LECTURE 5

Long range coherence
Vortices

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WYDZIAŁ FIZYKI
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BEC OF POLARITONS - BEC OF ATOMS

CHARACTERISTICS:

- Solid state system
- Disordered environment
- Long range spatial coherence
- Non equilibrium
- Steady state is characterized by incoming and outgoing flow of particles
- Emitted light is linearly related to polaritons

	atoms	**polaritons**
m	Rb: $10^4 m_e$	$10^{-4} m_e$
T	10^{-7}K	>100K
N	$10^{14}/cm^3$	$<10^{11}/cm^2$
t	∞	1 ps

FUNDAMENTAL DIFFERENCES:

- Condensation in a disordered medium
- Interacting Bose gas
- **Out of equilibrium**
- **Non-isolated system**

Sources: J. Kasprzak *et al* Bose–Einstein condensation of exciton polaritons. Nature 443: 409-414, (2006)
V. Savona *et al.* Optical Properties of Microcavity Polaritons. *Phase Transitions* **68** 169-279 (1999) and
V. Savona *et al.* Quantum-Well Excitons in Semiconductor Microcavities - Unified Treatment of Weak and Strong-Coupling Regimes. *Sol. St. Com.* **93** 733-739 (1995)

Exciton polaritons

dispersion - energy-wave vector dependence

Exciton dispersion in quantum well

$$E_X(k) = E_g - E_b + \frac{\hbar^2 k^2}{2m_X}$$

photon dispersion in quantum well

$$E(\vec{k}) = \frac{\hbar c}{n}|\vec{k}| = \frac{\hbar c}{n}\sqrt{\left(\frac{2\pi}{L_C}\right)^2 + k_{II}^2}$$

because we are interested in small wave-vectors $k_{||}$ we can make the following approximation:

$$\sqrt{\varepsilon^2 + a^2} \approx a + \frac{\varepsilon^2}{2a}$$

and derive the energy in a form:

$$E(\vec{k}) \approx \frac{\hbar c}{n}\left[\frac{2\pi}{L_C} + \frac{k_{II}^2 L_C}{4\pi}\right] = E_0 + \frac{\hbar^2 k_{II}^2}{2m_{ph}^*}$$

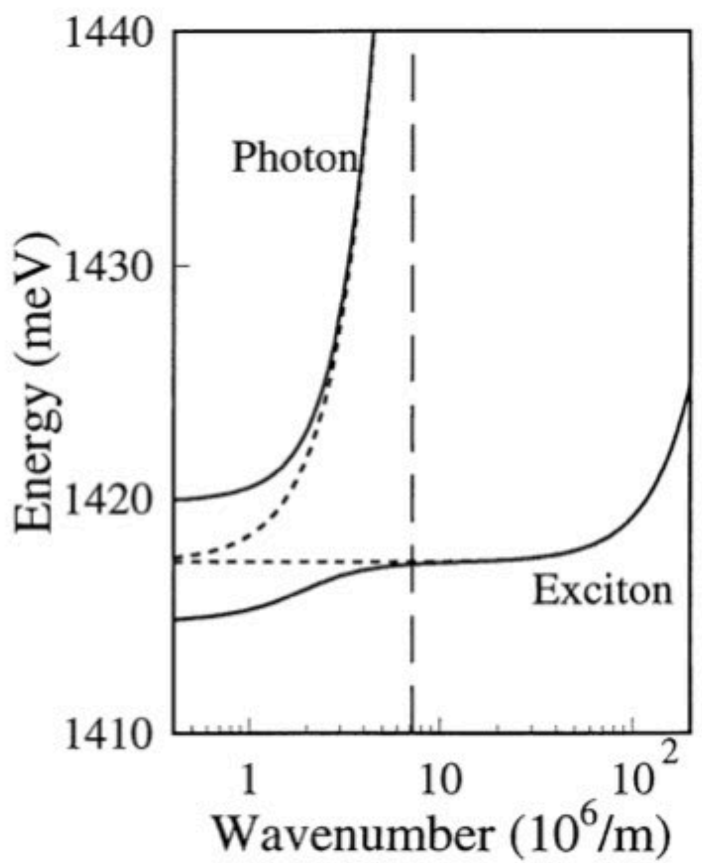

image after: M. S. Skolnick et al. Semicond. Sci. Technol. 13, 645 (1998)

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conclusions:

in a cavity photon gain an effective mass:

$$m_C^* = \frac{\hbar k_z n}{c} = \frac{hn^2}{c\lambda_0}$$

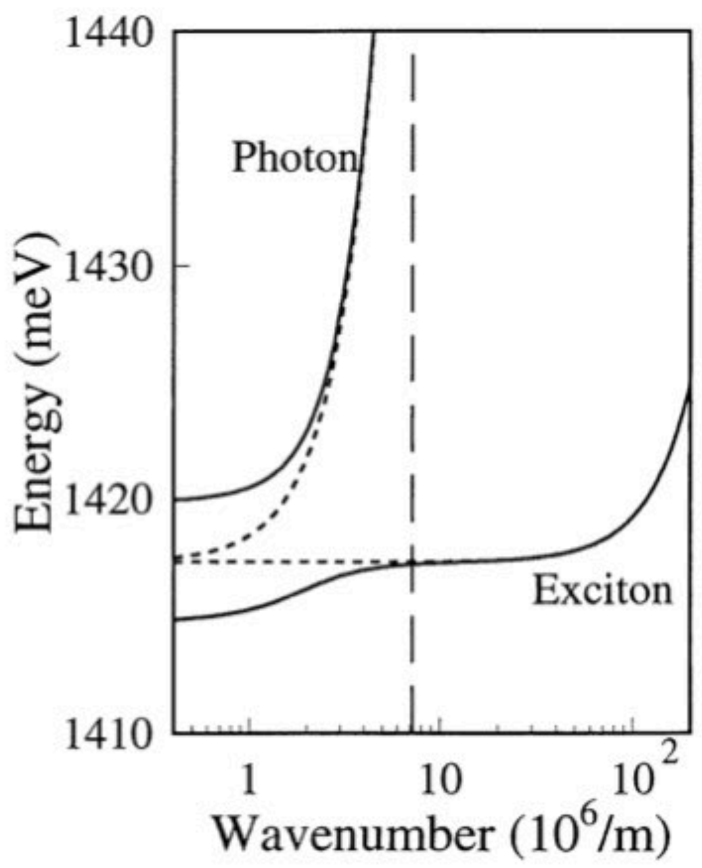
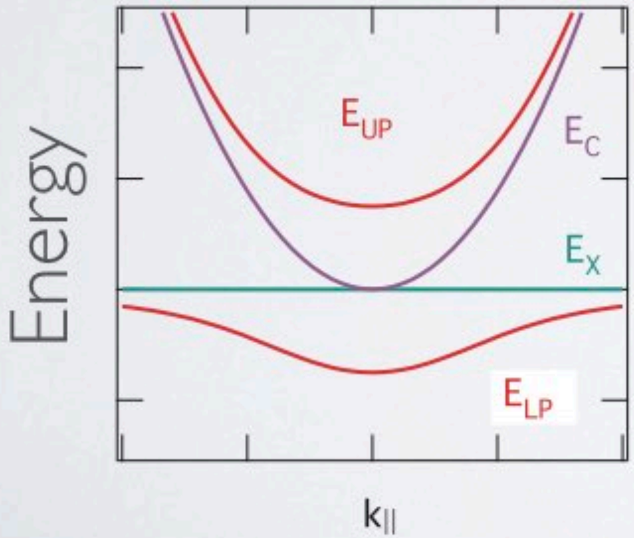


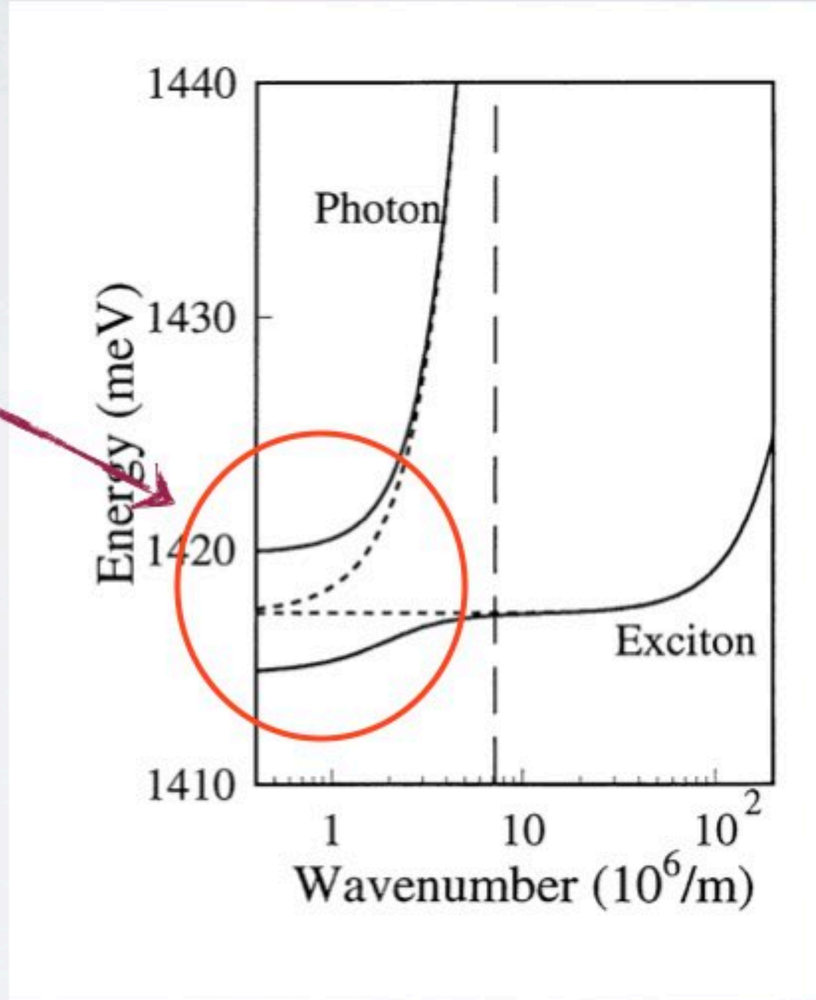
image after: M. S. Skolnick et al. Semicond. Sci. Technol. 13, 645 (1998)

Exciton polaritons

dispersion - energy-wave vector dependence

experimental access





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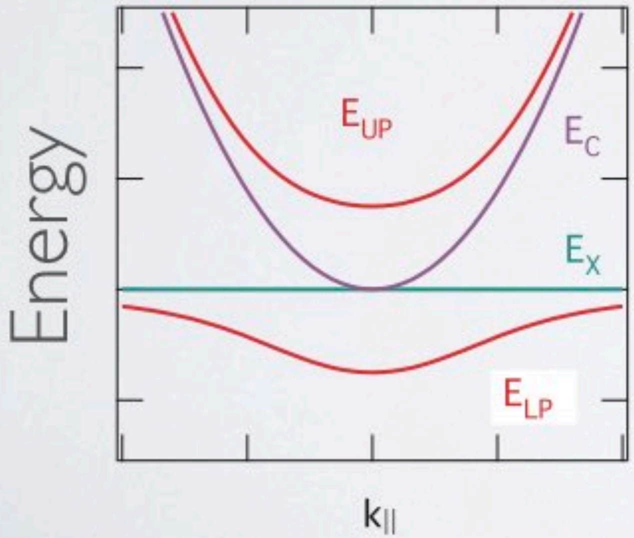
Semicond. Sci. Technol. 13, 645 (1998)

Exciton polaritons
dispersion - energy-wave vector dependence



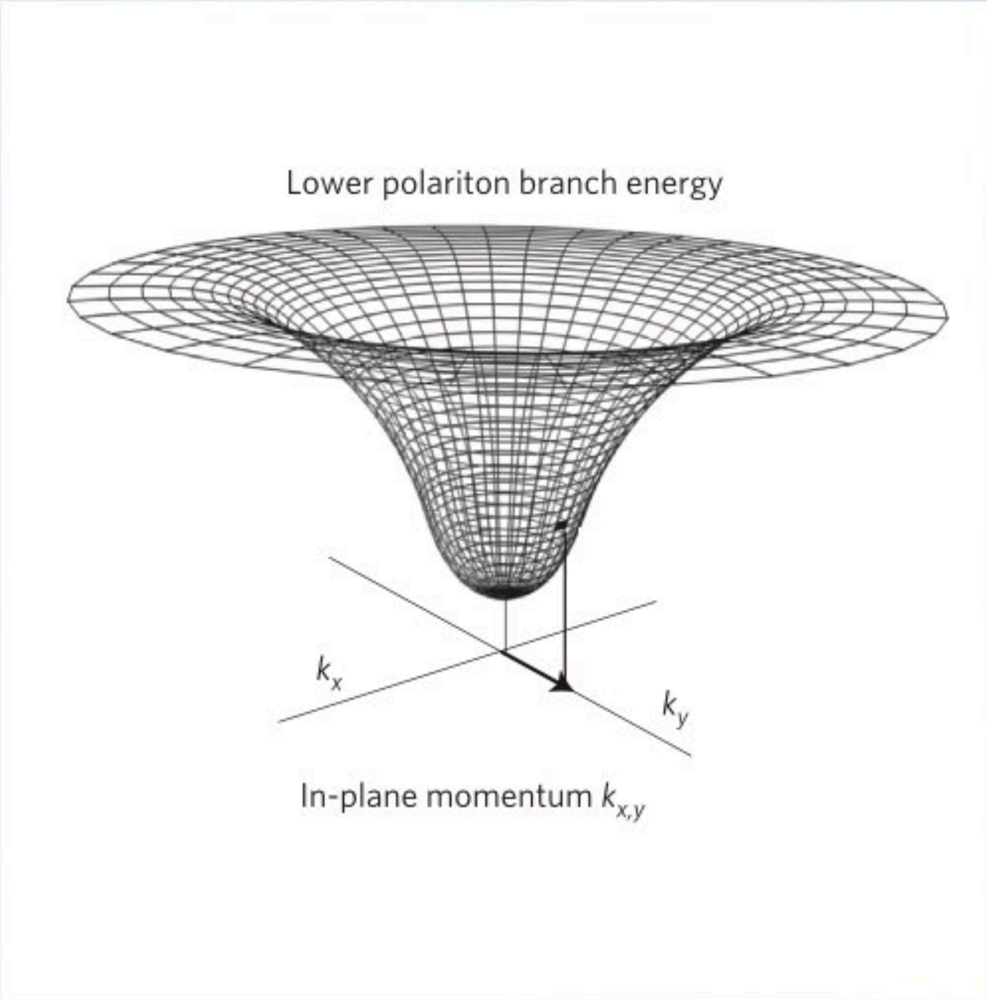
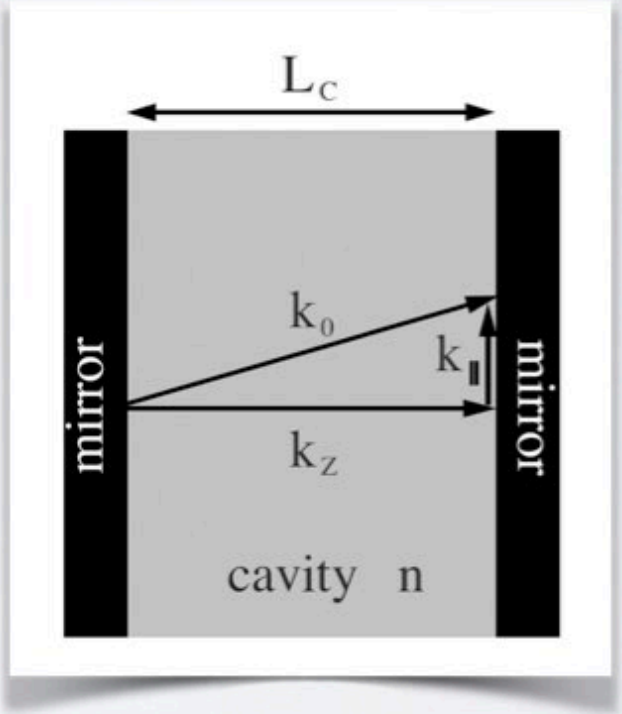


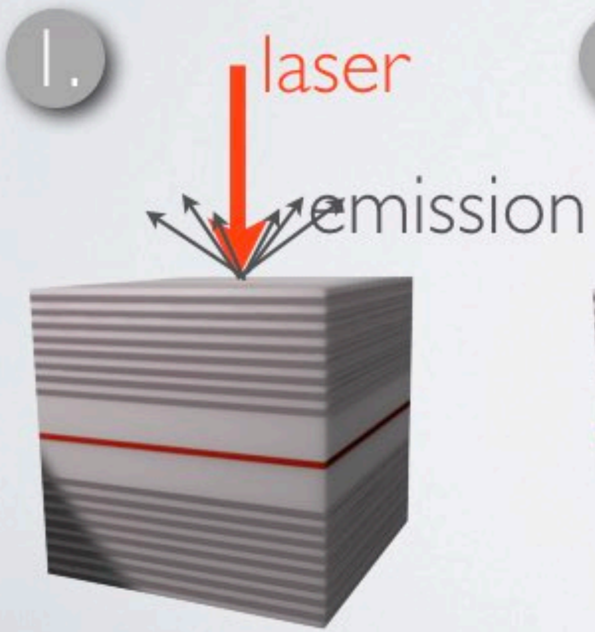
image after: D. Sanvitto et. al.

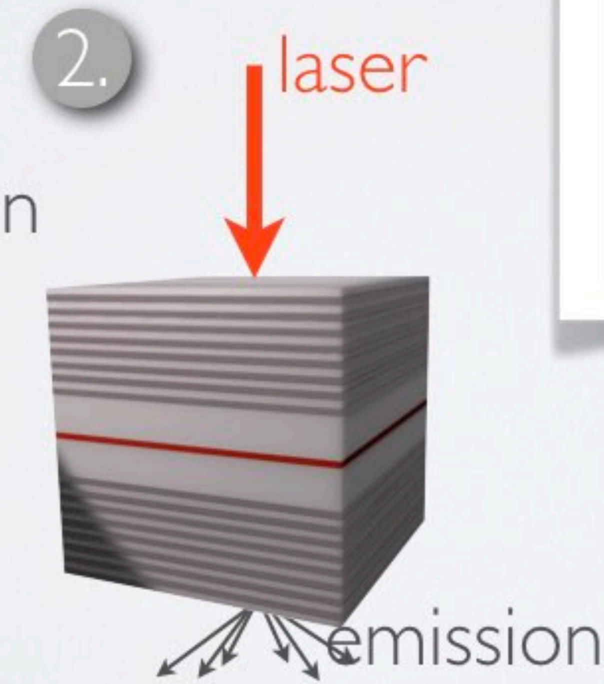
Exciton polaritons

typical experimental setup

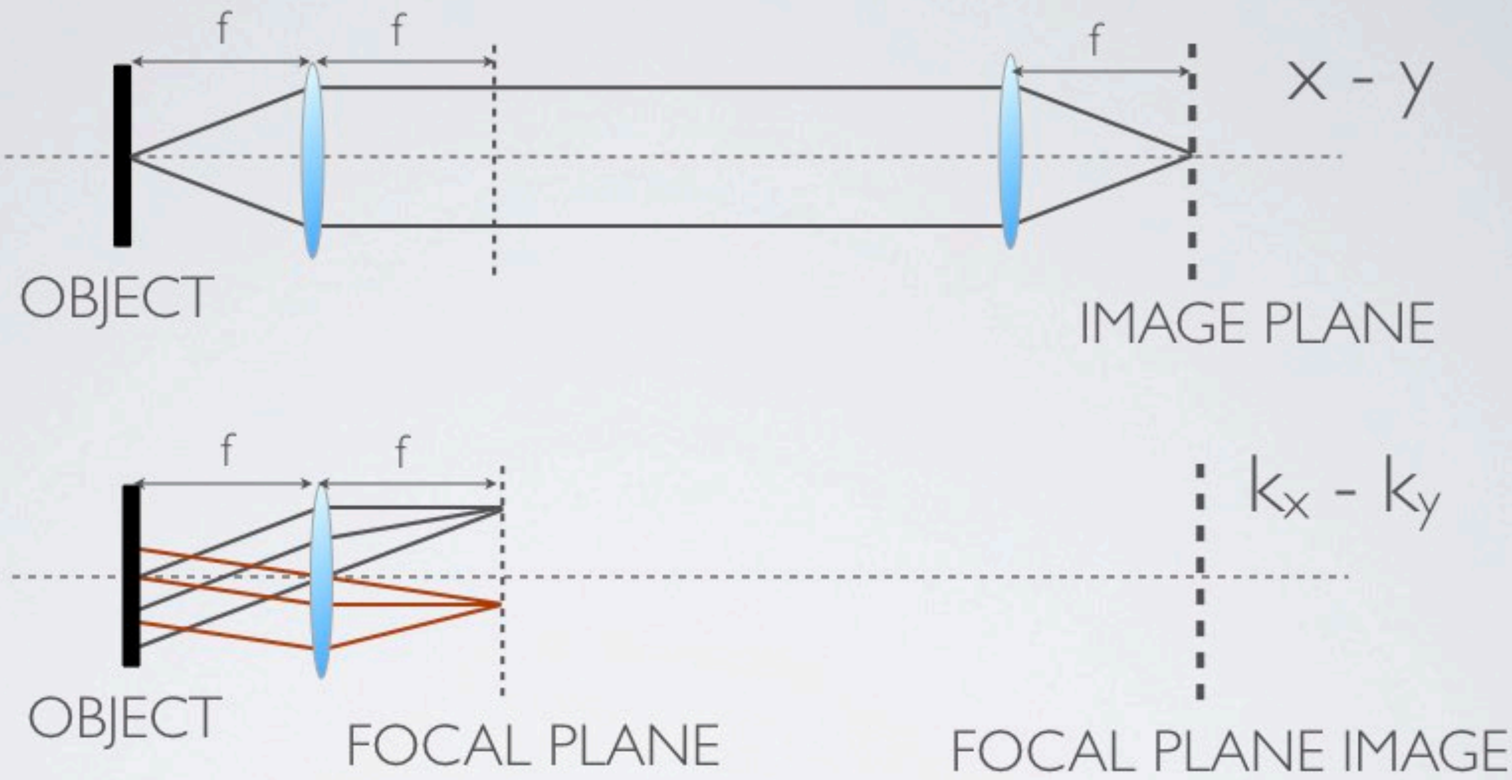
$$k_{\parallel} = k \sin\theta_{ext} = \frac{E(\theta_{ext})}{\hbar c} \sin\theta_{ext}$$



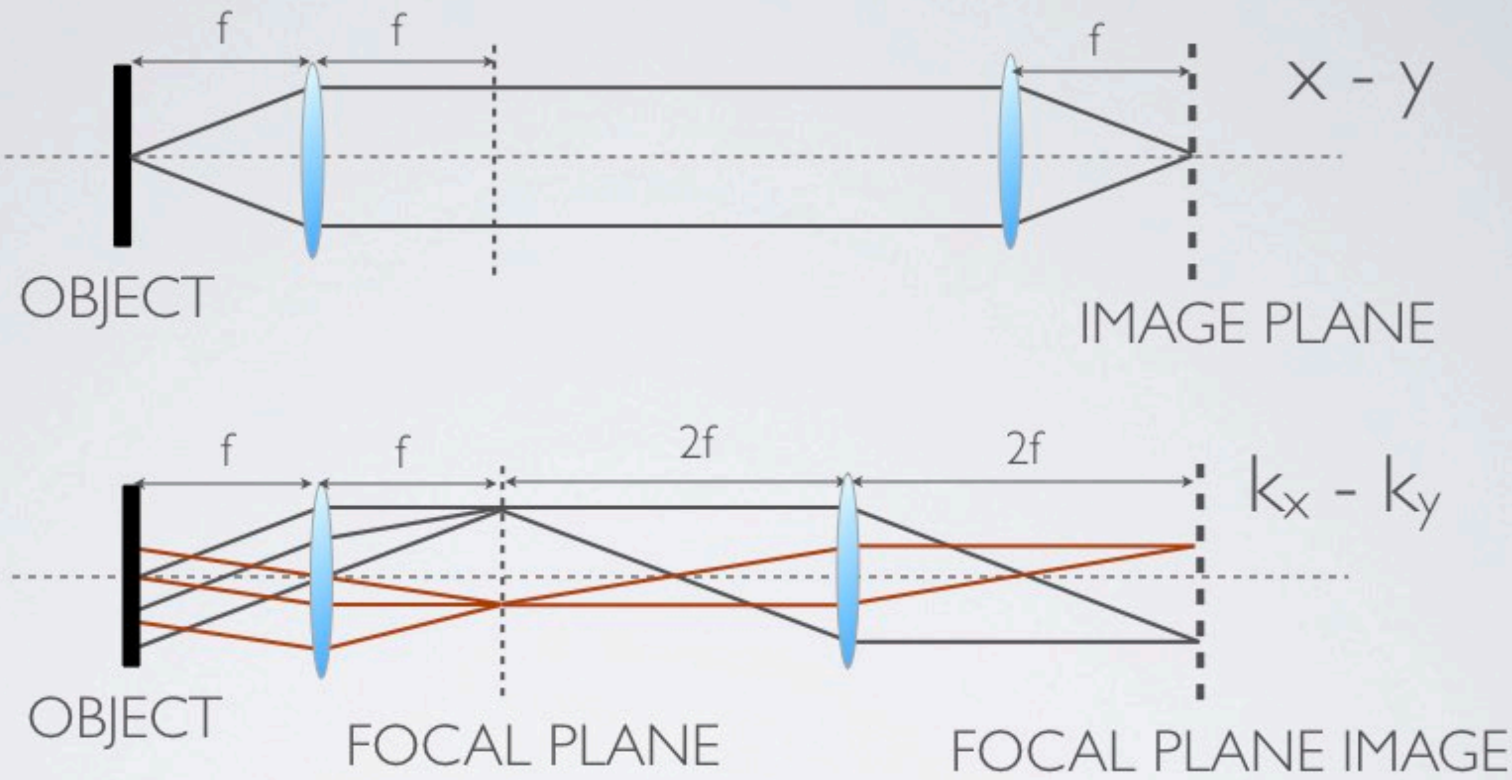




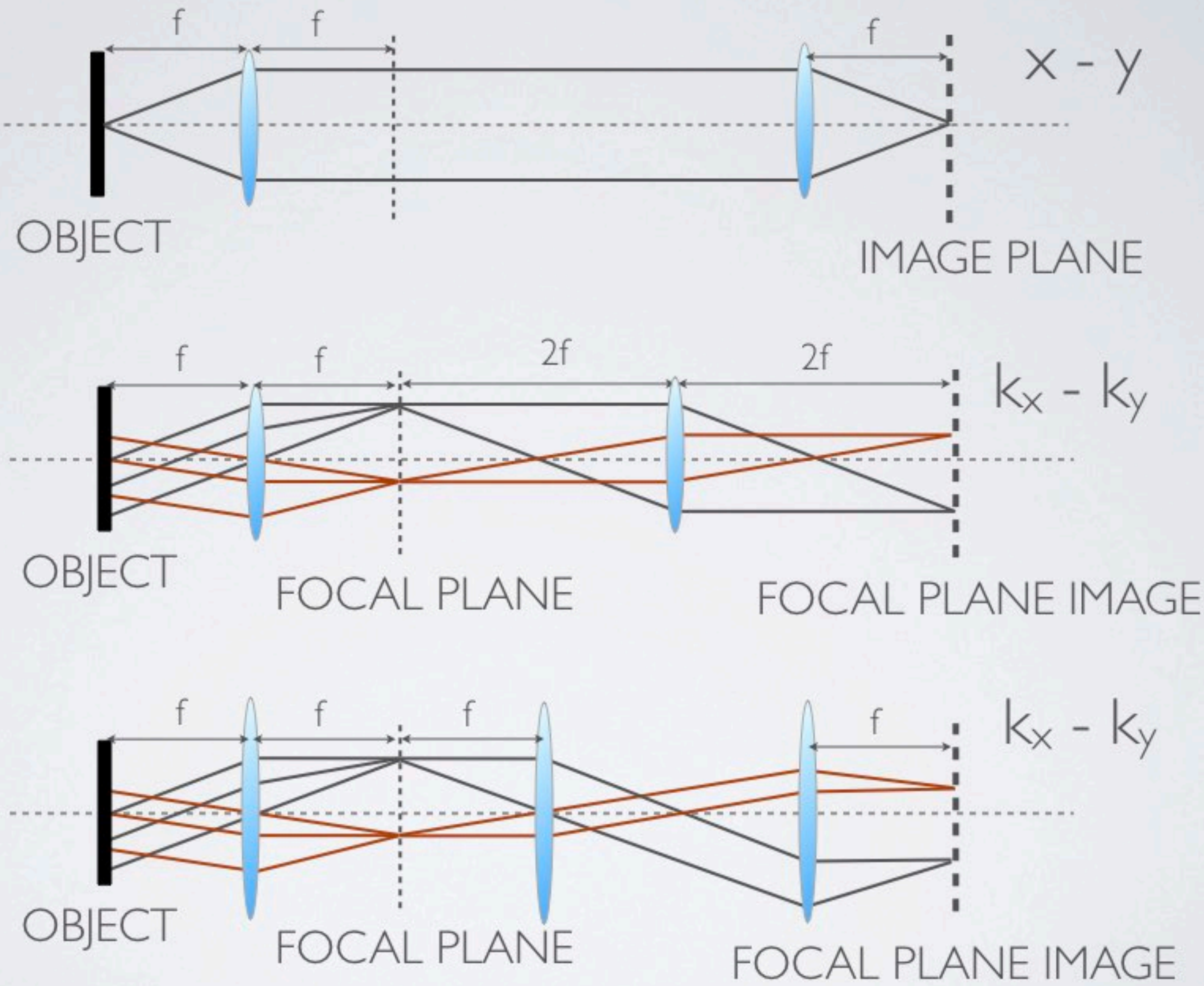
REAL (x) & MOMENTUM (k) SPACE



REAL (x) & MOMENTUM (k) SPACE



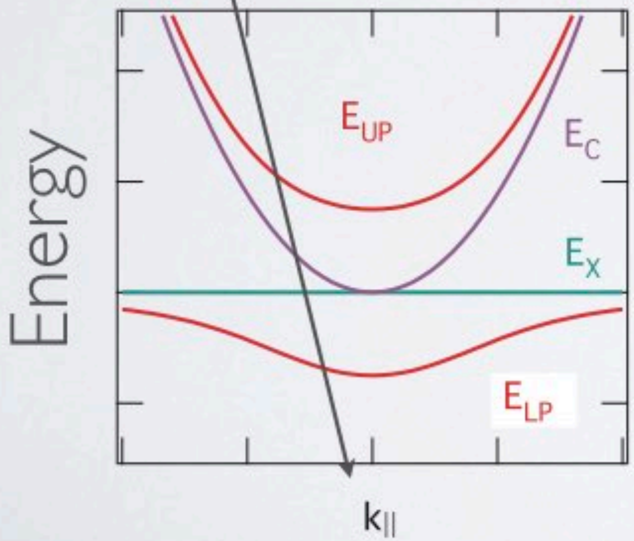
REAL (x) & MOMENTUM (k) SPACE

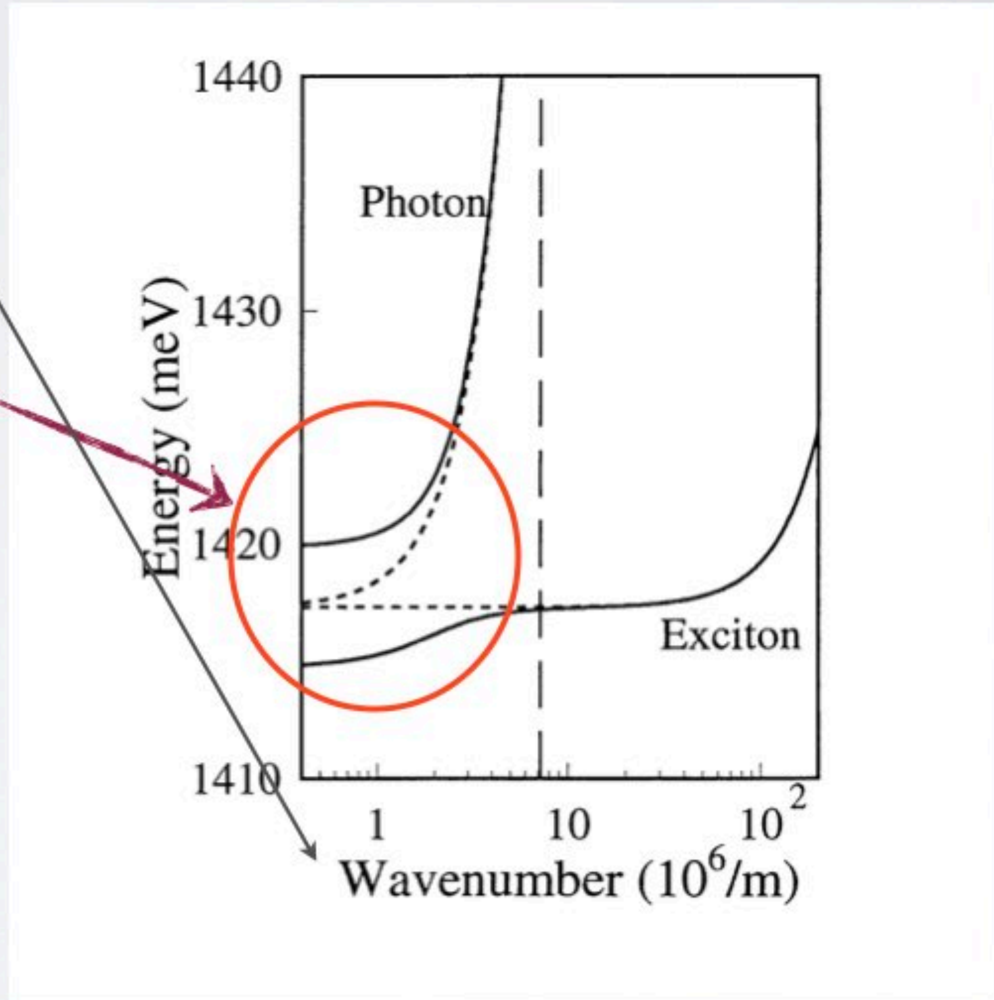


Exciton polaritons
typical experimental setup

x axis :
!!! emission angle !!!
= polariton momentum

experimental access





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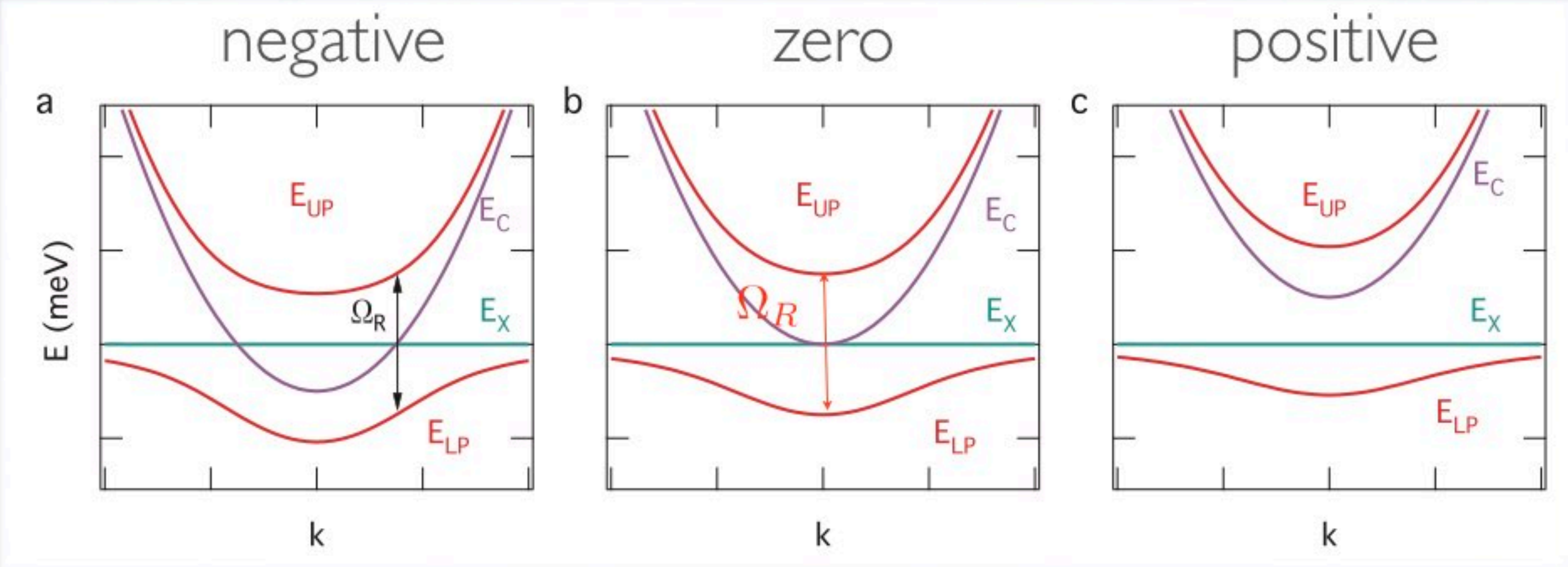
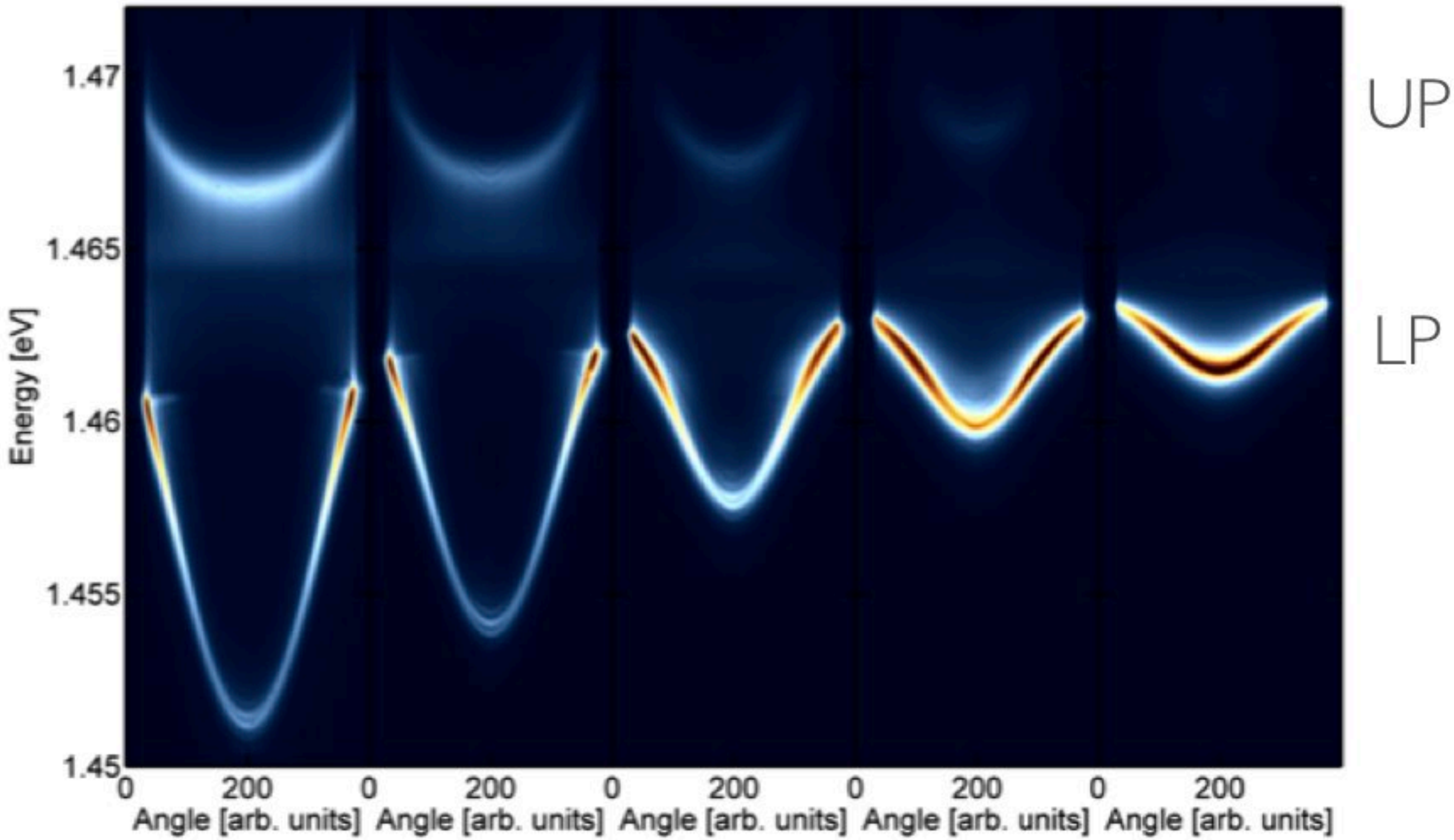
Exciton polaritons

dispersion shape and detuning



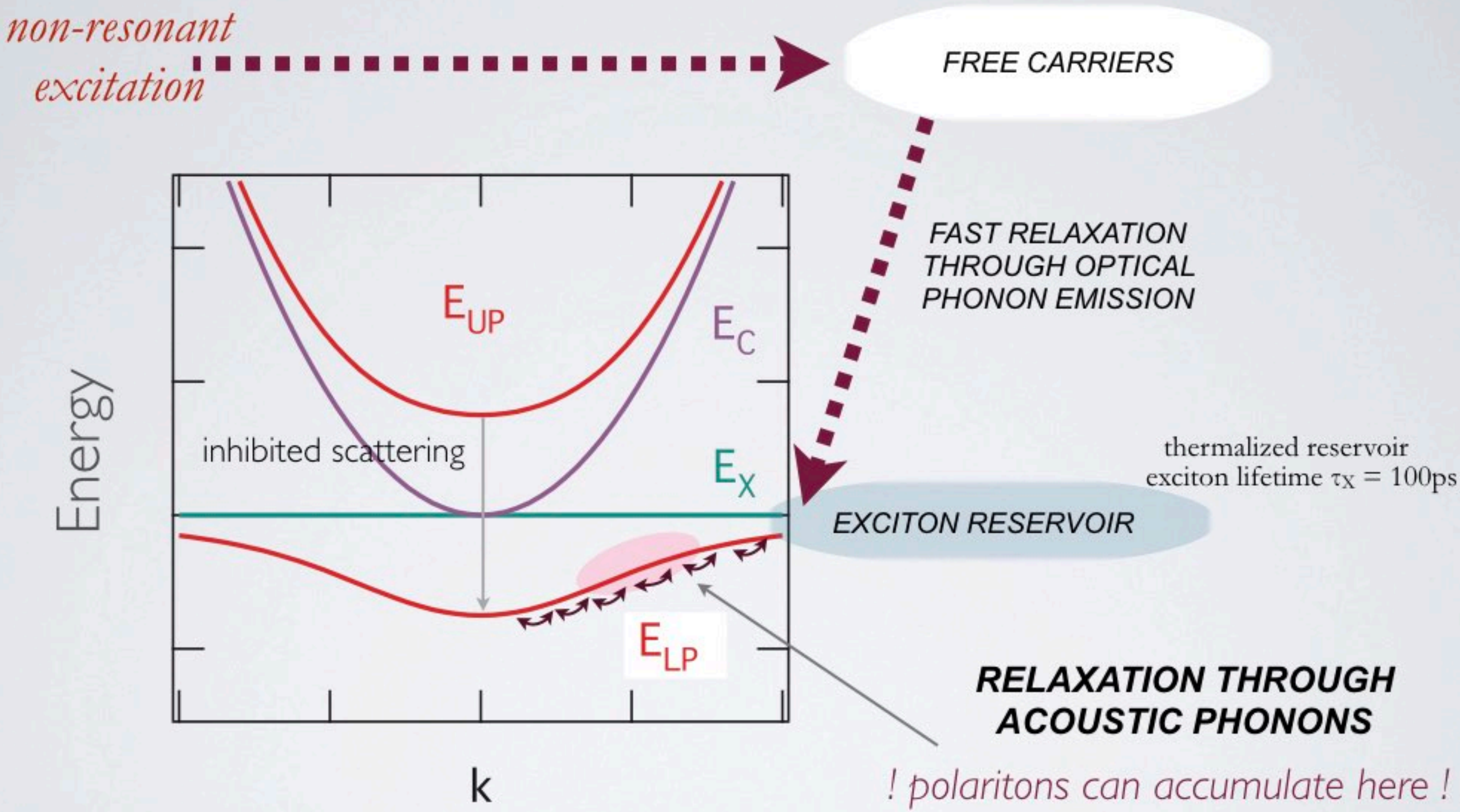
Exciton polaritons
dispersion shape and detuning

image: K. Lekenta, R. Mirek



Exciton polaritons

formation of polariton population



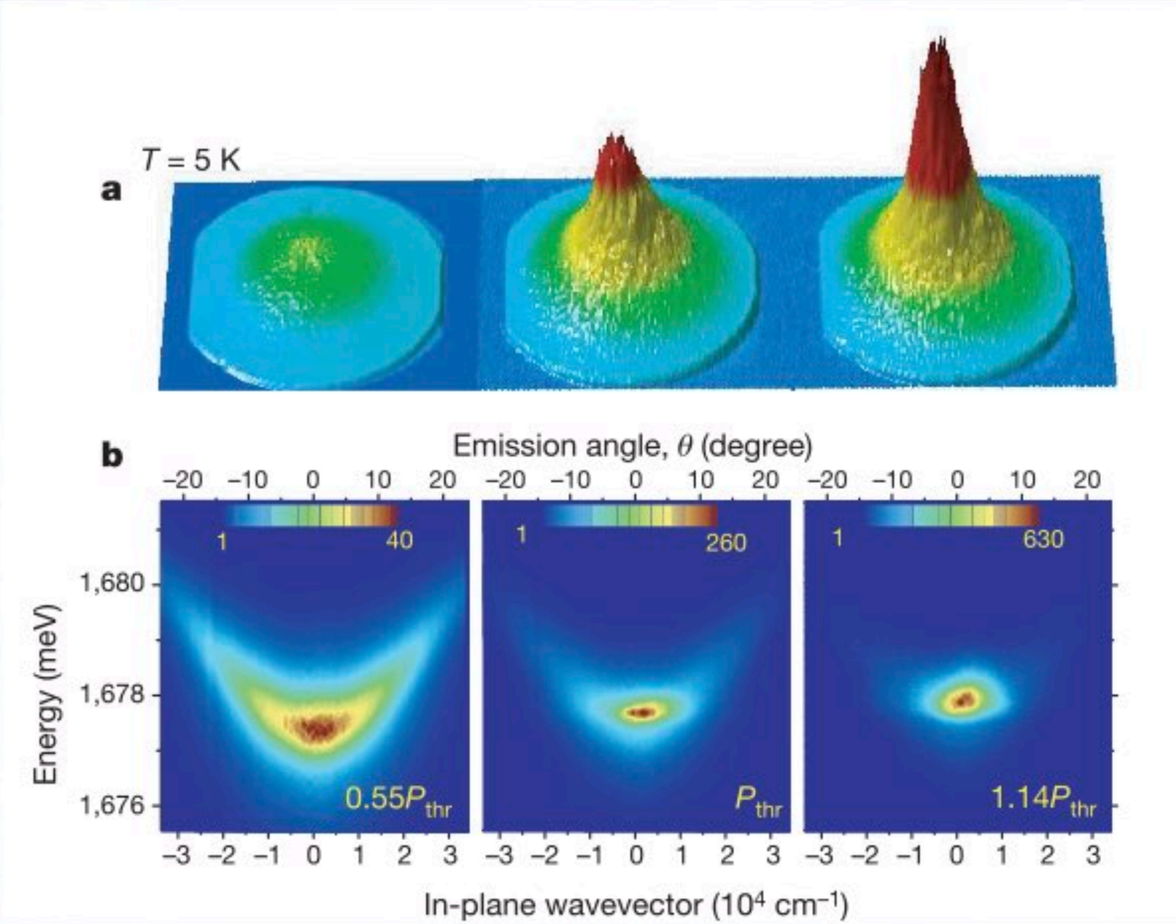
IN NON-LINEAR REGIME:

- FFS final state stimulation - polariton - polariton scattering

Bose–Einstein condensation of exciton polaritons

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]

nature



increased polariton density

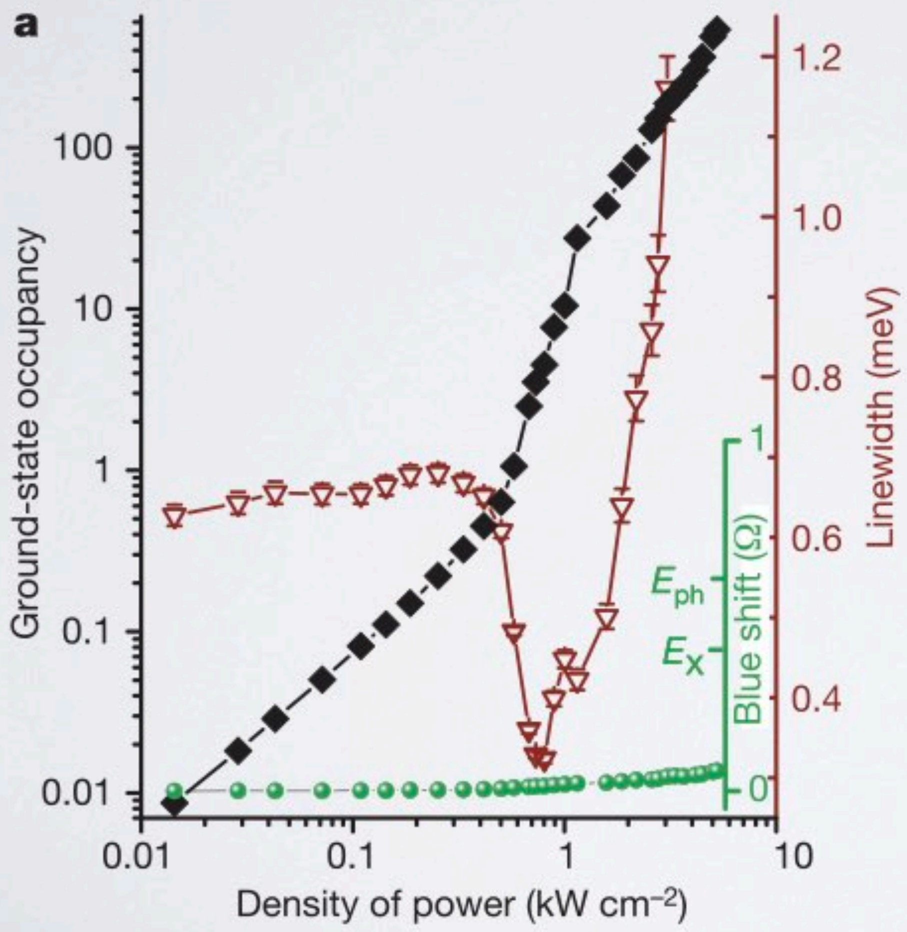
MACROSCOPIC OCCUPATION OF GROUND STATE

J. Kasprzak, et. al *Nature* **443**, 409 (2006)

Bose–Einstein condensation of exciton polaritons

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]

nature



INTENSITY INCREASE VS LINE-WIDTH NARROWING

- INCREASE OF TEMPORAL COHERENCE !

J. Kasprzak, et. al
Nature **443**, 409 (2006)

nature

Bose–Einstein condensation of exciton polaritons

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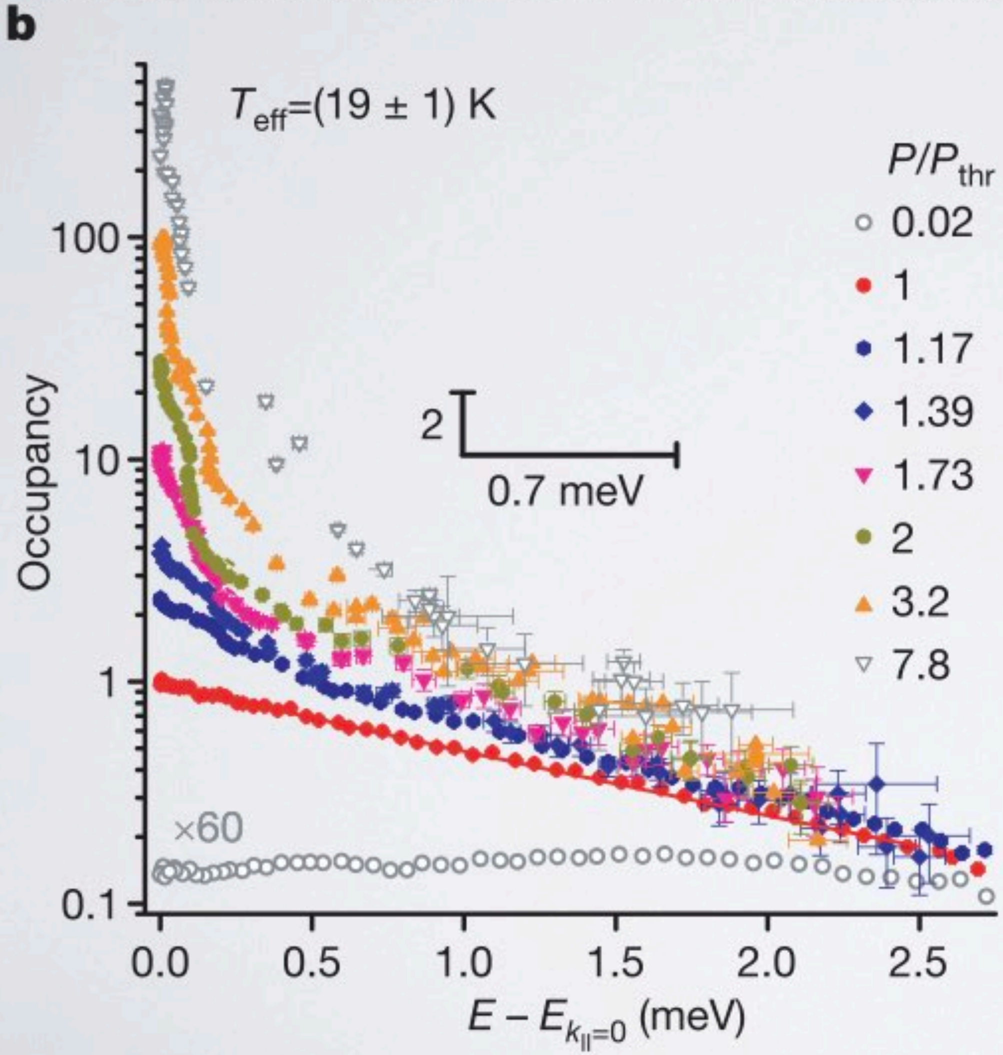


POPULATION DISTRIBUTION OVER EXCITED STATES

Maxwell - Boltzmann distribution at thermal equilibrium

$$n_j = \frac{N}{Z} e^{-\frac{\varepsilon_j}{kT}}$$

Bose- Einstein distribution in condensate

$$n_i = \frac{g_i}{Be^{\frac{\varepsilon_i}{kT}} - 1}$$

J. Kasprzak, et. al *Nature* **443**, 409 (2006)

QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

- One body density matrix express the probability amplitude to annihilate a particle at location **r'** and to create one at location **r**

$$\rho(\mathbf{r},\mathbf{r}') = \langle \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r}')\rangle$$

- for **r = r'**, this describes the local density of the system

$$n(\mathbf{r}) = \rho(\mathbf{r},\mathbf{r}) = \langle \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\rangle$$

- density matrix is normalized, such that the total number of particles is *N*

$$\int \mathrm{d}\mathbf{r}\rho(\mathbf{r},\mathbf{r}) = N$$

- for a pure state the average is the quantum mechanical expectation

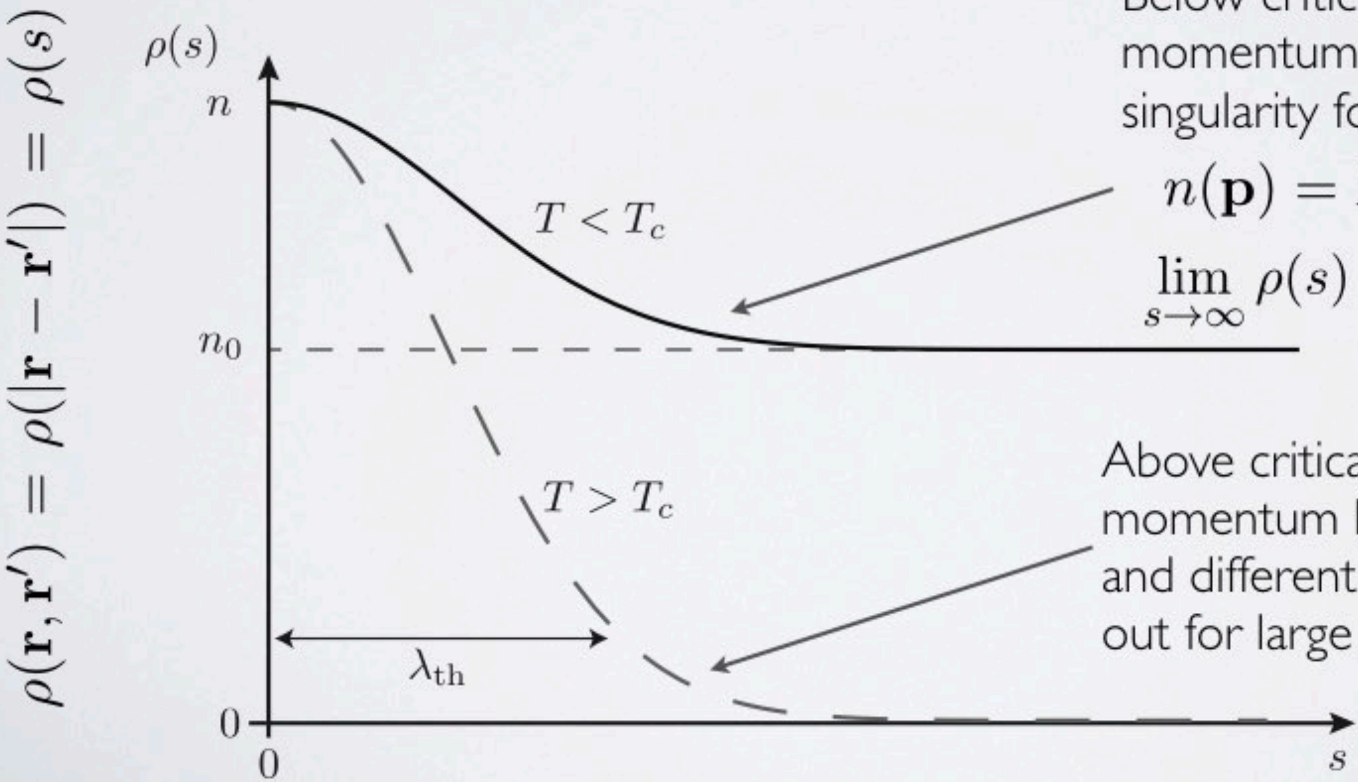
$$\rho(\mathbf{r},\mathbf{r}') = N\int \mathrm{d}\mathbf{r}_2\mathrm{d}\mathbf{r}_3\ldots\mathrm{d}\mathbf{r}_N\Psi^*(\mathbf{r},\mathbf{r}_2,\mathbf{r}_3,\ldots\mathbf{r}_N)\Psi(\mathbf{r}',\mathbf{r}_2,\mathbf{r}_3,\ldots\mathbf{r}_N)$$

QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

- Bose - Einstein condensation is a phenomenon taking place in momentum space
- The density matrix can be expressed also by the momentum distribution of the system

$$n(\mathbf{p}) = \langle \hat{\Psi}^\dagger(\mathbf{p})\hat{\Psi}(\mathbf{p})\rangle$$

- the one-body density matrix depend only on the relative distance s = |**r'-r**|



Below critical temperature the momentum distribution has a singularity for **p** = 0

$$n(\mathbf{p}) = N_0\delta(\mathbf{p}) + \tilde{n}(\mathbf{p})$$

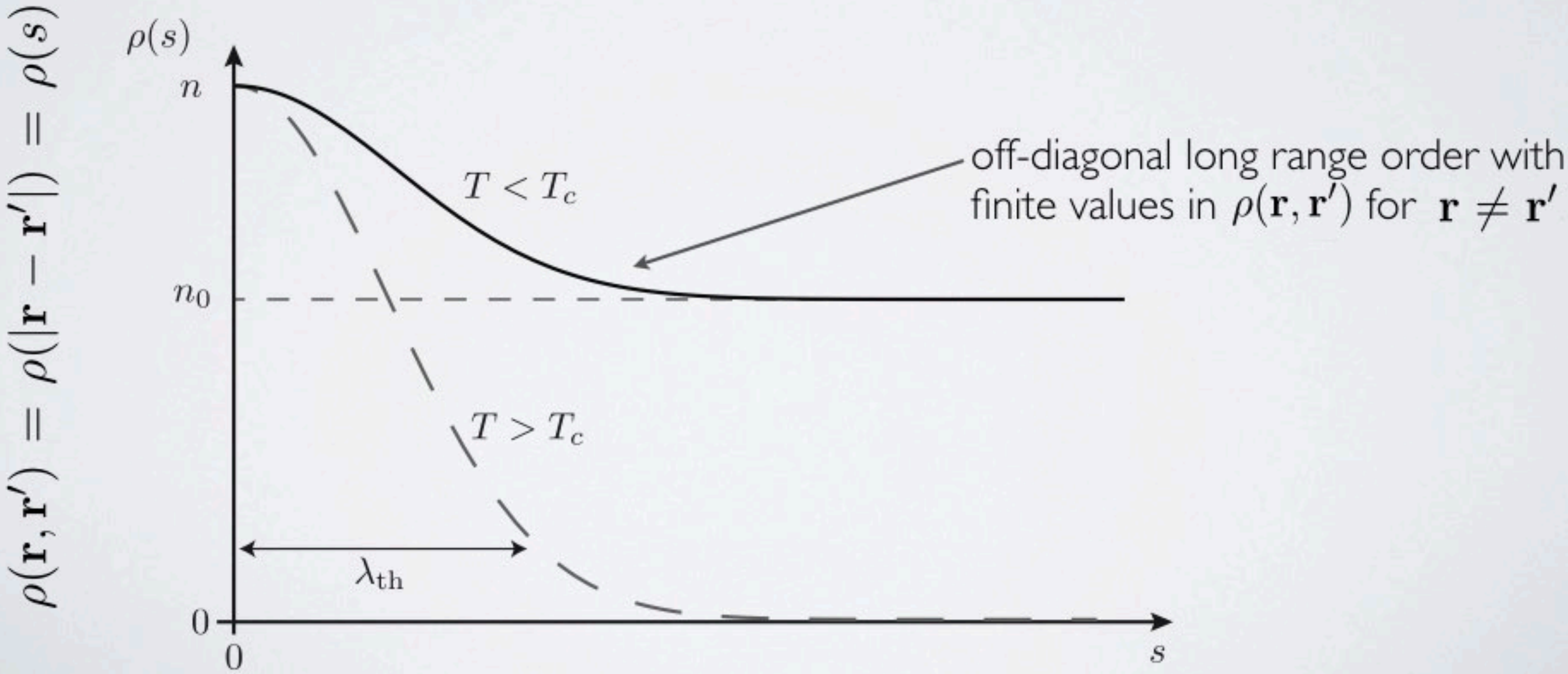
$$\lim_{s\to\infty}\rho(s) = n_0 = \frac{N_0}{V}$$

Above critical temperature momentum has a smooth distribution and different contributions average out for large s

$$\lim_{s\to\infty}\rho(s) = 0$$

QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

- The one-body density matrix for a BEC is decaying over a distance given by the thermal de Broglie length to a finite value determined by the condensate fraction



QUANTUM CORRELATIONS ON THE MACROSCOPIC SCALE

- density matrix has finite off-diagonal elements

- referred to as **O**ff-**D**iagonal **L**ong **R**ange **O**rder (**ODLRO**)

  concept introduced first by Penrose and Onsager in 1956

- **can be measured through the first order coherence function**

$$G^{(1)}(\mathbf{r},\mathbf{r}') = \rho(\mathbf{r},\mathbf{r}')$$

- in normalized form

$$g^{(1)}(\mathbf{r},\mathbf{r}') = \frac{G^{(1)}(\mathbf{r},\mathbf{r}')}{\sqrt{G^{(1)}(\mathbf{r},\mathbf{r})}\sqrt{G^{(1)}(\mathbf{r}',\mathbf{r}')}}$$

- perfect correlations correspond to $g^{(1)}(\mathbf{r},\mathbf{r}') = 1$

- higher order coherence further characterize the state and distinguishes it from a thermal mixture

ORDER PARAMETER AND CONDENSATE WAVE-FUNCTION

- Applying field operator to BEC where the ground state is macroscopically populated

$$\hat{\Psi}(\mathbf{r}) = \sum_i \phi_i(\mathbf{r})\hat{a}_i = \phi_0(\mathbf{r})\hat{a}_0 + \sum_{i\neq 0} \phi_i(\mathbf{r})\hat{a}_i$$

condensate non-condensed part

$$\hat{\Psi}(\mathbf{r}) = \psi(\mathbf{r}) + \delta\hat{\Psi}(\mathbf{r})$$

$$\psi(\mathbf{r}) = \sqrt{N_0}\phi_0(\mathbf{r})$$

For a pure BEC the field operator is described by a wave-function thus a classical object

$$\psi(\mathbf{r}) = |\psi(\mathbf{r})|e^{i\varphi(\mathbf{r})}$$

diagonal therm - density

off-diagonal density - coherence

- Condensate wave-function is therefore the order parameter of the normal to condensed phase transition

ORDER PARAMETER

zero before the phase transition and becomes determined after phase transition

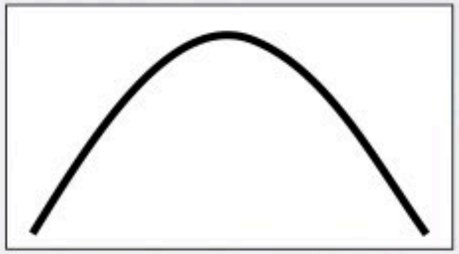
$$g^{(1)}(\mathbf{r},\mathbf{r}',\tau,t) = \langle \psi^*(\mathbf{r},\tau,t)\psi(\mathbf{r}',\tau,t)\rangle$$

(density matrix)

FIRST ORDER CORRELATION FUNCTION

Phase coherence!!!

spatial and temporal:



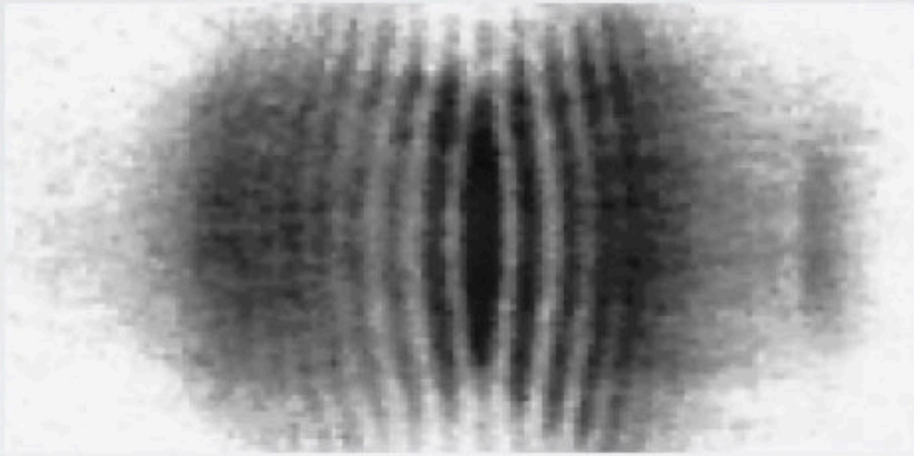
T=0:
Pure Bose condensate
"Giant matter wave"

- Coherence in time
- Long-range order = spatial coherence

during the phase transition

interference !!

ability to form interference fringes



Interference between two atomic BEC

M. R. Andrews et al., Science **275,** 637 (1997)

Exciton polaritons

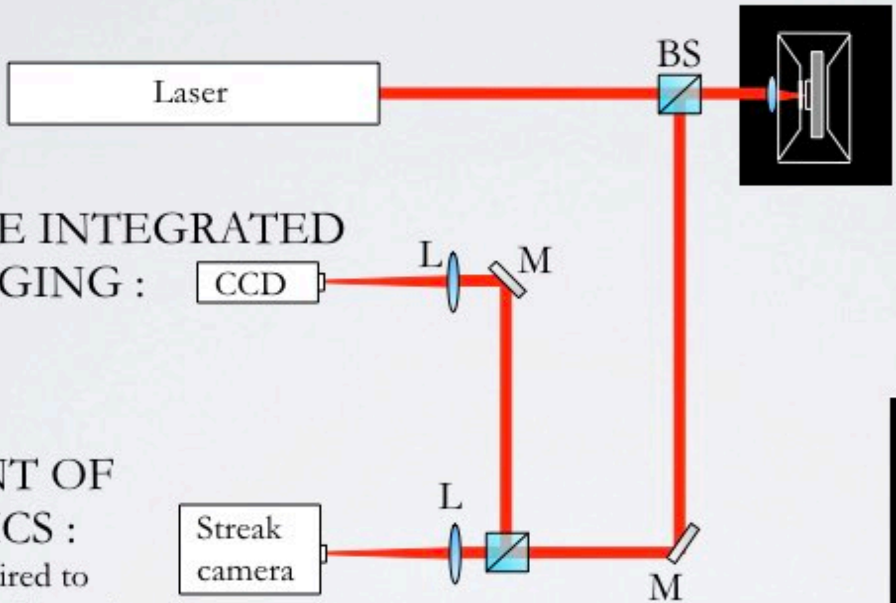
more advanced experimental setup

EXCITATION

- Pulsed OR
- Cw
- Non-resonant excitation

MICROSCOPE OBJECTIVE

- for spatial imaging
- condensate sizes from a few to hundreds of μm



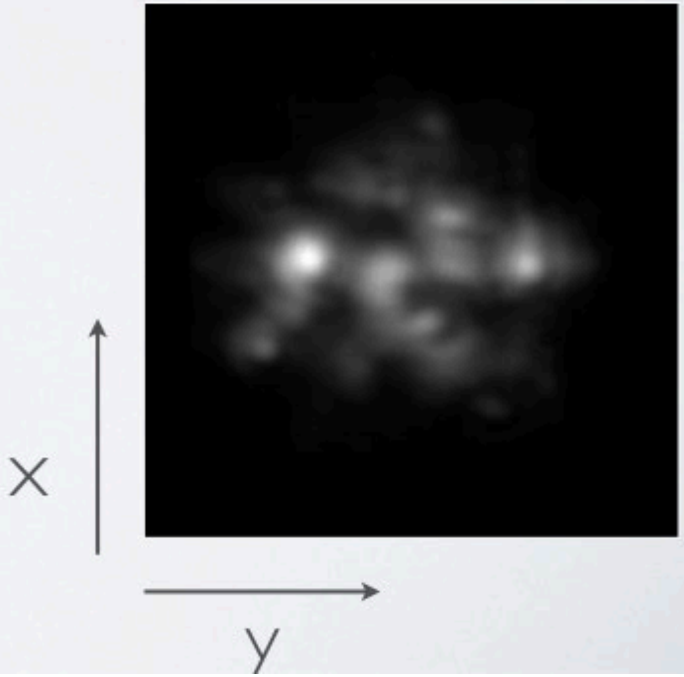
Sample cooled to max. 40 K

- typically 4.2K

TIME INTEGRATED IMAGING :

MEASUREMENT OF TIME DYNAMICS :

- ps resolution required to capture the condensate dynamics



- Disorder
- Different spots on the sample give different images corresponding to single or multiple localized condensates

Exciton polaritons
more advanced experimental setup

EXCITATION
- Pulsed OR
- Cw
- Non-resonant excitation

MICROSCOPE OBJECTIVE
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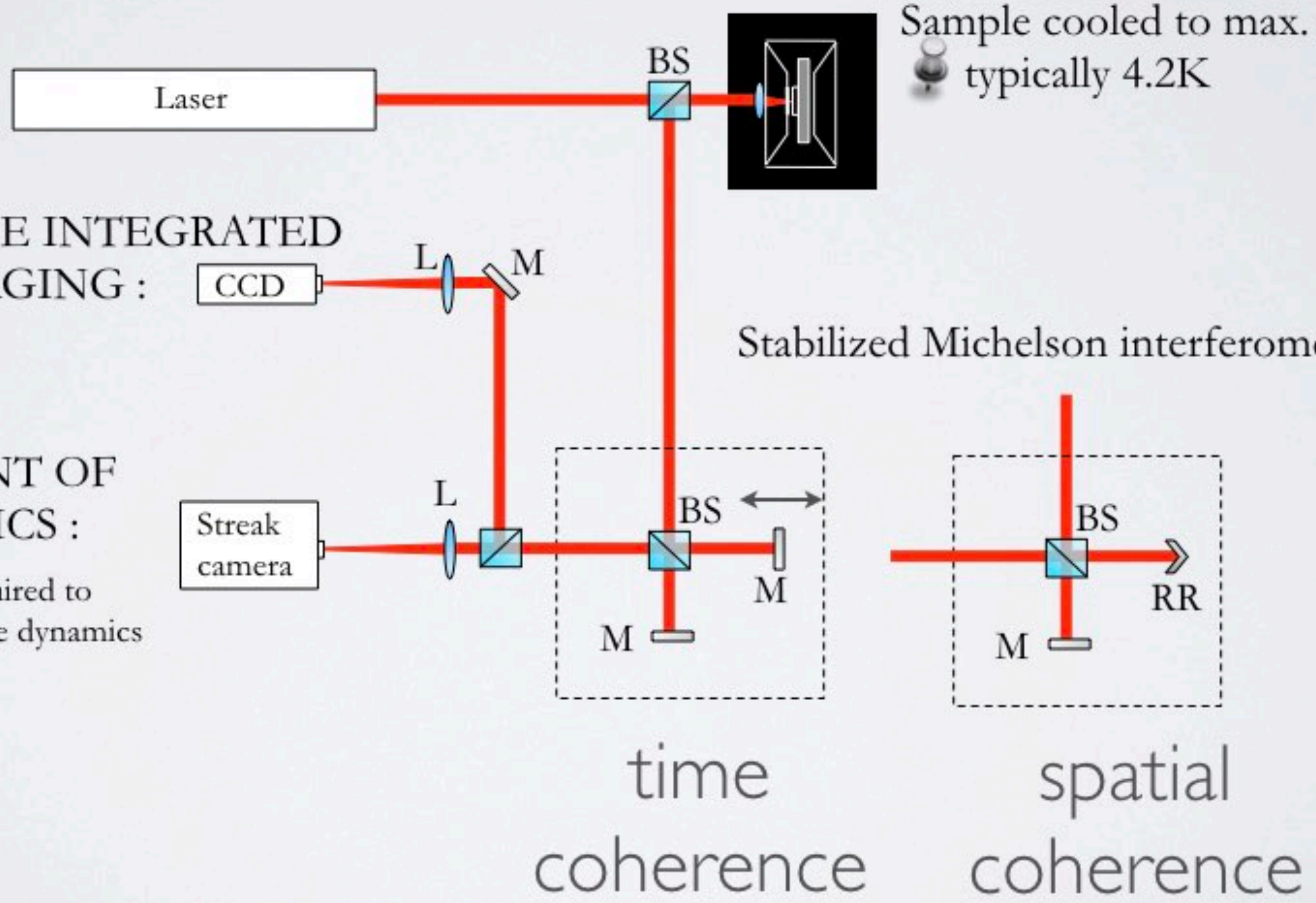
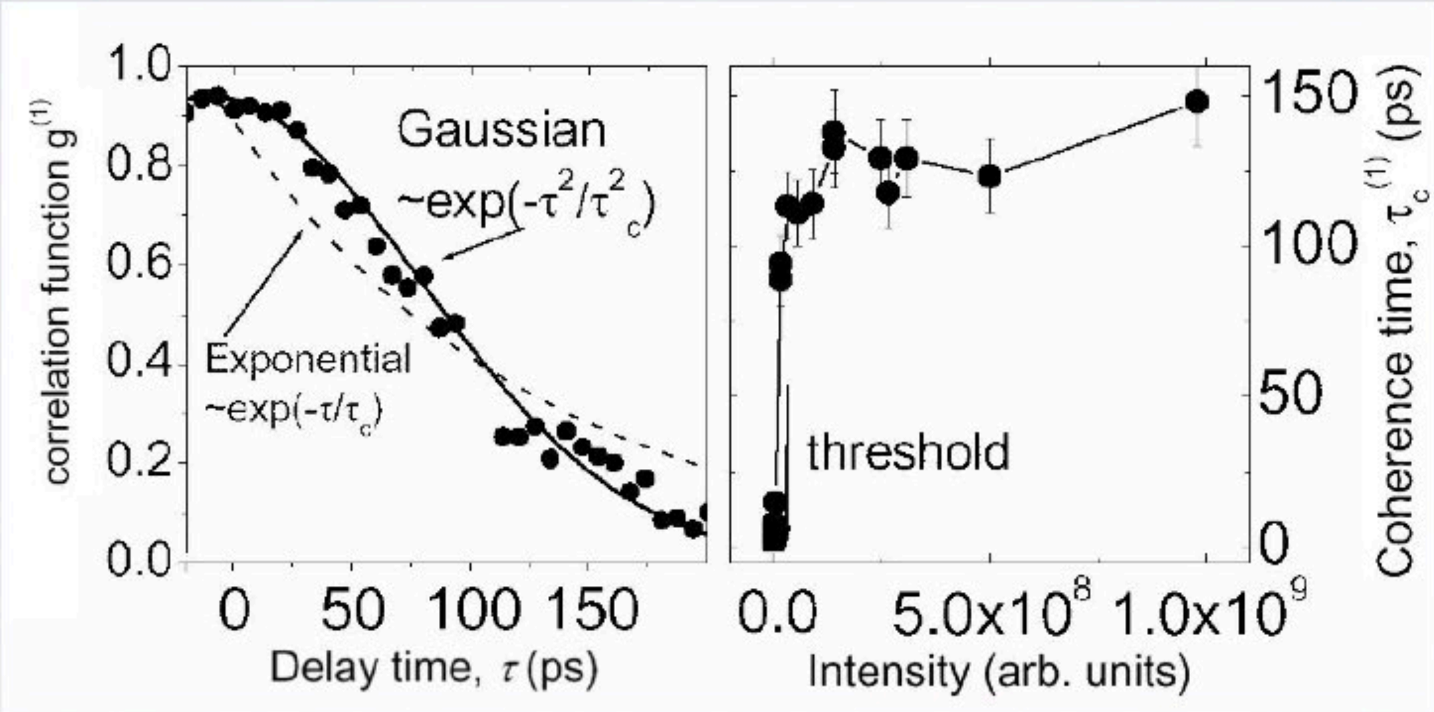


Exciton polaritons
first order correlation function



Temporal decay time ~150ps

A.P. D. Love et al, Phys. Rev. Lett. 101, 067404 (2008)

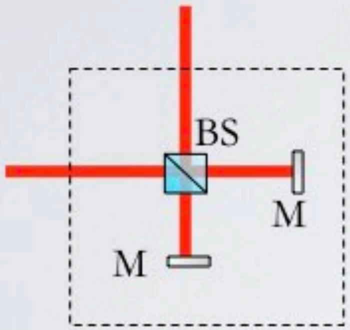
Spatial coherence

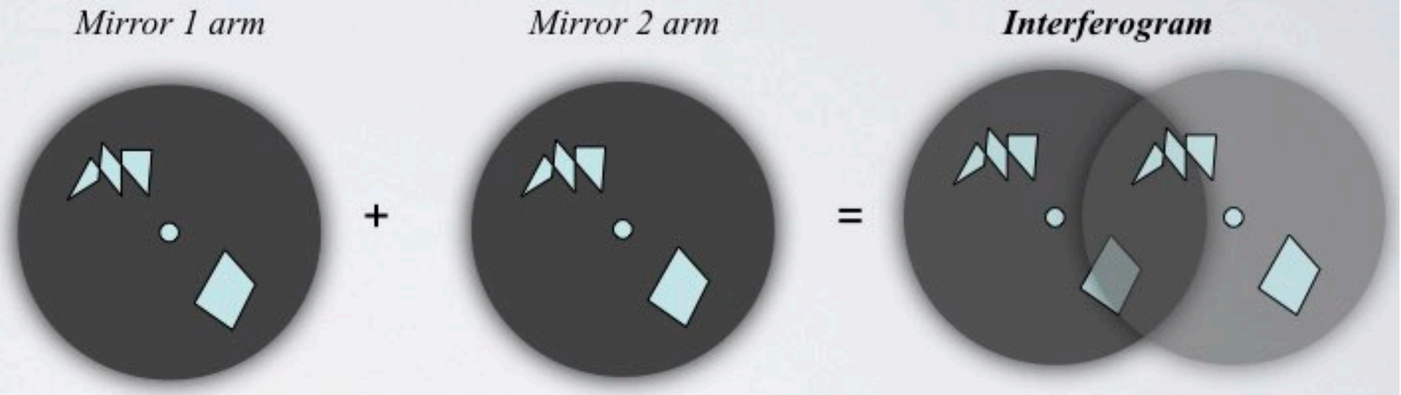
more advanced experimental setup

Operation principle

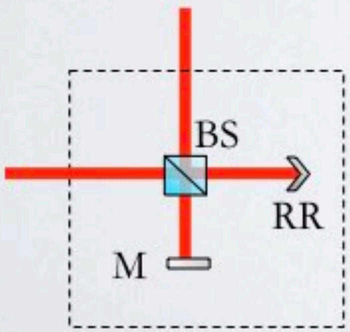
Stabilised Michelson interferometer

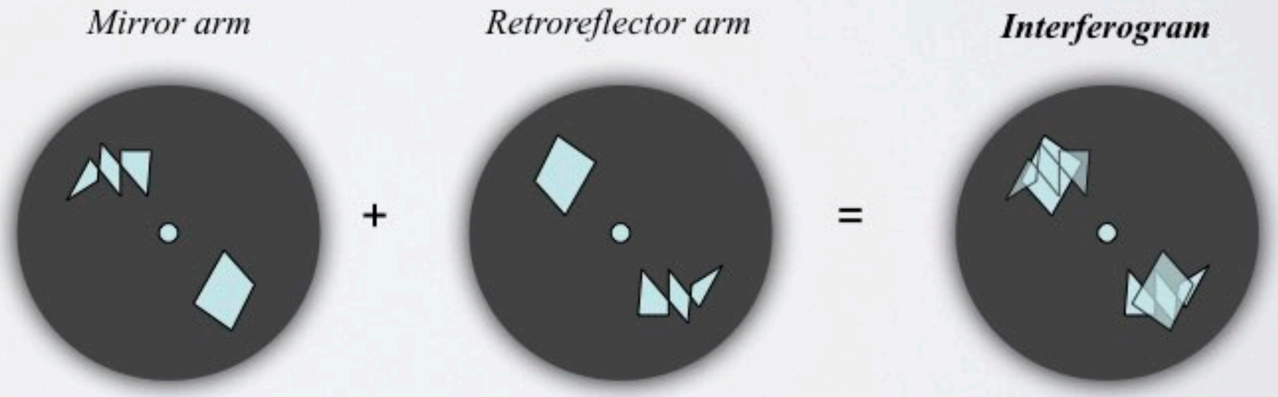
@ mirror- mirror configuration





@ mirror - retroreflector configuration

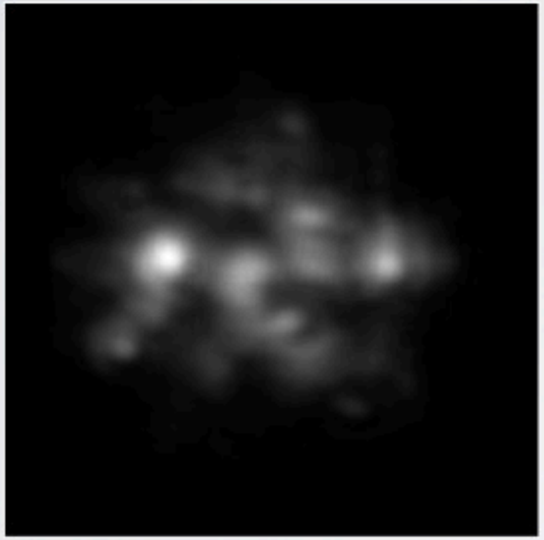




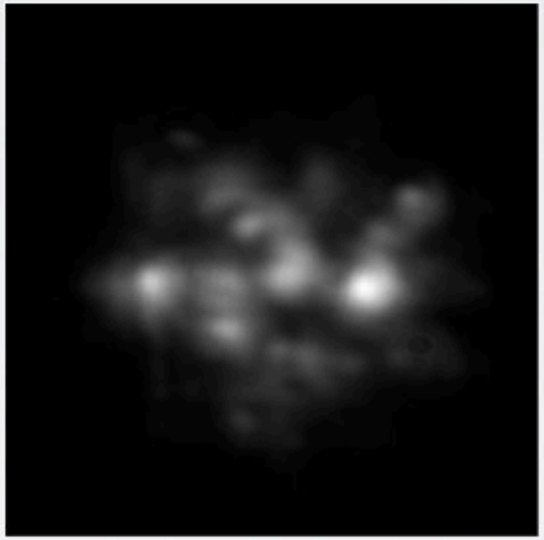
Spatial coherence
exciton-polariton condensed state

- Experimental realisation:

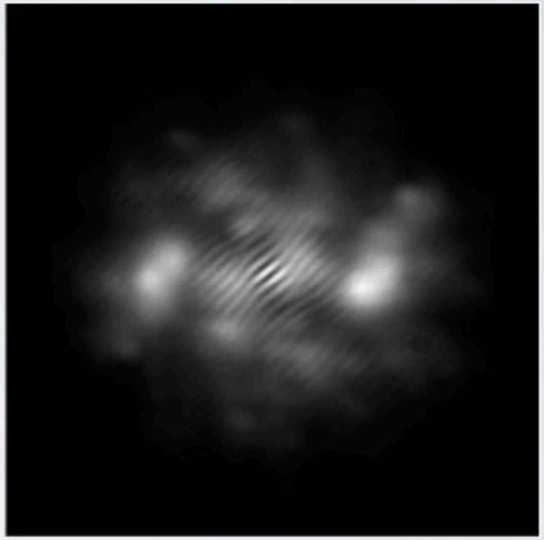
Mirror arm real space



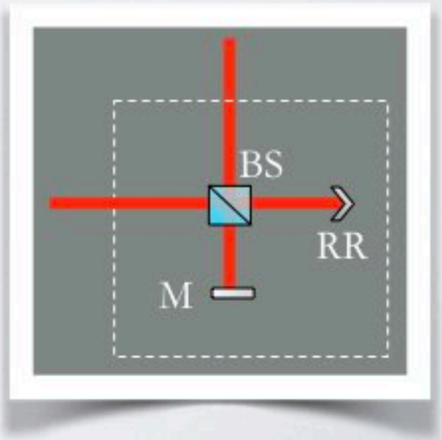
Retroreflector arm real space



Interferogram



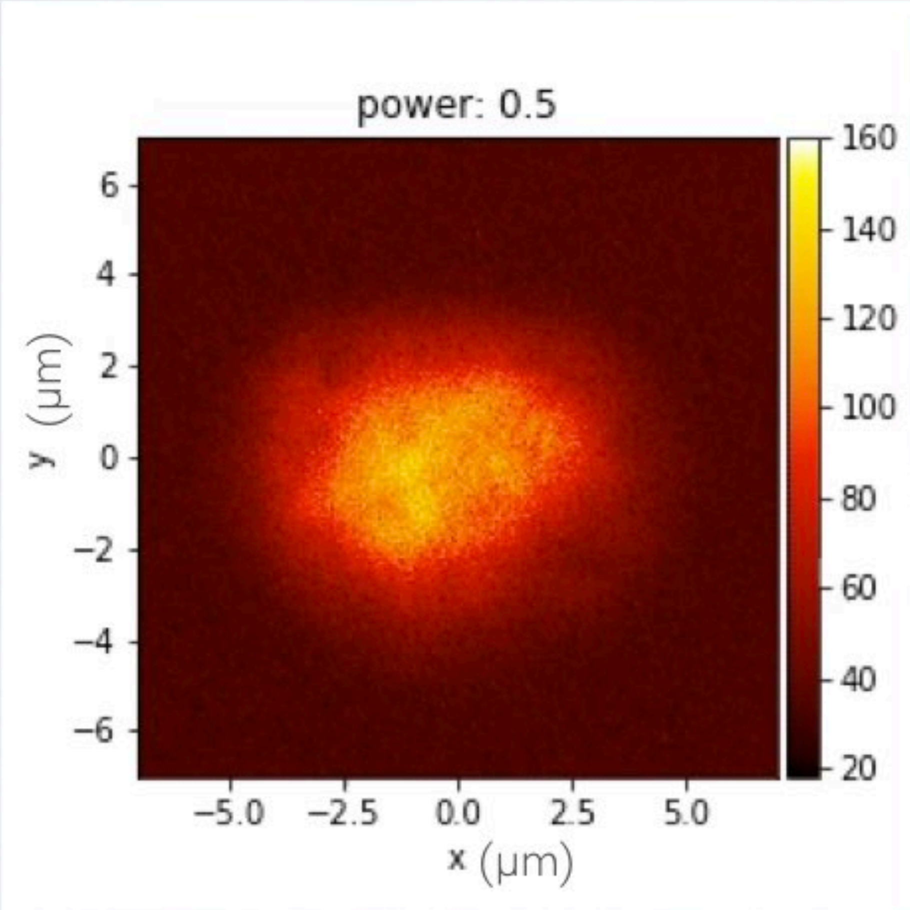
- Disorder in the sample
- Different positions will give
- different interferograms

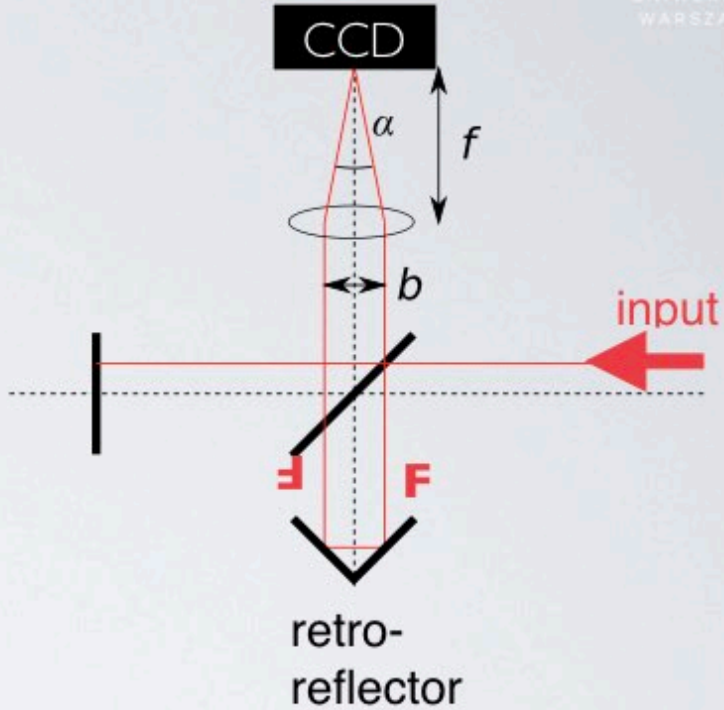


images: K. Lagoudakis, EPFL

Phase detection of polariton condensate

Modified Michelson interferometer

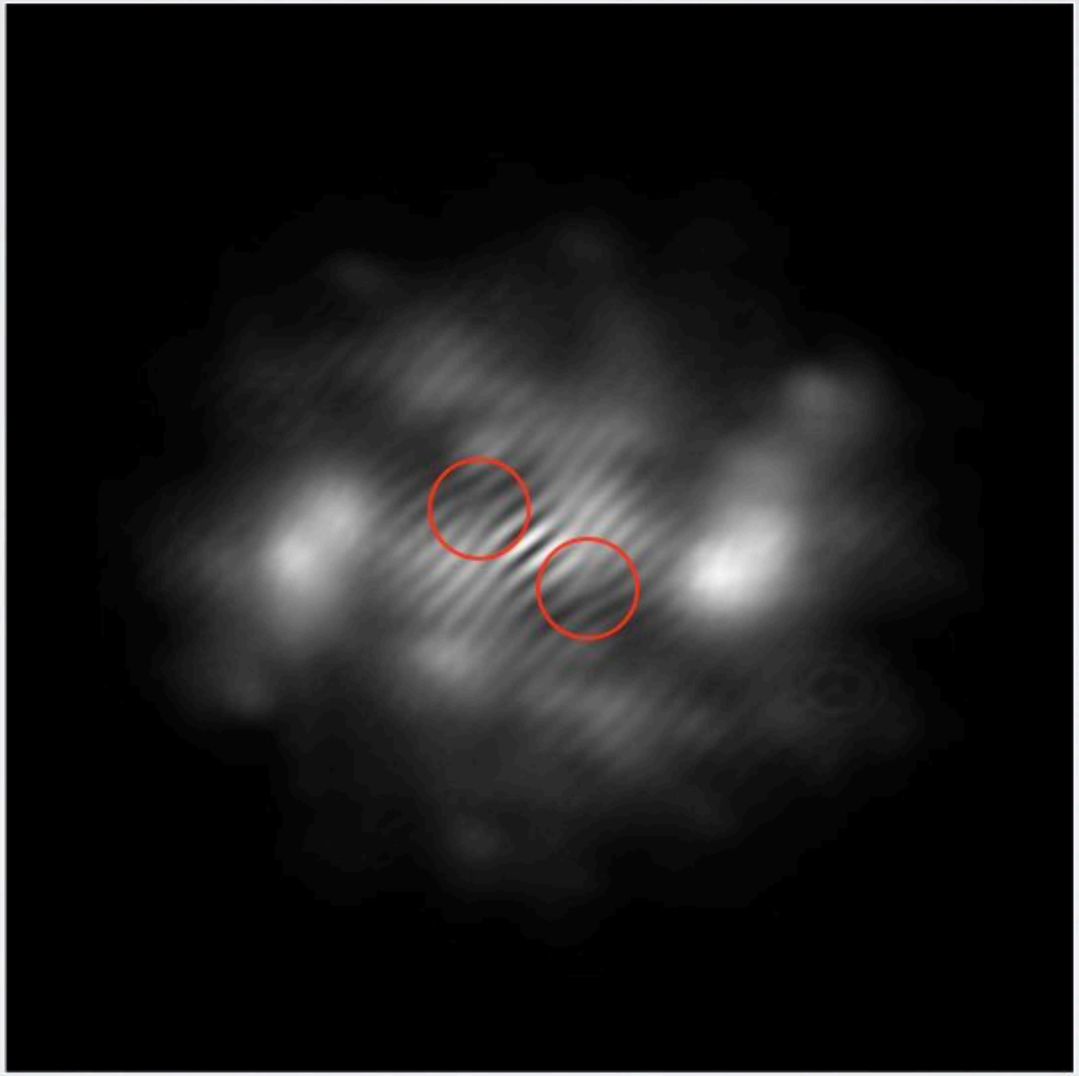




Proof of long-range coherence

Quantum vortices
detected in the interferogram

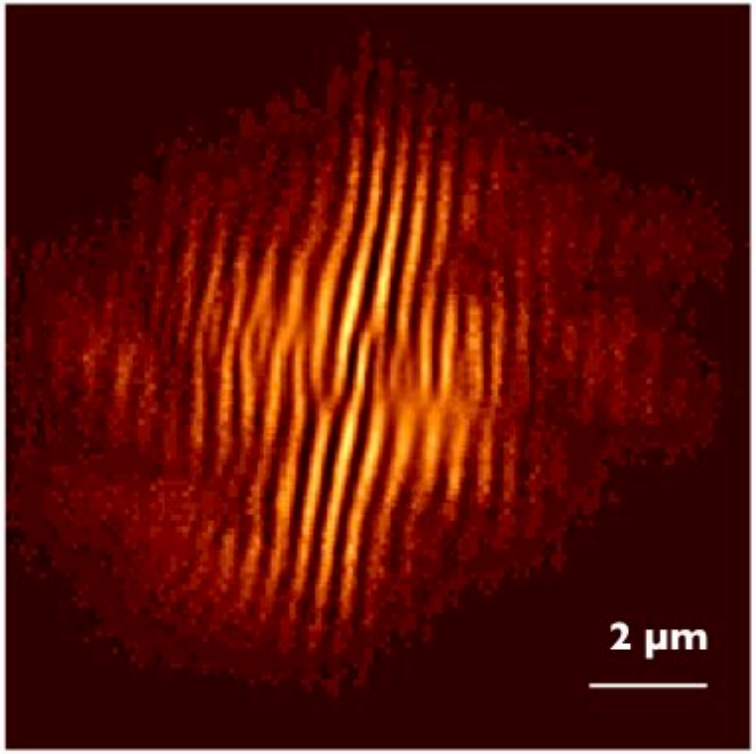
Interferogram



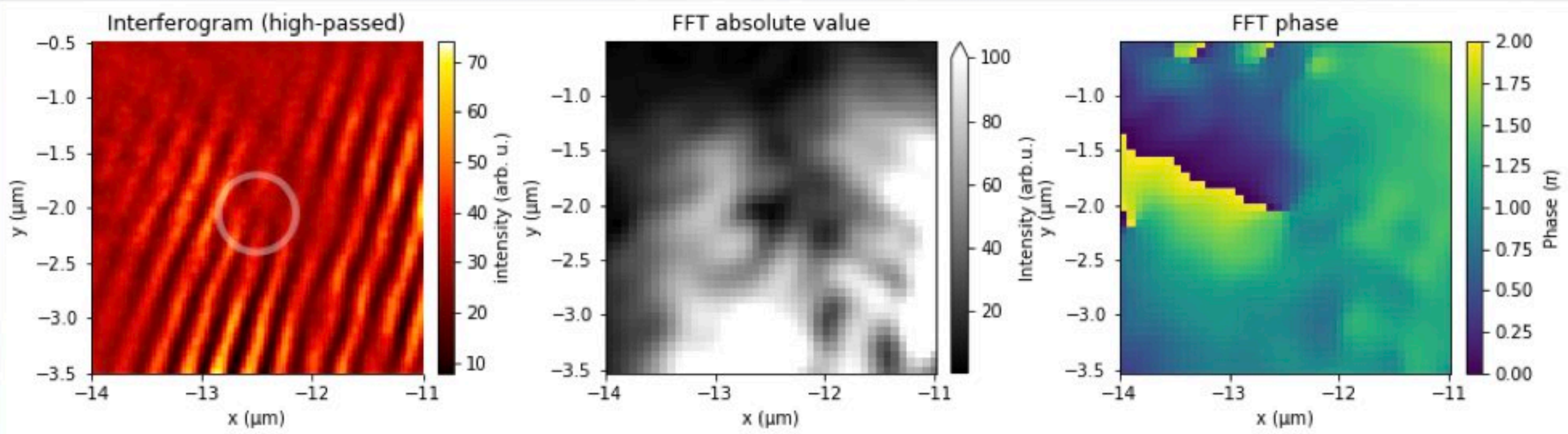
images: K. Lagoudakis, EPFL

Phase detection of polariton condensate

Phase singularity



- Fork-like dislocation in the interferogram is a signature of a **quantum vortex** with 2π phase shift.
- Phase extraction procedure - off-axes digital holography



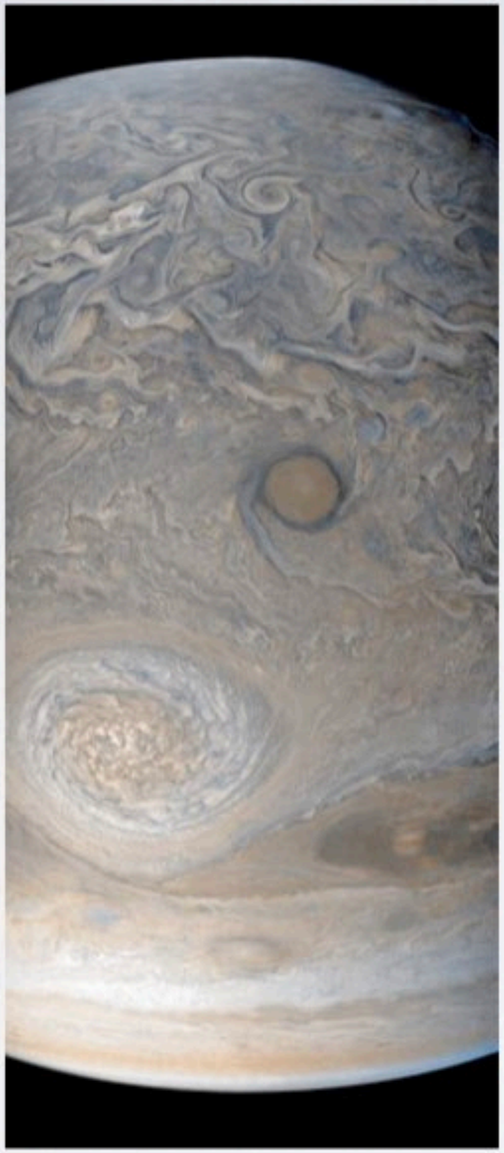
Classical vortices
classical vortices

- Many examples of vortices in nature (tornados, whirlpools etc.)

Jupiter

13.07.2017
NASA opublikowała serię pierwszych takich zdjęć Jowisza. Fotografie wykonała bezzałogowa sonda Juno, wystrzelona kilka lat temu. Przeleciała ona w odległości zaledwie kilku tysięcy kilometrów od planety.

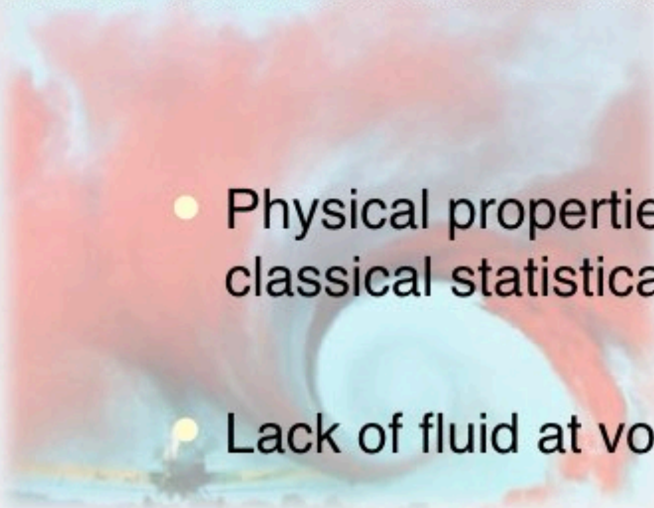
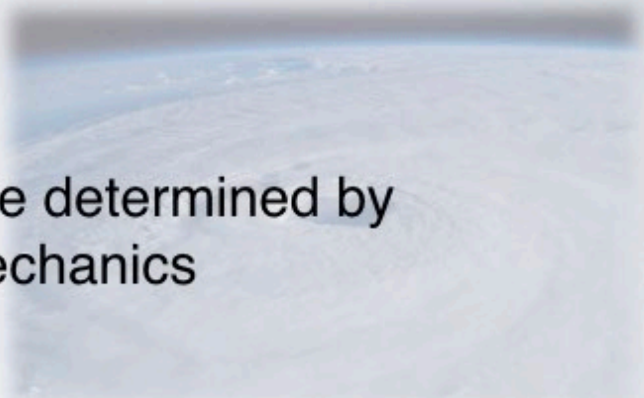
Więcej: http://wiadomosci.gazeta.pl/wiadomosci



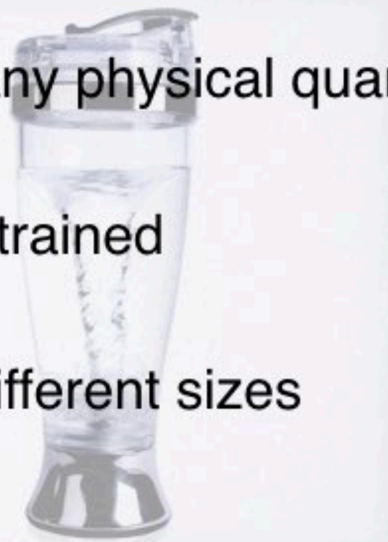
Antycyklon o średnicy większej niż średnica Ziemi

Classical vortices
classical vortices

- A vortex is a spinning, often turbulent, flow of fluid.
- Any spiral motion with closed streamlines is vortex flow.

- Physical properties are determined by classical statistical mechanics
- Lack of fluid at vortex core
- No quantisation of any physical quantity
- Vortices are unconstrained
- Vortices can have different sizes



Classical vs Quantum vortices

classical vortices

Vorticity in the classical vortex region

$$\overrightarrow{\omega} = \overrightarrow{\nabla} \times \overrightarrow{v} \neq 0$$

Angular velocity, thus a curl of a velocity change gradually with increasing the distance from the vortex

Irrotational flow

continuity equation

Time-dependent Gross - Pitaevskii equation describes the zero-temperature properties of a BEC:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) + U_0|\psi(\mathbf{r},t)|^2\psi(\mathbf{r},t)$$

multiply the time-dependent GPE with ψ^* and using

$$\boldsymbol{\nabla}^2\psi\psi^* = \boldsymbol{\nabla}(\psi^*\boldsymbol{\nabla}\psi + \psi\boldsymbol{\nabla}\psi^*)$$

leads to:

$$\frac{\partial}{\partial t}|\psi|^2 + \frac{\hbar}{2mi}\boldsymbol{\nabla}(\psi^*\boldsymbol{\nabla}\psi + \psi\boldsymbol{\nabla}\psi^*) = 0$$

comparing with the continuity equation:

$$\frac{\partial}{\partial t}n + \nabla\cdot(n\mathbf{v}) = 0$$

velocity field of the condensate:

$$\mathbf{v} = \frac{\hbar}{2mi}\frac{\psi^*\boldsymbol{\nabla}\psi + \psi\boldsymbol{\nabla}\psi^*}{|\psi|^2}$$

Irrotational flow

velocity field of the condensate:

$$\mathbf{v} = \frac{\hbar}{2mi}\frac{\psi^*\boldsymbol{\nabla}\psi + \psi\boldsymbol{\nabla}\psi^*}{|\psi|^2}$$

using $$\boldsymbol{\nabla}(fe^{i\phi}) = e^{i\phi}\boldsymbol{\nabla}f + fi\boldsymbol{\nabla}\phi e^{i\phi}$$

condensate flows with particle velocity

$$\mathbf{v} = \frac{\hbar}{m}\nabla\theta$$

fundamental equation defining relation for the superfluid velocity with the phase gradient !

The BEC behaves like a perfect fluid and can be described by the local density and local velocity only.

The flow of the BEC is rotation-less, as long as phase contains no singularity, as e.g. in a vortex.

Irrotational flow - current density

Superfluid velocity:

$$\mathbf{v} = \frac{\hbar}{m}\nabla\theta$$

superfluid velocity is therefore a gradient of a scalar thus super-flow is irrotational

$$\nabla \times \mathbf{v} = 0$$

$$\vec{\omega} = \vec{\nabla} \times \vec{v} = \frac{\hbar}{m}\vec{\nabla} \times \vec{\nabla}\phi = 0$$

since the net current of particles equals a density times a velocity:

$$\boldsymbol{j}(\boldsymbol{r}) = n_0\mathbf{v}$$

current density defines the number of particles flowing per unit area per second:

$$\boldsymbol{j}(\boldsymbol{r}) = \frac{\hbar}{m}n_0\nabla\theta$$

Quantum vortices in superfluid

phase singularities

Considering the flow along a closed tube. Circulation:

$$\Gamma = \oint_C \mathbf{v}\mathrm{d}\mathbf{l} = \frac{\hbar}{m}\oint_C \boldsymbol{\nabla}\boldsymbol{v}\mathrm{d}\mathbf{l} = \frac{\hbar}{m}\int_A \boldsymbol{\nabla}\times\boldsymbol{\nabla}\phi = 0$$

The circulation must vanish for a non-singular phase.

The motion of the condensate must be irrotational - as if a superfluid could not be put into rotation.

If we allow for phase singularity !

$$\Gamma = \oint \mathbf{v}\mathrm{d}\mathbf{l} = \frac{\hbar}{m}\oint \boldsymbol{\nabla}\phi\mathrm{d}\mathbf{l} = \frac{\hbar}{m}(\phi_2 - \phi_1) = 2\pi l\frac{\hbar}{m} = l\frac{h}{m}$$

total phase difference, $\Delta\phi$, **phase winding**, of the wave-function on going around the path and returning to the initial point must be equal to $2\pi l$

Quantum vortices in superfluid

phase singularities

Macroscopic rotation of a superfluid is impossible.

But a vortex in the form

$$\mathbf{v} = \frac{nh}{m}\frac{1}{2\pi r}\mathbf{e}_\phi$$

vector in the azimuthal direction in polar coordinates

satisfies the irrotational condition: $\nabla \times \mathbf{v} = 0$

In the vicinity of the vortex core:

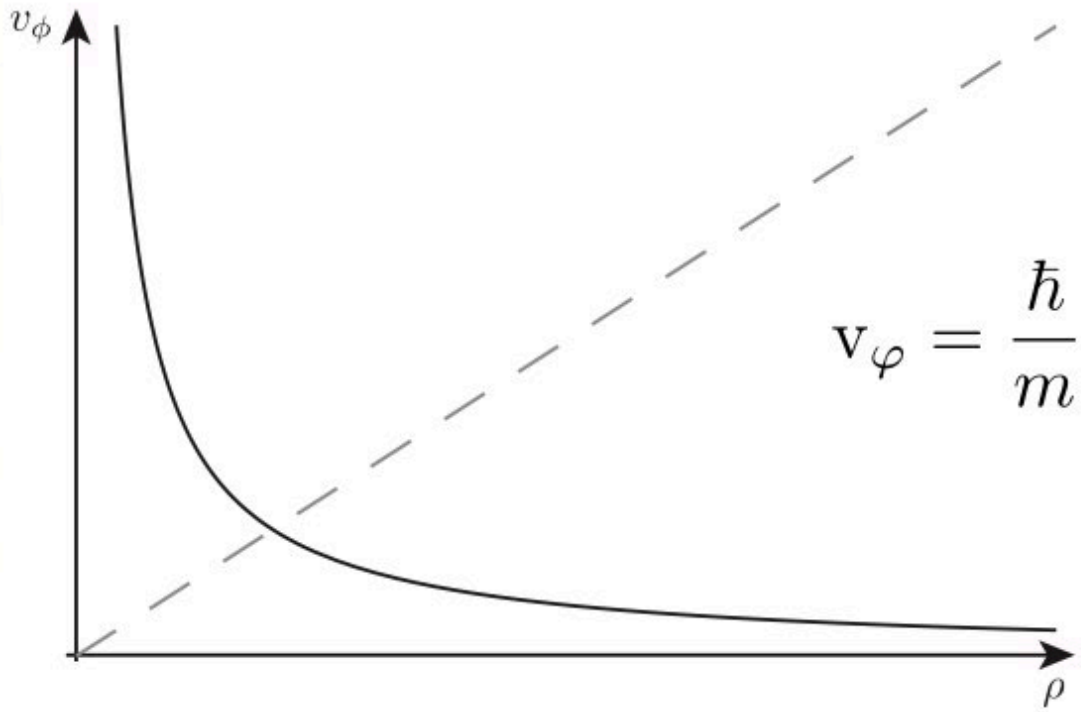
$$\vec{\omega} = \vec{\nabla} \times \vec{v} = \frac{\hbar}{m}\vec{\nabla} \times \vec{\nabla}\phi = \hat{z}\frac{\hbar}{m}l \cdot \delta^2(\rho)$$

$$\rho = (x, y)$$

In a rotating condensate, macroscopic rotation is build up from a vortex array.

Quantum vortices in superfluid

phase singularities



$$\mathrm{v}_\varphi = \frac{\hbar}{m}\nabla\phi = \frac{\hbar}{m}\frac{1}{\rho}\frac{\partial}{\partial\varphi}\phi = \frac{\hbar}{m}\frac{l}{\rho}$$

Azimuthal velocity of a vortex. The velocity field for a normal rotating fluid behaves like a rigid body, where the azimuthal velocity increases linearly with increasing distance from the symmetry axis. A superfluid shows a non-intuitive velocity pattern, where the azimuthal velocity diverges for decreasing distance from the rotation axis. In order not to let the kinetic energy of the system diverge, the density of the superfluid must go to zero for $\rho \to 0$.

Quantum vortices in superfluid

structure of a single vortex

In cylindrical coordinates: $\psi(\mathbf{r}) = f(\rho, z)e^{il\varphi}$

G-P equation takes the form:

$$-\frac{\hbar^2}{2m}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial f}{\partial\rho}) + \frac{\partial^2 f}{\partial z^2}\right) + \frac{\hbar^2 l^2}{2m\rho^2}f + V(\rho, z)f + U_0 f^3 = \mu f$$

Kinethic energy due to the azimuthal velocity field of a vortex $\frac{\hbar^2 l^2}{2m\rho^2}$

Cross over between small and large distance behaviour, thus size of a vortex: $-\frac{\hbar^2}{2m\xi^2} = U_0 n$

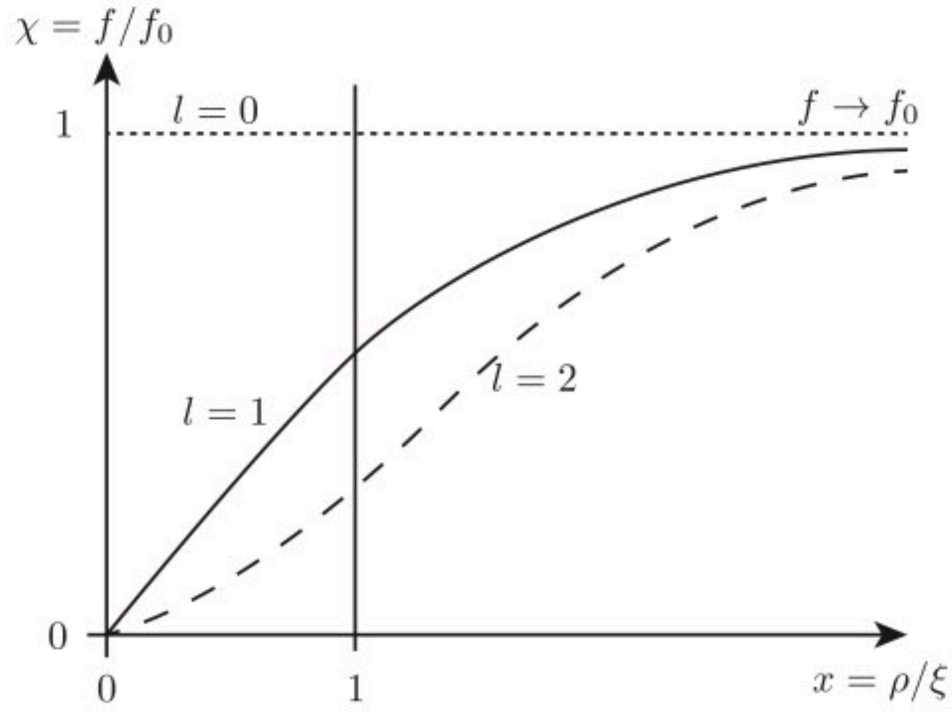
kinethic energy equals interaction strength

Healing length:

$$\xi \approx \sqrt{\frac{\hbar^2}{2mU_0 n}}$$

Quantum vortices in superfluid

structure of a single vortex



$$x = \frac{\rho}{\xi}$$

$$\chi = \frac{f}{f_0}$$

Density profile of a vortex. Shown are the density profiles for the system without a vortex ($l = 0$), and for a singly and doubly charged vortex ($l = 1, 2$). The short-range behavior is given by $f \propto \rho^l$, while the asymptotic behavior of the wave function always approaches the wave function of the unperturbed system. The cross over between the short-range and the long-range behavior takes place around the healing length $\rho = \xi$.

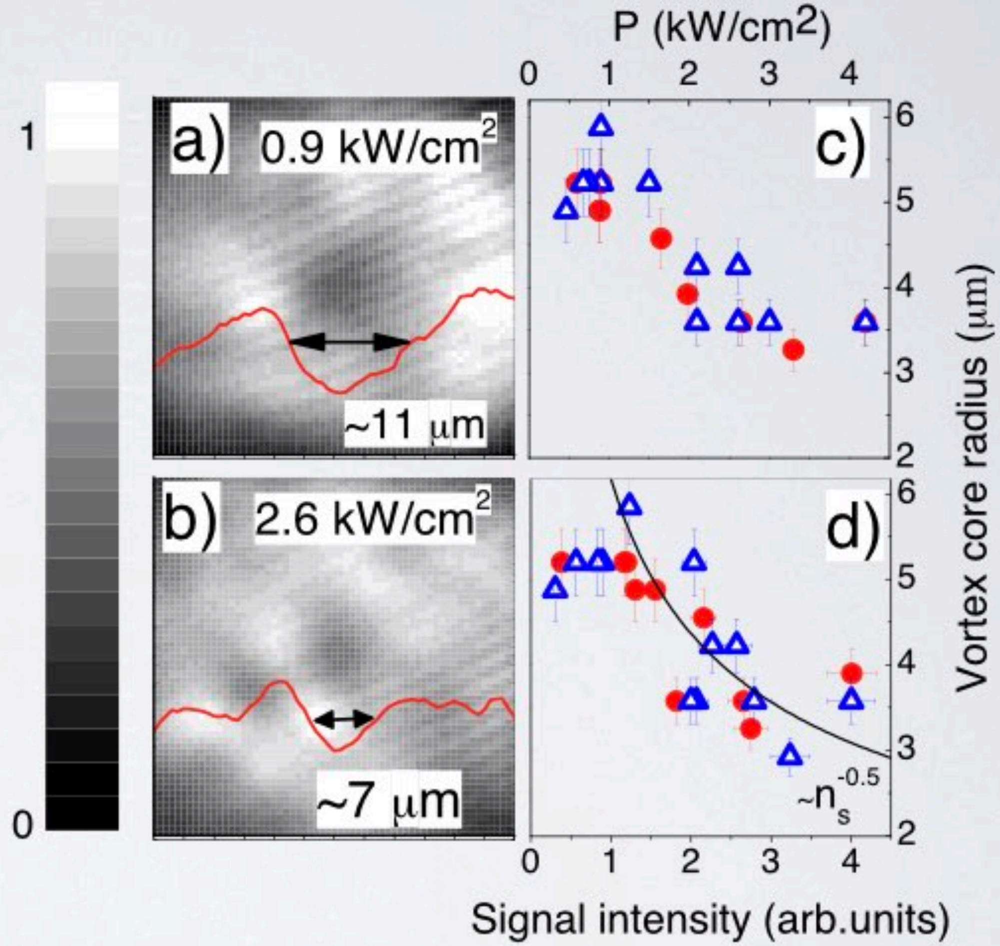
Quantum vortices in superfluid

phase singularities

- Size of the vortex core - the particle density has to increase from zero to the bulk value over the distance given by the healing length:

$$\xi = \frac{\hbar}{\sqrt{2mE_{int}}}$$

- In atomic BEC the size depends on the atom size $\sim 10^{-8}$cm $= 10^{-4}$μm



D. Krizhanovskii, et al. Phys. Rev. Lett. 3, 126402 (2010)

Quantum vortices - summary

phase singularities ⟶ response of the system to the perturbation

VORTICES ARE THE EXCITATIONS OF THE SYSTEM

- zero particle density in the vortex core
- quantum vortices have a quantized phase

$$\delta\phi = 2\pi \cdot l$$ phase winding is quantized

$$\overrightarrow{v} = \frac{\hbar}{m}\overrightarrow{\nabla}\phi$$ velocity of the superfluid is proportional to the phase gradient

ℓ - topological vortex charge

Quantum vortices
characteristics

Scalar quantum fluid vortices

	Full Vortices	Half Vortices
Phase shift:	2π	π
Polarisation rotation:	0	π
Density @ core:	Minimum	Minimum in σ^+ Maximum in σ^-
Quantum numbers:	$m=1,2,\ldots$	$(k\,,\ m)=(\pm½,\pm½)$
Observation:	Superfluids Condensates Superconductors Exciton-polaritons	High T_c superconductors Exciton-polaritons

K. Lagoudakis et al., Nat. Phys. 4, 706 (2008)

Phase detection of polariton condensate

Modified Michelson interferometer