Lorentz Invariance of Vacuum in the Closed Time Path Formalism
Manuscript DW10822 by Adam Bednorz

The author claims there is not a strict proof that in the zero-temperature limit the closed-time-path Green's functions are Lorenz invariant. He demonstrates in Sec. III that the finite temperature path integral can be computed using any path in the complex time plane that begins at t_i and ends at $t_i - i\beta$.

The proof that the limit $\beta \to \infty$ gives Lorentz invariant results is contained mostly in Sec. IV-C and IV-D, where the author resorts to analysis of the propagator and vertices. The propagator in this limit is given in Eq. (42) with the times contour-ordered in the complex plane. The author argues that this is Lorentz invariant.

Sec V argues that the expressing the propagators in the energy-momentum representation will not give a satisfactory proof. For this the author uses the Keldysh contour, which gives the propagator in Eq.(49).

Comment

Hundreds of calculations in finite temperature field theory have been published. To my knowledge, none of these calculations have ever conflicted with Lorentz invariance in the limit $\beta \to \infty$.

Simple Proof

Instead of using the Keldysh contour in Sec V, it is far better to use the "symmetric" contour in which the time runs along the real axis and then returns anti-parallel to the real axis but shifted down by $\beta/2$. I believe this form of the propagator is used in Refs. 13 and 14 (and probably in many of the other references). It is described in the the book *Thermal Field Theory* by Michel Le Bellac, Cambridge Univ Press, 1996. From page 55 of Le Bellac the matrix form the propagator is

$$G(p) = \begin{pmatrix} \frac{i}{p^2-m^2+i\epsilon} & 0 \\ 0 & \frac{-i}{p^2-m^2-i\epsilon} \end{pmatrix} + \frac{2\pi\delta(p^2-m^2)}{e^{\beta|p^0|}-1} \begin{pmatrix} 1 & e^{\beta|p^0|/2} \\ e^{\beta|p^0|/2} & 1 \end{pmatrix}$$

When $\beta \to \infty$ the second contour decouples from the first contour. The vertices do not couple the two contours. This is all that is necessary for the proof.

Conclusion

This paper does not provide anything new or useful to the readers of Physical Review. I do not recommend publication.