

Polaritons and polariton condensation: A theorist's point of view

Michał Matuszewski

Institute of Physics, Polish Academy of Sciences, Warsaw



Exciton-polariton BEC Research Group



**Institute of Physics
Polish Academy of Sciences**

$$i \frac{\partial \psi}{\partial t} = \left[-\frac{\partial^2}{\partial x^2} + \frac{i}{2} (Rn_R - \gamma) + g|\psi|^2 \right] \psi$$

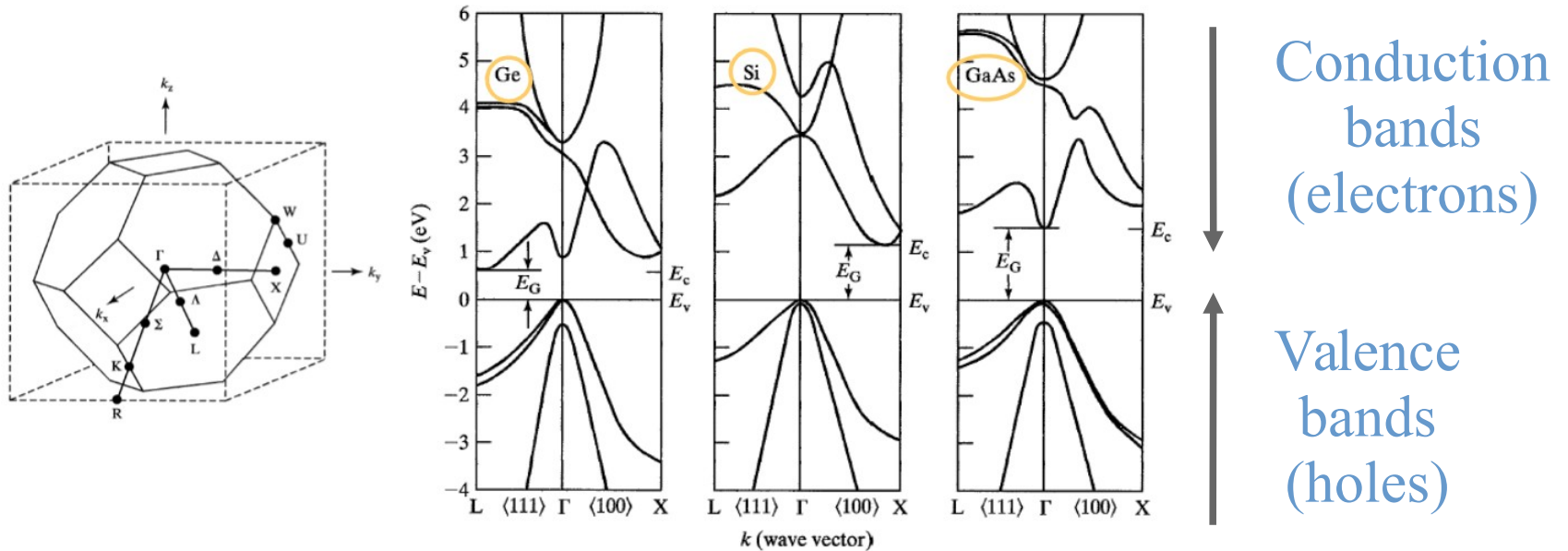
$\frac{\partial n_R}{\partial t} =$

Band structure of semiconductors

Schrödinger equation for an electron in a crystal lattice

$$\frac{-\hbar^2}{2m_0} \nabla^2 \Psi_{\vec{k},n} + V(\vec{r}) \Psi_{\vec{k},n} = E_{\vec{k},n} \Psi_{\vec{k},n}$$

Solutions result in a band structure, valence and conduction bands

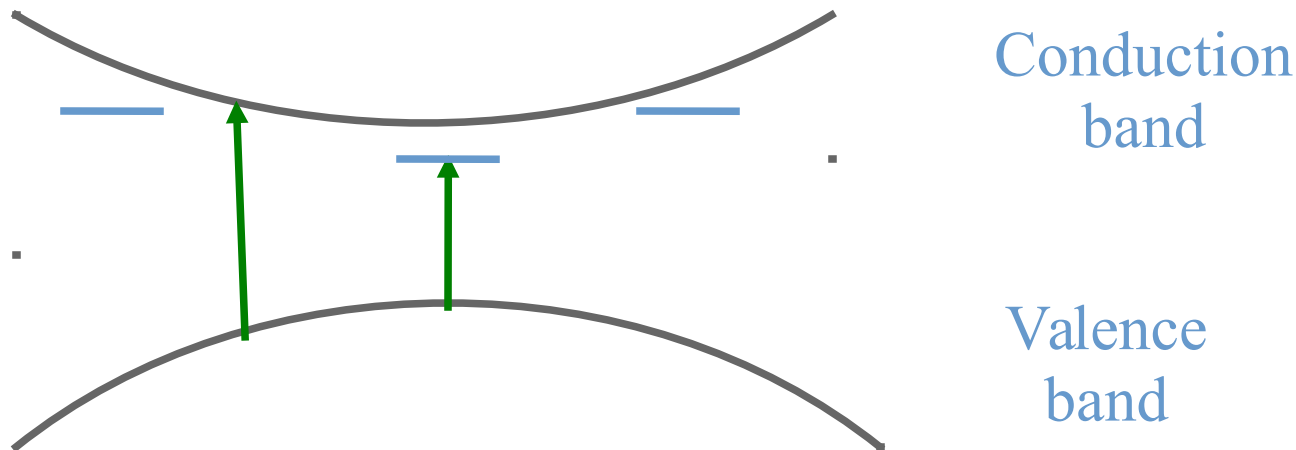


Excitons

Interaction between electrons and holes

$$\sum \left(\frac{-\hbar^2}{2m_0} \nabla_i^2 \Psi + V(\vec{r}_i) \Psi \right) + \sum V(\vec{r}_i - \vec{r}_j) \Psi = E \Psi(r_1, \dots, r_N)$$

results in bound states of electrons and holes – excitons



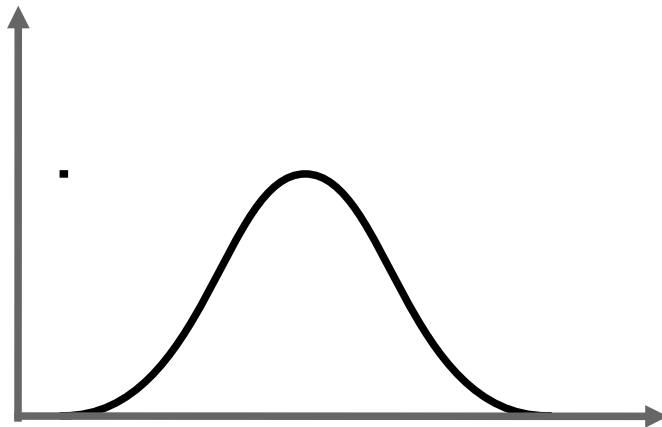
Exciton energy is lower than E_g by the interaction energy

Quantum superposition

- In quantum mechanics, addition of states means that the particle is in a superposition of these states, eg. the Schrödinger cat state

$$|\psi\rangle = \alpha|\text{electron here}\rangle + \beta|\text{electron there}\rangle + \dots$$

- The probability of finding the particle in a given state is $|\psi|^2$

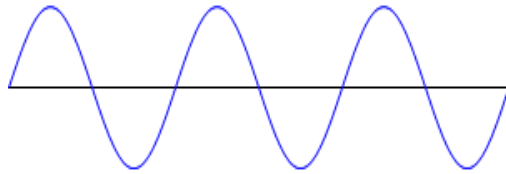


Real and Fourier space – Heisenberg uncertainty principle

Real space

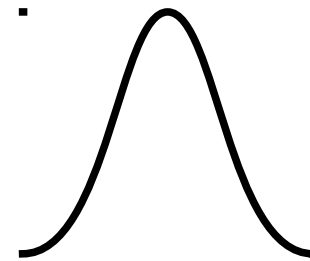
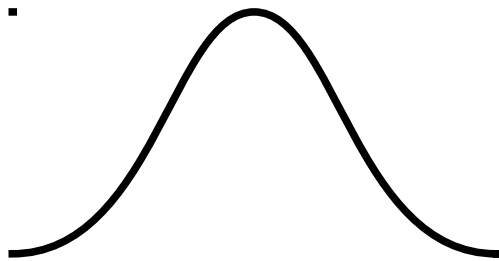
Fourier space

$$e^{ik_0x}$$

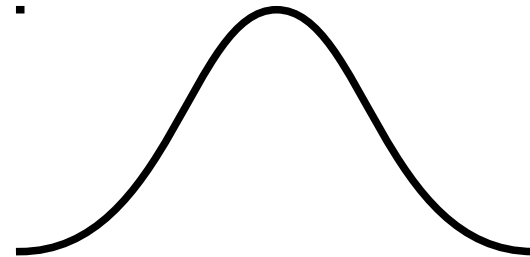
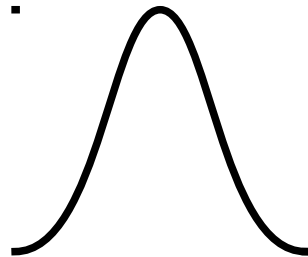


$$\delta(k - k_0)$$

$$e^{-\frac{1}{2}\left(\frac{x}{W}\right)^2}$$



$$e^{-\frac{1}{2}(kW)^2}$$



Excitons

The exciton operator is a “wave packet” made of electrons and holes

$$X_\nu(\mathbf{k}) \equiv \sum_{\mathbf{p}} \varphi_\nu(\mathbf{p}) h_{\mathbf{k}/2-\mathbf{p}} e_{\mathbf{k}/2+\mathbf{p}}$$

How to understand this?

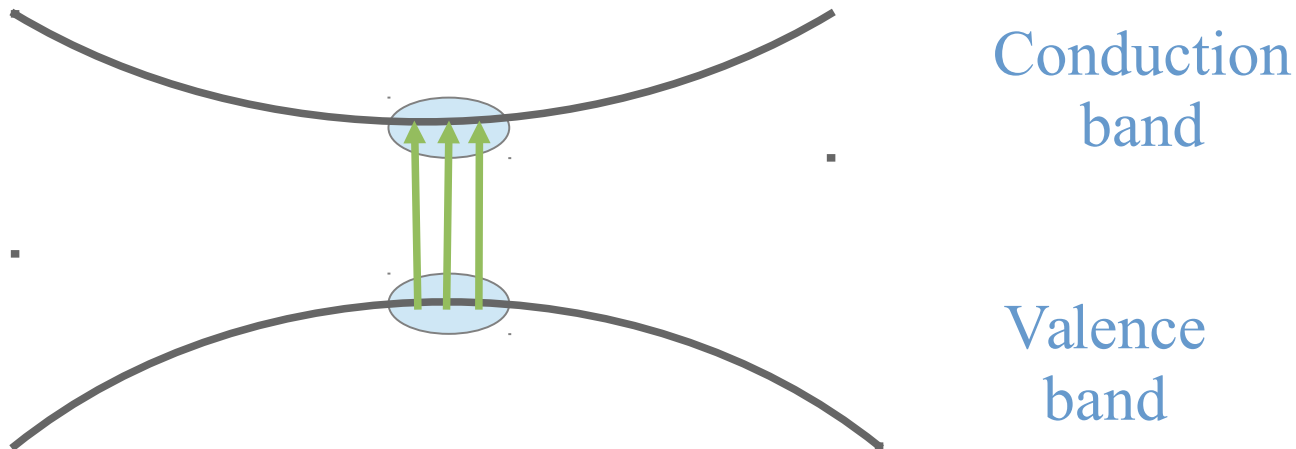
$|G\rangle$ - Ground state (all electrons in the valence band)

$$X^+(0) |G\rangle = \alpha |\text{electron in state } k_1, \text{ hole in state } -k_1\rangle + \\ + \beta |\text{electron in state } k_2, \text{ hole in state } -k_2\rangle + \dots$$

Excitons

$$X^+(0) |G\rangle = \alpha |\text{electron in state } k_1, \text{ hole in state } -k_1\rangle + \beta |\text{electron in state } k_2, \text{ hole in state } -k_2\rangle + \dots$$

$$X_\nu(\mathbf{k}) \equiv \sum_{\mathbf{p}} \varphi_\nu(\mathbf{p}) h_{\mathbf{k}/2-\mathbf{p}} e_{\mathbf{k}/2+\mathbf{p}}$$

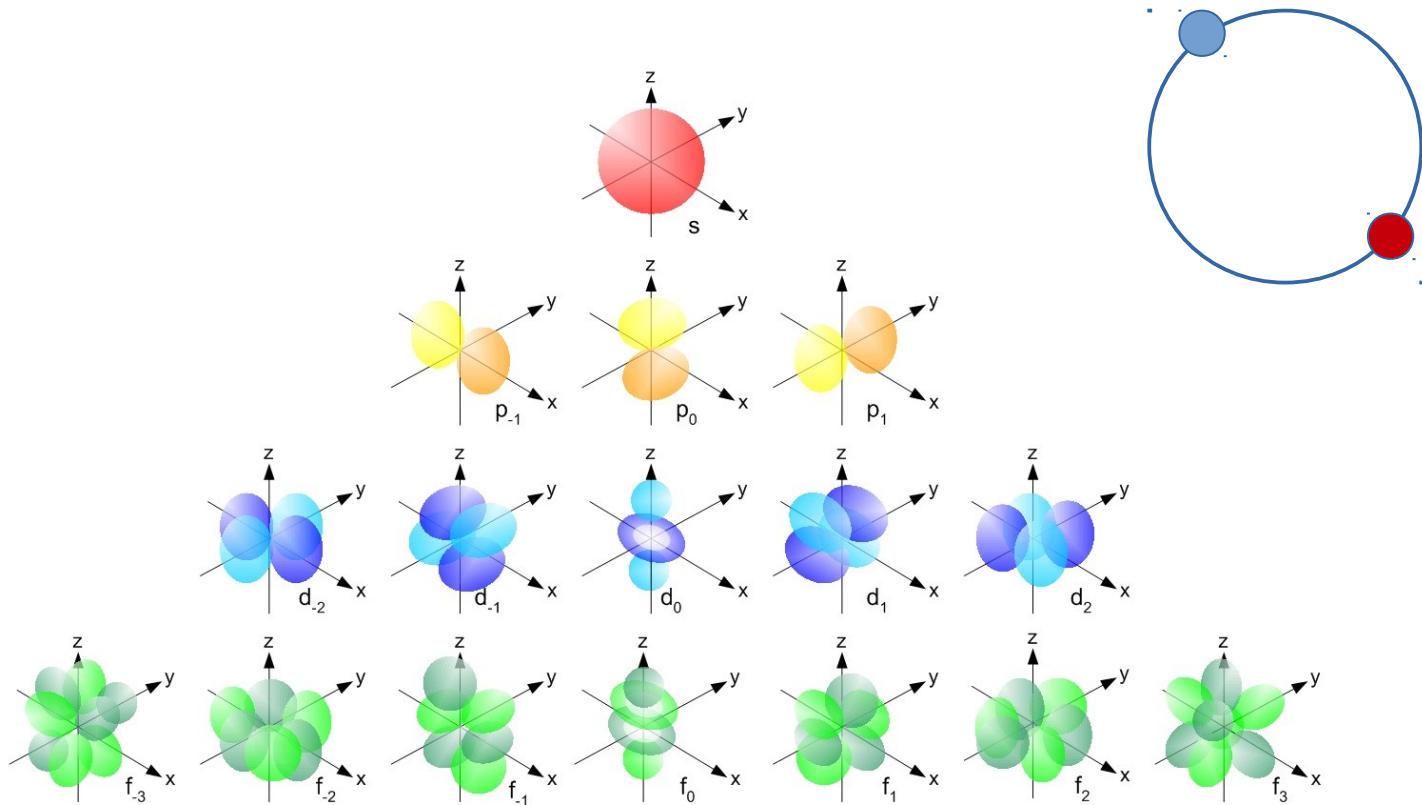


A more correct picture! $h_{\mathbf{k}} = e_{v,-\mathbf{k}}^\dagger$

Exciton state

The relative electron-hole motion is analogous to the hydrogen atom

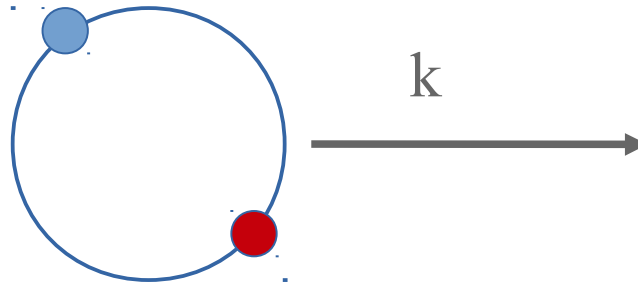
$$\Psi_{rel}(\vec{r}_e, \vec{r}_h) \approx \Psi_{nlm}(\vec{r}_e - \vec{r}_h)$$



Exciton state

The motion of exciton as a whole

$$\Psi(\vec{r}_e, \vec{r}_h) \approx \Psi_{rel}(\vec{r}_e - \vec{r}_h) \exp(i \vec{k} \cdot \vec{R}), \quad \vec{R} = \frac{\vec{r}_e m_e + \vec{r}_h m_h}{m_e + m_h}$$



Are excitons bosons or fermions?

Electron and hole have half-integer spin, exciton has integer spin

The commutation relation of true bosons

$$[X_\nu(\mathbf{k}), X_\mu^\dagger(\mathbf{q})] = \delta_{\nu,\mu} \delta_{\mathbf{k},\mathbf{q}}$$

For excitons we have

$$[X_\nu(\mathbf{k}), X_\mu^\dagger(\mathbf{q})] = \delta_{\nu,\mu} \delta_{\mathbf{k},\mathbf{q}} - \sum_{\mathbf{p}} |\varphi_{1s}(\mathbf{p})|^2 (c_{\mathbf{k}}^\dagger c_{\mathbf{q}} + h_{-\mathbf{k}}^\dagger h_{-\mathbf{q}})$$

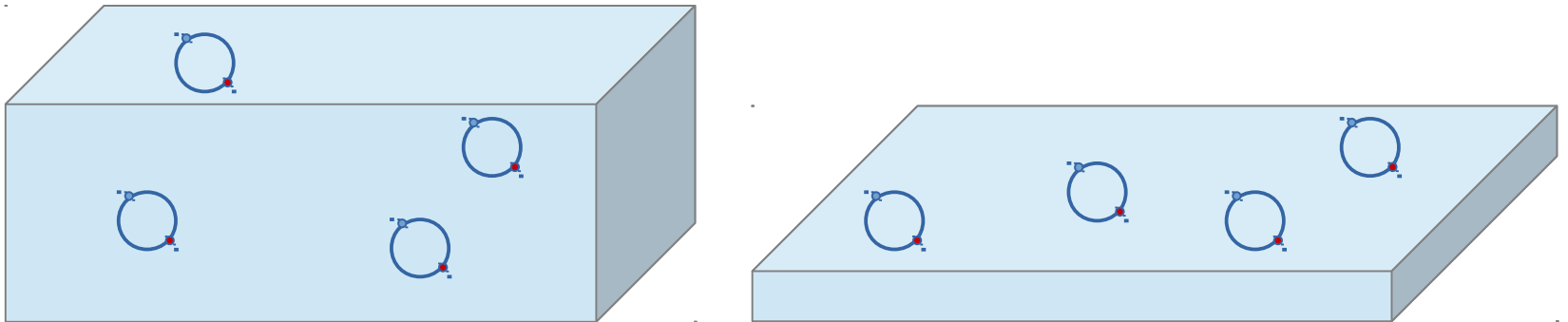
Excitons are well approximated by bosons in the low density limit

$$Na_0^2 \ll 1$$

Are excitons bosons or fermions?

The exciton is in a superposition of being all over the place!
But the electron and hole are bound together

$$\Psi(\vec{r}_e, \vec{r}_h) \approx \Psi_{rel}(\vec{r}_e - \vec{r}_h) \exp(i\vec{k} \cdot \vec{R}) \quad Na_0^2 \ll 1$$

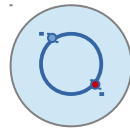


This is true for bulk (3D) and quantum well (2D) excitons.

They behave as bosons, because we can put many of them in the same state.

Are excitons bosons or fermions?

In small quantum dots (0D), the excitons behave as fermions:
we cannot put more than one in the same state



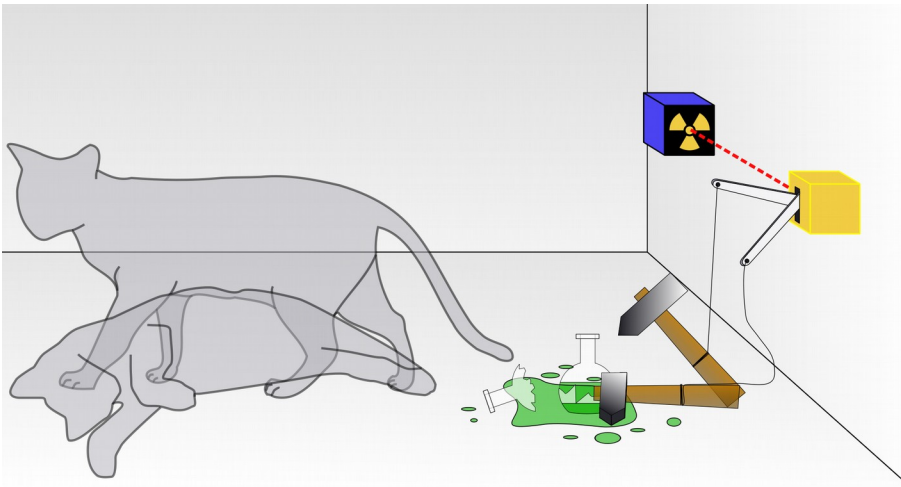
$$Na_0^2 \approx 1$$

In this limit, we „see” the fermionic nature of the constituents

Polaritons

The excitons are no longer correct quantum states if the strong coupling of excitons to photons is introduced.

Polariton is a superposition of an exciton and a photon (a Schrödinger cat)

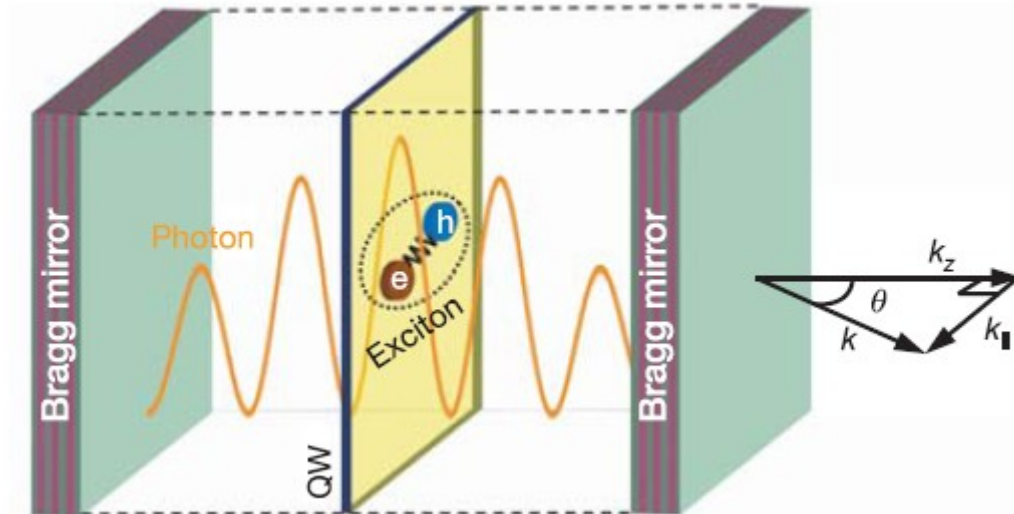
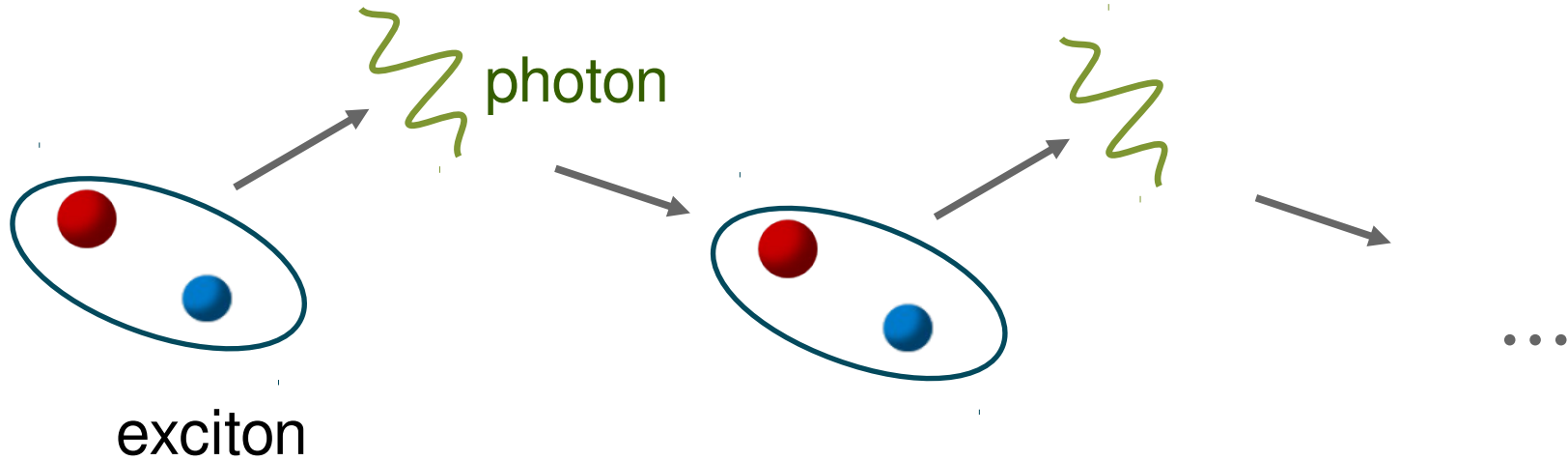


$$\frac{1}{\sqrt{2}}|\text{cat sitting}\rangle + \frac{1}{\sqrt{2}}|\text{cat lying}\rangle$$

$$|\text{polariton}\rangle = \alpha |\text{exciton}\rangle + \beta |\text{photon}\rangle$$

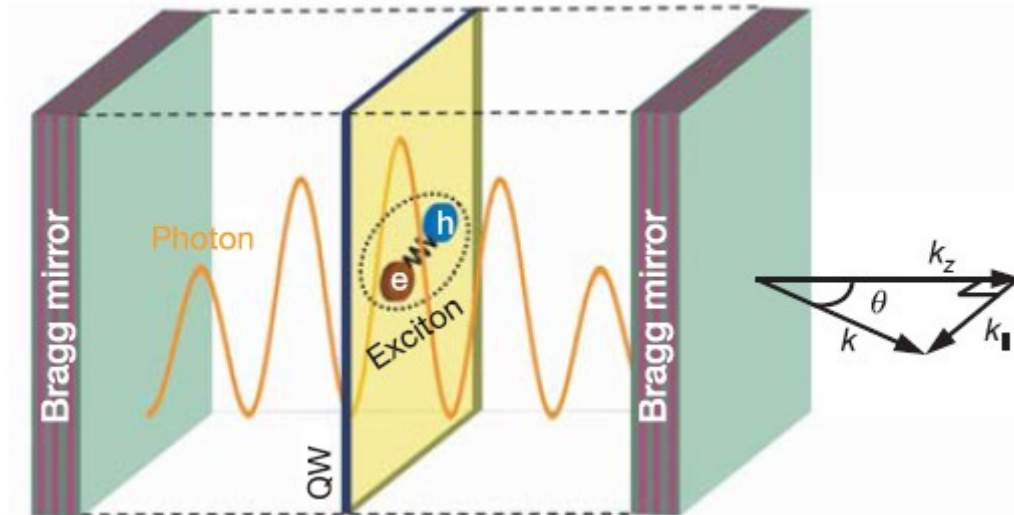
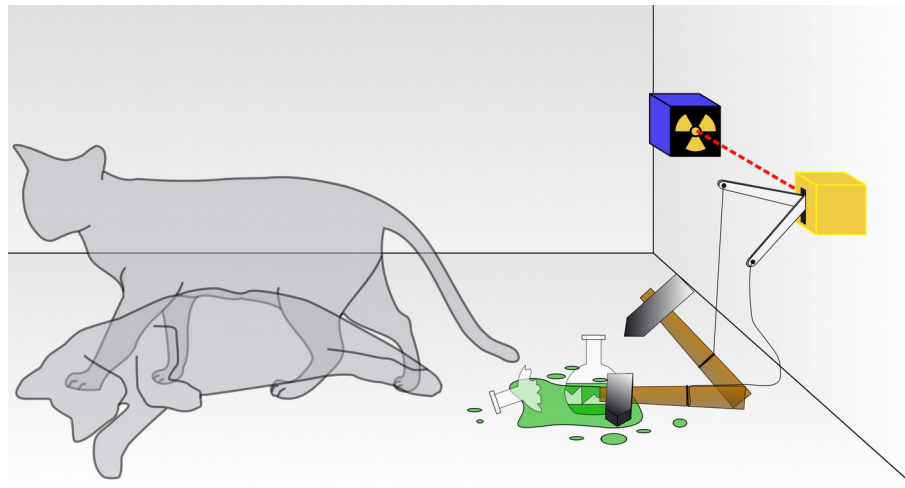
Polaritons

„Classical picture”: exciton periodically transforms into photon (Rabi)



Polaritons

„Quantum picture”: there is a superposition of the two possibilities



Polaritons

Exciton: $a_{\mathbf{k}}$, photon: $c_{\mathbf{k}}$ coupling: Ω

$$\hat{H} = \sum_{\mathbf{k}} E_X(\mathbf{k}, B) \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + E_C(\mathbf{k}) \hat{c}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \frac{1}{2} \Omega(B, \mathbf{k}) (\hat{a}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \hat{c}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}})$$

Eigenstates consist of lower and upper polaritons, mixed states of excitons and photons

Do polaritons exist without pumping?

Polaritons are correct quantum eigenstates even in the vacuum

The vacuum fluctuations provide virtual polaritons even without pumping!

$$\hat{H} = \sum_{\mathbf{k}} E_X(\mathbf{k}, B) \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + E_C(\mathbf{k}) \hat{c}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \frac{1}{2} \Omega(B, \mathbf{k}) (\hat{a}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \hat{c}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}})$$

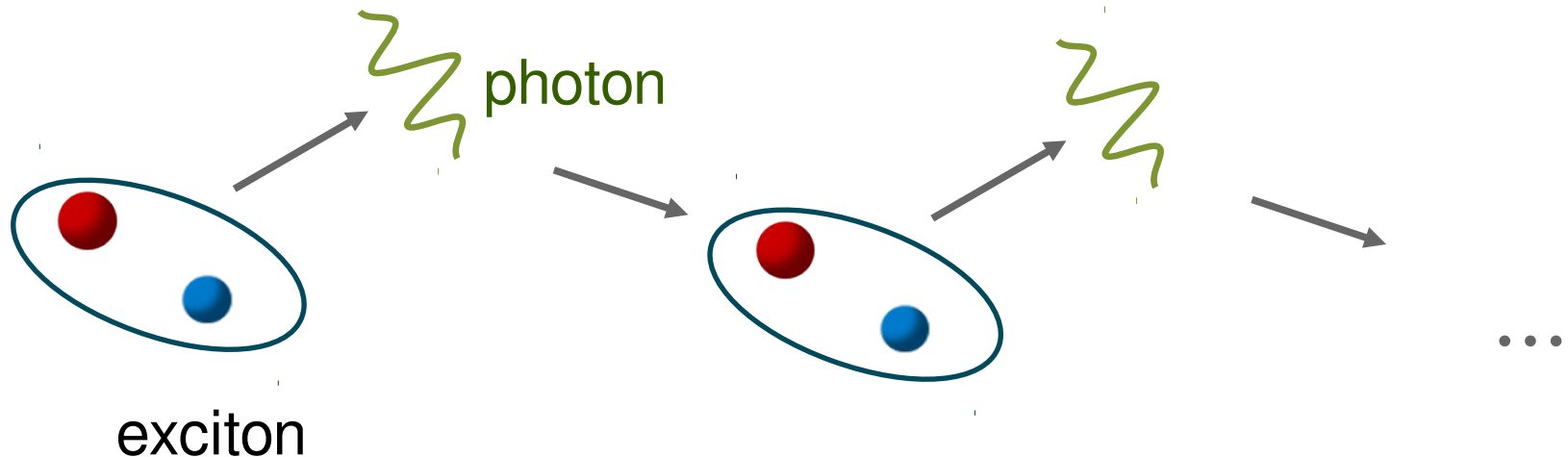
In the basis of states $|1,0\rangle$ and $|0,1\rangle$

$$\hat{H} = \begin{pmatrix} E_X & \Omega/2 \\ \Omega/2 & E_C \end{pmatrix} \quad |\text{polariton}\rangle = \alpha |1,0\rangle + \beta |0,1\rangle$$

Polaritons: weak and strong coupling

Strong coupling means that the decoherence rate is lower than the conversion rate (Rabi oscillations)

Possible sources of decoherence: finite lifetime, interactions with the crystal lattice, between polaritons etc.



$$\hat{H} = \sum_{\mathbf{k}} E_X(\mathbf{k}, B) \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + E_C(\mathbf{k}) \hat{c}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \frac{1}{2} \Omega(B, \mathbf{k}) (\hat{a}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \hat{c}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}})$$

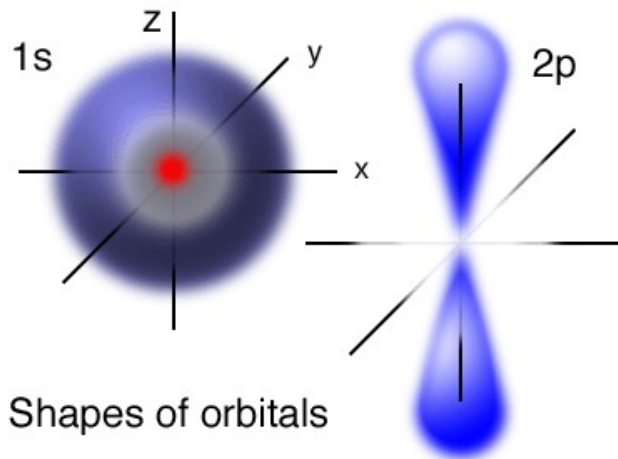
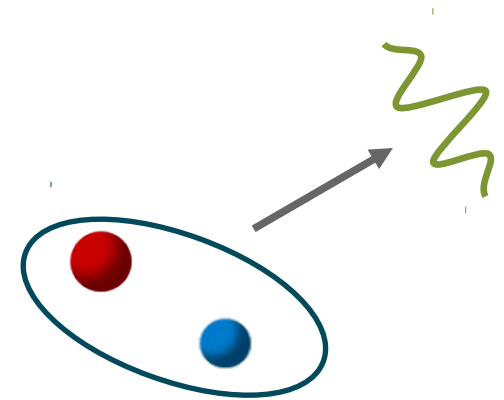
$$|\text{polariton}\rangle = \alpha |\text{exciton}\rangle + \beta |\text{photon}\rangle$$

Polariton coupling strength

Coupling strength is proportional to the probability of conversion of a photon to an exciton and vice versa

This can occur only when electron and hole are “in the same place”

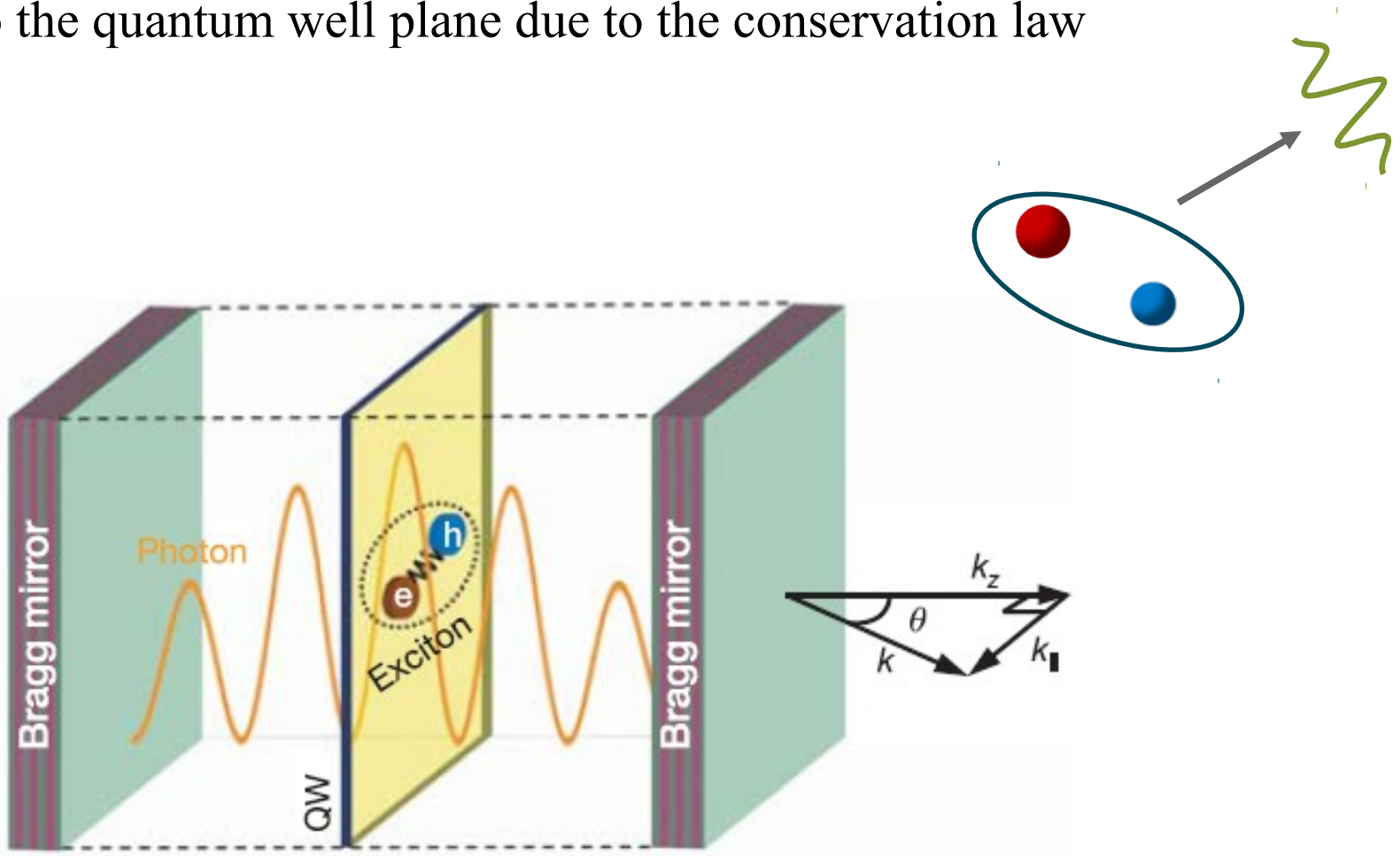
Selection rules: only s states are active



$$\Psi_{rel}(\vec{r}_e, \vec{r}_h) \approx \Psi_{nlm}(\vec{r}_e - \vec{r}_h)$$

Polariton momentum

Excitons and photons share the same momentum parallel to the quantum well plane due to the conservation law



Bosons, fermions and...

Bosons



Classical particles

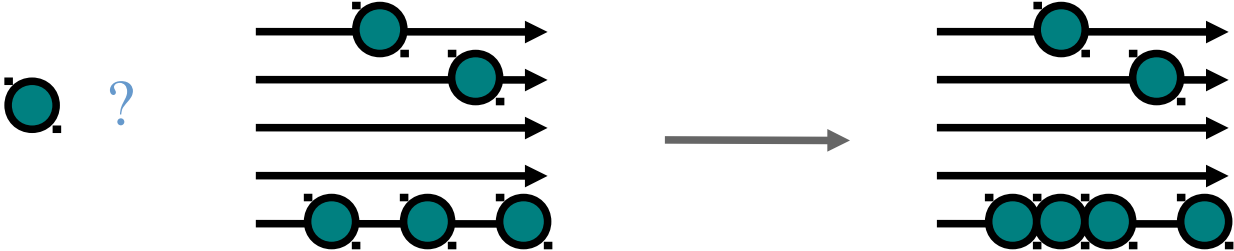


Fermions



Bosonic enhancement

Bosonic particles prefer to be where the others are



This is the most probable choice!

Thermal equilibrium distributions

In thermal equilibrium, the probability of finding the **system** in a certain state A is proportional to (Boltzmann distribution)

$$p_A \sim e^{-\frac{E_A - \mu N}{k_B T}}$$

The probability of finding a **particle** in the state a is

$$p_A \sim \frac{1}{e^{\frac{E_a - \mu}{k_B T}} - 1}$$

Bosons

$$p_A \sim e^{-\frac{E_a - \mu}{k_B T}}$$

Classical particles

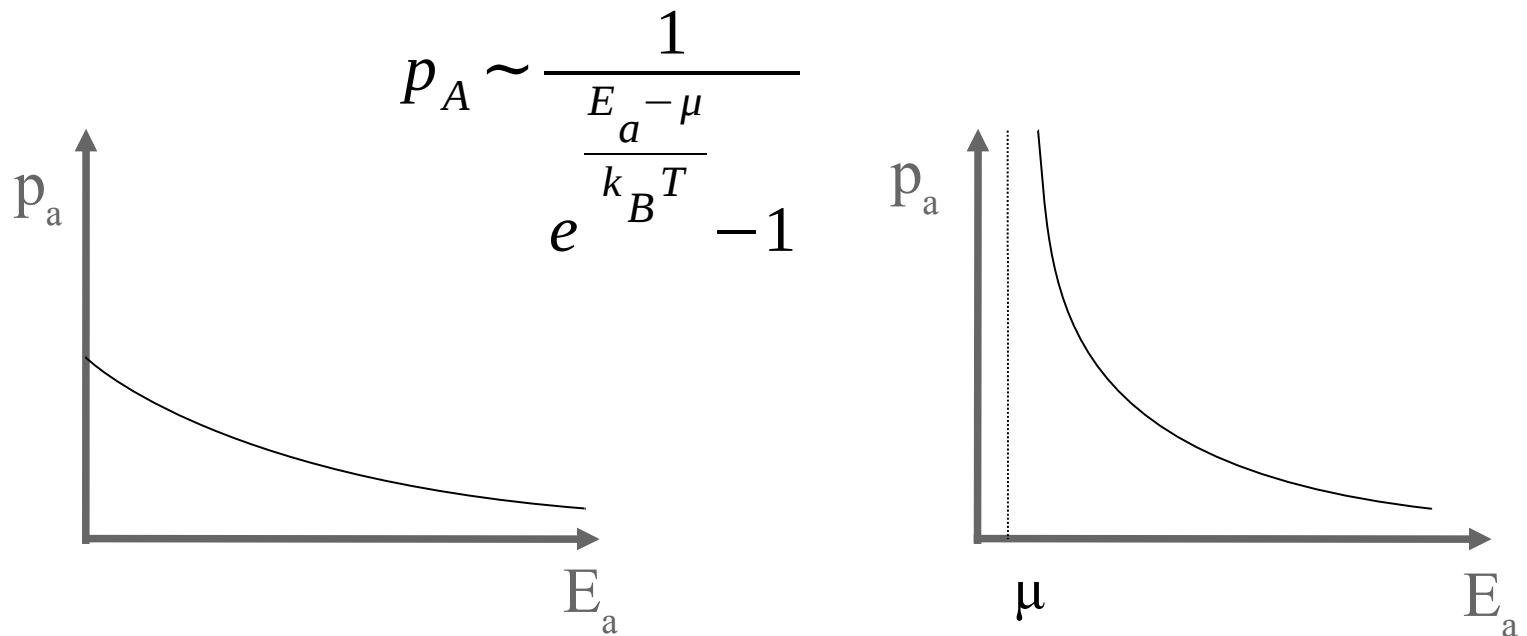
$$p_A \sim \frac{1}{e^{\frac{E_a - \mu}{k_B T}} + 1}$$

Fermions

Bose-Einstein condensation

At some point, divergence may occur.

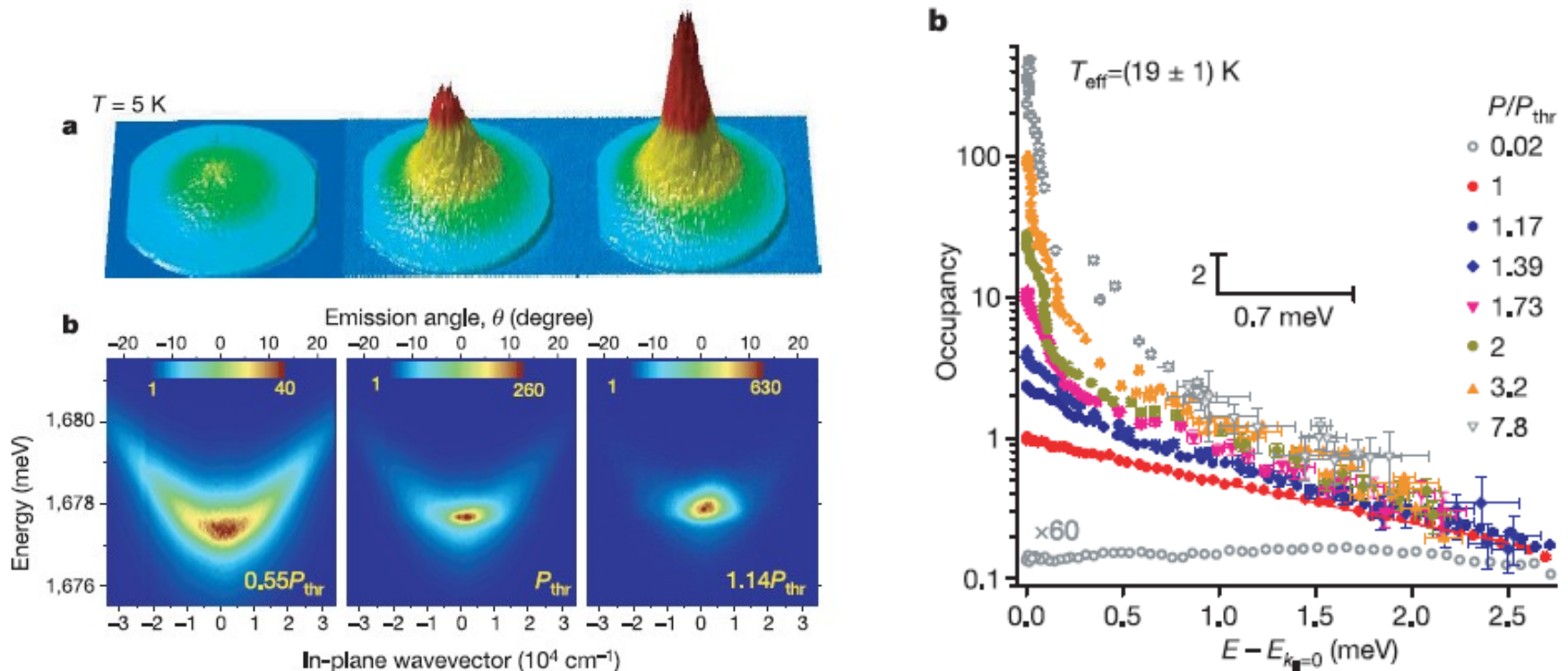
This means that the occupation of the lowest state becomes macroscopic



This can be achieved in two ways: decreasing T or increasing μ (number of particles)

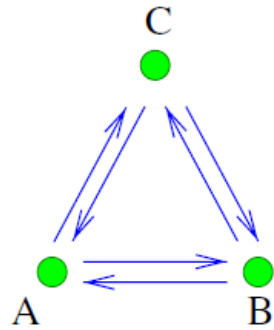
Bose-Einstein condensation

At some point, divergence may occur.
This means that the occupation of the lowest state becomes macroscopic

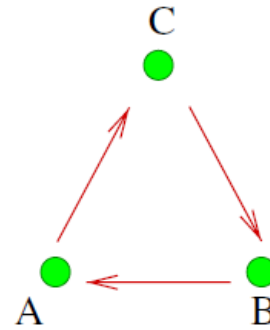


Above threshold pumping power, the ground state becomes massively occupied. The thermal cloud is found to be at 19K. (J. Kasprzak et al., Nature 2006)

Equilibrium and out of equilibrium



Equilibrium



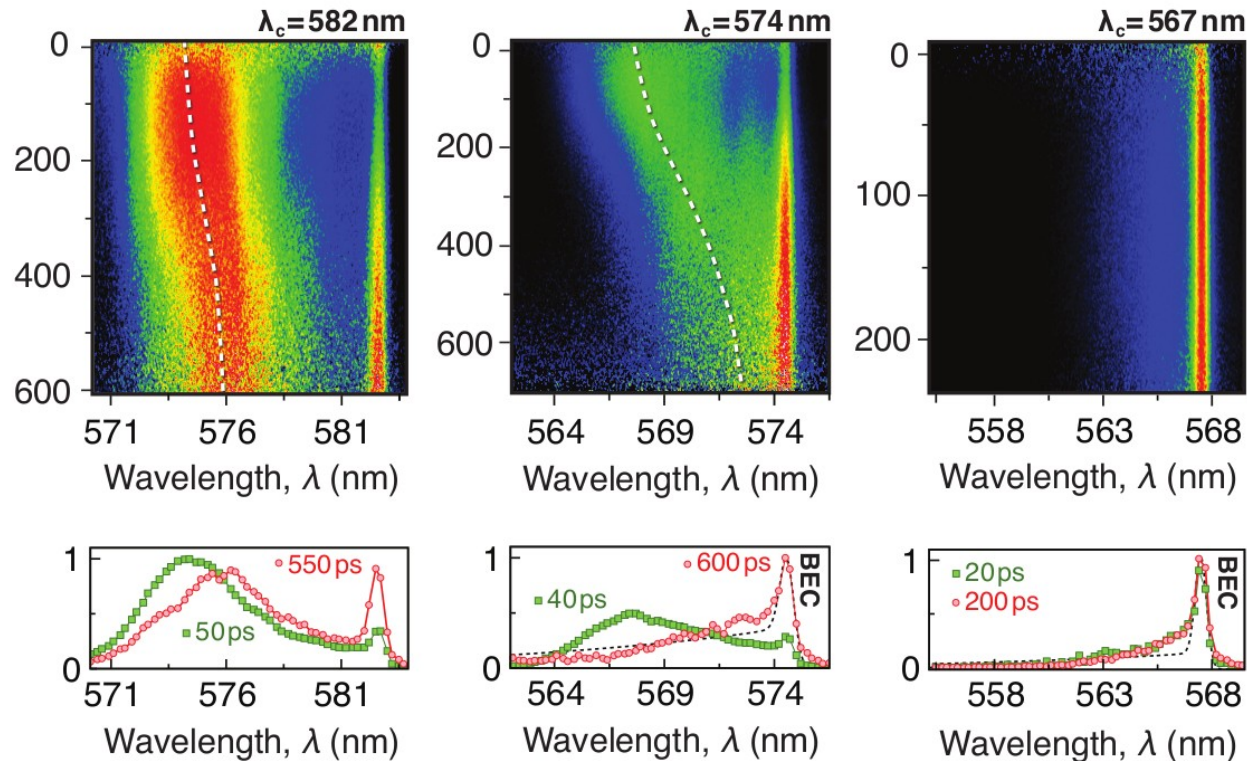
Out-of-equilibrium

- Close to equilibrium: tools of statistical mechanics (eg. Boltzmann distribution) can be applied
- Far from equilibrium: New description required

Lasers, Percolations, growing surfaces, traffic jams, social networks...

Nonequilibrium condensation

Bosonic enhancement works even away from equilibrium
Transition from equilibrium to nonequilibrium condensation



M. Weitz et al., PRA 92, 011602(R) (2015)

Phase diagram

