LECTURE 3

Examples of atomic BEC
Quasi-particles in solid
Light-matter coupling
Exciton-polaritons

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EXPERIMENTAL PROCEDURE

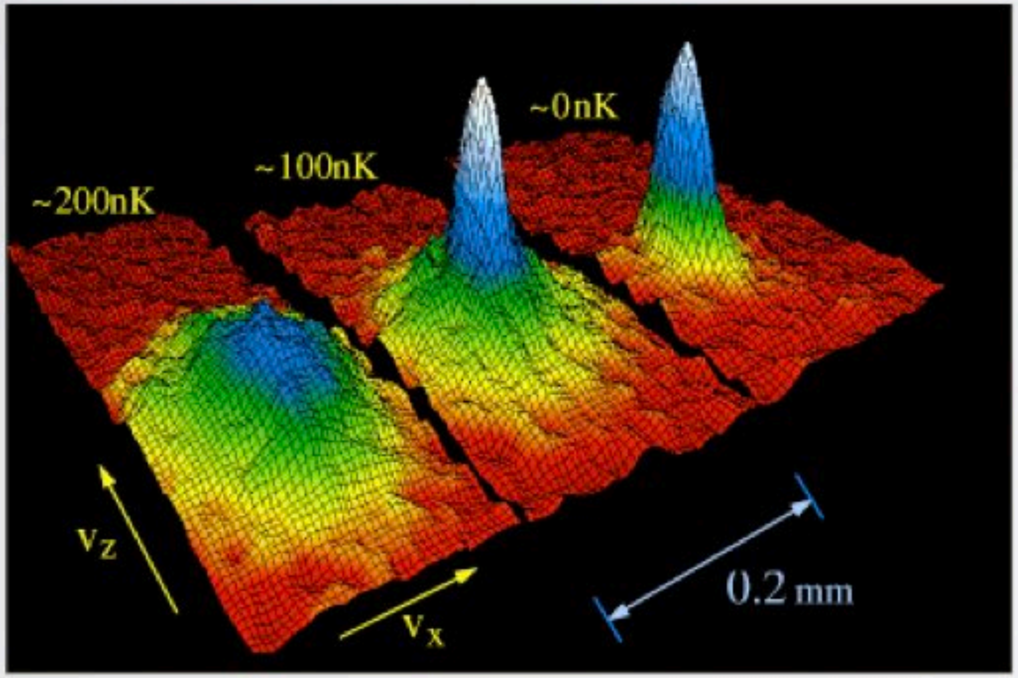
1. Atoms are released from a source. $N = 10^{10}$ atoms.
2. Atoms are trapped in a magnetooptical trap. (MOT)
3. Doppler cooling. $N = 10^{9}$ atoms. $T = 100$ mikroK.
4. Atoms are trapped in a magnetic trap.
5. Evaporative cooling. $N = 10^{7}$ atoms. $T = 100$ nanoK.

HISTORY

experimental results

^{87}Rb

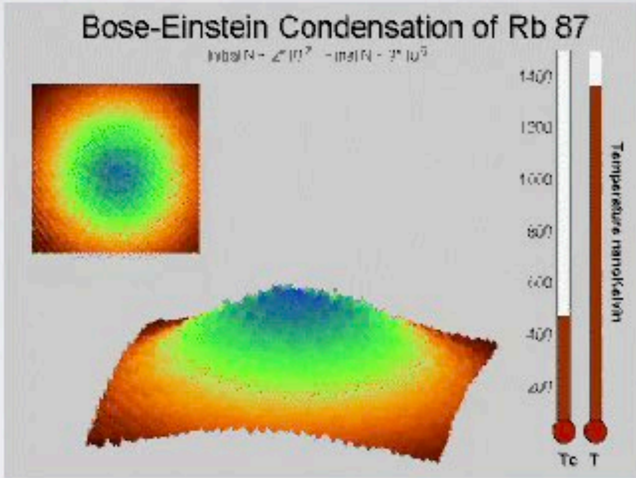
2 D velocity distributions



Velocity distribution of gas of Rb atoms.

M. H. Anderson et al., Science **269**, 198 (1995)

E. Cornell et al. JILA, 1995

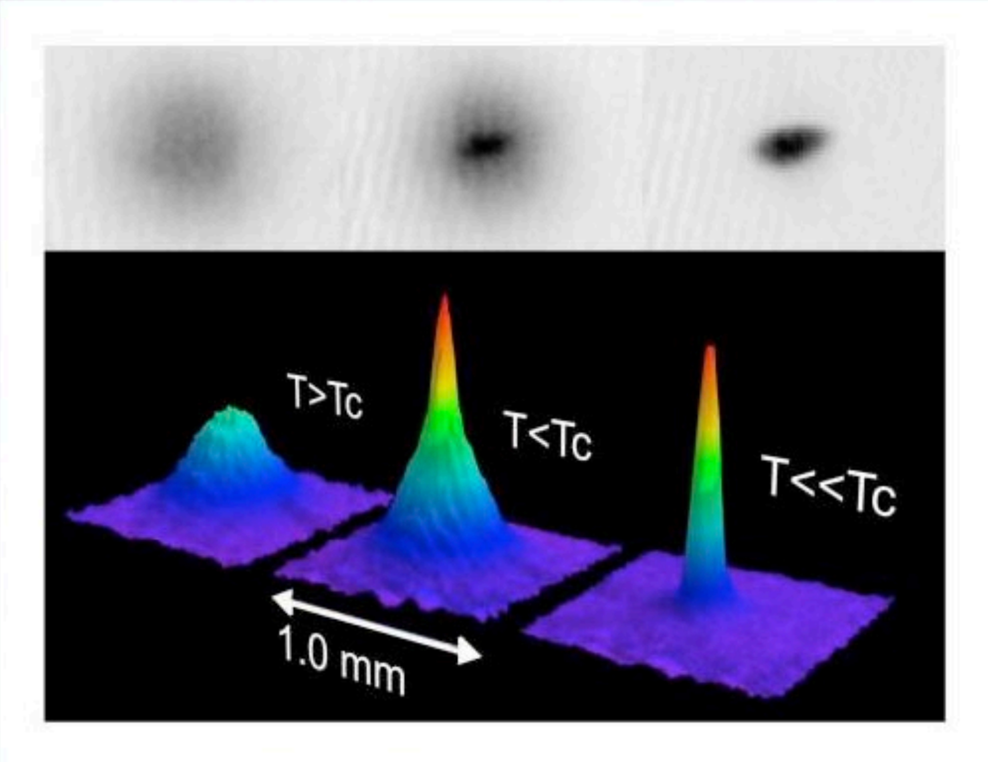


http://www.colorado.edu/physics/2000/bec/images/evap2.gif

- Produced in vapor of ^{87}Rb atoms
- Fraction of condensed atoms first appear near $T = 170$ nK & $n = 2.5 \cdot 10^{12}$ cm^{-3}
- Could be preserved for more than 15 seconds
- BEC on top of broad thermal velocity
- Fraction of atoms that were in this low-velocity peak increases abruptly
- Non-thermal, anisotropic velocity distribution expected of minimum-energy quantum state of magnetic trap

ABSORPTION IMAGING

^{23}Na



Expanding cloud cooled to just above the transition point; middle: just after the condensate appeared; right: after further evaporative cooling has left an almost pure condensate.

The "sharp peak" is the Bose-Einstein condensate, characterized by its slow expansion observed after 6 msec time of flight.

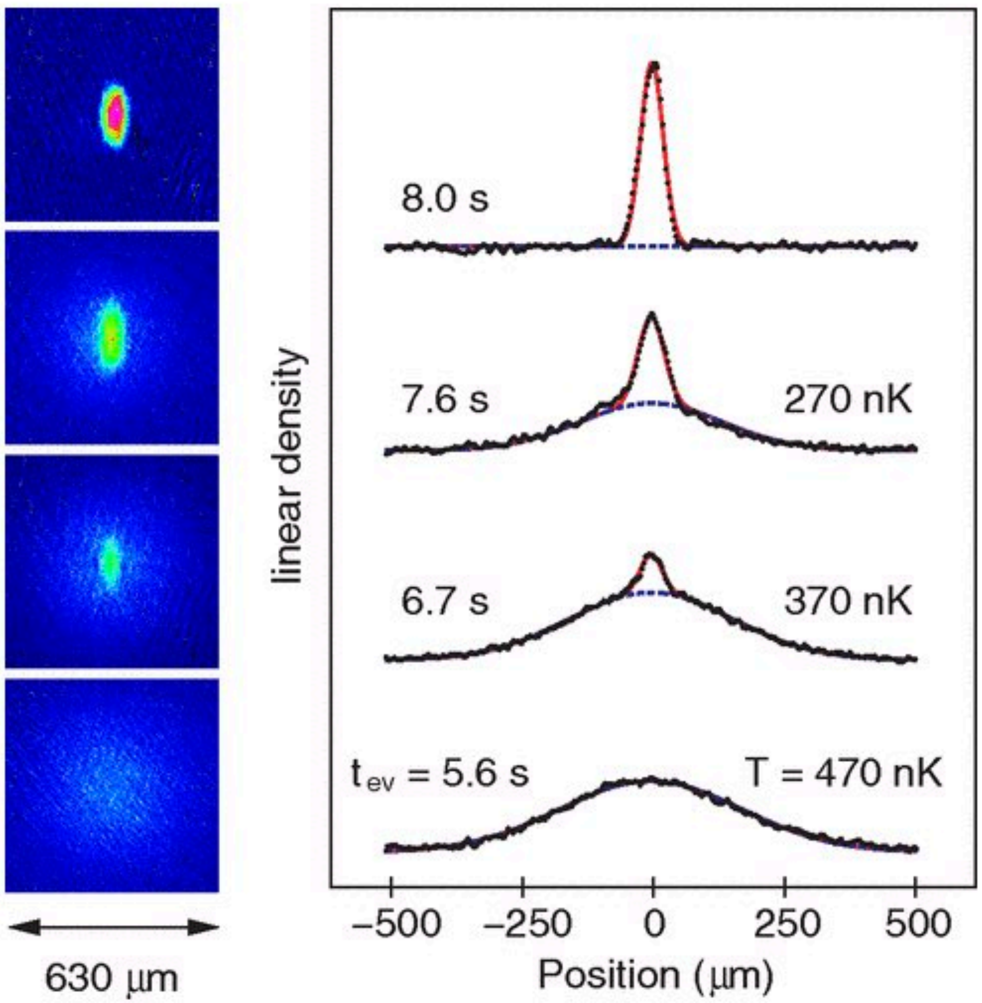
The width of the images is 1.0 mm. The total number of atoms at the phase transition is about 700,000, the temperature at the transition point is 2 microkelvin.

W. Ketterle et al. MIT, 1995

http://cua.mit.edu/ketterle_group/Popular_papers/Ultralow_temperatures.htm

Bose-Einstein Condensation of Strontium

Simon Stellmer, Meng Khoon Tey, Bo Huang, Rudolf Grimm, and Florian Schreck
Phys. Rev. Lett. **103**, 200401 – Published 9 November 2009



Absorption images and integrated density profiles showing the BEC phase transition for different times of the evaporative cooling ramp.

The images are along the vertical direction, 25 ms after release from the trap. The solid line represents a fit with a bimodal distribution, while the dashed line shows the Gaussian-shaped thermal part, from which the given temperature values are derived.

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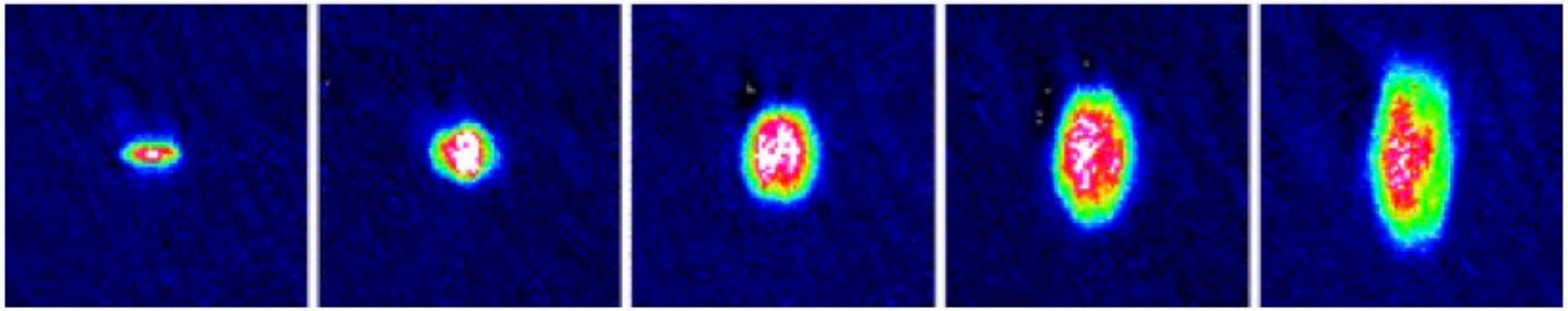
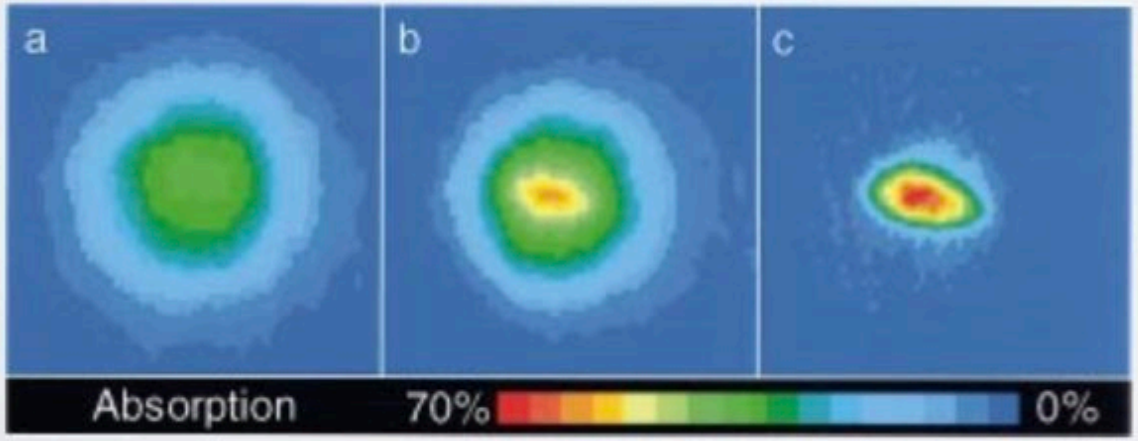


Inversion of the aspect ratio during the expansion of a pure BEC. The images field of view 250 μm×250 μm. The first image is an in situ image recorded at the time of release. The further images are taken 5, 10, 15, and 20 ms after release.

ABSORPTION IMAGING

- Switching off trap ⇒ condensate falling down (gravity) and

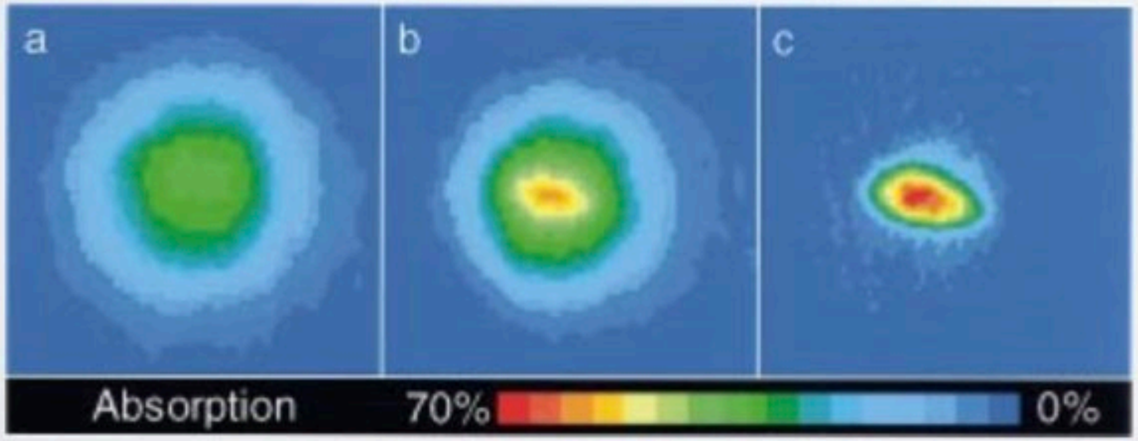
  ballistically expands
- Illuminating atoms with nearly resonant laser beam and imaging shadow cast on charge-coupled device camera (CCD-camera)
- Cloud heats up by absorbing photons (about one recoil energy per photon)
- Single destructive image
- Provides reliable density distributions of which properties of condensates and thermal clouds can be inferred



Stefan Kienzle
Technische Universitat Munchen

ABSORPTION IMAGING

- 2D probe absorption images after 6 ms time of flight Width of images is 870 μm
- Velocity distribution of cloud just above transition point
- Shows difference between isotropic thermal distribution and elliptical core attributed to expansion of dense condensate
- Almost pure condensate (after further evaporative cooling)



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Technische Universitat Munchen

Order parameter

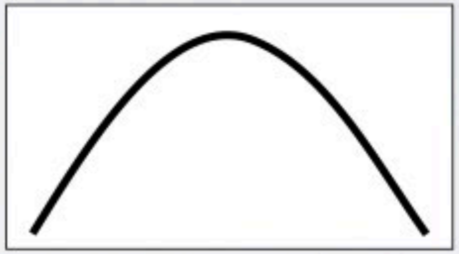
zero before the phase transition and becomes determined after phase transition

ORDER PARAMETER

$$\psi(\vec{r},t) = \sqrt{n(\vec{r},t)}\, e^{i\theta(\vec{r},t)}$$

Phase coherence!!!

spatial and temporal:



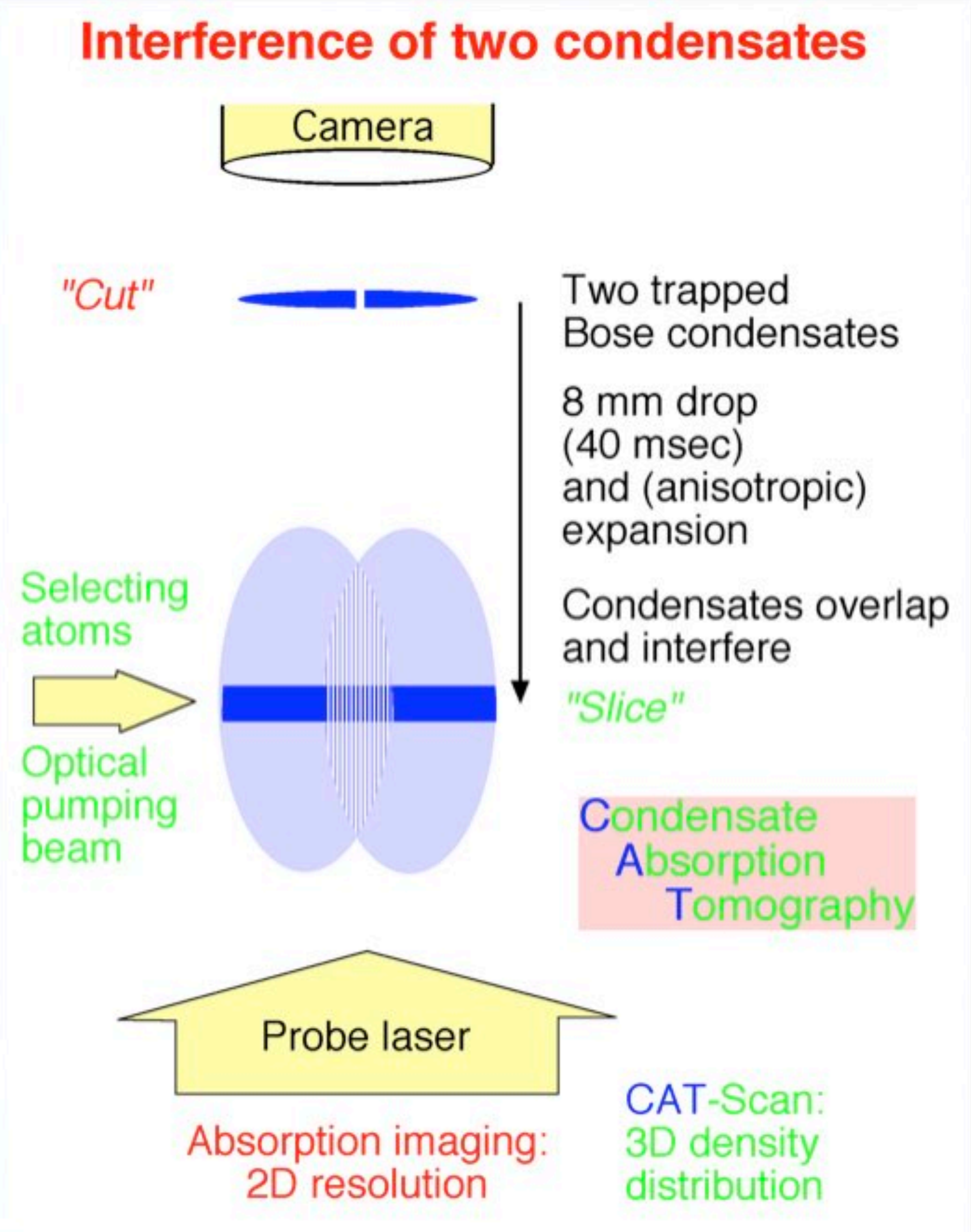
T=0:
Pure Bose condensate
"Giant matter wave"

- Coherence in time
- Long-range order = spatial coherence

during the phase transition

interference !!

INTERFERENCE BETWEEN TWO BEC



Evidence for coherence of BEC's

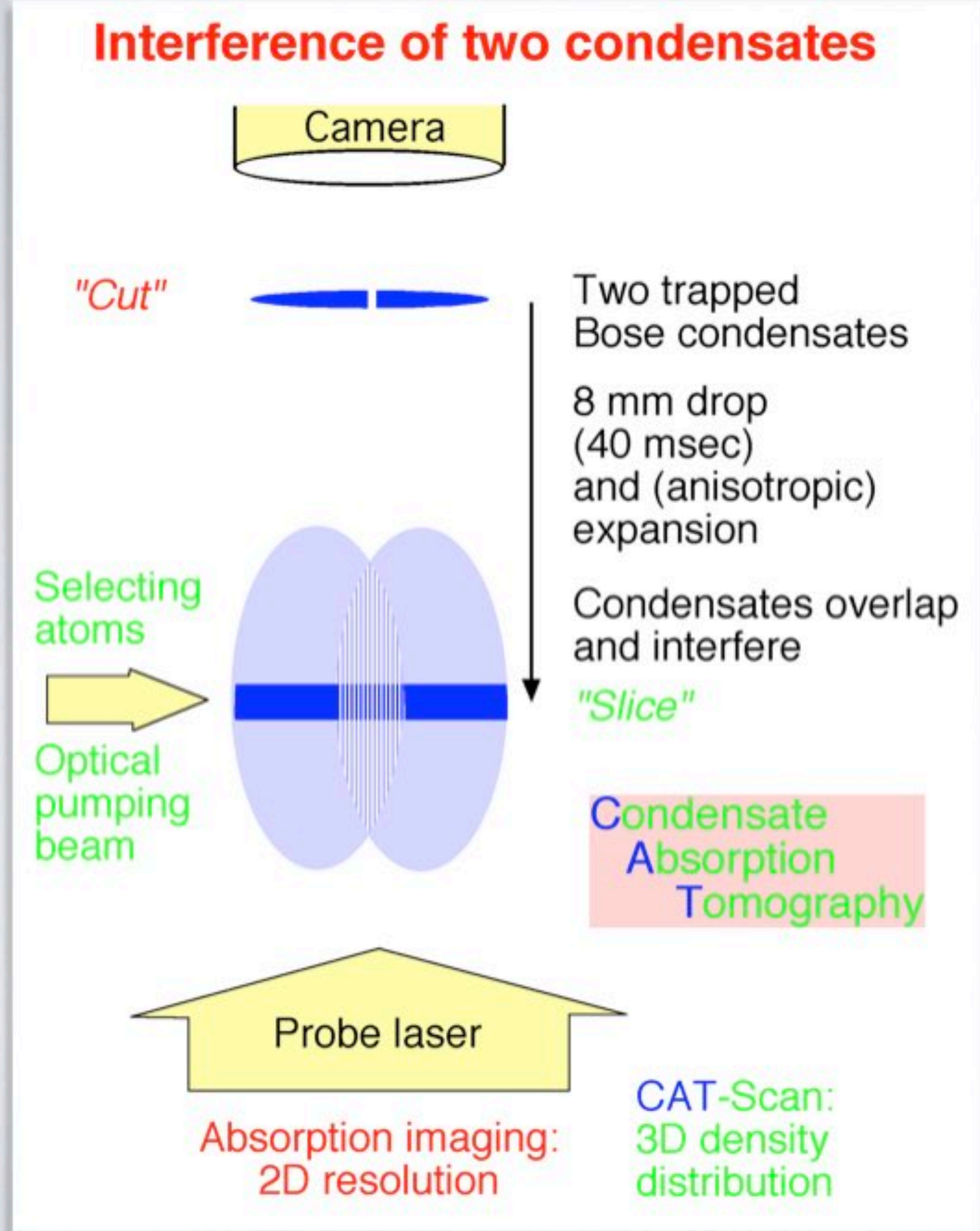
Cut atom trap in half (double-well potential) by focusing far-off-resonant laser light into center of magnetic trap

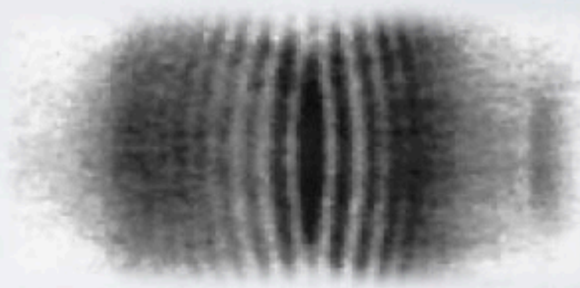
Cool atoms in these two halves to form two independent condensates

Quickly turn off laser and magnetic fields, allowing atoms to fall and expand freely

Both condensates start to overlap and interfere with each other

INTERFERENCE BETWEEN TWO BEC





Interference between two atomic BEC

M. R. Andrews et al., Science **275,** 637 (1997)

! Interference of a matter wave !

Quasi-particles in solid
Light-matter coupling

Griffin, Snoke, Stringari, *Bose Einstein Condensate*

Particle	Composed of	In	Coherence seen in
Cooper pair	e^-, e^-	metals	superconductivity
Cooper pair	h^+, h^+	copper oxides	high-T_c superconductivity
exciton	e^-, h^+	semiconductors	luminescence and drag-free transport in Cu_2O
biexciton	$2(e^-, h^+)$	semiconductors	luminescence and optical phase coherence in CuCl
positronium	e^-, e^+	crystal vacancies	(proposed)
hydrogen	e^-, p^+	magnetic traps	(in progress)
^{4}He	$^4He^{2+}$, $2e^-$	He-II	superfluidity
^{3}He pairs	$2(^3He^{2+}, 2e^-)$	^{3}He-A,B phases	superfluidity
cesium	$^{133}Cs^{55+}$, $55e^-$	laser traps	(in progress)
interacting bosons	nn or pp	nuclei	excitations
nucleonic pairing	nn or pp	nuclei neutron stars	moments of inertia superfluidity and pulsar glitches
chiral condensates	$\langle \bar{q}q \rangle$	vacuum	elementary particle structure
meson condensates	pion condensate $= \langle \bar{u}d \rangle$, etc. kaon condensate $= \langle \bar{u}s \rangle$	neutron star matter	neutron stars, supernovae (proposed)
Higgs boson	$\langle \bar{t}t \rangle$ condensate (proposed)	vacuum	elementary particle masses

Probability distribution

The probability of filling the quantum state of the energy E
E_F – chemical potential

Fermions:

$$f_0 = \frac{1}{e^{\frac{E-E_F}{k_B T}} + 1}$$

Electrons
Holes
Trions (charged excitons)

Bosons:

$$f_0 = \frac{1}{e^{\frac{E-E_F}{k_B T}} - 1}$$

Polaritons
Phonons
Magnons
Excitons, biexcitons
Plazmons

Boltzman distribution:

$$f_0 = \frac{1}{e^{\frac{E-E_F}{k_B T}} \pm 1} \approx e^{-\frac{E-E_F}{k_B T}}$$

$$E_F = \frac{\partial F}{\partial n_i}$$

$$F = U - TS$$

Anyons – np. composite fermions $|\psi_1 \psi_2\rangle = e^{i\theta} |\psi_2 \psi_1\rangle$
Slave fermions (chargon, holon, spinon) = fermion+bozon in spin-charge separation

Few remarks up to now

Facts:

- crystals have highly ordered microscopic structure
- in consequence, energy bands are formed
- properties (electrical, optical, ...) are determined by electrons distributed over the bands
- electrons are fermions and obey Fermi-Dirac distribution

Optical properties of semiconductors

Optical absorption spectra are governed by the density of electronic states in the valence and conduction bands.

$$g(E) = \frac{\partial n}{\partial E}$$

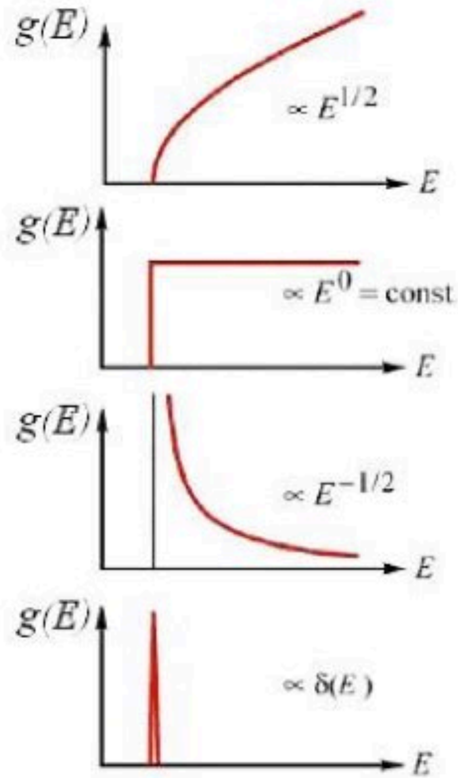
n - number of quantum states per unit area

3-D (bulk) $g(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E_g - E}$

2-D (slab) $g(E) = \frac{m^*}{\pi\hbar^2}\sigma(E_g - E)$

1-D (wire) $g(E) = \frac{m^*}{\pi\hbar}\sqrt{\frac{m^*}{2(E_g - E)}}$

0-D (dot) $g(E) = 2\delta(E_g - E)$

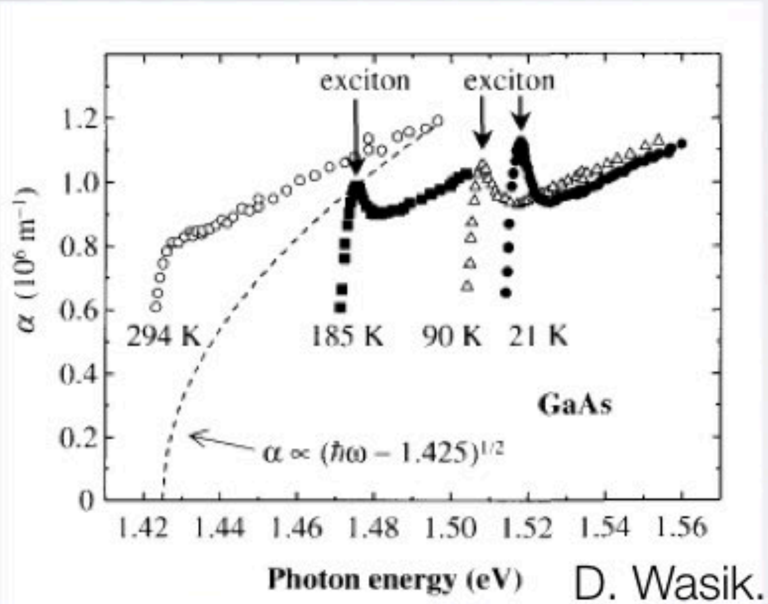


http://electrons.wikidot.com/density-of-states

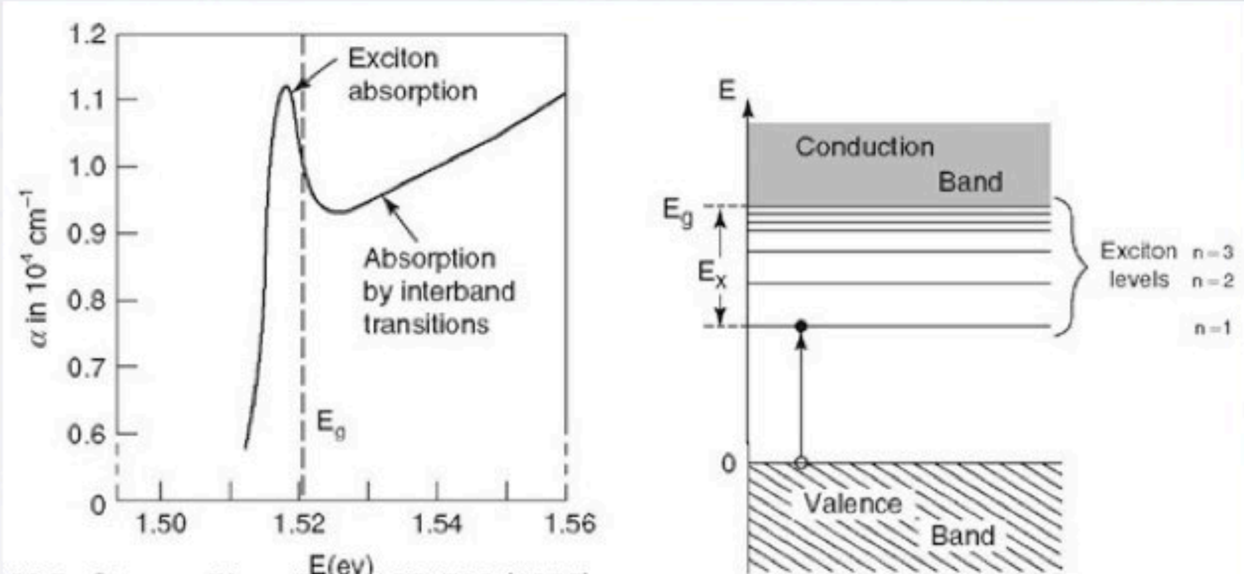
Optical properties of semiconductors

Excitons in bulk

Absorption spectra in semiconductors (at low temperature) exhibit sharp peaks below the edge of the inter-band absorption.



D. Wasik.



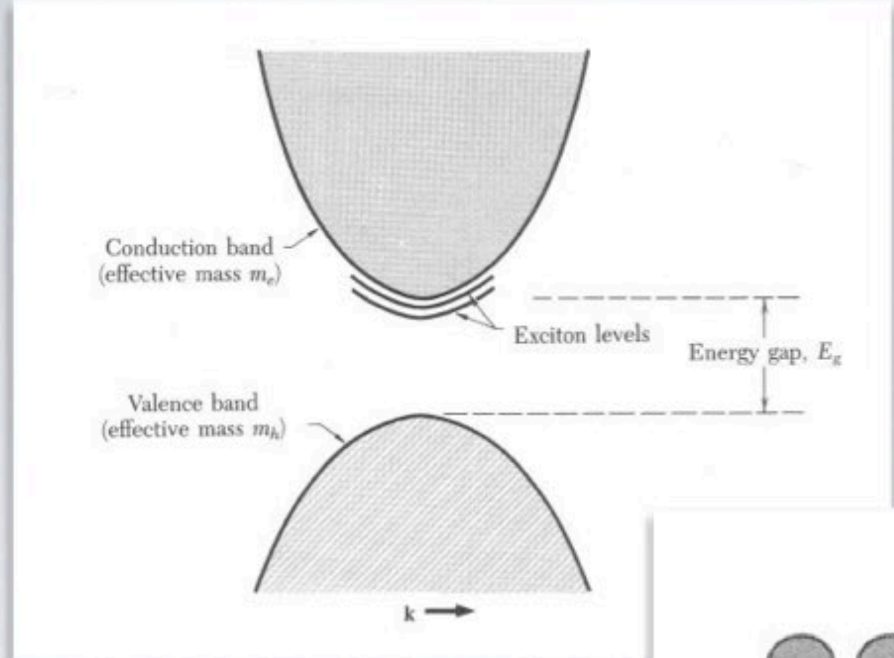
M.D. Sturge, Phys. Rev. 127, 768 (1962)

Manifestation of resonant light - matter coupling in semiconductor.

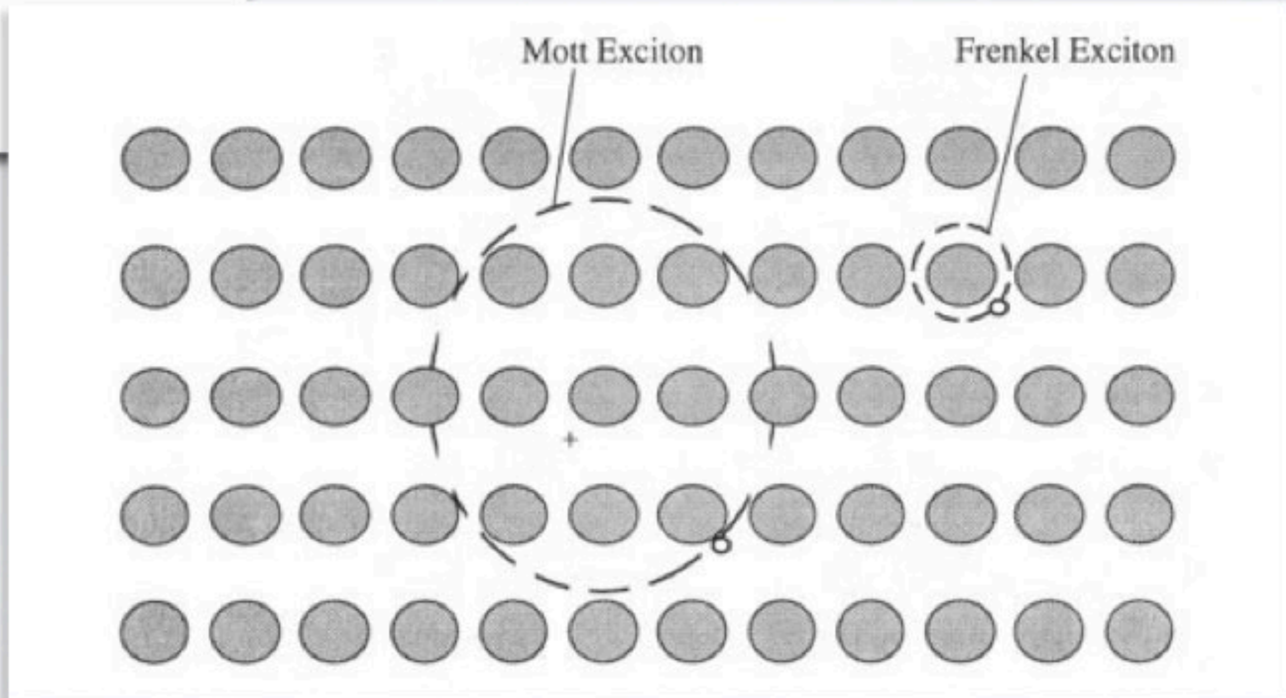
http://what-when-hov
materials/applications-opt

Excitons

An exciton is a bound state of an electron and an imaginary particle called an electron hole in an insulator or semiconductor.



- The overall charge for this quasiparticle is zero.
- It carries no electric current.
- ! It is a composite BOSON !



http://electrons.wikidot.com/solar-cells

Excitons - 3D

Consider an electron-hole pair bound by the coulomb interactions:

$$-\frac{\hbar^2}{2\mu}\nabla^2 f(r) - \frac{e^2}{4\pi\varepsilon\varepsilon_0 r} f(r) = E f(r)$$

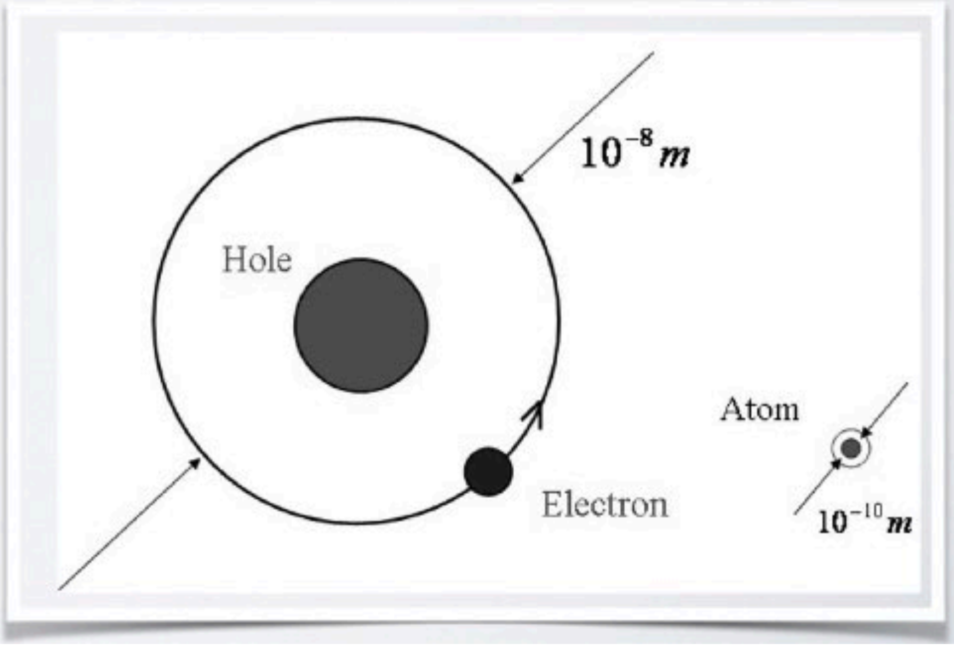
dielectric constant of a crystal

effective mass: $\mu = m_e m_h/(m_e + m_h)$

e-h distance: $r = \sqrt{x^2 + y^2 + z^2}$

Equation is analogous to Schrodinger equation for a hydrogen atom with the following renormalisations:

$$m_0 \to \mu, \qquad e^2 \to e^2/\varepsilon$$



A. Kavokin, et al. in *Microcavities*, Oxford Science Publications (2007)

Excitons - 3D

Consider an electron-hole pair bound by the coulomb interactions:

$$-\frac{\hbar^2}{2\mu}\nabla^2 f(r) - \frac{e^2}{4\pi\varepsilon\varepsilon_0 r} f(r) = E f(r)$$

dielectric constant of a crystal

Bohr radius : $$a_{\mathrm{B}} = \frac{4\pi\hbar^2\varepsilon\varepsilon_0}{\mu e^2}$$

Binding energy of a ground state: $$E_{\mathrm{B}} = \frac{\mu e^4}{(4\pi)^2 2\hbar^2\varepsilon\varepsilon^2} = \frac{\hbar^2}{2\mu a_{\mathrm{B}}^2}$$

Wavefunction of the 1*s* state: $$f_{1s} = \frac{1}{\sqrt{\pi a_{\mathrm{B}}^3}} e^{-r/a_{\mathrm{B}}}$$

A. Kavokin, et al. in *Microcavities*, Oxford Science Publications (2007)

Excitons - 3D

Semiconductor crystal	E_g (eV)	m_e/m_0	E_B (eV)	a_B (Å)
PbTe*	0.17	0.024/0.26	0.01	17 000
InSb	0.237	0.014	0.5	860
$Cd_{0.3}Hg_{0.7}Te$	0.257	0.022	0.7	640**
Ge	0.89	0.038	1.4	360
GaAs	1.519	0.066	4.1	150
InP	1.423	0.078	5.0	140
CdTe	1.606	0.089	10.6	80
ZnSe	2.82	0.13	20.4	60
GaN***	3.51	0.13	22.7	40
Cu_2O	2.172	0.96	97.2	38****
SnO_2	3.596	0.33	32.3	86****

Table 4.2 Strongly anisotropic conduction and valence bands, direct transitions far from the centre of the Brillouin zone.

* Strongly anisotropic conduction and valence bands, direct transitions far from the centre of the Brillouin zone.

** In the presence of a magnetic field of 5 T.

*** An exciton in hexagonal GaN.

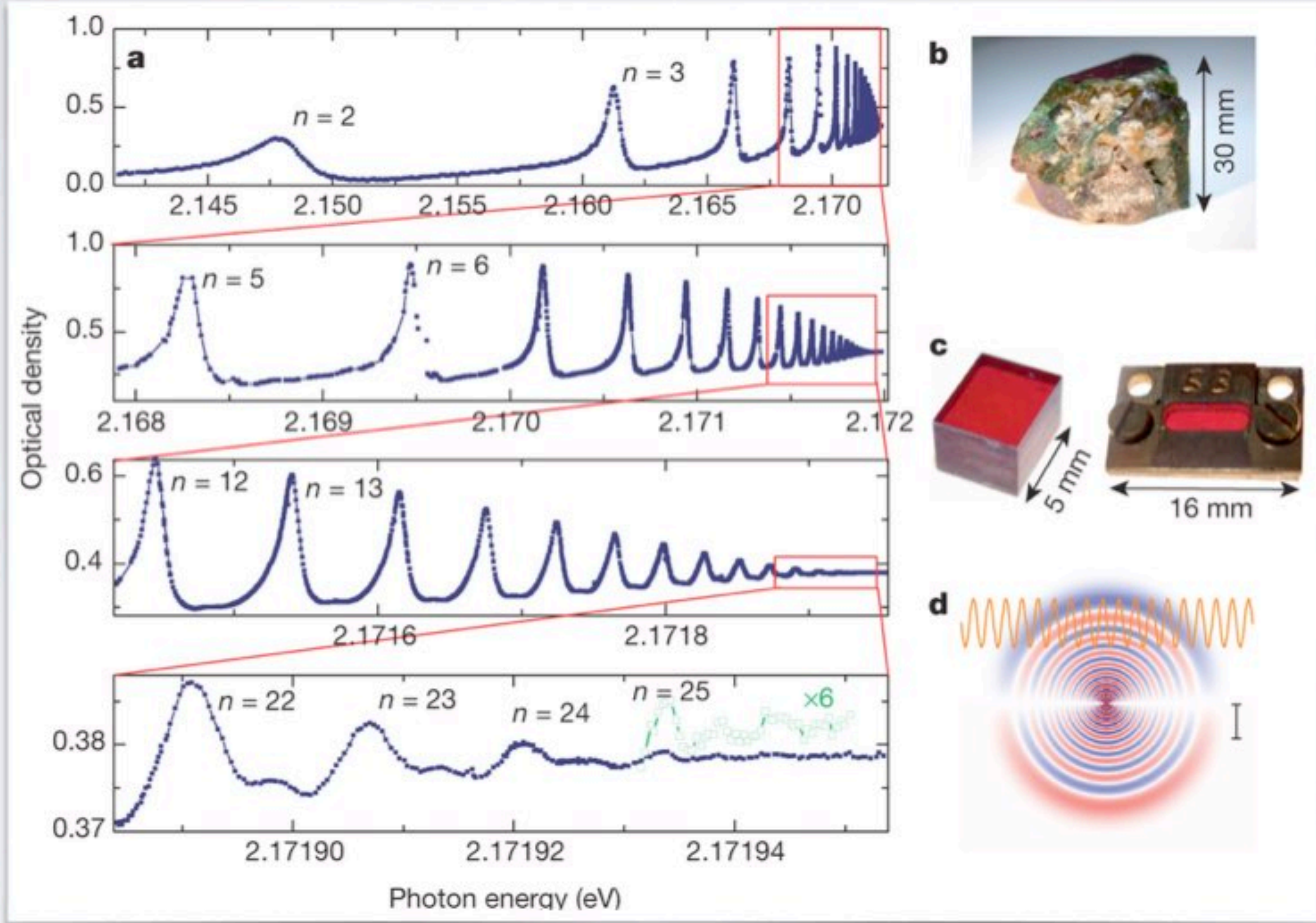
**** The ground-state corresponds to an optically forbidden transition, data given for $n = 2$ state.

A. Kavokin, et al. in *Microcavities*, Oxford Science Publications (2007)

Excitons - 3D

$$E_n = -\left(\frac{m^*}{m_0}\right)\frac{1}{\varepsilon_r^2}Ry\frac{1}{n^2}$$

Giant Rydberg excitons in the copper oxide Cu_2O



T. Kazimierczuk, D. Fröhlich, S. Scheel, H. Stolz & M. Bayer,
***Nature* 514, 343–347 (16 October 2014)**

Excitons - 2D (in quantum well)

The Schrödinger equation for an exciton in a quantum well (QW) reads:

$$\left(-\frac{\hbar^2}{2m_e}\nabla_e^2 - \frac{\hbar^2}{2m_h}\nabla_h^2 + V_e(z_e) + V_h(z_h) - \frac{e^2}{4\pi\varepsilon\varepsilon_0|\mathbf{r}_e - \mathbf{r}_h|}\right)\Psi = E\Psi$$

Solutions are again similar to 2D hydrogen atom:

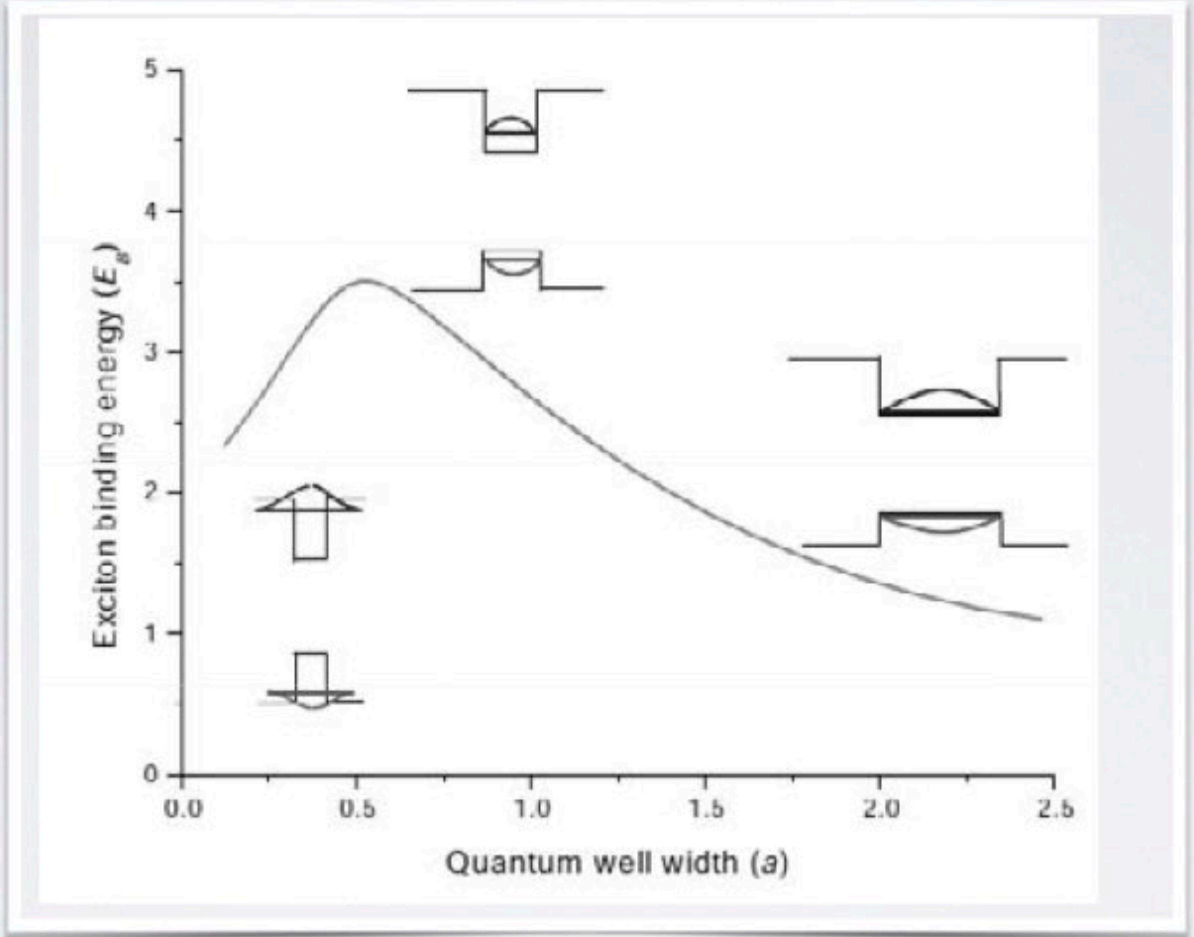
Bohr radius : $a_{\mathrm{B}}^{2\mathrm{D}} = \frac{a_{\mathrm{B}}}{2}$

Binding energy of a ground state: $E_{\mathrm{B}}^{2\mathrm{D}} = 4E_{\mathrm{B}}$

Wavefunction of the 1*s* state: $f_{1S}(\rho) = \sqrt{\frac{2}{\pi}}\frac{1}{a_{\mathrm{B}}^{2\mathrm{D}}}\exp(-\rho/a_{\mathrm{B}}^{2\mathrm{D}})$

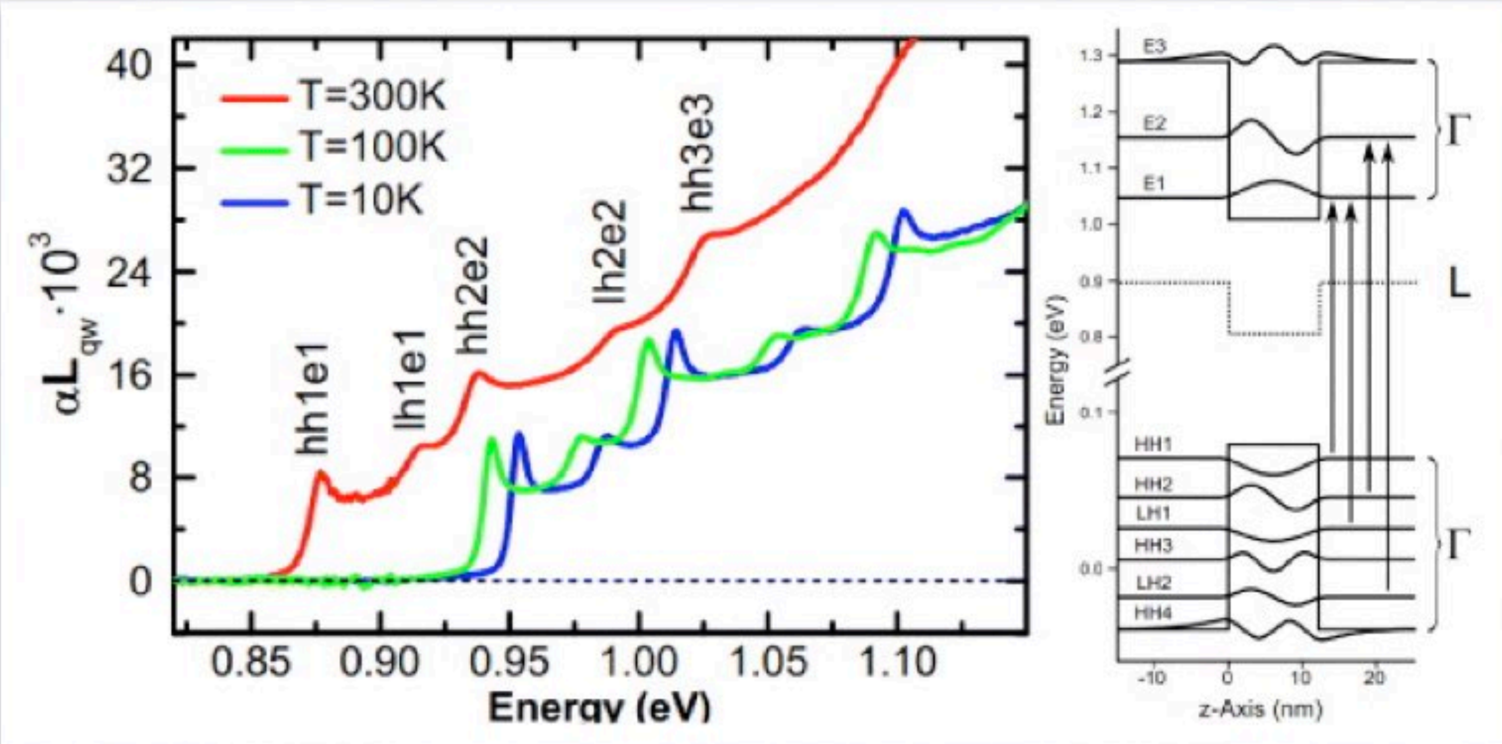
Energies of the excited states: $E_n = -\frac{Ry^*}{\left(n-\frac{1}{2}\right)^2}$

Excitons - 2D (in quantum well)



A. Kavokin, et al. in *Microcavities*, Oxford Science Publications (2007)

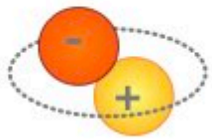
Optical properties of semiconductors
Excitons in quantum well



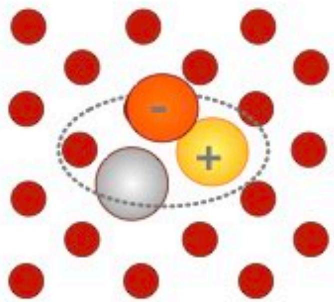
Linear absorption spectrum of the Ge multiple quantum well structure and a schematic sketch of the electronic structure.

N. S. Köster et al., New Journal of Physics, Volume 15, July 2013

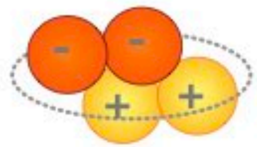
Exciton family



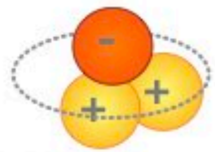
Exciton



Exciton bound on donor or acceptor



Biexciton



Trion - exciton positively charged

Trion - exciton negatively charged

Exciton condensation

particle mass significancy

$$T \sim \frac{(2\pi\hbar)^2}{2mk_B} n^{2/3}$$

Light mass implies Bose-Einstein effects at higher temperature !

	atoms	**EXCITONS**
m	Rb: $10^4 m_e$	$10^{-2} m_e$
T_C	10^{-7}K	~ 4 K possible
n	$10^{14}/cm^3$	limit: $10^{17}/cm^3$ or $10^{11}/cm^2$
lifetime	∞	typically ~100 ns up to 1 ms in specially designed samples

Exciton condensation
particle mass significancy

$$T \sim \frac{(2\pi\hbar)^2}{2mk_B} n^{2/3}$$

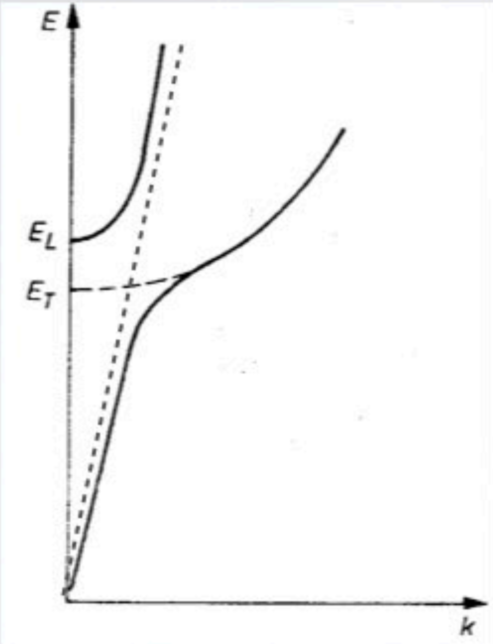
Light mass implies Bose-Einstein effects at higher temperature !

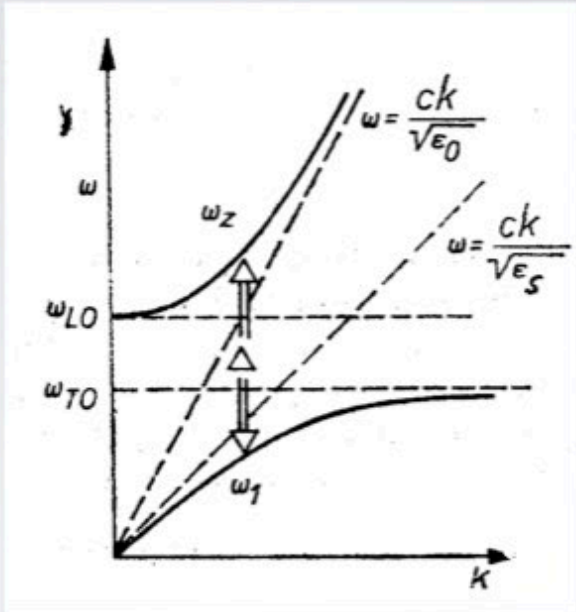
	atoms	**EXCITONS**	**EXCITON POLARITONS**
m	Rb: $10^{4}m_e$	$10^{-2}m_e$	$10^{-4}m_e$
T_C	10^{-7}K	~ 4 K possible	RT possible
n	$10^{14}/cm^3$	limit: $10^{17}/cm^3$ or $10^{11}/cm^2$	$<10^{11}/cm^2$
lifetime	∞	typically ~100 ns up to 1 ms in specially designed samples	10 ps

EXCITON POLARITON = EXCITON + PHOTON

SHORT HISTORY OF LIGHT-MATTER COUPLING IN SEMICONDUCTORS

1951 - Huang : interaction between the electromagnetic field with the crystal lattice excitation
(Maxwell equations + classical lattice vibrations)





1956 - Fano & 1958 Hopfield

interaction between the electromagnetic field with excitons (quantum approach)

phonon polaritons
exciton polaritons
magnon polaritons
plasmon polaritons

polaritons - coupled modes of electromagnetic waves and any excitation propagating in the material with complex dielectric function

overview after W.Wardzyński IFPAN

Light-matter interaction

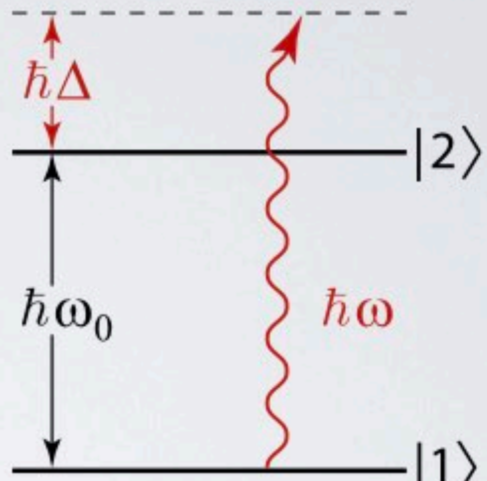
spontaneous emission	emission to the space of infinite number of modes
weak coupling	emissions in well-defined mode, but the space is open (dissipation, decoherence, cavity losses) reabsorption process is not possible
strong coupling	emission to well defined mode with strong reabsorption process

Light-matter interaction

	weak coupling	strong coupling
	perturbation theory with Fermi Golden rule	e-m field creates strong perturbation in the system
normal modes propagating through the crystal	excitons and photons	matter-field quasi-particles
	Purcell effect	**vacuum field Rabi splitting**
eigen states	exciton & photon	UP & LP polaritons
τ_C - exciton lifetime τ_X - photon lifetime	$\hbar\Omega < \tau_C , \tau_X$	$\hbar\Omega > \tau_C , \tau_X$
	resonant emission intensity increases	two new modes appear of exciton-photon mixed type

Light-matter interaction
two-levels in an external field

- $|1\rangle$ and $|2\rangle$ form an orthonormal basis for the system *i.e.* $\langle i|j\rangle = \delta_{ij}$ for $i, j = 1, 2$;
- Photon frequency : $\omega = \omega_0 + \Delta$;
- Detuning : Δ ;
- Resonant frequency : ω_0 .



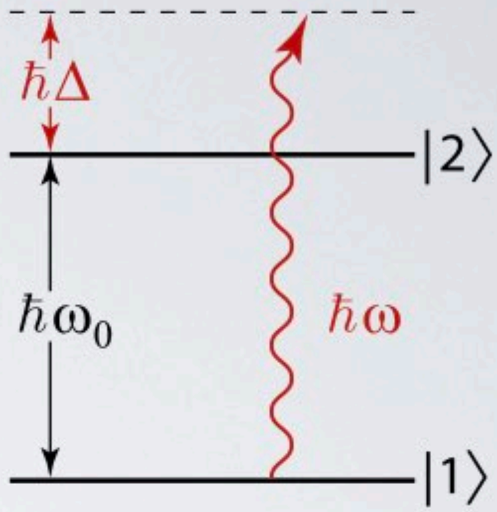
The bare hamiltonian:

$$H_0 = \hbar \begin{pmatrix} 0 & 0 \\ 0 & \omega_0 \end{pmatrix}$$

$$\hat{H}_0|1\rangle = 0\,|1\rangle,$$
$$\hat{H}_0|2\rangle = \hbar\,\omega_0|2\rangle$$

w.g. R. Anderson in *Two-level system: Rabi oscillations.*

Light-matter interaction
two-levels in an external field



$$\psi = \begin{pmatrix} \langle 1|\psi\rangle \\ \langle 2|\psi\rangle \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Any two-level quantum state can be expressed as $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$, where c_1 and c_2 are complex state amplitudes and $|c_1|^2 + |c_2|^2 = 1$. Such a state can be represented by a two-component vector;

The probability of finding the system in state $|i\rangle$ is $|\langle i|\psi\rangle|^2 = |c_i|^2$, (for $i = 1, 2$).

w.g. R. Anderson in *Two-level system: Rabi oscillations.*

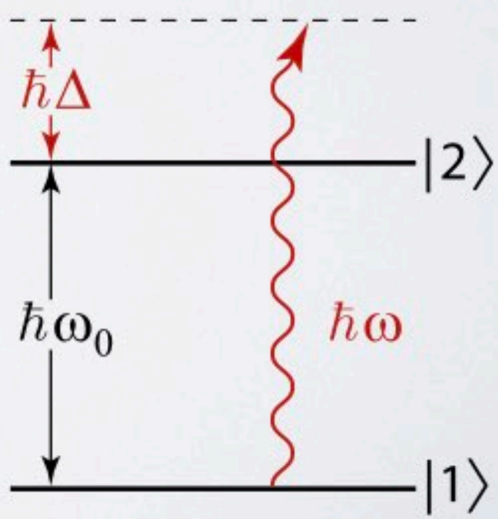
Light-matter interaction

two-levels in an external field

What about the driving field? What does it do? It induces a dipole (electric or magnetic) moment between the states $|1\rangle$ and $|2\rangle$. The electromagnetic field interacts with this dipole, resulting in an oscillatory perturbation. This perturbation is represented by the operator:

$$H_{int} = \hbar \begin{pmatrix} 0 & \Omega \cos(\omega t) \\ \Omega^* \cos(\omega t) & 0 \end{pmatrix}$$

the Rabi frequency $\Omega = \mathcal{E}\mu/\hbar$



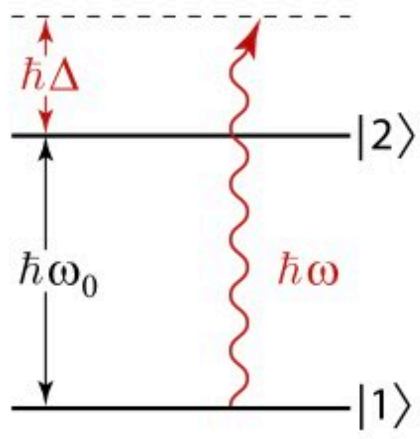
w.g. R. Anderson in *Two-level system: Rabi oscillations.*

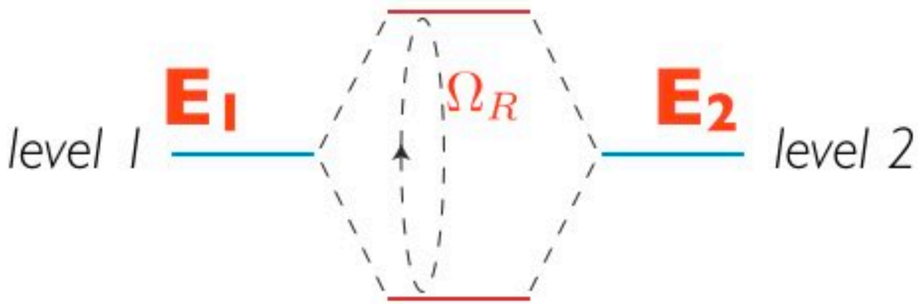
Light-matter interaction

two-levels in an external field

$$H_0 = \hbar \begin{pmatrix} 0 & 0 \\ 0 & \omega_0 \end{pmatrix}$$

$$H_{int} = \hbar \begin{pmatrix} 0 & \Omega \cos(\omega t) \\ \Omega^* \cos(\omega t) & 0 \end{pmatrix}$$





$$\psi(x,t) = c_1(t)\phi_1(x) + c_2(t)\phi_2(x)$$

The time dependent coefficients satisfy the Schrödinger equation in matrix form

$$i\hbar \frac{d}{dt} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} .$$

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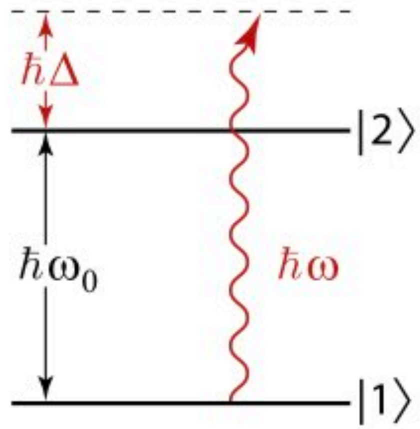

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = E \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\begin{vmatrix} H_{11} - E & H_{12} \\ H_{21} & H_{22} - E \end{vmatrix} = 0\,.$$

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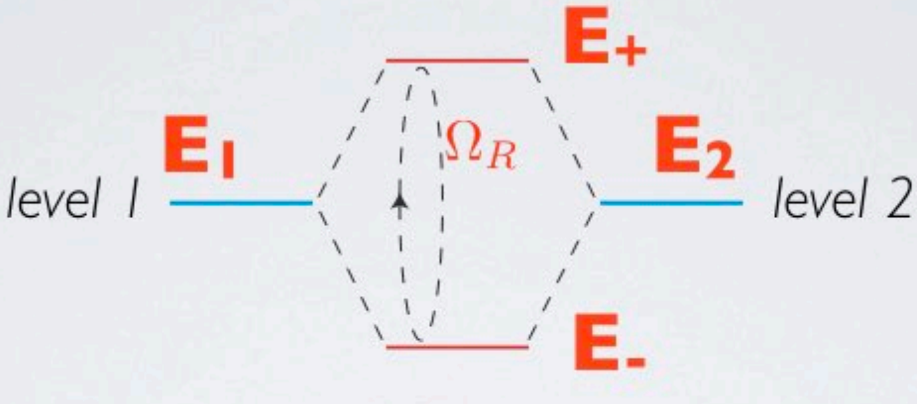
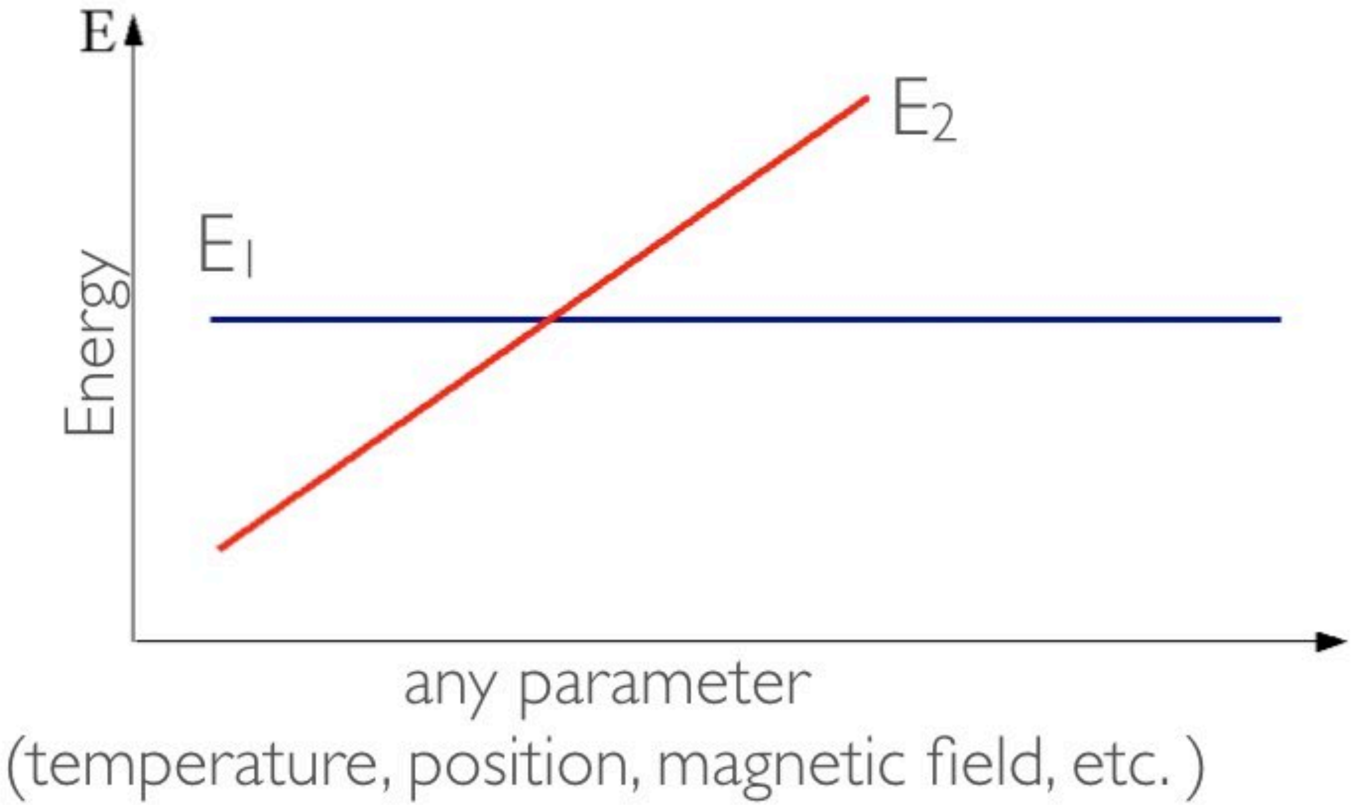


$$E_- = \frac{H_{11}+H_{22}}{2} - \sqrt{\left(\frac{H_{22}-H_{11}}{2}\right)^2 + |H_{12}|^2}$$

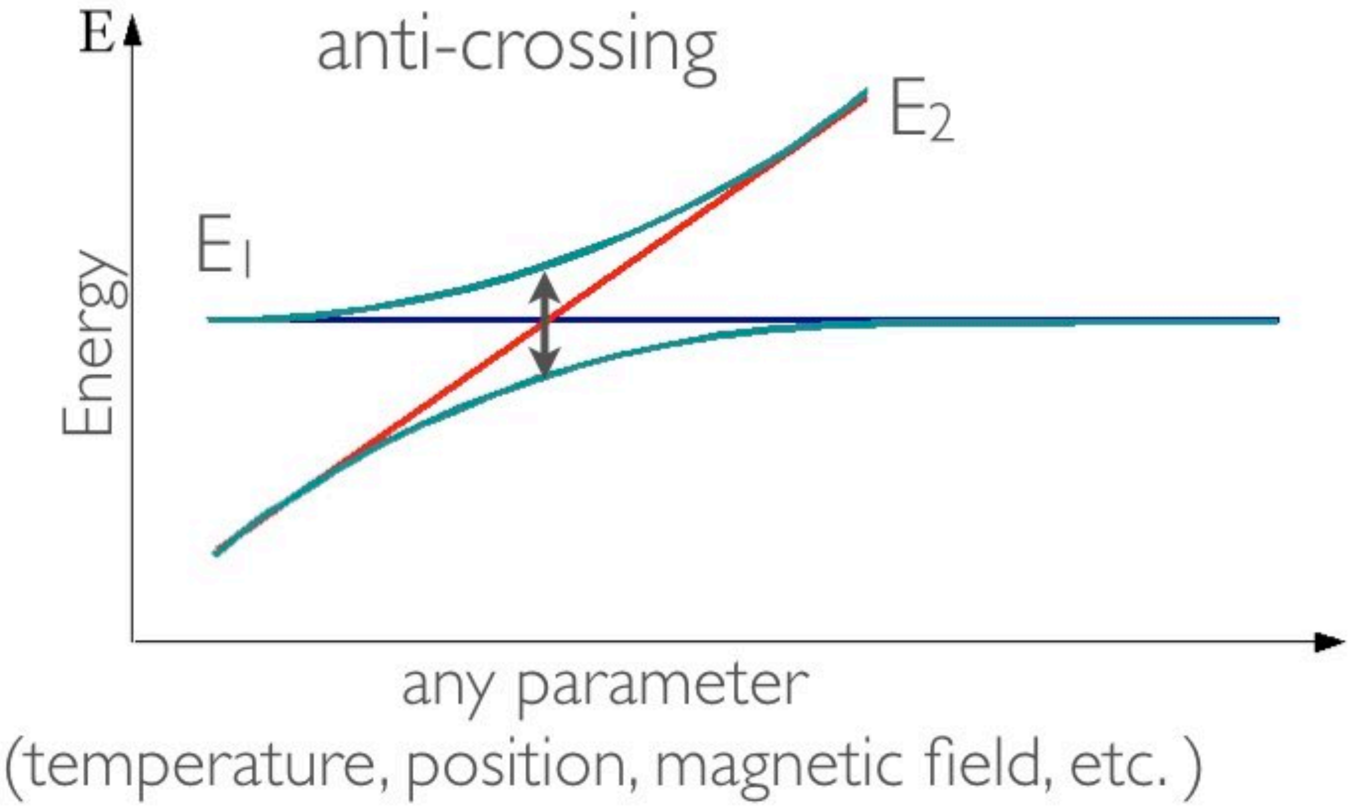
$$E_+ = \frac{H_{11}+H_{22}}{2} + \sqrt{\left(\frac{H_{22}-H_{11}}{2}\right)^2 + |H_{12}|^2}$$

Light-matter interaction

two-level interaction

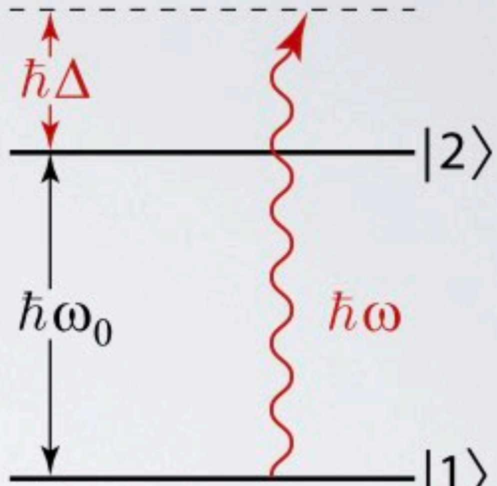


Light-matter interaction
two-level interaction



Light-matter interaction
two-levels in an external field

$$|c_1(t)|^2 = \frac{\Omega^2}{\Omega_R^2}\sin^2\left(\frac{\Omega_R t}{2}\right),$$
$$|c_2(t)|^2 = \frac{\Delta^2}{\Omega_R^2} + \frac{\Omega^2}{\Omega_R^2}\cos^2\left(\frac{\Omega_R t}{2}\right),$$
$$\Omega_R^2 \equiv \Omega^2 + \Delta^2 .$$

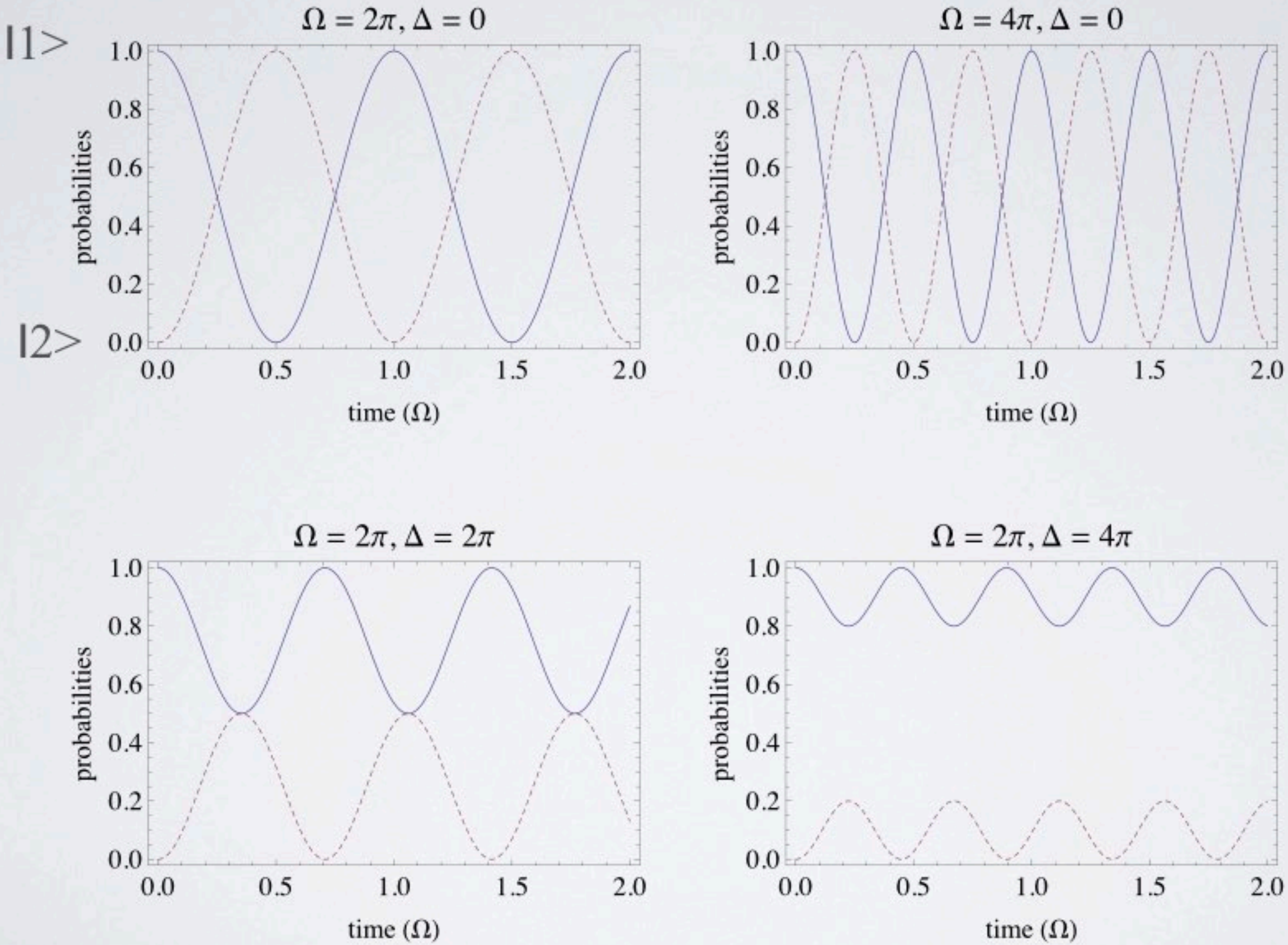


This means that the probabilities to be in state $|1\rangle$ or $|2\rangle$ oscillate with the frequency Ω_R defined above, the *total Rabi frequency*. From this result, it is clear that states $|1\rangle$ and $|2\rangle$ are no longer stationary states of the system. It is remarkable that the dynamic behaviour of the system is governed (at this point) by only two parameters. These parameters are the coupling strength Ω (proportional to the electromagnetic field strength) and the detuning Δ (how far the field is away from resonance).

w.g. R. Anderson in *Two-level system: Rabi oscillations.*

Light-matter interaction

two-levels in an external field



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