Wstęp do Optyki i Fizyki Materii Skondensowanej

1100-3003

Optyka 3

Wydział Fizyki UW

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Klasyczny model współczynnika załamania

Fala w ośrodku (różnym)

$$\frac{d^2\vec{x}}{dt^2} + \gamma\frac{d\vec{x}}{dt} + \omega_0^2\vec{x} = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

Model Lorentza

$$\frac{d^2\vec{x}}{dt^2} + \gamma\frac{d\vec{x}}{dt} + \omega_0^2\vec{x} = 0$$

Widmo emisji

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

Fale plazmowe

the steady state solution:

$$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$$

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Plasma waves

the steady state solution:
$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$

Swobodne nośniki: $\vec{j} = \sigma\,\vec{E}$

$$\vec{H} = \frac{1}{\mu\mu_0}\vec{B} \approx \frac{1}{\mu_0}\vec{B}$$

$$\vec{D} = \varepsilon_0\varepsilon_L\vec{E}$$

$$\frac{d^2\vec{x}}{dt^2} = \frac{q}{m}\vec{E}(t) = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

$$\vec{j} = Nq\frac{d\vec{x}}{dt} = Nq\int\frac{q}{m}\vec{E}_0 e^{i\omega t}dt = \frac{Nq^2}{m\,i\omega}\vec{E}_0 e^{i\omega t} = \frac{Nq^2}{m\omega^2}(-i)\omega\vec{E}_0 e^{i\omega t} = -\frac{Nq^2}{m\omega^2}\frac{\partial\vec{E}}{\partial t}$$

Klasyczny model współczynnika załamania

$$\frac{d^2\vec{x}}{dt^2} + \boxed{0} + \boxed{0} = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

Plasma waves

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$$\begin{cases} \nabla \times \vec{E} = rot\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} = rot\,\vec{H} = \dfrac{\partial \vec{D}}{\partial t} + \vec{j} \end{cases}$$

$$\vec{j} = -\frac{Nq^2}{m\omega^2}\frac{\partial \vec{E}}{\partial t}$$

Klasyczny model współczynnika załamania

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$$\nabla \times \vec{B} = rot\,\mu_0 \vec{H} = \mu_0 \varepsilon_0 \varepsilon_L \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j}$$

Klasyczny model współczynnika załamania

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

Plasma waves

the steady state solution:
$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$

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$$\vec{j} = -\frac{Nq^2}{m\omega^2}\frac{\partial\vec{E}}{\partial t}$$

$$\nabla \times \vec{B} = rot\,\mu_0\vec{H} = \mu_0\varepsilon_0\varepsilon_L\frac{\partial\vec{E}}{\partial t} + \mu_0\vec{j} = \frac{\varepsilon_L}{c^2}\frac{\partial\vec{E}}{\partial t} - \frac{1}{c^2\varepsilon_0}\frac{Nq^2}{m\omega^2}\frac{\partial\vec{E}}{\partial t} = \frac{1}{c^2}\left(\varepsilon_L - \frac{Nq^2}{\varepsilon_0 m\omega^2}\right)\frac{\partial\vec{E}}{\partial t}$$

Klasyczny model współczynnika załamania

$$\frac{d^2\vec{x}}{dt^2} + \boxed{0} + \boxed{0} = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

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$$\nabla\times\nabla\times\vec{E} = \nabla\left(\nabla\vec{E}\right) - \Delta\vec{E}$$

$$\nabla\times\nabla\times\vec{E} = -\nabla\times\frac{\partial\vec{B}}{\partial t} = -\frac{\partial}{\partial t}\nabla\times\vec{B}$$

Klasyczny model współczynnika załamania

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

Plasma waves

the steady state solution:
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$$\nabla\times\vec{B} = rot\,\mu_0\vec{H} = \mu_0\varepsilon_0\varepsilon_L\frac{\partial\vec{E}}{\partial t} + \mu_0\vec{j} = \frac{\varepsilon_L}{c^2}\frac{\partial\vec{E}}{\partial t} + \frac{1}{c^2\varepsilon_0}\frac{Nq^2}{m\omega^2}\frac{\partial\vec{E}}{\partial t} = \frac{1}{c^2}\left(\varepsilon_L - \frac{Nq^2}{\varepsilon_0 m\omega^2}\right)\frac{\partial\vec{E}}{\partial t}$$

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$$\nabla\times\nabla\times\vec{E} = -\nabla\times\frac{\partial\vec{B}}{\partial t} = -\frac{\partial}{\partial t}\nabla\times\vec{B}$$

$$\boxed{\nabla\left(\nabla\vec{E}\right) - \Delta\vec{E} = \frac{1}{c^2}\left(\varepsilon_L - \frac{Nq^2}{\varepsilon_0 m\omega^2}\right)\frac{\partial^2\vec{E}}{\partial t^2}}$$

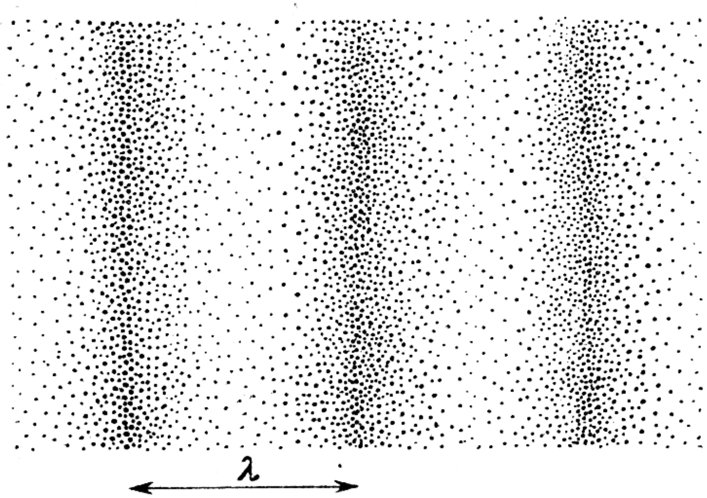
Rozwiązanie w postaci fal biegnących $\vec{E}(t) = \vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}$

Klasyczny model współczynnika załamania

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

Plasma waves

the steady state solution:
$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$



Free carriers: $\vec{j} = \sigma\,\vec{E}$

$$-\vec{k}(\vec{E}_0\vec{k}) + k^2\vec{E}_0 = \frac{\omega^2}{c^2}\left(\varepsilon_L - \frac{Nq^2}{\varepsilon_0 m\omega^2}\right)\vec{E}_0$$

- ionized gases (eg. in gas lamps, ionosphere in the atmospheres of stars and planets),
- plasma,
- plasma in a solid - the gas free carriers in metals or semiconductors,
- liquids - as electrolytes or molten conductors.

Fale plazmowe

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

Plasma waves

the steady state solution:
$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$

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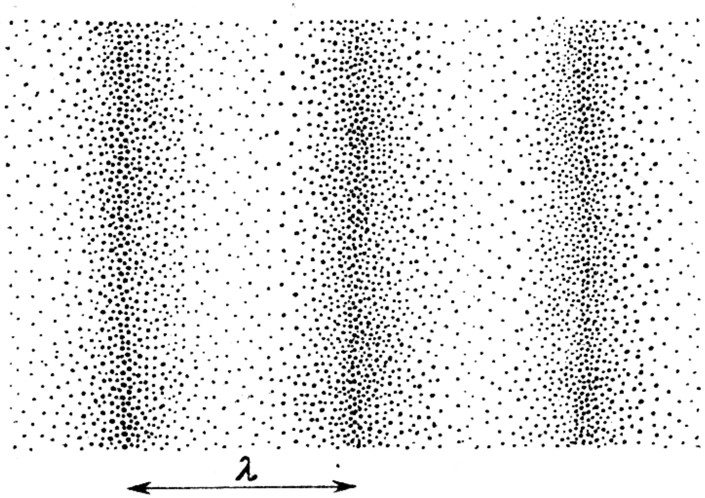


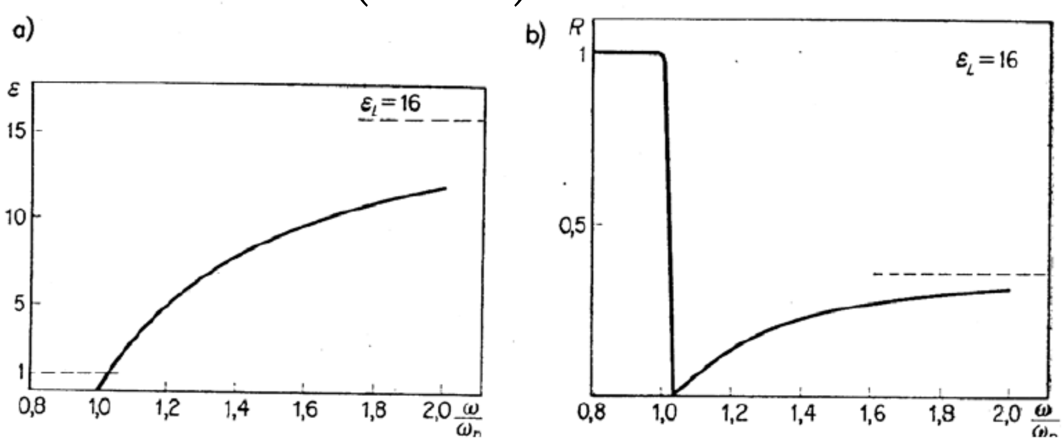
Longitudinal wave (*fala podłużna*): $\vec{k} \parallel \vec{E}$

$$-\vec{k}(\vec{E}_0\vec{k}) + k^2\vec{E}_0 = 0 \qquad \omega_p^2 = \frac{Nq^2}{\varepsilon_0\varepsilon_L m}$$

The transverse wave (*fala poprzeczna*): $\vec{k} \perp \vec{E}$

$$-\vec{k}(\vec{E}_0\vec{k}) + k^2\vec{E}_0 = \frac{\omega^2}{c^2}\varepsilon_L\left(1 - \frac{\omega_p^2}{\omega^2}\right)\vec{E} = \frac{\omega^2}{c^2}\varepsilon_L\varepsilon(\omega)$$

$$R = \left|\frac{n-1}{n+1}\right|^2 = \left|\frac{\sqrt{\varepsilon(\omega)}-1}{\sqrt{\varepsilon(\omega)}+1}\right|^2$$



Fale plazmowe

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

$$-\vec{k}\left(\vec{E}_0\vec{k}\right) + k^2\vec{E}_0 = -\frac{\omega^2}{c^2}\left(\varepsilon_L - \frac{Nq^2}{\varepsilon_0 m\omega^2}\right)\vec{E}$$

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$$R = \left|\frac{n-1}{n+1}\right|^2 = \left|\frac{\sqrt{\varepsilon(\omega)}-1}{\sqrt{\varepsilon(\omega)}+1}\right|^2$$

$$R = \left|\frac{\tilde{n}-1}{\tilde{n}+1}\right|^2 = \frac{(n-1)^2+\kappa^2}{(n+1)^2+\kappa^2}$$ (with damping)

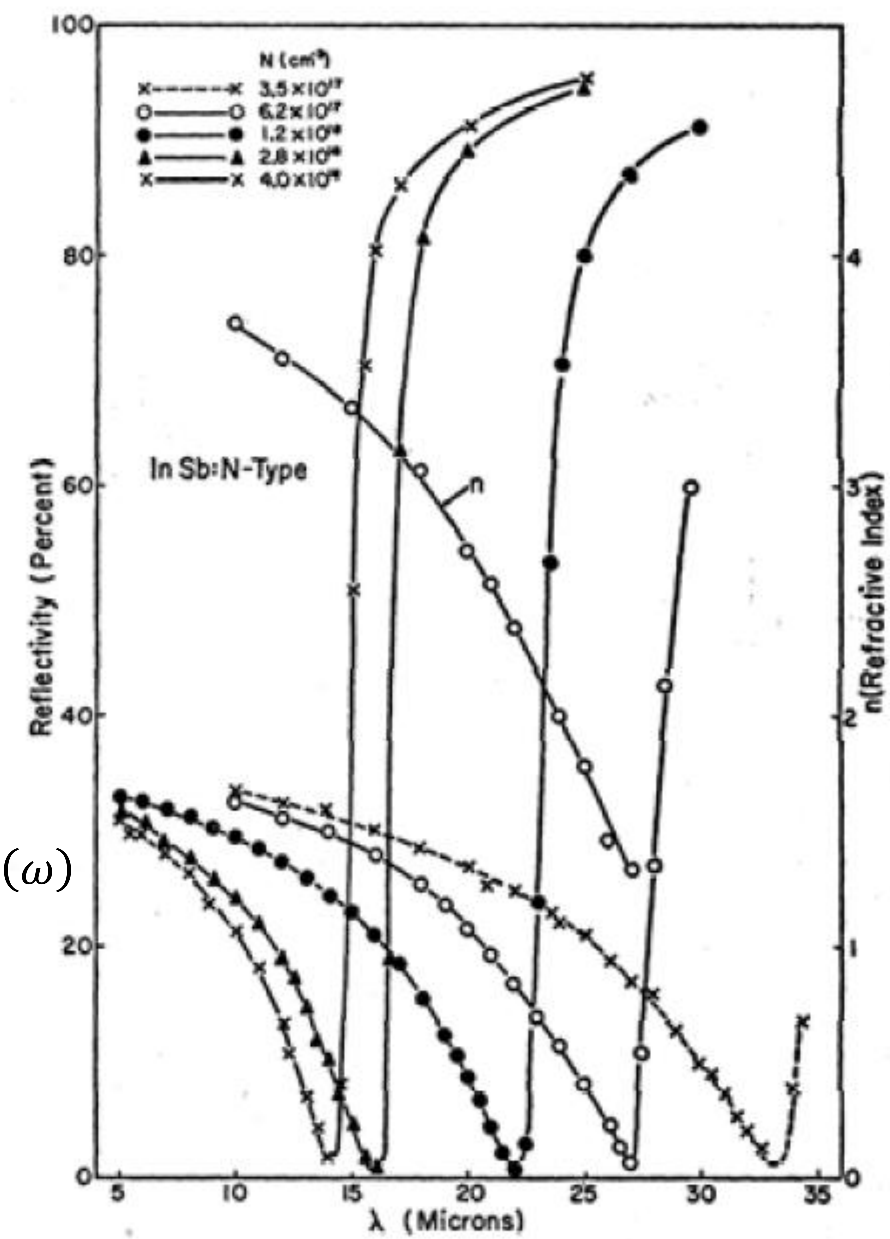
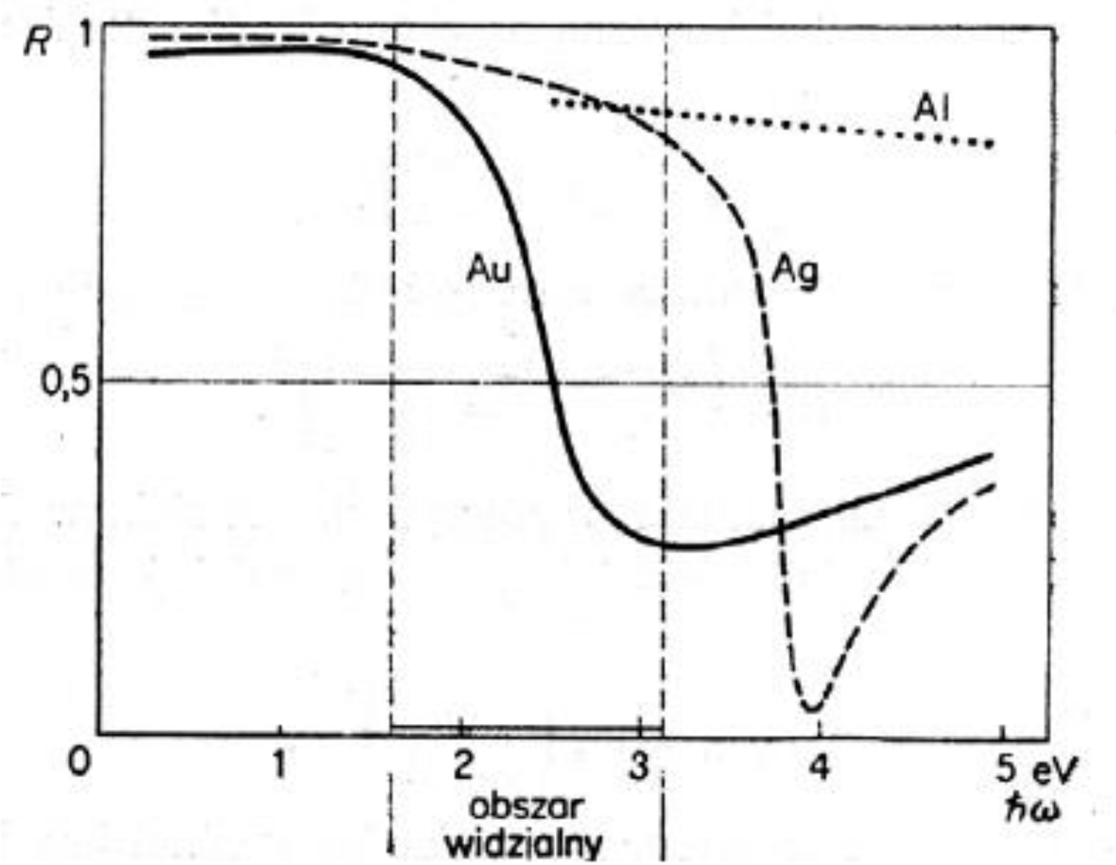
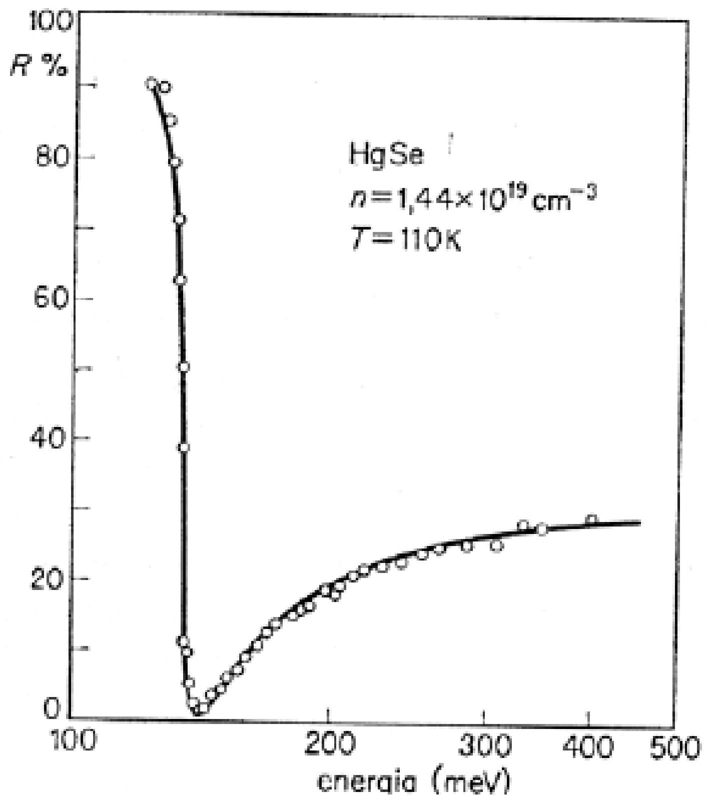


FIG. 8. Reflectivity *vs* wavelength for five *n*-type indium antimonide samples. The refractive index curve labeled *n* is for the sample with $N = 6.2\times10^{17}$ cm^{-3}.

Fale plazmowe

$$\frac{d^2\vec{x}}{dt^2} + \boxed{0} + \boxed{0} = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

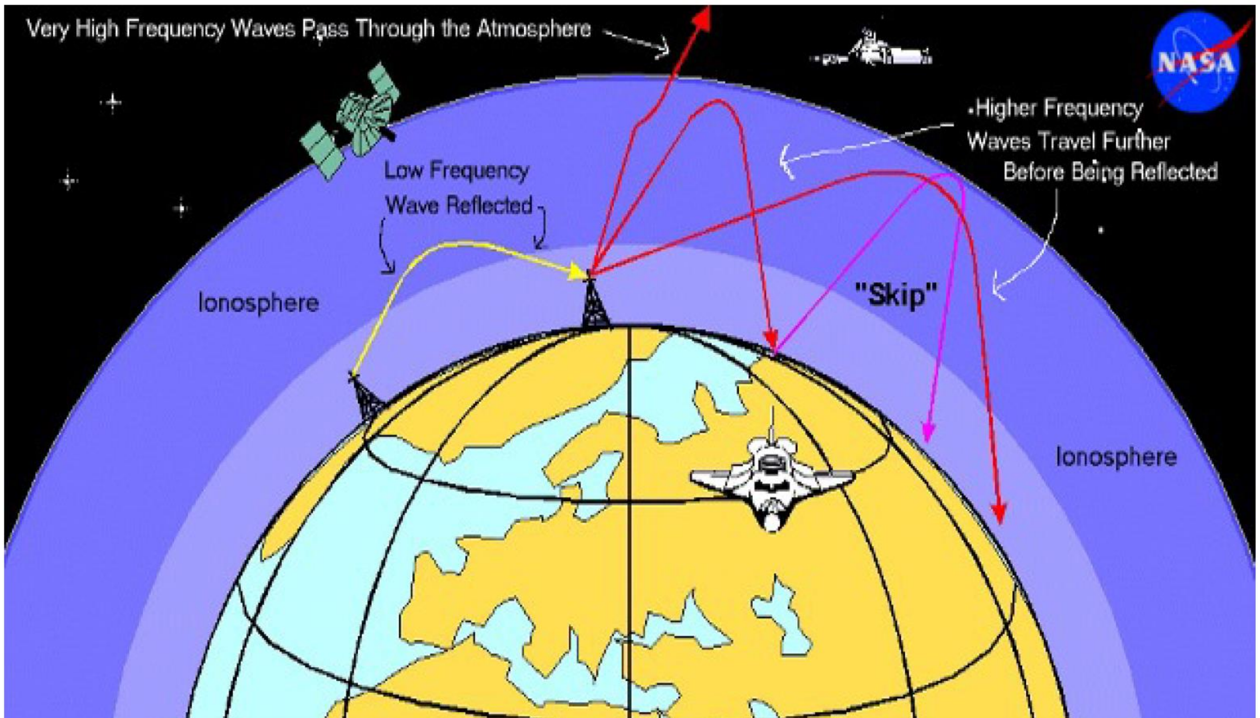




$$R = \left|\frac{n-1}{n+1}\right|^2 = \left|\frac{\sqrt{\varepsilon(\omega)}-1}{\sqrt{\varepsilon(\omega)}+1}\right|^2$$

Fale plazmowe

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$



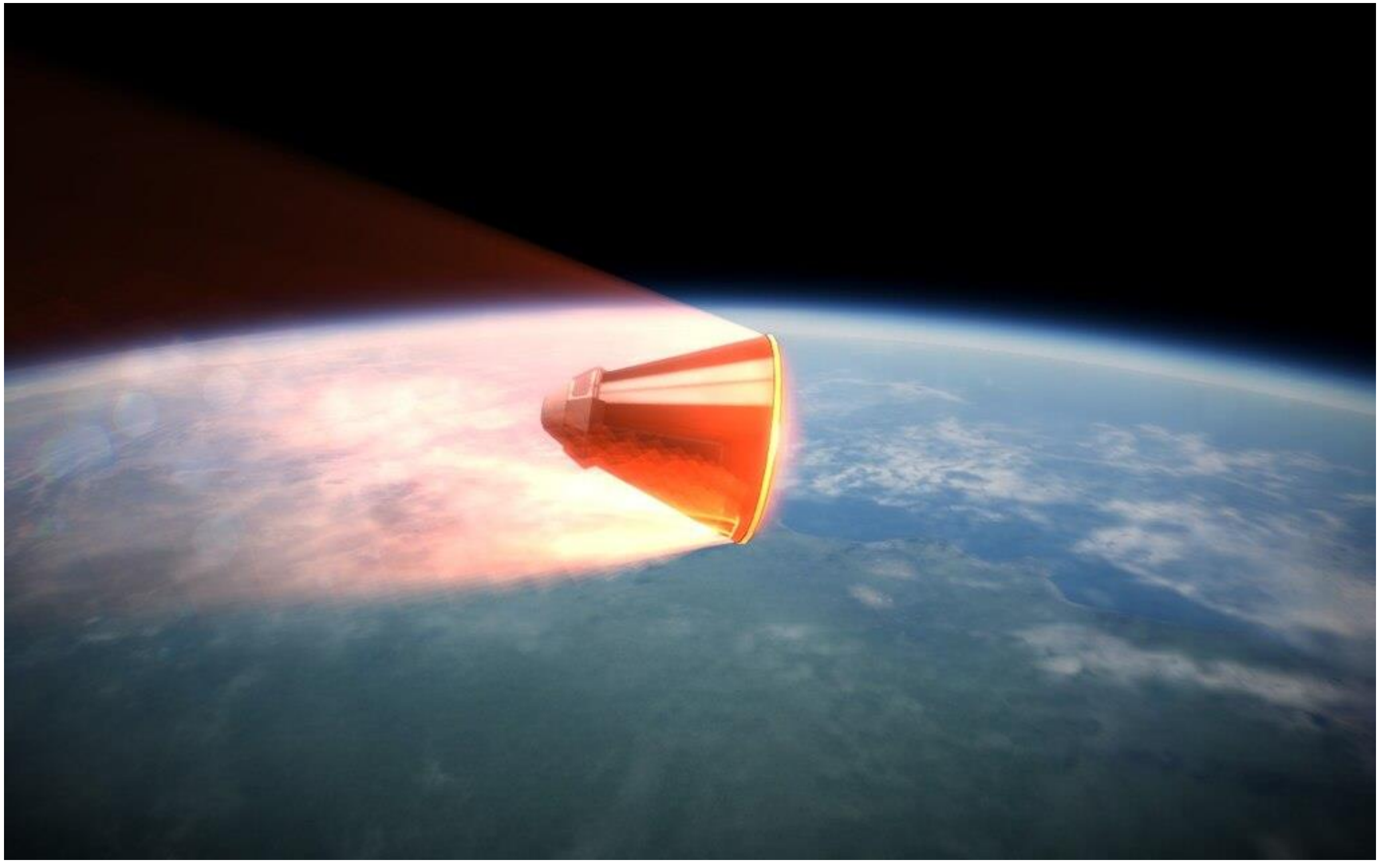
(c) 2007 NASA -- Comical NASA diagram to illustrate ionospheric radiowave propagation

Neatly weaving in four kooky NASA fantasies -

(i) Space Shuttle;
(ii) International Space Station;
(iii) the geostationary man-made satellite; and
(iv) higher frequency radiowaves "pass through" the ionosphere - the Big Lie to prop up the satellite hoax!

Fale plazmowe

$$\frac{d^2\vec{x}}{dt^2} + \boxed{0} + \boxed{0} = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$



Klasyczny model współczynnika załamania

Fala w ośrodku (różnym)

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Widmo emisji

$$\frac{d^2\vec{x}}{dt^2} + 0 + 0 = \frac{q}{m}\vec{E}_0 e^{i\omega t}$$

Fale plazmowe

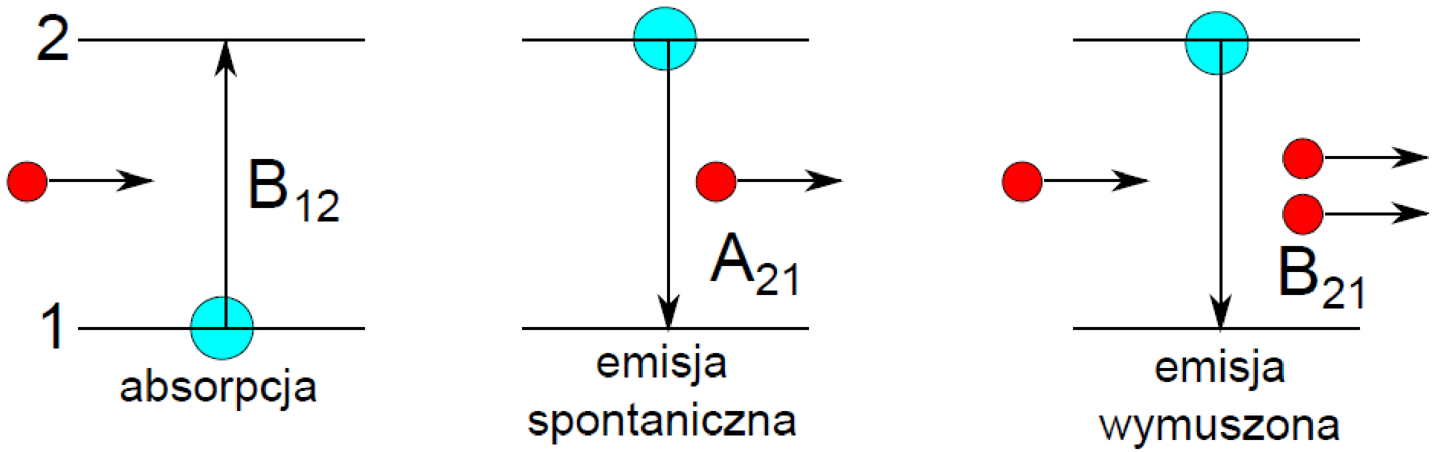
the steady state solution:
$\vec{x}(t) = \vec{x}_0 e^{i\omega t}$

Odbicie, transmisja, absorpcja

$$\boldsymbol{T + R + A = 1}$$

https://en.wikipedia.org/wiki/Fresnel_equations

Absorpcja i emisja światła



Relacje między współczynnikami Einsteina:

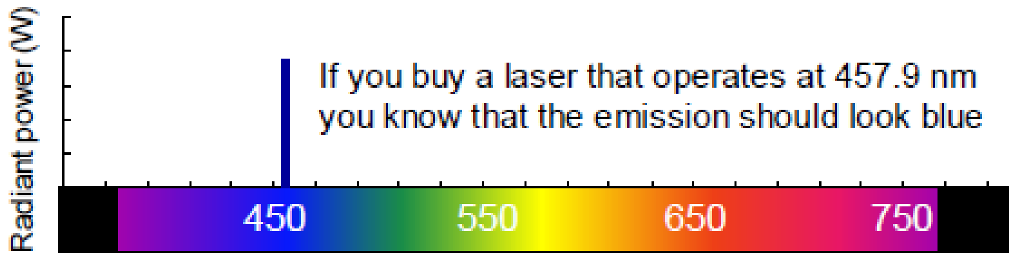
$$B_{12} = B_{21}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

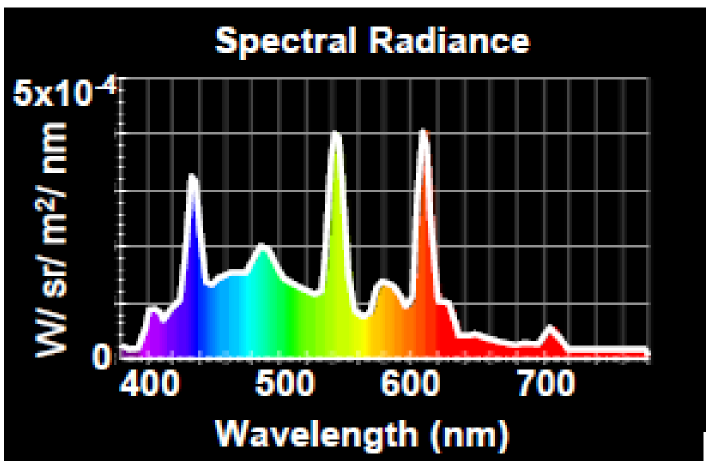
(Wyprowadzenie - na ćwiczeniach)

Trzy kolory

Monochromatic / spectral colours have a single wavelength:

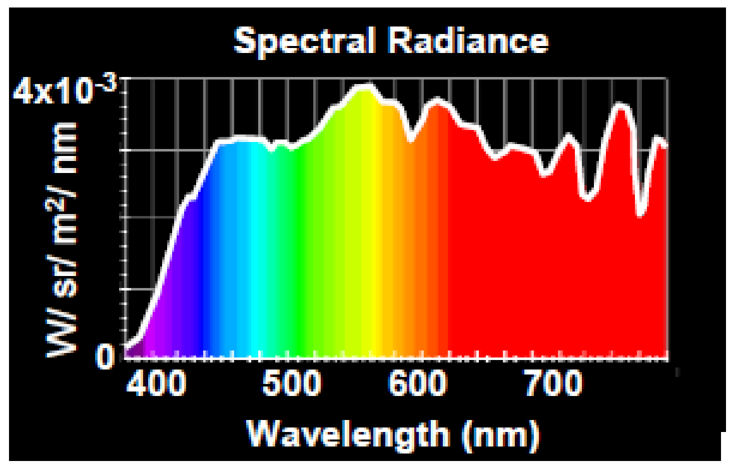


Colour of non-monochromatic light is more difficult to quantify:



Light spectrum of D65 fluorescent lamp

≡

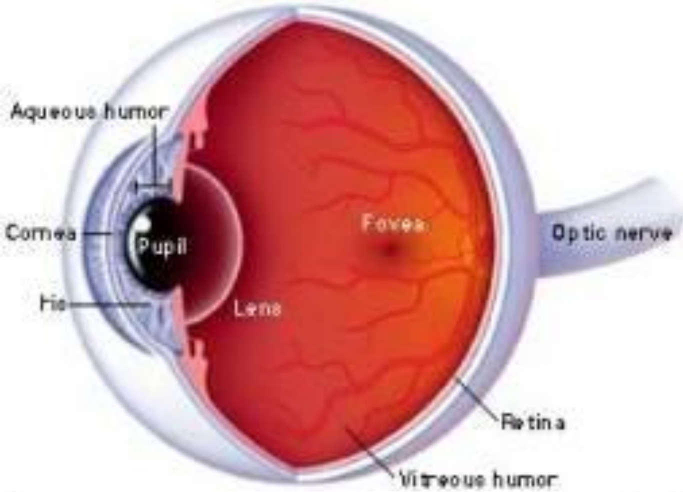


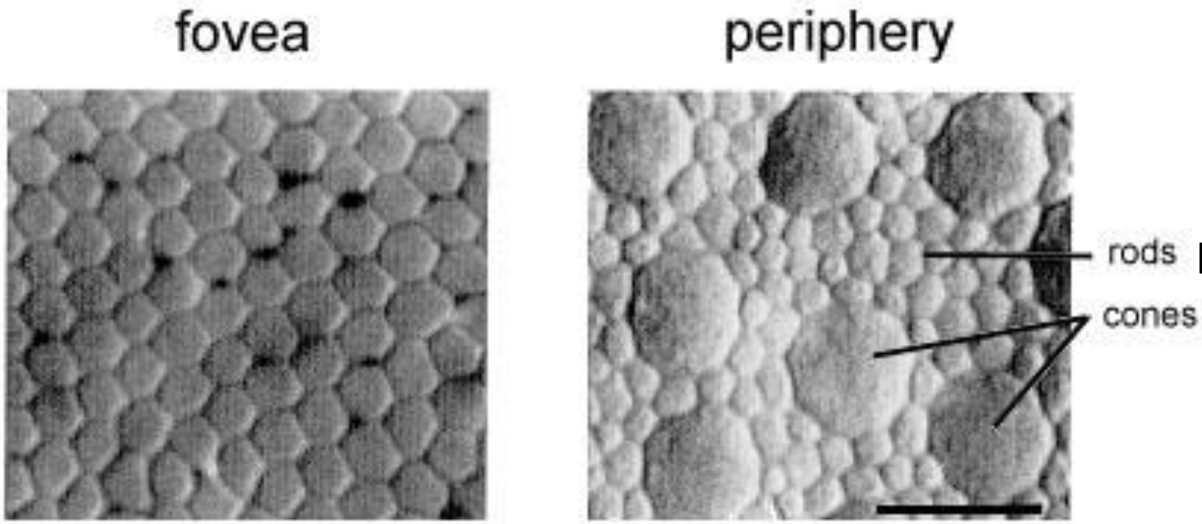
Light spectrum of late afternoon daylight

Taka sama biel!

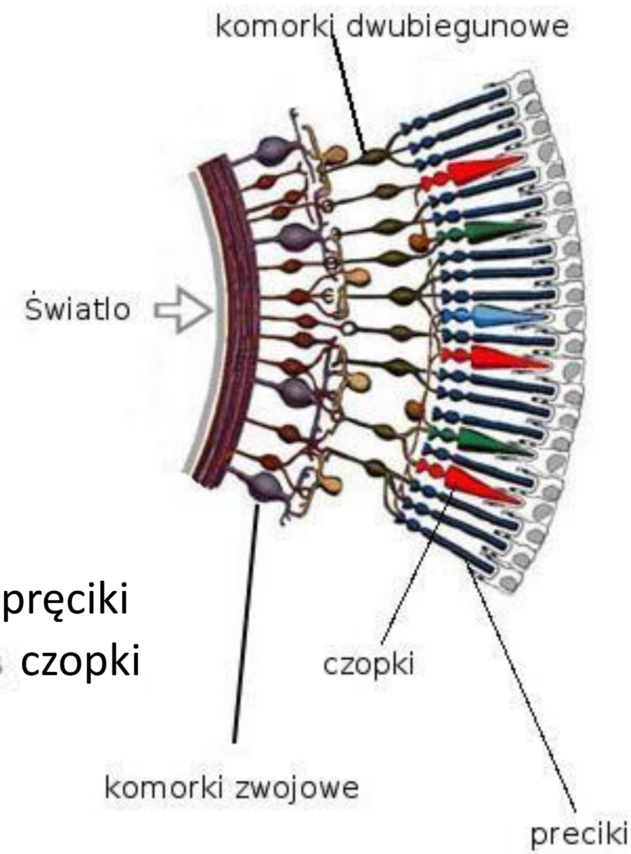
Prof. Thomas Anthopoulos

Trzy kolory



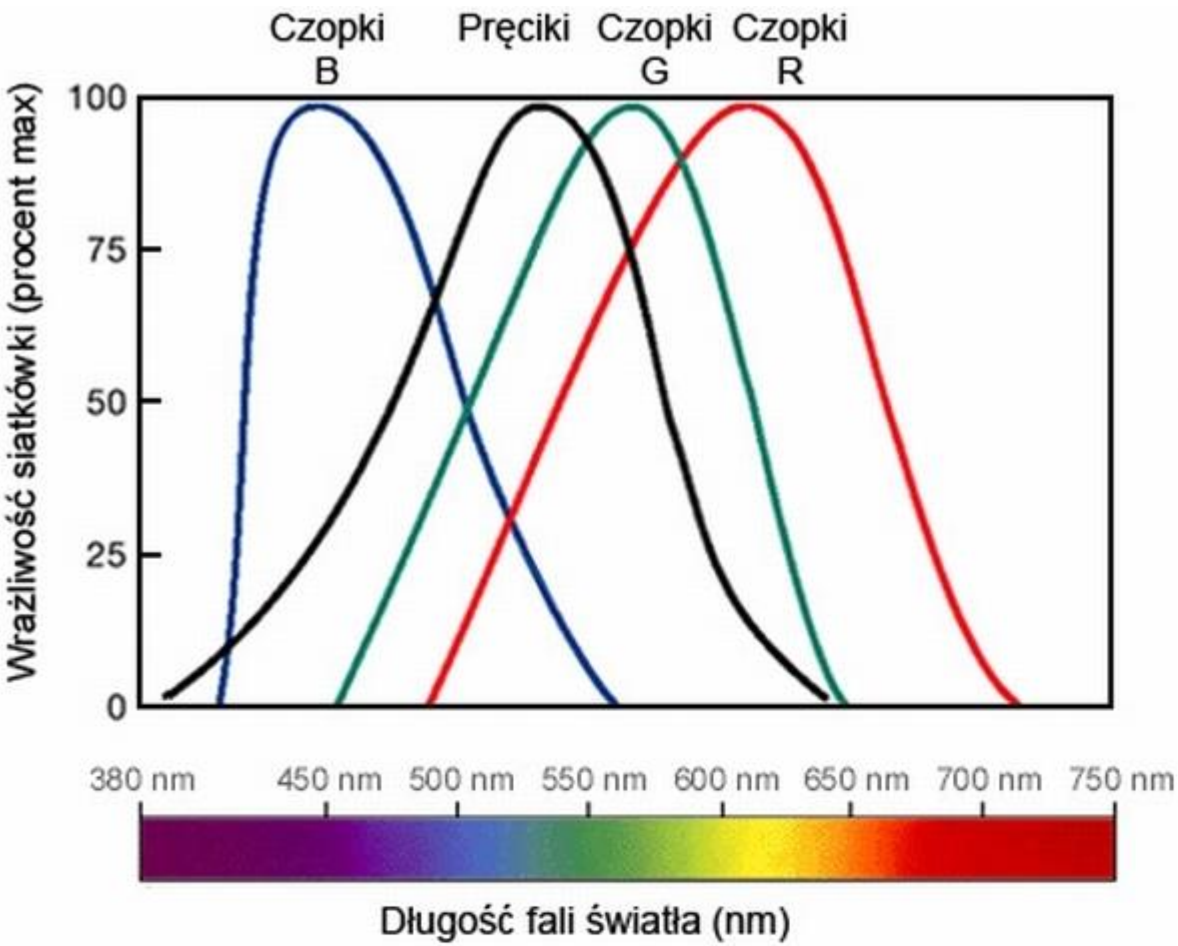


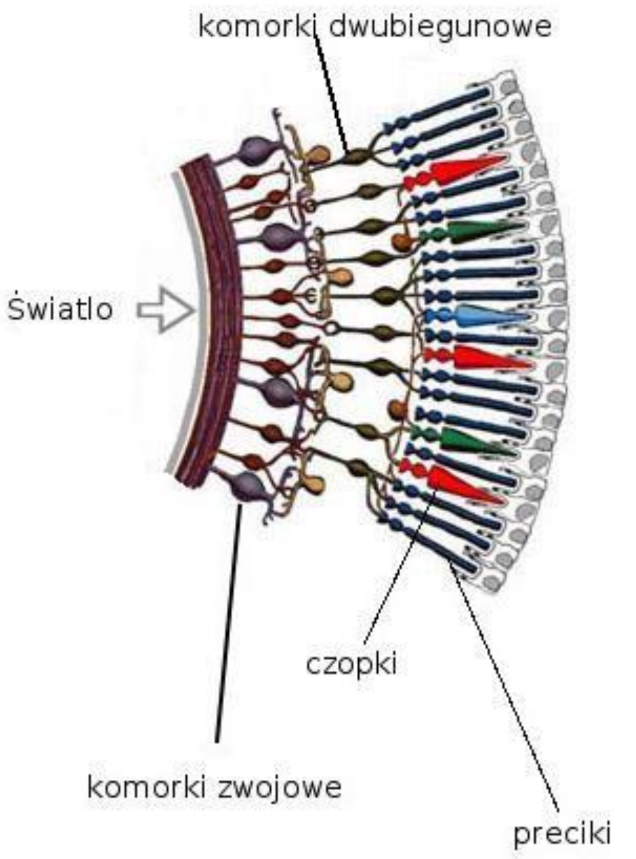
pręciki
czopki



http://195.117.188.199/rozdzial_1_12.htm

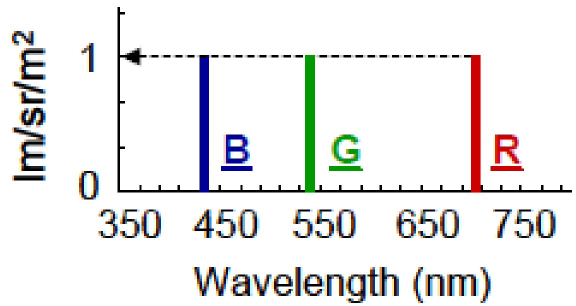
Trzy kolory


Czopki B
Pręciki
Czopki G
Czopki R
Wrażliwość siatkówki (procent max)
100
75
50
25
0
380 nm
450 nm
500 nm
550 nm
600 nm
650 nm
700 nm
750 nm
Długość fali światła (nm)


komorki dwubiegunowe
Światlo
czopki
komorki zwojowe
preciki

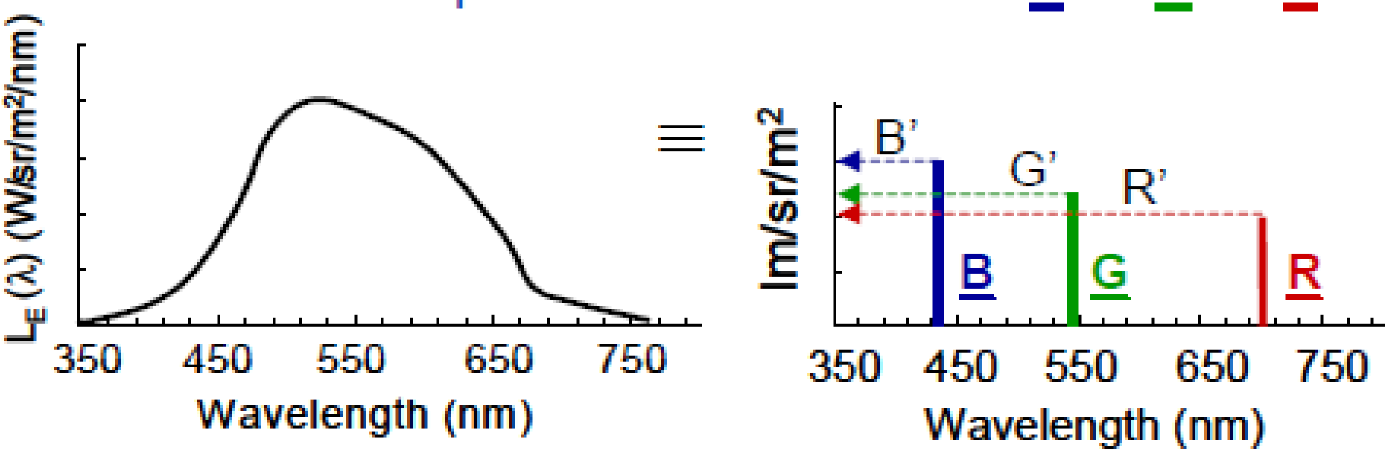
Trzy kolory

- Early experiments used **B** (435.8 nm), **G** (546.1 nm) and **R** (700 nm) as "unit amounts" of blue, green and red primary colours in luminance ($lm/sr/m^2$)



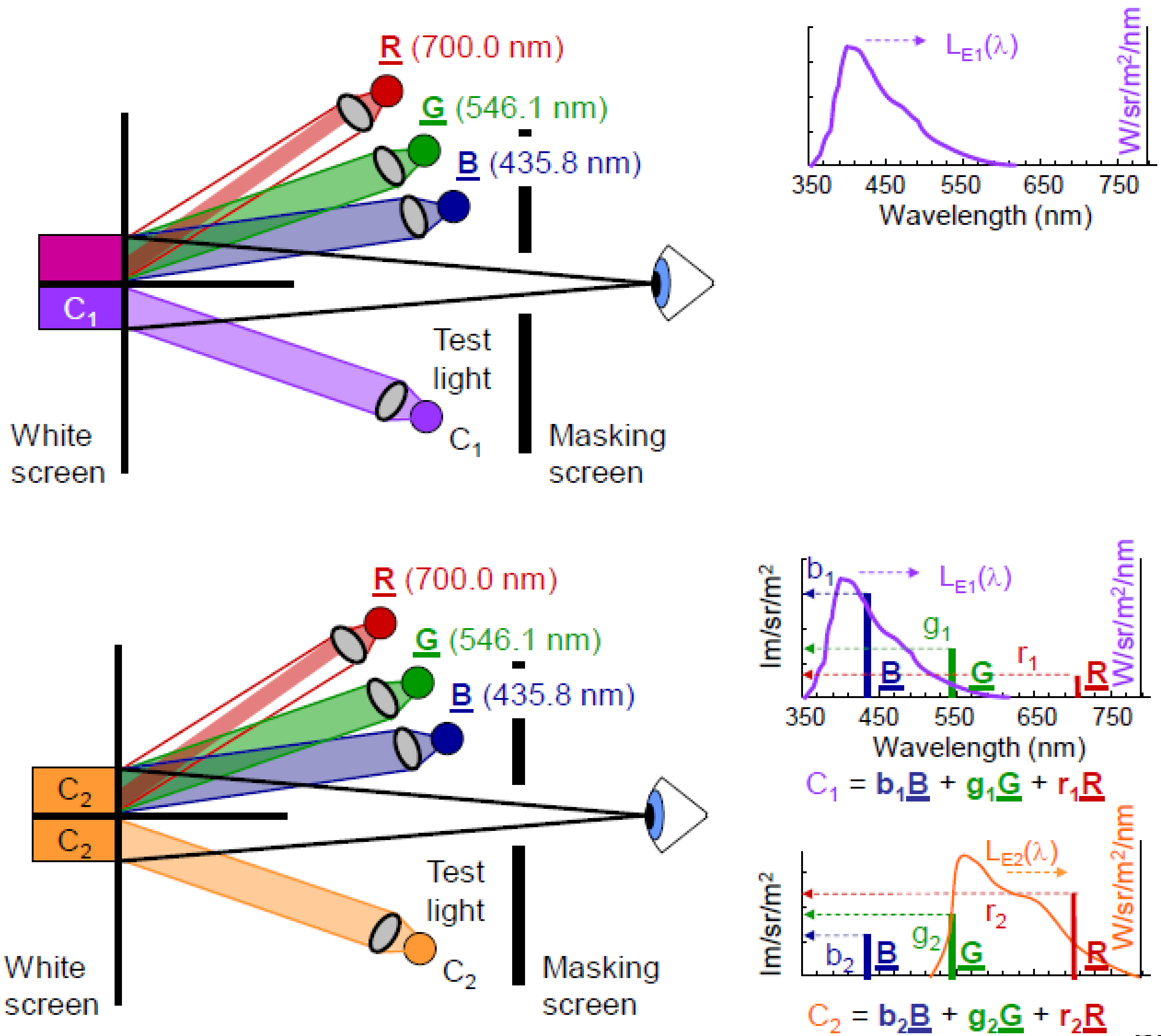
- Note that **B**, **G** and **R** are of different size in radiometric power units ($W/sr/m^2$) because the sensitivity of the eye will be different for different colours

Colour of emission spectrum ≡ Colour of: B'**B** + G'**G** + R'**R**



Prof. Thomas Anthopoulos

Trzy kolory



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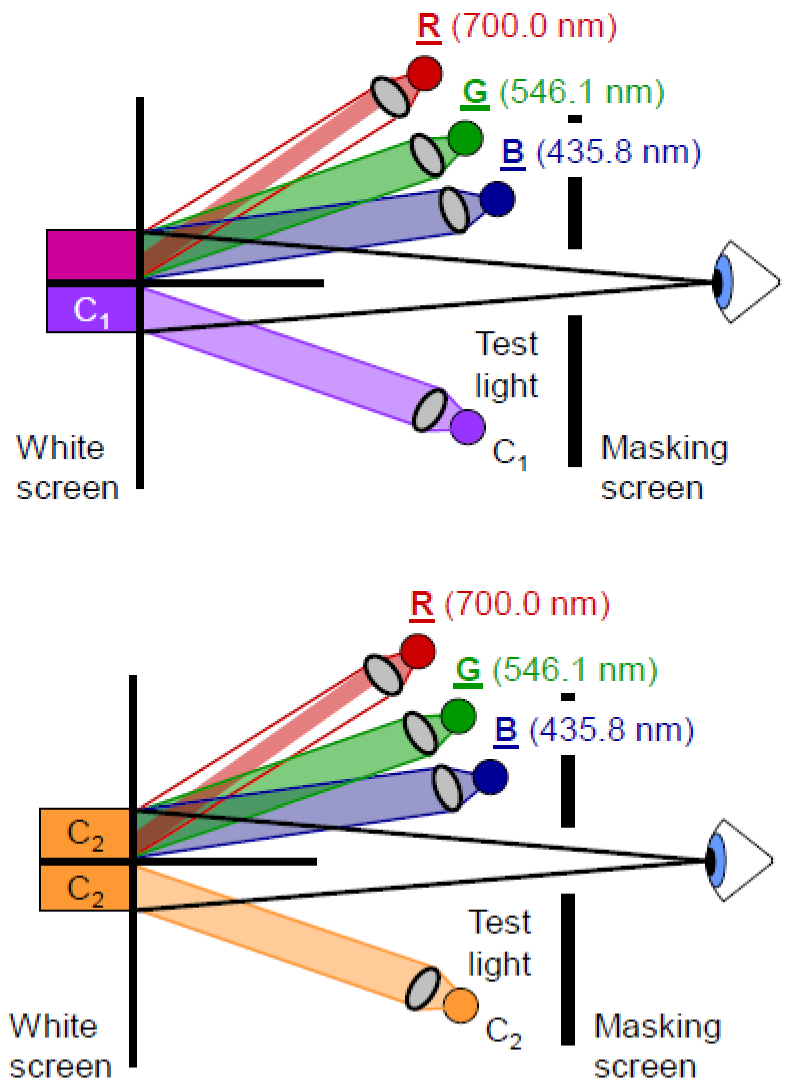
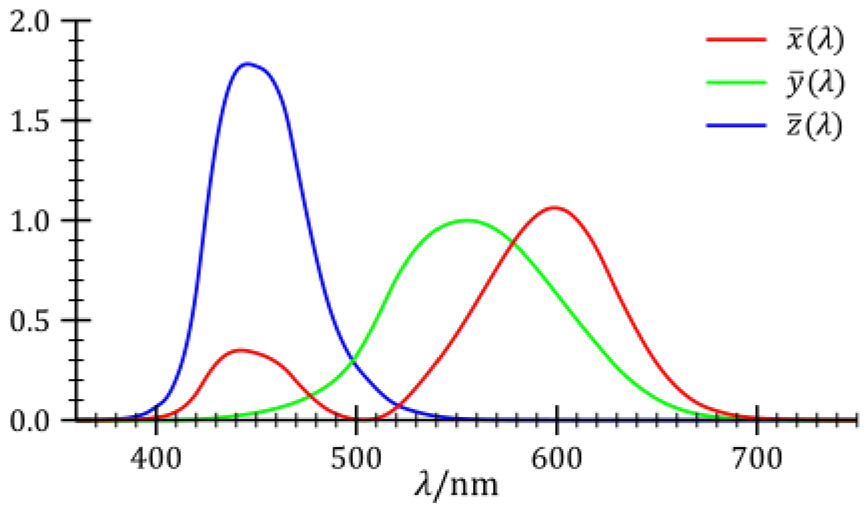
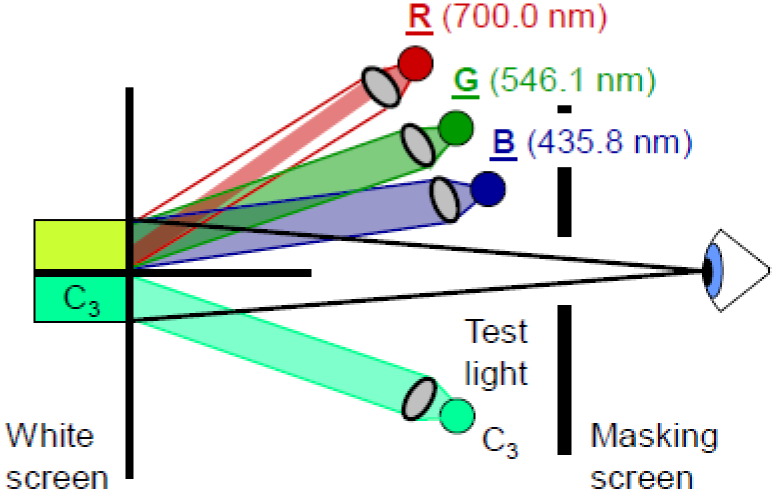


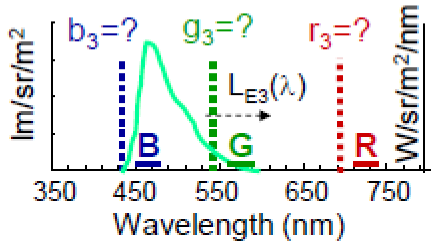


Trójchromatyczne składowe widmowe

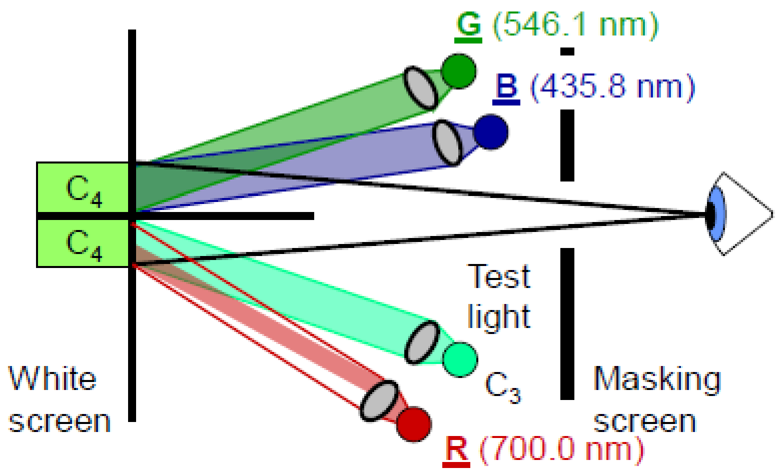
Wiki

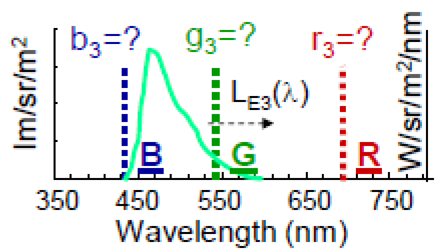
Trzy kolory





C_3 = ?B + ?G + ?R



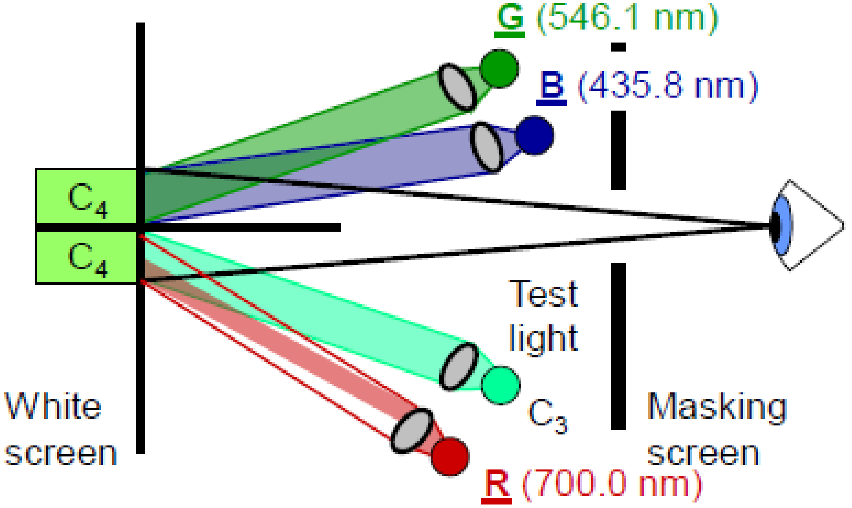


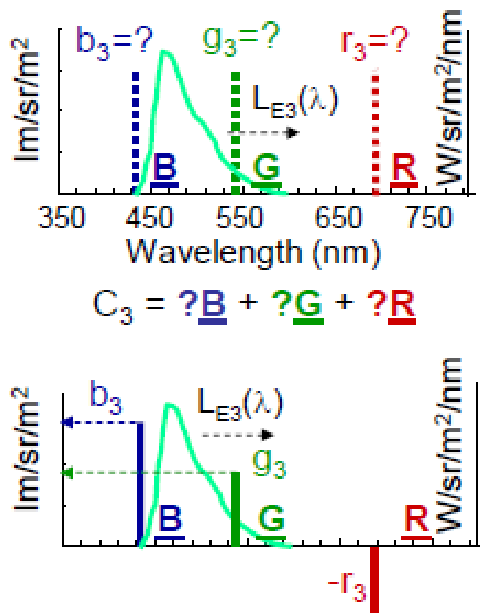
C_3 = ?B + ?G + ?R

Niektórych kolorów NIE DA się otrzymać!

Prof. Thomas Anthopoulos

Trzy kolory





$$C_4 = (C_3 + r_3\underline{R}) = b_3\underline{B} + g_3\underline{G}$$

$$C_3 = b_3\underline{B} + g_3\underline{G} - r_3\underline{R}$$

Niektórych kolorów NIE DA się otrzymać!

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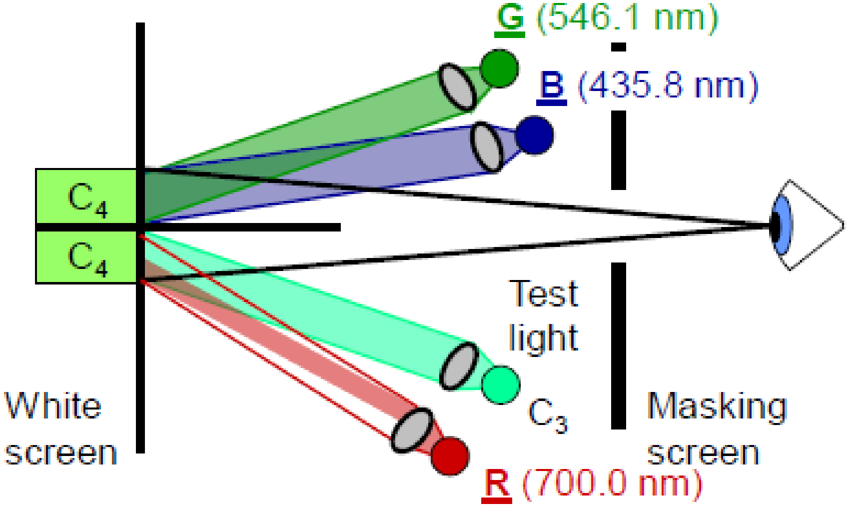
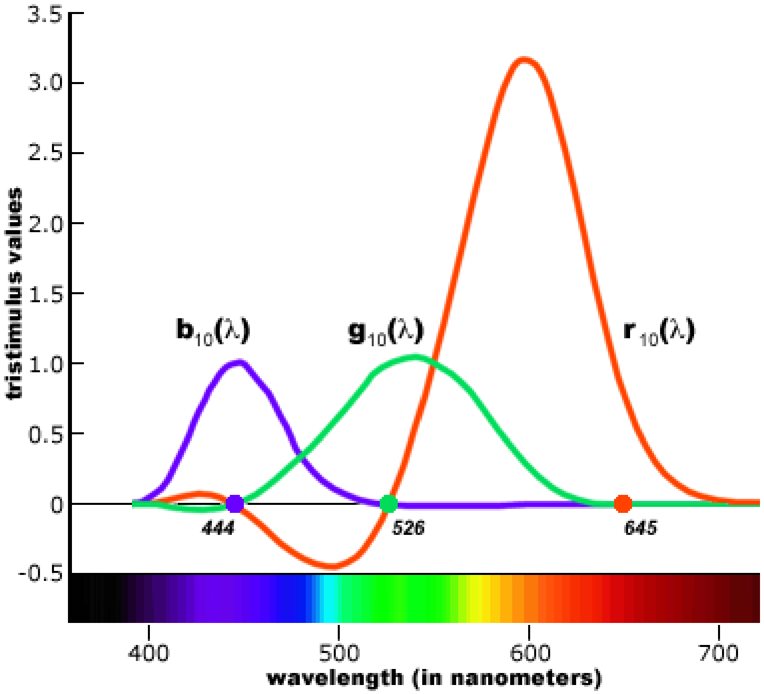
Trzy kolory





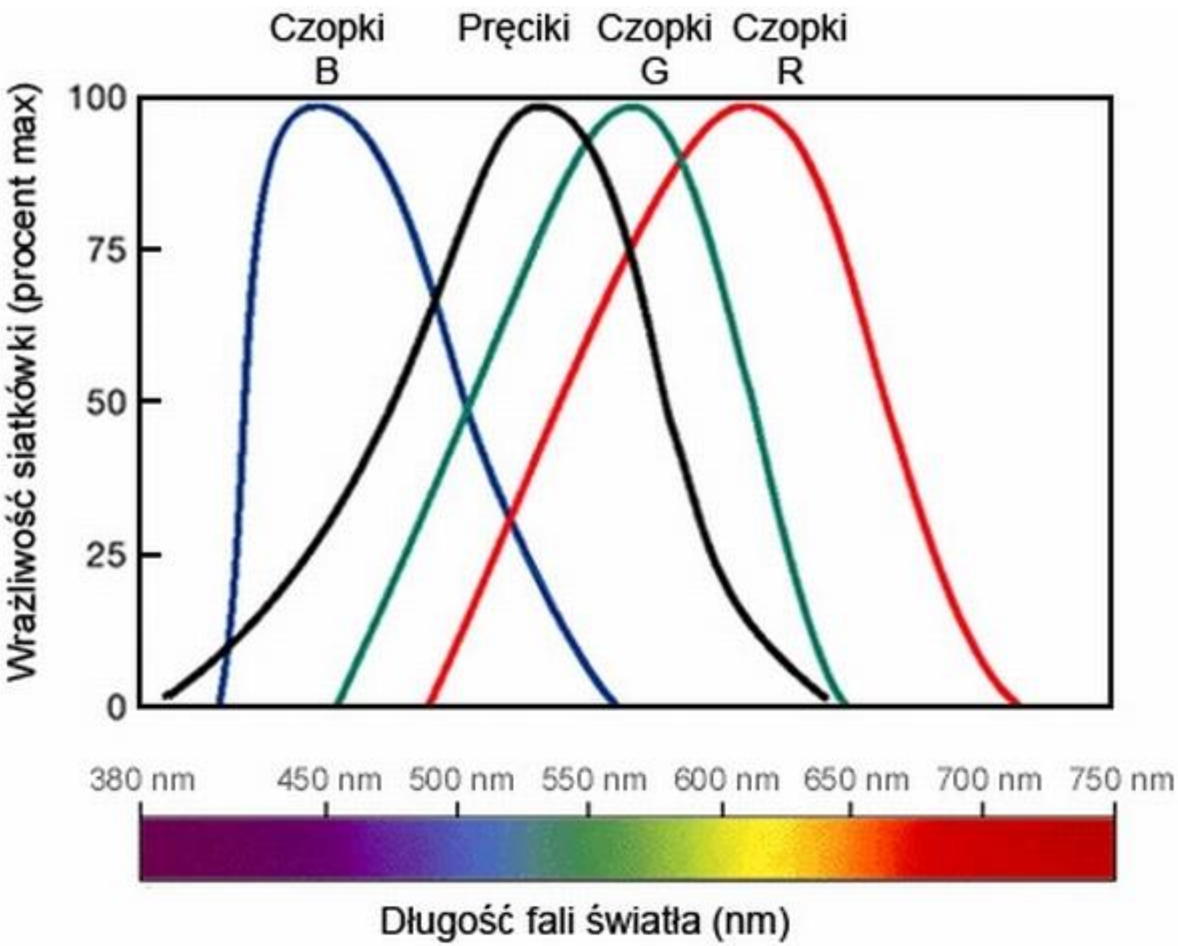
RGB color matching functions

Stiles-Burch 10° color matching functions averaged across 37 observers

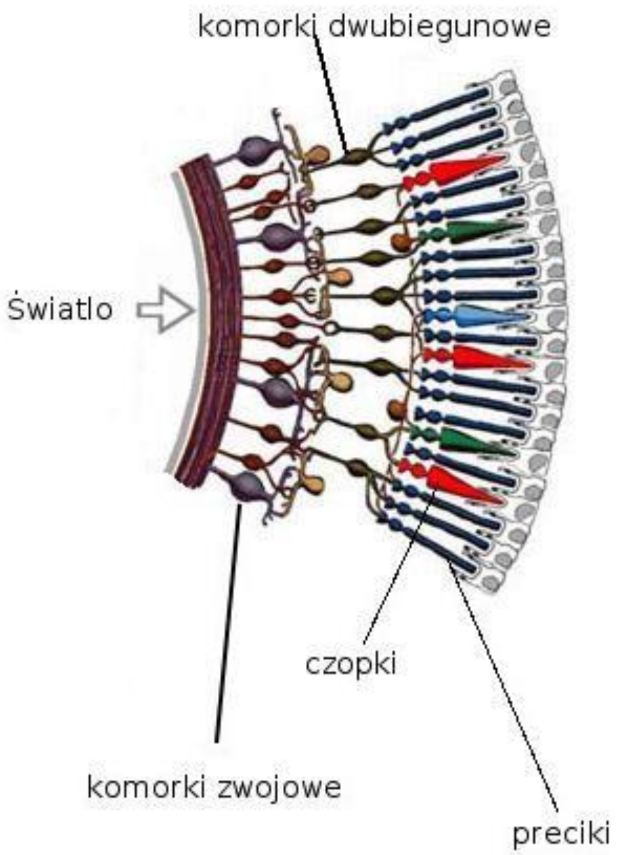
(adapted from Wyszecki & Stiles, 1982)

http://www.handprint.com/HP/WCL/color6.html

Trzy kolory



Czopki B
Pręciki
Czopki G
Czopki R
Wrażliwość siatkówki (procent max)
100
75
50
25
0
380 nm
450 nm
500 nm
550 nm
600 nm
650 nm
700 nm
750 nm
Długość fali światła (nm)



komorki dwubiegunowe
Światlo
czopki
komorki zwojowe
preciki

Enchroma glasss

TRY NOT TO CRY CHALLENGE #2, EnChroma glasses

Vrrr Tube • 1,2 mln wyświetleń • 1 rok temu

Watch these amazing videos of colorblind people seeing more of the spectrum of color for the very first time.

Max sees color! Enchroma 12/23/17

Kandra Jones • 140 tys. wyświetleń • 10 miesięcy temu

The best gift I received this Christmas was JOY! A joy I have never before experienced. A joy so wide and deep that my heart felt ...

COLORBLIND WOMEN see color for the first time! (Enchroma Glasses Compilation)

Pepper • 491 tys. wyświetleń • 1 rok temu

Colorblindness is rarer in women - about 1 in 200 has it. But colorblind women should see themselves represented too! Watch as ...

Our Favorite Reactions of 2015

EnChroma, Inc. • 971 tys. wyświetleń • 2 lata temu

Take the Color Blindness Test: http://enchroma.com/test/instructions/ Shop Color Blindness Glasses: http://enchroma.com/shop/ ...

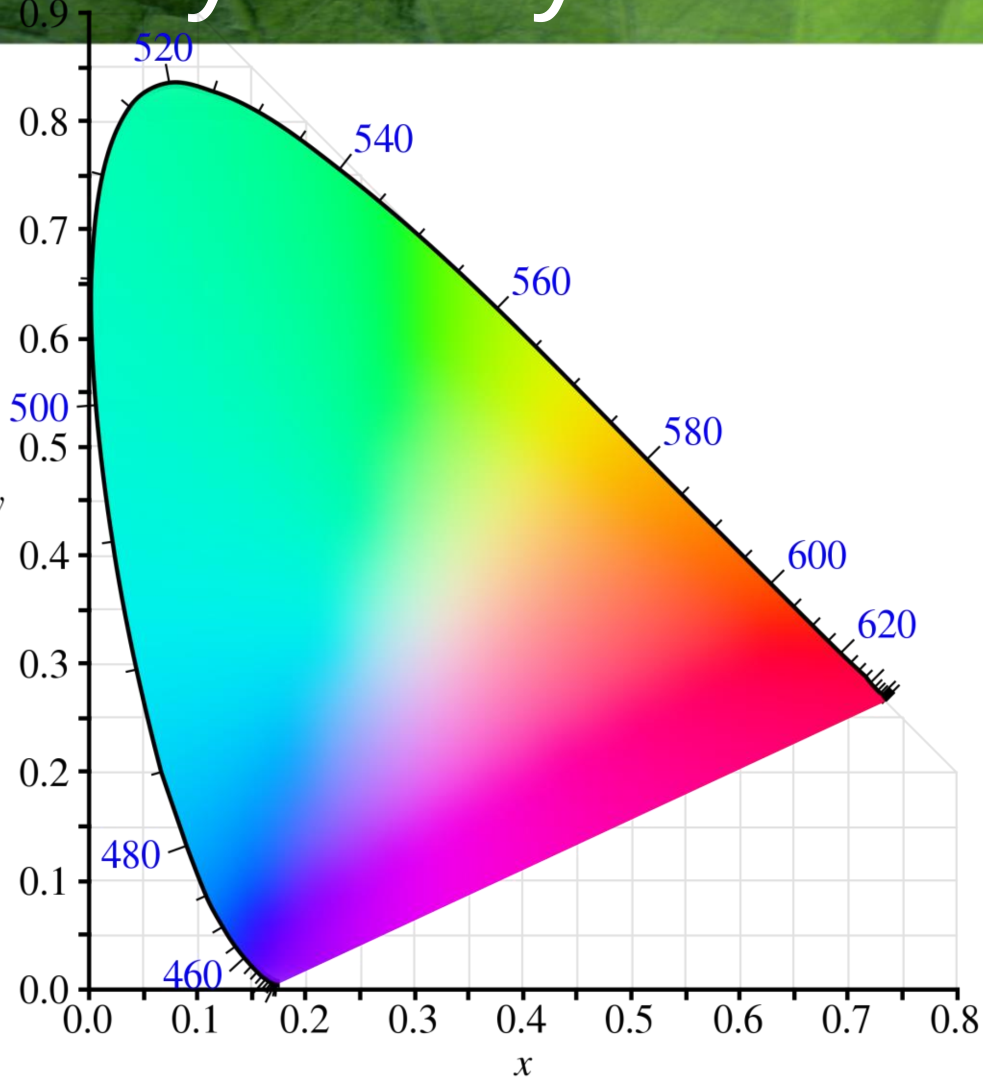
napisy

This Is What Color Blind People See With These Viral Glasses

Tech Insider • 489 tys. wyświetleń • 8 miesięcy temu

These glasses bring more color to the color blind by helping them see more hues and differentiate colors. You have seen the viral ...

Trzy kolory



Kolory CIE (1931) – przestrzeń barw

$$L(\lambda) = Z'\boldsymbol{Z} + Y'\boldsymbol{Y} + X'\boldsymbol{X}$$

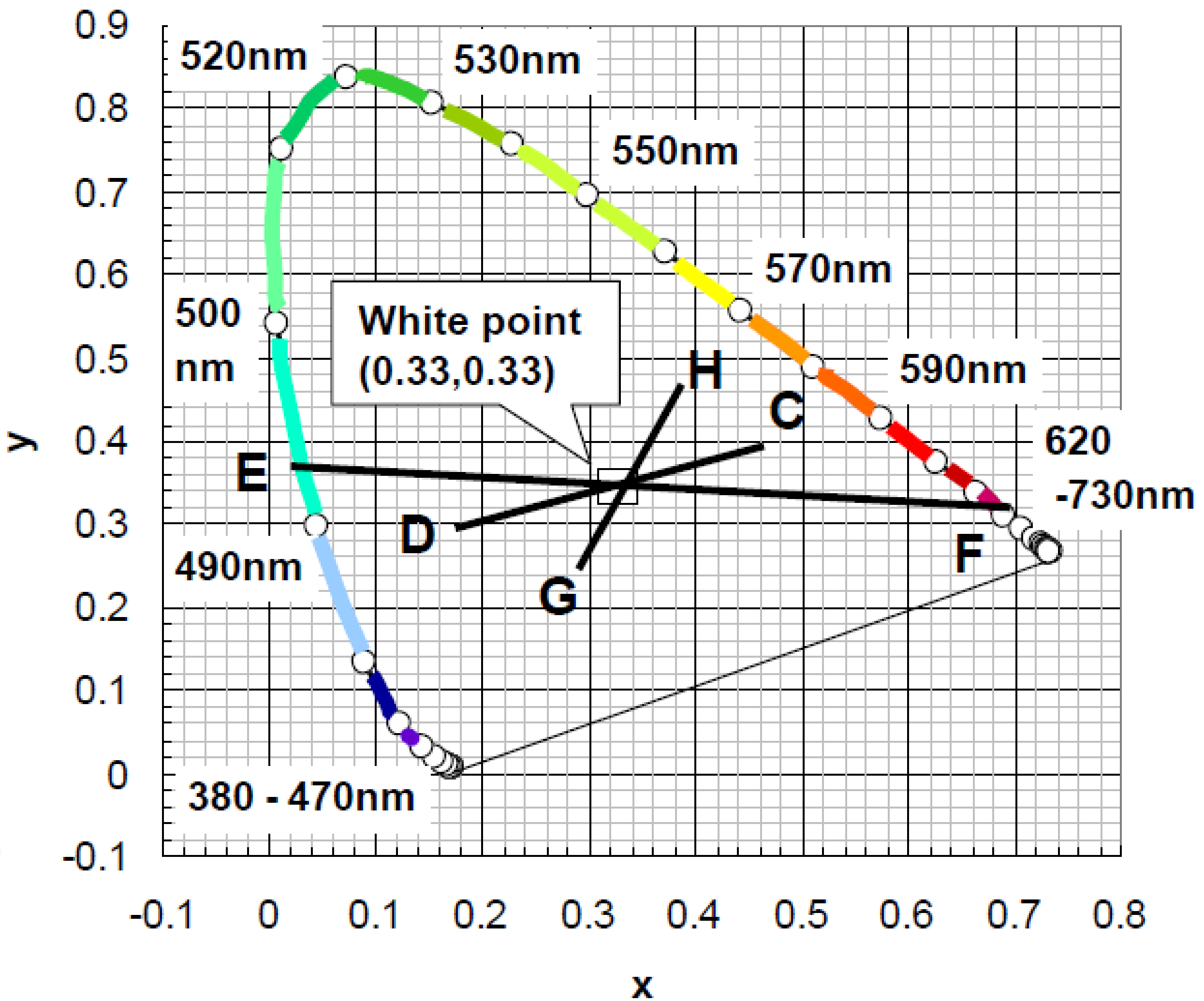
$$x = \frac{X'}{X' + Y' + Z'}$$

$$y = \frac{Y'}{X' + Y' + Z'}$$

$$z = \frac{Z'}{X' + Y' + Z'} = 1 - x - y$$

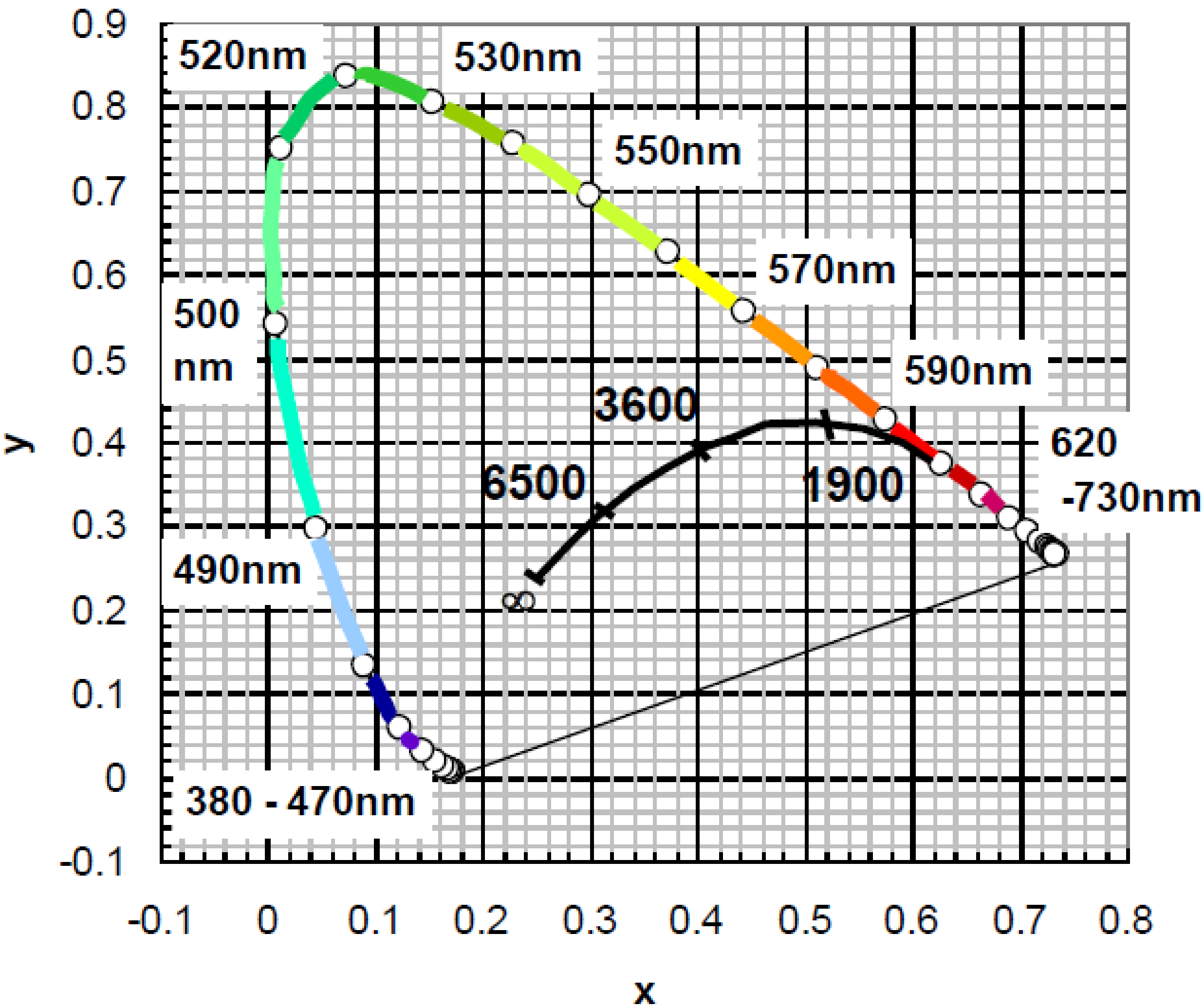
Tylko dwie zmienne (x i y) są niezależne, więc wykres 2D wystarcza do reprezentowania wszystkich barw w tzw. Współrzędnych trójchromatycznych

Trzy kolory



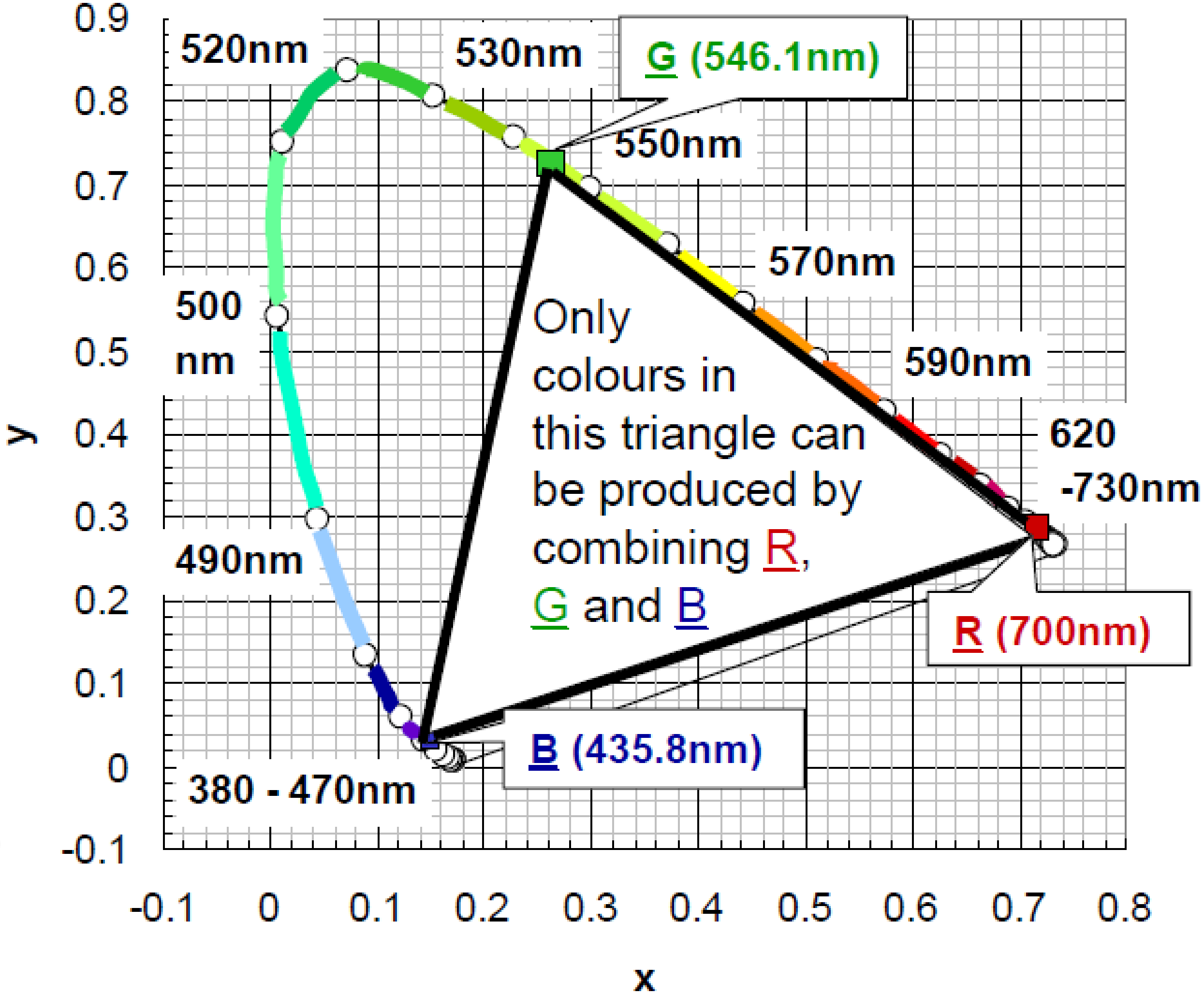
Prof. Thomas Anthopoulos

Trzy kolory



Prof. Thomas Anthopoulos

Trzy kolory

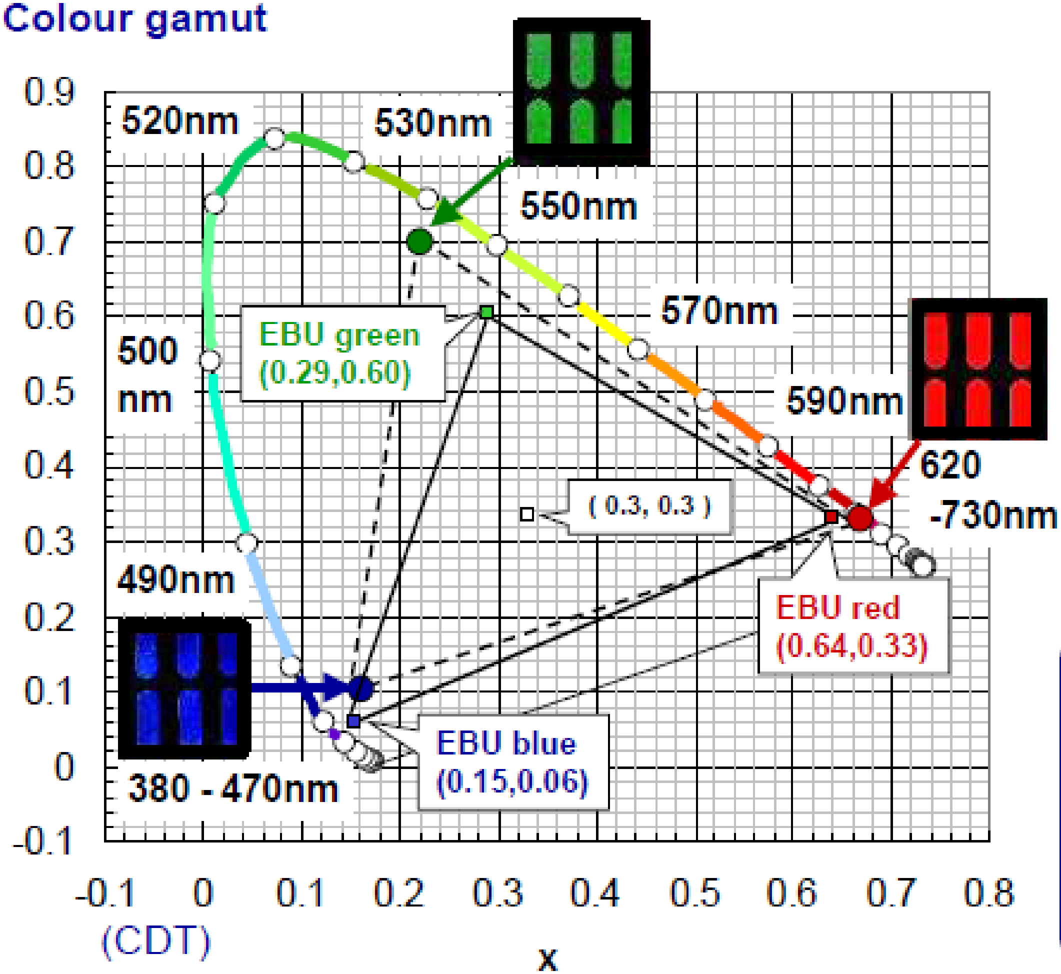


Prof. Thomas Anthopoulos

Trzy kolory

Why use organic LEDs for displays?

Colour gamut



- Coordinates shown are for solution deposition, but powders give similar coordinates, apart from green which is slightly better for the solutions

- Range of emission colours obtained by using organic semiconductors with different chemical structures
- Good colour gamut

Prof. Thomas Anthopoulos


DO PRACY
ZGŁASZAJ SIĘ
TRZEŹWY i WYPOCZĘTY

Atomy

Atom wodoru

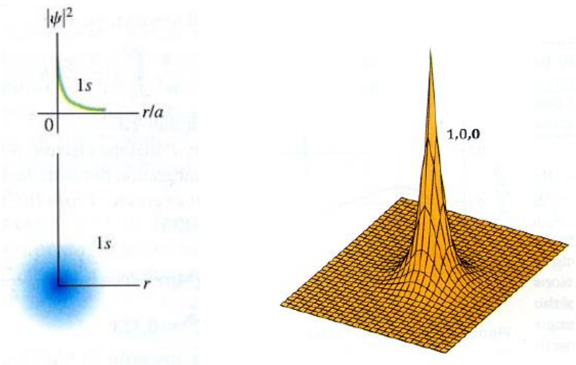
Eigenstates (stany własne) of L :

$$\psi_{1s} = \frac{1}{\sqrt{\pi a^3}} \exp(-\frac{r}{a})$$

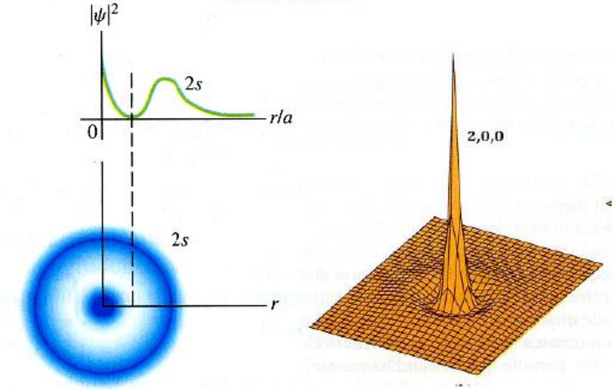
$$\psi_{2s} = \frac{1}{4\sqrt{2\pi a^3}} (2-\frac{r}{a}) \exp(-\frac{r}{2a})$$

$$\psi_{2po} = \frac{1}{4\sqrt{2\pi a^3}} \frac{r}{a} \exp(-\frac{r}{2a}) \cos\theta$$

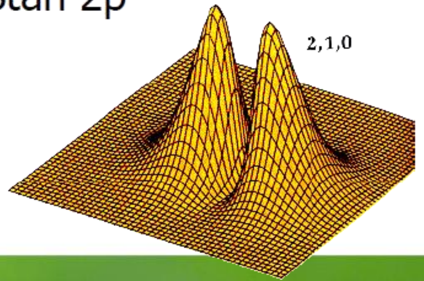
$$\psi_{2p\pm} = \frac{1}{8\sqrt{\pi a^3}} \frac{r}{a} \exp(-\frac{r}{2a}) \sin\theta \exp(\pm i\varphi)$$



Stan 2s



Stan 2p



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$$\psi_{2p\pm} = \frac{1}{8\sqrt{\pi a^3}} \frac{r}{a} \exp(-\frac{r}{2a}) \sin\theta \exp(\pm i\varphi)$$

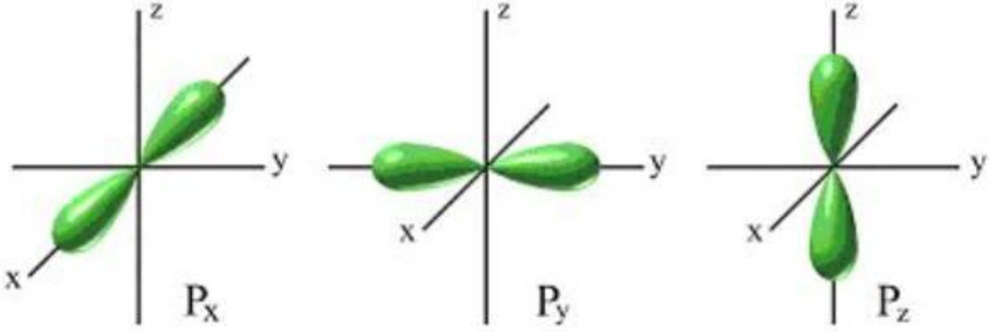
Real functions (funkcje rzeczywiste):

$$\psi_{1s} = \frac{1}{\sqrt{\pi a^3}} \exp(-\frac{r}{a})$$

$$\psi_{2px} = \frac{1}{4\sqrt{2\pi a^3}} \frac{x}{a} \exp(-\frac{r}{2a}) = \frac{1}{\sqrt{2}} \left(\psi_{2p+1} + \psi_{2p-1}\right)$$

$$\psi_{2py} = \frac{1}{4\sqrt{2\pi a^3}} \frac{y}{a} \exp(-\frac{r}{2a}) = \frac{1}{\sqrt{2}} \left(\psi_{2p+1} - \psi_{2p-1}\right)$$

$$\psi_{2pz} = \frac{1}{4\sqrt{2\pi a^3}} \frac{z}{a} \exp(-\frac{r}{2a}) = \psi_{2p0}$$



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$$\psi_{2p\pm} = \frac{1}{8\sqrt{\pi a^3}} \frac{r}{a} \exp(-\frac{r}{2a}) \sin\theta \exp(\pm i\varphi)$$

Spherical harmonics (harmoniki sferyczne) Y_{lm}:

$$\Psi_{nlm}(r,\theta,\varphi) = R_{nl}(r) Y_{lm}(\theta,\varphi)$$

$$R_{20}(r) = \left(\frac{Z}{2a}\right)^{3/2} 2\left(1 - \frac{Zr}{2a}\right) \exp\left(-\frac{Zr}{2a}\right)$$

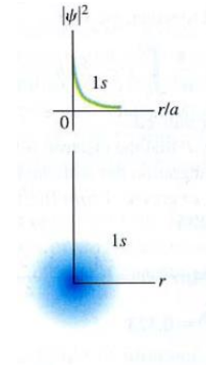
$$R_{21}(r) = \left(\frac{Z}{2a}\right)^{3/2} \frac{2}{\sqrt{3}} \left(\frac{Zr}{2a}\right) \exp\left(-\frac{Zr}{2a}\right)$$

$$Y_{00}(\theta,\varphi) = \sqrt{\frac{1}{4\pi}}$$

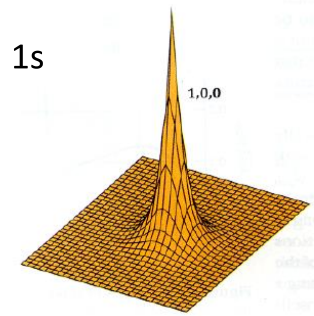
$$Y_{10}(\theta,\varphi) = \sqrt{\frac{3}{4\pi}} \cos(\theta)$$

$$Y_{1\pm1}(\theta,\varphi) = \sqrt{\frac{3}{8\pi}} \sin(\theta) \exp(\pm i\varphi)$$

Atomy



Stan 1s



Eigenstate:

E.g. hydrogen wavefunction

$$\Psi = R_{n,l}(r)\Theta_{l,m}(\theta)\Phi_m(\phi)$$

$$R_{n,l}(r) = \sqrt{\frac{(n-l+1)!}{2n(n+l)!}}\left(\frac{2Z}{na_0}\right)^{3/2} e^{-\rho/2}\rho^l G_{n-l-1}^{2l+1}(\rho)$$

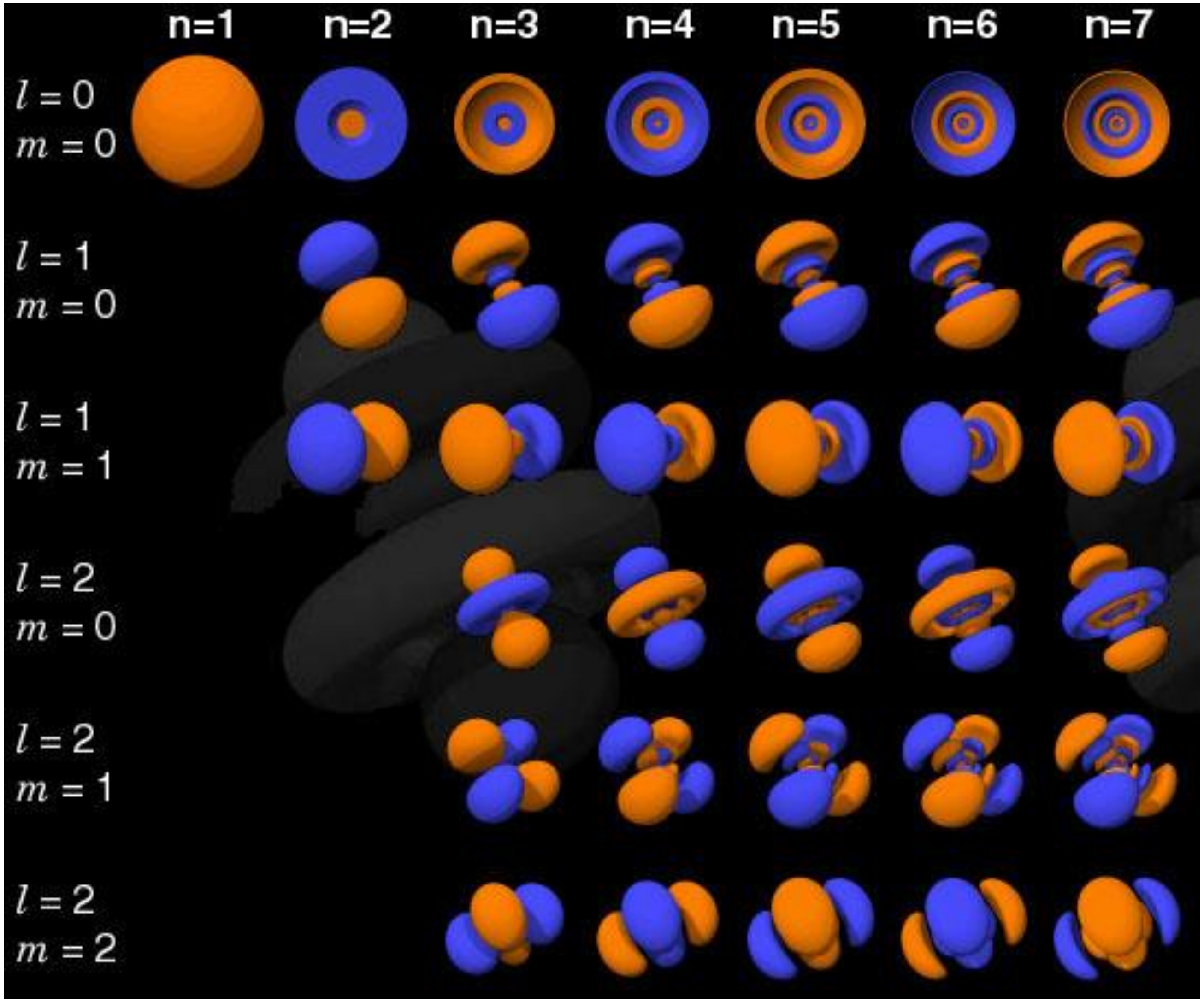
$$\Theta_{l,m}(\theta) = (-1)^m \sqrt{\frac{2l+1}{2\pi}\frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta)$$

$$\Phi_m(\phi) = Ce^{im\phi}$$

Quantum numbers!

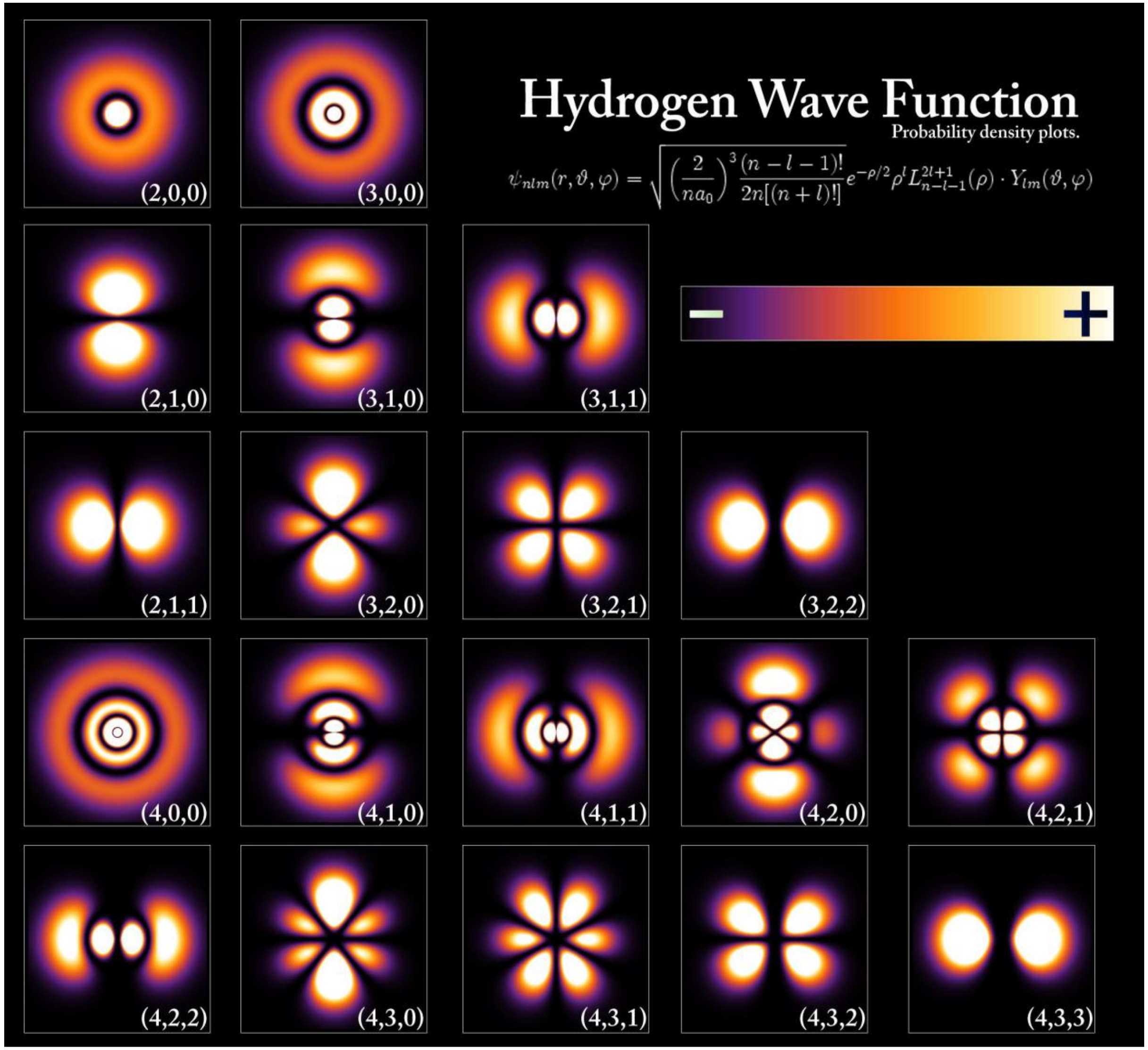
$$\boxed{\Psi = R_{n,l}(r)\Theta_{l,m}(\theta)\Phi_m(\phi) = |n, l, m\rangle}$$

Atomy



http://chemistry.stackexchange.com

Atomy



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Perturbation theory (rachunek zaburzeń)

Time-independent perturbation theory (rach. zab. bez czasu)

rach.zab.bez.czas.

$$\hat{H} = \hat{H}_0 + \lambda \hat{H}'$$

perturbation

Known solutions of unperturbed Hamiltonian $\hat{H}_0 \psi_n^{(0)} = E_n^{(0)} \psi_n^{(0)}$

We are looking for the function ψ_n: $(\hat{H}_0 + \lambda \hat{H}')\psi_n = E_n \psi_n$

we can write E_n and ψ_n as power series in λ::

$$\psi_n = \psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + \ldots$$
$$E_n = E_n^0 + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ldots$$

Thus:

$$(\hat{H}_0+\lambda\hat{H}')(\psi_n^{(0)}+\lambda\psi_n^{(1)}+\lambda^2\psi_n^{(2)}+\ldots) = (E_n^0+\lambda E^{(1)}+\lambda^2 E_n^{(2)}+\ldots)(\psi_n^{(0)}+\lambda\psi_n^{(1)}+\lambda^2\psi_n^{(2)}+\ldots)$$

comparing coefficients of each power of λ

$$\hat{H}_0 \psi^{(0)} = E_n^{(0)} \psi_n^{(0)}$$
$$\hat{H}_0 \psi_n^{(1)} + \hat{H}' \psi_n^0 = E_n^{(0)} \psi_n^{(1)} + E_n^{(1)} \psi_n^0$$
$$\hat{H}_0 \psi_n^{(2)} + \hat{H}' \psi_n^{(1)} = E_n^{(0)} \psi_n^{(2)} + E_n^{(1)} \psi_n^{(1)} + E_n^{(2)} \psi_n^{(0)}$$

http://pl.wikibooks.org/wiki/Mechanika_kwantowa/Rachunek_zaburzeń_dla_równania_Schrödingera_niezależnego_od_czasu

Perturbation theory (rachunek zaburzeń)

Time-independent perturbation theory

Eigenfunction

$$E_n = E_0 + \lambda E_n^{'} = E_0 + \lambda H_{nn}^{'}$$

$$\psi_n = \psi_n^{(0)} + \lambda \psi_n^{(1)} = \psi_n^{(0)} + \lambda \sum_{k, k \neq n} \frac{\hat{H}_{kn}^{'}}{E_n^{(0)} - E_k^{(0)}} \psi_k^0$$

a solution exists only when its determinant : $\det\left(\lambda H' - \hat{I}E\right) = 0$

Perturbation

$$\widehat{H}_{ij}' = \langle \psi_i | \widehat{H}' | \psi_j \rangle$$

$$\begin{vmatrix} \lambda H_{11}' - E^{'} & \lambda H_{12}' & \cdots & \lambda H_{1k}' \\ \lambda H_{21}' & \lambda H_{22}' - E^{'} & \cdots & \lambda H_{2k}' \\ \vdots & \vdots & \ddots & \vdots \\ \lambda H_{k1}' & \lambda H_{k2}' & \cdots & \lambda H_{kk}' - E^{'} \end{vmatrix} = 0$$

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Hydrogen-like atom

Alkali metal atoms (*wodoropodobne*):

perturbation theory $\hat{H} = \hat{H}_0 + \hat{H}'$:

$$\hat{H}_0 \psi_i = E_i \psi_i$$

perturbation $\hat{H}'_{ij} = \langle \psi_i | \hat{H}' | \psi_j \rangle$

the method: $\det\left| \hat{H}' - E\hat{I} \right| = 0$

a) $$V(r) = -\frac{3e^2}{4\pi\varepsilon_0 r} + A\delta(r-a)$$

b) $$V(r) = \begin{cases} -\dfrac{Ze^2}{4\pi\varepsilon_0 r}; r \le b \\ -\dfrac{e^2}{4\pi\varepsilon_0 r}; r > b \end{cases}$$

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$$\hat{H}_0\psi_i = E_i\psi_i$$

b) $V(r) = \begin{cases} -\frac{Ze^2}{4\pi\varepsilon_0 r}; r \le b \\ -\frac{e^2}{4\pi\varepsilon_0 r}; r > b \end{cases}$

perturbation $\hat{H}'_{ij} = \langle\psi_i|\hat{H}'|\psi_j\rangle$

the method: $\det\left|\hat{H}' - E\hat{I}\right| = 0$

a) $V'(r) = A\delta(r-a)$

b) $V'(r) = \begin{cases} 0 \quad ; r \le b \\ -\frac{(Z-1)e^2}{4\pi\varepsilon_0 r}; r > b \end{cases}$

Pole elektryczne

Stark effect of hydrogen atom

electric field E

$$H' = \vec{p}\vec{E} = ezE_z$$

dipole moment p

Eigenfunctions of hydrogen atom for 2p state:

$$\psi_{200}, \psi_{21-1}, \psi_{210}, \psi_{211}$$

Perturbation $\quad \widehat{H}'_{ij} = \langle \psi_i | \widehat{H}' | \psi_j \rangle$

$$\Psi_{nlm}(r,\theta,\varphi) = R_{nl}(r) Y_{lm}(\theta,\varphi)$$

$$R_{20}(r) = \left(\frac{Z}{2a}\right)^{3/2} 2\left(1 - \frac{Zr}{2a}\right)\exp\left(-\frac{Zr}{2a}\right)$$

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Eigenfunctions of hydrogen atom for 2p state

$\psi_{200}, \psi_{21-1}, \psi_{210}, \psi_{211}$

Exercises !

Perturbation $\widehat{H}'_{ij} = \langle \psi_i | \widehat{H}' | \psi_j \rangle$

$$\ldots_{n}(r,\theta,\varphi) = R_{nl}(r)Y_{lm}(\theta,\varphi)$$

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