LECTURE 5

Long range coherence
Vortices

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BEC OF POLARITONS - BEC OF ATOMS

CHARACTERISTICS:

- Solid state system
- Disordered environment
- Long range spatial coherence
- Non equilibrium
- Steady state is characterized by incoming and outgoing flow of particles
- Emitted light is linearly related to polaritons

	atoms	**polaritons**
m	Rb: $10^4 m_e$	$10^{-4} m_e$
T	10^{-7}K	>100K
N	$10^{14}/cm^3$	$<10^{11}/cm^2$
t	∞	1 ps

FUNDAMENTAL DIFFERENCES:

- Condensation in a disordered medium
- Interacting Bose gas
- **Out of equilibrium**
- **Non-isolated system**

Sources: J. Kasprzak *et al* Bose–Einstein condensation of exciton polaritons. Nature 443: 409-414, (2006)
V. Savona *et al.* Optical Properties of Microcavity Polaritons. *Phase Transitions* **68** 169-279 (1999) and
V. Savona *et al.* Quantum-Well Excitons in Semiconductor Microcavities - Unified Treatment of Weak and Strong-Coupling Regimes. *Sol. St. Com.* **93** 733-739 (1995)

Exciton polaritons

dispersion - energy-wave vector dependence

Exciton dispersion in quantum well

$$E_X(k) = E_g - E_b + \frac{\hbar^2 k^2}{2m_X}$$

photon dispersion in quantum well

$$E(\vec{k}) = \frac{\hbar c}{n}|\vec{k}| = \frac{\hbar c}{n}\sqrt{\left(\frac{2\pi}{L_C}\right)^2 + k_{II}^2}$$

because we are interested in small wave-vectors $k_{\parallel}$ we can make the following approximation:

$$\sqrt{\varepsilon^2 + a^2} \approx a + \frac{\varepsilon^2}{2a}$$

and derive the energy in a form:

$$E(\vec{k}) \approx \frac{\hbar c}{n}\left[\frac{2\pi}{L_C} + \frac{k_{II}^2 L_C}{4\pi}\right] = E_0 + \frac{\hbar^2 k_{II}^2}{2m_{ph}^*}$$

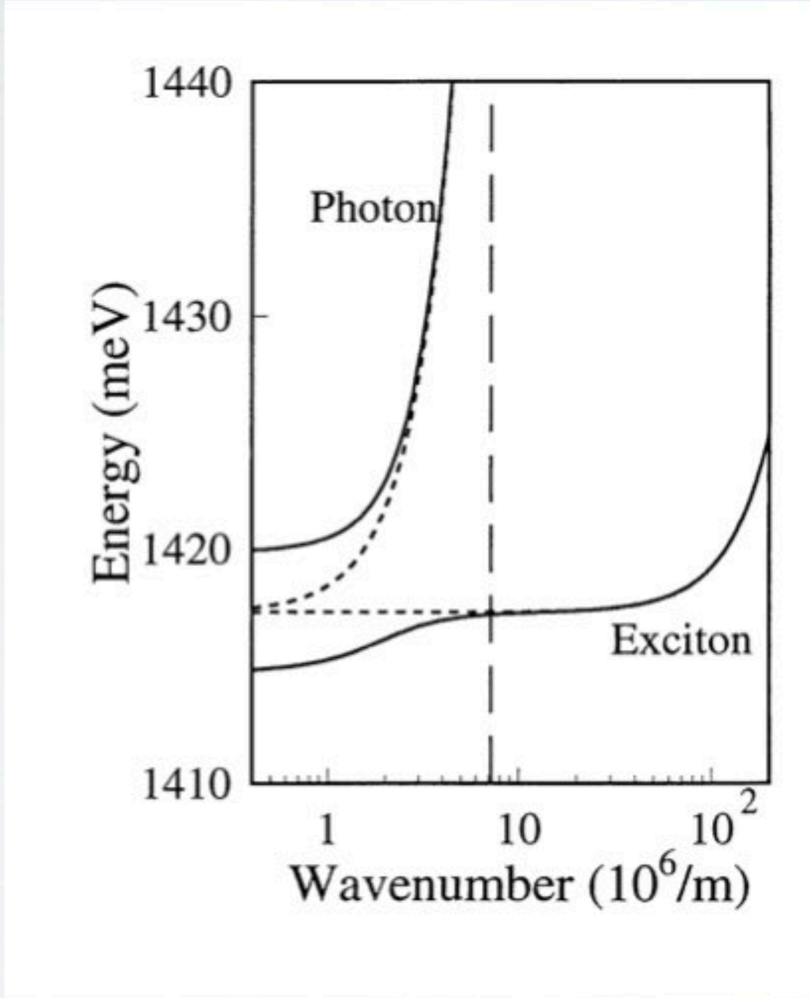

image after: M. S. Skolnick et al. Semicond. Sci. Technol. 13, 645 (1998)

Exciton polaritons
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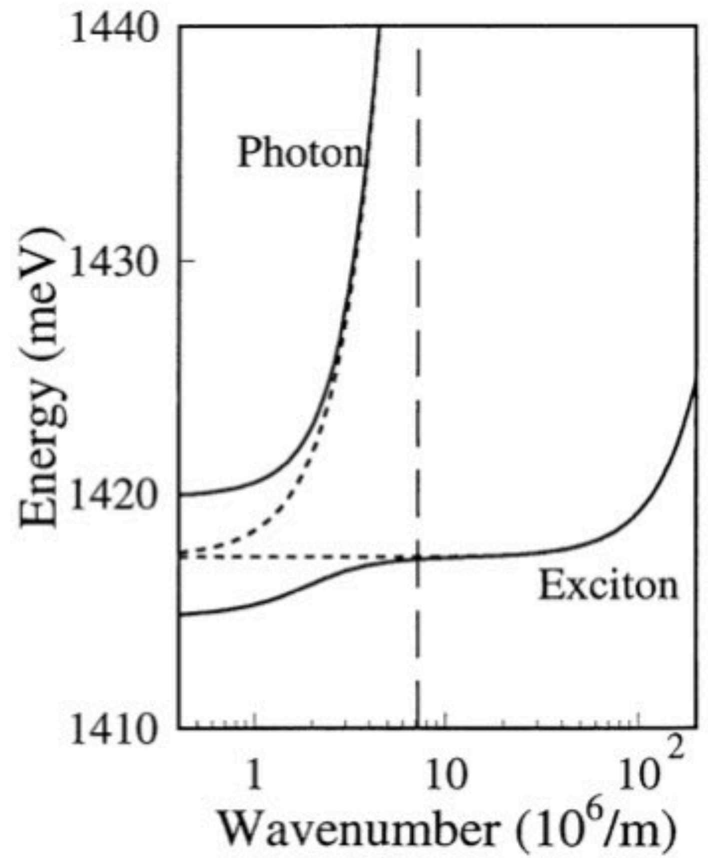
conclusions:

in a cavity photon gain an effective mass:

$$m_C^* = \frac{\hbar k_z n}{c} = \frac{h n^2}{c\lambda_0}$$



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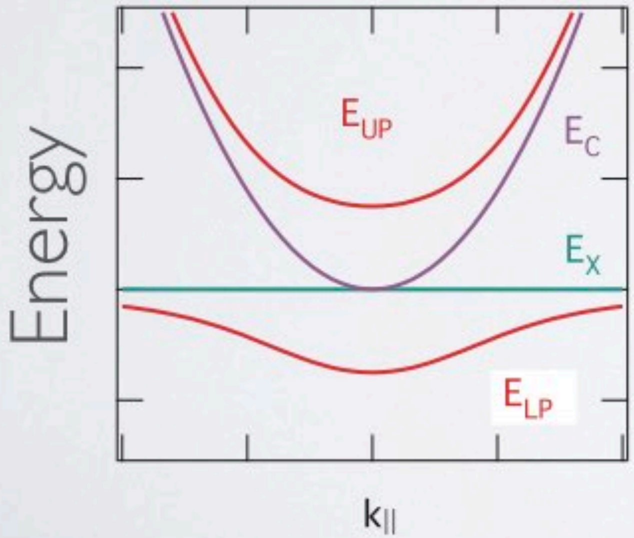
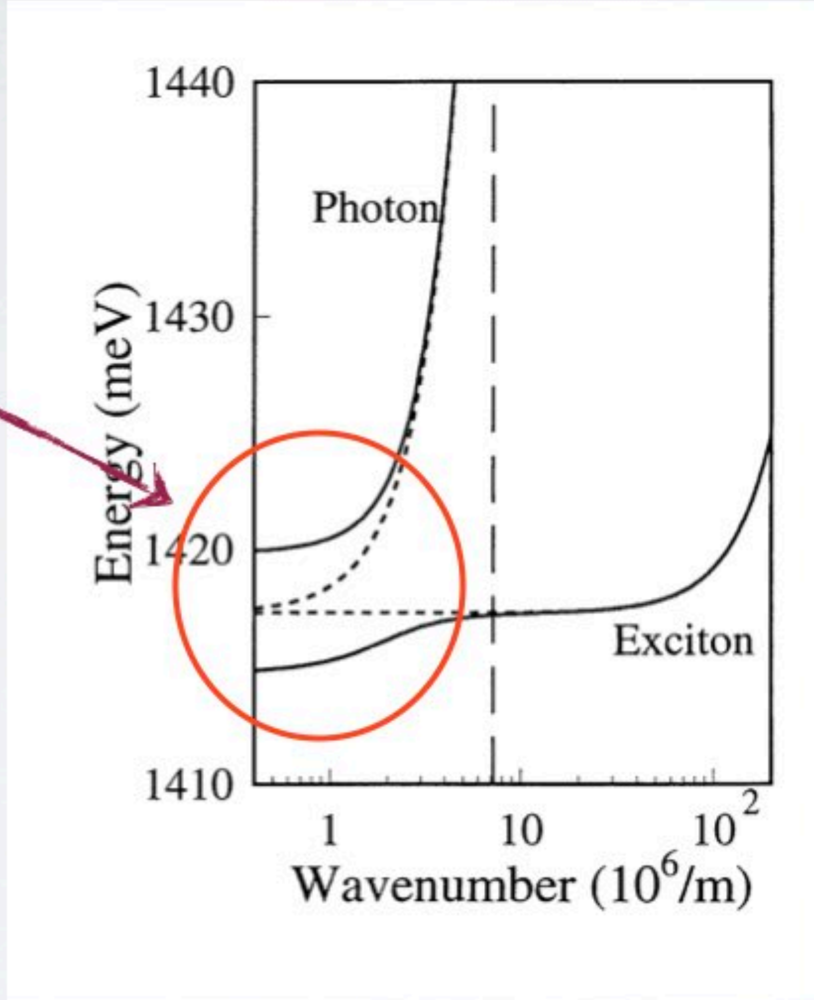
Exciton polaritons

dispersion - energy-wave vector dependence

experimental access





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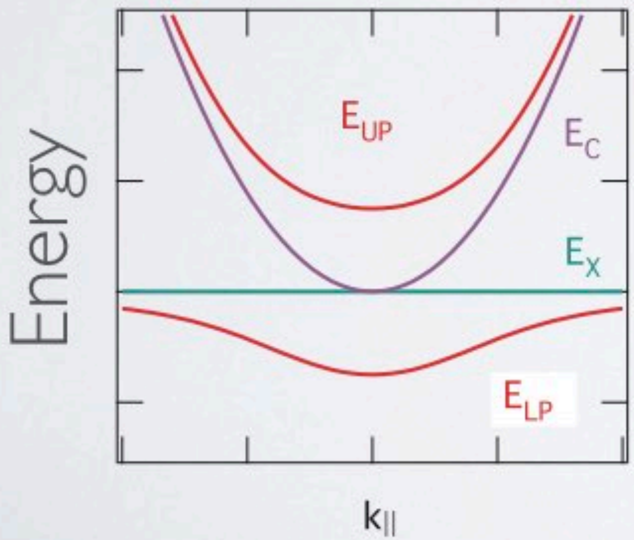
Exciton polaritons
dispersion - energy-wave vector dependence



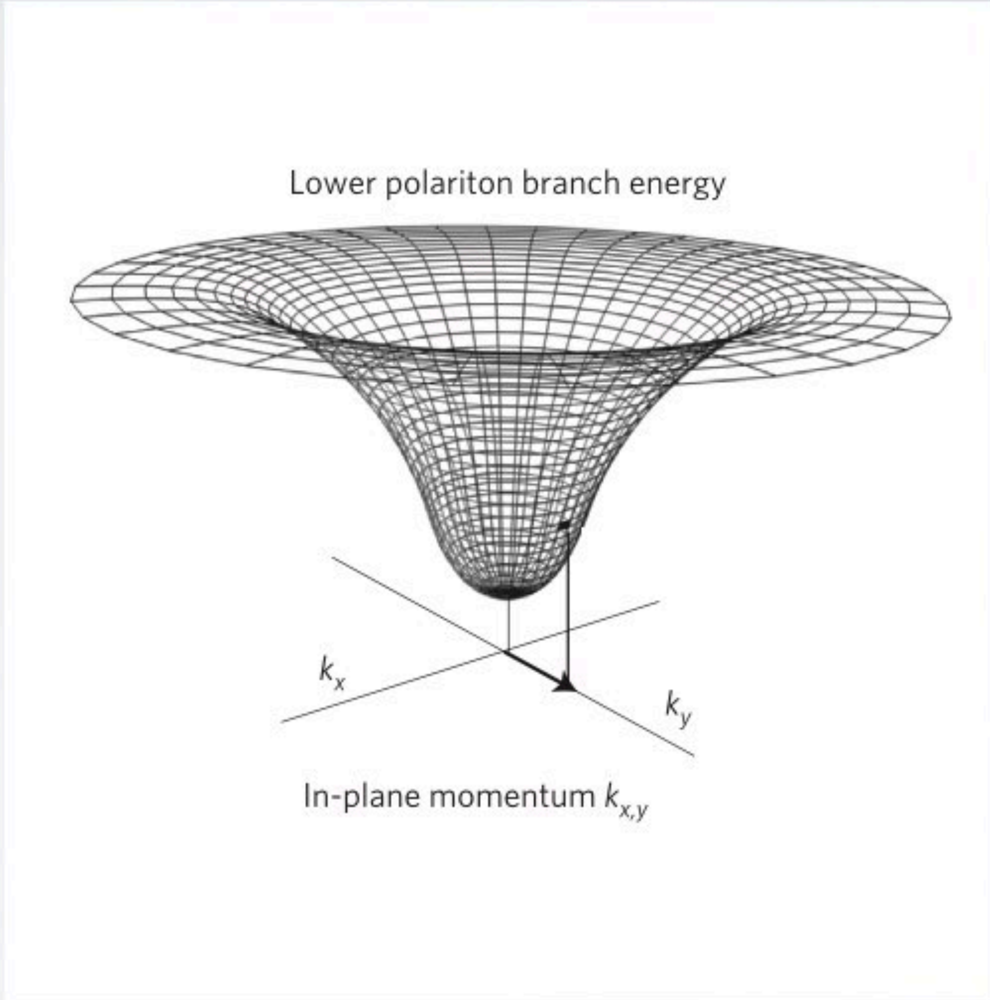
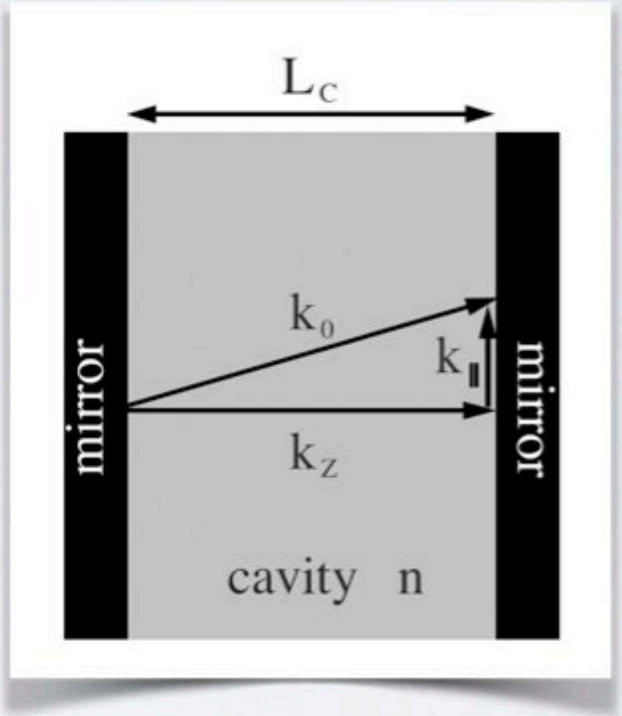
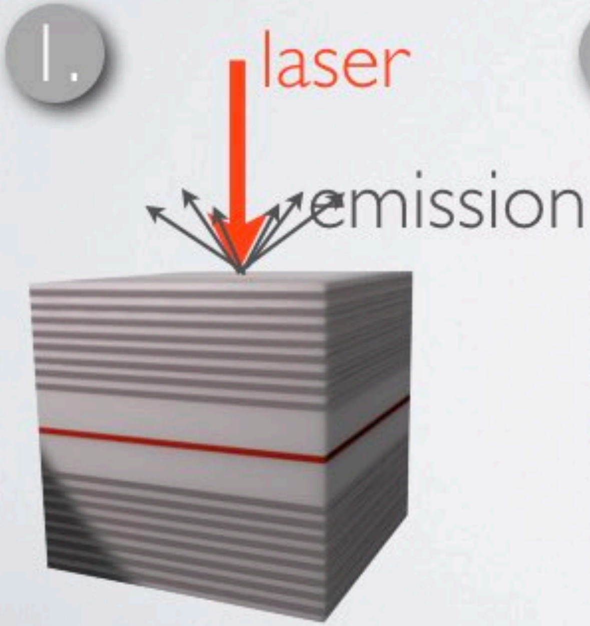


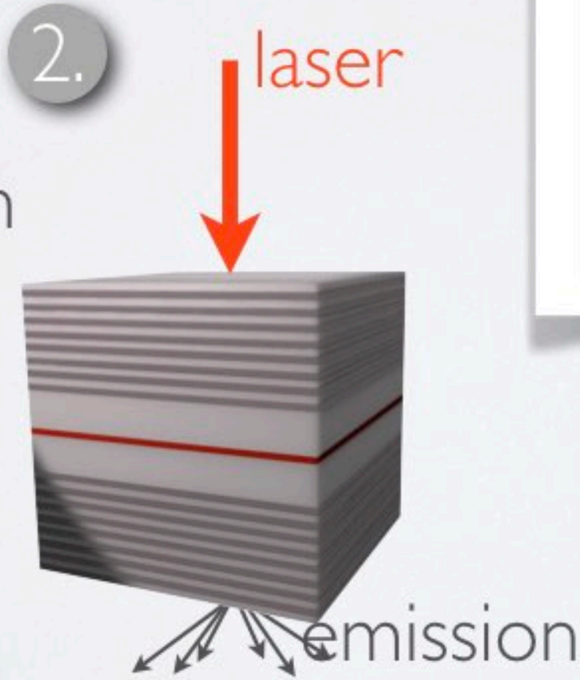
image after: D. Sanvitto et. al.

Exciton polaritons
typical experimental setup

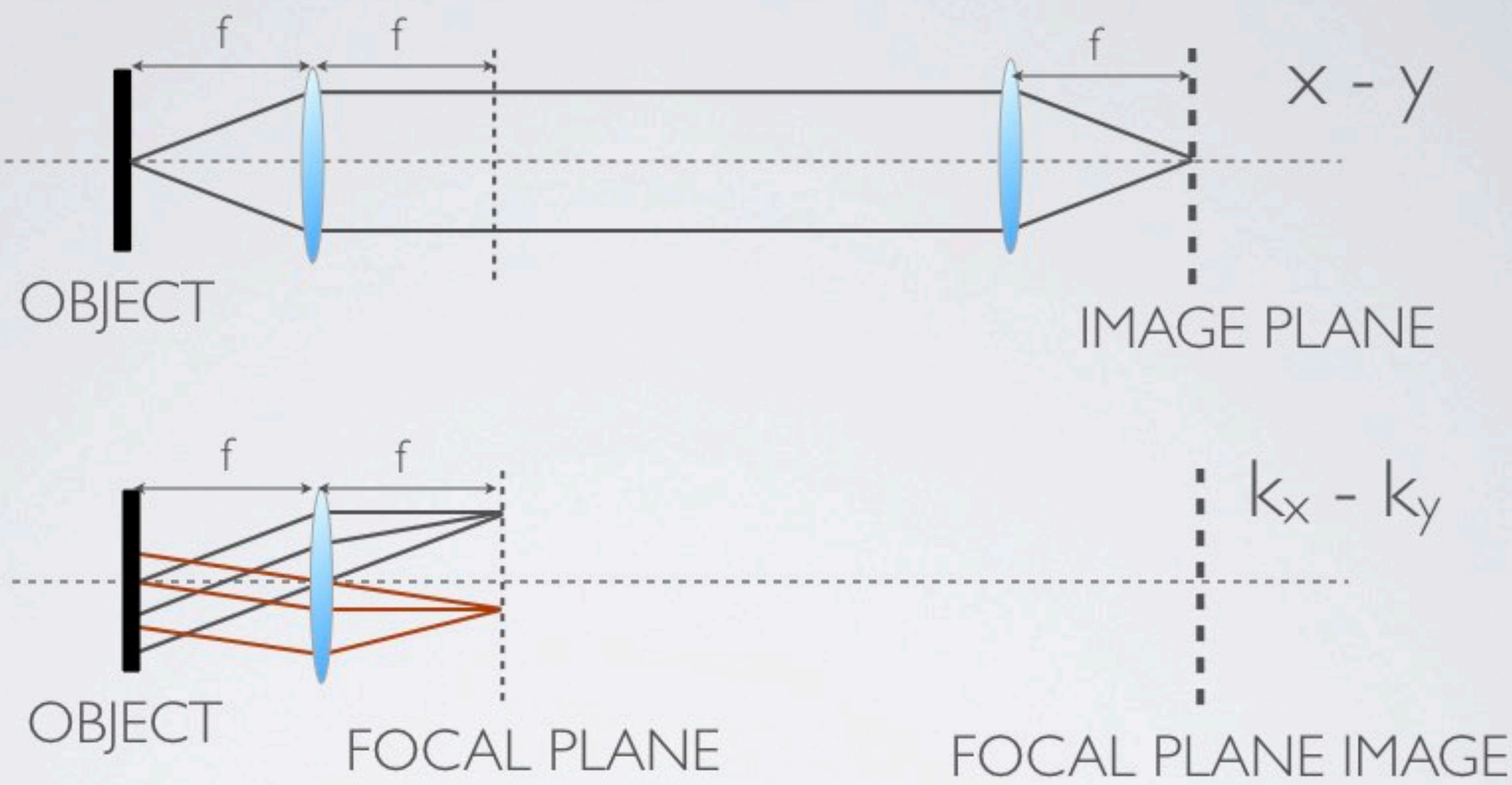
$$k_{\parallel} = k \sin\theta_{ext} = \frac{E(\theta_{ext})}{\hbar c} \sin\theta_{ext}$$







REAL (x) & MOMENTUM (k) SPACE



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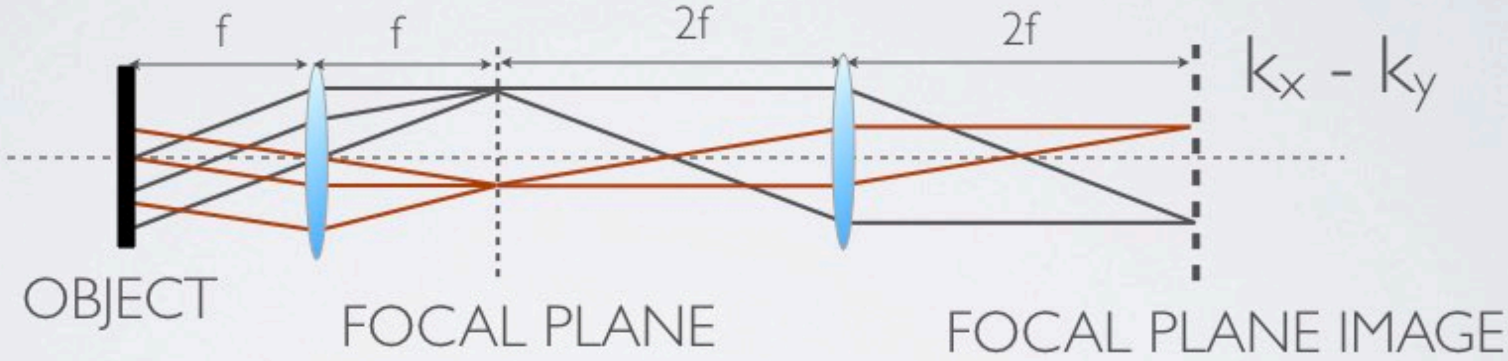

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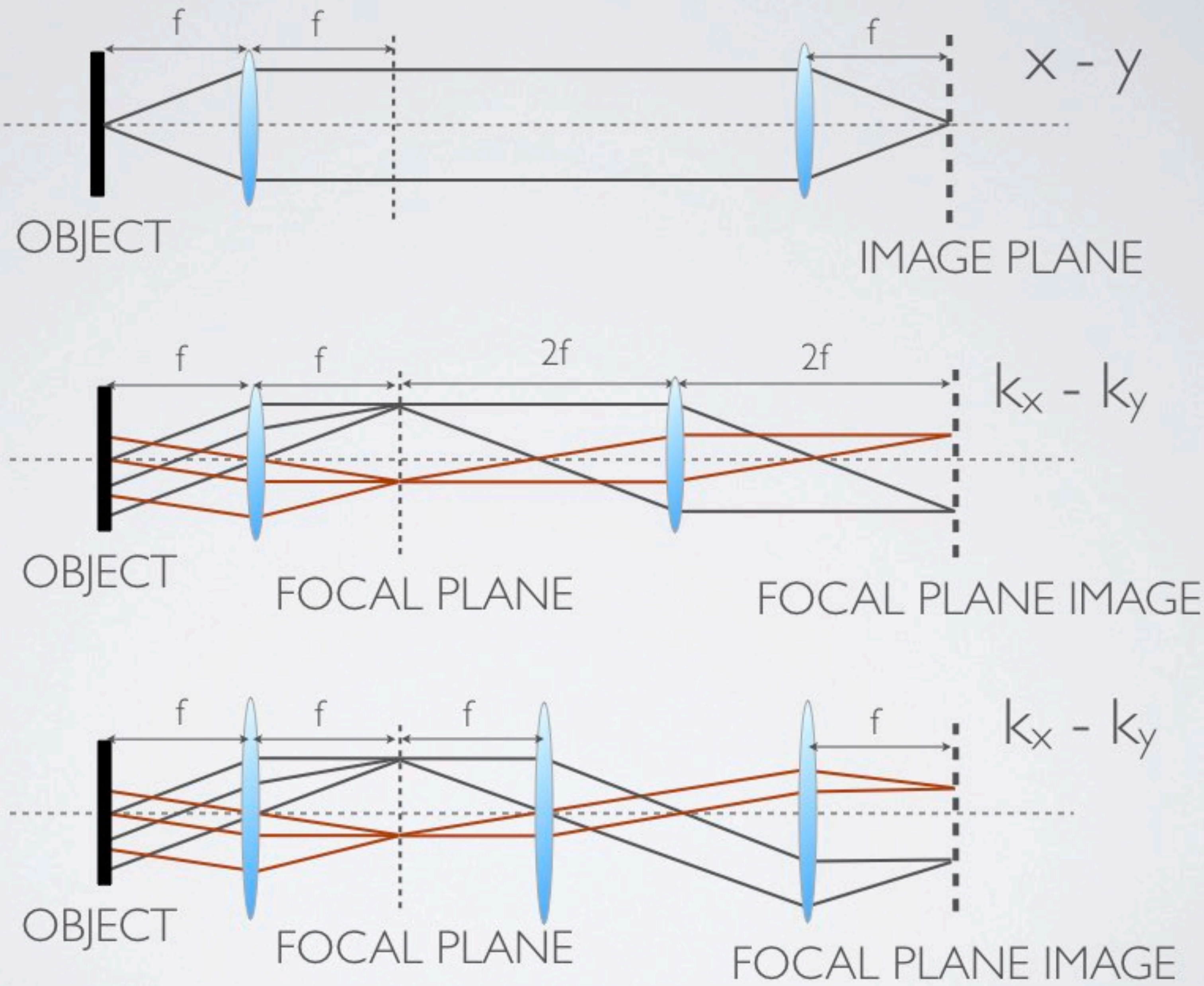


Exciton polaritons
typical experimental setup

x axis :
!!! emission angle !!!
= polariton momentum

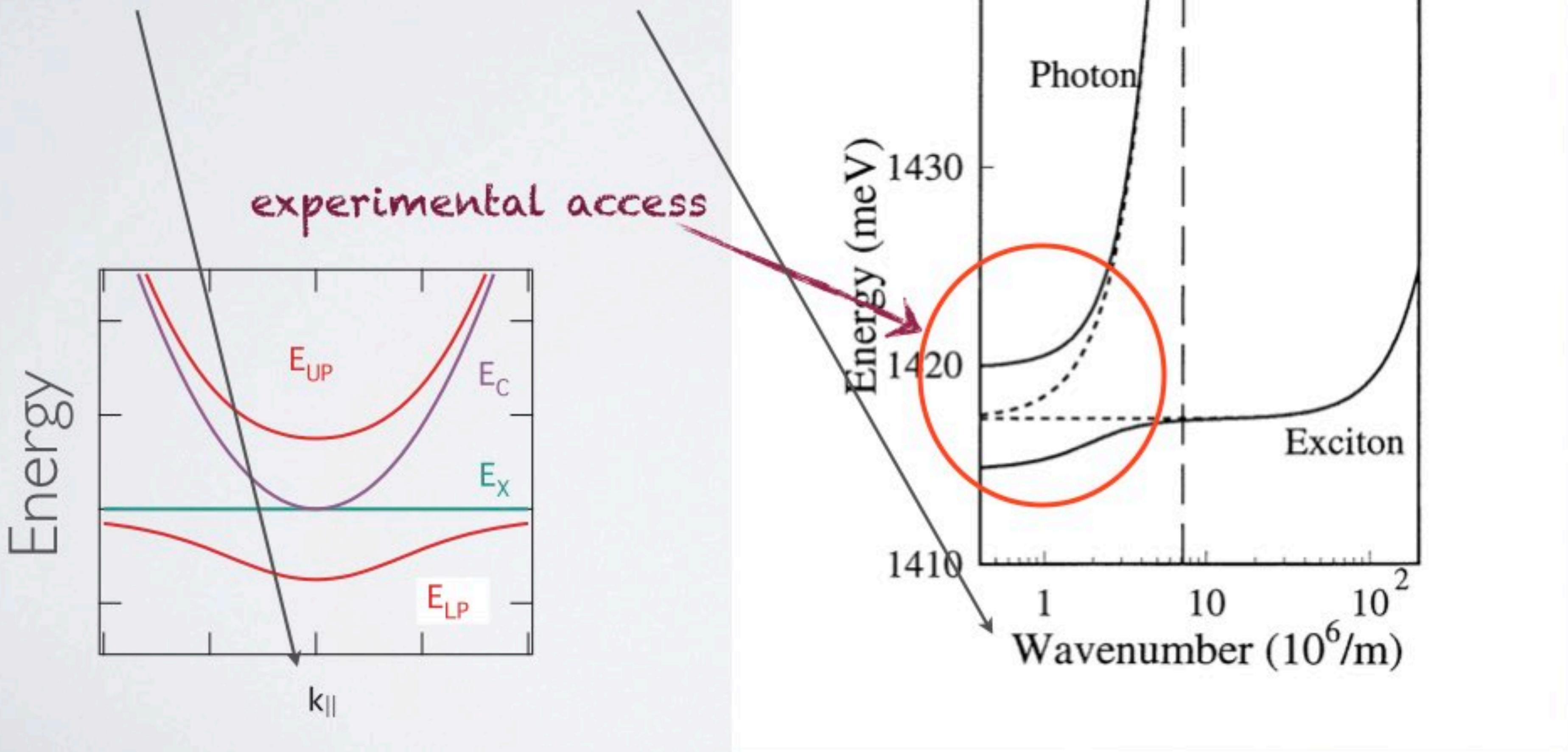
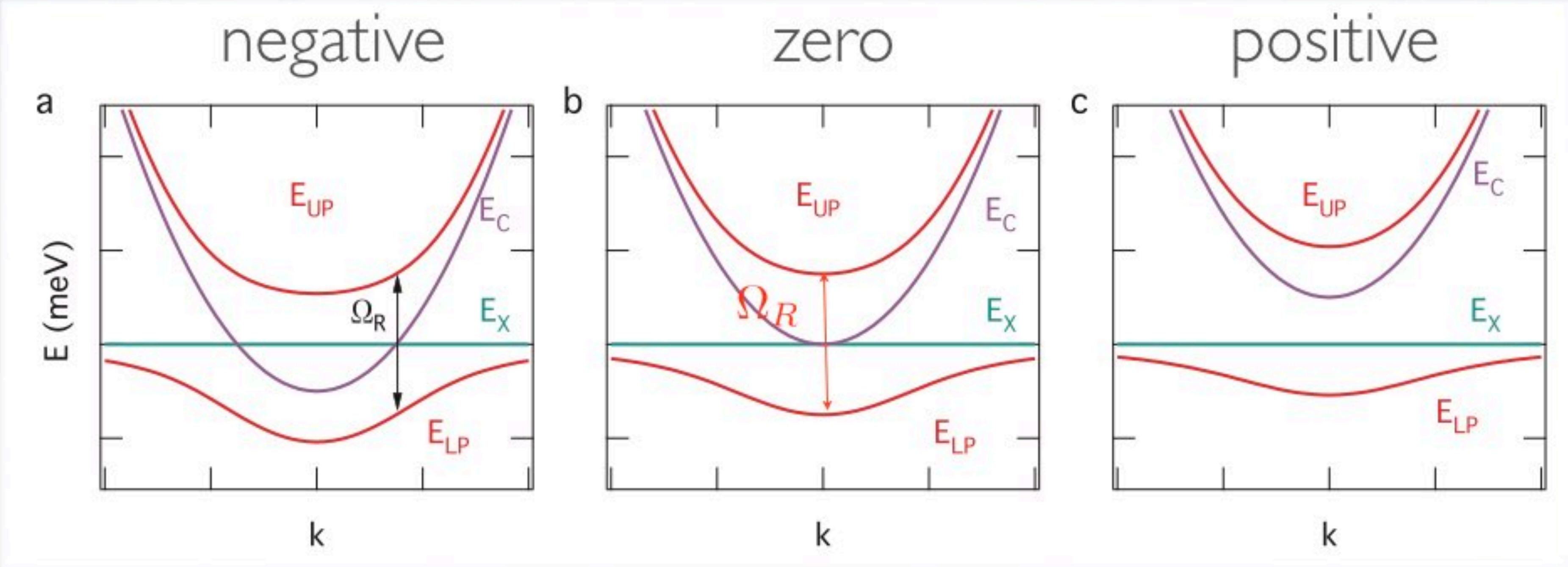


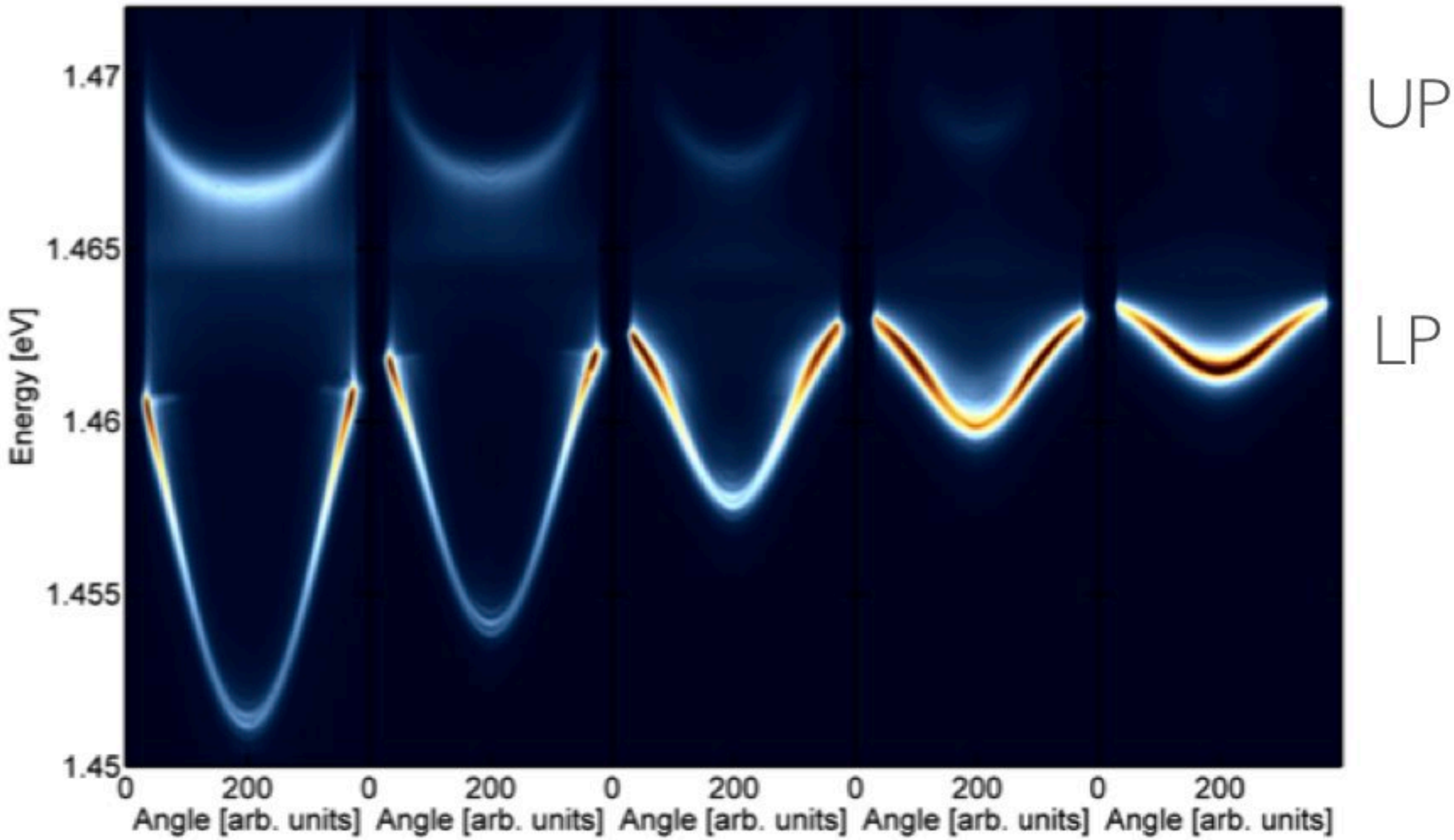
image after: M. S. Skolnick et al. Semicond. Sci. Technol. 13, 645 (1998)

Exciton polaritons
dispersion shape and detuning



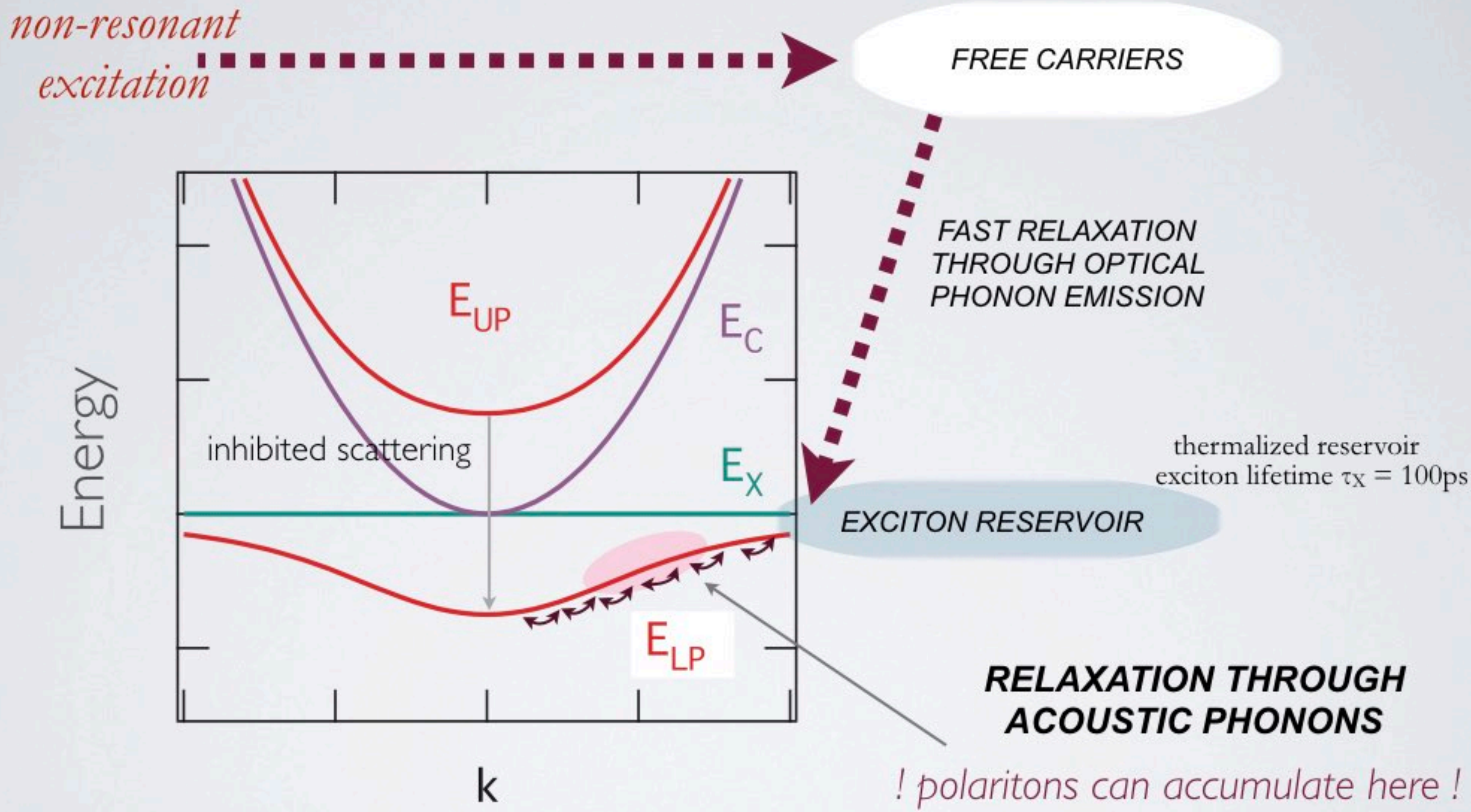
Exciton polaritons
dispersion shape and detuning

image: K. Lekenta, R. Mirek



Exciton polaritons

formation of polariton population



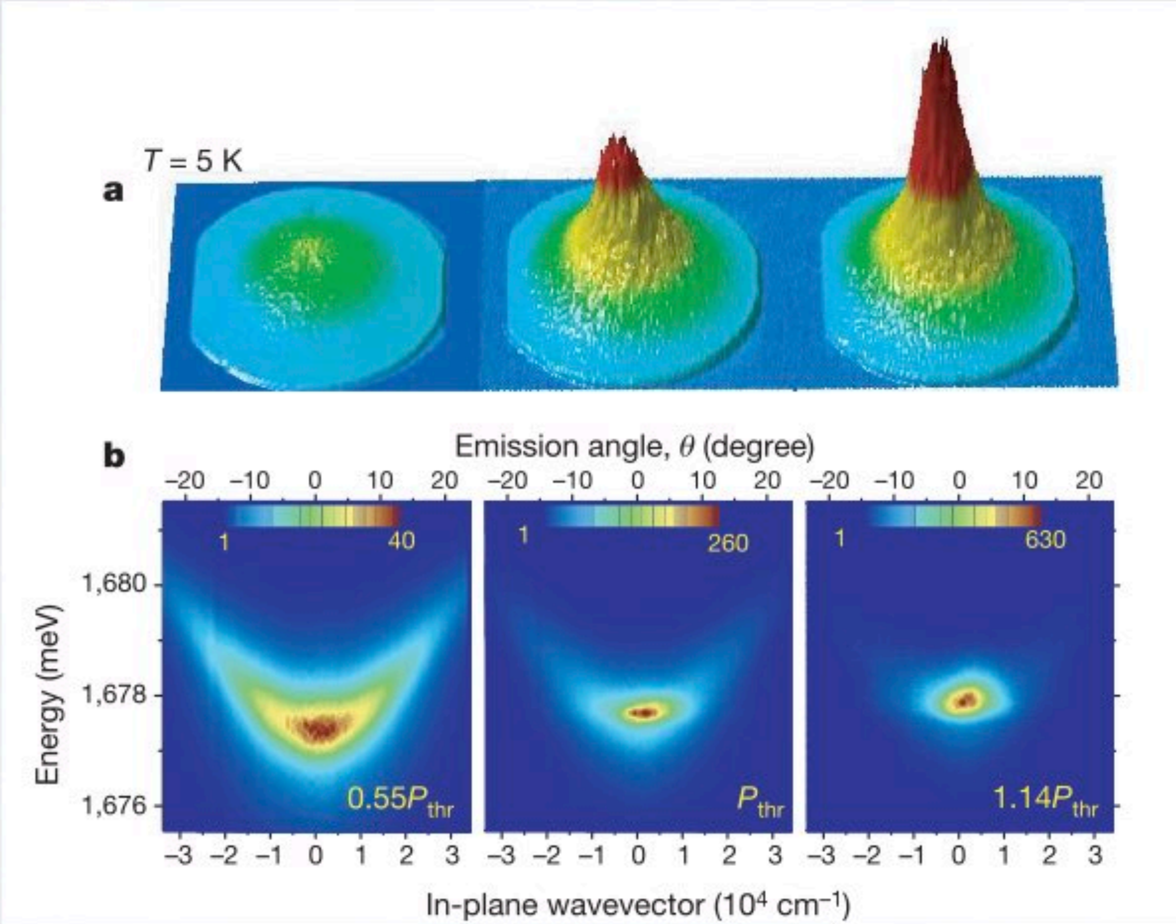
IN NON-LINEAR REGIME:

- FFS final state stimulation - polariton - polariton scattering

Bose–Einstein condensation of exciton polaritons

nature

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]



increased polariton density

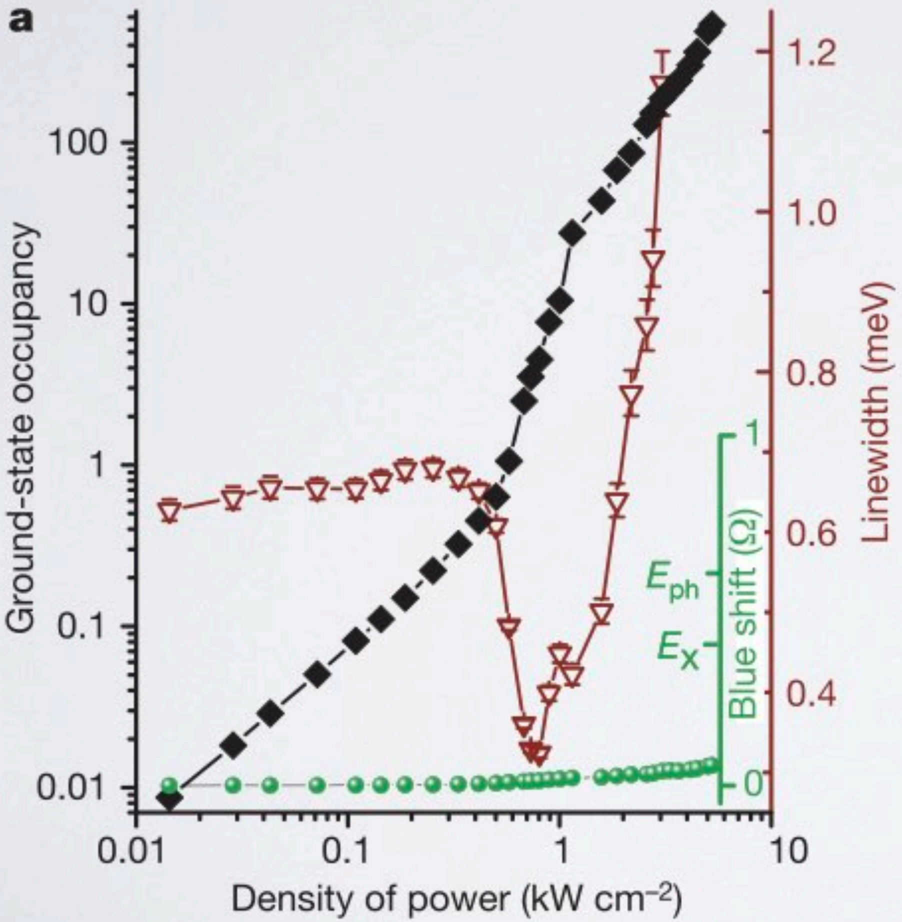
MACROSCOPIC OCCUPATION OF GROUND STATE

J. Kasprzak, et. al *Nature* **443**, 409 (2006)

Bose–Einstein condensation of exciton polaritons

nature

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]



INTENSITY INCREASE VS LINE-WIDTH NARROWING

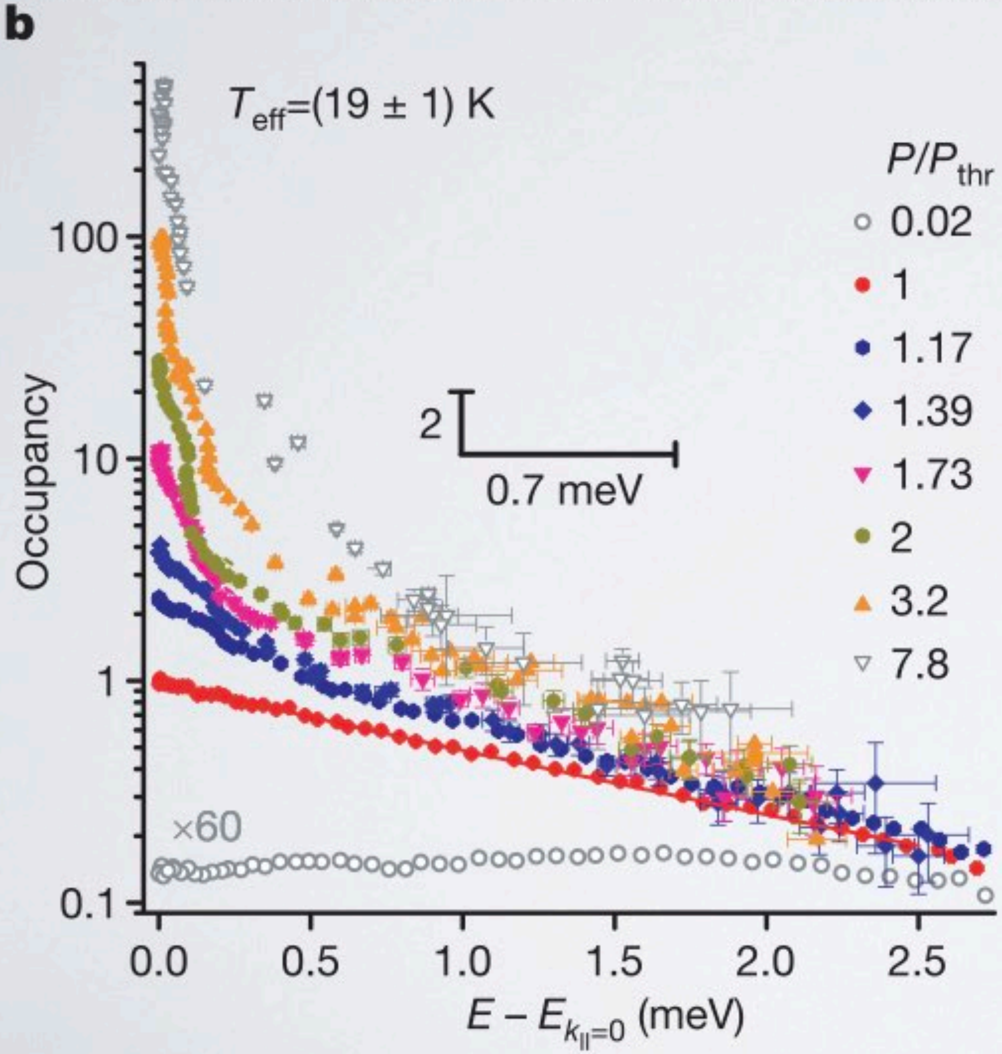
- INCREASE OF TEMPORAL COHERENCE !

J. Kasprzak, et. al *Nature* **443**, 409 (2006)

nature

Bose–Einstein condensation of exciton polaritons

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]



POPULATION DISTRIBUTION OVER EXCITED STATES

Maxwell - Boltzmann distribution at thermal equilibrium

$$n_j = \frac{N}{Z} e^{-\frac{\varepsilon_j}{kT}}$$

Bose- Einstein distribution in condensate

$$n_i = \frac{g_i}{Be^{\frac{\varepsilon_i}{kT}} - 1}$$

J. Kasprzak, et. al *Nature* **443**, 409 (2006)

QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

- One body density matrix express the probability amplitude to annihilate a particle at location **r'** and to create one at location **r**

$$\rho(\mathbf{r},\mathbf{r}') = \langle \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r}')\rangle$$

- for **r = r'**, this describes the local density of the system

$$n(\mathbf{r}) = \rho(\mathbf{r},\mathbf{r}) = \langle \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\rangle$$

- density matrix is normalized, such that the total number of particles is *N*

$$\int \mathrm{d}\mathbf{r}\rho(\mathbf{r},\mathbf{r}) = N$$

- for a pure state the average is the quantum mechanical expectation

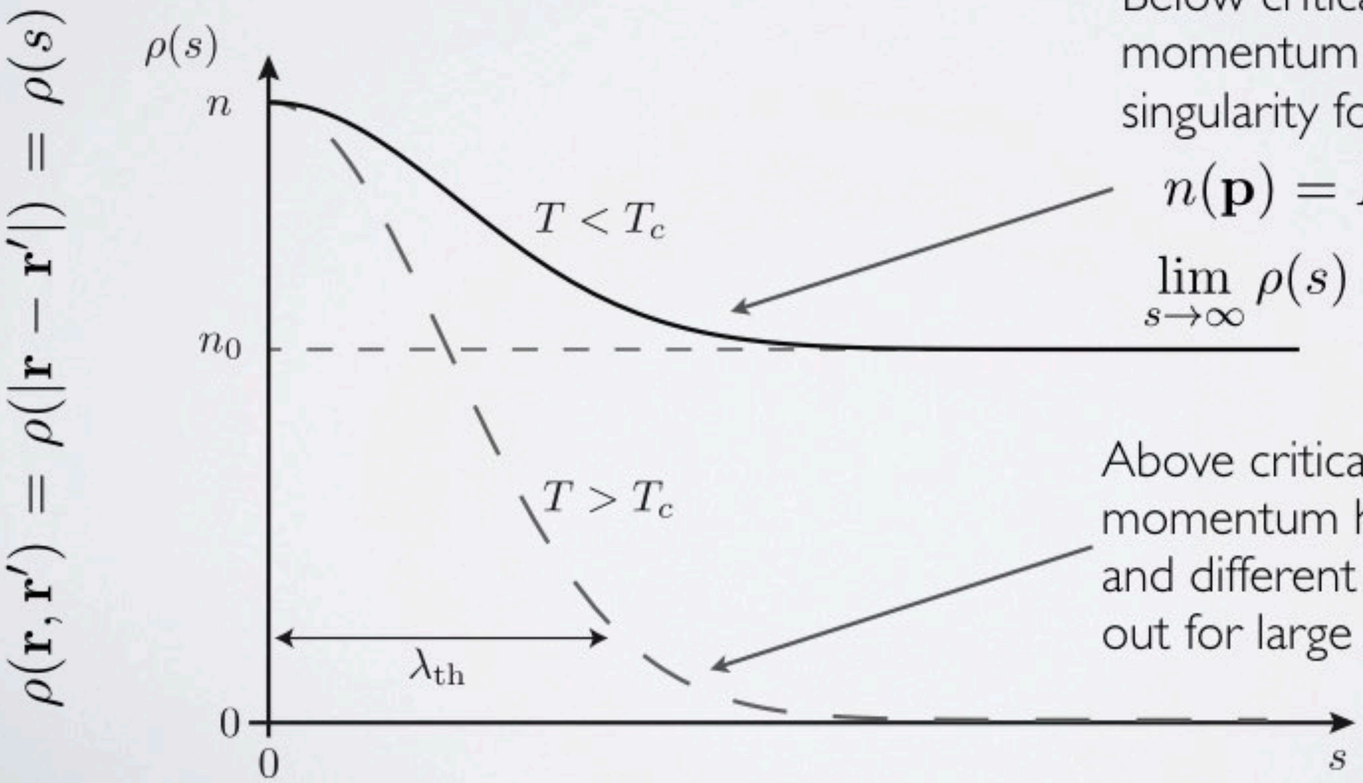
$$\rho(\mathbf{r},\mathbf{r}') = N\int \mathrm{d}\mathbf{r}_2\mathrm{d}\mathbf{r}_3\ldots\mathrm{d}\mathbf{r}_N\Psi^*(\mathbf{r},\mathbf{r}_2,\mathbf{r}_3,\ldots\mathbf{r}_N)\Psi(\mathbf{r}',\mathbf{r}_2,\mathbf{r}_3,\ldots\mathbf{r}_N)$$

QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

- Bose - Einstein condensation is a phenomenon taking place in momentum space
- The density matrix can be expressed also by the momentum distribution of the system

$$n(\mathbf{p}) = \langle \hat{\Psi}^\dagger(\mathbf{p}) \hat{\Psi}(\mathbf{p}) \rangle$$

- the one-body density matrix depend only on the relative distance *s* = |**r'-r**|



Below critical temperature the momentum distribution has a singularity for **p** = 0

$$n(\mathbf{p}) = N_0\delta(\mathbf{p}) + \tilde{n}(\mathbf{p})$$

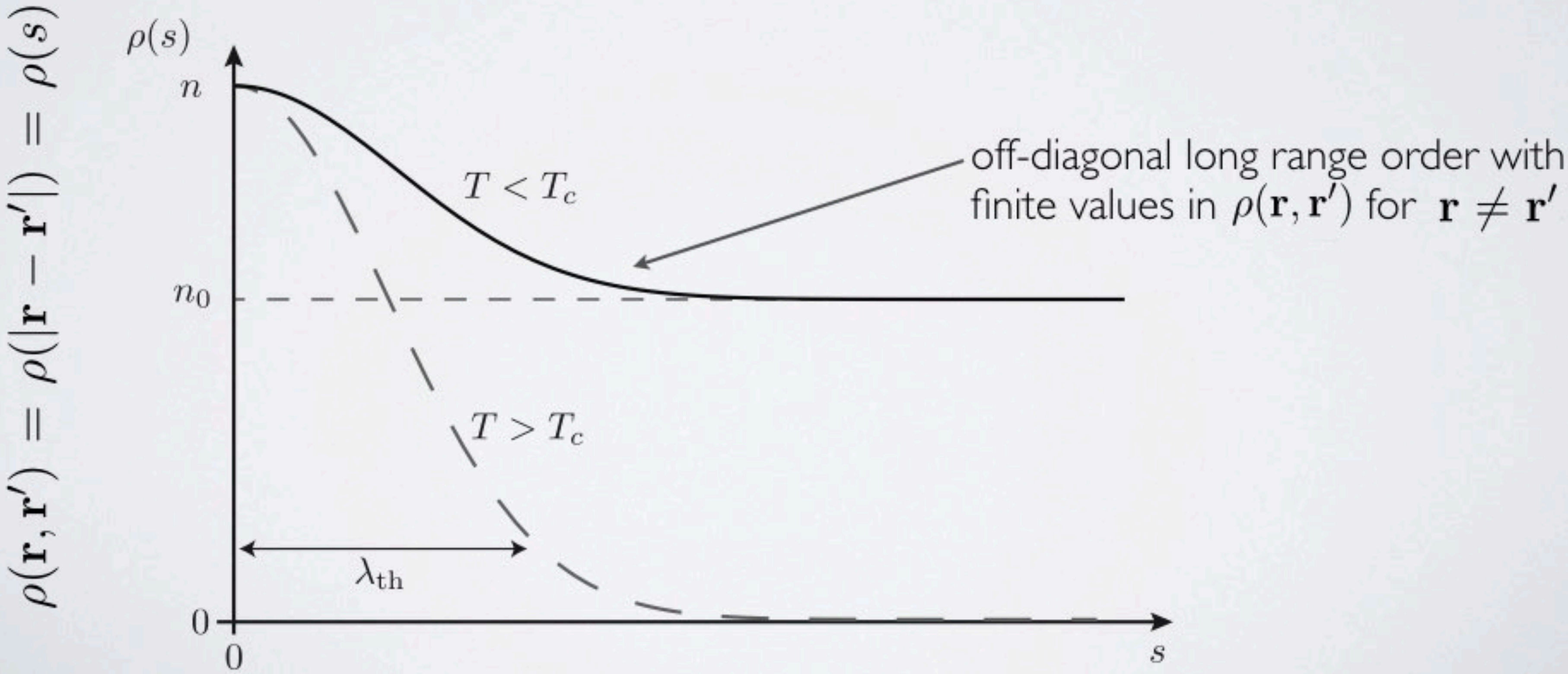
$$\lim_{s\to\infty} \rho(s) = n_0 = \frac{N_0}{V}$$

Above critical temperature momentum has a smooth distribution and different contributions average out for large *s*

$$\lim_{s\to\infty} \rho(s) = 0$$

QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

- The one-body density matrix for a BEC is decaying over a distance given by the thermal de Broglie length to a finite value determined by the condensate fraction



QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

- density matrix has finite off-diagonal elements

- referred to as **O**ff-**D**iagonal **L**ong **R**ange **O**rder (**ODLRO**)

  concept introduced first by Penrose and Onsager in 1956

- **can be measured through the first order coherence function**

$$G^{(1)}(\mathbf{r},\mathbf{r}') = \rho(\mathbf{r},\mathbf{r}')$$

- in normalized form

$$g^{(1)}(\mathbf{r},\mathbf{r}') = \frac{G^{(1)}(\mathbf{r},\mathbf{r}')}{\sqrt{G^{(1)}(\mathbf{r},\mathbf{r})}\sqrt{G^{(1)}(\mathbf{r}',\mathbf{r}')}}$$

- perfect correlations correspond to $g^{(1)}(\mathbf{r},\mathbf{r}') = 1$

- higher order coherence further characterize the state and distinguishes it from a thermal mixture

ORDER PARAMETER AND CONDENSATE WAVE-FUNCTION

- Applying field operator to BEC where the ground state is macroscopically populated

$$\hat{\Psi}(\mathbf{r}) = \sum_i \phi_i(\mathbf{r})\hat{a}_i = \underbrace{\phi_0(\mathbf{r})\hat{a}_0}_{\text{condensate}} + \underbrace{\sum_{i\neq 0} \phi_i(\mathbf{r})\hat{a}_i}_{\text{non-condensed part}}$$

$$\hat{\Psi}(\mathbf{r}) = \psi(\mathbf{r}) + \delta\hat{\Psi}(\mathbf{r})$$

$$\psi(\mathbf{r}) = \sqrt{N_0}\phi_0(\mathbf{r})$$

For a pure BEC the field operator is described by a wave-function thus a classical object

$$\psi(\mathbf{r}) = |\psi(\mathbf{r})|e^{i\varphi(\mathbf{r})}$$

diagonal therm - density

off-diagonal density - coherence

- Condensate wave-function is therefore the order parameter of the normal to condensed phase transition

ORDER PARAMETER

zero before the phase transition and becomes determined after phase transition

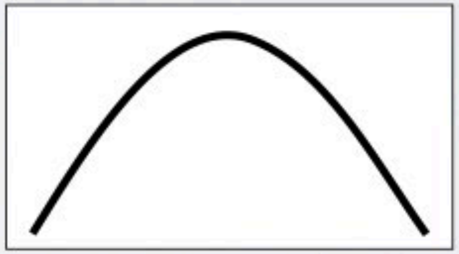
$$g^{(1)}(\mathbf{r},\mathbf{r}',\tau,t) = \langle \psi^*(\mathbf{r},\tau,t)\psi(\mathbf{r}',\tau,t)\rangle$$

(density matrix)

FIRST ORDER CORRELATION FUNCTION

Phase coherence!!!

spatial and temporal:



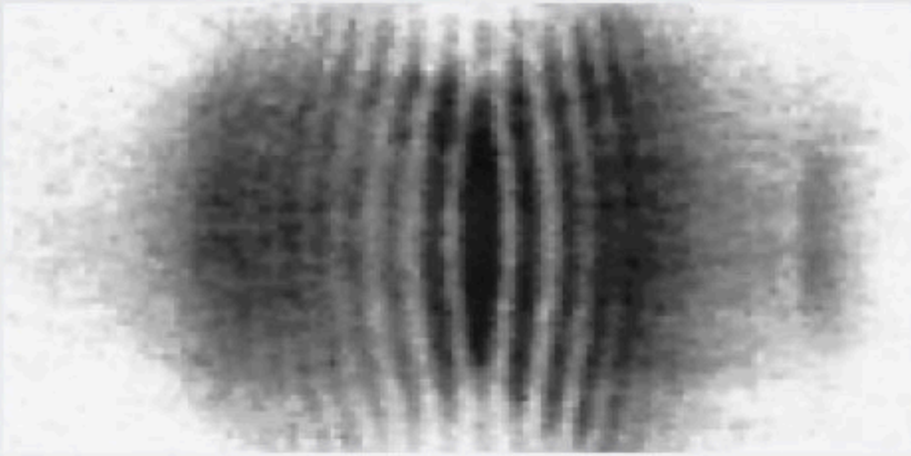
T=0:
Pure Bose condensate
"Giant matter wave"

- Coherence in time
- Long-range order = spatial coherence

during the phase transition

interference !!

ability to form interference fringes



Interference between two atomic BEC

M. R. Andrews et al., Science **275,** 637 (1997)

Exciton polaritons

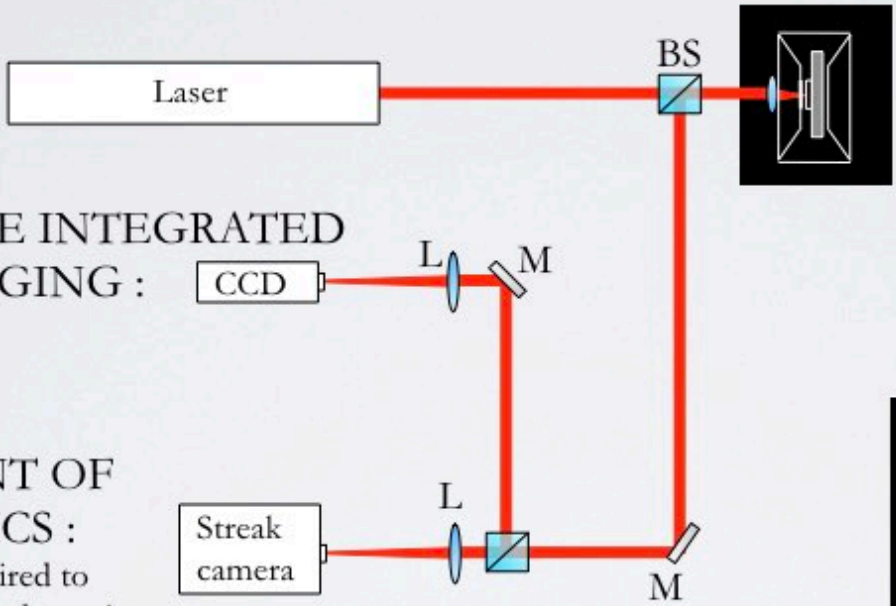
more advanced experimental setup

EXCITATION

- Pulsed OR
- Cw
- Non-resonant excitation

MICROSCOPE OBJECTIVE

- for spatial imaging
- condensate sizes from a few to hundreds of μm



Sample cooled to max. 40 K

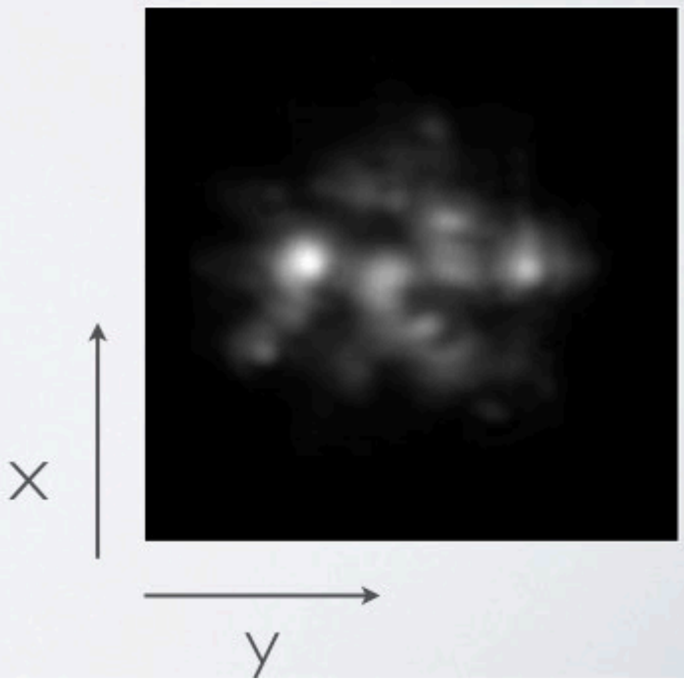
- typically 4.2K

TIME INTEGRATED IMAGING :

MEASUREMENT OF TIME DYNAMICS :

- ps resolution required to capture the condensate dynamics

- Disorder
- Different spots on the sample give different images corresponding to single or multiple localized condensates



Exciton polaritons

more advanced experimental setup

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TIME INTEGRATED IMAGING :

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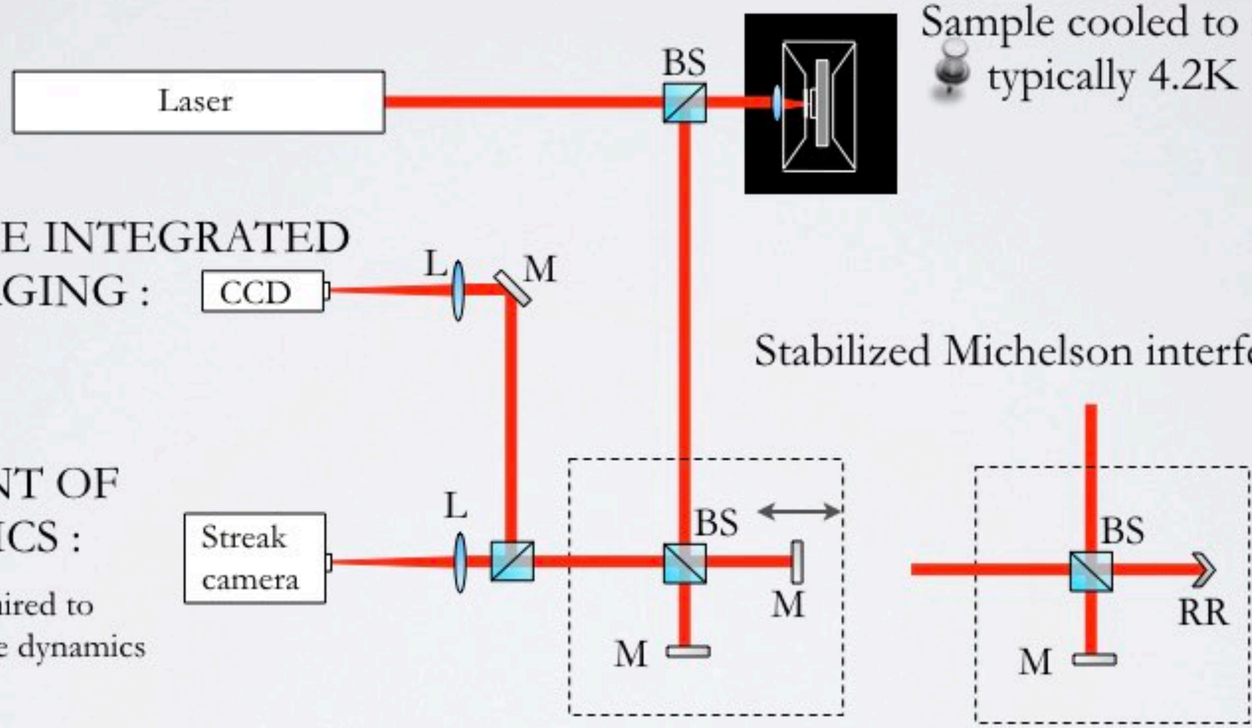
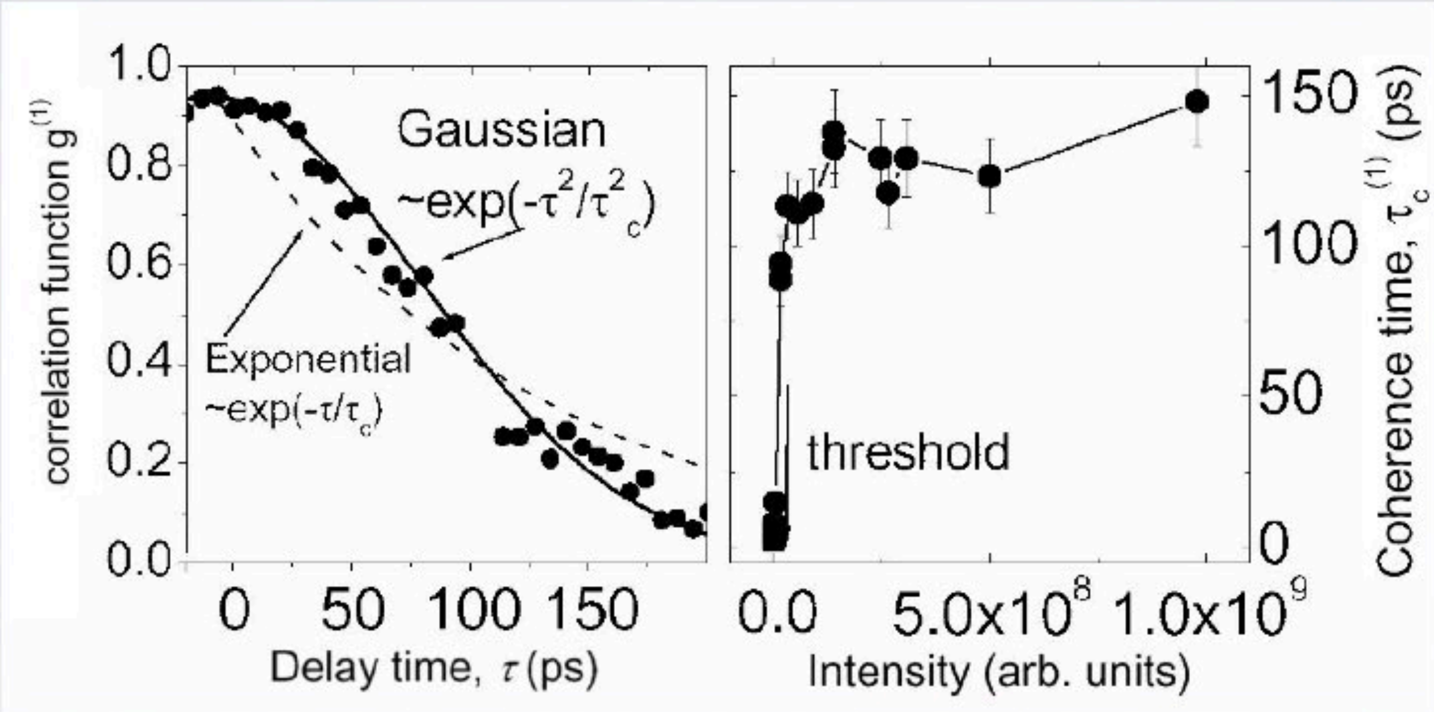
Stabilized Michelson interferometer



time coherence

spatial coherence

Exciton polaritons
first order correlation function



Temporal decay time ~150ps

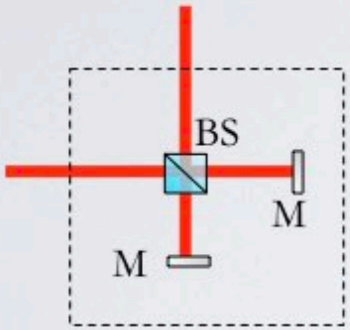
A.P. D. Love et al, Phys. Rev. Lett. 101, 067404 (2008)

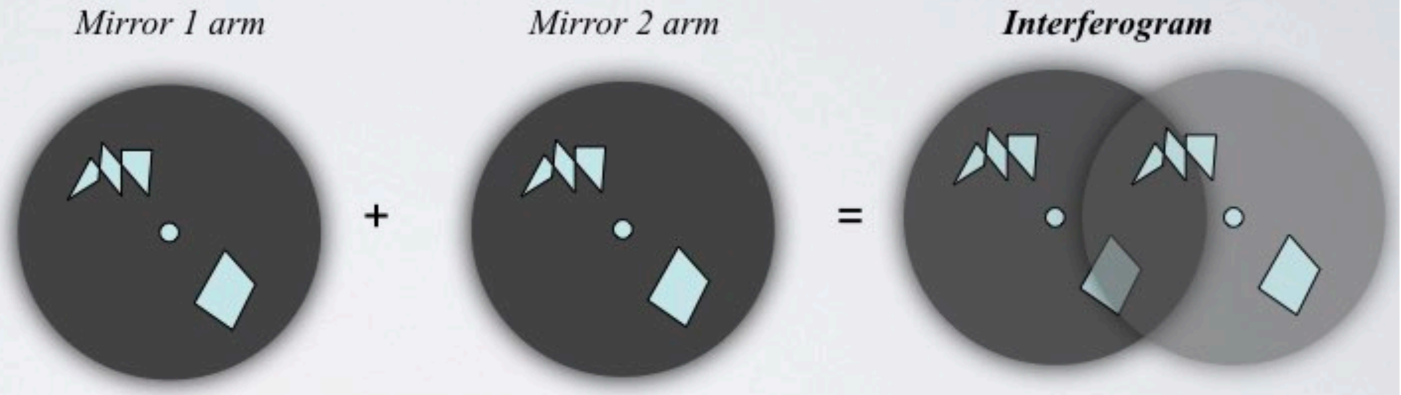
Spatial coherence
more advanced experimental setup

Operation principle

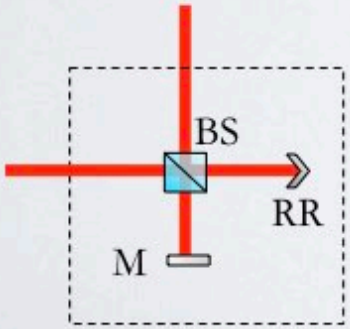
Stabilised Michelson interferometer

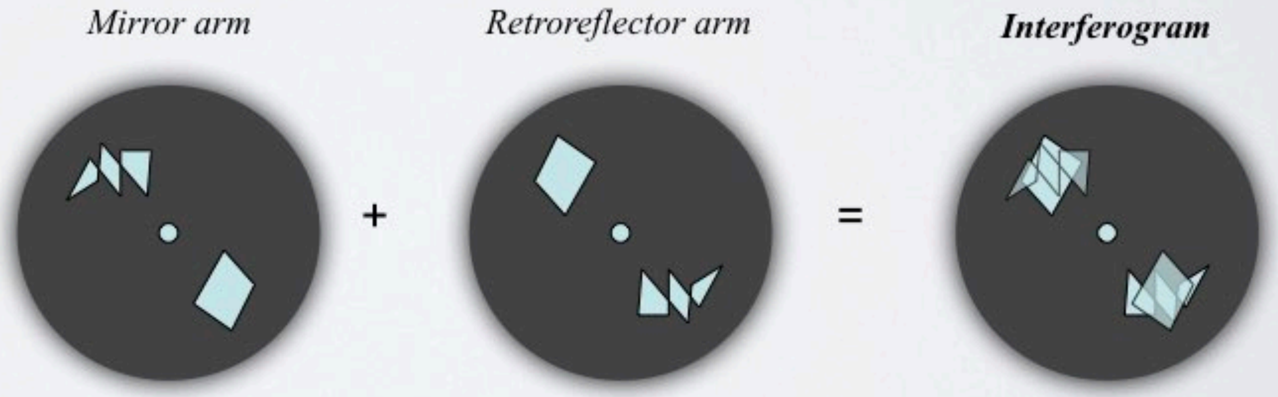
@ mirror- mirror configuration





@ mirror - retroreflector configuration

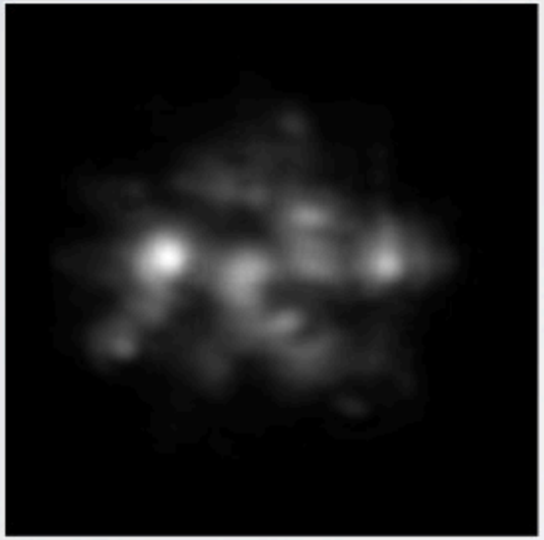




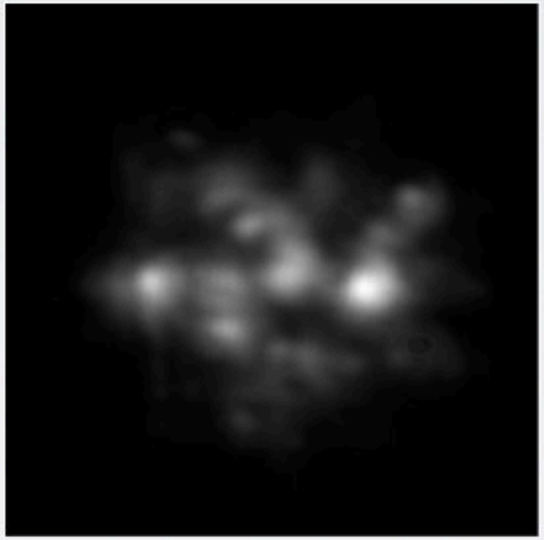
Spatial coherence
exciton-polariton condensed state

- Experimental realisation:

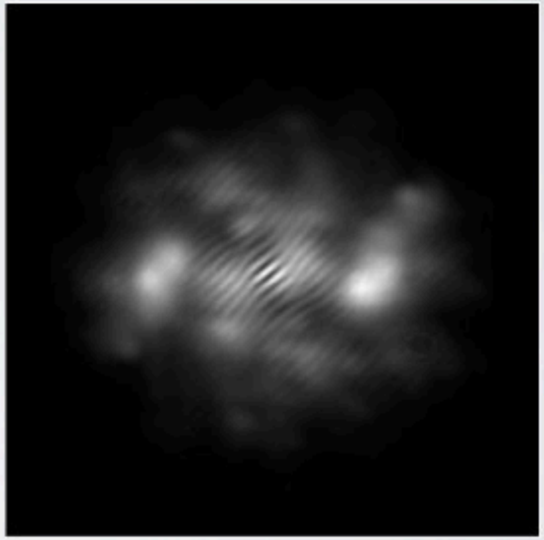
Mirror arm real space

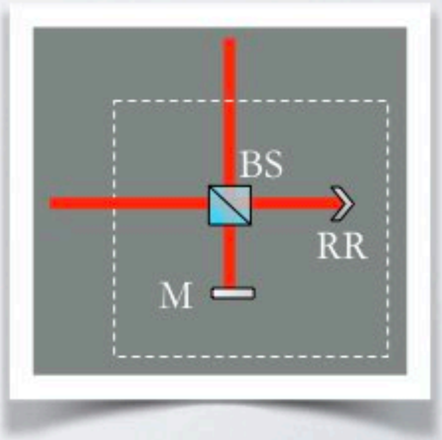


Retroreflector arm real space



Interferogram





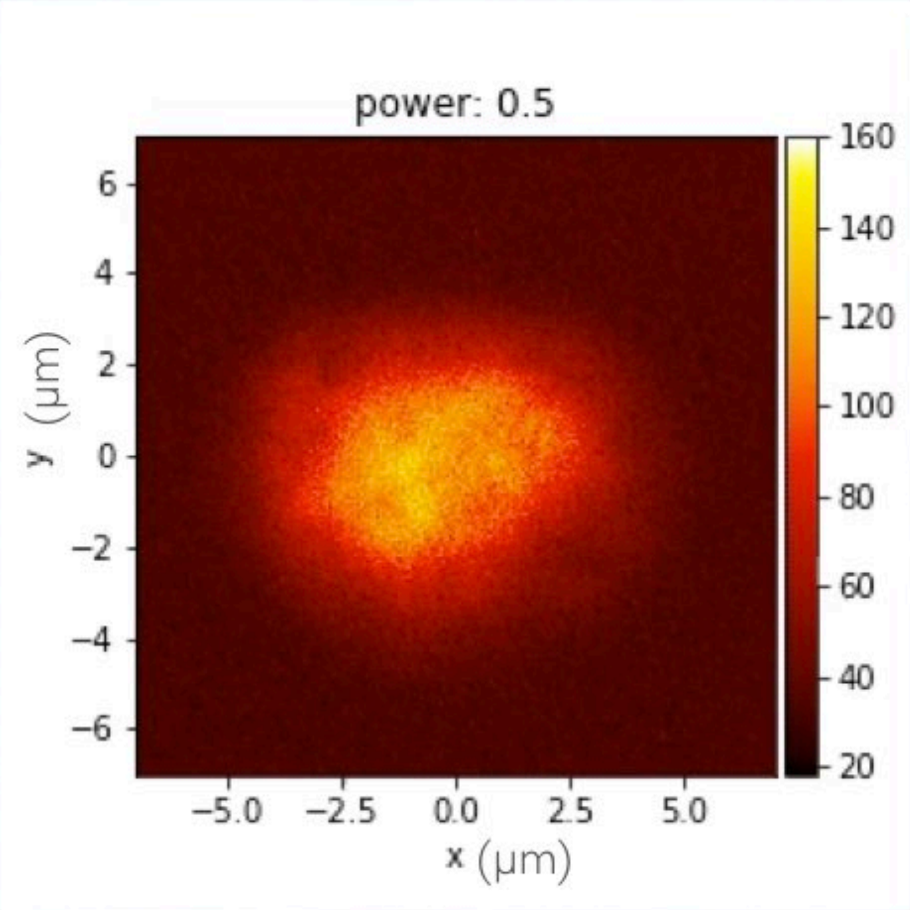
- Disorder in the sample
- Different positions will give
- different interferograms

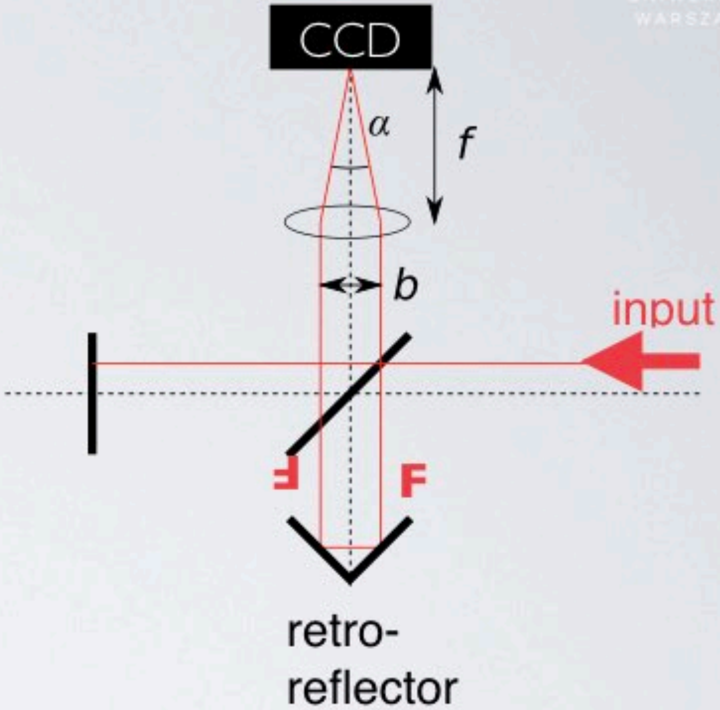
images: K. Lagoudakis, EPFL

Phase detection of polariton condensate

Modified Michelson interferometer

WYDZIAŁ FIZYKI UNIWERSYTET WARSZAWSKI

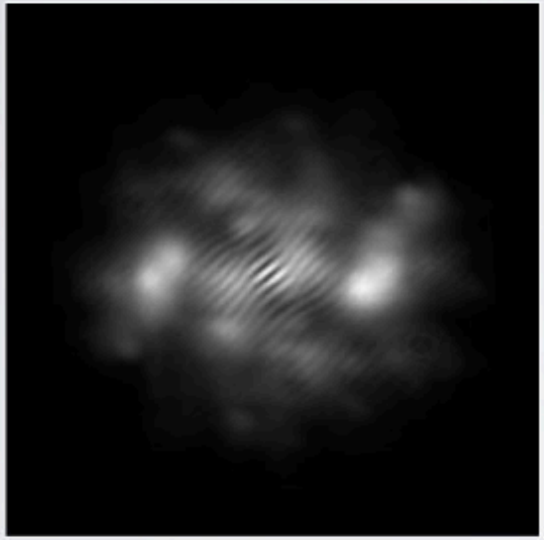




Proof of long-range coherence

Spatial coherence
what is hidden in the interferogram

Interferogram



images: K. Lagoudakis, EPFL

Fast Fourier Transform (FFT)

- Information about amplitude
- Information about phase

$$C(\mathbf{r},\mathbf{d}) = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{I(\mathbf{r})I(\mathbf{r}+\mathbf{d})}}{I(\mathbf{r}) + I(\mathbf{r}+\mathbf{d})} g^{(1)}(\mathbf{r},\mathbf{r}+\mathbf{d})$$

contrast

correlation function

ORDER PARAMETER

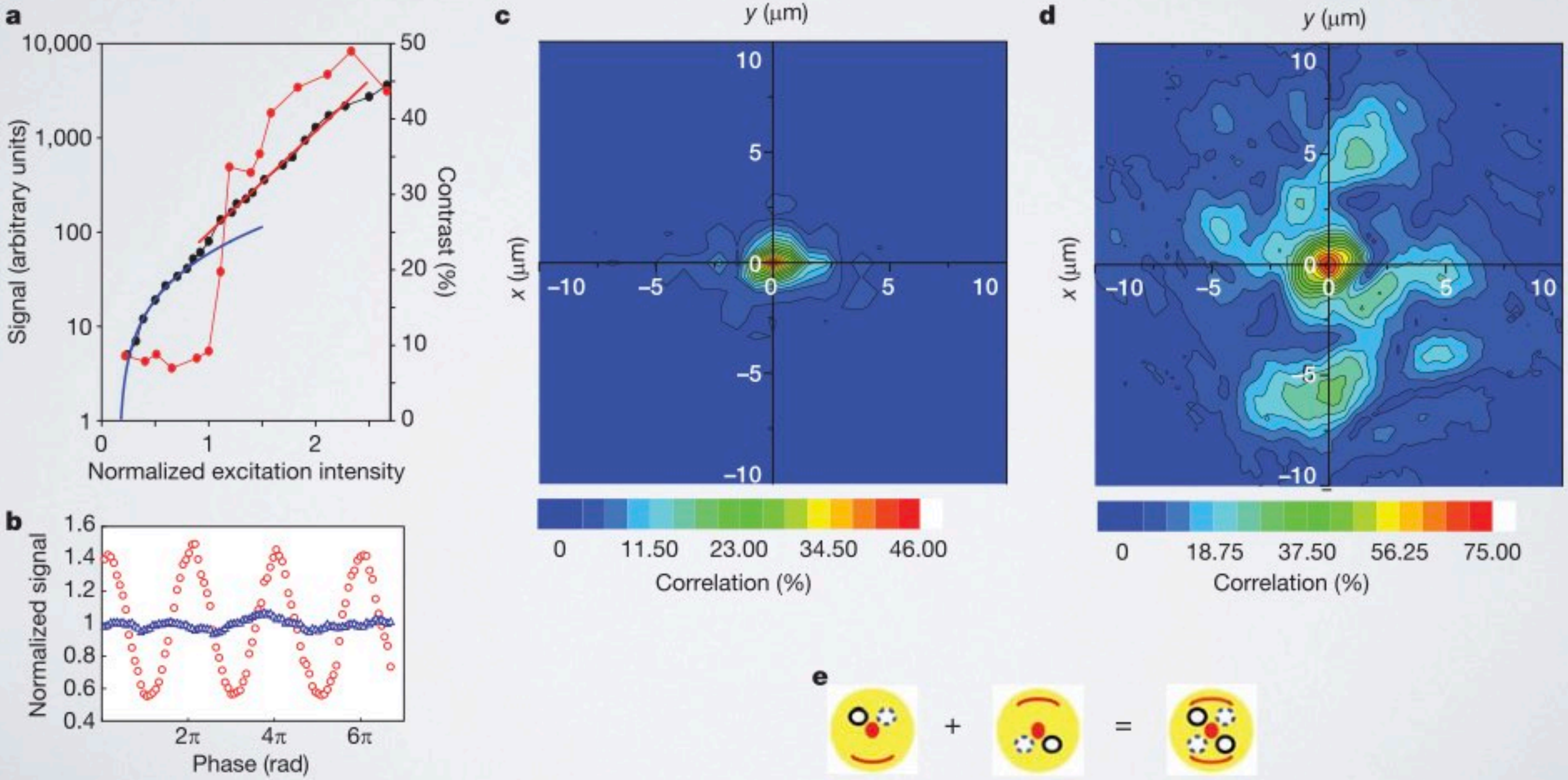
$$g^{(1)}(\mathbf{r},\mathbf{r}',\tau,t) = \langle \psi^*(\mathbf{r},\tau,t)\psi(\mathbf{r}',\tau,t)\rangle$$

nature

Bose–Einstein condensation of exciton polaritons

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]

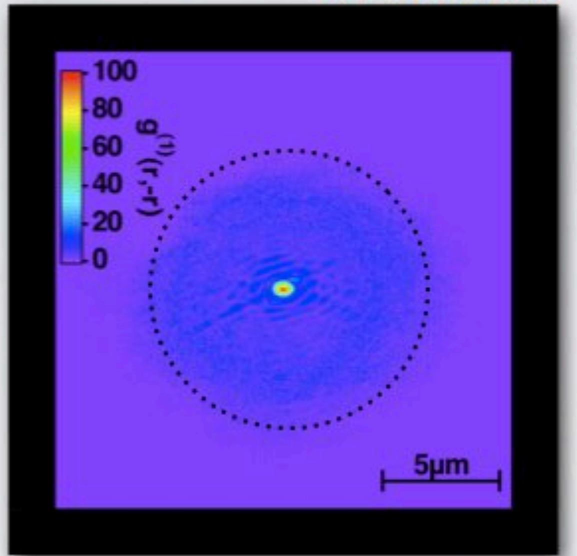
LONG RANGE SPATIAL COHERENCE



Spatial coherence
mapping of spatial coherence

$$g^{(1)}(\mathbf{r},\mathbf{r}',\tau=0)=\left\langle \psi^*(\mathbf{r})\psi(\mathbf{r}')\right\rangle$$

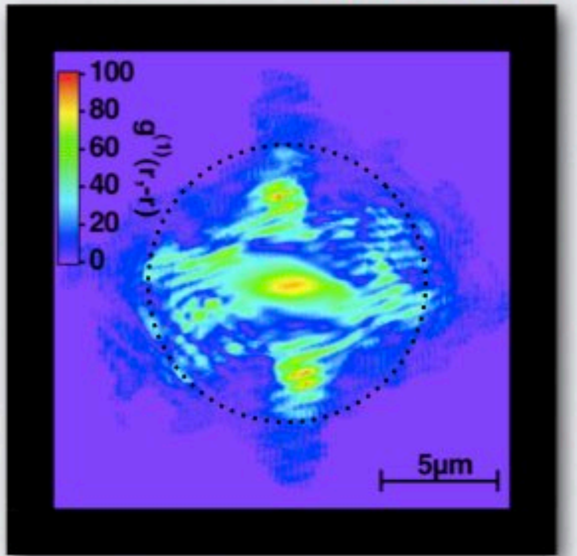
BELOW THE CONDENSATION THRESHOLD



Correlations at the de Broglie wavelength (λ)

Correlations are extended over the whole condensate (25λ)

ABOVE THE CONDENSATION THRESHOLD



WAVE-FUNCTION OF UNCORRELATED POLARITONS

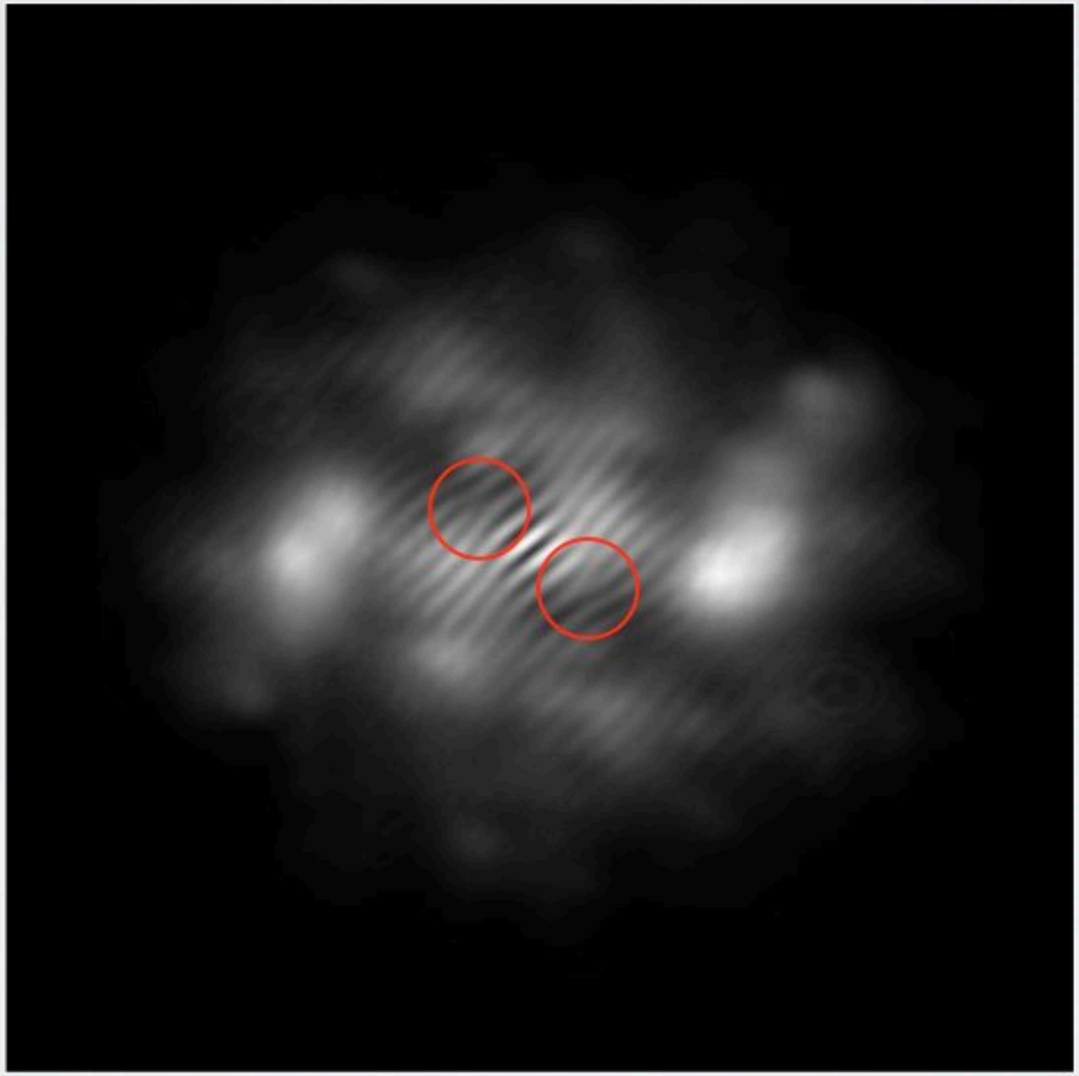
WAVE-FUNCTION OF THE CONDENSATE

COLLECTIVE DELOCALIZED STATE

images: K. Lagoudakis, EPFL

Quantum vortices
detected in the interferogram

Interferogram



images: K. Lagoudakis, EPFL