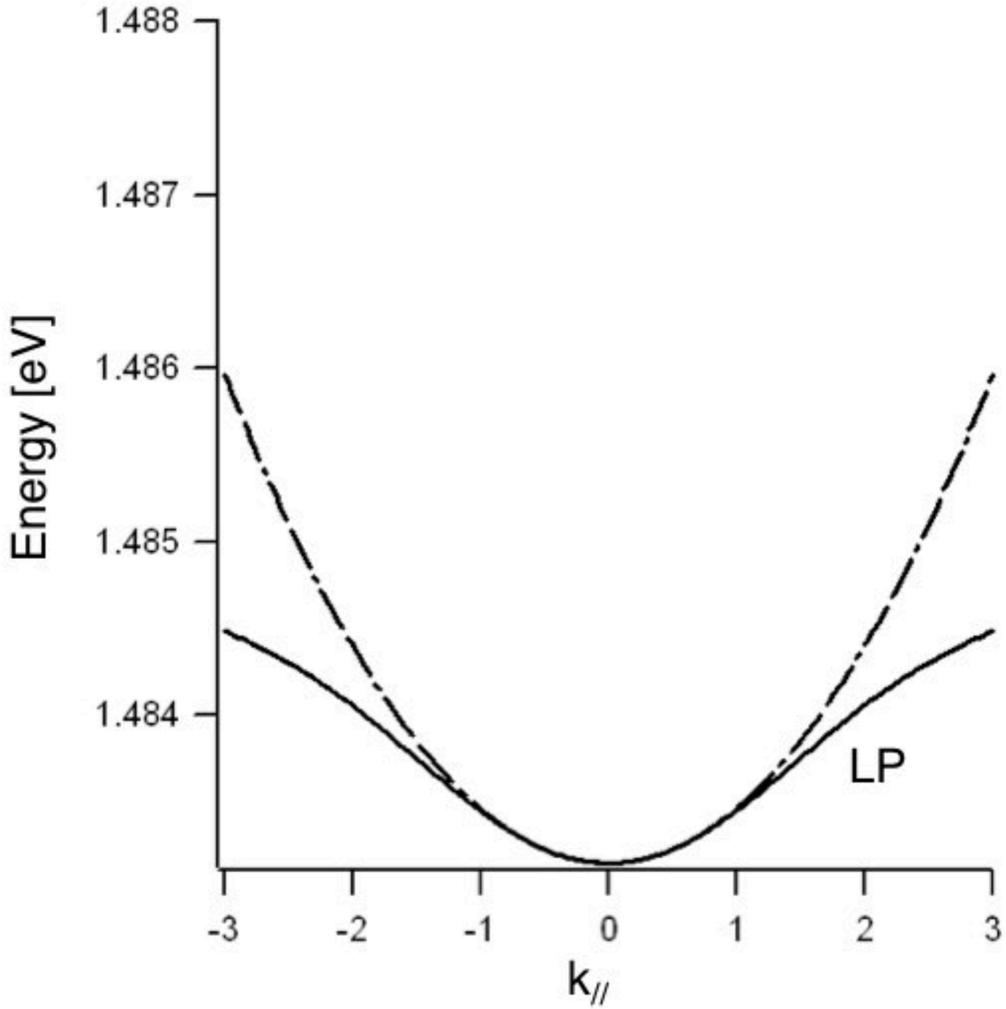


A.Amo, et al. *Nature* **457**, 291 (2009)

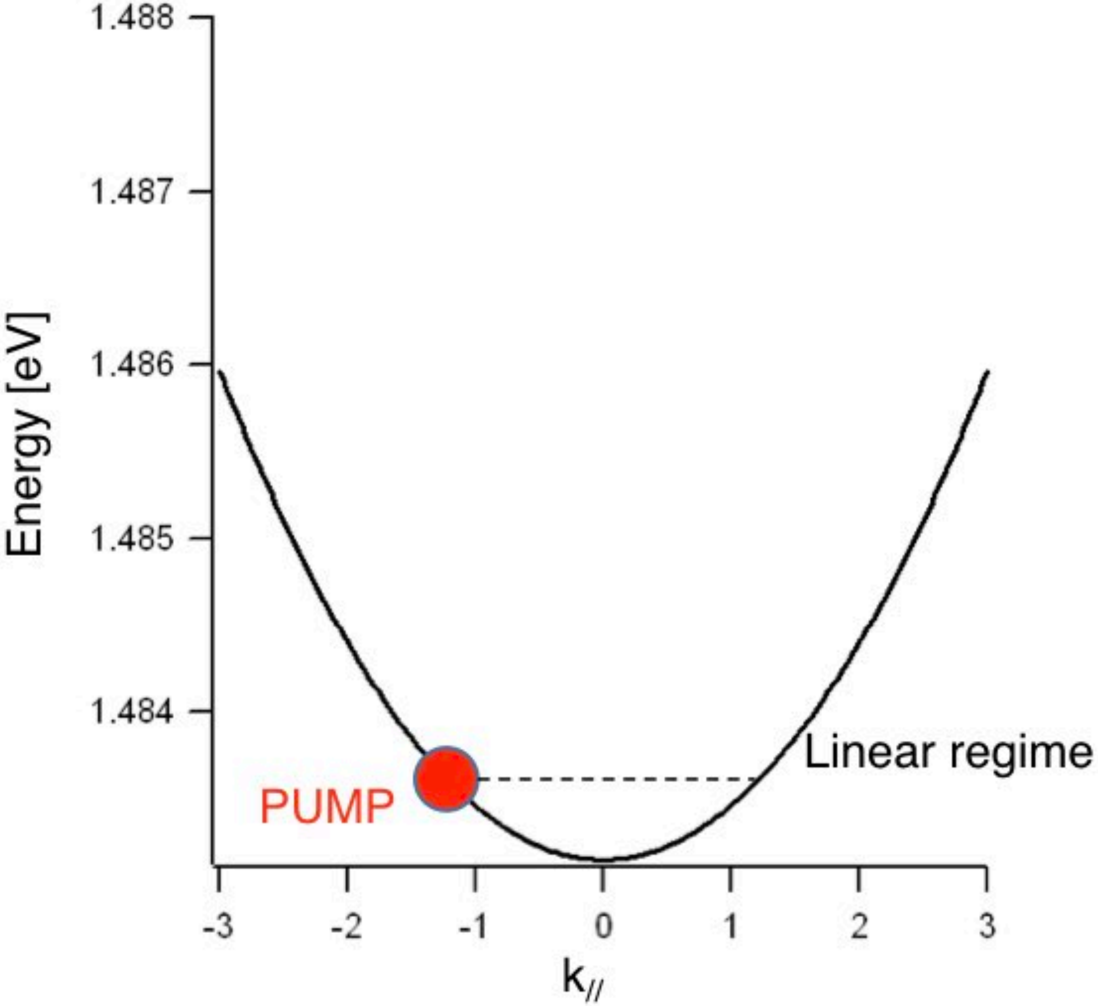
TO UNDERSTAND POLARITON SUPERFLUIDITY

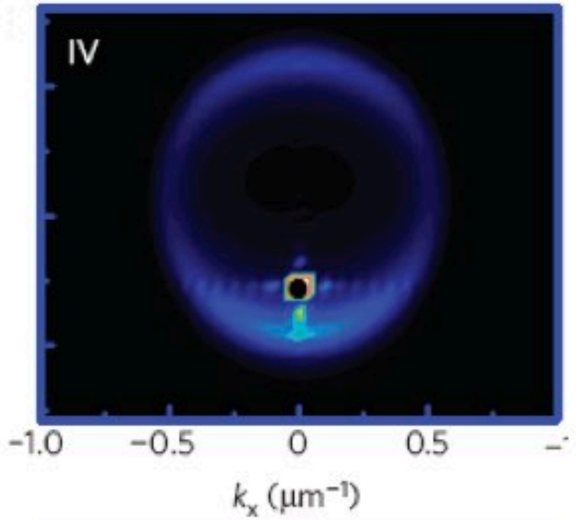


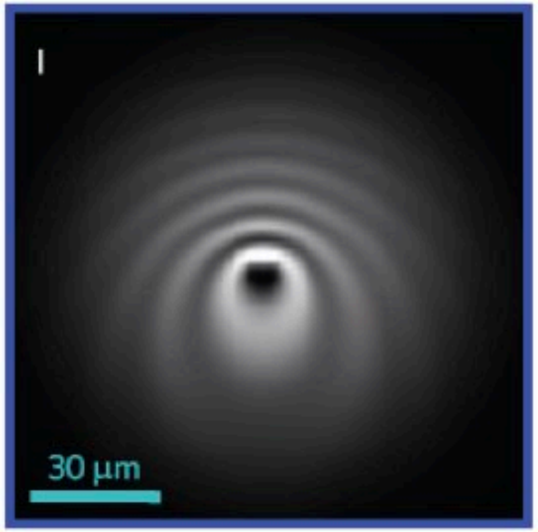
TO UNDERSTAND POLARITON SUPERFLUIDITY



TO UNDERSTAND POLARITON SUPERFLUIDITY

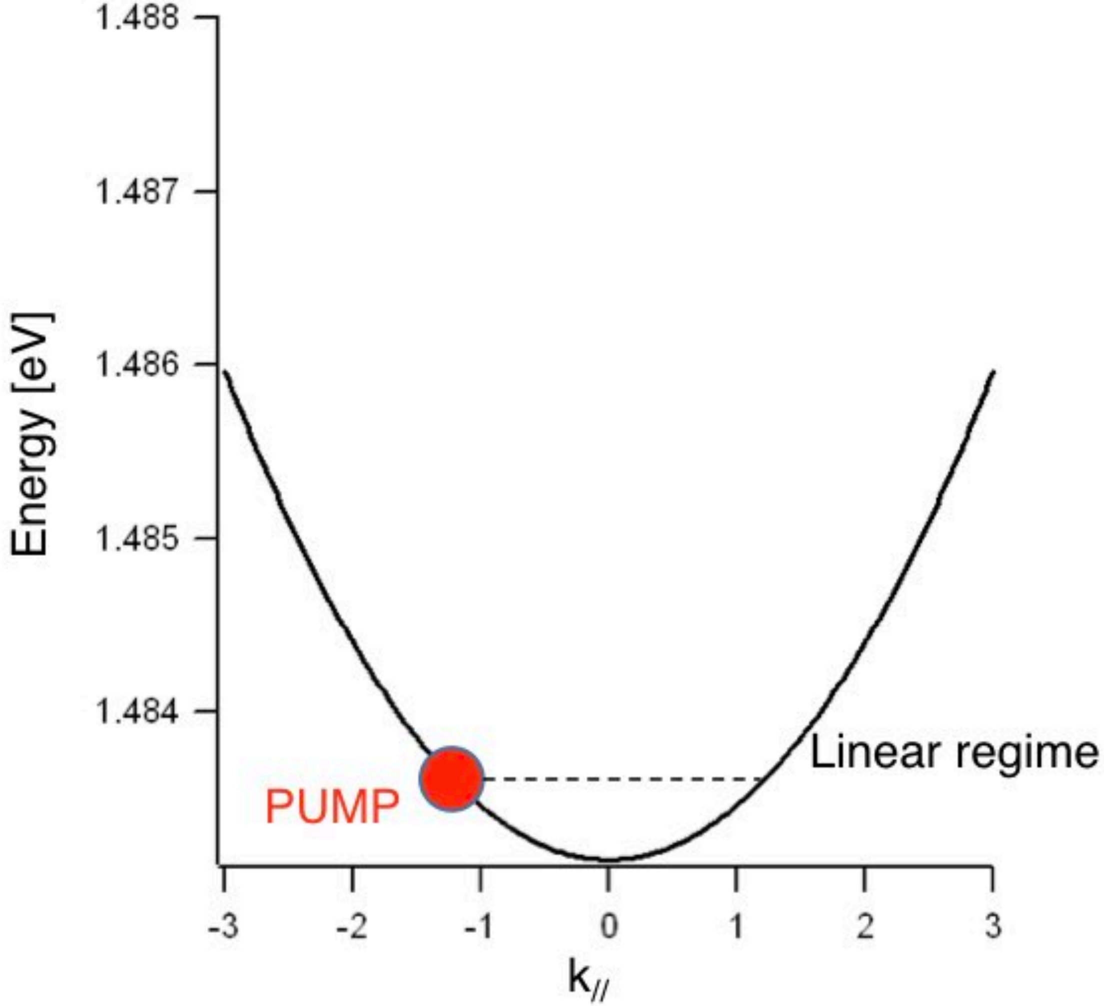


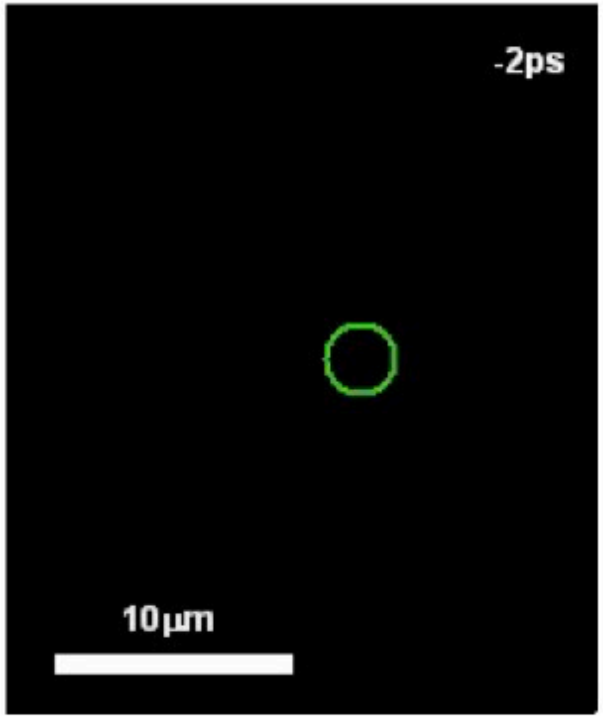




Theory: Carusotto & Ciuti, PRL **93**, 166401 (2004)
Theory + experiments: Amo *et al,* Nature Phys. **5**, 805 (2009)

TO UNDERSTAND POLARITON SUPERFLUIDITY



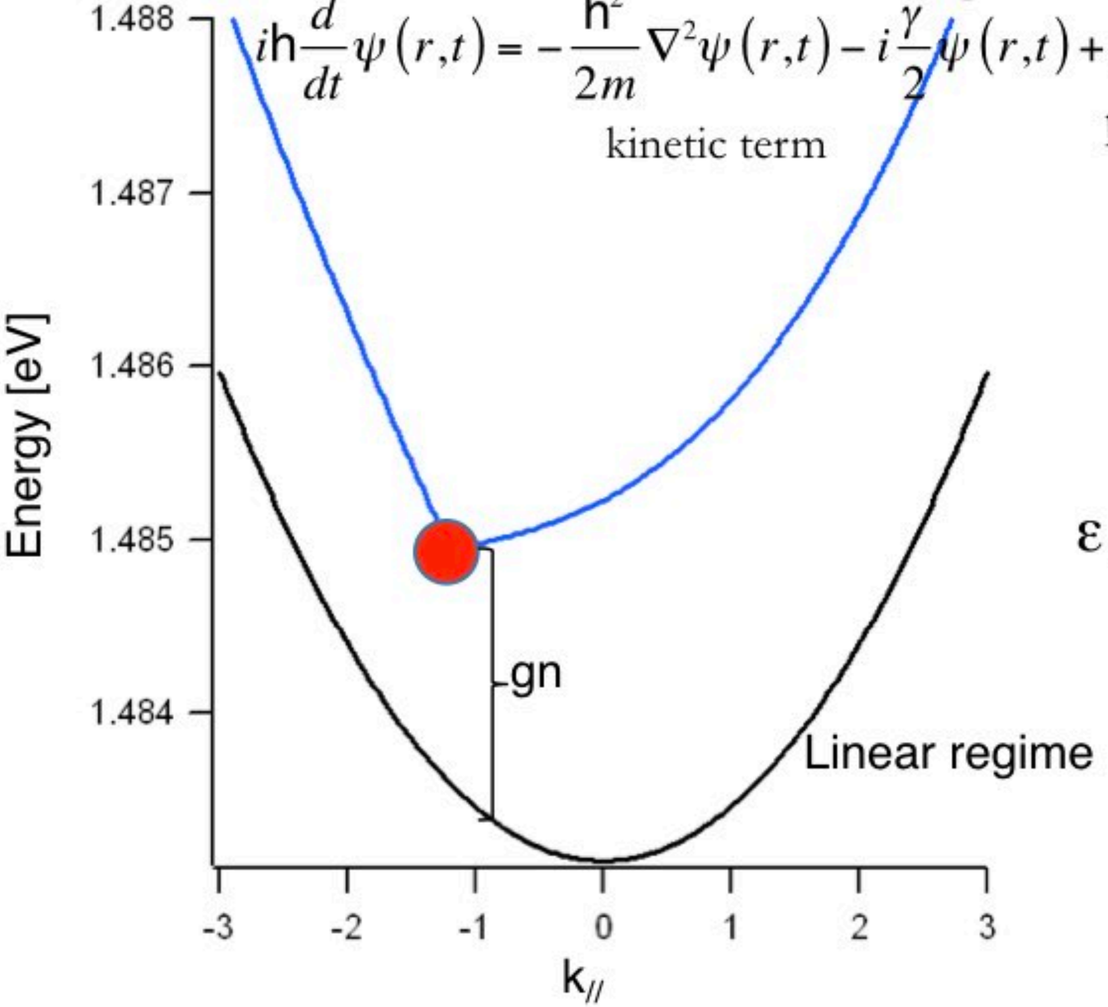


TO UNDERSTAND POLARITON SUPERFLUIDITY

dissipation, obstacle

$$ih\frac{d}{dt}\psi(r,t) = -\frac{h^2}{2m}\nabla^2\psi(r,t) - i\frac{\gamma}{2}\psi(r,t) + g|\psi(r,t)|^2\psi(r,t) + V\psi(r,t) + F_p(r,t)$$

kinetic term, pol-pol interactions, driving field



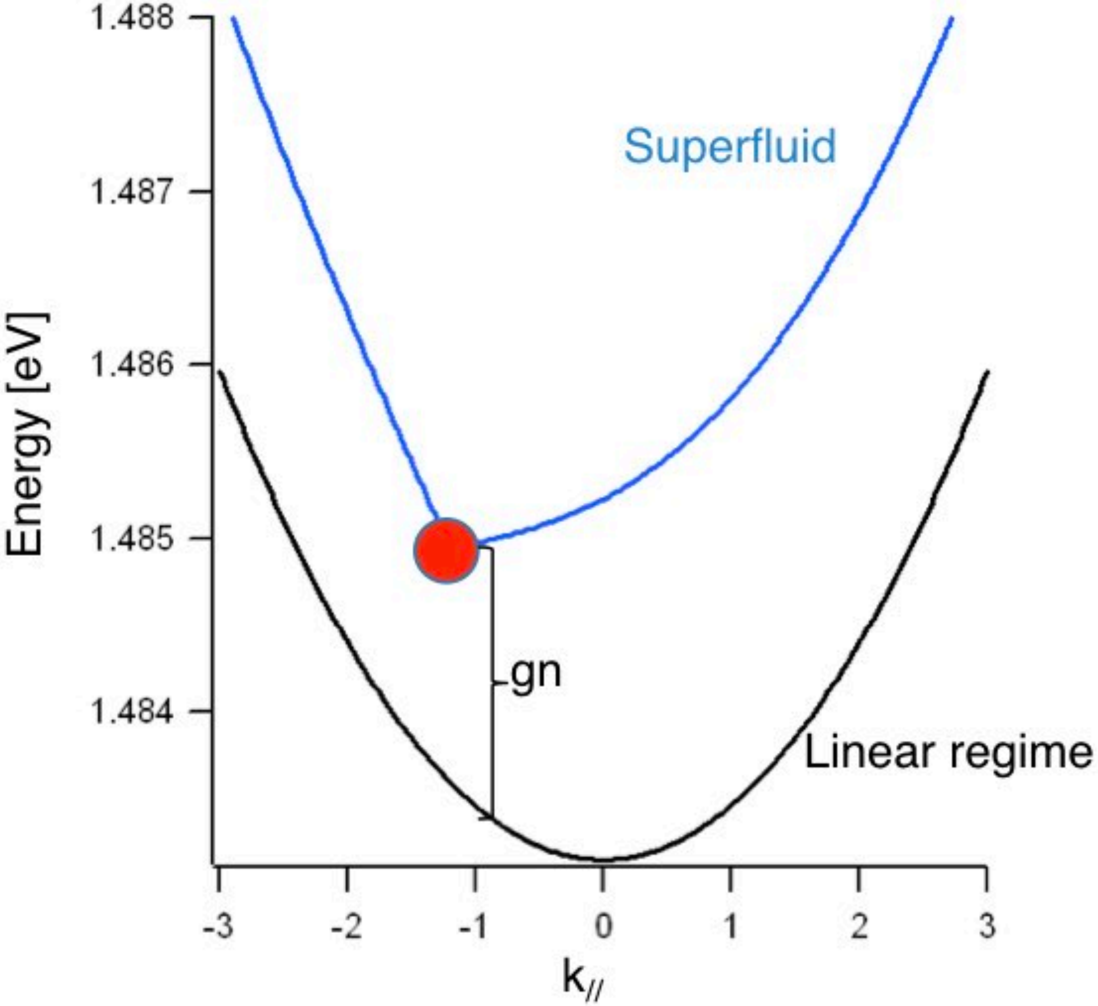
Bogoliubov

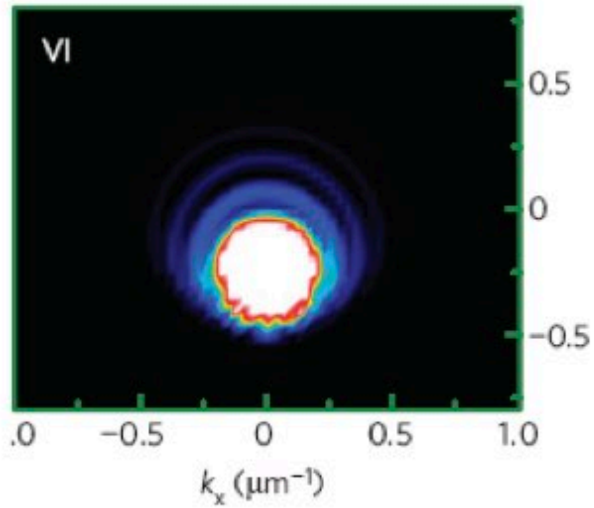
$$\varepsilon_B(k) = \sqrt{\varepsilon(k)(\varepsilon(k)+2gn)} + gn$$

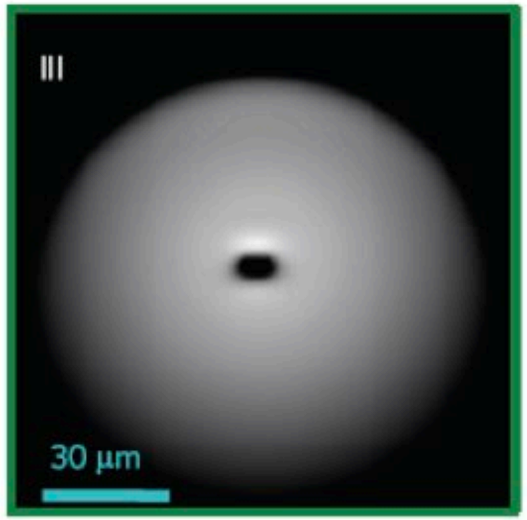
ε(k) : parabolic dispersion
n: density of polariton
g: interaction const

Theory: Carusotto & Ciuti, PRL **93**, 166401 (2004)
Theory + experiments: Amo *et al,* Nature Phys. **5**, 805 (2009)

TO UNDERSTAND POLARITON SUPERFLUIDITY







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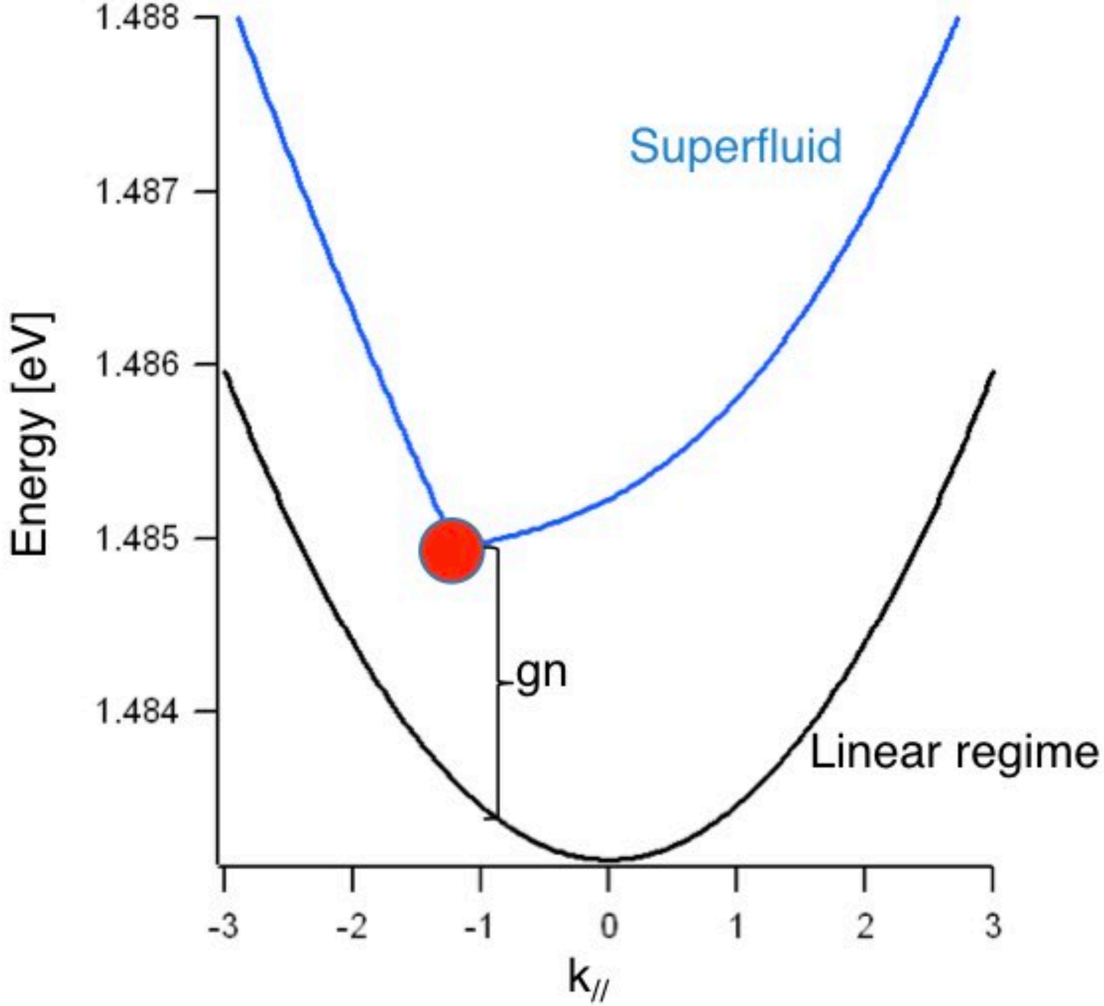
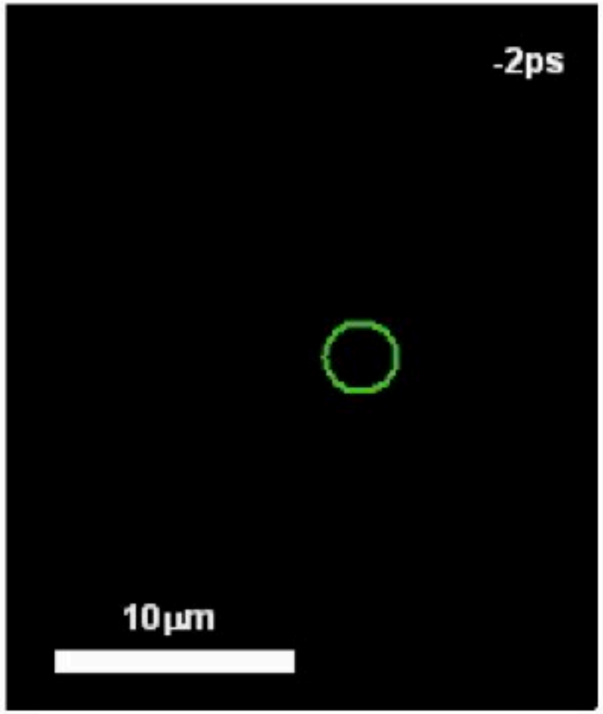
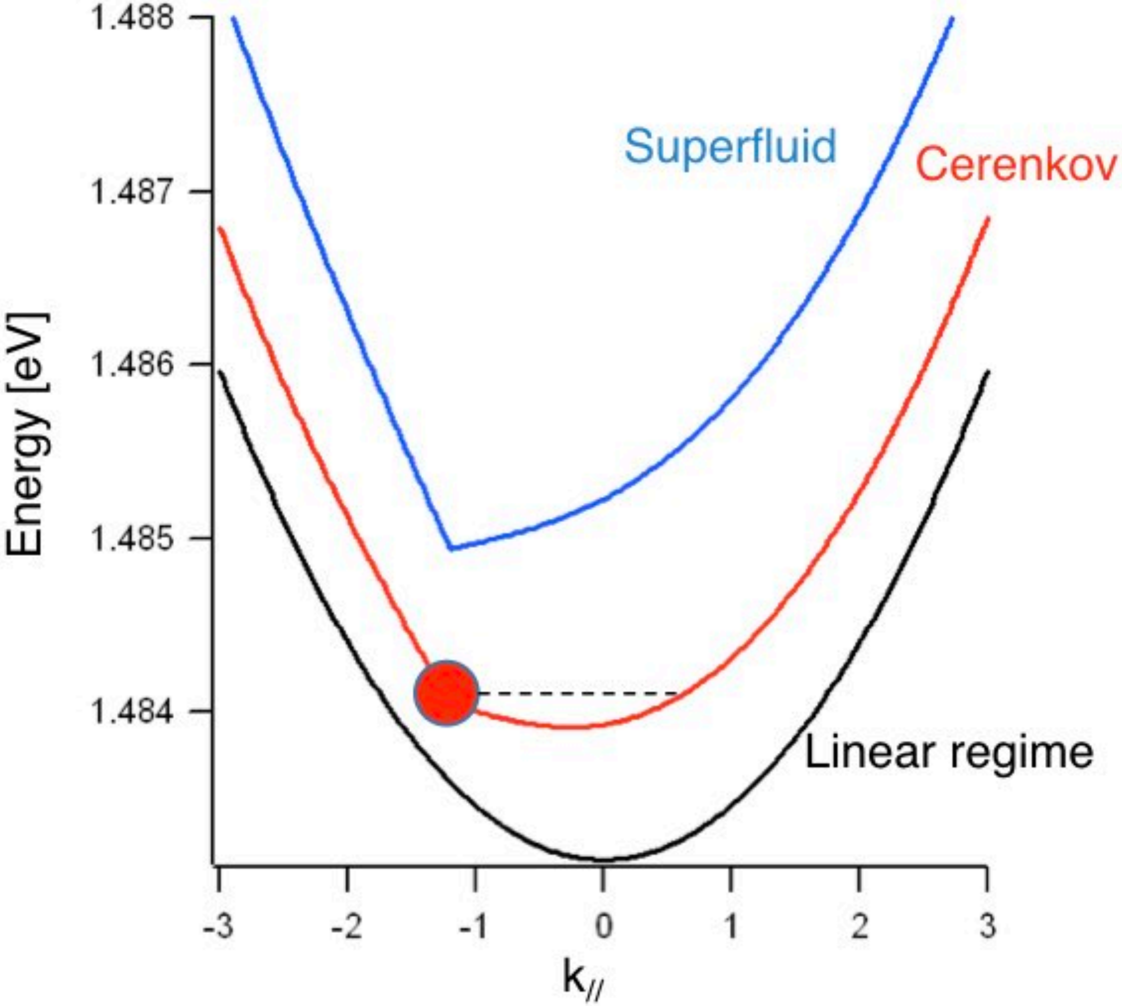
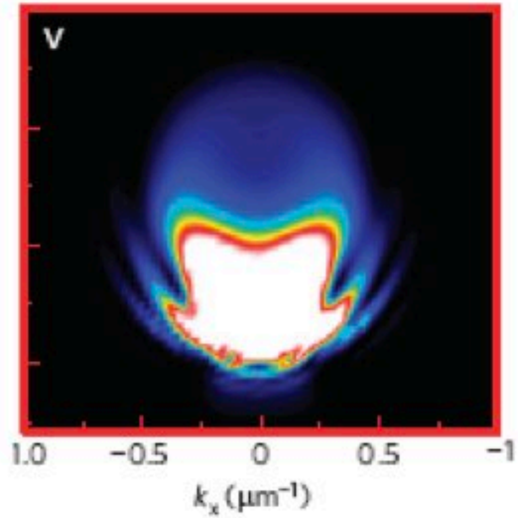
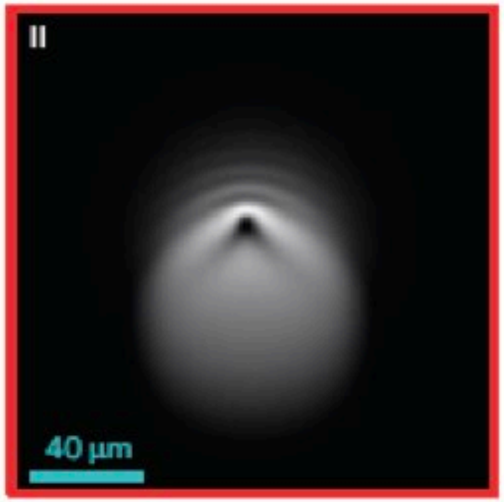




TO UNDERSTAND POLARITON SUPERFLUIDITY

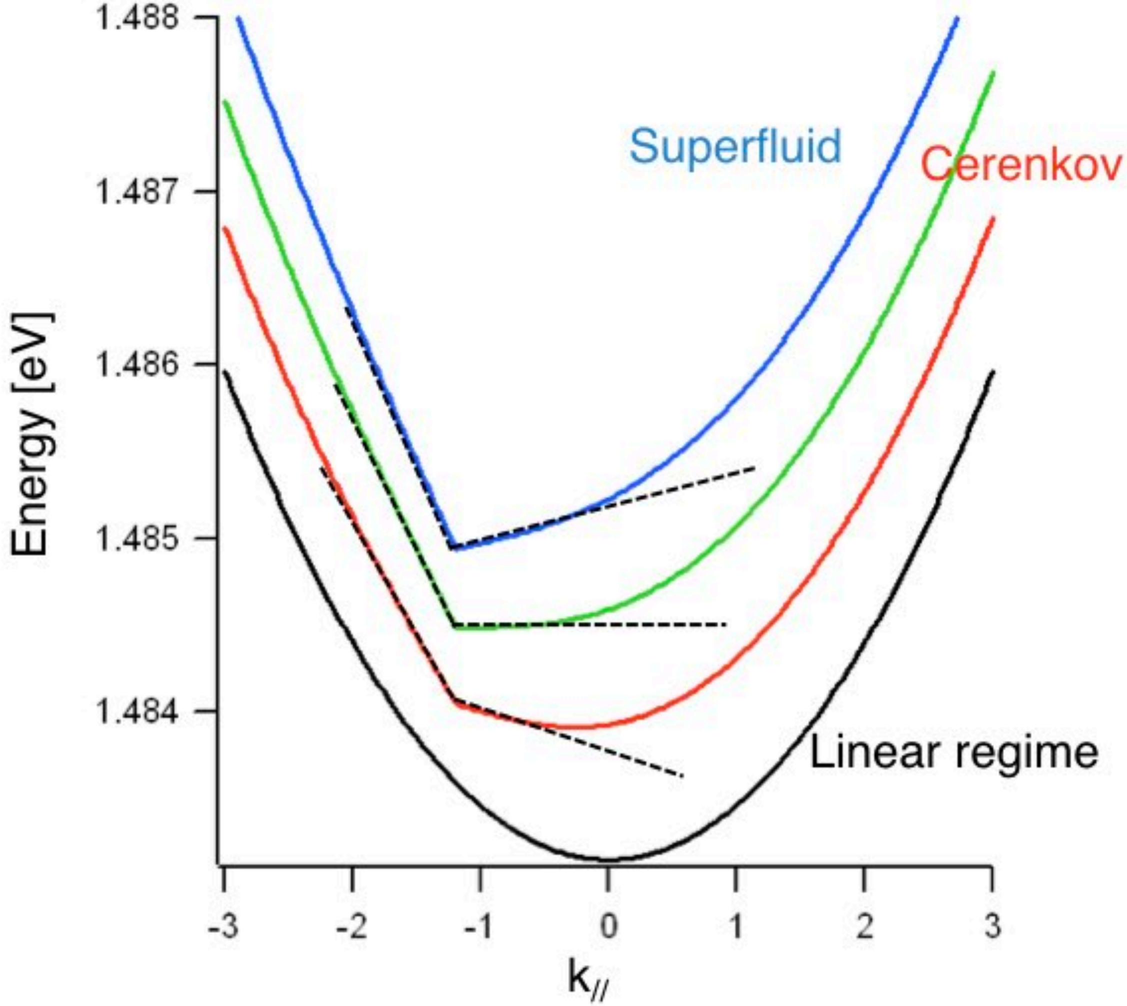






Theory: Carusotto & Ciuti, PRL **93**, 166401 (2004)
Theory + experiments: Amo *et al,* Nature Phys. **5**, 805 (2009)

TO UNDERSTAND POLARITON SUPERFLUIDITY



$$v < c_s$$

$$v > c_s$$

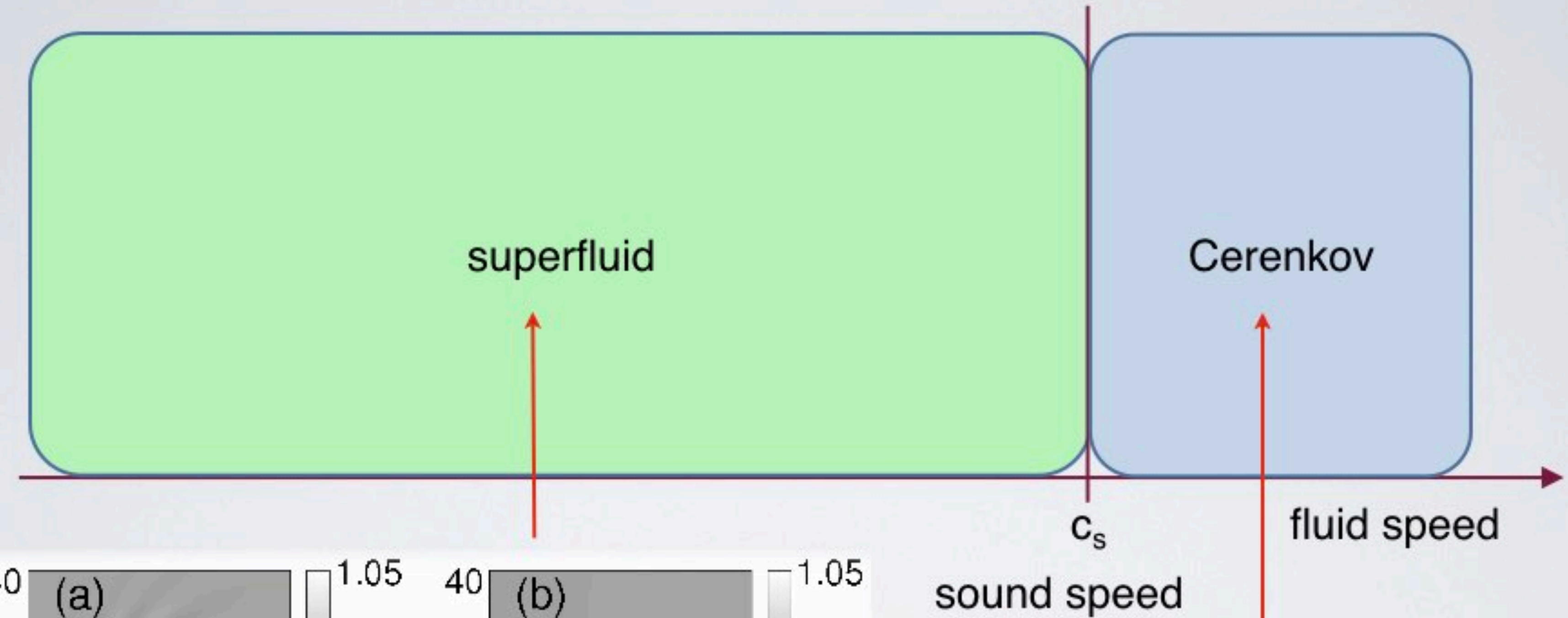
$$v = c_s$$

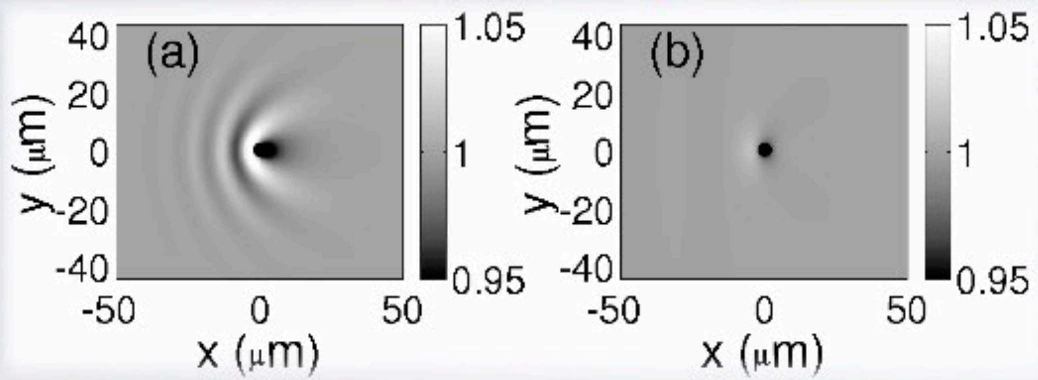
$$v = \frac{\partial \omega}{\partial k} = \frac{\hbar k}{m_{LP}}$$

$$c_s = \sqrt{\frac{gn}{m_{LP}}}$$

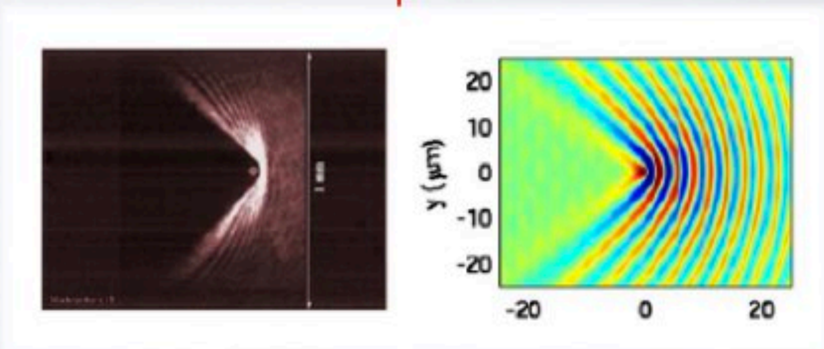
Theory: Carusotto & Ciuti, PRL **93**, 166401 (2004)
Theory + experiments: Amo *et al,* Nature Phys. **5**, 805 (2009)

DIFFERENT REGIMES vs FLUID SPEED



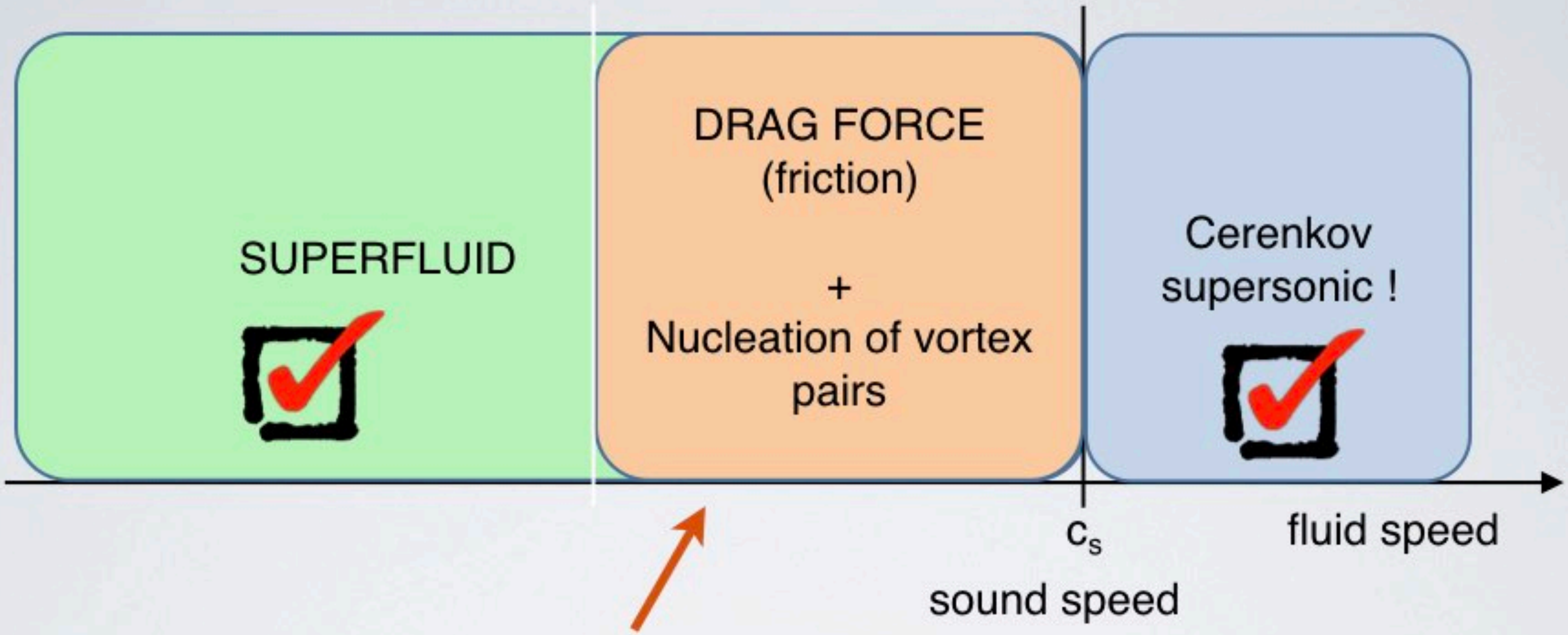


Carusotto *et al*,
PRL **93**, 166401 (2004)



Carusotto *et al*,
PRL **97**, 260403 (2006)

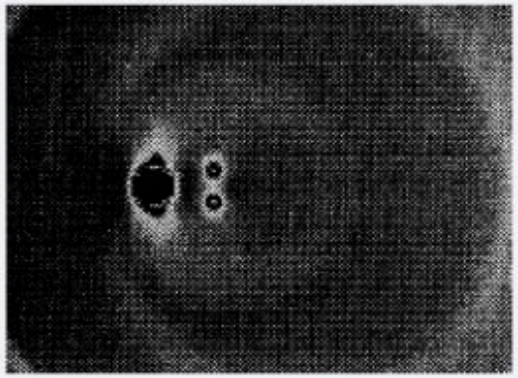
DIFFERENT REGIMES vs FLUID SPEED



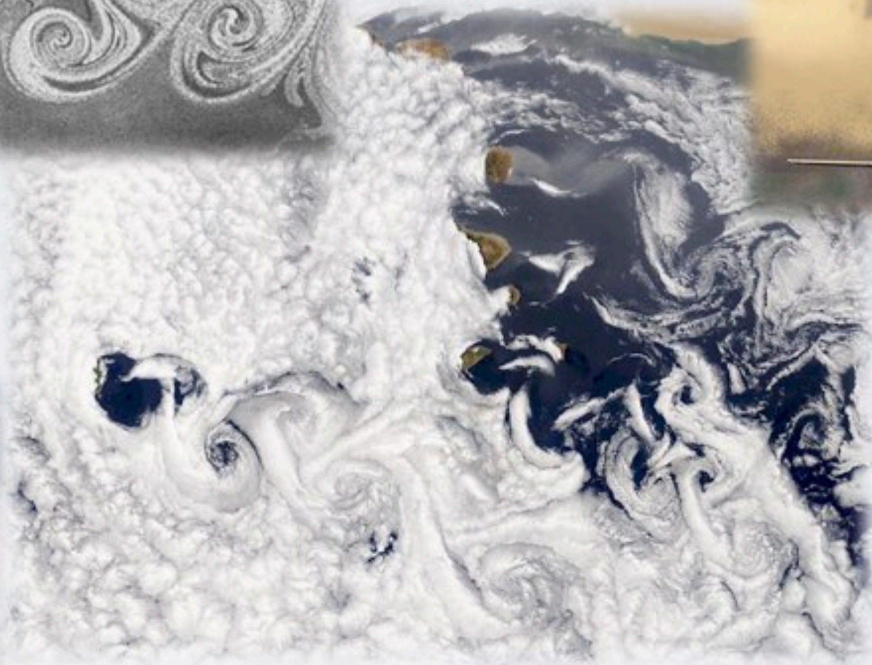
THEORETICAL WORK

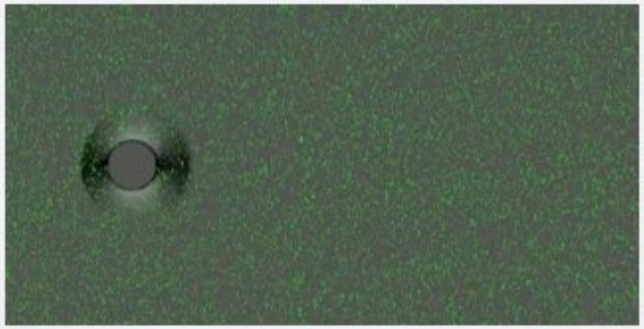
Frisch *et al*, PRL **69**, 1644 (1992)

nucleation of vortices due to the friction



VORTEX STREETS IN CLASSICAL FLUIDS

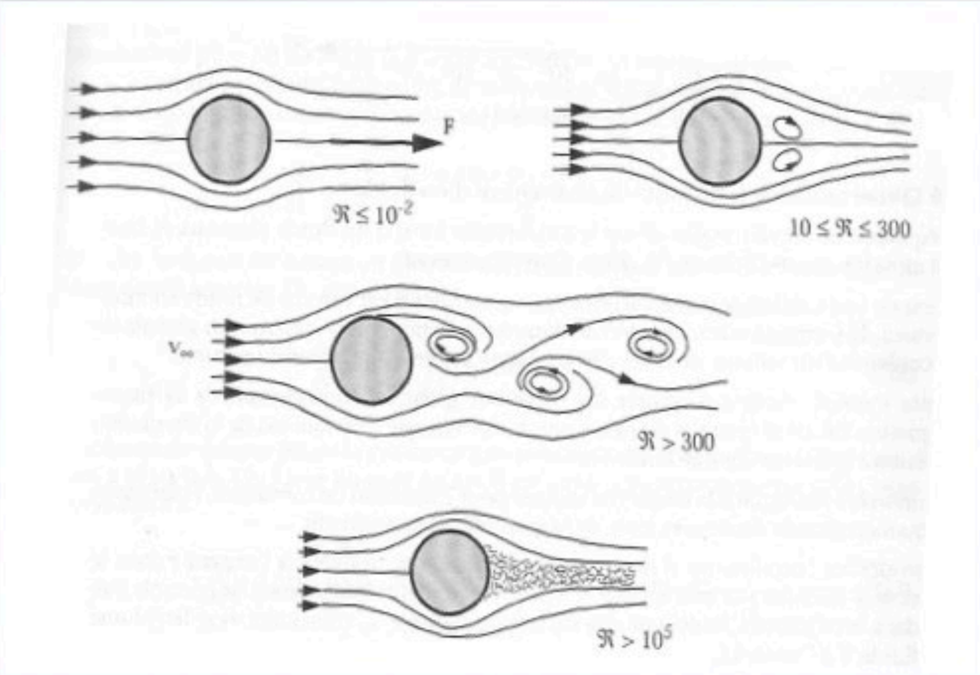




VON KARMAN VORTEX STREET

image source:http://www.youtube.com/watch?v=qpDKRrS9aqE

VORTICES IN CLASSICAL FLUIDS



R_e

UNSEPARATED FLOW

A PAIR OF VORTICES FIXED IN A WAKE

BENARD-VON KARMAN VORTEX STREET - LAMINAR VORTEX STREET

TURBULENT FLOW

$$R_e = \frac{Fluid\_Velocity \cdot Obstacle\_diameter}{Kinematic\_viscosity} = \frac{predkosc \cdot promien}{lepkosc}$$

Reynolds number sets the border between the laminar flow and the different regimes of turbulent flow for classical fluid

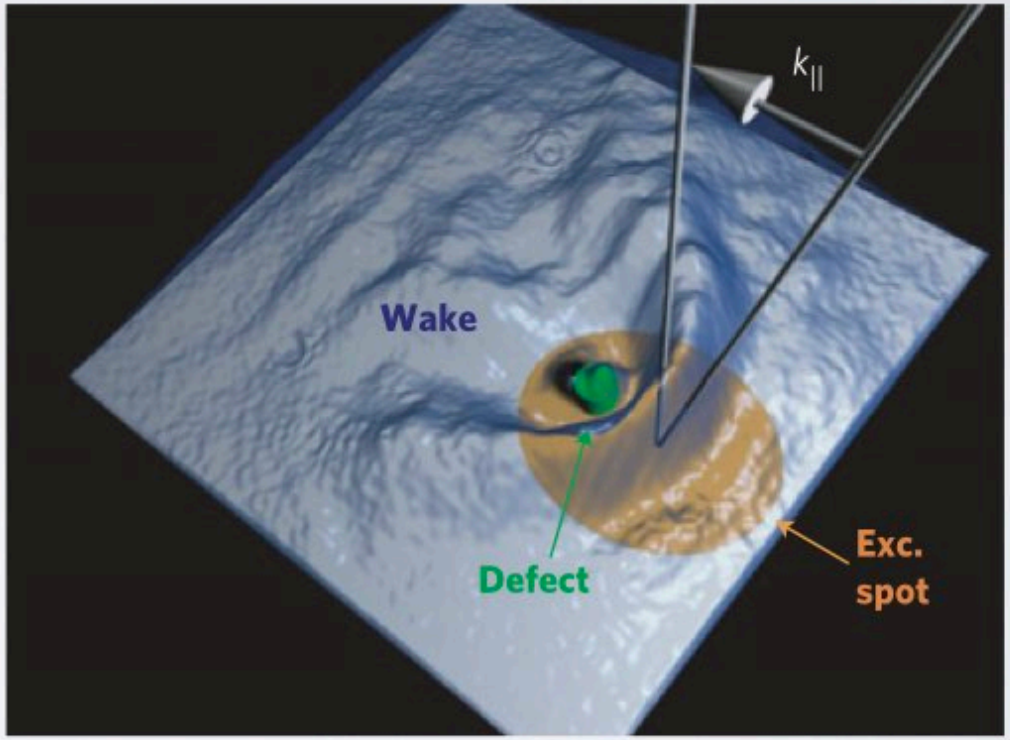
Experiment in the classical form

The same experiment in a quantum form :

nature physics
ARTICLES
PUBLISHED ONLINE: 3 APRIL 2011 | DOI: 10.1038/NPHYS1959

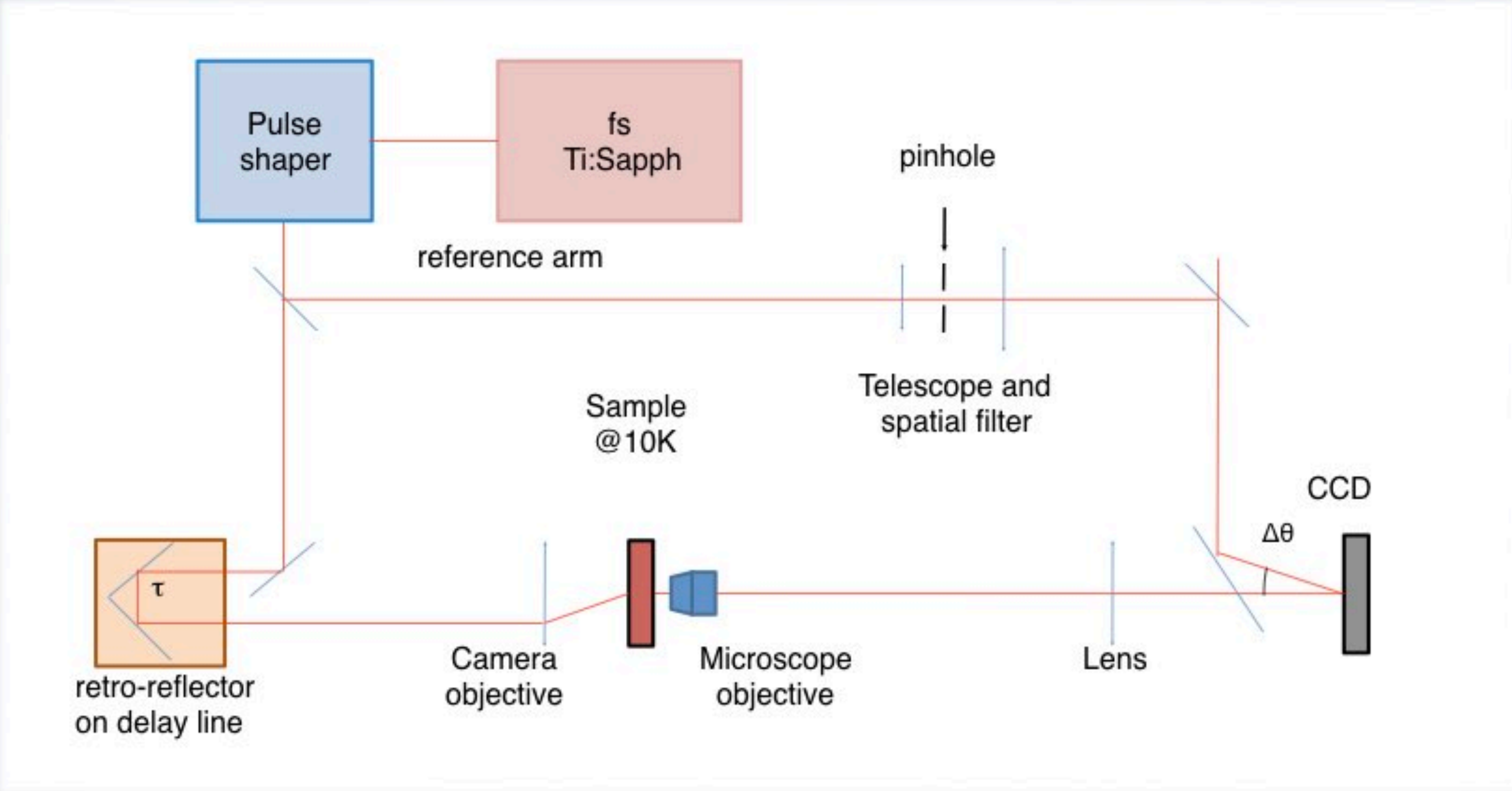
Hydrodynamic nucleation of quantized vortex pairs in a polariton quantum fluid

G. Nardin, et al. *Nature* Physics (2010)

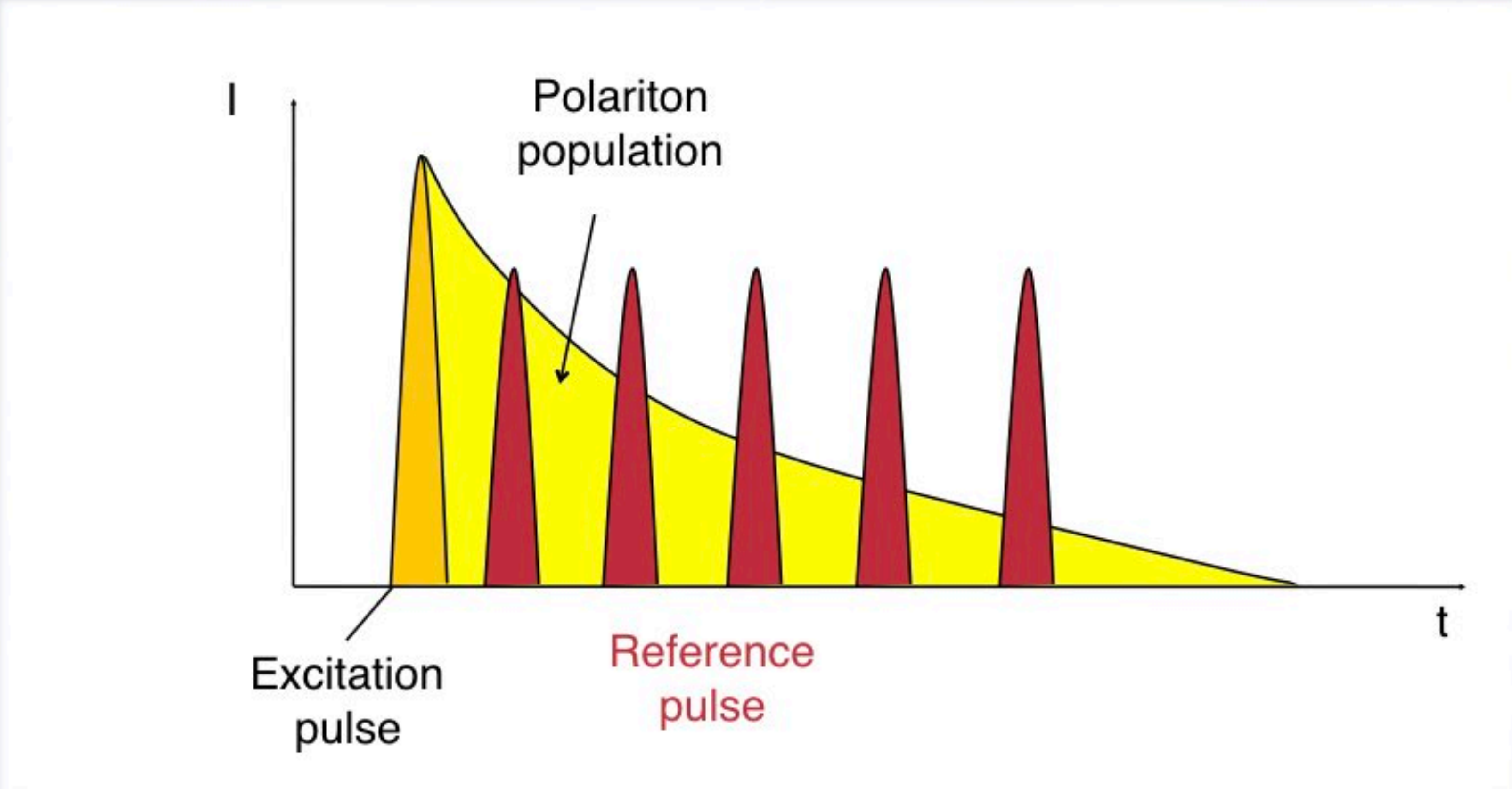


IPEQ, Ecole Polytechnique Fédérale de Lausanne (EPFL) Switzerland

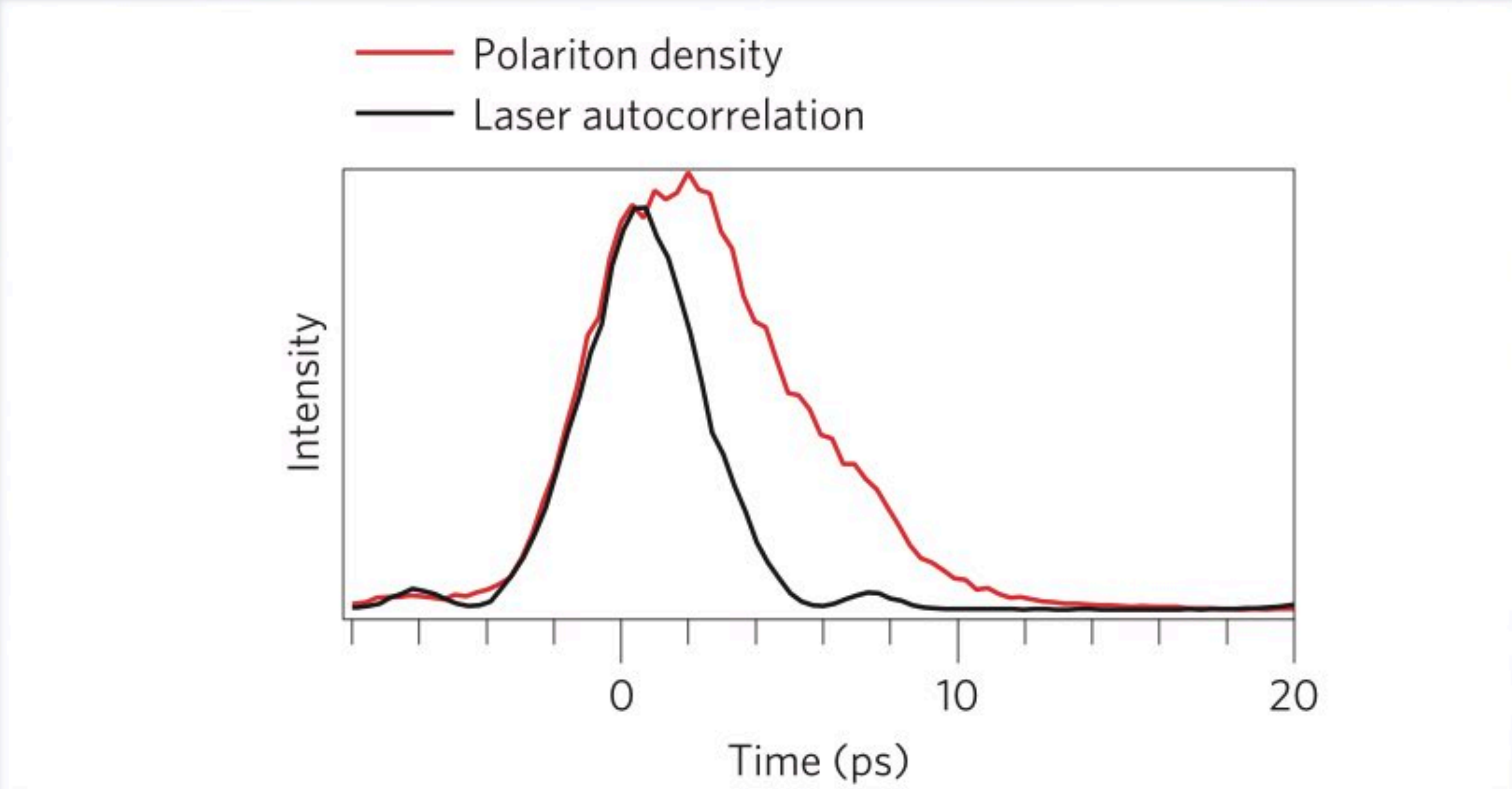
Phase and time resolved imaging setup



EXPERIMENTAL PRINCIPLE



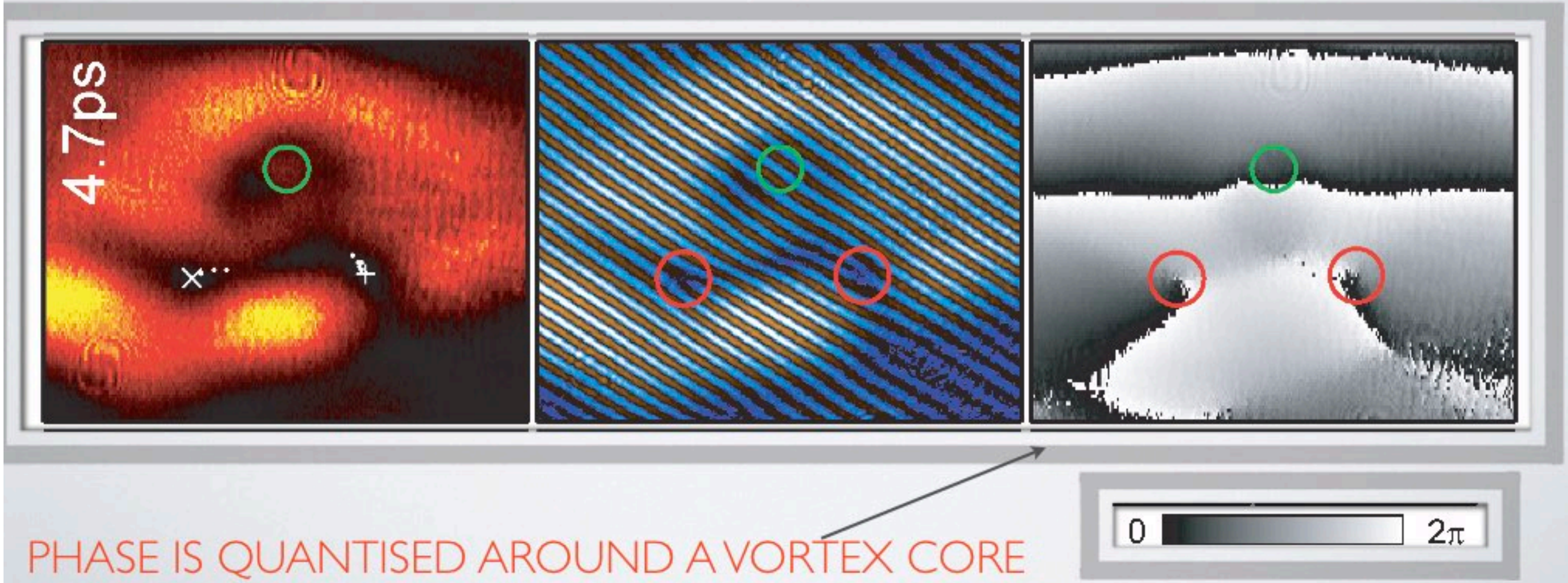
DISTRIBUTION OF POLARITON POPULATION



Interferogram
reminder

1. observe interferogram
2. calculate FFT (fast Fourier transform)
3. distinguish amplitude and phase

amplitude ⟵ *interferogram* ⟶ *phase*



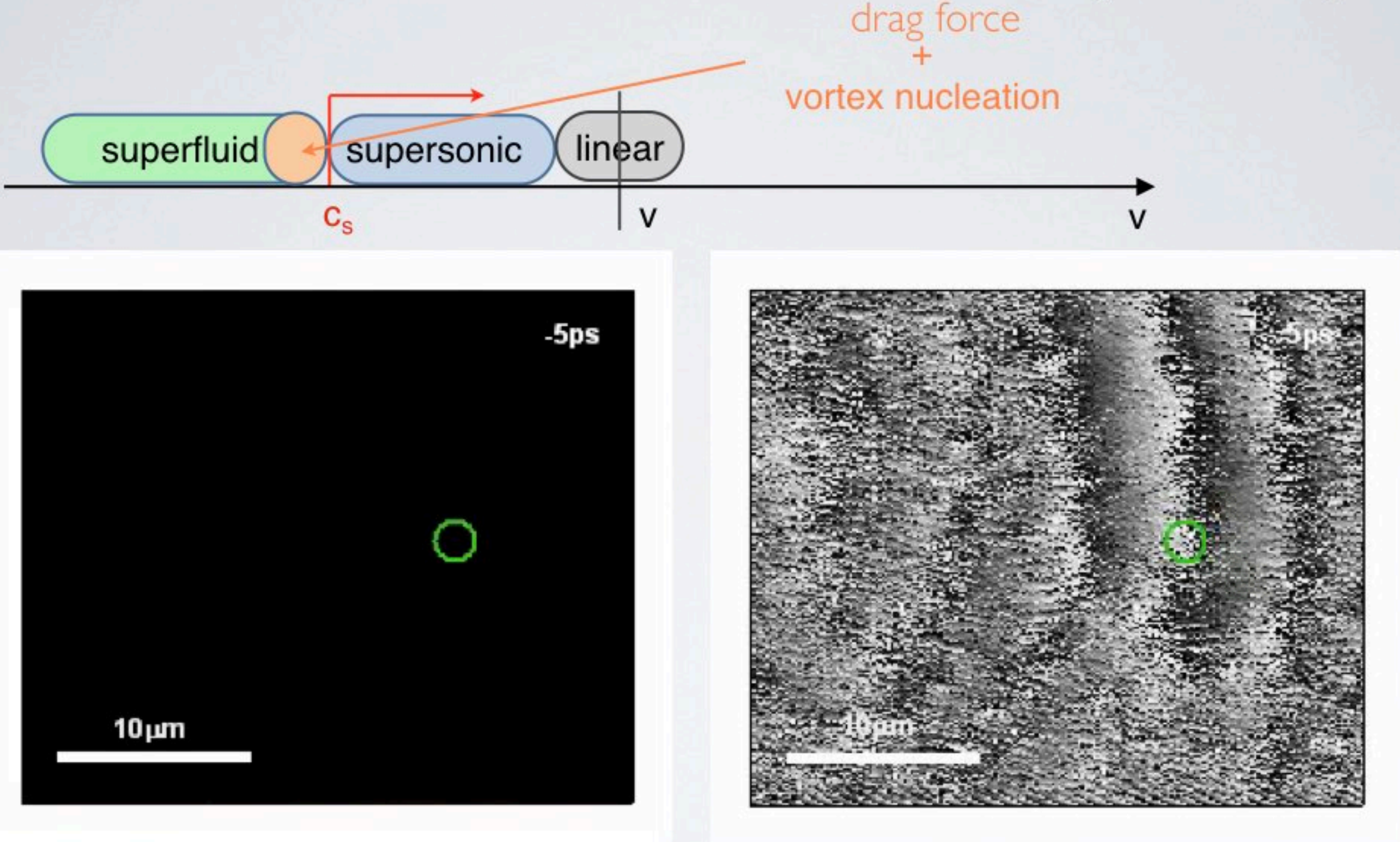
PHASE IS QUANTISED AROUND A VORTEX CORE

Linear regime

low polariton densities - low sound speed

elastic scattering

supersonic speeds



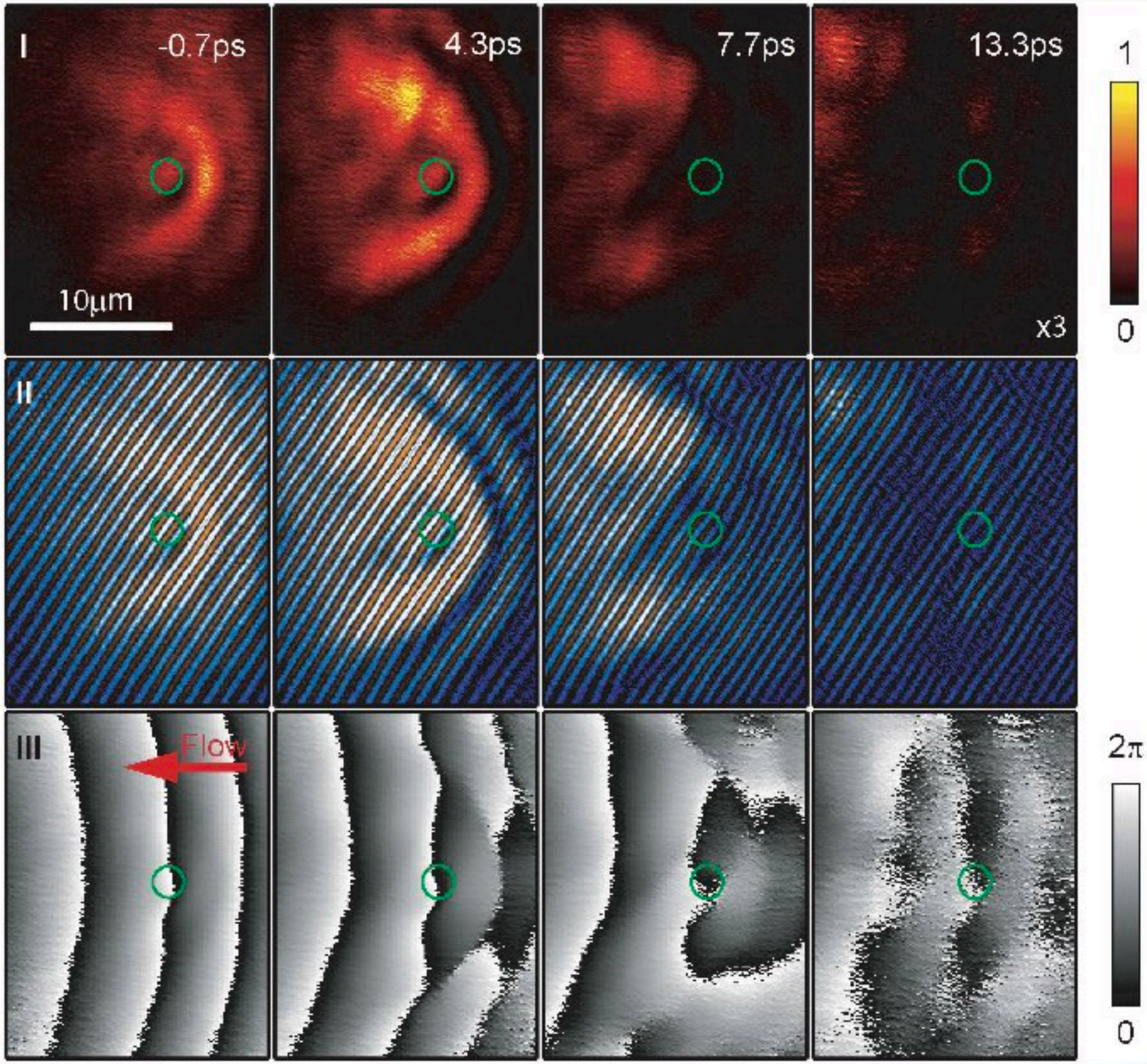
polariton flow

$$v = 1.13 \frac{\mu m}{ps}$$

Linear regime

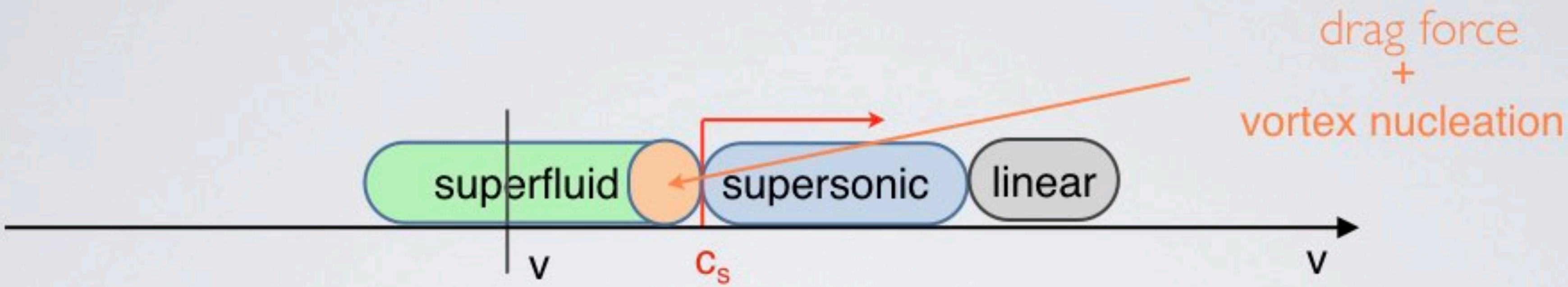
low polariton densities

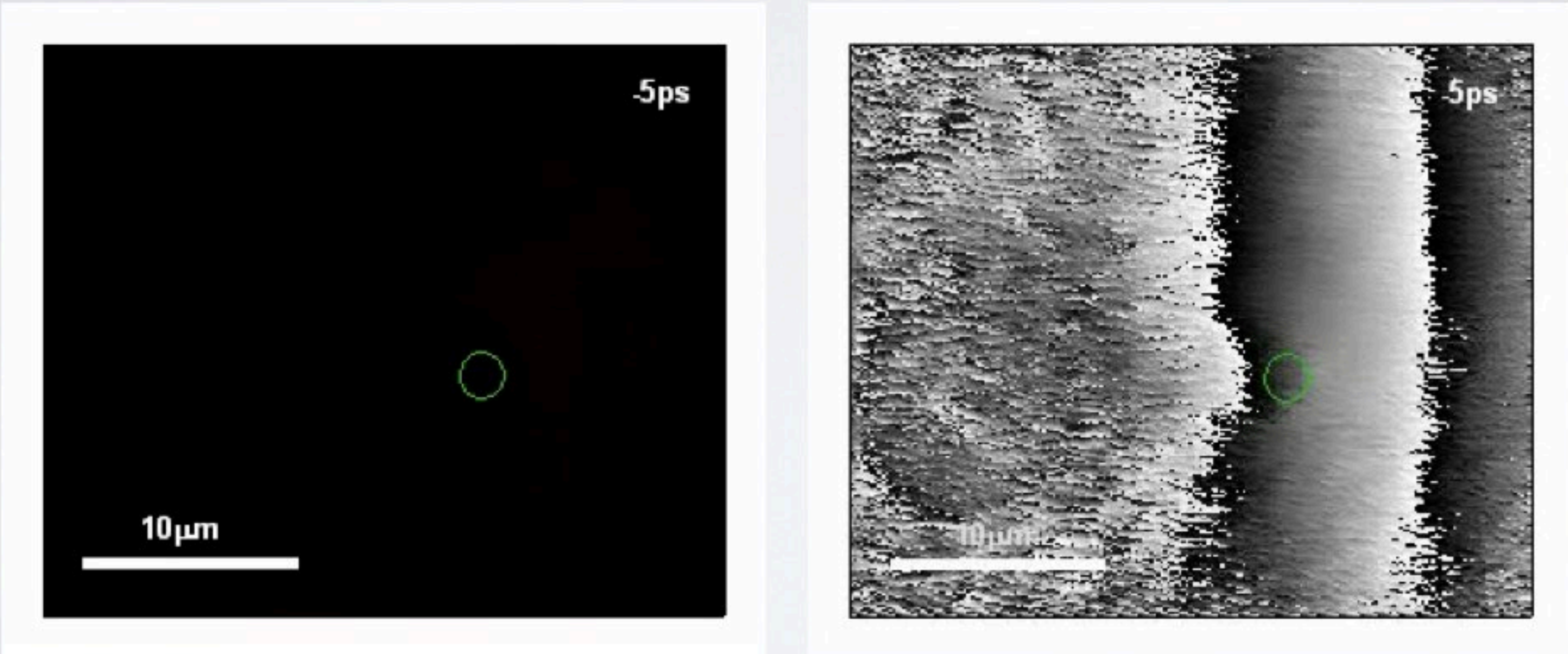
- low sound speed



Superfluid regime

high polariton densities - high sound speed



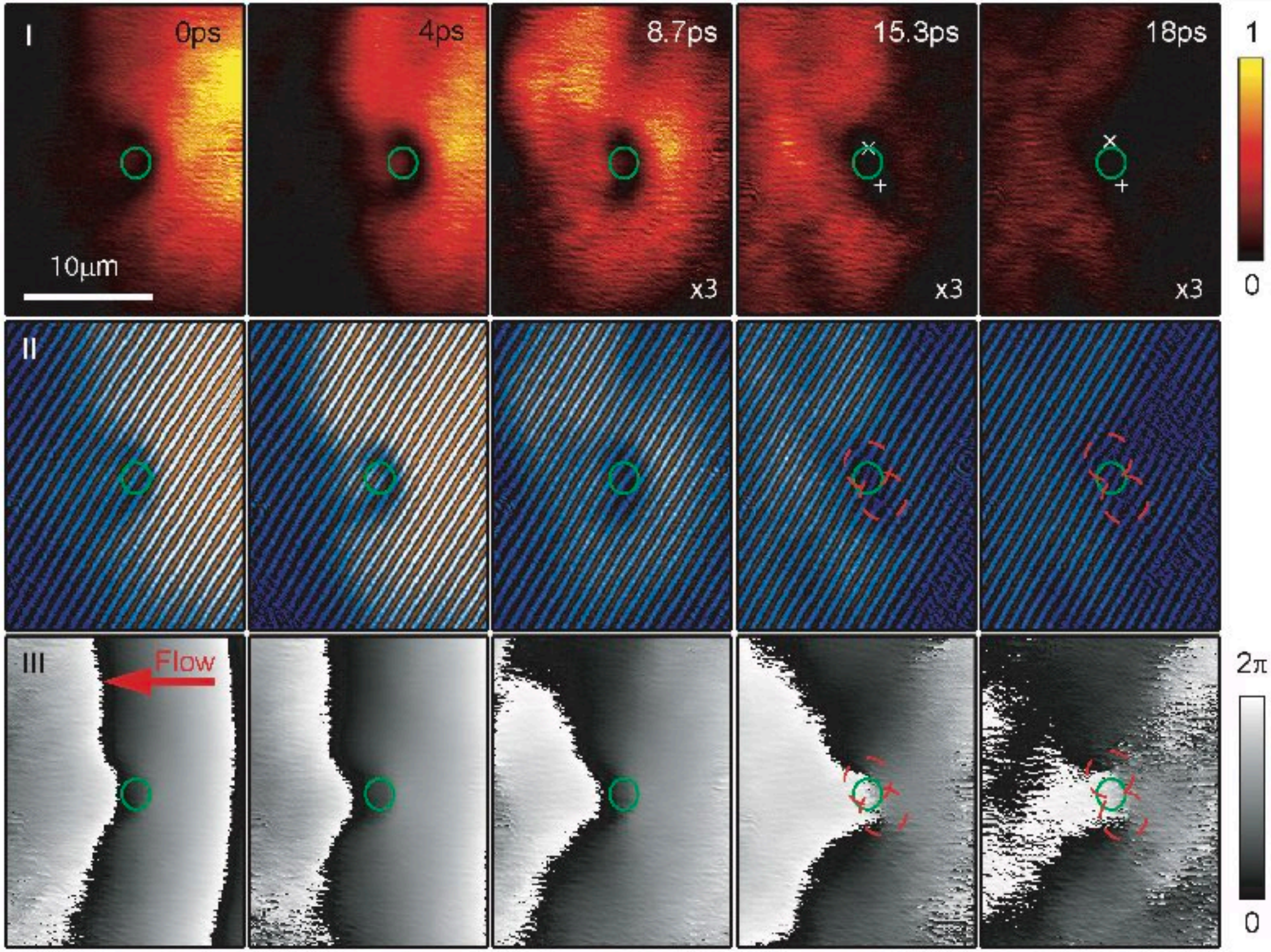


polariton flow

$$v = 0.6\,\frac{\mu m}{ps}$$

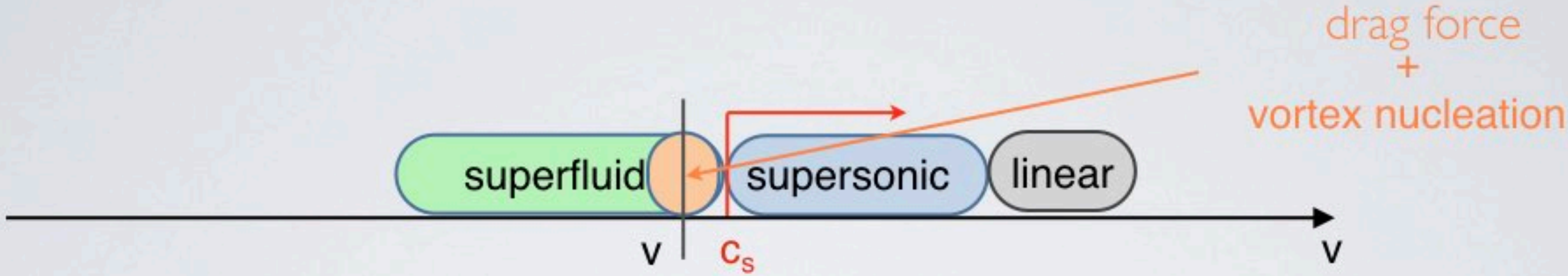
Superfluid regime

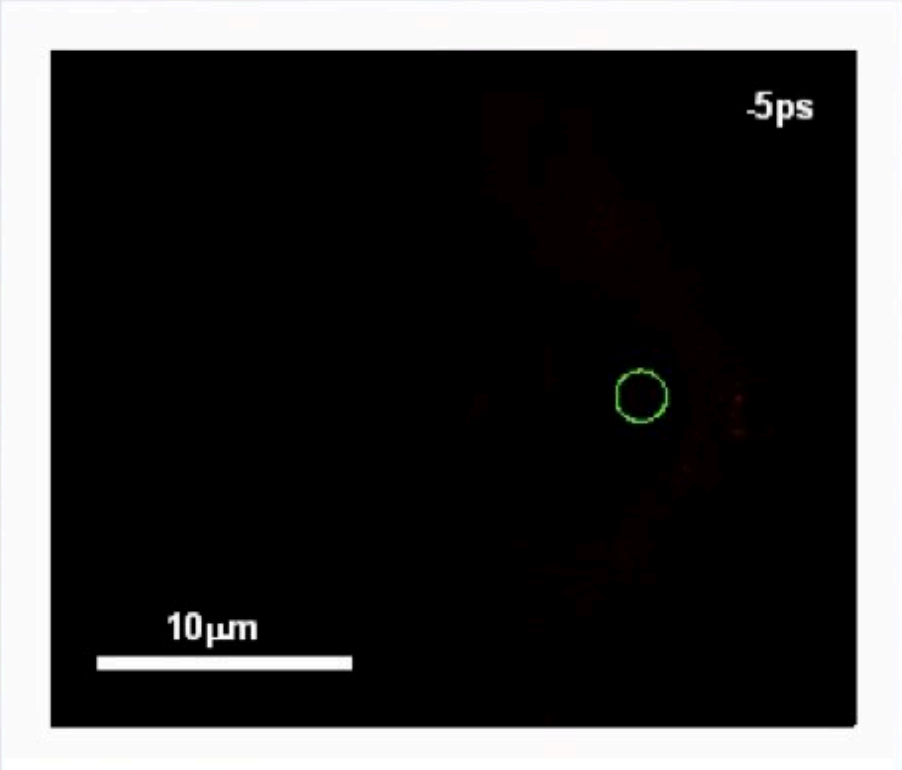
high polariton densities - high sound speed

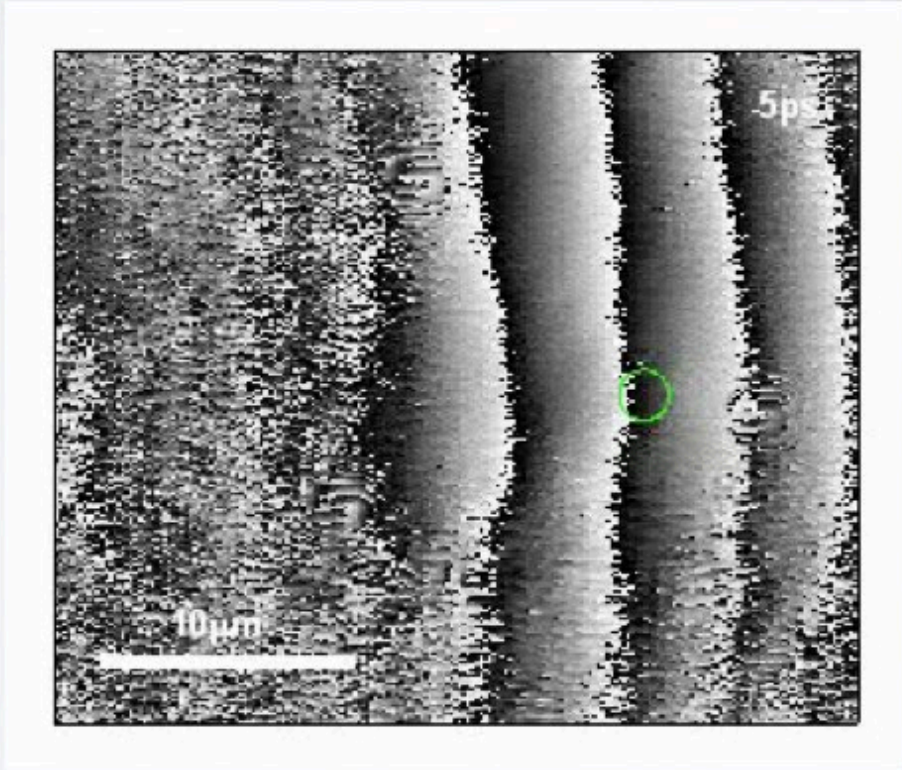


Vortex nucleation regime

high polariton densities - high sound speed





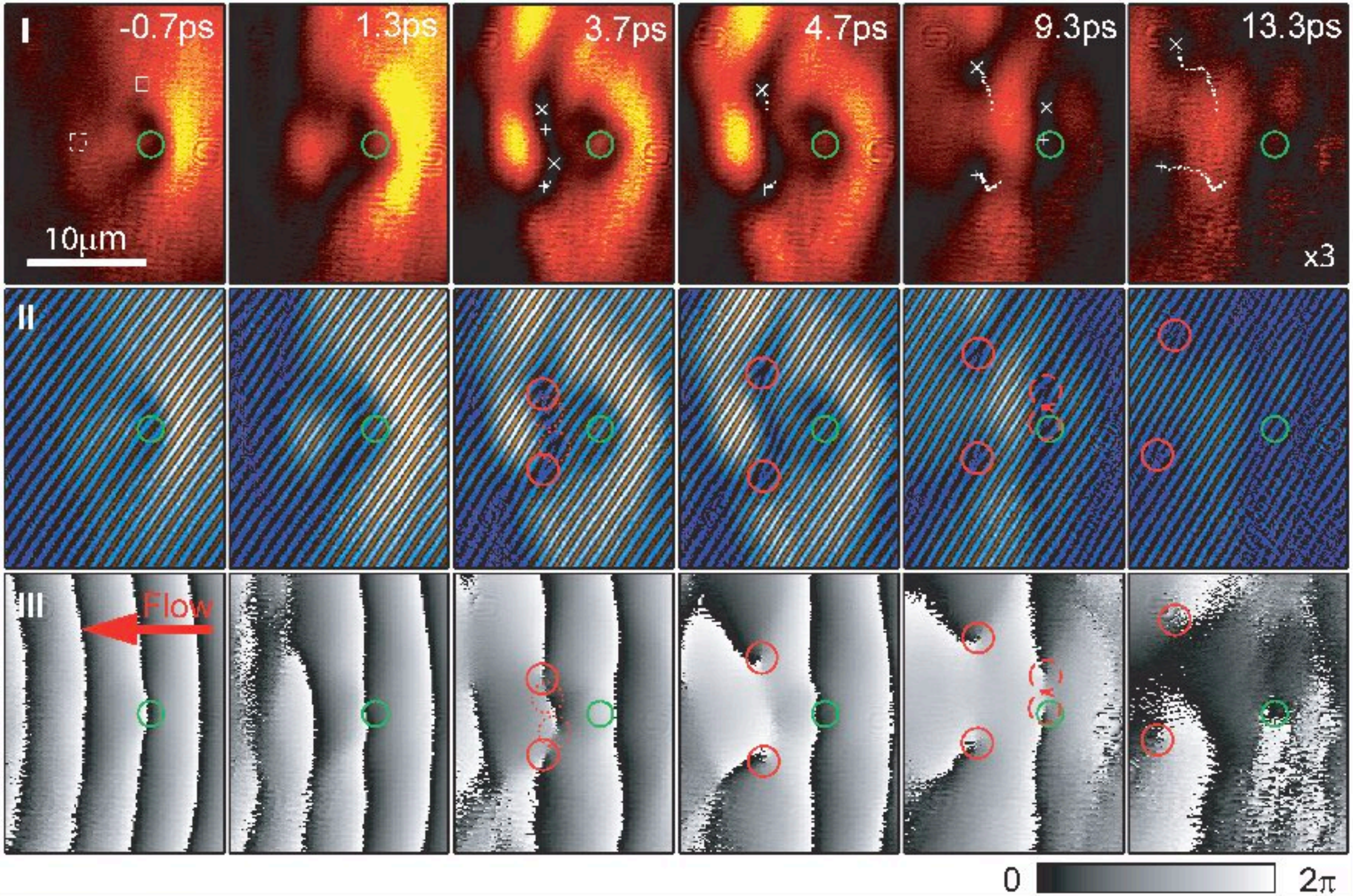


← polariton flow

$$v = 1.13\frac{\mu m}{ps}$$

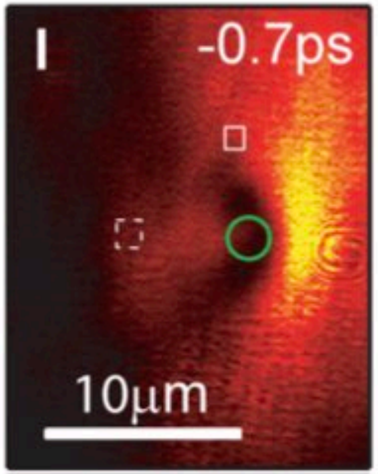
Vortex nucleation regime

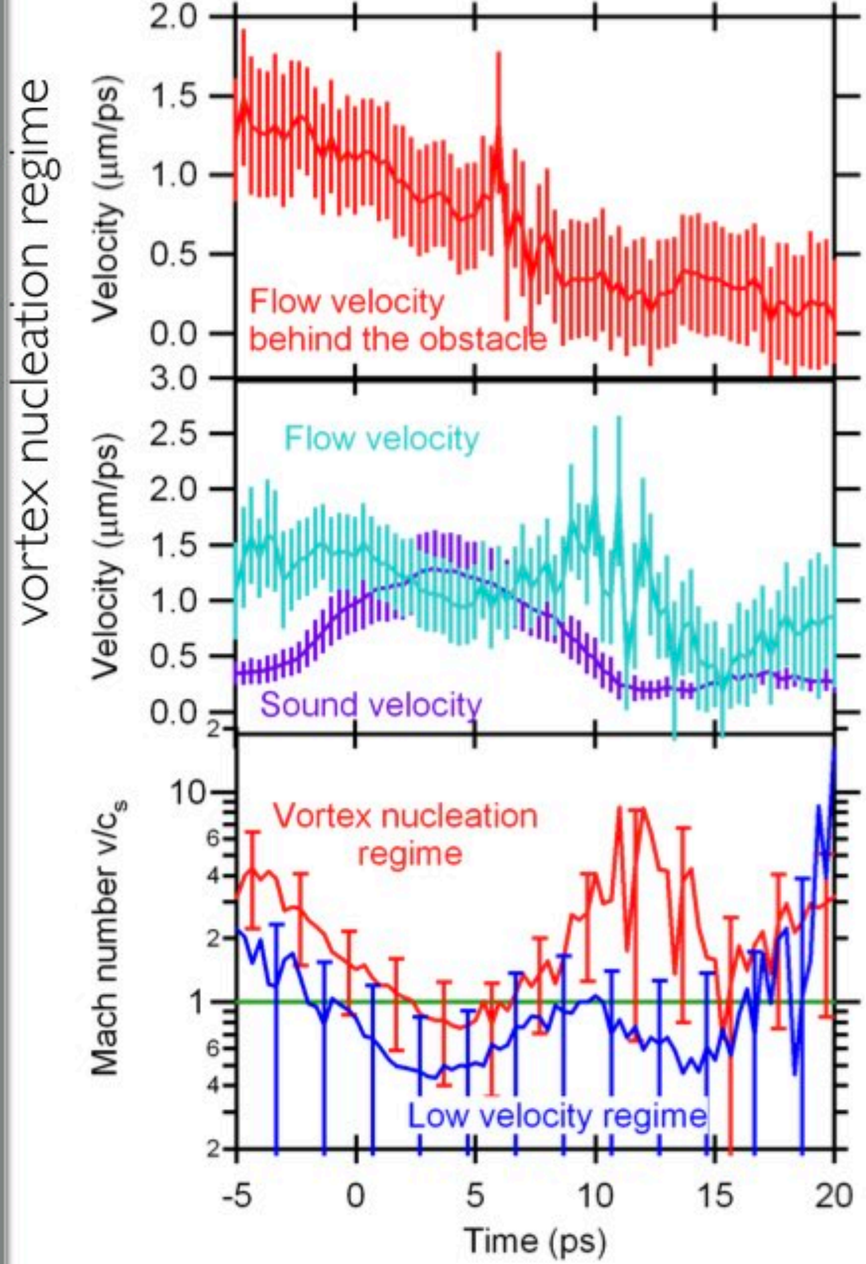
high polariton densities - high sound speed



quantitative analysis

$$\vec{v} = \frac{\hbar}{m}\vec{\nabla}\phi \qquad c_s = \sqrt{\frac{ng}{m}}$$





NUMERICAL SIMULATIONS

Gross-Pitaevskii equation:

obstacle $V = 1meV, 2\mu m \times 5\mu m$

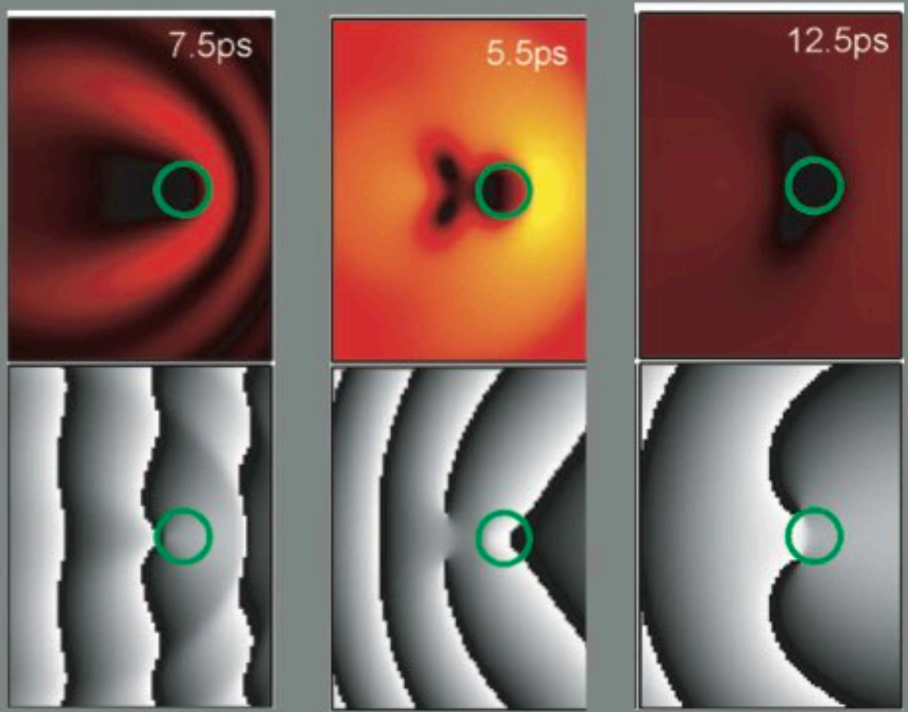
$$i\hbar\frac{d}{dt}\psi(r,t) = -\frac{\hbar^2}{2m}\nabla^2\psi(r,t) - i\frac{\gamma}{2}\psi(r,t) + g|\psi(r,t)|^2\psi(r,t) + V\psi(r,t) + F_p(r,t)$$

kinetic term $m = 0.7meV \cdot ps^2 \cdot \mu m^{-2}$

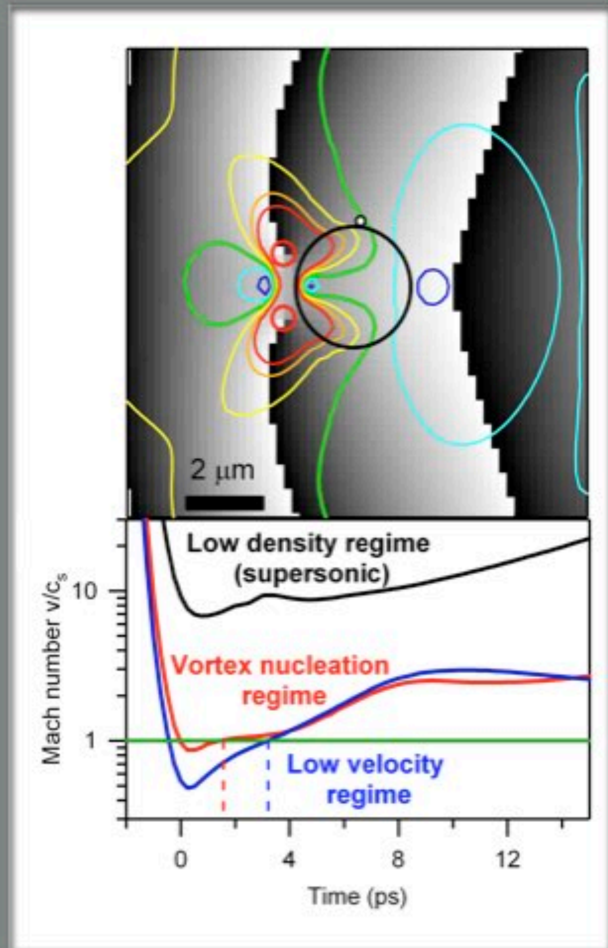
dissipation $\gamma = \hbar / 15ps$

pol-pol interactions $g = 0.01meV \cdot \mu m^2$

driving field



low density regime | vortex nucleation regime | low velocity regime



Quantum fluid
& classical fluid

Classical fluid

physical properties are purely determined by the laws of classical statistical mechanics

Quantum fluid

remains fluid (i.e. gas or liquid) at such low temperatures that the effects of quantum mechanics play a dominant role

→ **quantum fluids of light**