Quantum Key Distribution

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No cloning - states (keys) are secret

$$|\psi\, 0\rangle \longrightarrow |\psi\, \psi\rangle \text{ ?}$$

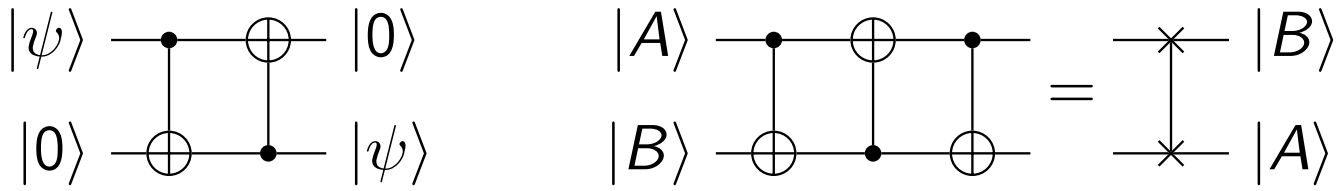
NO! $|00\rangle \rightarrow |00\rangle$, $|10\rangle \rightarrow 11\rangle$ so

$$(|00\rangle + |10\rangle)/\sqrt{2} \rightarrow (|00\rangle + |11\rangle)/\sqrt{2} \neq (|00\rangle + |11\rangle + |01\rangle + |11\rangle)/2$$

BUT we can transmit the state by swapping or ...

Distribution by SWAP

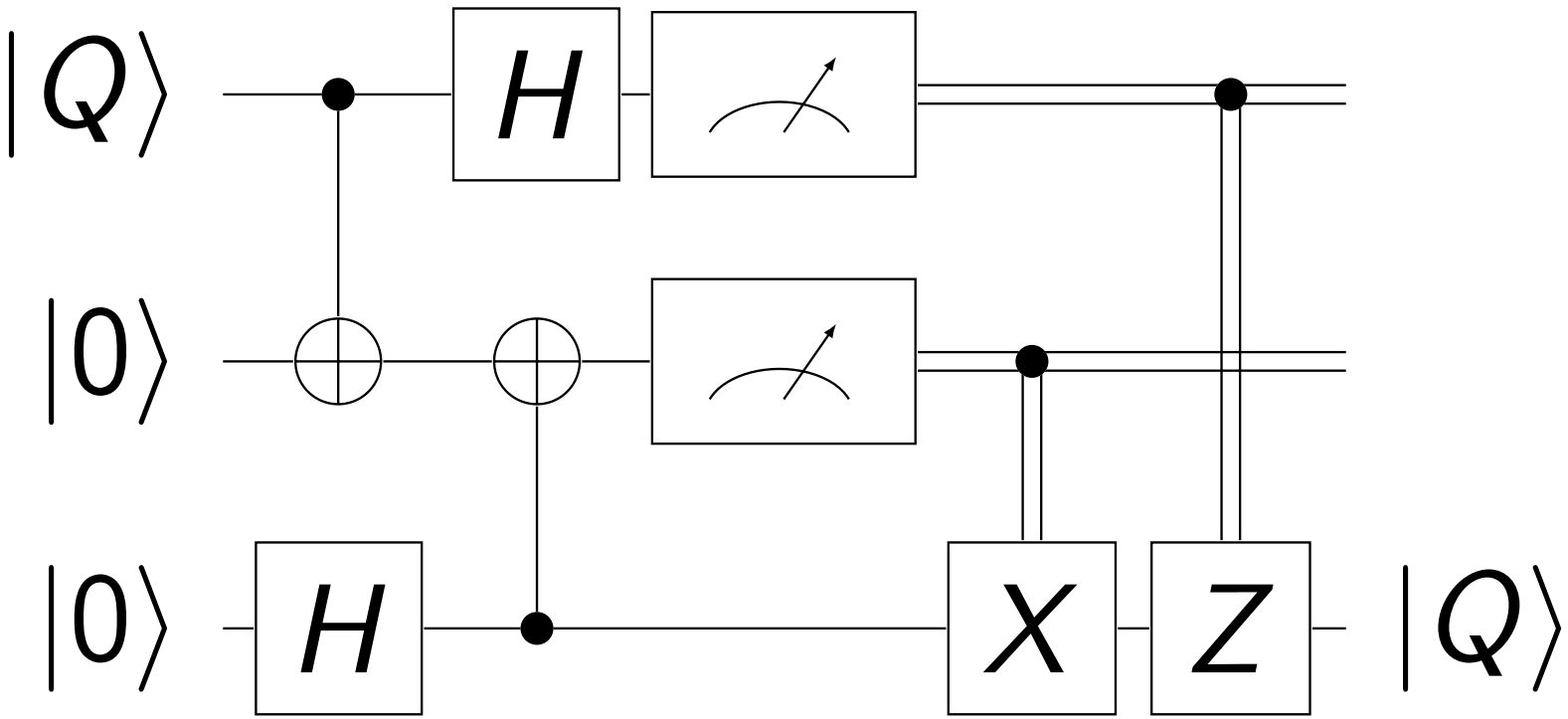
Swap with $|0\rangle$ and full swap (self reverse)



TELEPORTATION $|Q\rangle \rightarrow |R\rangle$

- $|Q\rangle = \alpha|0\rangle + \beta|1\rangle$
- $CX|QA\rangle$ with $|A\rangle = |0\rangle$ to $\alpha|00\rangle + \beta|11\rangle$
- rotate Q with H to
  $(\alpha|00\rangle + \alpha|10\rangle + \beta|01\rangle - \beta|11\rangle)/\sqrt{2}$
- measure Q:
  $0 \rightarrow |A\rangle = \alpha|0\rangle + \beta|1\rangle$, $1 \rightarrow |A\rangle = \alpha|0\rangle - \beta|1\rangle$
- $|R\rangle = |0\rangle$ rotated by H to $(|0\rangle + |1\rangle)/\sqrt{2}$
- Apply $CX|RA\rangle$ to
  $0: \ (\alpha|00\rangle + \beta|01\rangle + \alpha|11\rangle + \beta|10\rangle)/\sqrt{2}$
  $1: \ (\alpha|00\rangle - \beta|01\rangle + \alpha|11\rangle - \beta|10\rangle)/\sqrt{2}$
- Measure A (outcome notation AQ):
  $00 \rightarrow \alpha|0\rangle + \beta|1\rangle$, $10 \rightarrow \alpha|1\rangle + \beta|0\rangle$
  $01 \rightarrow \alpha|0\rangle - \beta|1\rangle$, $11 \rightarrow \alpha|1\rangle - \beta|0\rangle$
- Apply to R: X if $A = 1$, Z if $Q = 1$ (order arbitrary)

Circuit



Alternative circuits