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Deconstruction

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Contents

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Chapter 1

Introduction

The Standard Model (SM) provides a microscopic description of three fundamental forces of nature: the strong, the weak and the electromagnetic interactions [1]. Since 1967, when it was proposed by Weinberg (and, independently by Salam), its numerous predictions have been confirmed in accelerator experiments. Recently, experiments LEP and LEP2 have established its applicability up to energies of order 100 GeV. Only one key ingredient of the SM - an elementary scalar particle called the Higgs boson - has not been discovered. Nevertheless, the electroweak precision measurements strongly suggest the existence of such a scalar with mass not greater than 1 TeV, which means that the Higgs boson should be accessible to the next generation of accelerators.

Despite the experimental success, it has been known for a long time that the SM cannot be the ultimate fundamental theory of short distance physics. First of all, it does not include gravity. In the low energy world gravitational interactions are much weaker than the other three, therefore they do not affect experiments performed in the present accelerators. However, at energies of the order of the Planck scale, $M_{PL} \sim 10^{18}$ GeV, the strength of gravity should become comparable to the strength of the SM interactions. Thus the Planck scale determines a maximal possible range of applicability of the SM. But it is very likely that the SM ceases to be valid at much lower energies. Firstly, the peculiar quantum numbers of the quarks and leptons suggest that the $SU(3) \times SU(2) \times U(1)$ group is a part of a larger group, broken at some high scale (called the unification scale) to the SM group. Renormalization group (RG) evolution of the gauge couplings points toward the unification scale somewhere between 10^{13} and 10^{16} GeV.

Secondly, the SM suffers from the hierarchy problem. More generally speaking, in generic quantum field theories it is not natural to have two largely different mass scales. This is because quantum corrections tend to erase the scale difference, unless there is some symmetry which protects it. In the SM no symmetry protects the Higgs boson mass. The technical consequence of this fact is that the Higgs boson mass receives one-loop correction that are quadratically divergent. It should be stressed that this is not an inconsistency of the SM itself. The SM is a renormalizable theory, which means that divergences can be simply removed by a redefinition of the parameters of the theory. However we believe that the SM is just an effective theory derived from a more fundamental one (e.g. from a unified theory) and is valid only below some cutoff M_*. In such a case, the quadratic divergence in the computation of the Higgs boson mass is a signal that the natural order of magnitude of this mass parameter is M_*. As the Higgs boson mass should be lighter than 1 TeV, the TeV scale is expected to be the cutoff scale to the SM. The exciting consequence of this fact is that new physics beyond the SM could be visible in the next generation of accelerators, including the Large Hadron Colider (LHC) which starts operating in 2007.

Thirdly, the SM physics could in principle explain why the excess of baryons over anti-baryons has appeared in the early universe (the problem of the so-called baryogenesis). But concrete mechanisms require a fairly light Higgs boson, already excluded by the LEP data. Thus the very fact that we exist is the evidence in favour of physics beyond the SM. Baryogenesis does not point at one concrete scale where this physics should take place, but the idea seems to work well in the context of unified theories.

There are more unanswered questions in the SM, including that about the origin of the hierarchy of fermion masses and of the structure of the CKM matrix, about the origin of the electroweak breaking (of the negative mass parameter of the Higgs field), why there are three generations and so on. These problems are rather of aesthetical nature. It is conceivable that these peculiar structures of the SM are simply consequences of some random physics at some high scale. Also, these problems do not point at any particular scale at which the SM should break down. Nevertheless, it is the common lore that the ultimate theory of fundamental interactions should contain as few arbitrary parameters as possible and therefore all the parameters of the SM should be determined by the dynamics of the underlying theory.

Actually, in recent years, experimental evidence in favour of physics beyond the SM has grown considerably. Strangely enough, this evidence comes in most part not from accelerators. The first thing to mention is the discovery of neutrino oscillations. It is now rather well established that these oscillations imply the existence of neutrino masses of order $10^{-2} - 10^{-1}$ eV. Without introducing new fields, the guiding principles of the SM - the gauge symmetry and renormalizability - do not allow for a neutrino mass. One widely accepted way to account for the experimental data is to extend the SM by adding new singlet neutrinos ν_R with a large Majorana mass M, coupled to the SM neutrinos ν_L via ordinary Yukawa terms $\sim H\nu_L\nu_R$. Then the so called see-saw mechanism implies the existence of light neutrinos with masses $m_\nu \sim v_H^2/M$, where v_H is the vev of the SM Higgs boson. The correct order of magnitude for neutrino masses is obtained for $M \sim 10^{15}$ GeV.

This is another, apart from the approximate gauge coupling unification, strong indication for the existence of a high scale at which the SM is replaced by a more fundamental theory.

More evidence is provided by recent astrophysical data. A consensus has been reached that majority of non-relativistic matter in the universe is non-baryonic (the so-called dark matter), that is, it cannot be accounted by any of the SM fields. Even more puzzling discovery is that the total energy of the universe is today dominated by some unknown form of matter with negative pressure. This could be a tiny cosmological constant or a slowly evolving scalar field (quintessence). If the latter hypothesis turns out to be true, it would require the existence of a very light scalar particle coupled very weakly to the known particles. It has been also claimed that there is the evidence for a slow time variation of the electromagnetic coupling constant. If this were confirmed, this would also point towards a slowly evolving scalar field.

For all these reasons searching for physics beyond the SM has become one of the main activities in high energy physics. From the theoretical side, the guiding principle is that the new physics should solve the hierarchy problem. This is because, of many other motivations we have mentioned, only the hierarchy problem is an indication that new physics should appear already at TeV energies. Generally, one can distinguish two strategies. One relies on introducing new symmetries that stabilize the Higgs sector, by forbidding quadratically divergent loop corrections to the Higgs boson mass. The examples of such symmetries are supersymmetry, conformal symmetry or higher dimensional Lorentz invariance. The other strategy assumes that at energies of order 1 TeV some strongly coupled physics enters. In this context one should mention technicolor and large extra dimensions.

To date, the best motivated extension of the SM is the Minimal Supersymmetric Standard Model (MSSM). Roughly speaking, the MSSM is based on the SM to which one more organizing principle - supersymmetry - has been added. Supersymmetry requires that each SM field has its superpartner, that is, the field in the same representation of the gauge group but with different statistics. In particular, the gauge fields are accompanied by spin 1/2 gauginos, while the SM fermions come in pairs with sfermions - squarks and sleptons. If supersymmetry were an exact symmetry, superpartners would have the same masses as the corresponding SM particles, which is of course experimentally excluded. Breaking of supersymmetry in the MSSM is parametrized by 105 new parameters beyond those of the SM, the so-called soft breaking terms. This is the main source of uncertainty in physical predictions of the MSSM. The main consequence of supersymmetry breaking is that the superpartners acquire masses of order $100 - 1000$ GeV.

The presence of supersymmetry solves the hierarchy problem of the SM. More precisely, it does not provide an explanation why the electroweak scale is so much smaller than the cutoff scale, but it renders the scale difference stable against quantum corrections. In the limit of exact supersymmetry the contribution of the superpartners cancels the contribution of the SM fields in the loop corrections to the Higgs boson mass. Supersymmetry is broken in the MSSM but the breaking is soft (soft means that it does not reintroduces quadratic divergences). In consequence the cancellation is not exact and the result is of the order of the mass splitting between the SM fields and the superpartners. This is why superpartners must have masses at most of order TeV; otherwise the MSSM would not provide a solution to the hierarchy problem.

However the MSSM creates problems of his own. The most serious problem is certainly the fact that almost all volume of the MSSM parameter space is already excluded by experimental data! This is because, in general, superpartner couplings are new sources of flavor and CP violation. The soft breaking parameters must therefore have very special values, otherwise one would have already found deviations from the SM predictions in various precision measurements. In particular, the sfermion mass matrices must be diagonal to a high degree of precision (in the fermion mass eigenstate basis). One should also mention the so-called μ-problem. The problem is that breaking the electroweak symmetry in the MSSM requires the supersymmetric mass parameter μ of the Higgs bosons, $W \sim \mu H_U H_D$, to be less than 1 TeV. Otherwise, in order to match the experimental value of the Z-boson mass, $m_Z = 91$ GeV, cancellation between μ and the soft breaking Higgs boson masses would have to occur. Such cancellation, called fine-tuning, is considered unlikely. There is also a lower bound on the μ-paremeter, $\mu \gtrsim 100$ GeV, derived from the bounds on superpartner (charginos and neutralinos) masses. Hence, without any apparent theoretical reason, the μ-parameter has to be of the same order of magnitude as the supersymmetry breaking scale.

Last but not least, no superpartners have been yet discovered, despite tremendous experimental efforts. This is slowly becoming a problem. Present bounds on superpartner and the lightest Higgs boson masses imply that certain amount of fine-tuning in electroweak symmetry breaking is inevitable in the MSSM. Thus the shape of physics beyond the SM is still an open question and it is important to investigate other possible solutions to the SM hierarchy problem. This is the idea standing behind this thesis.

An alternatitive to softly broken supersymmetry that has received considerable attention is the idea that the space-time has more than four dimensions. A very strong motivation to consider such theories is provided by String Theories. Recent research is however dominated by a more phenomenological bottom-up approach. In other words, theories with extra dimension are constructed ad-hoc in order to solve the hierarchy problem or some other problems of the SM. The question about the fundamental origin of these theories is usualy not addressed.

One concrete realization of this idea is provided by theories in which the effects of extra space-time dimensions become visible already at the TeV scale. Two approaches that have received most attention are large extra dimensions [2] of Arkani-Hamed, Dimopoulos and Dvali and warped extra dimensions [3] of Randall and Sundrum. Both approaches attempt to explain the huge gap between the Planck scale and the electroweak scale by means of gravitational interactions propagating in extra dimensions. The technical hierarchy problem (in the sense of divergent corrections to the Higgs mass) is circumvented simply because at the TeV energies physics becomes strongly coupled. Beyond that scale new physics (presumably string theory) enters.

A different approach to theories with extra dimensions is based on the assumption that the physics above the TeV scale (at least up to 10 TeV or so) is still described in terms of perturbative gauge theories. Then the solution to the hierarchy problem must involve some new symmetry that protects the Higgs boson mass. As we shall demonstrate, one candidate for such symmetry could be the higher dimensional Poincare invariance. Alternatively, the hierarchy problem can be solved by supersymmetry. The advantage of such an approach over the traditional 4d supersymmetry is that, typically, the presence of extra dimensions constrains the possible form of low energy supersymmetry. In particular, extra dimensions could justify the very special pattern of the soft breaking terms of the MSSM.

But, again, gauge theories with extra dimensions create a number of new problems. One of the most serious is that these theories have dimensionful couplings, which is a sign that they are non-renormalizable. Thus they have to be defined as theories with a cutoff. Generic observables will then be sensitive to the precise value of the cutoff, as well as to the nature of the UV completion of the theory. This is the point where deconstruction - the keyword of this thesis - enters the stage. Recently it was noticed [4, 5] that the physics of extra dimensions can be reproduced in purely four dimensional setups. In this approach the 4d theory is a gauge theory with a product gauge group G^N. This enlarged gauge group is intended to mimic the local gauge symmetry G acting in higher dimensional space-time. The product gauge group is subsequently broken to the diagonal group G by vacuum expectation values (vevs) of certain scalar fields. It turns out that, at low energies, the spectrum and interactions of such theory can be analogous to the spectrum and interactions of some compactified higher dimensional gauge theories. The idea that 4d theories can approximate higher dimensional ones is called *deconstruction.*

What makes deconstruction so interesting is the fact that it presents us with the possibility of studying higher dimensional theories using the familiar tools of 4d gauge theories. In particular, the 4d theory can be made renormalizable. Then even those questions that do not have unambiguous answers in the higher dimensional framework, can be asked in deconstruction and the answers can be put to test in experiment. But in the course of research it has become clear that deconstruction should not be viewed merely as a regularization of higher dimensional gauge theories. Extended gauge symmetry of the product group can be used as *the symmetry* solving the hierarchy problem or some other problems of the SM. From this point of view, deconstruction is simply a new direction in the search for physics beyond the SM.

The objective of this thesis is to provide concrete examples in which the virtues of deconstruction, mentioned above, are made explicit. On one hand we show that deconstruction is a very efficient tool for understanding various aspects of theories with extra dimension. It is possible to construct 4d analogues of many different higher dimensional theories and, in certain energy range, the tree-level predictions of the two theories are the same. Moreover, those observables of higher dimensional theories that are calculable at the quantum level, can be also studied in deconstruction and the results coincide. But the real advantage of deconstruction is that it allows to address the questions that are not well defined in the higher dimensional formulation. On the other hand, we show that the deconstruction setup can be used to construct phenomenologically interesting models, even without any reference to extra dimensions. We will provide explicit examples of theories, in which the symmetries of deconstruction can be explored in order to solve the hierarchy problem of the SM.

This thesis is organized as follows. The next two chapters are a compilation of previously published material. They are intended to provide the necessary background for dealing with physics of extra dimension and of deconstruction. In Chapter 2 we concentrate on gauge theories with extra dimensions. We present a technical introduction to 5d gauge theories and their various compactifications. We also discuss two examples of physical applications of such theories. Deconstruction is introduced in Chapter 3. Simplest deconstruction models are described in some detail and the relation between deconstruction and higher dimensional gauge theories is established.

Chapters 4 and 5 are based on the new material and are the central part of this thesis. In Chapter 4 we discuss 4d models intended to be regularizations of certain interesting 5d constructions. The model in Section 4.1 investigates, from the point of view of deconstruction, a rather theoretical question of RG running in 5d gauge theories in the AdS_5 background. The model in Section 4.2 provides a new insight, from the deconstruction perspective, into the 5d construction of Barbieri, Hall and Nomura [6], that addresses the hierarchy and supersymmetry breaking problems. In Chapter 5 we stress the point that the role of deconstruction should not be restricted to regularizing gauge theories with extra dimensions. We present two interesting models that aim at solving the hierarchy problem. Although these models involve a similar setup as deconstruction, they do not strictly correspond to any model with extra dimensions. The one in Section 5.1 is based on supersymmetry and the deconstruction setup is used to generate the large hierarchy between the supersymmetry breaking scale and the fundamental scale. In the model of Section 5.2 we derive certain non-supersymmetric theories with a softened UV behaviour.

Chapter 6 contains conclusions and discusses some possible future directions of research. The full actions of the 5d Yang-Mills theory compactified on S_1 and of the corresponding 4d deconstruction model are given in Appendix A and Appendix B, respectively.

In Appendix C we summarize the notation and conventions.

The main results of this thesis are based on three published papers ref. [7], ref. [8] and ref. [9], and on one yet unpublished preprint, ref. [10]. Part of the results obtained in refs. [11, 12] have also been included.

Chapter 2

Gauge Theories with Extra Dimensions

Although there is no experimental evidence for the existence of more than four space-time dimensions, theories with extra dimensions are very interesting from a theoretical point of view. The idea dates back to early 20th century when Kaluza (and later Klein) noticed that five-dimensional (5d) setups allow for unification of gravity and electromagnetism [13]. Ever since the birth of String Theories, which can be consistently formulated only in 26 space-time dimensions (or 10 for Superstrings), theories with extra dimensions occupy a central position in high energy physics.

Recently, the idea has received a new stimulus. It was observed that, if gravity propagates in extra dimensions, the huge gap between the Planck and the electroweak scales can be explained. In other words, the hierarchy problem can be solved. One concrete realization of this idea is the Arkani-Hamed, Dimopoulos, Dvali (ADD) model of ref. [2]. The hallmark of this scenario is that the extra compact dimensions form a flat manifold, whose volume V is large in terms of the fundamental length scale $1/M_*$. The scale of observed gravitational interaction (the 4d Planck mass M_{PL}) is connected to the fundamental scale M_* of the $4+n$ dimensional world by the formula:

$$M_{PL}^2 = M_*^{n+2} V \, . \tag{2.1}$$

If V is much bigger than $(1/M_*)^n$ then also the 4d Planck scale is much larger than the true fundamental scale M_*. In order to solve the hierarchy problem, the authors of ref. [2] assumed that the fundamental scale is close to the TeV scale, $M_* \sim 1$ TeV. Thus, inherently, gravity is not weaker than the other forces of nature. It only appears to be weak to the 4d observer because it 'leaks' into the large extra dimensions. Of course, the large volume of extra dimensions introduces another, this time geometrical, hierarchy problem and any complete model should address this issue.

In the ADD model the gauge and matter fields of the SM cannot propagate in the extra dimensions. Otherwise the excitations of the SM fields connected with motion along extra dimensions, so-called Kaluza-Klein (KK) modes, would have been discovered in accelerator experiments. For example, in the ADD model with two extra dimensions the mass of the lightest KK modes is only of order 10^{-4} eV. Thus, the SM fields have to be confined to a 4d hyper-surface (a *brane*) embedded in the (4+n)-dimensional space-time (the *bulk*). Therefore, up to energies of order M_*, physics of gauge and matter fields is analogous as in ordinary 4d theories. On the other hand, the existence of light KK modes of the graviton does not contradict experiment as these KK modes interact very weakly with matter.

Another interesting solution to the hierarchy problem is provided by the Randall-Sundrum (RS) model of ref. [3]. The authors give up the assumption of flat, Minkowski geometry of space-time. Instead, they assume a 5d space-time whose metric is of the warped product form,

$$ds^2 = a^2(y) dx^\mu dx_\mu - dy^2 \,, \tag{2.2}$$

with the fifth dimension being an interval of finite length, $y \in [0, \pi R]$. More concretely, the action of the RS model contains a negative cosmological constant in the bulk, $\Lambda = -6k^2$, so that the vacuum solution is of the form (2.2) with $a(y) = \exp(-ky)$. Hence the geometry is that of a slice of the 5d anti-de-Sitter space-time (AdS_5). The cosmological constant is taken to be of the order of the fundamental scale M_*. It is possible to show that in the RS model the scale of 4d gravity is also of order M_* and thus M_* should be comparable to the 4d Planck scale M_{PL}. Now, the presence of the warp factor $a(y)$ implies that an observer at the point y experiences all physical scales rescaled by $\exp(-ky)$. The authors of ref. [3] assume that the SM matter fields are confined to a brane located at $y = \pi R$. The scale M of local physics is connected to the fundamental scale by the formula:

$$M = M_* \exp(-k\pi R) \tag{2.3}$$

It suffices to choose $\pi R \sim 30/k$ to explain the gap between M_{PL} and the electroweak scale.

In the RS model the KK modes have masses of order 1 TeV. Therefore it is consistent to assume that the SM gauge and/or matter fields propagate in the bulk. Only the Higgs boson has to be localized at $y = \pi R$ in order to allow for a successful solution to the hierarchy problem.

In these two examples the key role in solving the hierarchy problem is played by gravity. This creates some theoretical problems. At energies above TeV gravitational interactions become strong and their classical description is not expected to be valid. But the quantum theory of gravity is still missing. This is the reason why many authors consider extra dimensional theories which are not warped and which have relatively small volumes of the extra dimensions, at least $V < (1\text{TeV})^{-n}$. Hence, the physics at scales comparable to the compactification scale is under control. This class of theories is what we will discuss in the remainder of this chapter. In such theories gravity cannot play a prominent role in solving the hierarchy problem. Nevertheless, if gauge and/or matter fields are allowed to propagate in extra dimension, these theories offer a number of interesting possibilities for the physics beyond the SM. Below we discuss these possibilities in some more detail.

1. **Breaking of symmetries.** In four dimensional field theories symmetries can be broken either explicitly or spontaneously, that is by a vacuum expectation value (vev) of some field transforming non-trivially under these symmetries. Extra dimensions offer novel, elegant tools for reducing symmetries of a theory. One possibility consists in the assumption that the extra dimensions have a non-trivial topology. It often happens that a symmetry generator can be defined only locally, but not globally in the entire space time. One well-known example is the compactification of 10d type I supergravities on a 6d Calabi-Yau manifold, which leaves unbroken only one fourth of the original supersymmetries. Another possibility is offered by compactifications on orbifolds. An orbifold can be defined as a smooth manifold divided by a discrete symmetry transformation. If the discrete transformation does not act freely, the resulting orbifold contains singular points (or hypersurfaces). Defining a field theory in such a space-time it is necessary to specify the behaviour of fields near these singularities. Some of the degrees of freedom of the theory can be projected out in this manner and the symmetries involving the projected fields can be broken there. Although the bulk space-time preserves the symmetry, from the point of view of the effective low energy theory (in which extra dimensions are integrated out) such symmetry is completely broken.

2. **Stabilizing the electroweak scale.** In the SM the Higgs boson mass receives quadratically divergent corrections, and this fact is the origin of the hierarchy problem. In general, we expect mass parameters of scalar fields to receive quadratically divergent corrections unless there is some symmetry which forbids this. The most popular solution is supersymmetry. In models with extra dimensions there exists another possibility. Namely, the SM Higgs c boson can be identified with a component of a higher dimensional vector field tangent to extra dimensions. It is seen as a scalar field from the point of view of the low energy theory but nevertheless its mass is protected by the higher dimensional gauge invariance.

3. **Generating small numbers.** The SM provides us with a number of dimensionless parameters that differ by many orders of magnitudes. For example, the ratio of the up and top Yukawa couplings is approximately 10^{-5}. Although this hierarchy is technically not disturbing (in the sense that it is not spoiled by quantum corrections), one would like to gain some understanding about the origin of the peculiar pattern

of fermion masses and mixing. Furthermore, in grand unified theories one would often like to suppress certain couplings between quarks and leptons to avoid a too large rate of proton decay and other exotic processes. Extra dimensions offer a way to understand the origin of small numbers in a geometrical language. One assumes that different fields are localized at different positions in extra dimensions and the smallness of couplings between them comes from the small overlap of their wave functions.

Further on in this chapter we will discuss specific models which address some of the aforementioned questions. But first we present a technical introduction to gauge theories with extra dimensions. For simplicity and concreteness we will concentrate on theories with just one extra dimension.

2.1 Gauge Theories in Five Dimensions

Among gauge theories with extra dimensions the simplest setup is offered by theories formulated in five space-time dimensions. The fifth dimension is space-like and it is parametrized by the coordinate y. A 5d gauge field A_α has five components, $\alpha = 0 \dots 3, 5$ and transforms in the adjoint representation of the gauge group G, $A_\alpha = A_\alpha^a T^a$, where T^a are the generators of G. Under the 4d Lorentz transformations it splits into a 4d vector field A_μ, $\mu = 0 \dots 3$ and a scalar A_5. The action for the gauge fields has the usual Yang-Mills form,

$$S = -\frac{1}{2} \int d^4x dy \ \mathrm{Tr} \left\{ F_{\alpha\beta} F^{\alpha\beta} \right\} , \tag{2.4}$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + i g_5 [A_\alpha, A_\beta]$ and g_5 is the gauge coupling. Note that A_α has the dimension $[\mathrm{mass}]^{3/2}$ while g_5 has dimension $[\mathrm{mass}]^{-1/2}$. Alternatively, one can rescale the gauge fields by $A_\alpha \to \frac{1}{g_5} A_\alpha$ so that the gauge coupling appears only as a multiplicative factor in the kinetic term.

Five dimensional scalar fields can be added straightforwardly. For example, for a complex scalar Φ in the fundamental representation of the gauge group the action reads:

$$\begin{aligned} S = \textstyle\int d^4x dy \left[(D_\alpha \Phi)^\dagger D_\alpha \Phi - m^2 \Phi^\dagger \Phi \right] , \\ D_\alpha \Phi = \partial_\alpha \Phi + i g_5 A_\alpha \Phi , \end{aligned} \tag{2.5}$$

where m is the 5d mass. As for the fermion fields, it should be remembered that the 5d Lorentz symmetry does not admit chiral fermions. Chiral projectors, $P_{L,R} = \frac{1}{2}(\mathbf{1} \pm \gamma_5)$ contain the γ_5 matrix which enters the 5d Clifford algebra. In consequence, the notion of chirality depends on the coordinate system. The minimal fermion field is represented by a four-component spinor of the Dirac type (alternatively, by a pair of symplectic Majorana spinors). For example, the action for a fermion Ψ with mass M, in the fundamental representation reads:

$$
\begin{aligned}
S = \int d^4x dy \left[\overline{\Psi}(i\Gamma^\alpha D_\alpha - M)\Psi \right] , \\
D_\alpha \Psi = \partial_\alpha \Psi + i g_5 A_\alpha \Psi
\end{aligned}
\tag{2.6}
$$

The 5d gamma matrices Γ^α are related to the 4d ones by $\Gamma^\mu = \gamma^\mu$, $\Gamma^5 = i\gamma^5$ so that they satisfy the equations of the 5d Clifford algebra, $\{\Gamma^\alpha, \Gamma^\beta\} = 2\eta^{\alpha\beta}$. In the following we shall always choose a coordinate frame in which $\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and label the two upper (lower) components of Ψ by Ψ_L (Ψ_R).

Since there is no chirality in 5d, the minimal supersymmetry has twice as much supercharges as the minimal supersymemtry in 4d and it corresponds to $N=2$ supersymmetry from the 4d point of view. The gauge multiplet consists of a gauge field A_α, a Dirac gaugino λ, and a real scalar field Σ, all in the adjoint representation. The 5d lagrangian reads:

$$
S = \int d^4x dy \mathrm{Tr} \Big[-\frac{1}{2} F_{\alpha\beta}^2 + D_\alpha \Sigma D_\alpha \Sigma + 2\overline{\lambda} i \Gamma^\alpha D_\alpha \lambda - 2 g_5 \overline{\lambda}[\Sigma, \lambda] \Big] . \tag{2.7}
$$

A 5d matter multiplet is called a *hypermultiplet* and consists of two scalar fields, H^i, $i = 1, 2$, and a Dirac fermion Ψ. The lagrangian for a hypermultiplet interacting with the vector multiplet is given by

$$
\begin{aligned}
\mathcal{L} &= |D_\alpha H_i|^2 + i\overline{\Psi}\Gamma^\alpha D_\alpha \Psi - (i\sqrt{2} H^{\dagger 1} \overline{\lambda} \Psi_L + i\sqrt{2} H^{\dagger 2} \overline{\lambda} \Psi_R + \mathrm{h.c.}) - g_5 \overline{\Psi} \Sigma \Psi \\
&- g_5^2 H^{\dagger i} \Sigma^2 H_i - \frac{g_5^2}{2} [H^{\dagger i} (\sigma^m)_i^j T^a H_j]^2 ,
\end{aligned}
\tag{2.8}
$$

where σ^m are the Pauli matrices.

In 4d $N=1$ supersymmetry it is often very convenient to work in the superspace formalism. This formalism is also suitable for 5d $N=2$ theories [14], despite the fact that it obscures $N=2$ supersymmetry and 5d Lorentz invariance. Below we present the superspace action only for the case of $U(1)$ gauge group. More involved formulae for non-abelian gauge groups are given in refs [14, 15].

The 5d off-shell vector multiplet can be split into the $N=1$ vector multiplet $V = (A_\mu, \chi, D)$ and a chiral multiplet $\Phi = (\frac{1}{\sqrt{2}}\Sigma + i\frac{1}{\sqrt{2}}A_5, \Psi_\chi, G)$, singlet under the $U(1)$ group. The 5d abelian gauge theory in a flat background in the $N=1$ superspace formalism reads:

$$
S_{5d} = \int d^4x dy \left[\frac{1}{2} W^\alpha W_\alpha + \mathrm{h.c.} \right]_F + \left[\left(\partial_5 V - \frac{1}{\sqrt{2}} (\Phi + \Phi^\dagger) \right)^2 \right]_D \tag{2.9}
$$

Writing down the component action and integrating out the auxiliary fields one finds that the superspace action eq. (2.9) leads to the same component action as that given in eq. (2.7) with $\lambda_L \to \Psi_\chi$, $\lambda_R \to \chi$.

A 5d fundamental hypermultiplet can be split into two chiral multiplets, $H = (H, \psi, F)$ and $\tilde{H} = (\tilde{H}, \tilde{\psi}, \tilde{F})$ in the fundamental and anti-fundamental representation of the gauge group. The action for a hypermultiplet with the charge q under $U(1)$ is given by

$$S_5 = \int d^4x dy \left[H^\dagger e^{2g_5 qV} H + \tilde{H}^\dagger e^{-2g_5 qV} \tilde{H}\right]_D + \int d^4x dy \left[W + \text{h.c.}\right]_F$$
$$W = \sqrt{2} g_5 \tilde{H} \Phi H + \tilde{H} \partial_5 H, \tag{2.10}$$

Integrating out the auxiliary fields one finds the same action as that given in eq. (2.8) with $H^1 \to H$, $H^2 \to \tilde{H}^\dagger$, $\Psi_L \to \psi$, $\Psi_R \to \tilde{\psi}^\dagger$.

The idea behind theories with extra dimensions is that they are a relevant description of physics at some high energy scale. To make contact with phenomenology we should derive the low energy effective action and integrate out the degrees of freedom associated with the extra dimensions. This procedure is called *compactification* and, in first approximation, it consists in performing the integral over the extra dimensions in the action. The form of the low energy effective theory depends on the topological properties of the space-time. Below we review the basic features of several simplest (and most widely used) compactifications.

2.1.1 Compactification on a circle

Compactification on a circle consists in the assumption that the 5d space-time can be factorized into the four dimensional Minkowski spacetime M_4 and the circle S_1 of radius R. Since the gauge field has to be periodic in the fifth coordinate, $A_\alpha(x^\mu, y+2\pi R) = A_\alpha(x^\mu, y)$, it can be decomposed into Fourier modes (in this context called KK modes),

$$A_\mu(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \left(\eta_{(n)} \cos(\frac{ny}{R}) A_\mu^{(n)}(x^\mu) + \sin(\frac{ny}{R}) A_\mu^{(-n)}(x^\mu)\right) \tag{2.11}$$

$$A_5(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \left(-\sin(\frac{ny}{R}) A_5^{(n)}(x^\mu) + \eta_{(n)} \cos(\frac{ny}{R}) A_5^{(-n)}(x^\mu)\right) . \tag{2.12}$$

In the above $\eta_{(n)} = 1/\sqrt{2}$ for $n = 0$ and $\eta_{(n)} = 1$ otherwise. Plugging this decomposition back into the action of eq. (2.4) and performing the integral over y we get

$$S = \int d^4x \left[\textstyle\sum_{n=-\infty}^{\infty} \left((\partial_\mu A_\nu^{(n)} - \partial_\nu A_\mu^{(n)})^2 + (\frac{n\pi}{R})^2 A_\mu^{(n)} A_\mu^{(n)}\right) + \text{interaction terms}\right] . \tag{2.13}$$

The compactified theory contains one massless vector field, $A_\mu^{(0)}$ (corresponding to the constant KK mode of the 5d gauge field). Thus there is an unbroken 4d gauge symmetry G and the gauge coupling is

$$g_0 = \frac{g_5}{\sqrt{2\pi R}}. \tag{2.14}$$

The remaining KK modes $A_\mu^{(n)}$ become massive vector fields in the adjoint representation of the gauge group. Their masses are given by the formula

$$m_{(n)}^2 = \frac{n^2}{R^2}\,. \tag{2.15}$$

There are two vector fields at each massive level. The masses of the KK modes can be readily interpreted as momenta along the fifth dimension. Because the fifth coordinate is periodic, the momenta must be quantized. In the de-compactification limit $R \to \infty$ the spectrum of KK modes becomes continuous. The doubling of states at each massive level originates from the fact that there are left- and right-moving modes along S_1.

From the 4d point of view $A_5^{(n)}$ with $|n| > 0$ are not physical scalars and can be removed from the spectrum by a gauge transformation. Only the zero mode $A_5^{(0)}$ remains in the physical spectrum as a massless scalar in the adjoint representation. The full action, including the interactions of the KK modes, is given in Appendix A.

If supersymmetry is present in 5d it is not broken by compactification on S_1. The 4d low energy theory possesses $N = 2$ supersymmetry. This is too restrictive from the phenomenological point of view as in such theories chirality is absent and all Yukawa couplings are set by the single gauge coupling. Thus it is interesting to study compactifications on singular manifolds as it enables to break unwanted symmetries.

2.1.2 Compactification on the orbifold S_1/Z_2

A standard tool to reduce a number of symmetries and degrees of freedom is a compactification on orbifolds. In the case of five dimensional theories on a circle S_1 one can impose a discrete Z_2 symmetry which acts as $y \to -y$ on the fifth coordinate (it follows that $\partial_5 \to -\partial_5$). There are two fixed points of the Z_2 action, 0 and πR. The resulting space-time is the product of the 4d Minkowski space-time M_4 and the finite interval $[0, \pi R]$. Instead of working on a manifold with boundaries (downstairs picture) it is technically more convenient to define the action on the full circle $y \in [-\pi R, \pi R]$ but represent the Z_2 symmetry on the fields of the theory (upstairs picture). In other words, the field values for $y < 0$ are uniquely determined by the field values for $y > 0$. The Z_2 action has to be such that the Lagrangian is invariant under these Z_2 transformations.

It is instructive to consider first a simpler case when the Z_2 action commutes with the gauge symmetry transformations. For example, the 5d YM lagrangian is invariant under the Z_2 action defined as

$$A_\mu(x^\mu, y) = A_\mu(x^\mu, -y) \qquad\qquad A_5(x^\mu, y) = -A_5(x^\mu, -y). \tag{2.16}$$

The consequence of imposing the Z_2 parity of eq. (2.16) is that only the KK modes corresponding to positive (negative) parity of the wave function remain in the KK decomposition of A_μ (A_5),

$$\begin{aligned} A_\mu(x^\mu, y) &= \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \eta_{(n)} \cos\left(\frac{ny}{R}\right) A_\mu^{(n)}(x^\mu) \\ A_5(x^\mu, y) &= \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \sin\left(\frac{ny}{R}\right) A_5^{(n)}(x^\mu). \end{aligned} \tag{2.17}$$

The relative minus sign in the Z_2 action on A_5 is necessary for the invariance of the lagrangian (this can be seen e.g. from the presence of the term $\partial_\mu A_5 \partial_5 A_\mu$). The compactified theory contains a massless gauge field $A_\mu^{(0)}$ which signifies that gauge symmetry survives at low energies. On top of the massless mode there is an infinite tower of massive vector fields $A_\mu^{(n)}$, one at every mass level, with masses given by the same formula as for the S_1 compactification,

$$m_{(n)} = \frac{n}{R}. \tag{2.18}$$

Similarly as for the compactification on S_1, the masses of the KK modes can be interpreted as momenta along the fifth dimension. On the orbifold there is no degeneracy of the massive levels. The left- and right-moving modes must be combined so as to form a standing wave.

The massless mode of A_5 is absent, because the constant zero-mode wave function is not compatible with the negative parity assignment, eq. (2.16). All the modes of A_5 with $n > 0$ can be gauged away so no scalars remain in the physical spectrum.

Imposing the Z_2 parity has very interesting consequences for bulk fermion fields because it allows to generate chirality. Consider a 5d fermion Ψ whose action is given by eq. (2.6). Note that the presence of the kinetic term $\overline{\Psi}\gamma_5\partial_5\Psi$ forces the two upper (left) and two lower (right) components of Ψ to have opposite Z_2 assignments, for example

$$\Psi_L(x_\mu, y) = \Psi_L(x_\mu, -y) \qquad\qquad \Psi_R(x_\mu, y) = -\Psi_R(x_\mu, -y) \tag{2.19}$$

Thus Ψ_L has the KK decomposition analogous to A_μ in eq. (2.17), while the KK decomposition of Ψ_R is analogous to A_5. There is a constant mode $\Psi_L^{(0)}$ which is a chiral massless fermion in the low energy theory. Thus compactification on S_1/Z_2 is a way to generate chirality from a non-chiral 5d theory. The remaining KK modes $\Psi_L^{(n)}$ and $\Psi_R^{(n)}$ form Dirac spinors of mass $m_{(n)} = n/R$.

Another consequence of Z_2 is that the bulk mass term $M\overline{\Psi}\Psi$ is not allowed. However the so called kink-mass term, $M\epsilon(y)\overline{\Psi}\Psi$, is allowed. It involves the step function $\epsilon(y)$ defined as $\epsilon(y) = (-1)^{\mathrm{sgn}(y)}$. Such a kink-mass term can be generated, e.g., from the Yukawa interaction $\Phi\overline{\Psi}\Psi$ when the Z_2 odd scalar Φ acquires a vev. Solving the 5d equations of motion in the presence of the kink-mass term one finds that the massless fermionic mode is given by

$$\Psi^{(0)} = \sqrt{\frac{M}{2(e^{-M\pi R} - 1)}} \exp(-M|y|)\psi_L(x_\mu) \tag{2.20}$$

where $\psi_L(x_\mu)$ is an arbitrary left-handed 4d spinor. Thus orbifold compactification allows to generate chiral localized fermions.

Let us now be more general. One can define the Z_2 action on the 5d bulk field $\phi(x^\mu, y)$ in the fundamental representation as

$$\phi(x^\mu, -y) = P\phi(x^\mu, y). \tag{2.21}$$

where P is a matrix acting on the gauge indices, satisfying $P^2 = 1$. The Z_2 action on the gauge fields is given by:

$$\begin{aligned} A_\mu(x^\mu, -y) &= PA_\mu(x^\mu, y)P\,, \\ A_5(x^\mu, -y) &= -PA_5(x^\mu, y)P\,. \end{aligned} \tag{2.22}$$

By definition, P has eigenvalues 1 or -1. We will always assume that we work in the basis where P is diagonal. We denote a field with the eigenvalue 1 (-1) by ϕ_+ (ϕ_-). According to its Z_2 parity the field ϕ is called *even* or *odd*. The fields ϕ_+ and ϕ_- have the KK decomposition analogous to, respectively, A_μ and A_5 in eq. (2.17),

$$\phi_+(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \eta_{(n)} \phi_+^{(n)}(x^\mu) \cos\frac{ny}{R}, \tag{2.23}$$

$$\phi_-(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \sin\frac{ny}{R}, \tag{2.24}$$

and so $\phi_\pm^{(n)}(x^\mu)$ acquire mass n/R upon compactification. Note that ϕ_- must vanish at the fixed points, $\phi_-(x^\mu, y = 0) = 0$ and $\phi_-(x^\mu, y = \pi R) = 0$.

For P not proportional to the identity matrix, the gauge symmetry is broken. One can divide the group generators into those commuting with P, $PT^aP = T^a$, and those anticommuting with P, $PT^{\hat{a}}P = -T^{\hat{a}}$. Then the gauge fields corresponding to the generators T^a decompose as ϕ_+ and include a massless mode, while the gauge fields corresponding to the generators $T^{\hat{a}}$ decompose as ϕ_- and yield only heavy modes. Thus the zero modes do not form a full multiplet of G and the gauge symmetry is broken to the subgroup H generated by T^a. On the other hand, only the scalars $A_5^{\hat{a}}$ corresponding to the broken generators $T^{\hat{a}}$ have a massless mode.

One example of breaking gauge symmetry by orbifolding is that for $G = SU(5)$ and the matrix P chosen as $P = \mathrm{diag}(-1,-1,-1,1,1)$. At the zero-mode level only the gauge fields corresponding to the SM subgroup are present. This mechanism lies at the heart of the 5d SU(5) GUTs which we discuss further in this chapter. Another interesting example is that for $G = SU(3)$ and $P = \mathrm{diag}(1,1,0)$ which leaves the electroweak group $H = SU(2) \times U(1)$ unbroken. This orbifolding occurs in models of TeV electroweak unification [16].

If P in eq. (2.21) is proportional the identity matrix, the gauge symmetry stays unbroken. But even in such case the Z_2 orbifolding can be used for reducing symmetries, namely for breaking part of the supersymmetry. In supersymmetric theories on S_1/Z_2 the Z_2 action has to be chosen such that only $N = 1$ supersymmetry survives in four dimensions. This happens because invariance of the supersymmetric lagrangian eq. (2.7) under Z_2 requires that if $V = (A_\mu, \chi)$ is even then $\Phi = (\frac{1}{\sqrt{2}}\Sigma + \frac{i}{\sqrt{2}}A_5, \lambda)$ is odd under Z_2. Thus, at the zero mode level, only $N = 1$ vector multiplet V is present which signals that only $N = 1$ supersymmetry is present. As theories with $N = 1$ supersymmetry are phenomenologically interesting, the compactification of 5d supersymmetric theories on S_1/Z_2 is commonly encountered in literature.

2.1.3 Compactification on the orbifold $S_1/Z_2 \times Z_2'$

Sometimes orbifolding by a single Z_2 still leaves too many symmetries or degrees of freedom in the low energy theory. For example, breaking $SU(5)$ down to the SM group leaves also massless coloured scalars descending from A_5. Therefore, it may be advantageous to consider the fifth dimension to be the product of M_4 and the orbifold $S_1/(Z_2 \times Z_2')$. The orbifold $S_1/(Z_2 \times Z_2')$ is obtained by dividing S_1/Z_2 by another Z_2 transformation, denoted by Z_2'. It acts as $y' \to -y'$, where $y' = y + \frac{\pi R}{2}$. There are two fixed points $y = 0$ (of Z_2) and $y = \frac{\pi R}{2}$ (of Z_2') and the fifth dimension is an interval $[0, \pi R/2]$. As before, we will work in the upstairs picture where the fifth dimension is a circle, $y \in [-\pi R, \pi R]$, and the $Z_2 \times Z_2'$ symmetry is imposed on the fields.

The $Z_2 \times Z_2'$ action on fundamental matter fields in the fundamental representation is defined as

$$\begin{aligned} \phi(x^\mu, -y) &= P\phi(x^\mu, y), \\ \phi(x^\mu, -y') &= P'\phi(x^\mu, y'). \end{aligned} \qquad (2.25)$$

On the gauge it acts as

$$\begin{array}{ll} A_\mu(x^\mu,-y) = PA_\mu(x^\mu,y)P & A_5(x^\mu,-y) = -PA_5(x^\mu,y)P \\ A_\mu(x^\mu,-y') = P'A_\mu P'(x^\mu,y') & A_5(x^\mu,-y') = -P'A_5(x^\mu,y')P'. \end{array} \tag{2.26}$$

The fields with definite parity under both Z_2 transformations are denoted by ϕ_{++}, ϕ_{+-}, ϕ_{-+} and ϕ_{--} (the first subscript corresponds to the eigenvalue of P and the second to P'). Their KK decomposition is given by

$$\phi_{++}(x^\mu,y) = \sqrt{\frac{1}{\pi R}}\sum_{n=0}^{\infty}\eta_{(n)}\phi_{++}^{(n)}(x^\mu)\cos\frac{2ny}{R}, \tag{2.27}$$

$$\phi_{+-}(x^\mu,y) = \sqrt{\frac{1}{\pi R}}\sum_{n=0}^{\infty}\phi_{+-}^{(n)}(x^\mu)\cos\frac{(2n+1)y}{R}, \tag{2.28}$$

$$\phi_{-+}(x^\mu,y) = \sqrt{\frac{1}{\pi R}}\sum_{n=0}^{\infty}\phi_{-+}^{(n)}(x^\mu)\sin\frac{(2n+1)y}{R}, \tag{2.29}$$

$$\phi_{--}(x^\mu,y) = \sqrt{\frac{1}{\pi R}}\sum_{n=0}^{\infty}\phi_{--}^{(n)}(x^\mu)\sin\frac{(2n+2)y}{R}, \tag{2.30}$$

The fields $\phi_{++}^{(n)}(x^\mu)$, $\phi_{+-}^{(n)}(x^\mu)$, $\phi_{-+}^{(n)}(x^\mu)$ and $\phi_{--}^{(n)}(x^\mu)$ acquire masses $\frac{2n}{R}$, $\frac{2n+1}{R}$, $\frac{2n+1}{R}$ and $\frac{2n+2}{R}$, respectively. Note that a massless mode appears only in ϕ_{++}. Note also that some fields vanish at the fixed points $\phi_{+-}(x^\mu,-\frac{\pi R}{2}) = \phi_{--}(x^\mu,-\frac{\pi R}{2}) = 0$ and $\phi_{-+}(x^\mu,0) = \phi_{--}(x^\mu,0) = 0$.

In supersymmetric theories it is possible to define the $Z_2\times Z_2'$ action such that all supersymmetry is broken. More precisely, at each fixed point half of the supersymmetric multiplets is projected out, leaving $N=1$ supersymmetry multiplets. However different multiplets survive at the two fixed point (they correspond to 2 different supersymmetries of the bulk). In consequence, the 4d low energy has no supersymmetry whatsoever. The 'non-local' character of supersymmetry breaking leads to certain interesting finiteness properties of such theories. One model which takes advantage of these properties will be presented in Section 4.2.

2.1.4 Scherk-Schwarz compactification

Apart from orbifolding there exists another interesting mechanism of breaking symmetries in theories with compact extra dimensions. It is called the Scherk-Schwarz mechanism [17] and the idea behind it is to relax the periodicity condition imposed on the fields. For example, in 5d theories compactified on S_1 one assumes that under the $2\pi R$

translation along the extra dimension the field value changes by a symmetry transformation,

$$\phi(x^\mu,\, y+2\pi R) = e^{2\pi i\omega\mathcal{Q}}\phi(x^\mu,\, y)\ , \tag{2.31}$$

where $\mathcal{Q}$ is a generator of a global or local symmetry of the theory. As a consequence the KK decomposition of all non-singlet fields will be altered. Let us assume for simplicity that the 'twisted symmetry' is $U(1)$ and $\mathcal{Q} = \mathbf{1}$. Then ϕ decomposes as

$$\phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} e^{iy(n+\omega)/R}\phi_{(n)}(x^\mu). \tag{2.32}$$

Plugging this decomposition into the kinetic term in the action one finds that the mass of the $\phi_{(n)}$ mode is

$$m_n^2 = (\frac{n+\omega}{R})^2 \tag{2.33}$$

Thus all the KK levels (including the zero mode) are shifted by a constant parameter. This shift is the hallmark of the Scherk-Schwarz mechanism.

Now it is easy to see how the Scherk-Schwarz mechanism can be used to break supersymmetry. If the shift ω is different for bosons and fermions (which can happen e.g. when the twisted symmetry $\mathcal{Q}$ is embedded in the R symmetry) the mass levels of bosons and fermions are different in the low energy theory.

Note that the twisted periodicity condition (2.31) can be solved by:

$$\phi(x^\mu,\, y) = e^{i\omega\mathcal{Q}y/R}\tilde{\phi}(x^\mu,\, y)\ , \tag{2.34}$$

where $\tilde{\phi}$ satisfies the usual periodicity condition, $\tilde{\phi}(x^\mu,\, y+2\pi R) = \tilde{\phi}(x^\mu,\, y)$. If the symmetry generated by $\mathcal{Q}$ is a gauge symmetry it is possible to show that the Scherk-Schwarz mechanism is equivalent a vacuum expectation value of the internal component of the gauge field. Let us concentrate on the case of $U(1)$ gauge theory and take $\mathcal{Q} = \mathbf{1}$. The non-periodic factor in eq. (2.34) can be gauged away by a non-periodic gauge transformation with $U = e^{-i\omega y/R}$. But because under gauge transformations $A_5 \to A_5 + iU^{-1}\partial_5 U$, such transformation will generate the constant vev, $\langle A_5\rangle = \omega/R$.

If the gauge symmetry is non-abelian it will be broken by the vev of A_5 (thus the Scherk-Schwarz breaking of gauge symmetry is equivalent to the Hosotani mechanism [18]). For this reason Scherk-Schwarz breaking of a gauge symmetry is of spontaneous type, so that no new counterterms except those allowed by the unbroken theory are generated at loop level. It can be also shown that the Scherk-Schwarz breaking of local supersymmetry is spontaneous [19, 20].

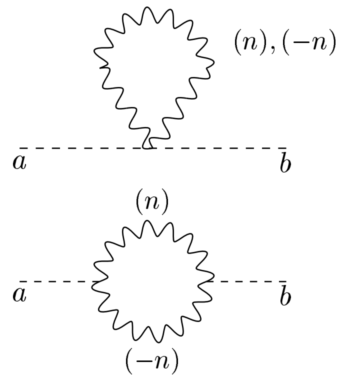
2.2 Physical applications

In the preceding section we discussed some technical aspects of gauge theories with extra dimensions. In this section we argue that these theories can provide frameworks for phenomenologically viable models of physics beyond the SM. We concentrate on two interesting examples. In the first, extra dimensions provide new symmetries which allow for stabilizing the electroweak scale. In the second, the extra dimensions are used as an efficient tool for reducing symmetries of a high energy theory.

2.2.1 Stabilizing scalar masses without supersymmetry

In 4d theories scalar masses receive quantum corrections that are quadratically divergent. The consequence of this fact is that generic theories with elementary scalars are not natural. This problem, in relation to the SM Higgs boson, is called the hierarchy problem. The common lore is that whenever we introduce an elementary scalar much lighter than the cut-off scale, we need to impose a symmetry which forbids quadratically divergent corrections. Possible solutions include softly broken supersymmetry, conformal symmetry and a spontaneously broken global symmetry (the Goldstone mechanism). Gauge theories with extra dimensions offer another possibility. The components of the gauge field tangent to extra dimensions (called the *internal components* of the gauge field) are seen as scalar fields in the 4d compactified theory. However in the full theory these are gauge fields and hence the loop corrections to their masses are still controlled by the higher dimensional gauge invariance. As we shall see, the effect of a higher dimensional gauge invariance is that the loop corrections are non-zero but finite. This observation opens a possibility of solving the hierarchy problem, by identifying the SM Higgs boson with an internal component of a higher dimensional gauge field.

Before we discuss a phenomenologically viable model of the SM Higgs sector, let us investigate in a toy-model example how the cancellation of the divergences works. The simplest setup to see this is the 5d $SU(K)$ theory compactified on the circle S_1 of radius R. As reviewed in Section 2.1.1, the 4d low energy theory contains a massless gauge boson $A_\mu^{(0)}$ and an infinite tower of massive gauge bosons $A_\mu^{(n)}$ with masses $m_{(n)}^2 = \frac{n^2}{R^2}$, $n \in (-\infty, \infty)$. Apart from that, there is one physical scalar $A_5^{(0)}$ in the adjoint representation. It is massless at the tree level but, since from the point of view of the 4d low energy theory it is a scalar field, its mass can be modified by loop corrections. The possible form of these correction is restricted by the gauge invariance of the 5d theory. The 5d theory is invariant under local transformations $\delta A_\alpha = D_\alpha \eta(x^\mu, y)$ which of course forbids any mass term for the gauge fields. However compactification on S_1 breaks some of the gauge invariance as only those generators which are periodic in the fifth coordinate, $\eta(x^\mu, y + 2\pi R) = \eta(x^\mu, y)$, are allowed. Under these residual transformations the operator $\mathrm{Tr}(\int_0^{2\pi R} dy A_5)^2$ is gauge invariant and it is precisely the mass term for $A_5^{(0)}$. However, this operator is non-local and cannot appear as a counterterm in a local theory. In consequence, loop corrections to $A_5^{(0)}$ mass should be finite, We also expect they should be proportional to $1/R$ so that they vanish in the de-compactification limit, $R \to \infty$.



$$3C(A)g_0^2\delta^{ab}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_n^2}$$

$$12C(A)g_0^2\delta^{ab}\int \frac{d^4k}{(2\pi)^4}\frac{m_n^2}{(k^2-m_n^2)^2}$$

Figure 2.1: One loop diagrams contributing to the mass of $A_5^{(0)}$

It is instructive to calculate explicitly one loop corrections to the mass of $A_5^{(0)}$. All interaction in the compactified theory are summarized in Appendix A. We work in the Landau gauge and so the derivative and ghost couplings of $A_5^{(0)}$ are not relevant here. The diagrams that contribute to the mass corrections are given in fig. 2.1 ($C(A)$ is the quadratic Casimir of the adjoint representation, $f^{acd}f^{bcd} = C(A)\delta_{ab}$. For $SU(K)$, $C(A) = K$). After the Wick rotation $k^0 = il^0$ and integration over the angular variables, $\int d\Omega = 2\pi^2$, one obtains

$$\delta_{m^2} = -\frac{6C(A)g_0^2}{(4\pi)^2}\left(-\int dll + 2R^2\sum_{n=1}^{\infty}\int_0^{\infty} dll^3\left(\frac{1}{R^2l^2+n^2} - \frac{2l^2R^2}{(R^2l^2+n^2)^2}\right)\right). \tag{2.35}$$

As it stands, the result is divergent and should be regularized. In general, the regulator should preserve the residual gauge symmetry of the 5d theory, which is the one which protects the mass of $A_5^{(0)}$. Note that cutting-off the infinite sum over the KK modes at some finite KK number leads to a quadratically divergent result. Obviously, a theory with a finite number of KK modes is not invariant under the full set of 5d gauge transformations. The simplest consistent regularization procedure consists in performing first the infinite sum over KK modes and is sometimes called KK-regularization. Using $cth(\pi x) = \frac{1}{\pi x} + \frac{2x}{\pi}\sum_{n=1}^{\infty}\frac{1}{x^2+n^2}$ and the derivative of this formula, the sum over the KK modes yields

$$\delta_{m^2} = \frac{6C(A)g_0^2}{(4\pi)^2}\int_0^{\infty} dl \frac{l^3\pi^2R^2}{\sinh^2(\pi lR)}. \tag{2.36}$$

This integral is exponentially damped in the UV and so is finite. The result is dominated by IR and reads:

$$\delta_{m^2} = \frac{9C(A)g_0^2\zeta(3)}{(4\pi)^2}\frac{1}{(\pi R)^2}\,. \tag{2.37}$$

As expected, the one loop mass of $A_5^{(0)}$ is finite and proportional $\frac{1}{R}$. More generally (see e.g. [21]), if fermion and or scalar fields are introduced in the theory, the one loop mass term of $A_5^{(0)}$ is given by

$$\delta_{m^2} = \frac{3g_0^2\zeta(3)}{(4\pi)^2}\frac{1}{(\pi R)^2}\left[3C(A) - 4C(F) + C(S)\right]\,, \tag{2.38}$$

where $C(F)$, $C(S)$ are the quadratic Casimirs of the representation of the bulk fermions and scalars, respectively.

This example shows that in extra dimensional gauge theories compactified to 4d there may exist scalar fields whose masses are protected. The question appears, if such scalars can be identified with the SM Higgs boson. The example presented above of course cannot be a realistic model of the electroweak Higgs sector. To construct a realistic model one has to take care the following things:

- The one loop Higgs mass parameter should come out negative. Otherwise, the minimum of the effective potential occurs at $\langle A_5^{(0)}\rangle = 0$ and breaking of gauge symmetry does not occur. By inspecting eq. (2.38) one can see that negative mass terms can be generated if fermion fields are present in the theory.

- The Higgs boson has to transform in the fundamental representation of the SM $SU(2)$ group. The internal components of the gauge field reside of course in the adjoint representation. However, it is possible to reduce the gauge group in the process of of compactification, e.g. using the mechanism of symmetry breaking by orbifolding. Then the adjoint representation splits into irreducible representations of the unbroken group which may include the fundamental representation. The simplest example is for $SU(3)$ broken to $SU(2)\times U(1)$, where we have $\mathbf{8} \to \mathbf{3}_0 \bigoplus \mathbf{1}_0 \bigoplus \mathbf{2}_{1/2} \bigoplus \mathbf{2}_{-1/2}$.

- The experimental fact that the Higgs boson mass is larger than ~ 110 GeV requires large (of order 1) coefficient of the Higgs quartic term. In the discussed toy model the quartic term is absent at tree level but it is generated radiatively in the same way as the mass term. But since it arises at one loop it is of order $\frac{\alpha}{4\pi}$, much too small to match the phenomenology. The way out is to consider theories with more than five dimensions. For example, in 6d, the kinetic term $F_{56}F_{56}$ yields quartic interactions of the internal gauge fields already at the tree-level.

- The SM quarks and leptons have to be included in the setup. It is of course possible to realize them as fermions living in extra dimensions. But then all the interactions with the Higgs boson are controlled by a single gauge coupling and it is difficult to reproduce the observed fermion masses and mixing. The alternative is to localize the SM fermions at the orbifold fixed points. The Yukawa couplings to the internal gauge bosons must be chosen in such a way so as not to reintroduce quadratic divergences. One possibility was pointed out in ref. [22]. One places the quark doublets at a different fixed point than the quark $SU(2)$ singlets. The Yukawa coupling between them goes via Wilson lines $Pe^{i\int A_i dx^i}$, where i runs over the compact coordinates.

Research along these lines started with early papers of Hosotani [18] and and has proceeded since then [23]. Recently a model based on the G_2 gauge group was constructed by Csaki, Grojean and Murayama [22], that yields correct electroweak breaking and also predicts the Weinberg angle close to the experimental value. Below we discuss this specific model in more detail.

The set-up is that of 6d gauge theory compactified on the orbifold T_2/Z_4, where T_2 is the torus $S_1 \times S_1$. The main role of the orbifolding is to break the higher dimensional gauge group G down to the SM electroweak group $SU(2) \times U(1)$. There is a well known fact [24] that abelian orbifolding cannot reduce the rank of the group, hence the list of possible choices for G is restricted to those of rank two. The authors of ref. [22] choose $G = G_2$.

G_2 belongs to the class of exceptional groups. It has fourteen generators and contains a maximal subgroup $SU(3)$. The adjoint representation decomposes under $SU(3)$ as

$$\mathbf{14} \to \mathbf{8} + \mathbf{3} + \bar{\mathbf{3}}\,. \tag{2.39}$$

The idea is to break G_2 by boundary conditions down to the subgroup $SU(2) \times U(1)$ embedded in the maximal subgroup $SU(3)$. The internal components of the 6d gauge boson are originally in the adjoint representation of G_2. In the compactified 4d theory they appear as scalars transforming in the representations of $SU(2) \times U(1)$, as dictated by the decomposition of $SU(3)$ representations **8** and **3** under its maximal subgroup $SU(2)\times U(1)$. Recall that the decomposition of **3** contains an $SU(2)$ doublet. This doublet can be identified with the SM Higgs boson.

The presence of the $SU(3)$ fundamental **3** in the decomposition of the adjoint, eq. (2.39), is what makes the G_2 group interesting in this setup. This is because embedding the Higgs doublet in **3** leads to the prediction of the Weinberg angle $\sin^2\theta_W = 1/4$, which is close to the experimental value $\sin^2\theta_W = 0.22$. To understand this it is enough to know that in breaking $SU(3) \to SU(2) \times U(1)$ the $U(1)$ factor is identified with the $SU(3)$ generator $\frac{1}{2\sqrt{3}}\mathrm{diag}(1,1,2)$ (that is $\frac{1}{2}\lambda_8$, where λ_8 is one of the Gell-Mann matrices). To normalize this $U(1)$ to the SM hypercharge Y (we use the normalization in which the

Higgs boson has $Y = 1/2$) one has to rescale the $U(1)$ gauge coupling by the factor $\sqrt{3}$, thus one has $g_{SU(2)}/g_{U(1)} = \sqrt{3}$. In consequence the value of the Weinberg angle at the compactification scale $1/R$ is

$$\sin^2\theta_W = \frac{g_{U(1)}^2}{g_{U(1)}^2 + g_{SU(2)}^2} = \frac{1}{4}\,. \tag{2.40}$$

The small difference between the predicted and the experimental value can be explained, e.g., by renormalization group running from the compactification scale to the Z-boson mass scale.

It remains to define the orbifold action in such a way, that the 6d gauge group G_2 is indeed broken down to the electroweak group, and the degrees of freedom identified with the Higgs boson survive in the massless spectrum. The torus can be represented by a $y - z$ plane on which the points are identified periodically, $y \to y + 2\pi R$, $z \to z + 2\pi R$. Ref. [22] chooses the Z_4 action as a $\pi/2$ rotation around the point $(0, 0)$. There are two fixed points, $(0, 0)$ and $(\pi R, \pi R)$. The Z_4 action on the gauge fields is defined as

$$\begin{aligned} A_\mu(x, -z, y) &= U A_\mu(x, y, z) U^\dagger; \\ A_y(x, -z, y) &= -U A_z(x, y, z) U^\dagger; \\ A_z(x, -z, y) &= U A_y(x, y, z) U^\dagger. \end{aligned} \tag{2.41}$$

$$U = \mathrm{diag}(i, i, -1, -i, -i, -1, 1). \tag{2.42}$$

One can choose a basis of the G_2 generators in which the generators of the $SU(3)$ subgroup are 7×7 matrices of the form $T^a = \frac{1}{2\sqrt{2}}\mathrm{diag}(\lambda_a, \lambda_a^T, 0)$, where λ_a are the Gell-Mann matrices. It is now straightforward to see that U commutes with the generators of the $SU(2) \times U(1)$ subgroup of G_2 (those with λ_1,λ_2,λ_3 and λ_8) and therefore the corresponding gauge bosons survive at the massless level. By direct calculation one can prove that no other generator of G_2 commutes with U. It is less trivial to find which components of the internal gauge bosons have massless modes, as the Z_4 action has to be diagonalized first. I turns out that there are two massless $SU(2)$ doublets: one doublet coming from $\mathbf{8}$ in eq. (2.39), H, which has hypercharge $3/2$ and the other one coming from $\mathbf{3}$ and $\bar{\mathbf{3}}$ in eq. (2.39), h, with hypercharge $1/2$. As discussed, it is the latter which should be identified with the SM Higgs boson, to retain a successful prediction for the Weinberg angle. The former doublet should acquire a large positive mass to avoid conflict with experimental data.

2.2.2 5d Grand Unified Theories

Grand Unification is an attractive idea. It consist in the assumption that the SM gauge group is a part of a larger (usually semi-simple) gauge group broken at some higher energy scale. The main reason to study Grand Unified Theories (GUTs), besides those of aesthetical nature, is that they allow to explain the peculiar quantum numbers of quarks and leptons and, in particular, to explain the quantization of electric charge. They also generically predict exotic processes violating the baryon and lepton number (for example proton decay). Occurrence of such processes in nature is necessary to explain the present excess of matter over antimatter - so called baryogenesis. GUTs are even more attractive in the context of low energy supersymmetry. Another generic prediction of GUTs is that the gauge couplings should unify at some energy scale and, indeed, it has been found that the three gauge couplings of the MSSM unify with amazing precision at the scale about $M_{GUT} = 2 \cdot 10^{16}$ GeV.

Successful as they are, GUTs are not free of serious problems. The first is of experimental nature. Current experimental limits on the proton decay rate are stringent enough to exclude some attractive models, for example the simplest supersymmetric SU(5) model. The second, purely theoretical, is called the doublet-triplet splitting problem and is another incarnation of the hierarchy problem. In unified models the MSSM Higgs doublets have to be embedded in some larger representation, e.g. $\mathbf{5}$ of $SU(5)$. The unified partners of the Higgs doublets (called the triplets for $SU(5)$) must acquire mass of the order of the unification scale, otherwise they would mediate proton decay at a too fast rate. However it is very difficult to find consistent models in which the MSSM Higgs doublets are light while their unified partners are super-heavy. Moreover, realistic models of spontaneous breaking of the unified symmetry require quite a complicated Higgs sector.

These are the reasons why the idea of realizing the grand unified symmetry in 5d theories, first put forward by Kawamura [25], have received so much attention. In this framework the unified gauge symmetry is realized in the 5d bulk, but is broken down to the SM subgroup at the fixed points of the orbifold. Consequently, only the SM gauge symmetry is present in the 4d effective theory. Breaking the unified symmetry is extremely simple and does not require any Higgs sector. Moreover, $SU(5)$ multiplets living in the bulk are automatically split in mass. This allows for an elegant solution to the doublet-triplet problem by assuming that the Higgs multiplet propagates in the bulk. It is also possible to avoid some problematic mass relations of conventional GUTs.

For concreteness, let us review one specific model proposed by Hall and Nomura in ref. [26]. The setup is that of 5d supersymmetric $SU(5)$ gauge theory compactified on the orbifold $S_1/Z_2 \times Z_2'$. The $N = 2$ vector supermultiplet, $V = (A_\alpha, \lambda, \Sigma)$, and two Higgs hypermultiplets (equivalent to four chiral multiplets), $(H^U, \tilde{H}^U)$ and $(H^D, \tilde{H}^D)$, propagate in the bulk. H^U is in the fundamental representation, $\mathbf{5}$, and H^D is in the anti-fundamental representation, $\bar{\mathbf{5}}$. In our convention tilded multiplets, $\tilde{H}$, always transform in the complex conjugated representation of H.

The Z_2 action is chosen so as to break half of the supersymmetry, leaving $N = 1$ supersymmetry in the low energy theory, while the Z_2' action is such that it breaks the

$SU(5)$ symmetry. More precisely, the Z_2 action is defined as

$$
\begin{aligned}
A_\mu(x^\mu,-y) &= PA_\mu(x^\mu,y)P^{-1} & H^U(x^\mu,-y) &= PH^U_{\mathbf{5}}(x^\mu,y)\\
A_5(x^\mu,-y) &= -PA_5(x^\mu,y)P^{-1} & \tilde{H}^U(x^\mu,-y) &= -P\tilde{H}^U(x^\mu,y)\\
\lambda_L(x^\mu,-y) &= -P\lambda_L(x^\mu,y)P^{-1} & H^D(x^\mu,-y) &= PH^D(x^\mu,y)\\
\lambda_R(x^\mu,-y) &= P\lambda_R(x^\mu,y)P^{-1} & \tilde{H}^D(x^\mu,-y) &= -P\tilde{H}^D(x^\mu,y)\\
\Sigma(x^\mu,-y) &= -P\Sigma(x^\mu,y)P^{-1} & &
\end{aligned}
\tag{2.43}
$$

and the Z_2' transformation are obtained by replacing y and P by y' and P'. The projecting matrices are chosen as $P = \mathrm{diag}(1,1,1,1,1)$ and $P' = \mathrm{diag}(-1,-1,-1,1,1)$, so that the $SU(5)$ gauge symmetry is reduced down to $G_{\mathrm{SM}} \equiv SU(3)\times SU(2)\times U(1)$. From the $N=2$ superpartners of the gauge field only one set of gauginos transforming in the adjoint of G_{SM} remains in the massless spectrum. This signals that $N=1$ supersymmetry is present in the low energy theory. The two MSSM Higgs doublets arise from the zero-modes of the two bulk Higgs hypermultiplets. The other components of these hypermultiplets, including dangerous triplets, have masses of the order of the compactification scale $1/R$. Thus the doublet-triplet problem is solved if $1/R$ is of the order of the GUT scale, $1/R \sim 10^{16}$ GeV.

There is one immediate problem with this setup. Note that at the fixed point at $y=\pi R/2$ the $SU(5)$ gauge symmetry is not realized because the boundary conditions project out the generators from the coset $SU(5)/SU(3)\times SU(2)\times U(1)$. Thus we can write down gauge kinetic terms for the three SM gauge fields localized at $y=\pi R/2$ and no symmetry requires that the coefficients multiplying each SM gauge field strength are universal. On the other hand at $y=0$ the full $SU(5)$ symmetry is operative. Thus the full set of gauge kinetic terms we can write down is

$$
S = \frac{1}{2}\,\mathrm{Tr}\left\{\int d^4x\,dy\,\left[\frac{1}{g_5^2}F^2 + \delta(y)\frac{1}{\tilde{g}^2}F^2 + \sum_i \delta(y-\pi R/2)\frac{1}{\tilde{g}_i^2}F_i^2\right]\right\},
\tag{2.44}
$$

where $i=1,2,3$ represents the standard model gauge groups. The MSSM gauge couplings at the compactification scale are given by the expression

$$
\frac{1}{g_i^2} = \frac{2\pi R}{g_5^2} + \frac{1}{\tilde{g}^2} + \frac{1}{\tilde{g}_i^2},
\tag{2.45}
$$

Thus, for generic values of $\tilde{g}_i$, the gauge couplings are not unified! The hallmark prediction of conventional 4d GUTs seems to be lost in the 5d models.

However, this problem can be solved by assuming that the extra dimension has a large volume. Since g_5 is dimensionful one can write it as $g_5^2 = \tilde{g}_5^2/M_*$, where $\tilde{g}_5$ is dimensionless and is expected to be of the same order as $\tilde{g}_a$ and $\tilde{g}$. Therefore, by making the radius R of the extra dimension large, $RM_* \gg 1$, the first term in eq. (2.45) dominates over the non-universal contribution from the brane kinetic term and gauge coupling unification is approximately recovered. In practice, it is enough to assume that RM_* is moderately large, of order 50.

The field content of the theory must of course include the quarks and leptons. Quite generally, there are three distinct ways one can introduce MSSM matter fields in this setup. Firstly, they can be realized as 4d $N=1$ multiplets localized at the brane at $y=\pi R/2$. However $SU(5)$ gauge symmetry is broken at this point by the boundary conditions. Such embedding could not explain why the quark and lepton quantum numbers fit into the $SU(5)$ representations, therefore this possibility is not used in ref. [26]. Secondly, matter fields can be localized at the other brane at $y=0$ where the $SU(5)$ symmetry is operative. Since only $N=1$ supersymmetry is preserved at this point, such localized matter can come in the form of $N=1$ chiral multiplets, in the $\overline{\mathbf{5}}$ or $\mathbf{10}$ representation. Finally, it is possible to realize matter fields as $N=2$ hypermultiplets propagating in the bulk. There is one subtlety involved in this choice. By construction, the orbifold projection acting on a bulk multiplet leaves only a SM representation at the massless level. On the other hand we want the MSSM matter fields to come in full $SU(5)$ representations. The way out is to assume that members of full $SU(5)$ representations, e.g. d and l, originate from different hypermultiplets F and F'. It will turn out that this fact is rather an advantage in model-building.

More precisely, the authors of ref. [26] assume that the first two generations are realized as two bulk hypermultiplets $(T_i,\,\tilde{T}_i)$, $(T'_i,\,\tilde{T}'_i)$ and $(F_i,\,\tilde{F}_i)$, $(F'_i,\,\tilde{F}'_i)$, $i=1,2$. Here T and F denote chiral multiplets respectively in the $\mathbf{10}$ and $\overline{\mathbf{5}}$ representation of $SU(5)$. The orbifold projection is defined so as to obtain a generation q,u,d,l,e from the zero modes of bulk fields as $T(u,e)$, $T'(q)$, $F(d)$, $F'(l)$. This can be done, for example, by giving F' the same Z_2 charges as for H in eq. (2.43), while the Z'_2 transformation of F should be given an additional minus sign.

The Yukawa interactions are not consistent with $N=2$ supersymmetry of the bulk. Thus the Yukawa couplings of the matter fields must be localized on the branes. On the $SU(5)$ preserving brane we can write:

$$\begin{aligned} S &= \int d^4x\; dy\; \delta(y)\Big[\int d^2\theta\Big(y_T^1 TTH^U + y_T^2 TT'H^U + y_T^3 T'T'H^U \\ &\quad + y_F^1 TFH^D + y_F^2 TF'H^D + y_F^3 T'FH^D + y_F^4 T'F'H^D\Big) + \text{h.c.}\Big]. \end{aligned} \tag{2.46}$$

The Yukawa interactions of massless matter fields in the low energy theory that follow are

$$S = \frac{1}{(2\pi R)^{3/2}}\int d^4x\,\Big[\int d^2\theta\Big(y_T^2 quh^U + y_F^2 leh^d + y_F^3 qdh^d\Big) + \text{h.c.}\Big]. \tag{2.47}$$

The pre-factor comes from the normalization factor in the KK decomposition of the bulk fields. The fact that d and l come from different hypermultiplets has the consequence that, unlike in conventional GUTs, the Yukawa couplings of leptons and down quarks are not correlated, hence their masses do not unify in general.

On the other hand, the third generation is realized as chiral multiplets T_3 and F_3 localized at the $y = 0$ brane. The Yukawa interactions are like in the conventional GUTs,

$$S = \int d^4x \; dy \; \delta(y) \Big[\int d^2\theta \Big(y'_T T_3 T_3 H^U + y'_F T_3 F_3 H^D \Big) + \text{h.c.} \Big]. \tag{2.48}$$

In the low energy theory the Yukawa interactions of the third generation are

$$S = \frac{1}{(2\pi R)^{1/2}} \int d^4x \; \Big[\int d^2\theta \Big(y'_T q_3 u_3 h^U + y'_F l_3 e_3 h^d + y'_F q_3 d_3 h^d \Big) + \text{h.c.} \Big]. \tag{2.49}$$

The pre-factor is different than for the first two generations because brane matter fields do not yield the normalization factor. Now, on dimensional grounds we expect $y \sim \frac{1}{M_*^{3/2}}$ and $y' \sim \frac{1}{M_*^{1/2}}$. Thus the low energy Yukawa couplings of the first two generations are suppressed by the factor $\frac{1}{RM_*}$ with respect to that of the third generation (recall that in this setup $R \sim 50 M_*^{-1}$). The third generation living on the brane is predicted to be heavier than the first two generations. Another thing to see from eq. (2.49) is that Yukawa couplings of the down quarks and leptons are correlated, which leads to the prediction of equality of b and τ masses at the unification scale. An interesting prediction of this setup is that the heavy generation should exhibit $SU(5)$ mass relations, while the lighter ones should not. This is what we seem to observe in experiment.

Thus we have shown that 5d setups allow for constructing interesting unified models. Of course, a realistic theory should also address such issues as supersymmetry breaking, electroweak breaking, fermion mass generation, and so on. This is discussed at length in a more recent paper of Hall and Nomura [27].

Chapter 3

Introducing deconstruction

In the preceding chapter we argued that gauge theories with extra dimensions provide a setup that can be useful for solving some of the long-standing problems of the SM. In this chapter we introduce 4d gauge theories which predict the same physics as that of higher dimensional theories. More precisely, below certain energy scale, the spectrum and interactions of these 4d theories are analogous to the spectrum and interactions of the KK modes of some higher dimensional theory. This class of 4d theories is called *deconstruction* [4, 5].

As outlined in the introduction, deconstruction is of physical interest because it is a novel tool to study the physics of extra dimensions. Higher dimensional gauge theories are non-renormalizable and therefore their predictions are sensitive to UV physics. On the other hand, deconstruction belongs to the familiar class of 4d gauge theories. In particular, deconstruction is compatible with renormalizability. Thus one can study its low energy predictions without detailed knowledge of the high energy theory.

The following sections give a technical account of deconstruction models. We shall describe in detail the simplest case, that is, a 4d model that mimics 5d gauge theories compactified on a circle. Then we discuss the 4d analogue of orbifold compactifications and how does supersymmetry fit in. Gradually, we shall introduce the dictionary and basic building blocks of deconstruction. This chapter reviews the material contained in refs. [4, 5, 28, 29, 10, 12].

3.1 Deconstruction of 5d gauge theories compactified on S_1

The best way to familiarize with deconstruction is to study one specific version - deconstruction of 5d $U(K)$ gauge theories compactified on the circle S_1. We choose to work with the $U(K)$ group because in this case the matching between the 5d and 4d theories is particularly transparent. Other 5d gauge groups are also possible to deconstruct and we will comment on it later on.

The 4d theory [4] we will analyze is an ordinary gauge theory with several $U(K)$ group factors and a peculiar matter content. It is very convenient to visualize such theories using the so called *quiver diagrams* [30, 31]. The quiver diagram for the presently analyzed model is given in fig. 3.1. Every circle denotes a single gauge group, an in-going line denotes a scalar field in the fundamental representation of the corresponding gauge group, an outgoing line denotes a scalar in the anti-fundamental representation. According to this interpretation, the lines joining two circles denote a scalar field in a bi-fundamental representation, that is, in the fundamental representation under the one, and in the anti-fundamental representation under the other gauge group. These scalars are called the *link-Higgs bosons* (or *links*, in short) and are the distinguishing feature of deconstruction. They provide communication between different gauge groups and it is their dynamics that generates extra dimensions.

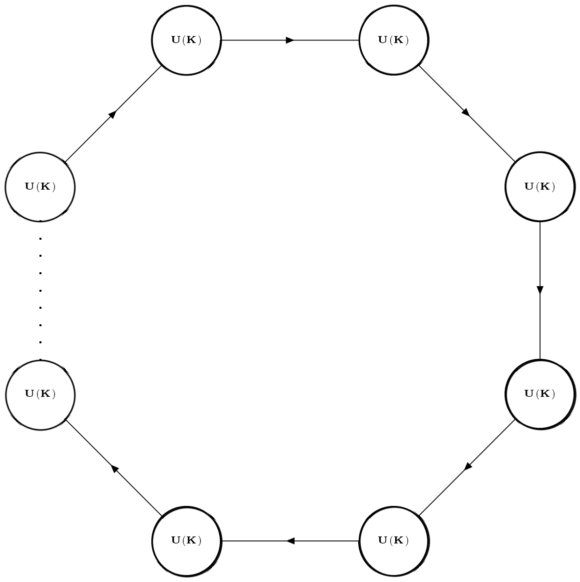


Figure 3.1: The periodic quiver diagram

We call the whole set of groups and links the *group product space*. The overall number of gauge groups is called the *replication number* and denoted by N. The gauge fields corresponding to the $U(K)_p$ gauge group will be represented by $K \times K$ hermitian matrices $A_{\mu,p} = A^a_{\mu,p} T^a$. The action for the gauge fields is simply the sum of N Yang-Mills actions,

$$\mathcal{L} = \sum_{p=1}^{N} \mathrm{Tr}\left\{-\frac{1}{2} F_{\mu\nu,p} F_{\mu\nu,p}\right\}, \tag{3.1}$$

where the field strength is defined as

$$F_{\mu\nu,p} = \partial_\mu A_{\nu,p} - \partial_\nu A_{\mu,p} - i g_p [A_{\mu,p}, A_{\nu,p}]. \tag{3.2}$$

This action is invariant under N separate YM transformations. In the infinitesimal form the transformations read

$$\delta A_{\mu,p} = \partial_\mu \eta_p + i g_p [\eta_p, A_{\mu,p}], \tag{3.3}$$

where the hermitian matrices η_p are the infinitesimal parameters of the transformations. Note that, for $K > 1$, the gauge symmetry does not allow for coupling the field strength tensors corresponding to different gauge groups. This is the group product space version of locality in 5d [1].

The present model contains N link-Higgs bosons, Φ_p. Each transforms in the $\mathbf{K}$ representation of $U(K)_p$ and in $\overline{\mathbf{K}}$ of $U(K)_{p+1}$, $\Phi_p \to U_p \Phi_p U^\dagger_{p+1}$. Thus Φ_p is represented by a general complex $K \times K$ matrix. In the infinitesimal form the gauge transformations are given by:

$$\delta \Phi_p = -i g_p \eta_p \Phi_p + i g_{p+1} \Phi_p \eta_{p+1}. \tag{3.4}$$

The minimal covariant kinetic term has the form

$$\mathcal{L} = 2 \sum_{p=1}^{N} \mathrm{Tr}\left\{D_\mu \Phi_p^\dagger D_\mu \Phi_p\right\}, \tag{3.5}$$

where the covariant derivative reads:

$$D_\mu \Phi_p = \partial \Phi_p + i g_p A_{\mu,p} \Phi_p - i g_{p+1} \Phi_p A_{\mu,p+1}. \tag{3.6}$$

Thus links provide coupling only between the gauge fields of the neighboring gauge group. This time locality in the group product space originates not only from the gauge symmetry but also from the assumed pattern of links. If we introduced the links joining groups with $\Delta p > 1$, the locality in the group product space would be lost.

In this model each pair of neighboring groups is linked, including the N-th and the first group. Therefore we call the models with such group product space *periodic*. The group product index p is identified modulo N, so that $p \equiv p + N$. Such structure is obviously related to the S_1 topology of the fifth dimension we want to deconstruct. Later

[1] For replicated U(1) the gauge symmetry does not forbid the coupling $F_{\mu\nu,p} F_{\mu\nu,r}$ with $p \neq r$ and the locality in the group product space must be imposed by hand. Moreover, because of the presence of the links, such 'nonlocal' couplings will be generated at the quantum level.

on we will formulate the relation between the group product and the physical space in a more precise way.

At this point our model does not yet resemble any 5d theories. The relation between the two theories can be seen only when the link-Higgs scalars acquire vevs. The vevs can be generated by adding an appropriety chosen scalar potential for the links. The energy range where the link vevs are not relevant is called the *unbroken phase*, while the energy range below the scale of the vevs is called the *deconstruction phase*. The form of the 4d/5d relation crucially depends on the precise pattern of the vevs (and on the gauge couplings as well).

In the following we will discuss the case with universal vevs. As we shall see, this case corresponds to 5d gauge theories in the flat (Minkowski) gravitational background (what happens when the vevs are not universal will be discussed in Section 4.1). More precisely, we assume that all links acquire a common vev of the form

$$\langle \Phi_p \rangle = \frac{1}{\sqrt{2}}\, v\, \mathbf{I}\,. \tag{3.7}$$

This can be achieved, e.g., by choosing the scalar potential as

$$V(\Phi_p) = \mathrm{Tr} \sum_{p=1}^{N} \left\{ (\Phi_p^\dagger \Phi_p - \frac{1}{2} v^2)^2 \right\} \,. \tag{3.8}$$

Moreover, we assume that all the gauge couplings have a common value, $g_p = g$. In other words we impose the discrete *shift symmetry* $p \to p + \alpha$ on the model.

Since the links transform under the product gauge group, their vevs lead to a spontaneous breaking of the gauge symmetry. However, the transformations (3.4) with $\eta_p = \eta_{p+1} \equiv \eta$ still leave the vacuum invariant. Thus the product of N $U(K)$ gauge groups is broken to a single $U(K)$ group called the *diagonal group*. The scale v at which this breaking occurs is called the *deconstruction scale*. The gauge coupling of the diagonal group is given by the formula

$$g_0 = \frac{g}{\sqrt{N}} \tag{3.9}$$

Below the deconstruction scale the gauge bosons acquire masses. The mass terms are

$$\mathcal{L} = g^2 v^2 \mathrm{Tr} \sum_{p=1}^{N} \left\{ (A_{\mu,p} - A_{\mu,p+1})^2 \right\} \tag{3.10}$$

and are not diagonal. The mass matrix is

$$\mathcal{M}_A^2 = g^2 v^2 \begin{bmatrix} 2 & -1 & 0 & \dots & -1 \\ -1 & 2 & -1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -1 & 2 & -1 \\ -1 & \dots & 0 & -1 & 2 \end{bmatrix} \tag{3.11}$$

It is possible to find eigenvalues and eigenvectors of this mass matrix for an arbitrary replication number N [4] (the similar problem emerges in coupled oscillator systems and the solution can be found in many textbooks). One finds that the unitary transformation which diagonalizes the mass terms is given by:

$$A_{\mu,p} = \sqrt{\frac{2}{N}} \sum_{n=0}^{(N-1)/2} \left\{ \eta_{(n)} \cos\left(\frac{2n\pi}{N} p\right) A_{\mu,(n)} + \sin\left(\frac{2n\pi}{N} p\right) A_{\mu,(-n)} \right\}, \tag{3.12}$$

where $\eta_{(n)} = \frac{1}{\sqrt{2}}$ for $n = 0$ and $\eta_{(n)} = 1$ for $n \neq 0$. The mass of the n-th mode is given by the formula

$$m_{(n)}^2 = 4 g^2 v^2 \sin^2\left(\frac{n\pi}{N}\right) . \tag{3.13}$$

The mass squared increases monotonically with $|n|$ (note that $|n| < N/2$), but the mass gaps are not equal as in 5d.

Now, let us concentrate on the large N limit and on the modes with $|n| \ll N$. In such case the formula (3.13) can be approximated by

$$m_{(n)}^2 \approx \frac{4 g^2 v^2 \pi^2}{N^2} n^2 . \tag{3.14}$$

This formula is analogous to that for the mass of the n-th KK mode in 5d gauge theory compactified on S_1 (in the flat background), $m_{(n)}^2 = (n/R)^2$. The relation between the parameters of the 4d and 5d theory is

$$\frac{2 g v \pi}{N} \to \frac{1}{R} . \tag{3.15}$$

Thus we have found that the spectra of the gauge bosons in the two theories are the same for large N and for the KK number $|n|$ much smaller than N. An equivalent way to formulate the latter condition is that the correspondence holds at energies much smaller than gv. Thus gv can be identified with the cutoff M_* of the 5d theory. Below gv the two theories have the same spectrum. At energies above gv deconstruction does not match any 5d theory and can be considered as an UV completion thereof.

So far we have shown only that the mass spectrum of the gauge bosons in the two theories can be matched. The correspondence extends also to the interactions. Interactions of the gauge bosons in the deconstruction phase of the 4d theory and in the compactified theory can be found in Appendix A and B. One can easily check that for small $|n|$ the interactions are indeed analogous. It is easy to see the underlying reason why the interactions of the two theories agree. Note namely that the mode decomposition in deconstruction, eq. (3.12), is a discretized version of the KK decomposition in 5d, eq. (2.11). More precisely, the two are analogous if we identify $(gv)^{1/2}A_{\mu,p}(x_\nu) \to A_\mu(x_\nu, y_p)$, $\frac{1}{gv}p \to y_p$. The coarseness of the discretization is measured by the lattice spacing $\Delta y = \frac{1}{gv}$. Note that $\Delta y \to \pi R/N$ thus the large N limit translates into small (compared to the length of the fifth dimension) lattice spacing. The scaling factor in the correspondence between the gauge fields is due to the fact that, in our conventions, the 5d gauge field is dimensionful. The rescaling goes via the natural unit of dimension - the cutoff M_* - which, recall, is mapped to gv in deconstruction.

So far we have ignored the link-Higgs degrees of freedom. Our next task is to investigate if they can be mapped onto some 5d objects. Doing so, it is convenient to assume that the link vevs v are real [2] and split the links as

$$\Phi_p = \frac{1}{\sqrt{2}}(v\,\mathbf{I} + \Sigma_p + iG_p)\,, \tag{3.16}$$

where $\Sigma_p = \Sigma_p^a T^a$, $G_p = G_p^a T^a$ and T^a are the hermitian generators of $U(K)$. The gauge transformations (3.4) with $\eta_p = \eta$ become

$$\delta\Phi_p = -ig[\eta, \Phi_p]\,, \tag{3.17}$$

which shows that the links transform in the adjoint representation of the unbroken diagonal gauge group.

The mass terms for the links clearly depend on the form of the scalar potential. If that of eq. (3.8) is assumed it is easy to see that all the Σ fields acquire the same mass, $m_\Sigma^2 = 2v^2$, while all the G fields stay massless. The masslessness of the latter can be readily understood by symmetry arguments. Note that the potential (3.8) has an accidental global $(U(K)^L \times U(K)^R)^N$ symmetry which acts as

$$\Phi_p \to L_p \Phi_p R_{p+1}^\dagger\,. \tag{3.18}$$

We will call it the *chiral symmetry* (though, of course, no chiral fermions are involved). It is spontaneously broken down to $U(K)^N$ (this is different $U(K)^N$ than the gauge group!) by

[2] In the periodic setup the gauge symmetry allows to gauge away all but one phase. The quantity $v_1 \ldots v_N = v^N e^{i\theta}$ is invariant under re-phasing. However, this nontrivial phase does not play any role in the following.

the vevs of links as only the transformations with $L_p = R_{p+1}$ leave the vacuum invariant. By the Goldstone theorem, breaking of N $U(K)$ groups should result in $N \cdot K^2$ massless Goldstone bosons and these are precisely the N fields G_p.

The vevs break also $N-1$ $U(K)$ gauge symmetries and, as usually for the Higgs mechanism, $N-1$ of the Goldstone bosons become longitudinal components of the massive gauge bosons. When the links are expanded around the minimum, eq. (3.16), the kinetic terms lead to the mixing of G_p with the gauge fields,

$$\mathcal{L} = -2\mathrm{Tr} \sum_{p=1}^{N} \{gv(A_{\mu,p+1} - A_{\mu,p})\partial_\mu G_p\} \ . \tag{3.19}$$

The mixing can be diagonalized by the mode decomposition

$$G_p = \sqrt{\frac{2}{N}} \sum_{n=0}^{(N-1)/2} \left\{ -\sin \frac{2n\pi(p+1/2)}{N} G^{(n)} + \eta_{(n)} \cos \frac{2n\pi(p+1/2)}{N} G^{(-n)} \right\} , \tag{3.20}$$

that brings the mixing to the form

$$\mathcal{L} = -2\mathrm{Tr} \sum_{n=0}^{(N-1)/2} \left\{ m_n A_{\mu,(n)} \partial_\mu G_{(n)} \right\} \ . \tag{3.21}$$

It is explicit that $G_{(n)}$ is eaten by the gauge boson $A_{\mu,(n)}$. Only $G_{(0)} = \frac{1}{\sqrt{N}}(G_1 + \ldots + G_N)$ remains in the physical spectrum as a massless (at the tree-level) scalar in the adjoint representation of the diagonal gauge group.

We have now all the necessary pieces of information to prove that the imaginary part of the links, G_p, can be mapped onto the internal component of the gauge boson in 5d, A_5. Firstly, the representation and the spectrum matches. Indeed, in both theories there is one physical massless scalar, $G_{(0)}$ in deconstruction and the zero-mode $A_5^{(0)}$ in 5d. The remaining degrees of freedom of A_5 and G_p become longitudinal components of the massive gauge bosons. Furthermore, the mode decomposition of G_p, eq. (3.20), is the discretized version of the mode decomposition of A_5, eq. (2.12). In particular both $A_5^{(0)}$ and $G_{(0)}$ are constant in the fifth dimension and in the group product space, respectively. This is the reason why interactions of $G_{(n)}$ and $A_5^{(n)}$ are the same for small $|n|$, which can be checked by consulting Appendix A and B.

What about Σ_p? In our model these are scalar fields in the adjoint representation of the diagonal group, with masses $m^2 = 2v^2$. Such objects clearly have no analogue among the five dimensional degrees of freedom. But, we have argued before that the spectrum of deconstruction matches that of 5d only below the deconstruction scale v. Since Σ_p have masses above that scale they do not destroy the correspondence at low energies. Thus Σ_p

can be considered as part of the UV completion of the 5d theory that ensure the gauge invariance and, possibly, renormalizability of the theory. Alternatively, at energies below v one can decouple Σ_p and work in the non-linear sigma model formalism. This steps amounts to substituting

$$\Phi_p = \exp(\frac{i}{v} G_p) \tag{3.22}$$

in the link kinetic term. The advantage of doing so is that one works only with low energy degrees of freedom, that match precisely those of the 5d theory. However the non-linear sigma model is non-renormalizable and should be treated as a theory with a cutoff of order $\Lambda \sim 4\pi v$. Thus it cannot be considered an UV completion of the 5d theory. Furthermore, we shall see that in supersymmetric models Σ_p do match some 5d degrees of freedom and should not be decoupled.

3.1.1 Relation to lattice gauge theories

We have managed to establish the correspondence between deconstruction and 5d gauge theories by comparing the spectrum and interactions of the 4d model in the deconstruction phase with that of the 5d compactified theory. The matching between the two theories becomes less surprising when we find that the 4d model is just a latticized 5d gauge theory. The gauge groups are the lattice sites while the links are Wilson lines of the fifth component of the gauge field. It is not our purpose here to introduce the whole complicated machinery of lattice gauge theories. Instead we will use a heuristic method that consist in comparing the tree-level action of a deconstruction model with a naively discretized 5d action. By 'naive discretization' we mean the 5d action in which the integral over the fifth coordinate has been replaced by a discrete sum. It should be clearly understood that this method is not rigorous. Firstly, the procedure is not unique - e.g. there are several ways one can discretize the terms with the fifth derivatives and the difference is not bound to vanish in the continuum limit. Secondly, the discretized action does not make sense as a quantum theory, it is not even gauge invariant. Nevertheless the method is a simple and efficient way to understand the matching between 5d and 4d theories.

First, consider the Yang-Mills action in deconstruction,

$$S_{4d} = \frac{1}{2}\mathrm{Tr} \int d^4x \sum_{p=1}^{N} \{F_{\mu\nu,p} F_{\mu\nu,p}\} \ . \tag{3.23}$$

By analyzing the mode decomposition we already found the the relation between the 4d and 5d gauge fields should be

$$\sqrt{gv} A_{\mu,p} \to A_\mu(y_p) \tag{3.24}$$

Performing the substitution in the 4d action we find

$$S_{4d} \to \frac{1}{2}\mathrm{Tr}\int d^4x \sum_{p=1}^{N} \{\Delta y F_{\mu\nu}(y_p)F_{\mu\nu}(y_p)\} \ , \tag{3.25}$$

where the *lattice spacing* Δy is defined as

$$\Delta y = \frac{1}{gv} \, . \tag{3.26}$$

The action of eq. (3.25) is the $F^2_{\mu\nu}$ part of the discretized 5d Yang-Mills action. The fifth dimension is divided into N intervals of equal length Δy. Note that the overall lengths match as, using eq. (3.15), $N\Delta y \to 2\pi R$.

To complete the comparison we must find a match for the $F^2_{\mu 5}$ part. Consider the link-Higgs kinetic terms with inserted link vevs,

$$\begin{gathered} S_{4d} = \int d^4x \mathrm{Tr} \textstyle\sum_{p=1}^{N} \left\{ F^\dagger_{\mu,p} F_{\mu,p} \right\} \\ F_{\mu,p} = (\partial_\mu G_p - i\partial_\mu \Sigma_p + gv(A_{\mu,p} - A_{\mu,p+1}) + gA_{\mu,p}(\Sigma_p + iG_p) - g(\Sigma_p + iG_p)A_{\mu,p+1}) \end{gathered} \tag{3.27}$$

We have already argued that Σ_p do not match any 5d degrees of freedom and therefore we omit it in the following formulae. The remaining terms in $F_{\mu,p}$ can be arranged as follows:

$$F_{\mu,p} = \partial_\mu G_p + gv(A_{\mu,p} - A_{\mu,p+1}) + ig[A_{\mu,p}, G_p] - igG_p(A_{\mu,p+1} - A_{\mu,p}) \, . \tag{3.28}$$

When we identify

$$\begin{aligned} g &\to \frac{1}{\sqrt{\Delta y}} g_5 \, , \\ G_p &\to \sqrt{\Delta y} A_5(y_p) \, , \end{aligned} \tag{3.29}$$

we obtain

$$\begin{gathered} F^\dagger_{\mu,p} F_{\mu,p} \to \Delta y \left| \partial_\mu A_5(y_p) - \tfrac{1}{\Delta y}(A_\mu(y_{p+1}) - A_\mu(y_p)) \right. \\ \left. + ig_5[A_\mu(y_p), A_5(y_p)] - i\Delta y g_5 A_5(y_p)(A_\mu(y_{p+1}) - A_\mu(y_p)) \right|^2 \, . \end{gathered} \tag{3.30}$$

Thus we recover the discretized $F^2_{\mu 5}$ term (where the fifth derivative is discretized as $\partial_5 f(y) \to \partial_5^d f(y_p) \equiv \frac{f(y_{p+1}) - f(y_p)}{\Delta y}$), up to the last term in eq. (3.30). However this term is of order $\mathcal{O}(\Delta y^2)$ and can be neglected for sufficiently small lattice spacing Δy

(which translates into large enough replication number N). In 5d interpretation it can be considered as a higher-derivative term suppressed by the appropriate power of the cutoff scale, $M_* \sim \frac{1}{\Delta y}$. This confirms our previous statement that, in deconstruction, the role of the cutoff is played by gv.

We conclude this subsection by giving the full dictionary between 4d and 5d parameters that holds in the periodic case with universal vevs and gauge couplings,

$$
\begin{aligned}
A_{\mu,p} &\to \sqrt{\Delta y} A_\mu(y_p) \\
G_p &\to \sqrt{\Delta y} A_5(y_p) \\
gv &\to M_* \\
\frac{1}{gv} &\to \Delta y \\
g &\to \frac{1}{\sqrt{\Delta y}} g_5(y_p) \\
\frac{2\pi gv}{N} &\to \frac{1}{R}
\end{aligned}
\tag{3.31}
$$

3.1.2 Bulk matter fields

In many phenomenological models with extra dimensions the bulk hosts not only gauge fields but also matter - fermions and/or scalars. In the following we discuss how such matter fields can be represented in 4d deconstruction models. The example that we study in detail is deconstructing a 5d fermionic field in the fundamental representation of the gauge group [5].

So far we have been able to keep our discussion quite general. The choice of the setup we has been clearly motivated by the properties of the higher dimensional theories we wanted to deconstruct. For example, the periodic group product space corresponded to the S_1 topology of the 5d theory, the shift symmetry corresponded to translational invariance, and so on. As we shall see soon, when it comes to deconstructing matter fields, certain amount of arbitrariness cannot be avoided. The parameter space of deconstruction seems to be much larger than that of higher dimensional theories. In the following we will have to choose special values of some parameters, which can be viewed as a fine-tuning. But we will not bother here what underlying principle could justify this fine-tuning. Our objective will be simply to construct a 4d setup which mimics the spectrum and interaction of matter fields propagating in extra dimensions. The arbitrariness will be reduced (but not removed) when we turn to supersymmetric models.

We expect that, to deconstruct a bulk field, we should introduce a set of 4d fields, each transforming under one gauge group of the product. But recall from Section 2.1 that in 5d there is no chirality and the minimal fermionic field is of the Dirac type. Thus in 4d we actually need to introduce a non-chiral pair of Weyl fermions at each site. We define the fermions ψ_p and $\tilde{\psi}_p$ to be two-component Weyl fermions, respectively in the fundamental and anti-fundamental representation of $U(K)_p$,

$$\psi_p \to U_p \psi_p \,, \qquad \tilde{\psi}_p \to \tilde{\psi}_p U_p^\dagger \,. \tag{3.32}$$

The most general renormalizable action consistent with the shift symmetry is given by

$$\mathcal{L} = \sum_{p=1}^{N} \left\{ i\psi_p \overline{\sigma}^\mu D_\mu \psi_p + i\tilde{\psi}\overline{\sigma}^\mu D_\mu \tilde{\psi}_p + \sqrt{2}Y \tilde{\psi}_p \Phi_p \psi_{p+1} + Y'\sqrt{2}\tilde{\psi}_{p+1}\Phi_p^\dagger \psi_p - m\tilde{\psi}_p\psi_p \right\} + \text{h.c.} \tag{3.33}$$

To proceed, we must further constrain the allowed pattern of Yukawa couplings. One reason is rather practical - it would be difficult to find the low energy spectrum of the fermionic fields except for special, fine-tuned values of Y and Y'. More physical reason is that, if both Y and Y' are non-zero, the mass squared matrix of fermions connects the sites with $\Delta p = 2$, which is at odds with our notion of locality in the group product space. Therefore in the following we set $Y' = 0$. Note that this is equivalent to imposing the chiral symmetry on the Yukawa interactions (3.33), under which the fermions transform as $\psi_p \to L_p \psi_p$, $\tilde{\psi}_p \to \tilde{\psi}_p R_p^\dagger$. In other words we insist on the chiral symmetry in the $m \to 0$ limit. This condition turns out to be crucial if deconstruction is to yield similar results as the 5d theory at the quantum level. For simplicity we shall also assume that all the parameters m and Y are real.

In the unbroken phase each fermion ψ_p, $\tilde{\psi}_p$ has a Dirac mass m. When the links acquire vevs the fermions living at the neighboring site do mix. Still, there are only Dirac mass term and the mass matrix is of the form

$$\mathcal{M}_F = \begin{bmatrix} 0 & \mathcal{M}_D \\ \mathcal{M}_D^\dagger & 0 \end{bmatrix} , \tag{3.34}$$

where

$$\mathcal{M}_D = \begin{bmatrix} m & -Yv & 0 & \dots & 0 \\ 0 & m & -Yv & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & m & -Yv \\ -Yv & 0 & \dots & 0 & m \end{bmatrix} . \tag{3.35}$$

To find the eigenvalues we need to analyse the mass squared matrix,

$$\mathcal{M}_D^\dagger \mathcal{M}_D = \begin{bmatrix} m^2+(Yv)^2 & -mYv & 0 & \dots & -mYv \\ -mYv & m^2+(Yv)^2 & -mYv & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -mYv & m^2+(Yv)^2 & -mYv \\ -mYv & 0 & \dots & -mYv & m^2+(Yv)^2 \end{bmatrix} . \tag{3.36}$$

At each mass level n there are two eigenvalues according to the equation

$$m_{(n)}^2 = (m - Yv)^2 + 4mYv \sin^2\left(\frac{n\pi}{N}\right), \qquad -\frac{N-1}{2} \leq n \leq \frac{N-1}{2}. \tag{3.37}$$

For $n \ll N$ the mass levels can be approximated by:

$$m_{(n)}^2 \approx (m - Yv)^2 + (1 + \frac{m - Yv}{Yv})\left(\frac{2Yv\pi}{N}\right)^2 n^2. \tag{3.38}$$

Comparing it with the 5d mass formula, $m_{(n)}^2 = M^2 + \frac{n^2}{R^2}$, it is apparent that the first term in eq. (3.38) can be identified with the bulk mass M, while the second term in eq. (3.38) resembles the KK tower term n^2/R^2. There is however one problem with such interpretation. The KK spacing of fermions, $\frac{2Yv\pi}{N}$, is in general different than the KK spacing of the gauge bosons, $\frac{2gv\pi}{N}$. Only when we fine-tune the Yukawa coupling,

$$Y = g, \tag{3.39}$$

the KK spacing in the two sectors coincide. We conclude that in a general periodic deconstruction model (with the shift symmetry) one does not necessarily recover 5d Lorentz symmetry in the $N \to \infty$ limit. The 5d Lorentz invariance emerges only when the Yukawa coupling linking the fermions at the neighboring sites is chosen as in eq. (3.39). In such case one can identify,

$$\begin{aligned} m - Yv &\to M \\ \frac{2gv\pi}{N} &\to \frac{1}{R} \\ gv &\to M_*, \end{aligned} \tag{3.40}$$

and the mass formula (3.38) becomes

$$m_{(n)}^2 \to M^2 + \frac{n^2}{R^2} + \frac{M}{M_*}\frac{n^2}{R^2}. \tag{3.41}$$

The low energy spectrum is thus analogous to the 5d one up to the last term. This term is negligible if the bulk mass is much smaller than the cutoff M_* (or, in deconstruction, if $|m - gv|$ is much smaller than gv). Thus another conclusion is that deconstruction can successfully mimic a 5d bulk fermion only if the bulk mass is much smaller than the cutoff. This could be expected, as deconstruction works well only far below the deconstruction scale gv (identified with the 5d cutoff M_*). If the fermion bulk mass is of the order of M_* the physics of the fermion takes place at the scale close to the cutoff where deconstruction is not a good description.

It is instructive to compare the action of deconstruction with the 5d discretized action. Recall, the 4d action after some rearrangement is,

$$S_{4d} = \int d^4x \sum_{p=1}^{N} \left\{ i\psi_p\overline{\sigma}^\mu D_\mu\psi_p + i\tilde{\psi}_p\overline{\sigma}^\mu D_\mu\tilde{\psi}_p + \sqrt{2}Y\tilde{\psi}_p\Phi_p\psi_{p+1} - m\tilde{\psi}_p\psi_p \right\} + \text{h.c.} \quad (3.42)$$

Decomposing the links as $\Phi_p = \frac{1}{\sqrt{2}}(v\mathbf{I} + \Sigma_p + iG_p)$ and dropping Σ_p,

$$\begin{aligned} S_{4d} = \textstyle\int d^4x \sum_{p=1}^{N} \Big\{ & i\psi_p\overline{\sigma}^\mu D_\mu\psi_p + i\tilde{\psi}_p\overline{\sigma}^\mu D_\mu\tilde{\psi}_p + Yv\tilde{\psi}_p(\psi_{p+1} - \psi_p) \\ & +(Yv - m)\tilde{\psi}_p\psi_p + Yi\tilde{\psi}_pG_p\psi_p + iY\tilde{\psi}_pG_p(\psi_{p+1} - \psi_p) \Big\} + \text{h.c.} \end{aligned} \quad (3.43)$$

We define the Dirac spinors $\Psi_p = (\psi_p, \tilde{\psi}_p^\dagger)$. Identifying $\Psi_p \to \sqrt{\Delta y}\Psi(y_p)$, the first line in eq. (3.43) corresponds to the kinetic term of the 5d discretized action,

$$S_{4d} \to \int d^4x \Delta y \sum_{p=1}^{N} \left\{ i\overline{\psi}\gamma^\mu\partial_\mu\psi + \frac{Y}{g}\overline{\psi}\gamma^5\partial_5^d\psi \right\} |_{y=y_p} \,, \quad (3.44)$$

where the discretized fifth derivative is defined as $\partial_5^d f(y_p) = \frac{f(y_{p+1}) - f(y_p)}{\Delta y}$. It is now apparent that the 5d Lorentz invariance (that is, the speed of light in the 5th direction equal to the speed of light in the remaining directions) is preserved in the continuum limit only for $Y = g$. In such case, identifying $m - gv \to M$, $gG_p \to g_5A_5(y_p)$ the second line in eq. (3.43) becomes

$$S_{4d} \to \int d^4x \Delta y \sum_{p=1}^{N} \left\{ -M\overline{\Psi}\Psi + ig_5\overline{\Psi}\gamma_5A_5\Psi + \Delta y g_5\overline{\Psi}\gamma_5A_5\partial_5^d\Psi \right\} |_{y_p} \,. \quad (3.45)$$

The last term is a higher derivative term suppressed by the cutoff $M_* = 1/\Delta y$. We recover the mass term and the Yukawa interaction of the fermion with A_5, the latter being exactly as required by the 5d gauge invariance.

In summary, the dictionary between the 4d and the 5d objects in the bulk fermionic sector is

$$\begin{aligned} \psi_p &\to \sqrt{\Delta y}P_L\Psi(y_p) \\ \tilde{\psi}_p^\dagger &\to \sqrt{\Delta y}P_R\Psi(y_p) \\ m - gv &\to M \end{aligned} \quad (3.46)$$

The matching holds for $Y = g$. For generic values of Y the 4d model correspond to some weird 5d theories where the speed of light of fermion along the fifth dimension is different than the speed of light along the remaining three spatial dimension.

Finally, we comment on deconstructing a complex bulk scalar in the fundamental representation [28]. The procedure is essentially the same as for deconstructing fermions. We introduce the *replicated scalar*, that is a set of N scalars in q_p in the fundamental representation of $U(K)_p$ and we choose their interaction terms as

$$\mathcal{L} = \sum_{p=1}^{N} \left\{ |g\sqrt{2}\Phi_p q_{p+1} - m q_p|^2 \right\} \tag{3.47}$$

This is not the most general set of couplings consistent with the shift symmetry but the virtue is that the chiral symmetry is recovered in the $m \to 0$ limit. We have also fine-tuned the coefficients of the triple and quartic terms to ensure 5d Lorentz invariance in the $N \to \infty$ limit. When the links acquire vevs the mass matrix of q_p takes the form:

$$\mathcal{M}_S^2 = \begin{bmatrix} m^2 + (gv)^2 & -mgv & 0 & \dots & -mgv \\ -mgv & m^2 + (gv)^2 & -mgv & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -mgv & m^2 + (gv)^2 & -mgv \\ -mgv & 0 & \dots & -mgv & m^2 + (gv)^2 \end{bmatrix} . \tag{3.48}$$

It is the same as the replicated fermion mass squared matrix, eq. (3.36). The eigenvalues are

$$m_{(n)}^2 = (m - gv)^2 + 4mgv \sin^2\left(\frac{n\pi}{N}\right) , \tag{3.49}$$

and thus, for $|m - gv| \ll gv$, the replicated scalar corresponds to a 5d bulk scalar with mass $M^2 = (m - gv)^2$.

3.2 Deconstructing gauge theories on S_1/Z_2

Most of the phenomenological applications of 5d gauge theories involves compactification on orbifolds. The main reason is that 5d theories compactified on S_1 are non-chiral and therefore they are of no direct relevance to the SM physics. In this section we describe how 5d theories compactified on the orbifold S_1/Z_2 can be represented by 4d gauge theories [5]. We concentrate on the simplest case when the gauge boson has the positive Z_2 parity (so that gauge symmetry survives at low energies) and when Z_2 commutes with the gauge symmetry.

In 5d one can derive an orbifold theory from the theory formulated on S_1 by imposing a Z_2 symmetry which acts on the fifth coordinate as $y \to -y$. The first thing to note is that the same trick does not work in deconstruction, as the action of the periodic model does not possess an analogue of the Z_2 symmetry. The reason is that the links

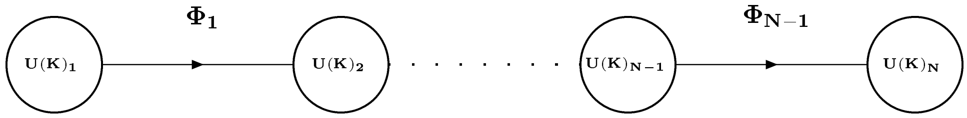


Figure 3.2: The aliphatic quiver diagram.

introduce an intrinsic orientation (the direction of arrows in the quiver diagram of fig. 3.1) in the group product space. The orientation originates from the fact that the fundamental and anti-fundamental representations of $U(K)$ are not equivalent (for $K \neq 2$). Two ways out seem to be possible. One could introduce another set of oppositely oriented link-Higgs fields $\tilde{\Phi}_p$, that is transforming as $\tilde{\Phi}_p \to U_{p+1}\tilde{\Phi}_p U_p^\dagger$, so that the Z_2 symmetry could be defined. Alternatively, one can use the group product space with boundaries to mimic the 5d orbifold in the downstairs picture. We shall follow the latter route here.

Consider a deconstruction model defined by the quiver diagram of fig. 3.2. The only difference between this and the periodic quiver diagram of fig. 3.1 is that the N-th link has been removed, so that $U(K)_N$ and $U(K)_1$ no longer communicate. Adopting chemistry nomenclature we call such group product space *aliphatic*. Given the relation between the group product space and the fifth dimension established in the periodic case this construction is very intuitive, as the 'topology' of the group product space resembles that of the orbifold S_1/Z_2.

The action is a sum of N YM actions for the gauge fields, $N-1$ covariant kinetic terms for the links and a scalar potential which produces the desired pattern of link vevs,

$$\mathcal{L} = \sum_{p=1}^{N} \mathrm{Tr}\left\{-\frac{1}{2}F_{\mu\nu,p}F_{\mu\nu,p}\right\} + \sum_{p=1}^{N-1} 2\,\mathrm{Tr}\left\{D_\mu\Phi_p^\dagger D_\mu\Phi_p\right\} + V(\Phi)\,. \tag{3.50}$$

Again, we expect that the relation between deconstruction and 5d theories will crucially depend on pattern of gauge couplings and link vevs. Below we consider the case with universal vevs and gauge couplings, $g_p = g$ and $v_p = v$. We shall prove that such 4d model possesses the analogous low energy spectrum and interactions as the 5d gauge theory (in the flat background) compactified on the orbifold S_1/Z_2.

Similarly as in the periodic case, the gauge group is broken down to the diagonal subgroup generated by the gauge transformations with $\eta_p = \eta$. The gauge coupling of the diagonal group is given by

$$g_0 = \frac{g}{\sqrt{N}}\,. \tag{3.51}$$

Below the deconstruction scale the gauge bosons acquire masses and the mass terms are

$$\mathcal{L} = g^2 v^2 \mathrm{Tr} \sum_{p=1}^{N-1} \left(A_{\mu,p} - A_{\mu,p+1}\right)^2 . \tag{3.52}$$

The difference to the periodic case is that the sum over the group product index ranges only to $N-1$. The mass matrix takes the form:

$$\mathcal{M}_A^2 = g^2 v^2 \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -1 & 2 & -1 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix} . \tag{3.53}$$

It is possible to find eigenvalues and eigenvectors of this mass matrix for arbitrary N. The matrix of eq. (3.53) is diagonalized by the following mode decomposition:

$$A_{\mu,p} = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \left\{ \eta_{(n)} \cos\left(\frac{n\pi}{N}(p-1/2)\right) A_{\mu,(n)} \right\} . \tag{3.54}$$

The mass of the n-th mode is given by the formula

$$m_{(n)}^2 = 4 g^2 v^2 \sin^2\left(\frac{n\pi}{2N}\right) . \tag{3.55}$$

There is only one gauge boson at each mass level. In the large N limit and for $n \ll N$ the formula (3.55) can be approximated by

$$m_{(n)}^2 \approx \frac{g^2 v^2 \pi^2}{N^2} n^2 . \tag{3.56}$$

This is the same formula as the one for the mass of the n-th KK mode in 5d gauge theories on S_1/Z_2 if one identifies

$$\frac{gv\pi}{N} \to \frac{1}{R} \tag{3.57}$$

Note the factor of two difference with the periodic case.

As for the links, to give them all universal vevs we can choose the analogous scalar potential as in the periodic case,

$$V(\Phi_p) = \mathrm{Tr} \sum_{p=1}^{N-1} \{(\Phi_p^\dagger \Phi_p - \frac{1}{2}v^2)^2\} \tag{3.58}$$

The potential possesses the global chiral symmetry $(U(K)_L \times U(K)_R)^{N-1}$ defined in eq. (3.18). The link vevs break it down to the diagonal subgroup with $L_p = R_{p+1}$ which guarantees $N-1$ massless Goldstone. Choosing all vevs to be real (which we can always do by using the $U(K)^N$ gauge invariance) we split the links as $\Phi_p = \frac{1}{\sqrt{2}}(v\mathbf{I} + \Sigma_p + iG_p)$. Just like in the periodic case, the Goldstone boson are the imaginary parts of the links, G_p. But now $N-1$ broken gauge group require all $N-1$ Goldstone bosons to be eaten by the massive gauge fields. Thus there is no massless physical scalar in the low energy spectrum. This can be checked by investigating the kinetic mixing of G_p with the gauge bosons,

$$\mathcal{L} = -2\mathrm{Tr}\sum_{p=1}^{N-1}\{gv(A_{\mu,p+1} - A_{\mu,p})\partial_\mu G_p\} \ . \tag{3.59}$$

The mixing can be diagonalized by the following mode decomposition

$$G_p = \sqrt{\frac{2}{N}}\sum_{n=1}^{N-1}\left\{\sin\left(\frac{n\pi}{N}p\right)G_{(n)}\right\} , \tag{3.60}$$

which brings the mixing to the form

$$\mathcal{L} = 2\mathrm{Tr}\sum_{n=1}^{N-1}\left\{m_n A_{\mu,(n)}\partial_\mu G_{(n)}\right\} \ . \tag{3.61}$$

We have all the necessary pieces of information to state that the spectrum of the aliphatic deconstruction model with universal vevs and gauge couplings matches that of the 5d gauge theory compactified on S_1/Z_2. The matching between the degrees of freedom is the same as in the periodic case, eq. (3.31), that is the gauge bosons $A_{\mu,p}$ match the 4d components of the 5d gauge boson, A_μ, while G_p matches the internal component of the 5d gauge boson, A_5. Σ_p have no 5d interpretation but the scalar potential gives them mass terms above the deconstruction scale. One can also check [5] that the interactions of the heavy modes in deconstruction match the interactions of the KK modes in 5d. Again, the reason is that the mode decomposition, eq. (3.54), is a discretized version of the KK decomposition in 5d, eq. (2.17).

3.2.1 Bulk matter fields

The next task is to deconstruct a bulk fermion field in the fundamental representation. The important feature of the 5d scenario is that the two chiralities of the fermion must have opposite Z_2 assignments, see eq. (2.19). In particular, since only Z_2 even fields can have a zero-mode, there exists one chiral massless fermionic mode. Thus starting with a non-chiral theory, the Z_2 projection (or, equivalently, the boundary conditions) yields a chiral theory after compactification.

In deconstruction we expect that, at least away from the boundary of the group product space, a bulk fermion should be represented analogously as in the periodic case. Therefore we introduce at each site a pair of fermions ψ_p and $\tilde{\psi}_p$ respectively in the fundamental and anti-fundamental representation of $U(K)_p$. We assume that the interactions of these fermions take the form:

$$\mathcal{L} = \sum_{p=1}^{N-1} \sqrt{2} g \tilde{\psi}_p \Phi_p \psi_{p+1} - \sum_{p=1}^{N} m \tilde{\psi}_p \psi_p + \text{h.c.}\,. \tag{3.62}$$

We have argued in the periodic case that the Yukawa coupling in eq. (3.62) has to be fine tuned to the gauge coupling in order to ensure 5d Lorentz invariance.

It is easy to convince oneself that, at this stage, the 4d model does not yet have a similar spectrum as the 5d model. In particular, since we have introduced an even number of fermionic fields it is not possible to get one chiral zero mode (we can have either none or a positive even number of zero modes). Thus it is necessary to modify the field content of the theory and remove one chiral degree of freedom. Apparently there are many possible choices which fermion to remove. But it turns out that only two of these choices yield a spectrum similar to some 5d model - either we can set $\tilde{\psi}_N \equiv 0$ or $\psi_1 \equiv 0$. Removing a fermionic degree of freedom at the boundary of the group product space is reminiscent to the Z_2 projection at the fixed point in 5d. However the analogy is not quite precise. In 5d Z_2 removes a chiral component at both fixed points, while in deconstruction we remove a fermion only at one boundary (if we did it at both boundaries, e.g. setting $\tilde{\psi}_N \equiv 0$ and $\psi_1 \equiv 0$, we would end up with an even number of chiral fermions and the original problem would remain). In deconstruction the two ends of the group product space are not equivalent. This is so because the presence of the link-Higgs scalars introduces an orientation of the group product space. In 5d there is no such orientation and the two fixed points are interchangeable.

If we decide on the choice $\tilde{\psi}_N = 0$ the mass matrix of the fermions takes the form

$$\mathcal{M}_F = \begin{bmatrix} 0 & \mathcal{M}_D^\dagger \\ \mathcal{M}_D & 0 \end{bmatrix}, \tag{3.63}$$

where $\mathcal{M}_D$ is now an $N-1 \times N$ matrix,

$$\mathcal{M}_D = \begin{bmatrix} m & -gv & 0 & \dots & 0 \\ 0 & m & -gv & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & m & -gv \end{bmatrix}. \tag{3.64}$$

The mass squared matrices are

$$\mathcal{M}_D^\dagger \mathcal{M}_D = \begin{bmatrix} m^2 & -mgv & 0 & \dots & 0 \\ -mgv & m^2+(gv)^2 & -mgv & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -mgv & m^2+(gv)^2 & -mgv \\ 0 & 0 & \dots & -mgv & (gv)^2 \end{bmatrix} , \tag{3.65}$$

$$\mathcal{M}_D \mathcal{M}_D^\dagger = \begin{bmatrix} m^2+(gv)^2 & -mgv & 0 & \dots & 0 \\ -mYv & m^2+(gv)^2 & -mgv & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -mgv & m^2+(gv)^2 & -mgv \\ 0 & 0 & \dots & -mgv & m^2+(gv)^2 \end{bmatrix} . \tag{3.66}$$

The two above matrices should of course have the same non-zero eigenvalues. In addition, the matrix (3.65) has a zero eigenvalue corresponding to the zero mode. Eigenvectors of the matrix (3.65) determine the mode decomposition of ψ_p, while eigenvectors of the matrix (3.66) determine the mode decomposition of $\tilde{\psi}_p$. On top of the massless mode there is a tower of fermions with masses

$$m_{(n)}^2 = g^2 v^2 \left[(1 - \frac{m}{gv})^2 + 4\frac{m}{gv} \sin^2\left(\frac{n\pi}{2N}\right)\right] , \tag{3.67}$$

It is clear that the spectrum matches that of the gauge bosons only if the mass term m is fine-tuned as $m = gv$. In such case the spectrum becomes

$$m_{(n)}^2 = 4 g^2 v^2 \sin^2\left(\frac{n\pi}{2N}\right) , \tag{3.68}$$

and matches the 5d formula $m_{(n)} = n/R$ if the radius is identified as in eq. (3.57). The mode decomposition is

$$\psi_p = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \left\{ \eta_{(n)} \cos\left(\frac{n\pi}{N}(p-1/2)\right) \psi_{(n)} \right\} , \tag{3.69}$$

$$\tilde{\psi}_p = \sqrt{\frac{2}{N}} \sum_{n=1}^{N-1} \left\{ \sin\left(\frac{n\pi}{N} p\right) \tilde{\psi}_{(n)} \right\} , \tag{3.70}$$

and is the discretized version of the 5d KK decomposition.

Recall that in the periodic case, $m \neq gv$ corresponds to the presence of a bulk fermion mass term in 5d. It turns out that in the aliphatic case $m \neq gv$ corresponds to a 5d kink-mass term (the ordinary mass term is not allowed on the orbifold).

We turn to bulk scalar fields, massless for definiteness. In 5d scalars can have either positive or negative Z_2 parity. In both cases the mass spectrum of the KK modes is $m_{(n)} = n/R$, but for negative parity the zero mode is absent. In deconstruction, for scalars we do not have chirality to guide us and thus we must simply guess what is the analogue of the orbifold projection. The goal is to get the same matrix as the matrices squared for fermions, eq. (3.65) for positive parity and eq. (3.66) for negative parity. It is easy to see that the former mass matrix is recovered for the choice

$$\mathcal{L} = \sum_{p=1}^{N-1} |g\sqrt{2}\Phi_p q_{p+1} - gvq_p|^2 \tag{3.71}$$

Interestingly enough, the mass term for the N-th scalar, $\mathcal{L} \sim q_N^\dagger q_N$, must be absent, otherwise the mass matrix would not yield a massless mode. The other mass matrix, that corresponding to negative scalars, is recovered if the interactions are of the form

$$\mathcal{L} = \sum_{p=1}^{N} |g\sqrt{2}\Phi_{p-1}^\dagger q_{p-1} - gvq_p|^2 \tag{3.72}$$

and, in addition, the N-th scalar is projected out, $q_N \equiv 0$. In both cases the analogue of the Z_2 projection in deconstruction is counterintuitive and cannot be derived by latticization methods. It is only the matching of the low-energy spectra that allows to establish the correspondence between the 5d theory on orbifold and deconstruction.

3.3 Deconstruction and supersymmetry

In this section we discuss how supersymmetric gauge theories in 5d can be matched in deconstruction. One consequence of wedding supersymmetry with 5d Lorentz invariance is that a minimal theory contains eight supercharges, which corresponds to $N = 2$ supersymmetry in 4d. However in deconstruction the correct procedure [29] is to start with just $N = 1$ supersymmetry. The second supersymmetry is generated dynamically along with the fifth dimension.

To visualize supersymmetric models we use similar quiver diagrams as in the non-supersymmetric case but we interpret them a little differently. Now every circle (site) denotes an $N = 1$ gauge multiplet, $V_p = (A_{\mu,p}, \chi_p)$ while every line denotes a chiral multiplet in the fundamental (outgoing line) or anti-fundamental (in-going line) representation of the corresponding gauge group. In particular, link-Higgs scalars are promoted to link-Higgs chiral multiplets, $\Phi_p = (\Phi_p, \Psi_p)$ transforming as $\Phi_p \to U_p \Phi_p U_{p+1}^\dagger$. It turns out that no more adjustment of the deconstruction setup is needed to match 5d theories.

For concreteness, let us concentrate on the periodic model of fig. 3.1, with the shift symmetry imposed (so that the gauge couplings are all equal). The first thing to note is that in supersymmetric models we cannot freely adjust the scalar potential of the link-Higgs scalar. If we stick to renormalizable interactions, the potential is fixed to be the D-term potential of the form:

$$V_D = \frac{1}{2} g^2 \sum_{p=1}^{N} \left\{ \left(\mathrm{Tr}(\Phi_p^\dagger T^a \Phi_p) - \mathrm{Tr}(\Phi_{p-1} T^a \Phi_{p-1}^\dagger) \right)^2 \right\} . \tag{3.73}$$

Luckily, this potential is minimized by universal vevs, $\langle \Phi_p \rangle = \frac{1}{\sqrt{2}} v \mathbf{I}$. More precisely, the deconstruction scale v is a flat direction. Thus in supersymmetric periodic model no particular scale is singled out, but at least arbitrary deconstruction scale is consistent with supersymmetry.

The link vevs break the product gauge group down to the diagonal U(K). The mass terms of the gauge bosons are not affected by supersymmetry, so the mode decomposition is given by eq. (3.12) and the mass tower is given by the formula

$$m_{(n)} = 2gv \sin(|n|\pi/N) . \tag{3.74}$$

The matching between 5d and 4d takes the analogous form as in the non-supersymmetric case.

Since supersymmetry is unbroken, there must be a tower of gauginos with the same masses as that of the gauge bosons. Indeed, supersymmetry implies that the lagrangian contains Yukawa interactions of the form:

$$\mathcal{L} = \sum_{p=1}^{N} \left\{ -ig2\sqrt{2}\mathrm{Tr}(\chi_{p+1} \Phi_p^\dagger \Psi_p) + ig2\sqrt{2}\mathrm{Tr}(\chi_p \Psi_p \Phi_p^\dagger) + \mathrm{h.c.} \right\} \tag{3.75}$$

These interactions are similar to those analyzed in relation to deconstructing a bulk fermion. Note that the fine-tuning of the Yukawa coupling necessary to recover 5d Lorentz invariance is an automatic consequence of supersymmetry. When the links acquire vevs the gauginos χ_p couple via Dirac mass terms to link-Higgsinos Ψ_p. After the unitary rotations,

$$\chi_p = \sqrt{\frac{2}{N}} \sum_{n=-(N-1)/2}^{(N-1)/2} \left\{ \eta_{(n)} \cos\left(\frac{2n\pi}{N} p\right) \chi_{(n)} + \sin\left(\frac{2n\pi}{N} p\right) \chi_{(-n)} \right\} , \tag{3.76}$$

$$i\Psi_p = \sqrt{\frac{2}{N}} \sum_{n=-(N-1)/2}^{(N-1)/2} \left\{ -\sin\left(\frac{2n\pi}{N}(p+1/2)\right) \Psi_{(n)} + \eta_{(n)} \cos\left(\frac{2n\pi}{N}(p+1/2)\right) \Psi_{(-n)} \right\} , \tag{3.77}$$

the Dirac spinor $\lambda_{(n)} = (\Psi_{(n)}, \chi^\dagger_{(n)})$ acquires mass equal to that of the n-th level gauge boson. In particular, there is one massless Dirac gaugino. At each massive level there are two Dirac gauginos. These fermions match the KK modes of the 5d gaugino. It is interesting to note that the left and right components of the 5d gaugino match different objects in deconstruction. The right component matches the 4d gaugino which is in the adjoint representation, while the left matches the link-Higgsino which is in the bi-fundamental representation. But from the point of view of the diagonal gauge group these two representations reduce to the adjoint - the correct representation for gauginos.

We move to discussing the degrees of freedom corresponding to the link-Higgs scalars. The D-term potential of eq. (3.73) also possesses the chiral symmetry, and the spontaneous breaking thereof results in N massless Goldstone bosons. These are imaginary parts of the links, G_p. $N-1$ become longitudinal components of the massive gauge bosons, while the constant mode $G_{(0)}$ is a physical scalar. So far this is all the same as in the non-supersymmetric case. The crucial difference lies in the scalars Σ_p corresponding to the real part of the link-Higgs scalars. Contrary to the non-supersymmetric case we cannot decouple them by giving them a large mass term. Instead, by expanding the potential (3.73) it is possible to check that the mass matrix of Σ_p has exactly the same form as that of the gauge bosons. In consequence, after diagonalization, the eigenvectors $\Sigma_{(n)}$ form a tower with masses given by eq. (3.74). Thus $\Sigma_{(n)}$ clearly correspond to the KK modes of the scalar component of the 5d vector multiplet (also denoted by Σ, see Section 2.1). Unlike non-supersymmetric models, the real part of the links is not an object we should get rid of to match a 5d theory. All the degrees of freedom involved in the model correspond to some 5d objects. Somehow, deconstruction is tailor-made for supersymmetry.

Let us summarize the low energy spectrum. At the massless level there is the gauge field, $A_{\mu,(0)}$, the Dirac gaugino, $\Psi_{(0)}$, and two massless scalars, $\Sigma_{(0)}$ and $G_{(0)}$, all in the adjoint representation of $U(K)$. These are exactly the components of an $N=2$ vector multiplet in 4d. Also at each massive level the spectrum matches that of massive $N=2$ supersymmetry. Although we started with just one supersymmetry, the second supersymmetry has appeared dynamically below the deconstruction scale. But this must be so, since we generate the fifth dimension and 5d Lorentz invariance is incompatible with $N=1$ supersymmetry. Above the deconstruction scale there is only one supersymmetry since there are chiral fermions (the link-higgsinos). and the links are in different representations than the gauge multiplet.

The presence of chiral fermions raises the question of anomalies. 5d gauge theories on S_1 are of course anomaly free, as all the fields in the gauge supermultiplet are in a

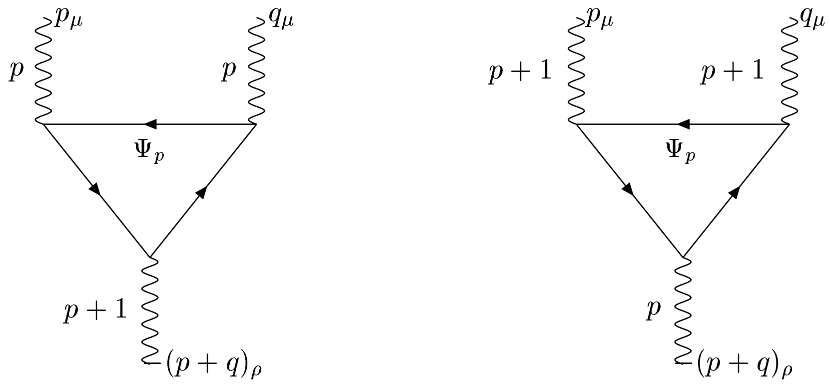


Figure 3.3: Triangle diagrams leading to mixed anomalies.

real (adjoint) representation. However deconstruction above the scale v does not match any 5d theory and thus anomalies can appear. Indeed, one can see that the periodic supersymmetric deconstruction model has mixed anomalies [10]. For simplicity, below we concentrate on the case of deconstructed $U(1)$ theories.

The anomalous triangle diagrams involve the chiral currents of the p-th and $p+1$-th group and the fermion circulating in the loop is the p-th link-higgsino, see fig. 3.3. The triangles with three p-th currents vanish because the links Ψ_p and Ψ_{p+1} contribute with opposite signs. It is well known that the form of the anomalous variation of the action depends on the way we regularize the triangle diagrams. In this context a natural regularizations is the one in which the anomalies are distributed symmetrically among all the currents.In such case the anomalous variation of the deconstruction action is given by:

$$\begin{aligned}\delta\mathcal{L}_{an} = \tfrac{1}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}\textstyle\sum_p \epsilon_p\ (F_{\mu\nu,p+1}F_{\rho\sigma,p+1} - F_{\mu\nu,p-1}F_{\rho\sigma,p-1}\\ -2F_{\mu\nu,p}F_{\rho\sigma,p+1} + 2F_{\mu\nu,p}F_{\rho\sigma,p-1})\ ,\end{aligned} \tag{3.78}$$

or, in the superspace formalism:

$$\begin{aligned}\delta\mathcal{L}_{an} = -\tfrac{i}{12\pi^2}\textstyle\sum_p \int d^2\theta\ \Lambda_p\ \Big(W^\alpha_{p+1}W_{\alpha,p+1} - W^\alpha_{p-1}W_{\alpha,p-1}\\ -2W^\alpha_p W_{\alpha,p+1} + 2W^\alpha_p W_{\alpha,p-1}\Big) + \text{h.c}\ ,\end{aligned} \tag{3.79}$$

In the case at hand it is possible to get rid of the anomaly by adding a Wess-Zumino (WZ) type term to the deconstruction action. Insisting on locality in the group product space, there are five various terms that have 'anomalous' variations under gauge transformations,

$$\begin{gathered}\mathcal{L}_{WZ} = \textstyle\sum_p \mathcal{L}_{WZ,p}\\ \mathcal{L}_{WZ,p} = -\tfrac{C}{12\pi^2}\epsilon_{\mu\nu\rho\sigma}\,[\\ \tfrac{1}{2i}\log(\tfrac{\Phi_p}{\Phi_p^\dagger})(a\partial_\mu A_{\nu,p}\partial_\rho A_{\sigma,p} + b\partial_\mu A_{\nu,p+1}\partial_\rho A_{\sigma,p+1} + c\partial_\mu A_{\nu,p}\partial_\rho A_{\sigma,p+1})\\ -(1+d)A_{\mu,p}\partial_\nu A_{\rho,p}A_{\sigma,p+1} + (1-e)A_{\mu,p+1}\partial_\nu A_{\rho,p+1}A_{\sigma,p}]\end{gathered} \tag{3.80}$$

In deconstruction we should also impose the condition that, in the continuum limit, $\mathcal{L}_{WZ}$ matches some 5d Lorentz invariant term. The obvious candidate [32] is the 5d Chern-Simons (CS) term,

$$\begin{aligned}\mathcal{L}_{CS} &= -\tfrac{C}{12\pi^2}\epsilon_{\alpha\beta\gamma\delta\epsilon}\left[A_\alpha\partial_\beta A_\gamma\partial_\delta A_\epsilon\right] \\ &= -\tfrac{C}{12\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[3A_5\partial_\mu A_\nu\partial_\rho A_\sigma - 2A_\mu\partial_\nu A_\rho\partial_5 A_\sigma\right]\end{aligned} \tag{3.81}$$

Using the dictionary of eq. (3.31) one can see that any of the terms in the first line of eq. (3.80) is a discretization of the term containing A_5 in the CS term (3.81), up to higher derivate terms. Similarly, any of the terms in the second line of eq. (3.80) is a discretization of the term containing ∂_5 in the CS term. Therefore, if $\mathcal{L}_{WZ}$ is to have a 5d Lorentz invariant limit we should impose the constraints: $a+b+c=3$ and $e=d$ in eq. (3.80). Then the variation of $\mathcal{L}_{WZ}$ is given by:

$$\begin{aligned}&\delta\mathcal{L}_{WZ,p} = -\tfrac{C}{12\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[-a\epsilon_p\partial_\mu A_{\nu,p}\partial_\rho A_{\sigma,p} + b\epsilon_{p+1}\partial_\mu A_{\nu,p+1}\partial_\rho A_{\sigma,p+1}\right.\\ &+(1-b-d)\epsilon_p\partial_\mu A_{\nu,p+1}\partial_\rho A_{\sigma,p+1} - (2-a-b-d)\epsilon_p\partial_\mu A_{\nu,p}\partial_\rho A_{\sigma,p+1}\\ &\left.-(1-a+d)\epsilon_{p+1}\partial_\mu A_{\nu,p}\partial_\rho A_{\sigma,p} + (2-a-b+d)\epsilon_{p+1}\partial_\mu A_{\nu,p}\partial_\rho A_{\sigma,p+1}\right]\end{aligned} \tag{3.82}$$

For $a=b$ the unmixed anomalies vanish when we sum over p and thus we assume $a=b$ from now on. One thing to note is that various discretizations of the CS term yield different result from the point of view of deconstruction. For example, for $a=b=1$ and $d=0$ $\mathcal{L}_{WZ}$ does not yield anomalies at all. In such form, the effect of $\mathcal{L}_{WZ}$ is very similar to that of the 5d CS term, whose variation is a total derivative that vanishes after integrating over y. But this is not what we want here. In order to cancel the mixed anomalies of eq. (3.78) we have to choose $d=0$ and $a=1-1/C$. Hence the most general 5d invariant WZ term we can choose is given by:

$$\begin{aligned}&\mathcal{L}_{WZ,p} = -\tfrac{C}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}\tfrac{1}{2i}\log\left(\tfrac{\Phi_p}{\Phi_p^\dagger}\right)\\ &\left[(1-1/C)F_{\mu\nu,p}F_{\rho\sigma,p} + (1-1/C)F_{\mu\nu,p+1}F_{\rho\sigma,p+1} + (1+2/C)F_{\mu\nu,p}F_{\rho\sigma,p+1})\right]\\ &+\tfrac{C}{24\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[A_{\mu,p}A_{\nu,p+1}(F_{\rho\sigma,p}+F_{\rho\sigma,p+1})\right]\end{aligned} \tag{3.83}$$

In order to supersymmetrize our WZ term we note that $\mathcal{L}_{WZ}$ can be derived from the superspace action,

$$\begin{aligned}&\mathcal{L}_{SWZ} = -\tfrac{1}{24\pi^2}\int d^2\theta\log\Phi_p\,\Big[\\ &(C-1)W_{\alpha,p}W_p^\alpha + (C-1)W_{\alpha,p+1}W_{p+1}^\alpha + (C+2)W_{\alpha,p}W_{p+1}^\alpha\Big]\\ &-\tfrac{C}{24\pi^2}\int d^4\theta\left[(V_pD_\alpha V_{p+1} - V_{p+1}D_\alpha V_p)W_p^\alpha + (V_pD_\alpha V_{p+1} - V_{p+1}D_\alpha V_p)W_{p+1}^\alpha\right]\\ &+\text{h.c.}\end{aligned} \tag{3.84}$$

The action of eq. (3.84) is not yet 5d invariant in the continuum limit. For example, the coefficient of the $F_{\mu\nu}^2$ term is not properly correlated with that of the $F_{\mu 5}^2$. The former is set by the holomorphic gauge kinetic function in eq. (3.84) and, in the continuum limit, we get (ignoring higher derivative terms):

$$\mathcal{L} = \sum_p \Delta y \frac{C}{8\pi^2} \Sigma(y_p) F_{\mu\nu}(y_p) F_{\mu\nu}(y_p) \rightarrow \frac{C}{8\pi^2} \int dy \Sigma F_{\mu\nu} F_{\mu\nu} \tag{3.85}$$

To preserve 5d Lorentz invariance we need the corresponding $F_{\mu 5}^2$ term, $\mathcal{L} = -\frac{C}{4\pi^2} \int dy \Sigma F_{\mu 5} F_{\mu 5}$ A discretized version of this term appears in deconstruction if the minimal Kahler potential is supplemented by a non-minimal term:

$$K = -\frac{C}{48\pi^2} g^2 v^2 (\chi_p + \bar{\chi}_p + 2V_p - 2V_{p+1})^3 \, , \tag{3.86}$$

where $\chi_p \equiv \log \frac{\sqrt{2}\Phi_p}{v}$. It is easy to see that
Summarizing, consistent deconstruction of the 5d supersymmetric $U(1)$ theory requires introducing WZ terms in the 4d action. In the continuum limit these WZ terms match the 5d CS term. Recall that in 5d theories on S_1 the CS term does not play any role in anomaly cancellation as its variation is a total derivative. However in deconstruction an analogue of this term must be present in order to cancel the mixed anomalies. Note that the particular choice $C = 0$ in eq. (3.84) corresponds to no CS term in 5d (while the WZ terms in 4d are still non-trivial); in fact we are able to deconstruct the 5d theory containing the CS term with an arbitrary coefficient.

Our next step is to deconstruct bulk matter fields. In 5d supersymmetric theories matter fields take the form of hypermultiplets which, in the $N = 1$ supersymmetric language, consist of a non-chiral pair of chiral multiplets. Thus, to mimic hypermultiplets in deconstruction [29] we introduce at each site two sets of chiral multiplets, $H_p = (H_p, \psi_p^H)$ and $\tilde{H}_p = (\tilde{H}_p, \tilde{\psi}_p^H)$ respectively in the fundamental and anti-fundamental representation of $U(K)_p$. The most general renormalizable superpotential consistent with the shift symmetry is the following:

$$W = \sum_{p=1}^{N} \left\{ \sqrt{2} Y \tilde{H}_p \Phi_p H_{p+1} - m \tilde{H}_p H_p \right\} \, . \tag{3.87}$$

The low energy spectrum of the fermionic fields ψ^H and $\tilde{\psi}^H$ is determined by the following interactions resulting from the superpotential (3.87):

$$\mathcal{L} = \sum_{p=1}^{N} \left\{ \sqrt{2} Y \tilde{\psi}_p^H \Phi_p \psi_{p+1}^H - m \tilde{\psi}_p^H \psi_p^H \right\} + \text{h.c.} \tag{3.88}$$

This are exactly the same interactions as that analyzed in Section 3.1.2 in the context of deconstructing bulk matter fields. First of all, we argued that the Yukawa coupling Y must be fine-tuned to the gauge coupling, $Y = g$, in order to recover the 5d Lorentz invariance in the large N limit. In supersymmetric theories we see that this fine-tuning is also the one to match $N = 2$ supersymmetry, as $N = 2$ requires that all Yukawa interactions in the theory are governed by the gauge couplings. Secondly, in the non-supersymmetric case we had to impose the chiral symmetry on Yukawa interactions to avoid the coupling $\tilde{\psi}_{p+1}\Phi_p^\dagger\psi_p$. In supersymmetry, the absence of such couplings is the consequence of holomorphicity of the superpotential. Thus supersymmetry reduces the degree of arbitrariness involved in deconstructing matter fields. Still however, the 5d Lorentz invariance in the continuum limit must be imposed by hand.

When the links acquire vevs the mass matrix of the fermions is diagonalized by the decomposition analogous to that of eq. (3.76) and eq. (3.77). The eigenvalues in the large N limit match the mass level of the massive bulk fermion, $m_{(n)}^2 = M^2 + (n/R)^2$, where the bulk mass term is identified as, $m - gv \to M$.

By supersymmetry, the scalars H and $\tilde{H}$ must have the same mass eigenvalues as fermions. Indeed, the relevant interactions following from the superpotential (3.87) are given by:

$$V = \sum_{p=1}^{N} \left\{ |mH_p - gvH_{p+1}|^2 + |m\tilde{H}_p - gv\tilde{H}_{p-1}|^2 \right\} . \tag{3.89}$$

It is straightforward to check that the mass matrix of both set of scalars is the same as the mass squared of fermions. Thus, in the deconstruction phase, at each mass level with $|n| > 0$ there are two Dirac fermions and four complex scalars. At the mass level corresponding to $n = 0$ there is one Dirac fermion and two complex scalars, forming an $N = 2$ hypermultiplet.

In the remainder of this section we comment on deconstructing supersymmetric gauge theories on orbifolds is in order. Subtleties that emerge are most transparently seen in the example of deconstructing 5d $U(1)$ gauge theory compactified on S_1/Z_2 [12, 10]. We consider a model defined by the quiver diagram of fig. 3.2 with $K = 1$. The links Φ_p are now charged as $(Q, -Q)$ under the p-th and $p + 1$-th gauge group, respectively. The first thing to notice is that the deconstruction scale v ceases to be a D-flat direction of the scalar potential (3.73). For example, the D-term of the first group takes the form $D_1 = gQ^2|\Phi_1|^2$ and it vanishes only for $\langle\Phi_1\rangle = 0$. All in all, the unique minimum of this potential is for $\langle\Phi_p\rangle = 0$ and thus there is no energy range where the model could match some 5d theory. The way out is to allow for Fayet-Iliopoulos (FI) terms. Adding the FI terms $\sum_p 2\xi_p[V_p]_D$ results in the scalar potential:

$$V = \frac{1}{2}g^2 \left[(Q|\Phi_1|^2 + \xi_1)^2 + (Q|\Phi_2|^2 - Q|\Phi_1|^2 + \xi_2)^2 + \ldots + (-Q|\Phi_{N-1}|^2 + \xi_N)^2 \right] . \tag{3.90}$$

The unique minimum with unbroken supersymmetry satisfies:

$$\begin{aligned}
Q\langle\Phi_1\rangle^2 &= -\xi_1\,, \\
Q\langle\Phi_2\rangle^2 &= -\xi_1-\xi_2\,, \\
&\ldots \\
Q\langle\Phi_{N-1}\rangle^2 &= -\xi_1-\xi_2-\ldots-\xi_{N-1}=\xi_N\,.
\end{aligned} \tag{3.91}$$

As we are interested here in models with universal link-Higgs vevs we must further constrain $\xi_2=\ldots=\xi_{N-1}=0$, thus we must forbid the appearance of FI terms in all except the boundary gauge groups. Furthermore, the existence of a supersymmetric minimum requires the (fine-tuning) condition on the FI terms and additional conditions on their signs:

$$\begin{aligned}
\xi_1+\xi_N &= 0\,, \\
Q\xi_1 &< 0\,.
\end{aligned} \tag{3.92}$$

(From the above it follows that $Q\xi_N>0$.) The former condition has an analogue in 5d, where the sum of fixed point localized FI terms has to vanish if the vacuum is to preserve supersymmetry. The latter has no corresponding condition in the 5d theory. This signals that the role of FI terms in deconstruction cannot be exactly mapped on the 5d theory. At this stage the FI terms are introduced 'by hand'. Once their magnitude is chosen, the deconstruction scale is unambiguously determined, thus the arbitrariness in choosing the FI terms translates into arbitrariness of choosing the cut-off scale of the 5d theory.

As in the periodic case, the presence of the links is a source of mixed anomalies that have to be canceled by WZ terms. But in the aliphatic case the anomalous variation of the action contains also the standard anomalies in the boundary groups,

$$\begin{aligned}
\delta\mathcal{L}_{an} = -\tfrac{i}{12\pi^2}Q^2\textstyle\sum_{p=1}^{N-1}\int d^2\theta\ \Lambda_p\ \Big(W^\alpha_{p+1}W_{\alpha,p+1}-W^\alpha_{p-1}W_{\alpha,p-1}-2W^\alpha_pW_{\alpha,p+1}+2W^\alpha_pW_{\alpha,p-1}\Big) \\
-\tfrac{i}{12\pi^2}Q^2\textstyle\int d^2\theta\ \left(\Lambda_1\ W^\alpha_1W_{\alpha,1}-\Lambda_N\ W^\alpha_NW_{\alpha,N}\right)+\text{h.c}\ ,
\end{aligned} \tag{3.93}$$

To cancel the mixed anomalies we need to choose any of the family of the WZ terms of eq. (3.84). In the aliphatic setup the gauge variation of the WZ term contains also 'boundary' terms,

$$\delta\mathcal{L}_{SWZ}=-\frac{i}{12\pi^2}(C-1)\int d^2\theta\ \left(\Lambda_1\ W^\alpha_1W_{\alpha,1}-\Lambda_N\ W^\alpha_NW_{\alpha,N}\right) \tag{3.94}$$

Simultaneous cancellation of the mixed and boundary anomalies uniquely sets $C=0$. Therefore the continuum limit of the deconstructed theory is a 5d theory with no CS term, as it should be in the absence of matter.

Another interesting example is that of the aliphatic model with two 'boundary' fields X_L and X_R with charges -1 under $U(1)_1$ and $+1$ under $U(1)_N$, respectively. In the

5d picture they correspond to two chiral multiplets of opposite charge living on the two different branes. Such 5d setup is inconsistent unless we add the Chern-Simons term of the form:

$$\mathcal{L}_{CS} = -\frac{1}{12\pi^2}\epsilon_{\alpha\beta\gamma\delta\epsilon}\left[A_\alpha \partial_\beta A_\gamma \partial_\delta A_\epsilon\right] . \tag{3.95}$$

In deconstruction we do not have boundary anomalies as the contribution from $X_{L,R}$ cancels the contribution from the corresponding links. Therefore in this case we need to choose a WZ term that cancels mixed anomalies and does not produce boundary anomalies, that is, we need $C = 1$. Such WZ term in the continuum limit becomes exactly the CS term of eq. (3.95). More generally, in the aliphatic setup the choice of the WZ term is unique for each concrete configuration of (deconstructed) matter fields. Moreover it is such that the continuum CS term is just appropriate for that configuration.

3.4 Some more comments

In the preceding sections we presented several examples that deconstruct certain aspects of higher dimensional theories. The list of possibilities we have presented is by no means exhaustive. For example, we are not bound to consider the $U(K)$ group in the bulk. Other gauge groups, in particular phenomenologically interesting $SU(K)$ groups, can also be deconstructed. One subtlety originates from the fact that the links transform under the diagonal group as a product of two representations. It happens for $U(K)$ that the product of the fundamental and anti-fundamental representation is just the adjoint representation. This fact allowed to interpret all the Goldstone bosons as the degrees of freedom corresponding to the internal component of the 5d gauge fields. But for $SU(K)$ we have,

$$K \times \overline{K} = \texttt{Adjoint} + \texttt{Singlet} \tag{3.96}$$

The Goldstone boson corresponding to the singlet, G_p^s, cannot be identified with any 5d degrees of freedom. Thus, for deconstructing $SU(K)$, the low energy theory contains massless scalars that spoil the correspondence. In practice, since G_p^s are singlets under the diagonal group, they can presumably be ignored from the point of view of low energy phenomenology. However such singlets can play some role in cosmological considerations.

Furthermore, one can deconstruct theories with more than five extra dimensions. Drawing on our experience about the correspondence between the physical and the group product space it is easy to guess how such deconstruction should look like. For example, deconstructing 6d theories involves the quiver diagram forming a two dimensional lattice, see fig. 3.4, deconstructing 7d the theories involves a three-dimensional group product space, and so on. Topological properties of the internal manifold can be matched by choosing an appropriate pattern of links. Deconstruction can also be straightforwardly generalized to product gauge groups, like e.g. the SM group. To deconstruct 5d theories

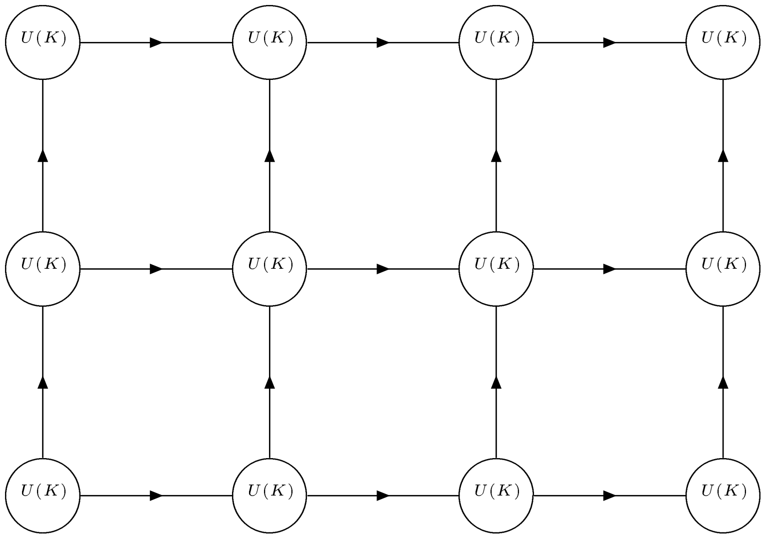


Figure 3.4: A quiver diagram for 6d models.

with the $G = SU(3) \times SU(2) \times U(1)$ gauge group one simply considers the replicated group G_p and introduces three sets of link-Higgs bosons $\Phi_p^{\tilde{3}}$, $\Phi_p^{\tilde{2}}$, $\Phi_p^{\tilde{1}}$, transforming in bi-fundamental representations [28].

Concluding this chapter, deconstruction is a setup which allows to study gauge theories with extra dimension using the well defined and well understood language of four dimensional gauge theories. The basic virtue is that one can work with a finite number of heavy fields (KK modes) and still the extended gauge symmetry can be preserved. This is not the case for $d > 4$ gauge theories with a cut-off. In deconstruction the analogue of the cut-off is realized as a scale of spontaneous symmetry breaking and so the physics above that scale is also defined. Thus deconstruction can serve as (one of many possible) UV completion to extra dimensional gauge theories. Any question, that can be unambiguously asked within the setup of higher imensional gauge theories, can also be asked (and, perhaps, claculated in a simpler fashion) in deconstruction. But another virtue is that the deconstruction setup can be made renormalizable. Thus even UV sensitive observables, in deconstruction can be dealt with using the standard techniques of renormalizable gauge theories. In Chapter 4 we will put this idea to work. We shall perform the 4d analogue of the calculations which, within the setup of extra dimensions, are not well defined or, at least, debatable.

But the role of deconstruction is not necessarily restricted to regularizing theories with extra dimensions. Such notions as 'locality in the group product space' can live their own life without any reference to extra dimensions. From this point of view deconstruction is just a new interesting direction for physics beyond the SM. Extended gauge symmetry is used as a tool for for constraining the possible form of interactions and counterterms in the theory. Certain ideas along this direction will be presented in Chapter 5.

Chapter 4

Deconstruction as an UV completion

As outlined in the introduction, deconstruction can serve as an UV completion to gauge theories with extra dimensions. In this chapter we illustrate this idea with three examples. One deals with the question of RG running in 5d gauge theories in the AdS_5 background. The second investigates certain surprising finiteness properties encountered in 5d supersymmetric theories compactified on $S_1/Z_2 \times Z_2'$. Deconstruction is used here as a tool which allows to clarify these aspects of 5d theories which are obscure or even ill-defined in the original formulation.

The first thing that comes to mind is that deconstruction could be used for modeling the evolution of gauge couplings in higher dimensional theories. In gauge theories with (flat) extra dimensions the couplings run according to the power-law, $\frac{1}{\alpha_5(Q)} \sim Q^{d-4}$. In ref. [33] it was proposed that one could take advantage of it, and construct phenomenologically viable models in which the SM (or MSSM) gauge couplings unify at a much lower scale than in conventional GUTs. The idea has received a lot of attention as it would allow to reconcile unification with theories with a low fundamental scale, for example with large extra dimensions. However, it turns out that in such models uncontrollable threshold corrections at the unification scale are of the same order of magnitude as the effects of the running. Therefore the predictions obtained in such theories cannot be trusted. On the contrary, in deconstruction we deal with a 4d gauge theory and the evolution of gauge couplings is perfectly controllable. Accelerated running of gauge couplings is simply an enhancement of the logarithmic running due to a large number of states in the theory. Thus, deconstruction offers an opportunity to construct models of low-scale unification which do not suffer from ambiguities associated with higher dimensional gauge theories. This issue was done in ref. [11]. Below, in Section 4.1, we shall concentrate on another problem associated with the running in higher dimensional gauge theories. We will try to understand why, contrary to naive expectations, gauge couplings do *not* run according to a power-law in 5d gauge theories defined in a highly curved background.

It should be stressed that the sole purpose of the models presented in this chapter is to provide an UV completion to certain specific higher dimensional theories. In order to match the tree-level spectra, the parameters of the 4d model must be fine-tuned. But we will not worry about the lack of naturalness of our deconstruction models as we do not treat them as self-standing theories. We just use them as a calculational tool to learn something about their higher dimensional analogues.

4.1 Running of gauge couplings in AdS_5

One of the most interesting recent attempts to solve the hierarchy problem is the Randall-Sundrum (RS) model [3]. Its setup consists of a slice of the AdS_5 space-time terminated by two branes, the UV brane at $y = 0$ and the IR brane at $y = \pi R$ where the SM matter is confined. The fact that the bulk is curved implies that the physics is different for observers at different positions y along the fifth dimension. Symmetries of AdS_5 (equivalent to the 4d conformal group) tell us that that a physical process occurring at $y = y_1$ at the scale M_1 is related to the same process at $y = y_2$ at the scale $M_1 e^{-k(y_2-y_1)}$, where k is the curvature of AdS_5. In particular, for the SM matter confined on the IR brane, the electroweak scale M_{EW} is connected to the scale of physics at the UV brane M_* by the formula $M_{EW} = M_* \exp(-k\pi R)$. M_* can be identified with the scale of 4d gravity M_{PL} (this is the strength with which the massless graviton mode couples to matter fields, irrespectively if they are localized on the UV or the IR brane). The parameters of the model are chosen such that the gap between M_* and the electroweak scale is explained by the smallness of the exponential (warp) factor $\exp(-k\pi R)$.

At first sight there seems no chance to reconcile the RS model with a perturbative gauge coupling unification. Indeed, the cut-off to the SM physics at the IR brane is close to TeV energies and there is no room for logarithmic running. However recently it has been realized that the RS model can be made compatible with the large-scale unification if the Standard Model gauge fields are allowed to propagate in the bulk AdS_5 space-time. This is because, unlike in gauge theories with flat extra dimensions, in gauge theories in the AdS_5 background the gauge couplings exhibit logarithmic running [34, 35], just like in 4d gauge theories. In ref. [35] the key ingredient leading to the logarithmic running is the cut-off scale depending on the position along the extra dimension.

The question of renormalization group running in AdS_5 was subsequently studied in refs [36]. Although all authors agree about the key result - the logarithmic running, there is some confusion as to what regulators are allowed and what is actually the definition of the gauge coupling exhibiting the logarithmic running. Below we will try to understand these problems from the point of view of deconstruction.

As a first step we need to find how 5d warped gauge theories can be realized in deconstruction [37, 9]. Recall that the so called warped geometry is described by the metric

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \tag{4.1}$$

where $\eta_{\mu\nu}$ is the 4d Minkowski metric and the function $a(y)$ is called the warp factor. We consider gauge theories in such fixed gravitational background. The YM action then reads

$$S = \int d^4x \int_0^{\pi R} dy \mathrm{Tr}\left(-\frac{1}{2}F_{\mu\nu}F_{\mu\nu} + a^2(y)F_{\mu 5}F_{\mu 5}\right), \tag{4.2}$$

The difference to 5d gauge theories in the flat background is that the $F^2_{\mu 5}$ part of the action is now multiplied by the warp factor. Of course the 5d Lorentz invariance, in particular, the translational invariance along the fifth dimension is not present. Recall that the analogue of the translational invariance in deconstruction is the shift symmetry that implies that all gauge couplings and vevs are universal. Therefore, to deconstruct warped gauge theories we must investigate the models with non-universal vevs and/or gauge couplings.

But for non-universal vevs it is generically not possible to find the low energy spectrum. Therefore, to establish a correspondence between deconstruction and 5d warped gauge theories we are forced to use the heuristic discretization method described in Section 3.1.1, adapted to the case of non-universal vevs. The 4d deconstruction action, with the link vevs inserted has the form

$$\begin{aligned} S_{4d} &= \textstyle\int d^4x \sum_{p=1}^N \mathrm{Tr}\left(-\frac{1}{2g_p^2}F_{\mu\nu,p}F_{\mu\nu,p} + F^\dagger_{\mu,p}F_{\mu,p}\right) \\ F_{\mu,p} &= (\partial_\mu G_p - i\partial_\mu\Sigma_p + v_p(A_{\mu,p} - A_{\mu,p+1}) + A_{\mu,p}(\Sigma_p + iG_p) - (\Sigma_p + iG_p)A_{\mu,p+1}) \ . \end{aligned} \tag{4.3}$$

We have rescaled the gauge fields as $g_pA_{\mu,p} \to A_{\mu,p}$ so that the gauge coupling appears only in front of the YM term. As before we drop Σ_p in the following formulae as these fields has no 5d interpretation. We shall assume that the scalar potential is chosen such that they are decoupled from the low energy theory. The remaining terms in $F_{\mu,p}$ can be arranged as

$$F_{\mu,p} = \partial_\mu G_p + v_p(A_{\mu,p} - A_{\mu,p+1}) + i[A_{\mu,p}, G_p] - iG_p(A_{\mu,p+1} - A_{\mu,p}) \tag{4.4}$$

Now we can do the following identification between the degrees of freedom in deconstruction and in 5d warped gauge theories: $A_{\mu,p} \to A_\mu(y_p)$, $G_p \to \frac{a(y_p)\sqrt{\Delta y}}{g_5(y_p)}A_5(y_p)$, $g_pv_p \to \frac{1}{\Delta y}a(y_p)$, $g_p \to \frac{1}{\sqrt{\Delta y}}g_5(y_p)$. After the substitution the action becomes

$$\begin{aligned} S_{4d} \to &\textstyle\int d^4x \,\mathrm{Tr} \sum_{p=1}^N \Delta y \left\{-\frac{1}{2g_5(y_p)^2}F_{\mu\nu}(y_p)F_{\mu\nu}(y_p)\right. \\ &\left.+\tfrac{a^2(y_p)}{g_5^2(y_p)}\left(\partial_\mu A_5(y_p) - \partial_5^d A_\mu(y_p) + i[A_\mu(y_p), A_5(y_p)] - \Delta y A_5(y_p)\partial_5^d A_\mu(y_p)\right)\right\} , \end{aligned} \tag{4.5}$$

where ∂_5^d denotes the discretized fifth derivative, $\partial_5^d f(y_p) \equiv \frac{f(y_{p+1})-f(y_p)}{\Delta y}$. In eq. (4.5) we recover the action of 5d warped gauge theories, eq. (4.2) up to the higher derivative term suppressed by Δy.

Coming back to our standard normalization of gauge bosons we need to rescale $A_{\mu,p} \to g_p A_{\mu,p}$, $A_\alpha \to g_5 A_\alpha$. So finally, the dictionary between 4d and 5d becomes

$$\begin{array}{rcl} A_{\mu,p} & \to & \sqrt{\Delta y} A_\mu(y_p) \\ G_p & \to & a(y_p)\sqrt{\Delta y} A_5(y_p) \\ g_p v_p & \to & \dfrac{1}{\Delta y} a(y_p) \\ g_p & \to & \dfrac{1}{\sqrt{\Delta y}} g_5(y_p) \\ N\Delta y & \to & \pi R \end{array} \tag{4.6}$$

There are two interesting things to note. Firstly, deconstruction with non-universal gauge couplings in the product group corresponds to 5d gauge theories with couplings varying along the fifth dimension. Secondly, if the quantity $g_p v_p$ depends on p, deconstruction matches 5d warped gauge theories. There is one subtlety involved, namely, we still need to determine at which scale the matching of eq. (4.6) should be performed. For the flat background it was clear that the matching should be done at the deconstruction scale, that corresponds to the cutoff scale of the 5d theory. In the warped case the cutoff scale depends on the position along the fifth dimension. It is then natural to assume, that the matching takes place at the local cut-off scale. From the point of view of deconstruction this means that we relate the two theories at the local (in the sense of the group product space) scale of spontaneous symmetry breaking v_p. In particular, a constant (y-independent) 5d gauge coupling g_5 translates into $g_p(Q = v_p) = g$.

Our goal here is deconstruction of gauge theories in the AdS_5 background. Choosing[1] the vevs as $v_p = v\epsilon^p$ and the constant gauge coupling we are able to mimic the physics of gauge bosons propagating in the AdS_5 background [37, 9, 38]. For definiteness we assume $\epsilon \le 1$. The 5d fundamental scale M_* can be identified with v - the scale of physics at the first site. As for the curvature scale, note that $\epsilon^N \to e^{-k\pi R}$. From this one can derive

$$\log(\frac{1}{\epsilon}) \to \frac{k}{M_*} \tag{4.7}$$

The choice of $\epsilon \sim 1$ corresponds to the curvature scale far below the cut-off scale. In such case we expect the physics to be analogous as in the flat space. On the other hand small

[1] As outlined at the beginning of this chapter, we do not specify the physics which could be responsible for such peculiar pattern of the link-Higgs vevs.

ϵ corresponds to the curvature scale above the cut-off scale, so that the curvature effects become important.

To investigate the running of gauge couplings we need to determine the spectrum of the gauge bosons. The mass matrix takes the form:

$$\mathcal{M}_A^2 = g^2 v_1^2 \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & 1+\epsilon^2 & -\epsilon^2 & 0 & 0 & \cdots & 0 \\ 0 & -\epsilon^2 & \epsilon^2+\epsilon^4 & -\epsilon^4 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 0 & -\epsilon^{2(N-3)} & \epsilon^{2(N-3)}+\epsilon^{2(N-2)} & -\epsilon^{2(N-2)} \\ 0 & \cdots & 0 & 0 & 0 & -\epsilon^{2(N-2)} & \epsilon^{2(N-2)} \end{bmatrix} \tag{4.8}$$

For $\epsilon \neq 1$, it is not possible to derive an analytical formula for the eigenvalues and the eigenvectors. However, for $\epsilon \ll 1$, we can use a trick which consists in diagonalizing this mass matrix step by step. In such case the following simple picture holds. At the scale v_1 the first two groups $U(K)_1 \times U(K)_2$ are broken to the diagonal group $U(K)_{(2)}$, at the scale v_2 $U(K)_{(2)} \times U(K)_3$ breaks to $U(K)_{(3)}$, and so on. We use the notation $U(K)_p$ for the p-th gauge group of the original product, $U(K)_{(p)}$ for the unbroken gauge group after the breaking of $U(K)_{(p-1)} \times U(K)_p$. The gauge coupling corresponding to $U(K)_{(p)}$ is denoted by $g_{(p)}$.

From now on we assume $\epsilon \ll 1$, so that the described procedure is a good approximation. As discussed, this is the most interesting limit in which the effects of the curved background are best visible. Thus the problem is reduced to diagonalizing 2×2 matrices. First we diagonalize the part involving $A_{\mu,1}$ and $A_{\mu,2}$ and the eigenvalues are $\mathcal{O}(gv\epsilon)$ and $\mathcal{O}(gv\epsilon^2)$. One state with $\mathcal{O}(gv\epsilon)$ eigenvalue is isolated and remains as a true eigenstate while the other with $\mathcal{O}(gv\epsilon^2)$ has a mixing term with $A_{\mu,3}$. We can do the same procedure iteratively and finally we obtain the spectrum $\mathcal{O}(gv\epsilon^{N-1})$ corresponding to the scale of symmetry breaking down to the diagonal group. Exact calculation shows that there is always a zero-mode eigenstate corresponding to the unbroken diagonal subgroup. Explicitly, the zero mode eigenvector is given by:

$$A_\mu^{(0)} = \frac{1}{\sqrt{N}} \sum_{i=p}^{N} A_{\mu,p} \,. \tag{4.9}$$

For higher eigenvalues, the eigenstates are given by:

$$
\begin{aligned}
A_\mu^{(N-1)} &= \frac{1}{\sqrt{2}}(A_{\mu,1} - A_{\mu,2}), \\
A_\mu^{(N-2)} &= \frac{1}{\sqrt{6}}(A_{\mu,1} + A_{\mu,2} - 2A_{\mu,3}), \\
& \cdots \\
A_\mu^{(N-j)} &= \frac{1}{\sqrt{j(j+1)}}(A_{\mu,1} + \cdots + A_{\mu,j} - jA_{\mu,j+1}), \\
& \cdots \\
A_\mu^{(1)} &= \frac{1}{\sqrt{(N-1)N}}(A_{\mu,1} + \cdots + A_{\mu,N-1} - (N-1)A_{\mu,N}).
\end{aligned}
\tag{4.10}
$$

Let us discuss how the physics changes with energy scale for observers located at different sites. For an observer at the N-th site (corresponding to the IR brane) the situation is simple. Below $v\epsilon^{N-1}$ he experiences a gauge theory with a single gauge group G. It is mediated by the zero-mode gauge field and the coupling strength is $g_0 = g/\sqrt{N}$. Above $v\epsilon^{N-1}$ he loses contact with the rest of the group product space, at least at the tree-level. The physics for him is just that of a single group G with the gauge coupling g. For large N the physics is presumably strongly coupled. From the point of view of deconstruction, the physics at this scale is one (of many possible) UV completion to the 5d theory and thus it cannot be used to conclude anything about the 5d theory. For an observer at the first site (corresponding to the UV brane) the physics looks differently. At each threshold he ceases to see what happens at some 'low energy site'. According to our step-by-step diagonalization procedure, at $v\epsilon^p$ his physics is also described a gauge theory with the group G, but the massless bosons is composed of the gauge fields up to the p-th site. Indeed, from eq. (4.9) it can be noted that all the KK modes except for the heaviest are peaked away from the first site and therefore have a small coupling to the matter at the first site.

We are now ready to address the running of gauge couplings in AdS_5 from the point of view of deconstruction [9, 38]. We find it convenient to use the bottom-up approach. We start with a gauge coupling measured at some low energy scale M_Z, $\alpha(M_Z)$. The running up to the scale of the lowest link-Higgs vev $v\epsilon^{N-1}$ proceeds as in an ordinary 4d gauge theory:

$$
\frac{1}{\alpha(Q)} = \frac{1}{\alpha(M_Z)} - \frac{b}{2\pi}\log\left(\frac{Q}{M_Z}\right) \qquad Q < v\epsilon^{N-1} \tag{4.11}
$$

and b is the beta-function coefficient corresponding to the contribution of massless $U(K)$ gauge fields.

At the scale $v\epsilon^{N-1}$ this coupling is identified with the coupling $\alpha_{(N)}$ of the diagonal group. In general, the matching equation at the p-th threshold is:

$$\frac{1}{\alpha_{(p+1)}(v\epsilon^p)} = \frac{1}{\alpha_{(p)}(v\epsilon^p)} + \frac{1}{\alpha_{p+1}(v\epsilon^p)} \,. \tag{4.12}$$

For $\epsilon \ll 1$ the heavy states of gauge bosons can be considered as decoupled between the thresholds. Also, we assumed that the links Σ_p are decoupled. Thus the running is given by

$$\frac{1}{\alpha_{(p+1)}(Q)} = \frac{1}{\alpha_{(p+1)}(v\epsilon^p)} - \frac{b}{2\pi}\log\left(\frac{Q}{v\epsilon^p}\right) \,, \qquad v\epsilon^p < Q < v\epsilon^{p+1} \,. \tag{4.13}$$

Putting together all these equations, the low-energy gauge coupling $\alpha(M_Z)$ depends on the high-energy couplings in the following way:

$$\begin{aligned}\frac{1}{\alpha(M_Z)} &= \frac{1}{\alpha_1(v)} + \frac{1}{\alpha_2(v\epsilon)} + \frac{1}{\alpha_3(v\epsilon^2)} + \cdots + \frac{1}{\alpha_N(v\epsilon^{N-1})} \\ &\quad + \frac{b}{2\pi}\log\left(\frac{v}{M_Z}\right)\end{aligned} \tag{4.14}$$

At this point there seem to be two plausible options for choosing the boundary conditions for the gauge couplings of the product group, α_p. One is to choose all the gauge couplings to be equal at the highest scale v. The other is to choose gauge couplings of the product group such that they have the same value at different scales, namely at the scale where the given group gets broken by the link-Higgs vev. The origin of this ambiguity is the already discussed question at which scale the matching between 4d and 5d degrees of freedom should be performed. Following the arguments below eq. (4.6), we choose the latter procedure. Thus the boundary conditions that corresponds to a constant gauge coupling in 5d take the form:

$$\alpha_1(v) = \alpha_2(v_1) = \cdots \alpha_N(v_{N-1}) \equiv \alpha. \tag{4.15}$$

and the low-energy gauge couplings depends on the high-energy ones as:

$$\frac{1}{\alpha(M_Z)} = \frac{N}{\alpha} + \frac{b}{2\pi}\log\left(\frac{v}{M_Z}\right) \tag{4.16}$$

Using the dictionary of eq. (4.6), the 5d interpretation of this equation is:

$$\frac{1}{\alpha(Q)} = \frac{\pi R}{\alpha_5} + \frac{b}{2\pi}\log(\frac{M_*}{Q}) \,. \tag{4.17}$$

The first term on the right hand side is just the tree-level formula for the gauge coupling. The second term shows the logarithmic running.

There are several things that can be clarified by investigating gauge theories in AdS_5 from the point of view of deconstruction. First thing to see is that the notion of a position dependent cutoff [35] is very naturally realized in deconstruction. In deconstruction the cutoff is replaced by the scale of spontaneous gauge symmetry breaking. At the scale $Q_p = v\epsilon^{p-1}$ the group product space is divided into two parts. One consists of the the first p groups where gauge symmetry is broken, $G^p \to G$. The other consists of $N-p$ groups which are unbroken at the scale Q_p. The latter region can be interpreted as an UV completion of the 5d theory at the scale Q_p. Note that in deconstruction the unbroken region can communicate to the spontaneously broken sector only at the higher loop level, via loops involving the link-Higgs fields. The 5d interpretation is that at the scale $Q_y = M_* e^{-ky}$ it still does make sense to calculate 5d correlators $\langle TA(0)A(y)\rangle$ and the results do not depend on the UV completion of the theory. This is why it is possible to study the evolution of gauge couplings above the scale of the IR brane.

Secondly, the coupling that runs logarithmically is by no means the zero-mode gauge coupling. In deconstruction the logarithmic running is exhibited by the coupling $\alpha_{(p)}$. This is defined at the scale Q_p as the coupling of the unbroken diagonal subgroup of the first p groups. Only below $v\epsilon^{N-1}$ (interpreted in 5d as the TeV scale of the IR brane) does this definition coincides with the zero-mode gauge coupling. In 5d the corresponding object defined at Q_y is $\alpha_5(Q_y) = \int_0^y dy' \frac{1}{g_5^2(Q_y')}$. The zero-mode coupling for $Q_y > 1$ TeV simply has no physical meaning, as the zero-mode wave function probes the IR brane where the physics is strongly coupled at this scale.

Furthermore, from eq. (4.9) one can see that heavy modes ($m_{(n)} \sim v$) are localized near the first site, while the light modes ($m_{(n)} \sim v\epsilon^{N-1}$) are localized near the N-th site. More generally, a mode with $m_{(n)} \sim v\epsilon^p$ is peaked near the p-th site. This is the origin of the logarithmic running in deconstruction. Although at higher energies there are many approximately massless modes, all but one do not contribute to the running of $\alpha_{(p)}$. The reason is that their wave functions have a tiny overlap with the first p sites for which $\alpha_{(p)}$ is defined. The 5d interpretation is clear. The relevant coupling at the scale Q_y is $\alpha_5(Q_y)$, defined only in the region $[0, y]$. The running of $\alpha_5(Q_y)$ is logarithmic because there is only one massless gauge boson which has some overlap with that region. This is what distinguishes the AdS_5 case from the flat extra dimensions, where the KK modes are not localized, and all modes lighter than Q_y contribute to the running.

There is also one interesting consequence if this setup is used for models of grand unification. The relations of eq. (4.15) tell us that in deconstruction the gauge couplings at different sites should unify at different energies. In particular, at the N-th site the couplings unify at $v\epsilon^{N-1}$ (identified with the TeV scale in 5d). But a low energy observer at the N-th site could not infer unification from running of gauge couplings. Below $v\epsilon^{N-1}$ the relevant

	$(+,+)$	$(+,-)$	$(-,+)$	$(-,-)$
Gauge multiplet	A_μ	λ_R	λ_L	A_5, Σ
Higgs multiplet	H	Ψ_H	$\tilde{\Psi}_H$	$\tilde{H}$
Matter multiplet	Ψ_M	ϕ_M	$\tilde{\phi}_M$	$\tilde{\Psi}_M$
KK tower	$m_{(n)} = \frac{2n}{R}$	$\tilde{m}_{(n)} = \frac{2n+1}{R}$	$\tilde{m}_{(n)} = \frac{2n+1}{R}$	$m_{(n)} = \frac{2n}{R},\ n > 0$

Table 4.1: The $Z_2 \times Z_2'$ assignemnets that break supersymmetry completely.

coupling for him is the zero-mode gauge coupling, which includes the contributions from all sites and therefore is far from being unified. Nevertheless, the observers at the N-th site should start to see some effects of unification already at $v\epsilon^{N-1}$. In particular, all fields at the N-th site should fill in full multiplets of the unified groups above $v\epsilon^{N-1}$, otherwise the equality of the gauge couplings of that group would be spoiled at higher energies. For example, for $SU(5)$ models the X and Y gauge bosons do appear at such low scale. Thus the 'desert' becomes populated in these models. This is of course a challenge for model builders, as generically the experimental proton decay bounds will not be satisfied. For recent models of AdS_5 unification see refs. [39].

4.2 Soft electroweak breaking from hard supersymmetry breaking

We mentioned in Section 2.1.2 that compactification may reduce a number of supersymmetries in the theory. In particular, compactification of 5d $N = 2$ supersymmetric theory on the orbifold S_1/Z_2 projects out a half of the supersymmetry, leaving $N = 1$ supersymmetry in the 4d effective theory. Furthermore, in Section 2.1.3 it was pointed out that supersymmetry can be completely broken by compactification on the orbifold $S_1/(Z_2 \times Z_2')$. This happens if bosons and fermions in a multiplet have different $(Z_2 \times Z_2')$ parities. Consider, for example, a 5d vector multiplet $(A_\alpha, \lambda, \Sigma)$. A consistent way to assign the parities is that displayed in Table 4.1. Then the bosonic fields (with parities $(+,+)$ and $(-,-)$) have KK modes with masses $m_{(n)} = 2n/R$ while the gauginos (that have parities $(+,-)$ and $(-,+)$) have KK modes with masses $\tilde{m}_{(n)} = (2n+1)/R$. Thus masses of bosons and fermions are different which indicates that supersymmetry is broken.

Recently, Barbieri, Hall and Nomura (BHN) attempted to construct a realistic model based on this setup [6]. They consider an $N = 2$ supersymmetric gauge theory on the orbifold $S_1/(Z_2 \times Z_2')$ with the SM gauge group $SU(3) \times SU(2) \times U(1)$. Apart from the 5d gauge multiplet they introduce two types of hypermultiplets in the bulk - the Higgs and the matter multiplet - that differ by parity assignments, see Table 4.1. The Higgs multiplet

H has a scalar zero-mode which is identified with the SM Higgs boson (there is only one Higgs doublet in the BHN model). As for the matter multiplets $Q_i, \overline{U}_i, \overline{D}_i, L_i, \overline{E}_i$, each has a chiral, fermionic zero-mode identified with the appropriate SM quark or lepton. The lightest superpartners of the SM fields have masses of the order of the compactification scale $\frac{1}{R}$. If the compactification scale $1/R$ is greater than, say, 200 GeV then there is no need to introduce soft breaking mass terms to explain why superpartners have not been observed. Thus the BHN model has one virtue from the start - it has much fewer parameters than the MSSM.

The Yukawa interactions in the BHN model are localized at the Z_2 and Z_2' fixed points. Recall that fields with the negative Z_2 parity vanish at $y = 0$, while the fields with the negative Z_2' parity vanish at $y = \pi R/2$. Thus at each fixed point half of a hypermultiplet is projected out and only a chiral multiplet remains dynamical there. We denote those components of the hypermuplit H which survive at $y = 0$ by $\hat{H}$ and those surviving at $y = \pi R/2$ by $\hat{H}'$. The authors of ref. [6] assume the following Yukawa interactions.

$$\begin{aligned} \mathcal{L} = \delta(y) \int d^2\theta \left(\lambda_{ij}^U \hat{Q}_i \hat{U}_j \hat{H}\right) + \text{h.c.} \\ +\delta(y - \tfrac{\pi}{2}R) \int d^2\theta' \left(\lambda_{ij}^D \hat{Q}_i' \hat{D}_j' \hat{H}^{\dagger\prime} + \lambda_{ij}^E \hat{L}_i' \hat{E}_j' \hat{H}^{\dagger\prime}\right) + \text{h.c.}\,. \end{aligned} \tag{4.18}$$

Such Yukawa interactions induce loop corrections to the zero-mode Higgs boson mass. In the SM, the one loop corrections proportional to the Yukawa couplings yield a quadratically divergent result. In the MSSM the quadratic divergence is cancelled by squark and slepton loops and only the logarithmic divergence remains. It turns out that in the BHN model at one loop a summed contribution from all the KK modes of the quarks and squarks is finite. In particular, the contribution from the top and stop loops is given by the formula:

$$m_H^2 = -\frac{21\,\zeta(3)}{8\pi^4}\frac{N_c Y_t^2}{R^2}, \tag{4.19}$$

where Y_t is the zero-mode top-Yukawa coupling and $N_c = 3$. Note that the one loop mass is negative which opens up the possibility of radiative electroweak symmetry breaking. But there is certain subtlety involved in deriving eq. (4.19). The contribution of each KK mode of top and stop is quadratically divergent and must be regularized. Ref. [6] uses the so called KK regularization (that is, summing first over the infinite tower of KK modes and then doing the 4d integrals). So far the situation is similar to that encountered during calculation of the one-loop mass $A_5^{(0)}$. However in that case we had a clear criterion for choosing the regulator, namely, that it should preserve the residual gauge invariance protecting the mass of $A_5^{(0)}$. In the case of BHN model no symmetry protects the Higgs boson mass as supersymmetry is completely broken by the Z_2 projections. This problem can be resolved by assuming that the model is a part of a locally supersymmetric lagrangian [40]. Indeed, breaking supersymmetry by boundary conditions in 5d supergravity can be reformulated as a spontaneous breaking due to an expectation value of the auxilary component of the radion multiplet [19]. Then one can impose the condition that the regulator should be consistent with the residual local supersymmetry that survives the orbifolding. Various regularization schemes that satisfy this condition yield the same result as in eq. (4.19), see e.g. [41, 42].

Since the Higgs boson transforms under the gauge group, loops involving the fields of the gauge multiplet are another source of potentially divergent corrections. If the bulk gauge group G did not contain a $U(1)$ factor one could calculate that at one loop the Higgs mass parameter receives the correction $\Delta m_H^2 = \frac{21\,\zeta(3)}{16\pi^4}\frac{g_0^2}{R^2}$ that is again finite. Since the gauge coupling is much smaller than the top Yukawa coupling this contribution is only a small shift to the negative top/stop contribution. However, in the real world the Higgs boson is charged under the $U(1)$ hypercharge group. It is well known that in supersymmetric $U(1)$ gauge theories a quadratically divergent Fayet-Iliopoulos(FI) term is generated at one-loop if the sum of $U(1)$ charges over the scalar spectrum is non-vanishing. Since in the BHN model the tree-level interactions are derived from a supersymmetric theory, a similar effect can occur. Now, bulk hypermultiplets contain an equal number of positively and negatively charged scalars and thus an FI term in the bulk cannot be induced. But at the fixed points $y = 0$, $y = \pi R/2$ half of the hypermultiplets are projected out and one can easily check that locally at the boundaries the sum of $U(1)$ charges of the scalars is non-zero. Thus one can expect that quadratically divergent brane localized FI terms will be induced at one loop. This was confirmed by direct calculations in ref. [43].

Summarizing, the BHN model is an interesting attempt at constructing a realistic model of physics beyond the SM. Supersymmetry breaking is realized in a straighforward way and does not require any soft breaking terms. If the top/stop loops is the leading source of the Higgs boson mass, the model exhibits radiative electroweak symmetry breaking. Note that the model has only one free parameter in the Higgs sector - the compactification scale $1/R$ and so such scenario would be very predictive. However the so far succesful picture is destabilized by $U(1)$, which introduces a quadratic cut-off dependence in the Higgs sector.

The physics of the BHN model was deconstructed in ref. [7]. One goal was to see into the origin of the finiteness properties, without invoking local supersymmetry. Another goal was to construct a realistic model which would not suffer from the quadratically divergent contribution of the gauge sector. In the following we concentrate on the former issue.

The first step is to understand how supersymmetry breaking by $Z_2 \times Z_2'$ projection can be be realized in deconstruction. We begin the analysis by deconstructing the 5d setup that contains only the $N = 2$ gauge multiplet. We expect that the effects of orbifolding are realized close to the boundary of the group product space. Therefore, we start from the supersymmetric aliphatic model defined by the quiver diagram of fig. 3.2, that is involving N gauge multiplets $V_p = (A_{\mu,p}, \chi_p)$ and $N-1$ link-Higgs multiplets $\Phi_p = (\Phi_p, \Psi_p)$ in bifundamental representations. Note that in 5d the bosonic fields have the same spectrum as in the S_1/Z_2 case, up to the rescaling of the radius, $R \to R/2$. Therefore the bosonic spectrum of the aliphatic model matches the 5d bosonic spectrum with

$$\frac{gv\pi}{2N} \to \frac{1}{R}\,. \tag{4.20}$$

(Note the factor of two difference as compared to the aliphatic case).

In the supersymmetric aliphatic model gauginos and link-Higgsinos combine into Dirac spinors that match the mass levels of the bosonic fields. In particular, there is one massless chiral gaugino. Its existence is guaranteed because the aliphatic setup contains an odd number of chiral fermions from the start. Here we do not want a massless gaugino, so we have to add one chiral gaugino to the setup. One choice that yields the correct spectrum is to introduce a fermion Ψ_N transforming in the adjoint representation of the group and add a mass term[2] of the form $\mathcal{L} = 2igv\mathrm{Tr}(\psi_N\chi_N + h.c.)$. When the link-Higgs bosons acquire universal vevs the full set of gaugino mass terms becomes

$$\mathcal{L} = 2\mathrm{Tr}\sum_{p=1}^{N}\left\{igv(\chi_p - \chi_{p+1})\Psi_p\right\}\,, \tag{4.21}$$

where it is understood $\chi_{N+1} \equiv 0$. These mass terms can be diagonalized by the mode decomposition

$$\begin{aligned}\chi_p &= \sqrt{\tfrac{2}{N+1/2}}\textstyle\sum_{n=1}^{N}\cos\left(\tfrac{(p-1/2)(n-1/2)\pi}{(N+1/2)}\right)\chi_{(n)}\\ -i\Psi_p &= \sqrt{\tfrac{2}{N+1/2}}\textstyle\sum_{n=1}^{N}\sin\left(\tfrac{p(n-1/2)\pi}{N+1/2}\right)\Psi_{(n)}\end{aligned} \tag{4.22}$$

There is no massless gauginos. At the n-th level $\chi_{(n)}$ and $\Psi_{(n)}$ combine into a Dirac spinor $\lambda_{(n)}$ with mass:

$$\tilde{m}_{(n)} = 2gv\sin\left(\frac{(n-1/2)\pi}{2(N+1/2)}\right)\,. \tag{4.23}$$

For large N this formula becomes:

$$\tilde{m}_{(n)} = \frac{gv\pi}{N+1/2}(n-1/2)\,. \tag{4.24}$$

[2]The resulting spectrum does not depend critically on the precise choice of the mass parameter. As long as this mass is of order gv, in the large N limit one obtains the same mass levels of gauginos. Also, instead of adding Ψ_N we could simply remove the N-th gaugino χ_N.

and matches the 5d spectrum of gauginos if the radius is identified as in eq. (4.20) (the factor $+1/2$ in the denominator is negligible for large N). Note also that the mode decomposition of eq. (4.21) is a discretized version of the KK decomposition of 5d $(+,-)$ and $(-,+)$ fields, see Section 2.1.3. This ensures matching of the interactions in the large N limit.

Note that the procedure we proposed is a hard breaking of supersymmetry. In particular, the number of fermionic and bosonic degrees of freedom is different. The supersymmetry breaking effects are inserted at the N-th site, which in the 5d interpretation corresponds to supersymmetry breaking by boundary conditions at $y = \pi R/2$. In deconstruction the model is obviously not natural in the t'Hooft sense. We started with only one supersymmetry acting at all sites and this supersymmetry is broken by introducing Ψ_N. Thus we are allowed to write supersymmetry breaking counterterms, also those involving fields at the first site. Still the model can be reasonable in the following sense. Imagine we start with a supersymmetric theory. At some scale the scalar partner of Ψ_N acquires a large mass and decouples from the theory. Because of locality in the group product space, divergent counterterms at the $p < N$-th site will be generated only at a higher loop level. If the cutoff of the theory is not too large, these divergences are suppressed by a many-loop factor and can be neglected. This reasoning shows that one should consider our construction as a model with a low cutoff, say, $M_* \sim 10$ TeV.

We continue deconstructing the content the BHN model. The next step is to deconstruct the two types of hypermultiplets - matter and Higgs. We choose the similar strategy as for the gauge multiplet. We start with the analogous construction as that for hypermultiplets in the aliphatic case and then we modify it at the N-th site so as to get the right spectrum. From the discussion in Section 3.3 we know that a massless hypermultiplet on S_1/Z_2 is realized in deconstruction by introducing the replicated multiplets, $Q_p = (q_p, \Psi_p^Q)$, $p = 1 \ldots N$ and $\tilde{Q}_p = (\tilde{q}_p, \tilde{\Psi}_p^Q)$, $p = 1 \ldots N-1$. The multiplet Q_p ($\tilde{Q}_p$) transforms as the (anti-)fundamental representation of the p-th gauge group. The analogous object that deconstructs the 5d Higgs hypermultiplet is denoted by $H_p = (h_p, \Psi_p^H)$, $\tilde{H}_p = (\tilde{h}_p, \tilde{\Psi}_p^H)$. The superpotential is chosen as:

$$W = \sum_{p=1}^{N-1} \left(\sqrt{2} g \tilde{Q}_p \Phi_p Q_{p+1} - g v \tilde{Q}_p Q_p \right) , \tag{4.25}$$

and similarly for H. The fermionic spectrum is already just right to match the fermions of the 5d matter hypermultiplet, if the 5d radius is matched as in eq. (4.20). Analogously, the scalar components of H match the scalars of the 5d Higgs hypermultiplet. More precisely, the fermions of the replicated matter multiplets decompose as

$$\Psi_p^Q = \sqrt{\tfrac{2}{N}} \textstyle\sum_{n=0}^{N-1} \eta_n \cos\left(\frac{(p-1/2)n\pi}{N}\right) \Psi_{(n)}^Q$$

$$\tilde{\Psi}^Q_p = \sqrt{\tfrac{2}{N}} \sum_{n=1}^{N-1} \sin\left(\tfrac{on\pi}{N}\right) \Psi^Q_{(n)} \tag{4.26}$$

There is a chiral zero-mode $\Psi^Q_{(0)}$. The n-th modes of Ψ_Q and $\tilde{\Psi}_Q$ get a Dirac mass

$$m_{(n)} = 2gv \sin\left(\frac{n\pi}{2N}\right) . \tag{4.27}$$

In the large N limit the tower of fermions matches the fermionic KK modes of the 5d matter hypermultiplet. The scalars of the Higgs replicated multiplet, h_p and $\tilde{h}_p$, decompose analogously as, respectively, Ψ^Q_p and $\tilde{\Psi}^Q_p$. There is a massless scalar, $h_{(0)}$, and two towers of massive scalars with masses given by $m_{(n)}$.

Now, in order to disturb the spectrum of the remaining degrees of freedom we shall modify the action at the N-th site, We add new degrees of freedom, a chiral fermion Ψ^H_N and a complex scalar $\tilde{q}_N$, thus introducing new sources of hard supersymmetry breaking. We also add interaction terms,

$$\mathcal{L} = |g\tilde{q}_{N-1}\Phi_{N-1} - gv\tilde{q}_N|^2 + g^2v^2|q_N|^2 - (gv\tilde{\Psi}^H_N\Psi^H_N + \text{h.c.}) \tag{4.28}$$

Again, this choice is of course arbitrary but the resulting spectrum does not depend critically on the precise value of the parameters in eq. (4.28). It is only important to have an even number of fermions in the replicated Higgs multiplet, otherwise there would be a fermionic zero mode. With such choice as in eq. (4.28) the mass matrices for the matter scalars (squarks) have the form:

$$\mathcal{M}^2_q = g^2v^2 \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -1 & 2 & -1 \\ 0 & \dots & 0 & -1 & 2 \end{bmatrix} \qquad \mathcal{M}^2_{\tilde{q}} = g^2v^2 \begin{bmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -1 & 2 & -1 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix} \tag{4.29}$$

The mass terms are diagonalized by the mode decomposition

$$\begin{aligned} q_p &= \sqrt{\tfrac{2}{N+1/2}} \sum_{n=1}^{N} \cos\left(\tfrac{(p-1/2)(n-1/2)\pi}{(N+1/2)}\right) q_{(n)} \\ \tilde{q}_p &= \sqrt{\tfrac{2}{N+1/2}} \sum_{n=1}^{N} \sin\left(\tfrac{p(n-1/2)\pi}{N+1/2}\right) \tilde{q}_{(n)} \end{aligned} \tag{4.30}$$

There is no massless mode. The mass of the n-th mode of both q and $\tilde{q}$ is given by

$$\tilde{m}_{(n)} = 2gv \sin\left(\frac{(n-1/2)\pi}{2(N+1/2)}\right) , \tag{4.31}$$

thus the mass levels are the same as that of the gauginos. For fermions of the Higgs replicated multiplet the mass squared matrices are the same as for squarks. Ψ_p^H decomposes analogously to q_p and $\tilde{\Psi}_p^H$ decomposes analogously to $\tilde{q}_p$.

To complete deconstruction of the salient features of the BHN model we need to find the 4d analogue of the brane localized Yukawa interactions. Let us first concentrate on the top-quark Yukawa interactions at $y = 0$. The intuitive analogue of such interactions in deconstruction is the superpotential

$$W = Y\bar{T}_1 Q_1^3 H_1 \,, \tag{4.32}$$

that involves only the first site members of the replicated multiplets. Here $T = (t_R, \psi_R^T)$ corresponds to the electroweak singlet top multiplet, $Q^3 = (t_L, b_L, \Psi_L^T, \Psi_L^B)$ corresponds to the 3rd generation quark doublet multiplet, and H is the Higgs boson. The superpotential (4.32) implies the following interaction of the Higgs multiplet scalar h_1,

$$\mathcal{L} = -Y^2 h_1^\dagger h_1 (t_{R,1}^\dagger t_{R,1} + t_{L,1}^\dagger t_{L,1}) + (Ygvh_1(t_{R,1}\tilde{t}_{L,1}^\dagger + t_{L,1}\tilde{t}_{R,1}^\dagger) + \text{h.c.}) - (Yh_1 \Psi_1^{T_R}\Psi_1^{T_L} + \text{h.c.}) \tag{4.33}$$

Quite generally, it is possible to prove that, as long as the hard supersymmetry breaking takes place away from the first site, the Yukawa interactions following from the superpotential (4.32) do not contribute any divergence to the Higgs boson mass. To see this we need to perform an orthogonal transformation to express the original fields on the first site in terms of the mass eigenstates: $q_p = \sum_n a_{p\,n}^q q_{(n)}$, $\tilde{q}_p = \sum_n a_{p\,n}^{\tilde{q}} \tilde{q}_{(n)}$, $\psi_{Q_p} = \sum_n b_{p\,n}^Q \psi_{Q_{(n)}}$. The zero-mode Higgs boson is constant in the group product space so we can substitute $h_1 \to \frac{1}{\sqrt{N}} h_{(0)}$. The Higgs mass receives one-loop radiative corrections through the diagrams depicted in Fig. 4.1, which results in:

$$\begin{aligned} -i\delta m^2 = 2\frac{Y^2}{N}\int \frac{d^4k}{(2\pi)^4} \Bigg(& \sum_n \frac{|a_{1\,n}^q|^2}{k^2 - m_{(n)}^{q\,2}} + \sum_{m,n} m_1^{q2} \frac{|a_{1\,n}^q|^2}{k^2 - m_{(n)}^{q\,2}} \frac{|a_{1\,m}^{\tilde{q}}|^2}{k^2 - m_{(m)}^{\tilde{q}\,2}} \\ & - k^2 \sum_{m,n} \frac{|b_{1\,n}^Q|^2}{k^2 - m_{(n)}^{Q\,2}} \frac{|b_{1\,m}^Q|^2}{k^2 - m_{(m)}^{Q\,2}} \Bigg) \end{aligned} \tag{4.34}$$

The divergences in this expression are proportional to:

$$\begin{aligned} \delta m^2 \sim \Lambda^2 & \left(\sum_n |a_{1\,n}^q|^2 - (\sum_n |b_{1n}^Q|^2)^2 \right) \\ + \ln\Lambda^2 & \left(-\sum_n m_{(m)}^{q\,2} |a_{1\,n}^q|^2 - m_1^{q2} (\sum_n |a_{1\,n}^q|^2)(\sum_n |a_{1\,n}^{\tilde{q}}|^2) + 2\sum_{m,n} m_{(m)}^{Q\,2} |b_{1\,n}^Q|^2 |b_{1\,m}^Q|^2 \right) \end{aligned} \tag{4.35}$$

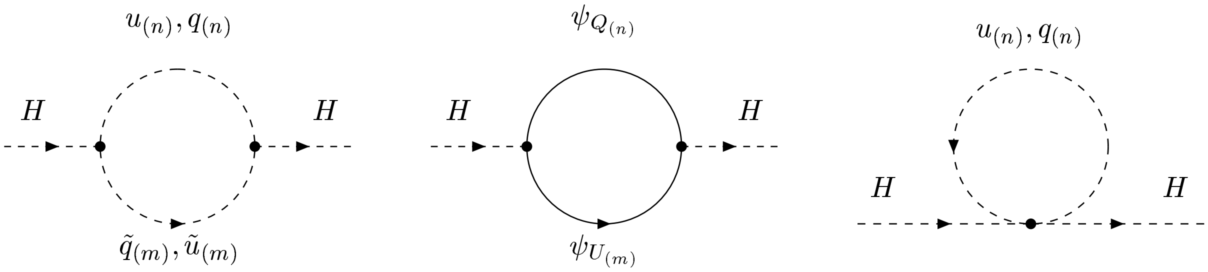


Figure 4.1: One-loop diagrams involving the top Yukawa coupling and contributing to the squared mass of the Higgs boson.

The coefficient of the quadratic divergence vanishes by the fact, that $a^q_{1\,n}$, $b^Q_{1\,n}$ are coefficients of the orthogonal transformation diagonalizing the squark and quark squared mass matrices. Furthermore, one can prove the orthogonality relations $\sum_n (m^q_{(n)})^2 |a^q_{1\,n}|^2 = (\mathcal{M}^q)^2_{11}$, $\sum_n m^{Q\,2}_{(n)} |b^q_{1\,n}|^2 = \left(\mathcal{M}^Q \mathcal{M}^{Q\dagger}\right)_{11}$. This leads to the conclusion that the Higgs mass gets a logarithmically divergent contribution proportional to the supertrace in the quark sector at the first site. But since we assumed that supersymmetry is not broken at the first site, this supertrace vanishes. Thus, in spite of the fact that the theory is non-supersymmetric, the Higgs mass can get, from the Yukawa interactions, only a finite one-loop correction to its mass. These conclusions hold even if the model has a different number of bosonic and fermionic degrees of freedom.

In the deconstructed model it is straightforward to see the origin of the finiteness properties. In the deconstruction phase the cancellation is not between any individual boson-fermion pair, but rather between the entire towers of top and stops KK modes. But divergences depend on the short distance physics. From the high energy point of view one can work in the unbroken phase. Then the cancellation is simply between one top and one stop living at the first site. Locality in the group product space forbids the supersymmetry breaking at the N-th site to modify the short distance physics close to the first site. Supersymmetry breaking can be only transmitted in IR, when the link-Higgs fields get vevs. Thus there can only be finite corrections to the Higgs boson mass. There is another interesting consequence of this reasoning. Our argument for vanishing of the divergences refers to the unbroken phase which, in the spirit of deconstruction, can be interpreted as an UV completion of the 5d theory. The key-point in our argument is the locality in the group product space. From this point of view, the finite result in the BHN model can be trusted as long as we assume that its UV completion is local. Local supersymmetry is not necessary.

Let us now restrict ourselves to a specific model in which the mode decomposition is given by eq. (4.26) and eq. (4.30). Plugging in the mode decomposition yields the following interactions of the Higgs zero mode $h_{(0)}$,

$$\begin{aligned}\mathcal{L} = &-2\tfrac{Y^2}{N(N+1/2)}h^\dagger_{(0)}h_{(0)}\textstyle\sum_{n,m=1}^N(t^\dagger_{R,(n)}t_{R,(m)}+t^\dagger_{L,(n)}t_{L,(m)})\cos(\tfrac{(n-1/2)\pi}{(2N+1)})\cos(\tfrac{(m-1/2)\pi}{(2N+1)})\\ &+2\tfrac{Y}{\sqrt{N}(N+1/2)}\textstyle\sum_{n,m=1}^N(h_{(0)}(t_{R,(n)}\tilde{t}^\dagger_{L,(m)}+t_{L,(n)}\tilde{t}^\dagger_{R,(m)})+\text{h.c.})\tilde{m}_m\cos(\tfrac{(n-1/2)\pi}{(2N+1)})\cos(\tfrac{(m-1/2)\pi}{(2N+1)})\\ &-2\tfrac{Y}{N\sqrt{N}}\textstyle\sum_{n,m=0}^{N-1}(h_{(0)}\overline{\Psi^T_{R,(n)}}\Psi^T_{L,(m)}+\text{h.c.})\eta_n\eta_m\cos(\tfrac{n\pi}{2N})\cos(\tfrac{m\pi}{2N})\end{aligned} \tag{4.36}$$

The above includes the usual SM Yukawa coupling of the zero-mode Higgs boson to the zero-mode top quarks. The SM top Yukawa coupling Y_T is given by

$$Y_T = \frac{Y}{N^{3/2}} \tag{4.37}$$

In addition, the Higgs boson couples to the entire tower of KK modes of top quarks and squarks and the coupling depends on the mass level. Sending N to infinity (and keeping λ_T fixed) we recover the top Yukawa interactions of the BHN model.

In this specific model we are able to determine the finite part of the one-loop correction to the Higgs boson mass from the top/stop loops. Summing up the diagrams of fig. 4.1 yields:

$$\begin{aligned}\delta m^2 = 4\frac{Y^2}{N}\int\frac{d^4l_E}{(2\pi)^4}\Bigg(&-\frac{1}{N+1/2}\sum_{n=1}^N\frac{\cos^2\tilde{\gamma}_n}{l_E^2+\tilde{m}^2_{(n)}}\\ &+\frac{2}{(N+1/2)^2}\sum_{m,n=1}^N\tilde{m}^2_{(n)}\frac{\cos^2\tilde{\gamma}_n}{l_E^2+\tilde{m}^2_{(n)}}\frac{\cos^2\tilde{\gamma}_m}{l_E^2+\tilde{m}^2_{(m)}}+\frac{2}{N^2}l_E^2\sum_{m,n=0}^{N-1}\frac{\eta_n^2\cos^2\gamma_n}{l_E^2+m^2_{(n)}}\frac{\eta_m^2\cos^2\gamma_m}{l_E^2+m^2_{(m)}}\Bigg),\end{aligned} \tag{4.38}$$

where l_E is the Euclidean momentum and $\gamma_n = \frac{n\pi}{2N}$, $\tilde{\gamma}_n = \frac{(n-1/2)\pi}{2(N+1/2)}$. Recall that $m_{(n)} = 2gv\sin\gamma_n$, $\tilde{m}_{(n)} = 2gv\sin\tilde{\gamma}_n$. Now we use the formulae:

$$\begin{aligned}\sum_{n=0}^{N-1}\eta_n^2\frac{1}{\sinh^2(x/2)+\sin^2\gamma_n} &= 2N\frac{\coth(Nx)}{\sinh(x)}-\frac{1}{2\coth^2(x/2)},\\ \sum_{n=1}^{N}\frac{1}{\sinh^2(x/2)+\sin^2\tilde{\gamma}_n} &= (2N+1)\frac{\tanh(Nx+x/2)}{\sinh(x)}-\frac{1}{2\coth^2(x/2)}.\end{aligned} \tag{4.39}$$

It follows:

$$\begin{aligned}\sum_{n=0}^{N-1}\eta_n^2\frac{\cos^2\gamma_n}{l_E^2+m_n^2} &= \frac{1}{4g^2v^2}N(\coth(Nx)\coth(x/2)-1)\,,\\ \sum_{n=1}^{N}\frac{\cos^2\tilde{\gamma}_n}{l_E^2+\tilde{m}_n^2} &= \frac{1}{4g^2v^2}(N+1/2)(\tanh(Nx+x/2)\coth(x/2)-1)\,,\end{aligned} \tag{4.40}$$

where x is related to the momentum as $l_E = 2gv\sinh(x/2)$. Now we are able to perform the sum over the KK modes in eq. (4.38). Note that in our model, contrary to 5d, the sum is over a finite number of KK modes so there is no mathematical ambiguity in this step. We obtain:

$$\begin{aligned}\delta m^2 &= -\frac{\lambda_T^2}{\pi^2}\left(\frac{gv}{2N}\right)^2 I(N)\,,\\ I(N) =& \;8N^4\int_0^\infty dx\sinh^3(x/2)\cosh^3(x/2)\\ &\Big((\tanh(Nx+x/2)-\tanh(x/2))^2-(\coth(Nx)-\tanh(x/2))^2\Big)\end{aligned} \tag{4.41}$$

The integral $I(N)$ is finite for $N>1$, which confirms our general arguments about the origin of finiteness. Numerical evaluation indicates that for $N\to\infty$ the integral converges to the limit $I(N)\to -\frac{21}{8}\zeta(3)$. Thus, in this limit, the Higgs mass parameter becomes:

$$\delta m^2 = -\frac{21}{8}\zeta(3)\frac{\lambda_T^2}{\pi^2}\left(\frac{gv}{2N}\right)^2 \tag{4.42}$$

With the matching of eq. (4.20) this formula is exactly the same as the corresponding one in the 5d BHN model. For finite N the result only slightly differs from the 5d one, e.g. for $N=2$ we get about 0.7 of the $N\to\infty$ limit result. This again illustrates the ability of deconstruction to regulate extra dimensional theories, even when the replication number is rather small.

Note that the contribution from the top Yukawa interactions to the Higgs boson mass parameter is negative. If this is the leading one, the model exhibits spontaneous electroweak breaking. But actually, to determine the physical Higgs boson mass we need to calculate the full one-loop effective potential,

$$V = \frac{1}{2}\,\mathrm{Tr}\int\frac{d^4k_E}{(2\pi)^4}\,\ln\frac{k_E^2+m_b^2(v_H)}{k_E^2+m_f^2(v_H)}\,, \tag{4.43}$$

where m_b and m_f are respectively the bosonic and fermionic mass matrices as functions of the vev of the Higgs field, v_H. We consider only the top-multiplet contribution and the dependence on the Higgs vev is coming from the Yukawa interactions localized on the first site.

For the stop sector, in the basis $(t_{(m)\,R}, \tilde{t}_{(n)\,L})$ and $(t_{(m)\,L}, \tilde{t}_{(n)\,R})$, we obtain the following $2N \times 2N$ squared mass matrix $(m, n, p, q = 1 \dots N)$:

$$\begin{pmatrix} \tilde{m}^2_{(m)}\delta_{mp} + a_b \cos\tilde{\gamma}_m \cos\tilde{\gamma}_p & b_b\,\tilde{m}_{(m)} \cos\tilde{\gamma}_m \cos\tilde{\gamma}_q \\ b_b\,\tilde{m}_{(n)} \cos\tilde{\gamma}_n \cos\tilde{\gamma}_p & \tilde{m}^2_{(n)}\delta_{nq} \end{pmatrix} \tag{4.44}$$

The two coefficients a_b and b_b are related to the Yukawa coupling as:

$$a_b = \frac{4Y^2 v_H^2}{(2N+1)N} \quad \text{and} \quad b_b = -\frac{4Y v_H}{(2N+1)\sqrt{N}} \tag{4.45}$$

Note that squarks mix with mirror squarks once the electroweak symmetry is broken.

Similarly in the top sector, in the basis $(\tilde{\psi}^T_{L\,(m)}, \psi^T_{R\,(n)})$ and $(\tilde{\psi}^T_{R\,(m)}, \psi^T_{L\,(n)})$, the $(2N-1) \times (2N-1)$ squared mass matrix reads $(m, p = 0 \dots N-1,\ n, q = 1 \dots N-1)$:

$$\begin{pmatrix} m^2_{(m)}\delta_{mp} + a_f\,\eta_m\eta_p \cos\gamma_m \cos\gamma_p & b_f\,\eta_m\, m_{(q)} \cos\gamma_m \cos\gamma_q \\ b_f\,\eta_p m_{(n)} \cos\gamma_n \cos\gamma_p & m^2_{(n)}\delta_{nq} \end{pmatrix} \tag{4.46}$$

The two coefficients a_f and b_f are given by:

$$a_b = \frac{2Y^2 v_H^2}{N^2} \quad \text{and} \quad b_b = -\frac{2Y v_H}{N\sqrt{N}} \tag{4.47}$$

Adding the tree-level Higgs self-coupling originating from the D-terms of replicated $SU(2)$ and (non-replicated) hypercharge U(1), we get:

$$V(v_H) = \frac{1}{8}(g_0^2 + g_0^{'2}) v_H^4 + \frac{3}{16\pi^2}\,\mathrm{STr}_{\,\mathrm{top}}\, M^4 \ln\left(\frac{M^2}{4g^2v^2}\right) \tag{4.48}$$

where the supertrace is over the $2N$ bosonic and $2N-1$ fermionic eigenvalues of the matrices (4.44)–(4.46) denoted collectively as M. These eigenvalues are given by solving the secular equations:

$$\begin{aligned} 1 - \tfrac{16Y^2 v_H^2}{N(2N+1)^2} M^2 \left(\textstyle\sum_{n=1}^N \frac{\cos^2\tilde{\gamma}_n}{\tilde{m}^2_{(n)} - M^2}\right)^2 &= 0\,, \\ 1 - \tfrac{4Y^2 v_H^2}{N^3} M^2 \left(\textstyle\sum_{n=0}^{N-1} \eta_n^2\, \frac{\cos^2\gamma_n}{m^2_{(n)} - M^2}\right)^2 &= 0\,. \end{aligned} \tag{4.49}$$

The solution can be found numerically which allows to evaluate the potential $V(v_H)$ and find its minimum. Note that the only dimensionful parameter in the potential is the deconstruction scale v. Once we adjust the position of the minimum to match the W-boson mass, $\langle h_{(0)} \rangle = v_H \sim 174$ GeV, the deconstruction scale is fixed in terms of other dimensionless parameters. Then the Higgs effective potential (in particular, the Higgs mass) becomes a function of well measured low energy parameters - the top Yukawa coupling and the electroweak gauge couplings - and the only free parameter which remains is the replication number N. Some numerical results for the Higgs boson mass can be found in ref. [7].

Let us now comment on deconstructing the bottom quark Yukawa couplings. In the BHN model they originate from the Z_2' fixed point at $y = \pi R/2$, see eq. (4.18). They involve different multiplets than the interactions at $y = 0$. In fact, the primed multiplets are multiplets under the second supersymmetry present in the bulk. It is now clear that it is difficult to deconstruct this aspect of the BHN model. Indeed, in deconstruction we deal with one supersymmetry, the second appears only in the $N \to \infty$ limit. It is then not natural, for finite N, to introduce multiplets under this second supersymmetry.

Nevertheless, still we can construct the multiplets $H_N' = (h_N^\dagger, \psi_{\tilde{H}_N})$, $Q_N' = (\tilde{q}_N^\dagger, \psi_{Q_N})$, $D_N' = (\tilde{d}_N^\dagger, \psi_{D_N})$ and write the Yukawa interactions *as if* resulting from the superpotential term $W = Y_D H_N' Q_N' D_N'$:

$$
\begin{aligned}
\mathcal{L} = -Y_D^2 &\left(\tilde{d}_N^\dagger \tilde{q}_N^\dagger \tilde{q}_N \tilde{d}_N + h_N^\dagger h_N \tilde{d}_N^\dagger \tilde{d}_N + \tilde{q}_N^\dagger h_N^\dagger h_N \tilde{q}_N \right) \\
&\lambda_D g_0 v (h_N^\dagger \tilde{q}_N^\dagger d_N + h_N^\dagger \tilde{d}_N^\dagger q_N + \tilde{q}_N^\dagger \tilde{d}_N^\dagger \tilde{h}_N + \text{h.c.}) \\
-\lambda_D &\left(h_N \psi_{Q_N} \psi_{D_N} + \tilde{q}_N \psi_{\tilde{H}_N} \psi_{D_N} + \tilde{d}_N \psi_{\tilde{H}_N} \psi_{Q_N} + \text{h.c.} \right)
\end{aligned}
\tag{4.50}
$$

It is possible to check that, similarly as for the Yukawa interactions (4.33) at the first site, the quark and squark loops resulting from (4.50) do not produce any divergences in the Higgs mass. But now we cannot use the argument of the locality in the group product space to argue that new counterterms appear only at higher loops. Hard breaking of the second supersymmetry is present everywhere (albeit suppressed by $1/N$) and so loop corrections will perturb the structure of interactions in eq. (4.50). But from the phenomenological point of view this not necessarily a disaster. Since we assume that the cutoff is low and since the bottom Yukawa coupling is much smaller than that of the top, absence of quadratic divergences at one loop is sufficient to ensure the stability of the Higgs boson mass.

It remains to investigate the corrections to the Higgs boson mass from the gauge interactions in deconstruction. As before, the divergences are determined by the structure of the model in the unbroken phase. The difference is that the gauge interactions take place at all sites, including the N-th site where supersymmetry is broken. Thus the locality arguments are not valid. But, similarly as for the bottom quark, from the phenomenological point of view it is important to ensure absence of quadratic divergences only at one loop.

The first potential source of quadratic divergences originates from the couplings of the Higgs field to the $N=1$ gauge multiplet (and to itself in the D-term scalar potential) and the relevant couplings are:

$$\mathcal{L} = \sum_p \left(g_p^2\, h_p^\dagger A_{\mu\, p} A_{\mu\, p} h_p \; - \frac{1}{2} g_p^2 \, \left(h_p^\dagger T^a h_p \right)^2 + \left(i\sqrt{2} g_p \, h_p^\dagger \chi_p \Psi_p^H + \text{h.c.} \right) \right) \tag{4.51}$$

The second source comes from the superpotential that links Higgs multiplets living at different sites, analogously as in eq. (4.25). The dangerous terms which can yield quadratic divergences are:

$$\mathcal{L} = -\sum_p \left(y_{p-1}^2 \, |h_p|^2 |\tilde{h}_{p-1}|^2 - y_{p-1}^2 \, h_p^\dagger \Phi_{p-1}^\dagger \Phi_{p-1} h_p - \left(y_{p-1} \tilde{\Psi}_{p-1}^H \Psi_{p-1} h_p + \text{h.c.} \right) \right) \tag{4.52}$$

In our model the couplings are fine-tuned, $g_p = y_p = g$, but it is more instructive to keep them general in the present discussion. To avoid quadratic divergences proportional to the gauge coupling g_p it is enough to ensure that at p-th site the full chiral multiplet (H_p, Ψ_p^H) interacts with the full $N=1$ gauge multiplet $(A_{\mu,p}, \chi_p)$. This requirement is satisfied in our model. Similarly, for quadratic divergences proportional to y_p to be absent, we need the full link (Φ_p, Ψ_p) and mirror multiplets $(\tilde{H}_p, \tilde{\Psi}_p^H)$ to be present at the p-th site. Note that the Higgs h_p at the p-th site always couples 'to the left', that is to links and mirrors at the $p-1$-th site. Adding or removing links-Higgs and mirror degrees of freedom at the N-th site has no consequence for the divergence of the Higgs mass because there is no h_{N+1}. Thus, adding, e.g., Ψ_N (to ensure the correct spectrum of gauginos) does not introduce any quadratic divergences. We conclude that in our model there is no quadratically divergent corrections at one loop.

Summarizing, in this section we concentrated on investigating the properties of deconstruction treated as an UV completion to the BHN model. Firstly, we showed that the peculiar spectrum and interactions of 5d $N=2$ supersymmetric theories on $S_1/Z_2 \times Z_2'$ can be readily mimicked in 4d gauge theories. The main objective was to see into the origin of the finiteness properties of the BHN model. Deconstruction is a perfect regulator when it comes to analyzing the 5d Yukawa interactions localized at the Z_2 fixed point at $y=0$. The finiteness of the loop corrections generated by these Yukawa interactions can be attributed to 5d locality which precludes the supersymmetry breaking at the Z_2' fixed point $y = \pi R/2$ to modify the short distance physics near $y=0$. 5d locality is realized in deconstruction as locality in the group product space. On the other hand deconstruction does not seem to be a good regulator when it comes to analyzing the 5d Yukawa interactions localized at the Z_2' fixed point at $y = \pi R/2$ or the gauge interactions of the Higgs multiplet. One can infer that for these interactions the second supersymmetry of the 5d theory plays an important role in ensuring the finiteness. This second supersymmetry is not realized in deconstruction with a finite replication number N. It seems plausible that a successful regulating theory in this case should involve local supersymmetry. Such theory is at the moment beyond the reach of deconstruction, as it is not yet known how to deconstruct gravity. Nevertheless, the deconstruction setup can still be useful if treated as an effective theory with a low cutoff. We showed that it was possible to avoid quadratic divergences at one loop, which ensured stability of the Higgs boson effective potential. The setup can then be used for constructing predictive models of radiative electroweak breaking, as in ref. [7].

Chapter 5

A new direction for physics beyond the SM

So far we have concentrated on proving that deconstruction can yield the same physics as extra dimensions. Consequently, in the previous chapter we used deconstruction to understand certain aspects of 5d gauge theories. In this chapter we change perspective. It turns out that models that mimic extra dimensions are just a subset of the deconstruction setup. Therefore we can forget about extra dimensions and simply consider deconstruction as a new, interesting setup for adressing the problems of the SM. For example, the hallmark properties of deconstruction - extended gauge symmetry and locality in the group product space - can be a useful tool to deal with the SM hierarchy problem.

Before we move to discussing the central results of this chapter let us summarize the most interesting developments in this direction. Recently, a considerable attention have been received by the Little Higgs models. These models represent a novel solution to the hierarchy problem. The inspiration [44] came from deconstructing higher dimensional gauge theories in which the Higgs boson is identified with an internal component of the gauge field. However, it turns out that phenomenologically more interesting are models with similar properties that do not strictly correspond to any higher dimensional theory. For example, the simplest model, the Littlest Higgs [45], has only two sites (two SM groups). The idea behind Little Higgs is that the SM Higgs boson is a pseudo-Goldstone boson associated with a global symmetry broken spontaneously by the link-Higgs vevs. These symmetries are not respected by the gauge and Yukawa interactions, which allows the Higgs boson to get a mass at a loop level. However, the structure of the group product space can be chosen such, that quadratically divergent corrections are forbidden at one loop. This can explain why the Higgs bosons is lighter (by an order of magnitude or so) than the cutoff scale. Technically speaking, the quadratic divergences in the SM are canceled due to appearance of new states at TeV energies. It is interesting that, contrary to supersymmetry, the cancellations are between the fields of the same statistics. For a review of the Little Higgs models, see [46].

The extended symmetry of deconstruction can also find some application in unified model-building. For example, one can assume that are several $SU(5)$ factors at the unification scale and that the SM is embedded in the diagonal subgroup. Such setup allows to solve the doublet-triplet problem [47] by assigning discrete symmetries forbidding the mass term for the Higgs doublet. For a more concrete realization of this idea, see [48]. The setup can also be used for constructing models of low scale unification that do not rely on a power-law like running [49, 11].

In this chapter we discuss two models that aim at solving the hierarchy problem. That of Section 5.1 provides an explanation why the supersymmetry breaking scale is so much lower than the fundamental scale. The technical hierarchy problem (in the sense of quadratic divergences) is solved by supersymmetry, like in the MSSM to which the model reduces at low energies . The model of Section 5.2 is non-supersymmetric. However, the structure of the theory is such that quadratic divergences do not appear at one loop. Note that, contrary to what was done in Chapter 4, we treat our models as self-standing 4d theories without any reference to extra dimensions. Any fine-tuning in the model building could no be justified. The models should be natural (also in the Dirac sense) and their structure should follow from some more fundamental theory. In Section 5.2 we show that setups similar to deconstruction arise in certain string theories.

5.1 Supersymmetry breaking and hierarchy from product U(1)

In this section we present a model that addresses the question of the origin of supersymmetry breaking and hierarchy in the MSSM. Although our setup is clearly inspired by deconstruction, a higher dimensional theory with analogous properties is not known. We need deconstruction just for one of its properties, namely, for the notion of locality (and non-locality) in the group product space.

We consider [10] supersymmetric theories with a product $U(1)$ gauge group and show that large hierarchy between the fundamental scale and the supersymmetry breaking scale can be generated. Our model is an extension of the scenario with supersymmetry breaking by a single, 'anomalous' $U(1)$ group [50, 51]. This scenario can be naturally linked to the Froggatt-Nielsen mechanism [52] of the fermion masses generation. It also allows for a solution to the supersymmetric flavor problem as the mass terms of squarks are also controlled .

Our setup is similar to deconstruction in the sense that it involves the product gauge group $U(1)_1 \times \ldots U(1)_N$ and $N-1$ links Φ_p charged under the neighboring groups. The quiver diagram is given in fig. 5.1. The link Φ_p has charge Q under $U(1)_p$ and

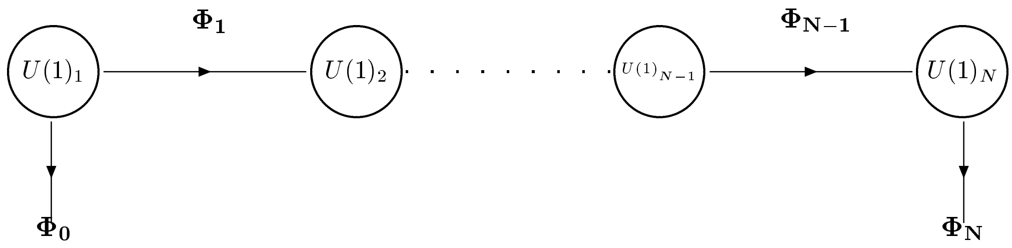


Figure 5.1: The quiver diagram of the model

$-Q$ under $U(1)_{p+1}$. The first and the N-th group are not linked. Furthermore, at the boundaries of the group product space we add chiral multiplets: Φ_0 with charge $-Q$ under $U(1)_1$ and Φ_N with charge Q under $U(1)_N$. In the following we normalize $Q = 1$. We shall also assume that some matter chiral multiplets (including the MSSM matter) live at the boundaries, that is they transform under $U(1)_1$ or $U(1)_N$.

The model as it stands is anomalous

In order to see how supersymmetry breaking and hierarchy can arise in this setup, let us first analyze a toy-model. We take the gauge couplings of each $U(1)$ to be universal, $g_p = g$. The key assumption we shall make here is that an FI term in the N-th group is generated. Moreover, we assume that its magnitude is slightly lower than that of the fundamental scale, $\xi_N = \epsilon^2 M_*^2 > 0$, where $\epsilon \sim 0.1$. Such situation can occur naturally, if we assume that the N-th group is anomalous. More precisely, we assume that the content of matter charged under $U(1)_N$ is such that $\mathrm{Tr(Q_N)} > 0$. It is well-known that in such case one loop calculations of the D-term tadpole yield a quadratically divergent result, which can be interpreted as an appearance of the FI term. Since it is a loop effect, the FI term is suppressed by a loop factor with respect to the fundamental scale, which justifies $\epsilon \sim 0.1$. Later on we specify the mechanism which cancels the anomaly (and other anomalies present in our toy-model).

The D- term potential takes the form:

$$\begin{aligned} V_D = \frac{g^2}{2} \big[&(|\Phi_1|^2 - |\Phi_0|^2)^2 + (|\Phi_2|^2 - |\Phi_1|^2)^2 + \dots \\ &+ (|\Phi_{N-1}|^2 - |\Phi_{N-2}|^2)^2 + (-|\Phi_{N-1}|^2 + |\Phi_N|^2 + \xi_N)^2 \big] \ . \end{aligned} \tag{5.1}$$

Note that (for N large enough) we cannot write any renormalizable superpotential for the links. Thus the D-term contribution is the leading one in the scalar potential. The model at this stage possesses a vacuum solution with a D-flat direction parametrized by the vev of Φ_N, $t^2 = \langle|\Phi_N|^2\rangle$.

$$\langle|\Phi_p|^2\rangle \;=\; \xi_N + t^2 \qquad\qquad p = 0 \ldots N-1 \tag{5.2}$$

If only the links had vev, the diagonal group would be unbroken. But also Φ_0 (which is charged under the diagonal group) has a vev and the product gauge symmetry is entirely broken in this background. There is one massless chiral multiplet (for $t = 0$ it is just Φ_0). The remaining degrees of freedom form a tower of $N = 2$ gauge multiplets with masses starting at $m^2 \sim \xi_N/N$. All the D-terms vanish. Since there is no F-terms at this stage, supersymmetry is unbroken.

Note however that the symmetries of the theory allow for a non-renormalizable superpotential of the form

$$W = \frac{\Phi_0 \ldots \Phi_N}{M_*^{N-2}}\,. \tag{5.3}$$

But since this new term comes from a non-renormalizable superpotential, it only slightly perturbs the zeroth order supersymmetric solution. As we shall see, the vacuum shift is suppressed by the small parameter κ defined as

$$\kappa^2 = \left(\frac{\xi_N + t^2}{M_*^2}\right)^{(N-2)}\,. \tag{5.4}$$

To find the vacuum solution in the presence of the operator (5.3) we investigate the equations of motion,

$$\begin{aligned}
\Phi_0\left(|\Phi_0|^2 - |\Phi_1|^2 + \sum_{r=1}^{N}\frac{1}{g^2|\Phi_0|^2|\Phi_r|^2}\left|\frac{\Phi_0\ldots\Phi_N}{M_*^{N-2}}\right|^2\right) &= 0 \\
\Phi_p\left(-|\Phi_{p-1}|^2 + 2|\Phi_p|^2 - |\Phi_{p+1}|^2 + \sum_{r\neq p}\frac{1}{g^2|\Phi_p|^2|\Phi_r|^2}\left|\frac{\Phi_0\ldots\Phi_N}{M_*^{N-2}}\right|^2\right) &= 0 \qquad p = 1\ldots N-2 \\
\Phi_{N-1}\left(-|\Phi_{N-2}|^2 + 2|\Phi_{N-1}|^2 - |\Phi_N|^2 - \xi_N + \sum_{r\neq N-1}\frac{1}{g^2|\Phi_{N-1}|^2|\Phi_r|^2}\left|\frac{\Phi_0\ldots\Phi_N}{M_*^{N-2}}\right|^2\right) &= 0 \\
\Phi_N\left(-|\Phi_{N-1}|^2 + |\Phi_N|^2 + \xi_N + \sum_{r=0}^{N-1}\frac{1}{g^2|\Phi_N|^2|\Phi_r|^2}\left|\frac{\Phi_0\ldots\Phi_N}{M_*^{N-2}}\right|^2\right) &= 0
\end{aligned} \tag{5.5}$$

We shall solve this equations to lowest order in κ^2. We expand the links around the zeroth-order vacuum solution,

$$\begin{aligned}
|\langle\Phi_p\rangle|^2 &= \xi_N + t^2 + a_p\,, \\
|\langle\Phi_N\rangle|^2 &= t^2 + a_N
\end{aligned} \tag{5.6}$$

where a_p are of order κ^2. The equations for a_p are derived from eqs. 5.5 by expanding these equations to the first order in κ^2. Note that the last terms on the l.h.s of eq. (5.5) are already suppressed by M_*^{2N-4} and in this order it is enough to insert the zeroth order vevs for Φ_p. We get:

$$\begin{aligned} \xi_N\left(a_0 - a_1 + \kappa^2(\xi_N + Nt^2)\right) &= 0\,, \\ \xi_N\left(-a_{p-1} + 2a_p - a_{p+1} + \kappa^2(\xi_N + Nt^2)\right) &= 0 \qquad p = 1\ldots N-1 \\ t^2\left(a_N - a_{N-1} + N\kappa^2(\xi_N + t^2)\right) &= 0 \end{aligned} \tag{5.7}$$

We encounter a difference equation of the form: $-a_{p-1} + 2a_p - a_{p+1} + X = 0$, with $X = \frac{\kappa^2}{g^2}(\xi_N + Nt^2)$. The general solution is given by $a_p = A + Bp + \frac{1}{2}Xp^2$, where A and B are arbitrary constants. As long as $\xi_N > 0$ this is the only possible solution of the equations of motiion for Φ_p with $p = 1\ldots N-1$. The first of eq. (5.7) acts as a 'boundary condition' for the difference equation, $a_{-1} = a_0$. This allows to determine the constant B, $B = X$. As for the last of eq. (5.7) we need to investigate two possibilities: either $t = 0$ or $a_N - a_{N-1} + N\kappa^2(\xi_N + t^2) = 0$. The latter equation is just another 'boundary condition', $a_{N+1} = a_N + (N-1)\kappa^2\xi_N$. It is easy to check that, for $\xi_N > 0$, this condition has no acceptable solutions (it requires $t^2 < 0$). Therefore we are left with the solution $t = 0$, which means that the flat direction of the zeroth order solution is lifted by the superpotential (5.3). The constant A is not determined. Thus the flat direction persists at the order κ^2 (to lift it we would need to solve the equations of motion to the order κ^4). However, the value of A is not important in what follows, in particular, the supersymmetry breaking parameters do not depend on A.

Summarizing, we have found the vacuum solution to lowest order in κ is given by:

$$\begin{aligned} |\langle\Phi_p\rangle|^2 &= \xi_N + A + \frac{1}{2g^2}\kappa^2 p(p+1)\xi_N\,, \qquad p = 0\ldots N-1 \\ |\langle\Phi_N\rangle|^2 &= A\,, \\ \kappa^2 &= \left(\frac{\xi_N}{M_*^2}\right)^{(N-2)} = e^{4N-2} \end{aligned} \tag{5.8}$$

In this shifted vacuum supersymmetry is broken. The expectation values of the D-terms are:

$$D_p = g^2(|\langle\Phi_p\rangle|^2 - |\langle\Phi_{p-1}\rangle|^2) = p\kappa^2\xi_N\,. \tag{5.9}$$

Note that all the D-terms are positive. The consequence of this fact is that the matter we assumed to be present at the boundaries has to have non-negative $U(1)_1$ or $U(1)_N$ charges.

Otherwise, the scalars of a negatively charged multiplet would get a tachyonic mass and render the model unstable.

There is also an F-term acquiring vev:

$$F_N = \frac{\partial W}{\partial \Phi_N} = \kappa \xi_N \,. \tag{5.10}$$

The F-terms corresponding to the links and Φ_0 all vanish.

The point of the whole constructions is that, in a natural way, the supersymmetry breaking scale is suppressed with respect to the fundamental scale M_*. Recall that $\kappa = \epsilon^{N-2}$, $\xi = \epsilon^2 M_*^2$. Defining the supersymmetry breaking scale as the scale of the D-term of the first group, $M_{\rm SUSY}^2 = D_1$, we get:

$$M_{\rm SUSY} = \epsilon^{N-1} M_* \,. \tag{5.11}$$

As $\epsilon \sim 0.1$, even for a moderate number of replications, say $N \sim 10$, it is easy to generate a huge hierarchy between the fundamental and the supersymmetry breaking scale. Hence the 'desert' above the TeV scale can simply be a consequence of the existence of a product $U(1)$ group at some high energy scale.

The origin of the hierarchy is the fact that supersymmetry breaking is triggered by a non-local object in the group product space - the superpotential of eq. (5.3). Thus hierarchy is not particularly sensitive to the technical assumptions we have made. For example, if we allow for an FI term in the first group, ξ_1, the solution remains qualitatively unchanged as long as $\xi_1 > -\xi_N$. One could also assume that there are equal FI terms in every gauge group, $\xi_p = \xi$. Then the zeroth order supersymmetric solution takes the form $|\Phi_p|^2 = (N-p)\xi$ but the rest of the discussion is similar. Another possible deformation of the model is to assume the boundary multiplets Φ_0 and Φ_N have charge $-a$, $+a$, respectively, where a is a natural number. Then for $a > 1$ the order parameter of supersymmetry breaking is:

$$\kappa^2 = \left(\frac{\xi_N}{M_*^2}\right)^{(a(N-1)-1)} = \epsilon^{2a(N-1)-2} \,, \tag{5.12}$$

and one needs even less replications N to get large hierarchy.

The model as it stands is inconsistent as the $U(1)$ gauge symmetries are anomalous. There are two kinds of anomalies in our model. The first are mixed anomalies of the neighboring groups, that are produced by the presence of the links. The second are the anomalies of the first and the N-th group produced by the matter multiplets living at the boundaries. The structure of anomalies can be conveniently described by the $N \times N$ anomaly matrix C defined as $C_{pq} \equiv \operatorname{Tr}\,(Q_p Q_q^2)$. The trace goes over all multiplets that transform simultaneously under p-th and q-th group. In our setup the anomaly matrix takes the form:

$$C_{pq} = \delta_{p,q-1} - \delta_{p,q+1} + \mathrm{Tr}(Q_1)^3\delta_{p,1}\delta_{q,1} + \mathrm{Tr}(Q_N)^3\delta_{p,N}\delta_{q,N} \ . \tag{5.13}$$

These anomalies can be canceled by the Green-Schwarz mechanism. To this end the theory should contain superfields M_k coupling to the gauge fields via the gauge kinetic functions,

$$f_p = \frac{1}{g_p^2} + \sum_k s_{pk} M_k \ . \tag{5.14}$$

Under the gauge transformations, the fields M_k must transform non-linearly

$$\begin{aligned} V_p &\to V_p + \frac{i}{2}(\Lambda_p - \bar{\Lambda}_p) \ , \\ M_k &\to M_k + \frac{i}{2}\sum_k \epsilon_{kp}\Lambda_p \ , \end{aligned} \tag{5.15}$$

where Λ_p are the infinitesimal parameters of the $U(1)^N$ gauge transformations written in the superfield formalism. To cancel all anomalies, the Green-Schwarz condition must be satisfied,

$$C_{pq} = \frac{1}{4\pi^2}\sum_k s_{pk}\ \epsilon_{kq} \ . \tag{5.16}$$

In the setup at hand, there is a natural solution to these constraints. Note that that the role of the moduli M_k can be played by the links itself. Indeed, we can define the object $\log(\Phi_{p-1}\Phi_p)$ that transforms non-linearly,

$$\log(\Phi_{p-1}\Phi_p) \ \to \ \log(\Phi_{p-1}\Phi_p) + i\ (\Lambda_{p+1} - \Lambda_{p-1}) \tag{5.17}$$

The boundary anomalies could also be cancelled solely by the links. But in such a case the boundary FI terms would depend on the links vev and the procedure of minimalizing the potential would become more involved. Therefore we choose to introduce new superfields, S_L and S_R, transforming as

$$\begin{aligned} S_L &\to S_L + \frac{i}{2}\ \mathrm{Tr}(Q_1^3)\Lambda_1 \\ S_R &\to S_R + \frac{i}{2}\ \mathrm{Tr}(Q_N^3)\Lambda_N \end{aligned} \tag{5.18}$$

Then a solution to eq. (5.16) is provided if the gauge kinetic function is chosen as:

$$f_1 = \frac{1}{g^2} - 2\pi^2\log\left(\frac{\Phi_1}{\xi_N}\right) + 4\pi^2 S_L\delta_{p,1}$$

$$
\begin{aligned}
f_p &= \frac{1}{g^2} - 2\pi^2 \log\left(\frac{\Phi_{p-1}\Phi_p}{\xi_N^2}\right) \\
f_N &= \frac{1}{g^2} - 2\pi^2 \log\left(\frac{\Phi_{N-1}}{\xi_N}\right) + 4\pi^2 S_R \delta_{p,N} \,. \quad (5.19)
\end{aligned}
$$

An interesting feature of this theory is that it becomes free in the unbroken phase $\langle\Phi_p\rangle = 0$. As $\frac{1}{g_p^2} = \langle f_p\rangle$, all the gauge couplings vanish in this phase. Therefore, the theory in the UV is a free theory.

We assume the Kahler potential of the form:

$$
K = \sum_{p=0}^{N} |\Phi_p|^2 e^{V_p - V_{p+1}} + \frac{1}{2}(S_L + \overline{S}_L - \frac{1}{4\pi^2}\mathrm{Tr}(Q_1)^3 V_1)^2 + \frac{1}{2}(S_R + \overline{S}_R - \frac{1}{4\pi^2}\mathrm{Tr}(Q_N)^3 V_N)^2 \,. \quad (5.20)
$$

In the above it is understood $V_0 = V_{N+1} = 0$. The links have a canonical kinetic term. The presence of vector multiplets in the kinetic term for S_L and S_R is necessary for gauge invariance. This generates FI terms at the boundaries of the group product space, that are proportional to the vacuum expectation values of S_L and S_R,

$$
\begin{aligned}
\xi_1 &= \frac{1}{4\pi^2}\langle S_L + \overline{S}_L\rangle \mathrm{Tr}(Q_1)^3 \\
\xi_N &= \frac{1}{4\pi^2}\langle S_R + \overline{S}_R\rangle \mathrm{Tr}(Q_N)^3 \quad (5.21)
\end{aligned}
$$

Supersymmetry breaking requires that at least $\xi_N > 0$. We will not try to construct an explicit superpotential that gives vevs to S_R but simply assume that $\langle S_R + \overline{S}_R\rangle \sim M_*$.

It is also important to discuss the consequences of embedding our model in local supersymmetry. This is because the physics of symmetry breaking takes place at energies close to the fundamental scale, where supergravity effects could be non-negligable. Recall that the scalar potential in $N = 1$ locally supersymmetric theories is given by:

$$
V = M_*^2 e^{G/M_*^2}(G_i (G^{-1})^i_j G^j - 3M_*^2) + \frac{1}{2}(\mathrm{Re} f^{-1})_{pq} D^p D^q \,. \quad (5.22)
$$

where $G = K + M_*^2 \ln(|W|^2/M_*^6)$, K is the Kahler potential, $G_i = \frac{\partial G}{\partial \phi_i}$, $G^i = \frac{\partial G}{\partial \phi_i^\dagger}$, $D^p = K_i Q_{\phi_i}\phi_i$. The superpotential we assume here has the form:

$$
W_{\mathtt{SUGRA}} = \hat{W}(S_L, S_R) + W(\Phi_p), \quad (5.23)
$$

where $W(\Phi_p)$ is the same as in the globally supersymmetric case, see eq. (5.3).

This is not our intention here minimalize the full supergravity potential. Instead, we give some heuristic arguments as to what the effects of supergravity corrections could be. Following this logic, we do not specify the precise form of $\hat{W}$ but simply assume that it stabilizes S_L, S_R (in the following denoted collectively as S) at the value close to the fundamental scale, $\langle S + S^\dagger \rangle \approx M_*$, and that $\langle M_* \frac{\partial \hat{W}}{\partial S} \rangle \approx \langle \hat{W} \rangle$. Then, to the leading order in the $|\xi_N / M_*^2|$ expansion the scalar potential takes the form

$$V \approx V_{\texttt{GLOBAL}} + N \frac{\langle \hat{W} \rangle}{M_*^2}(W + W^\dagger) - 2 \frac{\langle \hat{W} \rangle^2}{M_*^2} \tag{5.24}$$

To avoid a large cosmological constant, $\langle \hat{W} \rangle$ should cancel the positive vacuum energy generated by the globally supersymmetric part of the potential $V_{\texttt{GLOBAL}}$. The latter is dominated by the F-term F_N, where F_N is defined in eq. (5.10). Thus we need $F_S \approx F_N$. Of course, we do not have any explicit mechanism which could correlate F_S with F_N this way. Such mechanism would be a solution to the long-standing cosmological constant problem. We simply note that, if our setupis to be consistent with cosmological observations, these F-terms *have* to be related.

One consequence is that the gravitino mass can be estimated as

$$m_{3/2} = \frac{\langle \hat{W} \rangle}{M_*^2} \approx \frac{F_N}{M_*} . \tag{5.25}$$

Another consequence of the supergravity corrections is that the second term in eq. (5.24) modifies the equations of motion of the link fields Φ_p. The effect at lowest order is such that all vevs are shifted by the same amount and in particular the vev of Φ_N no longer vanishes:

$$\langle |\Phi_N| \rangle \approx \frac{\xi_N}{M_*} = \epsilon^2 M_* \tag{5.26}$$

The vev of Φ_N is suppressed with respect to the links vevs by the factor ϵ^1. Note that this vev will lead to non-zero F-terms $F_p = \frac{\partial W}{\partial \Phi_p}$, also suppressed with respect to F_N by the factor ϵ^1.

Let us summarize all scales relevant to the low energy phenomenology. Their orders of magnitude are determined in terms of three parameters: M_*, κ and ϵ. Recall, that we define the supersymmetry breaking scale $M^2_{\rm SUSY}$ as the magnitude of the D-term of the first group. The scales are given by:

$$\begin{array}{rcl} M_{\rm SUSY} & \approx & M_* \kappa \epsilon \\ D_p & \approx & p M^2_{\rm SUSY} \\ F_N,\, F_S & \approx & M_{\rm SUSY} M_* \epsilon \\ F_p & \approx & M_{\rm SUSY} M_* \epsilon^2 \\ \langle \Phi_p \rangle & \approx & M_* \epsilon \\ \langle \Phi_N \rangle & \approx & M_* \epsilon^2 \end{array} \tag{5.27}$$

It can be seen that the setup has led us to the so called hybrid scenario of supersymmetry breaking. The are two relevant supersymmetry breaking scales, that differ by an order of magnitude. One is the D-term scale, and the other is the F-term scale, with $\frac{F^2}{M_*^2} \sim \epsilon D$. This scenario can be very attractive for the superpartner spectroscopy in the MSSM.

The pattern of soft masses depends on how MSSM fields are embedded in the setup. A matter multiplet Q charged under $U(1)$ receive soft scalar mass terms from the appropriate D term,

$$m_Q^2 = q_p D_p \approx q_p p M_{\text{SUSY}}^2, \tag{5.28}$$

where q_p is the charge of Q under $U(1)_p$. For multiplets neutral under $U(1)$ the supersymmetry breaking is transmitted via gravitational interactions. That is, at the loop level the operator

$$\int d^4\theta \frac{\Phi_N^\dagger \Phi_N}{M_*^2} Q^\dagger Q \tag{5.29}$$

is generated. In consequence, the soft mass of neutral scalars can be estimated to be:

$$m_Q^2 \sim \frac{F_N^2}{M_*^2} \approx M_{\text{SUSY}}^2 \epsilon^2 \tag{5.30}$$

and so these soft masses pick up an additional factor ϵ. Thus we are able to generate a (small) hierarchy among soft breaking masses. This is an advantage as it allows us to address the supersymmetric flavor problem. Indeed, the diagonal squark mass entries can be controlled by choosing appropriate $U(1)$ charges, while the off-diagonal entries are suppressed by ϵ.

An interesting thing about our setup is that all the necessary ingredients to implement the Froggatt-Nielsen mechanism are already at hand. The MSSM matter fields need to have positive $U(1)$ charges (otherwise they would acquire negative mass squareds). In Yukawa interactions the excess charge has to be compensated by coupling to the appropriate power of the negatively charged field Φ_0. If $U(1)$ acts differently on the three generations, various Yukawa interactions are suppressed by various powers of the parameter ϵ. Note that ϵ has generically the order of magnitude of the Cabbibo angle. In the following we shall assume $\epsilon \approx 0.2$ Proceeding along the lines of [53, 54] we assume that all quarks are charged under the first group, $U(1)_1$. All quark masses come from supersymmetric interactions (in general non-renormalizable ones) with Φ_0:

$$W = \lambda_{ij}^U H_u Q_i U_j (\frac{\Phi_0}{M_*})^{h_u+q_i+u_j} + \lambda_{ij}^D H_d Q_i D_j (\frac{\Phi_0}{M_*})^{h_d+q_i+d_j} \, . \tag{5.31}$$

where we denote the $U(1)_1$ charges of the Higgs multiplets, left-handed quarks and right-handed quarks by h_u, h_d, q_i, u_i, d_i. For the sake of finding the mass ratios and mixing one can normalize $q_3 = u_3 = d_3 = 0$. The down quark mass matrix has the form

$$m_d = v_H \cos\beta \epsilon^{h_d} \begin{bmatrix} \epsilon^{d_1+q_1} & \epsilon^{d_1+q_2} & \epsilon^{d_1} \\ \epsilon^{d_2+q_1} & \epsilon^{d_2+q_2} & \epsilon^{d_2} \\ \epsilon^{q_1} & \epsilon^{q_2} & 1 \end{bmatrix} . \tag{5.32}$$

The matrix for the up-quarks is analogous with $d_i \to u_i$, $\cos\beta \to \sin\beta$ (for the charged leptons, $q_i \to l_i$, $d_i \to e_i$). It should be understood that all the entries are multiplied by a (possibly complex) factor of order 1. To find the eigenvalues one diagonalizes the mass matrix squared: $\mathrm{diag}(m_d, m_s, m_b) = D_L^\dagger m_d^\dagger m_d D_L$, $\mathrm{diag}(m_u, m_c, m_t) = U_L^\dagger m_u^\dagger m_u U_L$ and the CKM matrix is $V = U_L^\dagger D_L$. In general one expects the mass ratios to be of the following orders:

$$\begin{aligned} \frac{m_d}{m_s} &= \epsilon^{d_1-d_2+q_1-q_2} & \frac{m_s}{m_b} &= \epsilon^{d_2+q_2} \\ \frac{m_u}{m_c} &= \epsilon^{u_1-u_2+q_1-q_2} & \frac{m_c}{m_t} &= \epsilon^{u_2+q_2} \\ V_{12} \sim \epsilon^{q_1-q_2} \quad & V_{23} \sim \epsilon^{q_2} & V_{13} &\sim \epsilon^{q_1} \end{aligned} \tag{5.33}$$

Various charge assignments leading to acceptable mass and mixing patterns are summarized in ref. [54]. For example, a very good fit is obtained for $q_2 = 2$, $q_1 = 3$, $d_2 = 0$, $d_1 = 1$, $u_2 = 2$, $u_1 = 5$. Alternatively [55], the choice $q_1 = u_1 = 2$, $q_2 = u_2 = 1$, $q_3 = u_3 = 0$ and $d_1 = d_2 = d_3 = 1$ is consistent with $SU(5)$ unification and gives correct mass hierarchies. Furthermore, choosing lepton charges in accord with $SU(5)$ leads to correct charged leptons mass ratios and the anarchical neutrino mass matrix.

The mechanism of fermion mass generation affects also the squark mass pattern. In the flavor basis the diagonal entries are proportional to the corresponding $U(1)$ charges. The off-diagonal entries in the squark mass matrix originate from the Kahler potential and are expected to be of order $(m_F^2)_{ij} \sim M_{\mathrm{SUSY}}^2 \epsilon^{2+f_j-f_i}$ for $f_j \geq f_i$ and zero otherwise. But in the basis in which fermion masses are diagonal there are additional non-diagonal contributions, for example

$$(m_{D,LL}^2)_{ij} = M_{\mathrm{SUSY}}^2 D_L^\dagger \mathrm{diag}(q_1, q_2, q_3) D_L \sim V_{ij}(q_i - q_j) M_{\mathrm{SUSY}}^2 \tag{5.34}$$

The limits on flavor changing neutral currents constrain the quantity δ_{ij} defined as $\delta_{ij}^F = \sqrt{(\delta_{LL}^F)_{ij}(\delta_{RR}^F)_{ij}}$, where $(\delta_{LL}^F)_{ij} = (m_{F,LL}^2)_{ij}/M_{\mathrm{SUSY}}^2$. The most stringent bound is $\delta_{12}^d \leq 8 * 10^{-3}(M_{\mathrm{SUSY}}/1\mathrm{TeV})$. In our model $\delta_{12}^d \sim \sqrt{(d_1 - d_2)(q_1 - q_2) m_d/m_s}$ which suggests that phenomenologically viable models are those with $d_1 = d_2$.

Let us also comment on the issue of electroweak symmetry breaking. The set-up we presented is compatible with radiative electroweak breaking but, of course, any concrete realization depends on the soft masses of the MSSM fields (determined by their $U(1)$ charges) as well as on the value of the μ parameter. One attractive pattern of $U(1)$ charges is such that the third generation matter fields and the two Higgs fields are neutral. This choice is also the preferred one for the mechanisms of fermion masses generation, as then the Yukawa couplings of the third generation are not suppressed by powers of ϵ. In such case the corresponding scalar masses as well as the Higgs mixing mass b are of the order of $m_0^2 = \epsilon^2 M_{\rm SUSY}^2$. The soft masses of the first and the second generation (which are much larger) do not affect the electroweak breaking. For simplicity we assume that all soft masses of the third generation are exactly m_0 at the high scale. We have nothing new to say concerning the μ problem, but nevertheless we assume that $\mu \sim m_0$ as well. Starting with this boundary conditions at the scale of $U(1)$ symmetry breaking, the mass squared of H_u runs towards negative values due to large top/stop contributions. One can easily solve the RG equations, with the simplifying assumptions: 1) Only the gluino mass affects the running so we effectively set $g_1 = g_2 = 0$; 2) Only the top Yukawa coupling enters the RG equations and, in particular, we assume $y_b \sim y_\tau \ll 1$; 3) y_t and the gluino mass M_3 are constant; 4) The trilinear soft terms negligible. These assumptions lead to the expressions for the scale dependent Higgs and squark masses

$$m_{H_u}^2(Q) = (\frac{3}{2}m_0^2 - \frac{8}{9|y_t|^2}g_3^2 m_0^2)(\frac{Q}{M_*})^{\frac{3|y_t|^2}{2\pi^2}} - \frac{1}{2}m_0^2 + \frac{2}{3\pi^2}g_3^2 m_0^2(2\ln\frac{Q}{M_*} + \frac{4\pi^2}{3|y_t|^2}) \quad (5.35)$$

$$\begin{aligned} m_q^2(Q) &= \frac{1}{3}(\frac{3}{2}m_0^2 - \frac{8}{9|y_t|^2}g_3^2 m_0^2)(\frac{Q}{M_*})^{\frac{3|y_t|^2}{2\pi^2}} + \frac{1}{2}m_0^2 - \frac{4}{9\pi^2}g_3^2 m_0^2 t + \frac{8}{27|y_t|^2}g_3^2 m_0^2 \\ m_u^2(Q) &= \frac{2}{3}(\frac{3}{2}m_0^2 - \frac{8}{9|y_t|^2}g_3^2 m_0^2)(\frac{Q}{M_*})^{\frac{3|y_t|^2}{2\pi^2}} - \frac{2}{9\pi^2}g_3^2 m_0^2 t + \frac{16}{27|y_t|^2}g_3^2 m_0^2 \end{aligned} \quad (5.36)$$

For $\frac{Q}{M_*} \ll 1$ the Higgs mass is $m_{H_u}^2(Q) = -\frac{1}{2}m_0^2 + \frac{2}{3\pi^2}g_3^2 m_0^2(2\ln\frac{Q}{M_*} + 12)$ and is clearly negative. The gluino correction is important here as it helps the mass to become sufficiently negative $-m_{H_u}^2 > \mu^2 \sim m_0^2$. On the other hand the squark masses at low energies are $m_q^2 = \frac{1}{2}m_0^2 - \frac{4}{9\pi^2}g_3^2 m_0^2 t + \frac{8}{27|y_t|^2}g_3^2 m_0^2$ and $m_u^2 = -\frac{2}{9\pi^2}g_3^2 m_0^2 t + \frac{16}{27|y_t|^2}g_3^2 m_0^2$ and are positive, again in the latter case the gluino correction plays an important role, as otherwise the squarks would be too light.

In summary, in this section we presented a set up that generates the supersymmetry breaking scale hierarchically smaller than the fundamental scale. The set-up is inspired by deconstruction, but it has no known realizations among higher dimensional gauge theories. The low energy phenomenology is that of the hybrid scenario of supersymmetry breaking, with D-term domination. Such issues as the fermion masses and mixing, the supersymmetric flavor problem and electroweak supersymmetry breaking can be successfully addressed.

5.2 Custodial supersymmetry in non-supersymmetric quiver theories

In this section we discuss a class of models derived as a low energy limit of certain string theories. This class is known as the *quiver theories* [30, 56] and it describes dynamics of $D3$ branes close to the fixed points of the orbifolds R^6/Z_N. What matters to us is that these theories have a very similar structure to that of deconstruction, namely, they contain several $U(K)$ group factors linked by bi-fundamental matter fields. Thus, string theories may provide a possible derivation of, at least, some of the models of deconstructed dimensions. But here we will not discuss relations between the quiver theories and extra dimensions. Instead, we will point out several properties of the quiver theories that could make them interesting for phenomenology.

In the following we concentrate on non-supersymmetric setups. We show that this class of theories has some amazing finiteness properties, usually associated with supersymmetry [8]. Therefore, non-supersymmetric quiver theories may be relevant for solving the hierarchy problem.

Furthermore, we investigate the model in the deconstruction phase, that is when the bi-fundamental scalars acquire vevs. There comes another surprise - supersymmetry is recovered at low energies. More precisely, the massless sector exhibits $N = 4$ supersymmetry. As for the massive spectrum, at each mass level bosons and fermions are degenerate and their couplings to the zero-mode sector are of the supersymmetric form. We call this phenomenon the custodial supersymmetry. It is only interactions between the massive fields that do not preserve supersymmetry.

First, we must briefly describe the construction of quiver theories. These theories are obtained by 'orbifolding' the $N = 4$ globally supersymmetric $U(K \cdot N)$ YM theory (called the mother theory), where by orbifolding we understand removing certain degrees of freedom[1]. The $N = 4$ gauge theory is unique (up to a choice of the gauge group). Its field content is the gauge field, A, four fermions subject to the Majorana condition, ψ_i, $i = 1, 2, 3$, and $\psi_4 \equiv \lambda$ and three scalars ϕ_i, all in the adjoint representation of $U(K \cdot N)$. The lagrangian is:

$$\begin{aligned}\mathcal{L} = Tr\Big\{ &-\tfrac{1}{2}F_{\mu\nu}F_{\mu\nu} + i\overline{\lambda}\gamma^\mu D_\mu\lambda + 2D_\mu\phi_i^\dagger D_\mu\phi_i + i\overline{\psi_i}\gamma^\mu D_\mu\psi_i \\ &-2i\sqrt{2}g(\overline{\psi_i}[P_R\lambda, \phi_i] + \overline{\psi_i}[P_L\lambda, \phi_i^\dagger]) - i\sqrt{2}g\epsilon_{ijk}(\overline{\psi_i}[P_L\psi_j, \phi_k] + \overline{\psi_i}[P_R\psi_j, \phi_k^\dagger]) \\ &-g^2[\phi_i, \phi_i^\dagger][\phi_j, \phi_j^\dagger] + 2g^2[\phi_i, \phi_j][\phi_i^\dagger, \phi_j^\dagger]\Big\}. \end{aligned} \tag{5.37}$$

The prescription for obtaining the quiver theories (the daughter theories) from the mother theory is the following. One defines a discrete action of some Z_N symmetry

[1] It should be stressed that the orbifolding procedure described here is not the same as the orbifold compactification dicussed in Chapter 2.

group on the fields and retains in the spectrum only those fields which are invariant under this action. The interactions of the daughter theory are simply inherited from the mother theory, with all non-invariant fields and their interactions removed. The discrete group should be embedded simultaneously in the gauge group and in the R-symmetry group. If the embedding in the R-symmetry group is non-trivial, the daughter theory has fewer supersymmmetries than $N = 4$ and therefore it can be phenomenologically more interesting.

More precisely, the action of the Z_N group will be encoded in a $NK \times NK$ matrix γ. We choose this matrix in the form of a direct sum of N unit matrices of dimension $K \times K$, multiplied by consecutive Z_N elements $\omega^p = e^{\frac{2\pi i}{N}(p-1)}$, $p = 1 \ldots N$,

$$\gamma = \begin{bmatrix} \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} & & & \\ & \begin{bmatrix} \omega & & \\ & \ddots & \\ & & \omega \end{bmatrix} & & \\ & & \ddots & \\ & & & \begin{bmatrix} \omega^{N-1} & & \\ & \ddots & \\ & & \omega^{N-1} \end{bmatrix} \end{bmatrix} \tag{5.38}$$

The Z_N action on the gauge fields is the following:

$$A = \gamma A \gamma^{-1} \,. \tag{5.39}$$

(This is what we meant by saying that Z_N is embedded in the gauge group). To see the action explicitly, it is convenient to divide the gauge field A (which is a $NK \times NK$ hermitian matrix) into N^2 blocks A_{pr}. Then

$$\begin{bmatrix} [A_{11}] & [A_{12}] & \ldots & [A_{1N}] \\ [A_{21}] & [A_{22}] & \ldots & [A_{2N}] \\ \vdots & \vdots & \ddots & \vdots \\ [A_{N1}] & [A_{N2}] & \ldots & [A_{NN}] \end{bmatrix} \rightarrow \begin{bmatrix} [A_{11}] & [A_{12}]\,\omega^{-1} & \ldots & [A_{1N}]\,\omega^{-(N-1)} \\ [A_{21}]\,\omega^{1} & [A_{22}] & \ldots & [A_{2N}]\,\omega^{-(N-2)} \\ \vdots & \vdots & \ddots & \vdots \\ [A_{N1}]\,\omega^{(N-1)} & [A_{N2}]\,\omega^{(N-2)} & \ldots & [A_{NN}] \end{bmatrix} \tag{5.40}$$

Now it is easy to see that only the block-diagonal components are invariant under the Z_2 action. Thus, orbifolding amount to setting all the non-diagonal entries to zero,

$$A = \begin{bmatrix} [A_1] & & \\ & \ddots & \\ & & [A_N] \end{bmatrix}, \tag{5.41}$$

where $A_{pp} \equiv A_p$. Counting the gauge fields we conclude that the gauge group has been reduced to the product group $U(K)^N$.

The invariant components of the fermions and scalars satisfy the condition:

$$\begin{aligned} \psi_i &= \omega^{a_i}\gamma\psi_i\gamma^{-1}, \quad i = 1..4, \\ \phi_i &= \omega^{\tilde{a}_j}\gamma\phi_i\gamma^{-1}, \quad i = 1..3, \end{aligned} \tag{5.42}$$

where the integers (the so called shifts) a_i, $\tilde{a}_i$ obey the constraints:

$$a_1 + a_2 + a_3 + a_4 = 0, \qquad \tilde{a}_i = a_i + a_4. \tag{5.43}$$

These shifts are what we meant by embedding the discrete group in the R-symmetry group (the scalars and fermions of the $N = 4$ theory are charged under R-symmetry). Their practical effect is that each block is additionally multiplied by the Z_N element ω^{a_i}. If the shifts are non-zero the diagonal blocks are not Z_N invariant. But there are always N off-diagonal elements that are invariant and therefore survive the orbifold projection. For example, for $a_1 = 1$ the surviving components of the fermion ψ_1 are

$$\psi_1 = \begin{bmatrix} & [\psi_{1,1}] & & & \\ & & [\psi_{1,2}] & & \\ & & & \ddots & \\ & & & & [\psi_{1,N-1}] \\ [\psi_{1,N}] & & & & \end{bmatrix} \tag{5.44}$$

It can also be seen that for non-zero shifts the corresponding fields transform in bi-fundamental representations of the daughter theory product gauge group. A concise way to describe this block structure after orbifolding is the following:

$$\begin{aligned} A_{\mu,pr} &= A_{\mu,p}\delta_{p,r} \\ \phi^i_{pr} &= \phi_{i,p}\delta_{p,r-\tilde{a}_i} \\ \psi^i_{pr} &= \psi_{i,p}\delta_{p,r-a_i} \end{aligned} \tag{5.45}$$

where $p, r = 1 \ldots N$ label the consecutive blocks (that is the $U(K)$ gauge groups in the daughter theory). The gauge field $A_{\mu,p}$ transforms in the adjoint representation $(\mathbf{K_p}, \overline{\mathbf{K}}_\mathbf{p})$, while the fermions $\psi_{i,p}$ and the scalars $\phi_{i,p}$ transform respectively as the $(\mathbf{K_p}, \overline{\mathbf{K}}_{\mathbf{p+a_i}})$, $(\mathbf{K_p}, \overline{\mathbf{K}}_{\mathbf{p+\tilde{a}_i}})$ representations of the $(U(K))^N$ group.

An easy way to count unbroken supersymmetries in the daughter theory is to analyse the number of fermions in the adjoint representation in each factor of the gauge group, i.e., to count the gauginos. Supersymmetry is preserved when when at least one of the fermionic representation is the adjoint representation, that is, when $a_i = 0$ or $a_4 = 0$. Here, our main interest are the theories without supersymmetry, that is, $a_i \neq 0$ and $a_4 \neq 0$.

Now that we finished the construction of the model we are ready to investigate its UV properties. We are interested in quadratic divergences of the one-loop effective potential. To this end we need to calculate the mass matrices of the gauge bosons, scalars and fermions, with the expectation value of the scalar fields switched-on. We will show that, under certain conditions, the supertrace of $\mathcal{M}^2$ vanishes, hence the one-loop quadratic divergences are absent in the daughter theory.

Inserting the block decomposition (5.45) into the $\mathcal{F}$ lagrangian we find the daughter theory lagrangian:

$$
\begin{aligned}
\mathcal{L} = Tr \Big\{ & -\tfrac{1}{2} F_{\mu\nu,p} F_{\mu\nu,p} + i\overline{\lambda_p}\gamma^\mu D_\mu \lambda_p + 2 D_\mu \phi^\dagger_{i,p} D_\mu \phi_{i,p} + i\overline{\psi_{i,p}}\gamma^\mu D_\mu \psi_{i,p} \\
& -g\left[2i\sqrt{2}(\overline{\psi_{i,p}} P_L \lambda_{p+a_i}\phi^\dagger_{i,p} - \overline{\psi_i}\phi^\dagger_{i,p-a_4} P_L \lambda_{p-a_4}) + \text{h.c.}\right] \\
& -g\left[i\sqrt{2}\epsilon_{ijk}(\overline{\psi_{i,p}} P_L \psi_{j,p+a_i}\phi_{k,p-\tilde{a}_k} - \overline{\psi_{i,p}}\phi_{k,p+a_i} P_L \psi_{j,p-a_j}) + \text{h.c.}\right] \\
& -g^2(\phi_{i,p}\phi^\dagger_{i,p} - \phi^\dagger_{i,p-\tilde{a}_i}\phi_{i,p-\tilde{a}_i})(\phi_{j,p}\phi^\dagger_{j,p} - \phi^\dagger_{j,p-\tilde{a}_j}\phi_{j,p-\tilde{a}_j}) \\
& +4g^2(\phi_{i,p}\phi_{j,p+\tilde{a}_i}\phi^\dagger_{i,p+\tilde{a}_j}\phi^\dagger_{j,p} - \phi_{i,p}\phi_{j,p+\tilde{a}_i}\phi^\dagger_{j,p+\tilde{a}_i}\phi^\dagger_{i,p})\Big\}\,.
\end{aligned}
\tag{5.46}
$$

The covariant derivative acting on the scalars is $D_\mu \phi_{i,p} = \partial_\mu \phi_{i,p} + igA_p\phi_{i,p} - ig\phi_{i,p}A_{p+\tilde{a}_i}$. Hence the squared mass matrix of the gauge bosons is :

$$
\begin{aligned}
(\mathcal{M}^2_A)_{pr} = 2g_0^2 \Big[& \delta_{p,r} 1 \otimes \phi_{k,p}\phi^\dagger_{k,p} + \delta_{p,r}\phi^\dagger_{k,p-\tilde{a}_k}\phi_{k,p-\tilde{a}_k} \otimes 1 \\
& -\delta_{p+\tilde{a}_k,r}\phi_{k,p}\otimes\phi^\dagger_{k,p} - \delta_{p,r+\tilde{a}_k}\phi^\dagger_{k,p-\tilde{a}_k}\otimes\phi_{k,p-\tilde{a}_k}\Big]\,.
\end{aligned}
\tag{5.47}
$$

The tensor product defines the way the scalars are contracted with the gauge fields. The rule is $A_1(X \otimes Y)A_2 = A_1 X A_2 Y$.

In order to extract the scalar mass terms we substitute $\phi_{i,p} \to \phi_{i,p} + X_{i,p}$ in the scalar potential and calculate the terms which are quadratic in the fluctuations X. Moreover, it is convenient to separate the F-term and the D-term contributions. We define the mass matrix of scalars as:

$$
\mathcal{L} = -2\begin{pmatrix} X_{i,p} \\ X^\dagger_{i,p} \end{pmatrix}^\dagger \begin{pmatrix} \frac{1}{2}(\mathcal{M}^2_F + \mathcal{M}^2_D)^{ij}_{pr} & (M^2_F + M^2_D)^{ij}_{pr} \\ (M^{2\dagger}_F + M^{2\dagger}_D)^{ij}_{pr} & \frac{1}{2}(\mathcal{M}^2_F + \mathcal{M}^2_D)^{ij}_{pr} \end{pmatrix} \begin{pmatrix} X_{j,r} \\ X^\dagger_{j,r} \end{pmatrix}. \tag{5.48}
$$

The diagonal entries are:

$$
\begin{aligned}
(\mathcal{M}_F^2)^{ij}_{pr} = \quad & 2g_0^2\delta_{ij}[\delta_{p,r}\phi^\dagger_{k,p-\tilde{a}_k}\phi_{k,p-\tilde{a}_k}\otimes 1+\delta_{p,r}1\otimes\phi_{k,p+\tilde{a}_i}\phi^\dagger_{k,p+\tilde{a}_i} \\
& -\delta_{p,r+\tilde{a}_k}\phi^\dagger_{k,p-\tilde{a}_k}\otimes\phi_{k,p+\tilde{a}_i-\tilde{a}_k}-\delta_{p,r-\tilde{a}_k}\phi_{k,p}\otimes\phi^\dagger_{k,p+\tilde{a}_i}] \\
& +2g_0^2[-\delta_{r,p+\tilde{a}_i-\tilde{a}_j}\phi^\dagger_{j,p-\tilde{a}_j}\phi_{i,p-\tilde{a}_j}\otimes 1-\delta_{p,r}1\otimes\phi_{i,p+\tilde{a}_j}\phi^\dagger_{j,p+\tilde{a}_i} \\
& +\delta_{p,r-\tilde{a}_i}\phi_{i,p}\otimes\phi^\dagger_{j,p+\tilde{a}_i}+\delta_{p,r+\tilde{a}_j}\phi^\dagger_{j,p-\tilde{a}_j}\otimes\phi_{i,p}],
\end{aligned} \tag{5.49}
$$

$$
\begin{aligned}
(\mathcal{M}_D^2)^{ij}_{pr} = & \; g^2\delta_{ij}\delta_{p,r}[\phi_{k,p}\phi^\dagger_{k,p}\otimes 1+1\otimes\phi^\dagger_{k,p+\tilde{a}_i-\tilde{a}_k}\phi_{k,p+\tilde{a}_i-\tilde{a}_k} \\
& -\phi^\dagger_{k,p-\tilde{a}_k}\phi_{k,p-\tilde{a}_k}\otimes 1-1\otimes\phi_{k,p+\tilde{a}_i}\phi^\dagger_{k,p+\tilde{a}_i}] \\
& +g^2[\delta_{r,p+\tilde{a}_i-\tilde{a}_j}\phi_{i,p}\phi^\dagger_{j,p+\tilde{a}_i-\tilde{a}_j}\otimes 1+\delta_{p,r}1\otimes\phi^\dagger_{j,p}\phi_{i,p} \\
& -\delta_{p,r-\tilde{a}_i}\phi_{i,p}\otimes\phi^\dagger_{j,p+\tilde{a}_i}-\delta_{p,r+\tilde{a}_j}\phi^\dagger_{j,p-\tilde{a}_j}\otimes\phi_{i,p}].
\end{aligned} \tag{5.50}
$$

Finally, the squares of the fermion mass matrices are:

$$
\begin{aligned}
& (\mathcal{M}^\dagger_\Psi)^{ik}_{ps}(\mathcal{M}_\Psi)^{kj}_{sr}+(\mathcal{M}^\dagger_\Psi)^{i4}_{ps}(\mathcal{M}_\Psi)^{4j}_{sr} = \\
& 2g^2\delta_{ij}[\delta_{p,r}\phi^\dagger_{k,p-\tilde{a}_k}\phi_{k,p-\tilde{a}_k}\otimes 1+\delta_{p,r}1\otimes\phi_{k,p+a_i}\phi^\dagger_{k,p+a_i} \\
& -\delta_{p,r+\tilde{a}_k}\phi^\dagger_{k,p-\tilde{a}_k}\otimes\phi_{k,p+a_i-\tilde{a}_k}-\delta_{p,r-\tilde{a}_k}\phi_{k,p}\otimes\phi^\dagger_{k,p+a_i}] \\
& +2g^2[-\delta_{p,r+\tilde{a}_j-\tilde{a}_i}\phi^\dagger_{j,p-\tilde{a}_j}\phi_{i,p-\tilde{a}_j}\otimes 1-\delta_{p,r}1\otimes\phi_{i,p+a_j}\phi^\dagger_{j,p+a_i} \\
& \delta_{p,r+\tilde{a}_j-\tilde{a}_i}\phi_{i,p}\phi^\dagger_{j,p+\tilde{a}_i-\tilde{a}_j}\otimes 1+\delta_{p,r}1\otimes\phi^\dagger_{j,p-a_4}\phi_{i,p-a_4}],
\end{aligned} \tag{5.51}
$$

$$
\begin{aligned}
& (\mathcal{M}^\dagger_\Psi)^{4k}_{ps}(\mathcal{M}_\Psi)^{k4}_{sr} = \\
& 2g^2[\delta_{p,r}\phi_{k,p}\phi^\dagger_{k,p}\otimes 1+\delta_{p,r}1\otimes\phi^\dagger_{k,p-a_k}\phi_{k,p-a_k} \\
& -\delta_{p,r-\tilde{a}_k}\phi_{k,p}\otimes\phi^\dagger_{k,p+a_4}-\delta_{p,r+\tilde{a}_k}\phi^\dagger_{k,p-\tilde{a}_k}\otimes\phi_{k,p-a_k}].
\end{aligned} \tag{5.52}
$$

Taking into account the number of degrees of freedom and statistics of each field, the supertrace is:

$$
STr(\mathcal{M}^2)=3Tr(\mathcal{M}_A^2)+2Tr(\mathcal{M}_F^2)+2Tr(\mathcal{M}_D^2)-2Tr(\mathcal{M}^\dagger_\Psi\mathcal{M}_\Psi)^{44}-2Tr(\mathcal{M}^\dagger_\Psi\mathcal{M}_\Psi)^{ij}. \tag{5.53}
$$

Finally we find the supertrace of the mass matrix:

$$
\begin{aligned}
STr(\mathcal{M}^2) = 4g^2 \textstyle\sum_k \sum_p \delta_{\tilde{a}_k,0} \Big[\\
+ \Big(Tr(\phi^\dagger_{k,p})Tr(\phi_{k,p+a_4}) + Tr(\phi^\dagger_{k,p+a_4})Tr(\phi_{k,p}) - 2Tr(\phi^\dagger_{k,p})Tr(\phi_{k,p})\Big) \\
+ \textstyle\sum_i \Big(Tr(\phi^\dagger_{k,p})Tr(\phi_{k,p+a_i}) + Tr(\phi^\dagger_{k,p+a_i})Tr(\phi_{k,p}) \\
- Tr(\phi^\dagger_{k,p})Tr(\phi_{k,p+\tilde{a}_i}) - Tr(\phi^\dagger_{k,p+\tilde{a}_i})Tr(\phi_{k,p})\Big)\Big] \,.
\end{aligned}
\tag{5.54}
$$

One can check that (5.54) vanishes identically if at least one of the following conditions is satisfied:

- $a_4 = 0$ or $a_i = 0$, that is when at least $N = 1$ supersymmetry is preserved by the orbifolding,
- $\tilde{a}_1 \neq 0$, $\tilde{a}_2 \neq 0$ $\tilde{a}_3 \neq 0$, that is when there are no scalars in the adjoint representation of the $U(K)_p$ group.

In the first case the vanishing of the supertrace is of course guaranteed by unbroken supersymmetry of the daughter theory. Surprisingly, the absence of quadratic divergences can also occur if the daughter theory is completely non-supersymmetric. The only condition is that $\tilde{a}_i \neq 0$, in other words, all scalars are in bi-fundamental representations of the $U(K)^N$ gauge group. This condition is non-trivial as, for instance, the choice $\tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3 = 0$ and $N = 2$ leads to a non-supersymmetric theory with quadratic divergences.

Thus we have found that there exists a class of non-supersymmetric theories free from quadratic divergences. This makes the quiver theories interesting candidates for physics beyond the SM, an alternative to softly broken supersymmetry. But to make contact with the low energy world we must reduce the number of gauge symmetries. Inspired by deconstruction, we can assign to each bi-fundamental scalar a common expectation value. This breaks the product gauge group down to the diagonal $U(K)$. In the following we shall investigate properties of the quiver theories in the deconstruction phase.

We assume that the scalar fields acquire vevs proportional to the identity matrix and independent of the block indices,

$$
\langle \phi_{i,p} \rangle = \frac{1}{\sqrt{2}} v_i \mathbf{1} \,. \tag{5.55}
$$

For simplicity we assume that v_i's are real. Note that we still allow the vevs to depend on the generation index.

In general, such vevs as in (5.55) imply spontaneous breaking of $U(K)^N$ down to the diagonal $U(K)$ group[2]. The gauge bosons acquire mass terms,

$$\mathcal{L} = \mathrm{Tr} \sum_p \sum_{k=1}^{3} g^2 v_k^2 (A_{\mu,p} - A_{\mu,p+\tilde{a}_k})^2 \,. \tag{5.56}$$

These mass terms are diagonalized in an analogous way as in the periodic model [3], see Section 3.1:

$$A_{\mu,p} = \sqrt{\frac{2}{N}} \left(\sum_{n=0}^{(N-1)/2} \eta_n \cos\left(\frac{2n\pi}{N} p\right) A_\mu^{(n)} + \sum_{n=1}^{(N-1)/2} \sin\left(\frac{2n\pi}{N} p\right) \tilde{A}_\mu^{(n)} \right) . \tag{5.57}$$

where $\eta_0 = 1/\sqrt{2}$ and $\eta_n = 1$ for $n \neq 0$. Plugging in this decomposition we get:

$$\mathcal{L} = \mathrm{Tr} \sum_n \sum_k (m_k^{(n)})^2 (A_\mu^{(n)} A_\mu^{(n)} + \tilde{A}_\mu^{(n)} \tilde{A}_\mu^{(n)}) \qquad m_k^{(n)} \equiv 2 g v_k \sin\left(\frac{n\pi}{N} \tilde{a}_k\right) , \tag{5.58}$$

so that the n-th level gauge bosons have masses $(m^{(n)})^2 = \sum_k m_k^2$.

Scalar vevs also generate masses of the fermion and scalar fields. We decompose the complex scalars into real ones as $\phi_{i,p} = \frac{1}{\sqrt{2}} v_i \mathbf{I} + X_{i,p}$, $X_{i,p} = \frac{1}{\sqrt{2}}(B_{i,p} + i C_{i,p})$. Then we use the mode decomposition (5.57), with $p \to p + \tilde{a}_i/2$ on the r.h.s and we get :

$$\mathcal{L}_B = -\mathrm{Tr} \sum_i \sum_n (m^{(n)})^2 (B_i^{(n)} B_i^{(n)} + \tilde{B}_i^{(n)} \tilde{B}_i^{(n)}), \tag{5.59}$$

$$\mathcal{L}_C = -\mathrm{Tr} \sum_i \sum_j \sum_n (\delta_{ij} (m^{(n)})^2 - m_i^{(n)} m_j^{(n)}) (C_i^{(n)} C_j^{(n)} + \tilde{C}_i^{(n)} \tilde{C}_j^{(n)}). \tag{5.60}$$

The n-th level B-scalar is degenerate in mass with the n-th level gauge boson. As for the C-scalar, the mass matrix must be further diagonalized in the generation indices. This mass matrix has N zero eigenvalues corresponding to eigenvectors $C_0^{(n)} = \frac{1}{m^{(n)}} \sum_k m_k C_k^{(n)}$, $\tilde{C}_0^{(n)} = \frac{1}{m^{(n)}} \sum_k m_k \tilde{C}_k^{(n)}$. These eigenvectors are readily identified with the Goldstone bosons of the spontanously broken $U(K)^N$ symmetry. All but one of them are eaten by massive vector fields. The other two combinations of $C_i^{(n)}$ orthogonal to $C_0^{(n)}$ acquire masses equal to $m^{(n)}$.

[2]However, if all $\tilde{a}_k$ are divisors of N then the gauge group can be broken to some product of $U(K)$ groups.

[3]The decomposition is given for odd N. For even N the first sum goes to $N/2$ and the second to $N/2-1$.

Finally we investigate the fermion masses in the deconstruction phase. The mode decomposition is again analogous to (5.57) with $p \to p + a_4$ for gauginos and $p \to p + \tilde{a}_i/2$ for ψ_i. The mass terms take the form:

$$\mathcal{L} = -\mathrm{Tr} \sum_n \sum_k m_k^{(n)} (\overline{\tilde{\psi}_k^{(n)}} \lambda^{(n)} + \overline{\psi_k^{(n)}} \tilde{\lambda}^{(n)}) - \sqrt{2} \epsilon_{ijk} \overline{\psi_i^{(n)}} \tilde{\psi}_j^{(n)} m_k^{(n)}. \tag{5.61}$$

Similarly to the C-scalars we can define $\psi_0^{(n)} = \frac{1}{m^{(n)}} \sum_k m_k^{(n)} \psi_k^{(n)}$, $\tilde{\psi}_0^{(n)} = \frac{1}{m^{(n)}} \sum_k m_k^{(n)} \tilde{\psi}_k^{(n)}$ such that $\psi_0^{(n)}$ ($\tilde{\psi}_0^{(n)}$) combines with $\lambda^{(n)}$ ($\tilde{\lambda}^{(n)}$) to form a Dirac fermion of mass $m^{(n)}$. The two remaining linear combinations of $\psi_i^{(n)}$, which are orthogonal to $\psi_0^{(n)}$, combine with $\tilde{\psi}_i^{(n)}$ to form Dirac fermions of mass $m^{(n)}$.

We see that at the n-th level the spectrum is perfectly boson-fermion degenerate. This suggests that a part of the lagrangian in the deconstruction phase may possess some boson-fermion symmetry. Indeed, this is the case, and we shall call that approximate symmetry a custodial supersymmetry. A way to describe it is to rewrite the deconstruction phase lagrangian using the superspace formalism. We define the vector superfields in the Wess-Zumino gauge as:

$$V^{(n)}(y, \theta) = \frac{i}{2} (\overline{\theta} \gamma_5 \gamma_\mu \theta) A^{(n)} - i (\overline{\theta} \gamma_5 \theta)(\overline{\theta} \lambda^{(n)}) - \frac{1}{4} (\overline{\theta} \gamma_5 \theta)^2 D^{(n)}. \tag{5.62}$$

Similarly we define chiral superfields:

$$\Phi_i^{(n)}(y, \theta) = X_i^{(n)} - \sqrt{2} (\overline{\theta} P_L \psi_i^{(n)}) + F_i^{(n)} (\overline{\theta} P_L \theta). \tag{5.63}$$

Analogous expressions for the tilded fields hold.

First, we note that the self-couplings in the zero-mode sector are those of the $N = 4$ supersymmetric theory. Indeed, the interactions of the zero-modes can be found by making in (5.46) the replacement $\phi_{i,p} \to \frac{1}{\sqrt{N}} \phi_i^{(0)}$ (and similarly for fermion and gauge fields). Since all memory of the block indices is lost, as a result we obtain the lagrangian (5.37) with the gauge coupling $g_0 = \frac{g}{\sqrt{N}}$. Second, we have already shown that the mass pattern in the deconstruction phase is supersymmetric. It turns out that the custodial supersymmetry has a much wider extent and all the terms quadratic in the heavy modes (including triple and quartic interactions with the zero-modes) match the structure of a globally supersymmetric theory! As an example we present a superfield lagrangian which reproduces the Yukawa terms and the scalar potential of the daughter theory:

$$\begin{aligned} \mathcal{L} = \textstyle\sum_n \sum_k \mathrm{Tr} \Big[& 4 g v_k \sin\left(\tfrac{n\pi\tilde{a}_k}{N}\right) \left(\tilde{V}^{(n)} \Phi_k^{(n)} - V^{(n)} \tilde{\Phi}_k^{(n)}\right) \\ & + 2 g_0 \cos\left(\tfrac{n\pi\tilde{a}_k}{N}\right) \left([\Phi_k^{(0)\dagger}, \Phi_k^{(n)}] V^{(n)} + [\Phi_k^{(0)\dagger}, \tilde{\Phi}_k^{(n)}] \tilde{V}^{(n)} \right) \\ & + 2 g_0 \sin\left(\tfrac{n\pi\tilde{a}_k}{N}\right) \left(\{\Phi_k^{(0)\dagger}, \Phi_k^{(n)}\} \tilde{V}^{(n)} - \{\Phi_k^{(0)\dagger}, \tilde{\Phi}_k^{(n)}\} V^{(n)} \right) + \mathrm{h.c.} \Big]_D \\ & + [W]_F + [W^*]_F, \end{aligned} \tag{5.64}$$

where the superpotential is:

$$
\begin{aligned}
W = -i\sqrt{2}\sum_n \sum_{ijk} \epsilon_{ijk} \mathrm{Tr} \Big[& 4 g v_k \sin\left(\tfrac{n\pi\tilde{a}_k}{N}\right) \Phi_i^{(n)} \tilde{\Phi}_j^{(n)} \\
& - g_0 \cos\left(\tfrac{n\pi\tilde{a}_k}{N}\right) \left([\Phi_k^{(0)}, \Phi_i^{(n)}]\Phi_j^{(n)} + [\Phi_k^{(0)}, \tilde{\Phi}_i^{(n)}]\tilde{\Phi}_j^{(n)} \right) \\
& + g_0 \sin\left(\tfrac{n\pi\tilde{a}_k}{N}\right) \left(\{\Phi_k^{(0)}, \Phi_i^{(n)}\}\tilde{\Phi}_j^{(n)} - \{\Phi_k^{(0)}, \tilde{\Phi}_i^{(n)}\}\Phi_j^{(n)} \right) \Big] .
\end{aligned}
\tag{5.65}
$$

One immediate consequence of the custodial supersymmetry in the lagrangian is that universal vevs are a flat direction at one-loop. Indeed, for $\langle \phi_{i,p} \rangle = \frac{1}{\sqrt{2}} v_i \mathbf{I}$, $\mathrm{STr} M^{2q} = 0$ and the one-loop effective potential is zero for all such configurations. The presence of the custodial supersymmetry is also sufficient to ensure the vanishing of one-loop corrections to the zero-mode masses. A mass-splitting of the zero-mode multiplets can appear only at the two-loop level and we expect this mass splitting to be suppressed $M_{\mathrm{SUSY}} \ll v \ll \Lambda$. Supersymmetry is explicitly violated by triple and quartic self-interactions of the heavy modes. Thus, the heavy mode masses are not protected against logarithmically divergent corrections (earlier in the paper we have shown that quadratic divergences are absent at one-loop).

In summary, the non-supersymmetric quiver theories are very interesting from the theoretical point of view. They do not contain quadratic divergences at one loop. Therefore they may be suitable for solving the hierarchy problem in the weak sense, that is, explaining the stability of the SM Higgs sector when the UV cutoff is rather low, say $10-100$ TeV. Furthermore, in the deconstruction phase these theories exhibit a surprising phenomenon which we called the custodial supersymmetry. Its effect is that, from the low energy point of view, the theory seems supersymmetric, e.g., bosons and fermions are degenerate in mass. This degeneracy is removed only at higher loop levels and that is how hierarchy between the cutoff scale and the scale of superpartners may appear. Of course, at this stage we are still very far from constructing a realistic theory based on this setup, nevertheless we think that this direction deserves further study.

Chapter 6

Summary and outlook

There are two important messages we wanted to communicate in this thesis. Firstly, deconstruction is a useful tool for studying higher dimensional gauge theories. Its power is in using the well understood language of 4d gauge theories. Secondly, deconstruction is an interesting direction in search for physics beyond the SM, even without any reference to extra dimensions. For example, its extended symmetries lead to novel solutions to the hierarchy problem.

The main results of this thesis are contained in Chapter 4 and Chapter 5. In Chapter 4 we articulated the former message. It was shown that deconstruction can mimic the physics of extra dimensions not only at the tree-level but also at the quantum level. Moreover, we have demonstrated that it allows to address even those question that are not well-defined in the higher dimensional setup. This was illustrated by studying two models, that were intended to be 4d analogues of previously existing higher dimensional constructions. In Section 4.1 we used deconstruction to study the evolution of gauge couplings in 5d gauge theories defined in the curved AdS_5 background. The 4d setup, in which RG running is perfectly understood, could clarify the actual meaning of the logarithmic running in AdS_5.

In Section 4.2 deconstruction was used as an UV regulator for the model of Barbieri, Hall and Nomura [6]. This model is a 5d globally supersymmetric theory, in which supersymmetry is broken by boundary conditions. In deconstruction, the analogue of such breaking turned out to be hard supersymmetry breaking at one boundary of the group product space. In spite of this, it was shown that one loop corrections to scalar masses can be finite, if the interactions that generate these corrections are localized away (in the group product space) from the supersymmetry breaking sector. When the interactions and the supersymmetry breaking sectors overlap, one needs more subtle regulator than deconstruction, presumably one involving local supersymmetry.

As for the latter message, in Chapter 5 we presented two models that may turn out relevant for solving the hierarchy problem. The one in Section 5.1 was based on supersymmetry. Deconstruction was a mean to generate large hierarchy between the fundamental and the supersymmetry breaking scales. The model was shown to yield an interesting pattern of soft breaking terms, which could solve the flavor problem of the MSSM. Also, the Froggatt-Nielsen mechanism of fermion mass generation arose naturally in this context.

The model we introduced in Section 5.2 is at this point further from realistic phenomenology. Nevertheless, two interesting things could be learned. One is that some deconstruction setups can be derived as a low energy limit of certain string theories. The other fact is that there exists a class of 4d non-supersymmetric models (different than the Little Higgs models) in which the one loop effective potential is free from quadratic divergences.

In the remainder we would like to discuss some possible future directions of research. One thing that is obviously missing is deconstruction of gravity. So far only the physics of spin-1, spin-1/2 and spin-0 fields propagating in extra dimensions can be realized in 4d theories. An analogous construction involving spin-2 gravitons encounters some technical as well as conceptual problems. One important step in this direction was made in ref. [57], where consistent theories with extended general coordinate invariance were constructed. When a formalism for deconstructing gravity is finally worked out, even more fundamental issues than the hierarchy problem can be attacked. In particular, 4d gravity could be constructed on the base of lower dimensional gravities, that are softer in the UV. This may lead to a new, consistent theory of quantum gravity

Concerning the hierarchy problem, a very promising direction are the Little Higgs models [45]. These models aim at postponing the hierarchy problem up to energies of order 10 TeV, by assuming that the Higgs boson is a pseudo-Goldstone boson of some approximate global symmetry. Extended gauge symmetry of deconstruction setups can be then used to forbid quadratic divergences at low loop levels. Absence of quadratic divergences at the one loop level implies that, without fine-tuning, the Higgs bosons can be an order of magnitude lighter than the cutoff scale to which such models are valid. However it is still an open questions if there exist simple Little Higgs model consistent with all electroweak precision measurements [58].

Apart from that, deconstruction, or, more precisely, existence of an extended local symmetry group, can lead to interesting results in many distinct fields. Current directions of research include inflationary cosmology [59], grand unification [60] and neutrino physics [61]. All in all, deconstruction is an active field of research and we believe that the best is still to come ....

Appendix A

5d YM action compactified on S_1

We consider the 5d Yang-Mills action,

$$S = -\tfrac{1}{2}\int d^4x dy \ \mathrm{Tr}\left\{F_{\alpha\beta}F^{\alpha\beta}\right\},$$
$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + ig_5[A_\alpha, A_\beta]\,, \tag{A.1}$$

where $A_\alpha = A^a_\alpha T^a$ and T^a are the hermitian generators of the gauge group. If the fifth dimension is the circle of radius R, the KK decomposition of the 5d fields reads:

$$A_\mu(x^\mu, y) = \frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\left(\eta_{(n)}\cos(\frac{ny}{R})A^{(n)}_\mu(x^\mu) + \sin(\frac{ny}{R})A^{(-n)}_\mu(x^\mu)\right) \tag{A.2}$$

$$A_5(x^\mu, y) = \frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\left(-\sin(\frac{ny}{R})A^{(n)}_5(x^\mu) + \eta_{(n)}\cos(\frac{ny}{R})A^{(-n)}_5(x^\mu)\right). \tag{A.3}$$

In the above $\eta_{(n)} = 1/\sqrt{2}$ for $n = 0$ and $\eta_{(n)} = 1$ otherwise. The 4d effective action is obtained by plugging this decomposition back into the action of eq. (A.1) and performing the integral over y. The integrals that are needed read:

$$\int_0^{2\pi R} dy \cos\left(\frac{ny}{R}\right) = 2\pi R\,\delta_n\,,$$
$$\int_0^{2\pi R} dy \sin\left(\frac{ny}{R}\right) = 0\,, \tag{A.4}$$

where $\delta_n = 1$ for $n = 0$ and $\delta_n = 0$ for $n \neq 0$. From this, one can deduce the orthogonality relations:

$$\frac{1}{\pi R}\int_0^{2\pi R} dy \eta_n\eta_m \cos\left(\frac{ny}{R}\right)\cos\left(\frac{my}{R}\right) = \delta_{n-m}\,,$$

$$
\begin{aligned}
\frac{1}{\pi R}\int_0^{2\pi R} dy \sin\left(\frac{ny}{R}\right)\sin\left(\frac{my}{R}\right) &= \delta_{n-m}\,, \\
\frac{1}{\pi R}\int_0^{2\pi R} dy \cos\left(\frac{ny}{R}\right)\sin\left(\frac{my}{R}\right) &= 0\,.
\end{aligned}
\tag{A.5}
$$

To determine the interactions of the KK modes we will need the formulae involving three and four trigonometric functions. We introduce the 'three-point' tensors:

$$
\begin{aligned}
h_{nrs} &= \frac{\sqrt{2}}{\pi R}\int_0^{2\pi R} dy \eta_n \eta_r \eta_s \cos\left(\frac{ny}{R}\right)\cos\left(\frac{ry}{R}\right)\cos\left(\frac{sy}{R}\right) \\
\tilde{h}_{nrs} &= \frac{\sqrt{2}}{\pi R}\int_0^{2\pi R} dy \eta_n \cos\left(\frac{ny}{R}\right)\sin\left(\frac{ry}{R}\right)\sin\left(\frac{sy}{R}\right)
\end{aligned}
\tag{A.6}
$$

Explicitly, these tensors read

$$
\begin{gathered}
h_{nrs} = \frac{1}{\sqrt{2}}(\delta_{n-r-s} + \delta_{n-r+s} + \delta_{n+r-s}) \quad n,r,s > 0 \\
h_{0rs} = h_{r0s} = h_{rs0} = \delta_{r-s} \\
\tilde{h}_{nrs} = \frac{1}{\sqrt{2}}(-\delta_{n-r-s} + \delta_{n-r+s} + \delta_{n+r-s}) \quad n,r,s > 0 \\
\tilde{h}_{0rs} = \delta_{r-s} \qquad \tilde{h}_{n0s} = \tilde{h}_{nr0} = 0
\end{gathered}
\tag{A.7}
$$

Similarly, we define 'four-point' tensors:

$$
\begin{aligned}
k_{nmrs} &= \frac{2}{\pi R}\int_0^{2\pi R} dy \eta_n \eta_m \eta_r \eta_s \cos\left(\frac{ny}{R}\right)\cos\left(\frac{my}{R}\right)\cos\left(\frac{ry}{R}\right)\cos\left(\frac{sy}{R}\right) \\
\tilde{k}_{nmrs} &= \frac{2}{\pi R}\int_0^{2\pi R} dy \eta_n \eta_m \cos\left(\frac{ny}{R}\right)\cos\left(\frac{my}{R}\right)\sin\left(\frac{ry}{R}\right)\sin\left(\frac{sy}{R}\right) \\
\hat{k}_{nmrs} &= \frac{2}{\pi R}\int_0^{2\pi R} dy \sin\left(\frac{ny}{R}\right)\sin\left(\frac{my}{R}\right)\sin\left(\frac{ry}{R}\right)\sin\left(\frac{sy}{R}\right)
\end{aligned}
\tag{A.8}
$$

The explicit expressions for these tensors are easy to obtain but lengthy. Below we give only the results for the special cases in which two pairs of indices are equal,

$$
\begin{gathered}
k_{nnrr} = 1 + \frac{1}{2}\delta_{n-r} \quad n,r > 0 \\
k_{00rr} = 1 \\
\tilde{k}_{nnrr} = 1 - \frac{1}{2}\delta_{n-r} \quad n,r > 0 \\
\tilde{k}_{00rr} = 1 \\
\tilde{k}_{nrnr} = \frac{1}{2}\delta_{n-r} \quad n,r > 0
\end{gathered}
$$

$$\hat{k}_{nnrr} = 1 + \frac{1}{2}\delta_{n-r} \quad n, r > 0 \tag{A.9}$$

Now we are ready to give the full effective action derived from the 5d YM action compactified on S_1. We split the effective action as:

$$S_{4d} = \mathcal{L}_{kin} + \mathcal{L}_{AAA} + \mathcal{L}_{AAAA}\,. \tag{A.10}$$

The kinetic part is defined as:

$$\mathcal{L}_{kin} = -\frac{1}{4}\int dy(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + \frac{1}{2}\int dy(\partial_\mu A^a_5 - \partial_5 A^a_\mu)(\partial_\mu A^a_5 - \partial_5 A^a_\mu) \tag{A.11}$$

After integrating over y we get

$$\begin{aligned}\mathcal{L}_{kin} = -\tfrac{1}{4}\textstyle\sum_{n=-\infty}^{\infty}\Big[&(\partial_\mu A^a_{\nu,(n)} - \partial_\nu A^a_{\mu,(n)})(\partial_\mu A^a_{\nu,(n)} - \partial_\nu A^a_{\mu,(n)}) + \tfrac{1}{2}m^2_{(n)} A^a_{\mu,(n)} A^a_{\mu,(n)} \\ &+ \tfrac{1}{2}\partial_\mu A^a_{5,(n)}\partial_\mu A^a_{5,(n)} - m_{(n)} A^a_{\mu,(n)}\partial_\mu A^a_{5,(n)}\Big]\end{aligned} \tag{A.12}$$

Here $m_{(n)} = |n|/R$ is the mass of the n-th KK mode.

The trilinear interactions are defined as

$$\mathcal{L}_{AAA} = g_5 f^{abc} \textstyle\int dy\left[\partial_\mu A^a_\nu A^b_\mu A^c_\nu + \partial_5 A^a_\mu A^b_\mu A^c_5 + A^a_\mu \partial_\mu A^b_5 A^b_\mu A^c_5\right], \tag{A.13}$$

Performing the integration we get:

$$\begin{aligned}\mathcal{L}_{AAA} = g_0 f^{abc} \textstyle\sum_{n,r,s=0}^{\infty}\Big[\\ h_{nrs}(\partial_\mu A^a_{\nu,(n)} A^b_{\mu,(r)} A^c_{\nu,(s)} + A^a_{\mu,(n)}\partial_\mu A^b_{5,(r)} A^c_{5,(s)} + m_{(n)} A^a_{\mu,(-n)} A^b_{\mu,(r)} A^c_{5,(s)}) \\ +\tilde{h}_{nrs}(\partial_\mu A^a_{\nu,(n)} A^b_{\mu,(-r)} A^c_{\nu,(-s)} + A^a_{\mu,(n)}\partial_\mu A^b_{5,(-r)} A^c_{5,(-s)} + m_{(n)} A^a_{\mu,(-n)} A^b_{\mu,(-r)} A^c_{5,(-s)}) \\ +\tilde{h}_{rns}(\partial_\mu A^a_{\nu,(-n)} A^b_{\mu,(r)} A^c_{\nu,(-s)} + A^a_{\mu,(-n)}\partial_\mu A^b_{5,(r)} A^c_{5,(-s)} + m_{(n)} A^a_{\mu,(n)} A^b_{\mu,(r)} A^c_{5,(-s)}) \\ +\tilde{h}_{snr}(\partial_\mu A^a_{\nu,(-n)} A^b_{\mu,(-r)} A^c_{\nu,(s)} + A^a_{\mu,(-n)}\partial_\mu A^b_{5,(-r)} A^c_{5,(s)} + m_{(n)} A^a_{\mu,(n)} A^b_{\mu,(-r)} A^c_{5,(s)})\Big]\,.\end{aligned} \tag{A.14}$$

Here $g_0 = g_5/\sqrt{2\pi R}$ is the gauge coupling of the massless mode in the 4d effective theory. Finally, we come to the four-point interactions defined as:

$$\mathcal{L}_{AAAA} = g_5^2 f^{abe} f^{cde}\int dy\left[-\frac{1}{4}A^a_\mu A^b_\nu A^c_\mu A^d_\nu + \frac{1}{2}A^a_\mu A^b_5 A^c_\mu A^d_5\right] \tag{A.15}$$

Integrating over y we find:

$$
\begin{aligned}
\mathcal{L}_{AAAA} = -\tfrac{1}{4} g_0^2 \textstyle\sum_{n,m,r,s=0}^{\infty} \Big[\\
k_{nmrs}(A^a_{\mu,(n)}A^b_{\nu,(m)}A^c_{\mu,(r)}A^d_{\nu,(s)} - 2A^a_{\mu,(n)}A^b_{5,(m)}A^c_{\mu,(r)}A^d_{5,(s)}) \\
+\hat{k}_{nmrs}(A^a_{\mu,(-n)}A^b_{\nu,(-m)}A^c_{\mu,(-r)}A^d_{\nu,(-s)} - 2A^a_{\mu,(-n)}A^b_{5,(-m)}A^c_{\mu,(-r)}A^d_{5,(-s)}) \\
+\tilde{k}_{nmrs}(A^a_{\mu,(n)}A^b_{\nu,(m)}A^c_{\mu,(-r)}A^d_{\nu,(-s)} - 2A^a_{\mu,(n)}A^b_{5,(m)}A^c_{\mu,(-r)}A^d_{5,(-s)}) \\
+\tilde{k}_{nrms}(A^a_{\mu,(n)}A^b_{\nu,(-m)}A^c_{\mu,(r)}A^d_{\nu,(-s)} - 2A^a_{\mu,(n)}A^b_{5,(-m)}A^c_{\mu,(r)}A^d_{5,(-s)}) \\
+\tilde{k}_{nsmr}(A^a_{\mu,(n)}A^b_{\nu,(-m)}A^c_{\mu,(-r)}A^d_{\nu,(s)} - 2A^a_{\mu,(n)}A^b_{5,(-m)}A^c_{\mu,(-r)}A^d_{5,(s)}) \\
+\tilde{k}_{mrns}(A^a_{\mu,(-n)}A^b_{\nu,(m)}A^c_{\mu,(r)}A^d_{\nu,(-s)} - 2A^a_{\mu,(-n)}A^b_{5,(m)}A^c_{\mu,(r)}A^d_{5,(-s)}) \\
+\tilde{k}_{msnr}(A^a_{\mu,(-n)}A^b_{\nu,(m)}A^c_{\mu,(-r)}A^d_{\nu,(s)} - 2A^a_{\mu,(-n)}A^b_{5,(m)}A^c_{\mu,(-r)}A^d_{5,(s)}) \\
+\tilde{k}_{rsmn}(A^a_{\mu,(-n)}A^b_{\nu,(-m)}A^c_{\mu,(r)}A^d_{\nu,(s)} - 2A^a_{\mu,(-n)}A^b_{5,(-m)}A^c_{\mu,(r)}A^d_{5,(s)}) \\
\Big]
\end{aligned}
\tag{A.16}
$$

Appendix B

The periodic model of deconstruction

We consider the action:

$$\mathcal{L} = \sum_{p=1}^{N} \mathrm{Tr}\left\{-\frac{1}{2}F_{\mu\nu,p}F_{\mu\nu,p} + D_\mu\Phi_p^\dagger D_\mu\Phi_p - V(\Phi_p)\right\} . \tag{B.1}$$

The covariant derivative is given by[1]:

$$D_\mu\Phi_p = \partial_\mu\Phi_p + igA_p\Phi_p - ig\Phi_pA_{p+1} . \tag{B.2}$$

The potential should be chosen so as to give universal vevs to the link-Higgs fields, $\langle\Phi_p\rangle = \frac{1}{\sqrt{2}}v\mathbf{I}$. The simplest choice is a renormalizable potential of the form:

$$V(\Phi_p) = m^2\Phi_p^\dagger\Phi_p + \lambda(\Phi_p^\dagger\Phi_p)^2 , \tag{B.3}$$

with negative m^2, which forces the vev of the links to be $|v|^2 = -\frac{m^2}{\lambda}$. In the following we assume that v is real. Then it is convenient to split the links as $\Phi_p = \frac{1}{\sqrt{2}}(v\mathbf{I}+\Sigma_p+G_p)$. All the Σ fields acquire a common mass $M^2 = 2v^2\lambda$, while G_p are massless. There are two ways to proceed. One can keep all the degrees of freedom in the low-energy theory (linear sigma model) or decouple the heavy scalars Σ (non-linear sigma model). The former option has the advantage of preserving the renormalizability of the theory, so that the linear sigma model can be considered as a UV completion to the higher dimensional theory. It is also the only option in supersymmetric theories where Σ do not decouple but are degenerate in mass with the tower of the gauge bosons. The latter possibility allows us to work with the massless degrees of freedom only.

[1] In this chapter, to save some space, we suppress the Lorentz index of the gauge fields everytime it is obvious.

B.1 Useful formulae for the periodic models

In this section we present a number of formulae needed to derive the form of interactions in the periodic deconstruction model. For simplicity we assume an odd number of replications N, for even N the mode decomposition changes slightly.

The master formulae are those for 'one-point functions',

$$\begin{aligned} \sum_{p=1}^{N} \cos \frac{2np\pi}{N} &= N\,\delta_n\,, \\ \sum_{p=1}^{N} \sin \frac{2np\pi}{N} &= 0\,, \end{aligned} \tag{B.4}$$

where δ is now defined modulo N,

$$\delta_n = \begin{cases} 1 & n = 0 \mod N \\ 0 & n \neq 0 \mod N \end{cases} \tag{B.5}$$

We will also need more general 'one-point functions',

$$\begin{aligned} \sum_{p=1}^{N} \cos \frac{2n\pi(p+\alpha)}{N} &= N\,\delta_n \cos \frac{2n\pi\alpha}{N}\,, \\ \sum_{p=1}^{N} \sin \frac{2n\pi(p+\alpha)}{N} &= N\,\delta_n \sin \frac{2n\pi\alpha}{N}\,. \end{aligned} \tag{B.6}$$

For $\alpha = 1/2$, which is the only case of interest for this appendix, $\delta_n \cos \frac{2n\pi\alpha}{N} = (-1)^{n/N}\delta_n$, $\delta_n \sin \frac{2n\pi\alpha}{N} = 0$.

The remaining formulae can be be reduced to the 'one-point function' by using

$$\begin{aligned} \cos \frac{2r\pi p}{N} \cos \frac{2s\pi p}{N} &= \frac{1}{2}(\cos \frac{2(r+s)\pi p}{N} + \cos \frac{2(r-s)\pi p}{N}) \\ \sin \frac{2r\pi p}{N} \sin \frac{2s\pi p}{N} &= \frac{1}{2}(-\cos \frac{2(r+s)\pi p}{N} + \cos \frac{2(r-s)\pi p}{N}) \\ \cos \frac{2r\pi p}{N} \sin \frac{2s\pi p}{N} &= \frac{1}{2}(\sin \frac{2(r+s)\pi p}{N} - \sin \frac{2(r-s)\pi p}{N}) \\ \sin \frac{2r\pi p}{N} \cos \frac{2s\pi p}{N} &= \frac{1}{2}(\sin \frac{2(r+s)\pi p}{N} + \sin \frac{2(r-s)\pi p}{N}) \end{aligned} \tag{B.7}$$

Using the above, the 'two-point functions' read

$$\sum_{p=1}^{N} \cos \frac{2np\pi}{N} \cos \frac{2mp\pi}{N} = \frac{N}{2}\,(\delta_{n-m} + \delta_{n+m})$$

$$\sum_{p=1}^{N} \sin\frac{2np\pi}{N}\sin\frac{2mp\pi}{N} = \frac{N}{2}\left(\delta_{n-m} - \delta_{n+m}\right)$$

$$\sum_{p=1}^{N} \cos\frac{2np\pi}{N}\sin\frac{2mp\pi}{N} = 0 \tag{B.8}$$

The above relations are useful to prove orthogonality of the gauge boson mode decomposition. One can also prove analogous formulae for $p \to p + \alpha$, e.g.

$$\sum_{p=1}^{N} \cos\frac{2n\pi(p+\alpha)}{N}\cos\frac{2m\pi(p+\alpha)}{N} = \frac{N}{2}\left(\cos\frac{2(n-m)\pi\alpha}{N}\delta_{n-m} + \cos\frac{2(n+m)\pi\alpha}{N}\delta_{n+m}\right) \tag{B.9}$$

The case with $\alpha = 1/2$ is useful for proving the orthogonality of the link-Higgs mode decomposition.

For deriving 'three point functions', using the same trick we find

$$\sum_{p=1}^{N} \cos\frac{2n\pi p}{N}\cos\frac{2r\pi p}{N}\cos\frac{2s\pi p}{N} = \frac{N}{4}(\delta_{n-r-s} + \delta_{n+r+s} + \delta_{n-r+s} + \delta_{n+r-s})$$

$$\sum_{p=1}^{N} \cos\frac{2n\pi p}{N}\sin\frac{2r\pi p}{N}\sin\frac{2s\pi p}{N} = \frac{N}{4}(-\delta_{n-r-s} - \delta_{n+r+s} + \delta_{n-r+s} + \delta_{n+r-s})$$

$$\sum_{p=1}^{N} \cos\frac{2n\pi p}{N}\cos\frac{2r\pi p}{N}\sin\frac{2s\pi p}{N} = 0$$

$$\sum_{p=1}^{N} \sin\frac{2n\pi p}{N}\sin\frac{2r\pi p}{N}\sin\frac{2s\pi p}{N} = 0 \tag{B.10}$$

It is convenient to define the tensors

$$h_{nrs} \equiv \frac{2\sqrt{2}}{N}\sum_{p=1}^{N} \eta_{(n)}\eta_{(r)}\eta_{(s)} \cos\frac{2n\pi p}{N}\cos\frac{2r\pi p}{N}\cos\frac{2s\pi p}{N}$$

$$\tilde{h}_{nrs} = \frac{2\sqrt{2}}{N}\sum_{p=1}^{N} \eta_{(n)} \cos\frac{2n\pi p}{N}\sin\frac{2r\pi p}{N}\sin\frac{2s\pi p}{N} \tag{B.11}$$

where $\eta_{(n)} = 1/\sqrt{2}$ for $n = 0$ and $\eta_{(n)} = 1$ otherwise. Explicitly, these tensors read

$$h_{nrs} = \frac{1}{\sqrt{2}}(\delta_{n-r-s} + \delta_{n+r+s} + \delta_{n-r+s} + \delta_{n+r-s}) \quad n, r, s > 0$$

$$
\begin{aligned}
h_{0rs} = h_{r0s} = h_{rs0} &= \delta_{r-s} \\
\tilde{h}_{nrs} &= \frac{1}{\sqrt{2}}(-\delta_{n-r-s} - \delta_{n+r+s} + \delta_{n-r+s} + \delta_{n+r-s}) \quad n,r,s > 0 \\
\tilde{h}_{0rs} = \delta_{r-s} &\qquad \tilde{h}_{n0s} = \tilde{h}_{nr0} = 0
\end{aligned} \tag{B.12}
$$

Here it is understood that the KK indices n,r,s satisfy $n, r, s \in (0, \frac{N-1}{2})$. Note that in such case δ_{r+s} is not operative for r,s positive and so it was not displayed.

Analogously, one can define the tensors for the four-point vertices,

$$
\begin{aligned}
k_{nmrs} &= \frac{4}{N}\sum_{p=1}^{N} \eta_{(n)}\eta_{(m)}\eta_{(r)}\eta_{(s)} \cos\frac{2n\pi p}{N}\cos\frac{2m\pi p}{N}\cos\frac{2r\pi p}{N}\cos\frac{2s\pi p}{N} \\
\tilde{k}_{nmrs} &= \frac{4}{N}\sum_{p=1}^{N} \eta_{(n)}\eta_{(m)} \cos\frac{2n\pi p}{N}\cos\frac{2m\pi p}{N}\sin\frac{2r\pi p}{N}\sin\frac{2s\pi p}{N} \\
\hat{k}_{nmrs} &= \frac{4}{N}\sum_{p=1}^{N} \sin\frac{2n\pi p}{N}\sin\frac{2m\pi p}{N}\sin\frac{2r\pi p}{N}\sin\frac{2s\pi p}{N}
\end{aligned} \tag{B.13}
$$

The explicit expressions for those tensors are easy to obtain but lengthy. Below we give only the results for the special cases in which two two pairs of indices are equal,

$$
\begin{aligned}
k_{nnrr} &= 1 + \frac{1}{2}(\delta_{2n-2r} + \delta_{2n+2r}) \quad n,r > 0 \\
k_{00rr} &= 1 \\
\tilde{k}_{nnrr} &= 1 - \frac{1}{2}(\delta_{2n-2r} + \delta_{2n+2r}) \quad n,r > 0 \\
\tilde{k}_{00rr} &= 1 \\
\tilde{k}_{nrnr} &= \frac{1}{2}(\delta_{2n-2r} + \delta_{2n+2r}) \quad n,r > 0 \\
\hat{k}_{nnrr} &= 1 + \frac{1}{2}(\delta_{2n-2r} + \delta_{2n+2r}) \quad n,r > 0
\end{aligned} \tag{B.14}
$$

B.2 Interactions of the gauge fields

The action for the gauge fields is

$$
\mathcal{L}_{ym} = \mathrm{Tr}\sum_{p=1}^{N}\left\{-\frac{1}{2}F_{\mu\nu,p}F_{\mu\nu,p}\right\}, \tag{B.15}
$$

where

$$F_{\mu\nu,p} = \partial_\mu A_{\nu,p} - \partial_\nu A_{\mu,p} + ig[A_{\mu,p}, A_{\nu,p}] \tag{B.16}$$

This can be split as

$$\mathcal{L}_{ym} = \mathcal{L}_{kin} + \mathcal{L}_{AAA} + \mathcal{L}_{AAAA} \tag{B.17}$$

where

$$\mathcal{L}_{kin} = -\frac{1}{4}\sum_{p=1}^{N}\Big\{(\partial_\mu A^a_{\nu,p} - \partial_\nu A^a_{\mu,p})(\partial_\mu A^a_{\nu,p} - \partial_\nu A^a_{\mu,p})\Big\} \tag{B.18}$$

$$\mathcal{L}_{AAA} = \frac{1}{2} g f^{abc}\sum_{p=1}^{N}\Big\{(\partial_\mu A^a_{\nu,p} - \partial_\nu A^a_{\mu,p})A^b_{\mu,p}A^c_{\nu,p}\Big\} \tag{B.19}$$

$$\mathcal{L}_{AAAA} = -\frac{1}{4} g f^{abe} f^{cde}\sum_{p=1}^{N}\Big\{A^a_{\mu,p}A^b_{\nu,p}A^c_{\mu,p}A^d_{\nu,p}\Big\} \tag{B.20}$$

We insert the mode decomposition

$$A_p = \sqrt{\frac{2}{N}}\sum_{n=0}^{(N-1)/2}\left\{\eta_n \cos\frac{2n\pi p}{N}A^{(n)} + \sin\frac{2n\pi p}{N}A^{(-n)}\right\}, \tag{B.21}$$

which diagonalizes the gauge boson mass terms, see eq. (B.27). The kinetic terms remain diagonal,

$$\begin{aligned}\mathcal{L}_{gk} = -\tfrac{1}{4}\textstyle\sum_{n=0}^{(N-1)/2}\Big\{&(\partial_\mu A^a_{\nu,(n)} - \partial_\nu A^a_{\mu,(n)})(\partial_\mu A^a_{\nu,(n)} - \partial_\nu A^a_{\mu,(n)})\\ &+(\partial_\mu A^a_{\nu,(-n)} - \partial_\nu A^a_{\mu,(-n)})(\partial_\mu A^a_{\nu,(-n)} - \partial_\nu A^a_{\mu,(-n)})\Big\}\end{aligned} \tag{B.22}$$

The three point interactions take the form

$$\begin{gathered}\mathcal{L}_{AAA} = g_0 f^{abc}\textstyle\sum_{n,r,s=0}^{(N-1)/2}\{\\ h_{nrs}\partial_{[\mu}A^a_{\nu],(n)}A^b_{\mu,(r)}A^c_{\nu,(s)} + \tilde{h}_{nrs}\partial_{[\mu}A^a_{\nu],(n)}A^b_{\mu,(-r)}A^c_{\nu,(-s)}\\ +\tilde{h}_{rns}\partial_{[\mu}A^a_{\nu],(-n)}A^b_{\mu,(r)}A^c_{\nu,(-s)} + \tilde{h}_{snr}\partial_{[\mu}A^a_{\nu],(-n)}A^b_{\mu,(-r)}A^c_{\nu,(s)}\Big\}\end{gathered} \tag{B.23}$$

The four point interactions take the form

$$\begin{gathered}\mathcal{L}_{AAAA} = -\tfrac{1}{4}g_0^2\textstyle\sum_{n,r,s=0}^{\infty}[\\ k_{nmrs}A^a_{\mu,(n)}A^b_{\nu,(m)}A^c_{\mu,(r)}A^d_{\nu,(s)} + \hat{k}_{nmrs}A^a_{\mu,(-n)}A^b_{\nu,(-m)}A^c_{\mu,(-r)}A^d_{\nu,(-s)}\\ +\tilde{k}_{nmrs}A^a_{\mu,(n)}A^b_{\nu,(m)}A^c_{\mu,(-r)}A^d_{\nu,(-s)} + \tilde{k}_{nrms}A^a_{\mu,(n)}A^b_{\nu,(-m)}A^c_{\mu,(r)}A^d_{\nu,(-s)}\\ +\tilde{k}_{nsmr}A^a_{\mu,(n)}A^b_{\nu,(-m)}A^c_{\mu,(-r)}A^d_{\nu,(s)} + \tilde{k}_{mrns}A^a_{\mu,(-n)}A^b_{\nu,(m)}A^c_{\mu,(r)}A^d_{\nu,(-s)}\\ +\tilde{k}_{msnr}A^a_{\mu,(-n)}A^b_{\nu,(m)}A^c_{\mu,(-r)}A^d_{\nu,(s)} + \tilde{k}_{rsmn}A^a_{\mu,(-n)}A^b_{\nu,(-m)}A^c_{\mu,(r)}A^d_{\nu,(s)}\Big]\end{gathered} \tag{B.24}$$

B.3 Interactions of links - Linear Sigma Model

The interactions of the links follow from the covariant kinetic term:

$$\mathcal{L} = 2\ \mathrm{Tr}\left\{D_\mu \Phi_p^\dagger D_\mu \Phi_p\right\} = 2\ \mathrm{Tr}\left\{|\partial_\mu \Phi_p + +igv(A_p - A_{p+1}) + igA_p\Phi_p - ig\Phi_p A_{p+1}|^2\right\}\ . \tag{B.25}$$

This can be split as:

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_{\Phi\Phi A} + \mathcal{L}_{\Phi AA} + \mathcal{L}_{\Phi\Phi AA}\,. \tag{B.26}$$

The quadratic terms are given by:

$$\begin{aligned}\mathcal{L}_q = \textstyle\sum_{p=1}^N \Big\{\tfrac{1}{2}\partial_\mu \Sigma_p^a \partial_\mu \Sigma_p^a + \tfrac{1}{2}\partial_\mu G_p^a \partial_\mu G_p^a \\ +\tfrac{1}{2}g^2v^2(A_p^a - A_{p+1}^a)(A_p^a - A_{p+1}^a) - gv(A_{p+1}^a - A_p^a)\partial_\mu G_p^a\Big\}\end{aligned} \tag{B.27}$$

We choose the mode decomposition so as to diagonalize the gauge boson mass terms and the Goldstone-gauge boson kinetic mixing,

$$A_p = \sqrt{\frac{2}{N}}\sum_{n=0}^{(N-1)/2}\left\{\eta_n \cos\frac{2n\pi p}{N}A^{(n)} + \sin\frac{2n\pi p}{N}A^{(-n)}\right\}, \tag{B.28}$$

$$G_p = \sqrt{\frac{2}{N}}\sum_{n=0}^{(N-1)/2}\left\{-\sin\frac{2n\pi(p+1/2)}{N}G^{(n)} + \eta_n \cos\frac{2n\pi(p+1/2)}{N}G^{(-n)}\right\}. \tag{B.29}$$

With the scalar potential chosen as in eq. (B.3) the mass terms of Σ_p are diagonal. Nevertheless, it is convenient to define the mode decomposition of Σ_p analogously as for G_p (this is an orthogonal transformation so the mass terms remain diagonal). The kinetic terms become:

$$\mathcal{L}_q = \sum_{n=-(N-1)/2}^{(N-1)/2}\left\{\frac{1}{2}\partial_\mu\Sigma_{(n)}^a\partial_\mu\Sigma_{(n)}^a + \frac{1}{2}\partial_\mu G_{(n)}^a\partial_\mu G_{(n)}^a + \frac{1}{2}m_n^2 A_{(n)}^a A_{(n)}^a - m_n A_{(n)}^a\partial_\mu G_{(n)}^a\right\} \tag{B.30}$$

Next, there are trilinear term involving one gauge boson which read:

$$\begin{aligned}\mathcal{L}_{\Phi\Phi A} = -\tfrac{1}{2}g\textstyle\sum_{p=1}^N\Big\{f^{abc}(A_{p+1}^a + A_p^a)(\Sigma_p^b\partial_\mu\Sigma_p^c + G_p^b\partial_\mu G_p^c) \\ +d^{abc}(A_{p+1}^a - A_p^a)(\Sigma_p^b\partial_\mu G_p^c - G_p^b\partial_\mu\Sigma_p^c)\Big\}\end{aligned} \tag{B.31}$$

Inserting the mode decomposition we get

$$\begin{aligned}\mathcal{L}_{\Sigma\Sigma A} = -g_0 f^{abc}\cos\left(\tfrac{n\pi}{N}\right)\textstyle\sum_{n,r,s=0}^{(N-1)/2}\Big\{ \\ h_{nrs}A_{(n)}^a\Sigma_{(-r)}^b\partial_\mu\Sigma_{(-s)}^c + \tilde{h}_{nrs}A_{(n)}^a\Sigma_{(r)}^b\partial_\mu\Sigma_{(s)}^c \\ -\tilde{h}_{rns}A_{(-n)}^a\Sigma_{(-r)}^b\partial_\mu\Sigma_{(s)}^c - \tilde{h}_{snr}A_{(-n)}^a\Sigma_{(r)}^b\partial_\mu\Sigma_{(-s)}^c\Big\}\,,\end{aligned} \tag{B.32}$$

$$\mathcal{L}_{GGA} = -g_0 f^{abc} \cos\left(\tfrac{n\pi}{N}\right) \sum_{n,r,s=0}^{(N-1)/2} \Big\{ h_{nrs} A^a_{(n)} G^b_{(-r)} \partial_\mu G^c_{(-s)} + \tilde{h}_{nrs} A^a_{(n)} G^b_{(r)} \partial_\mu G^c_{(s)} - \tilde{h}_{rns} A^a_{(-n)} G^b_{(-r)} \partial_\mu G^c_{(s)} - \tilde{h}_{snr} A^a_{(-n)} G^b_{(r)} \partial_\mu G^c_{(-s)} \Big\}, \tag{B.33}$$

$$\mathcal{L}_{\Sigma GA} = -g_0 d^{abc} \sin\left(\tfrac{n\pi}{N}\right) \sum_{n,r,s=0}^{(N-1)/2} \Big\{ h_{nrs} A^a_{(-n)} \Sigma^b_{(-r)} \partial_\mu G^c_{(-s)} + \tilde{h}_{nrs} A^a_{(-n)} \Sigma^b_{(r)} \partial_\mu G^c_{(s)} + \tilde{h}_{rns} A^a_{(n)} \Sigma^b_{(-r)} \partial_\mu G^c_{(s)} + \tilde{h}_{snr} A^a_{(n)} \Sigma^b_{(r)} \partial_\mu G^c_{(-s)} - h_{nrs} A^a_{(-n)} G^b_{(-r)} \partial_\mu \Sigma^c_{(-s)} - \tilde{h}_{nrs} A^a_{(-n)} G^b_{(r)} \partial_\mu \Sigma^c_{(s)} - \tilde{h}_{rns} A^a_{(n)} G^b_{(-r)} \partial_\mu \Sigma^c_{(s)} - \tilde{h}_{snr} A^a_{(n)} G^b_{(r)} \partial_\mu \Sigma^c_{(-s)} \Big\}. \tag{B.34}$$

Trilinear terms involving two gauge bosons are:

$$\mathcal{L}_{\Phi AA} = \tfrac{1}{2} g^2 v \sum_{p=1}^{N} \Big\{ f^{abc} (A^a_{p+1} - A^a_p)(A^b_{p+1} + A^b_p) G^c_p) + d^{abc} (A^a_{p+1} - A^a_p)(A^b_{p+1} - A^b_p) \Sigma^c_p \Big\} \tag{B.35}$$

Inserting the mode decomposition we get:

$$\mathcal{L}_{GGA} = g_0 f^{abc} m_n \cos\left(\tfrac{r\pi}{N}\right) \sum_{n,r,s=0}^{(N-1)/2} \Big\{ h_{nrs} A^a_{(-n)} A^b_{(r)} G^c_{(-s)} - \tilde{h}_{nrs} A^a_{(-n)} A^b_{(-r)} G^c_{(s)} + \tilde{h}_{rns} A^a_{(n)} A^b_{(r)} G^c_{(s)} - \tilde{h}_{snr} A^a_{(n)} A^b_{(-r)} G^c_{(-s)} \Big\}, \tag{B.36}$$

$$\mathcal{L}_{\Sigma\Sigma A} = g_0 f^{abc} m_n \sin\left(\tfrac{r\pi}{N}\right) \sum_{n,r,s=0}^{(N-1)/2} \Big\{ h_{nrs} A^a_{(-n)} A^b_{(-r)} \Sigma^c_{(-s)} + \tilde{h}_{nrs} A^a_{(-n)} A^b_{(r)} \Sigma^c_{(s)} + \tilde{h}_{rns} A^a_{(n)} A^b_{(-r)} \Sigma^c_{(s)} + \tilde{h}_{snr} A^a_{(n)} A^b_{(r)} \Sigma^c_{(-s)} \Big\}, \tag{B.37}$$

Then there are quartic interactions,

$$\mathcal{L}_{\Phi\Phi AA} = \tfrac{1}{8} g^2 \sum_{p=1}^{N} \Big\{ f^{ace} f^{bde} (A^a_{p+1} + A^a_p)(A^b_{p+1} + A^b_p)(\Sigma^c_p + iG^c_p)(\Sigma^d_p - iG^d_p) + d^{ace} d^{bde} (A^a_{p+1} - A^a_p)(A^b_{p+1} - A^b_p)(\Sigma^c_p + iG^c_p)(\Sigma^d_p - iG^d_p) - i f^{ace} d^{bde} (A^a_{p+1} + A^a_p)(A^b_{p+1} - A^b_p)(\Sigma^c_p + iG^c_p)(\Sigma^d_p - iG^d_p) + i f^{ace} d^{bde} (A^a_{p+1} + A^a_p)(A^b_{p+1} - A^b_p)(\Sigma^c_p - iG^c_p)(\Sigma^d_p + iG^d_p). \Big\} \tag{B.38}$$

Inserting the mode decomposition yields a lengthy expression (even for the standard of this appendix :-). Below we concentrate only on those terms in which both gauge fields and both links have the same KK index,

$$
\begin{aligned}
\mathcal{L}_{\Phi\Phi AA} \subset \tfrac{1}{2}g_0^2 \textstyle\sum_{n,r=0}^{(N-1)/2} \Big\{ & \cos^2\left(\tfrac{n\pi}{N}\right) f^{ace} f^{bde} [\\
& k_{nnrr} A^a_{(n)} A^b_{(n)} (\Sigma^c_{(-r)}\Sigma^d_{(-r)} + G^c_{(-r)} G^d_{(-r)}) + \tilde{k}_{nnrr} A^a_{(n)} A^b_{(n)} (\Sigma^c_{(r)}\Sigma^d_{(r)} + G^c_{(r)} G^d_{(r)}) \\
& + \tilde{k}_{rrnn} A^a_{(-n)} A^b_{(-n)} (\Sigma^c_{(-r)}\Sigma^d_{(-r)} + G^c_{(-r)} G^d_{(-r)}) + \hat{k}_{nnrr} A^a_{(-n)} A^b_{(-n)} (\Sigma^c_{(r)}\Sigma^d_{(r)} + G^c_{(r)} G^d_{(r)})] \\
& + \sin^2\left(\tfrac{n\pi}{N}\right) d^{ace} d^{bde} [\\
& k_{nnrr} A^a_{(-n)} A^b_{(-n)} (\Sigma^c_{(-r)}\Sigma^d_{(-r)} + G^c_{(-r)} G^d_{(-r)}) + \tilde{k}_{nnrr} A^a_{(-n)} A^b_{(-n)} (\Sigma^c_{(r)}\Sigma^d_{(r)} + G^c_{(r)} G^d_{(r)}) \\
& + \tilde{k}_{rrnn} A^a_{(n)} A^b_{(n)} (\Sigma^c_{(-r)}\Sigma^d_{(-r)} + G^c_{(-r)} G^d_{(-r)}) + \hat{k}_{nnrr} A^a_{(n)} A^b_{(n)} (\Sigma^c_{(r)}\Sigma^d_{(r)} + G^c_{(r)} G^d_{(r)})] \Big\}
\end{aligned}
\tag{B.39}
$$

Finally we need to determine the ghost interactions. Working in the R_ξ gauge we choose the gauge fixing lagrangian as

$$
\mathcal{L}_{gf} = -\frac{1}{2\xi} 2\mathrm{Tr} \sum_{p=1}^{N} \left\{ (\partial_\mu A_p - \xi g v (G_p - G_{p-1}))^2 \right\}, \tag{B.40}
$$

so that the Goldstone-gauge kinetic mixing is removed and the Goldstone bosons $G_{(n)}$ acquire masses ξm_n. The gauge variation of the Goldstone G_p is

$$
\delta G_p = \frac{1}{2} g \left\{ 2v(\eta_p - \eta_{p+1}) + \{\eta_p - \eta_{p+1}, \Sigma_p\} + i[\eta_p + \eta_{p+1}, G_p] \right\} \tag{B.41}
$$

while the variation of the gauge field is

$$
\delta A_p = \partial_\mu \eta_p - ig[A_p, \eta_p]. \tag{B.42}
$$

Using the standard technique we get the ghost action

$$
\begin{aligned}
\mathcal{L}_{gh} = 2\mathrm{Tr} \textstyle\sum_{p=1}^{N} \{ & \partial_\mu \bar{c}_p \partial_\mu c_p + ig \partial_\mu \bar{c}_p [A_p, c_p] - \xi g^2 v^2 (\bar{c}_{p+1} - \bar{c}_p)(c_{p+1} - c_p) \\
& + \tfrac{1}{2} \xi g^2 v^2 (\bar{c}_{p+1} - \bar{c}_p) \{c_{p+1} - c_p, \Sigma_p\} - i\tfrac{1}{2}\xi g^2 v^2 (\bar{c}_{p+1} - \bar{c}_p)[c_{p+1} + c_p, G_p] \Big\}
\end{aligned}
\tag{B.43}
$$

The ghost mass terms are diagonalized by the mode decomposition analogous to that for the gauge fields,

$$
c_p = \sqrt{\frac{2}{N}} \sum_{n=0}^{(N-1)/2} \left\{ \eta_{(n)} \cos \frac{2n\pi p}{N} c_{(n)} + \sin \frac{2n\pi p}{N} c_{(-n)} \right\}, \tag{B.44}
$$

$$\bar{c}_p = \sqrt{\frac{2}{N}} \sum_{n=0}^{(N-1)/2} \left\{ \eta_{(n)} \cos \frac{2n\pi p}{N} \bar{c}_{(n)} + \sin \frac{2n\pi p}{N} \bar{c}_{(-n)} \right\}, \tag{B.45}$$

and the ghosts $c_{(n)}$ acquire masses ξm_n. The interactions are

$$\begin{aligned} \mathcal{L}_{ccA} = g_0 f^{abc} \textstyle\sum_{n,r,s=0}^{(N-1)/2} \{ \\ h_{nrs} A^a_{(n)} \partial_\mu \bar{c}^b_{(r)} c^c_{(s)} + \tilde{h}_{nrs} A^a_{(n)} \partial_\mu \bar{c}^b_{(-r)} c^c_{(-s)} \\ + \tilde{h}_{rns} A^a_{(-n)} \partial_\mu \bar{c}^b_{(r)} c^c_{(-s)} + \tilde{h}_{snr} A^a_{(-n)} \partial_\mu \bar{c}^b_{(-r)} c^c_{(s)} \} \end{aligned} \tag{B.46}$$

$$\begin{aligned} \mathcal{L}_{ccG} = g_0 m_r \cos\left(\tfrac{s\pi}{N}\right) f^{abc} \textstyle\sum_{n,r,s=0}^{(N-1)/2} \{ \\ h_{nrs} G^a_{(-n)} \bar{c}^b_{(r)} c^c_{(-s)} + \tilde{h}_{nrs} G^a_{(-n)} \bar{c}^b_{(-r)} c^c_{(-s)} \\ + \tilde{h}_{rns} G^a_{(n)} \bar{c}^b_{(r)} c^c_{(s)} + \tilde{h}_{snr} G^a_{(n)} \bar{c}^b_{(-r)} c^c_{(-s)} \} \end{aligned} \tag{B.47}$$

$$\begin{aligned} \mathcal{L}_{cc\Sigma} = g_0 m_r \sin\left(\tfrac{s\pi}{N}\right) d^{abc} \textstyle\sum_{n,r,s=0}^{(N-1)/2} \{ \\ h_{nrs} \Sigma^a_{(-n)} \bar{c}^b_{(-r)} c^c_{(-s)} + \tilde{h}_{nrs} \Sigma^a_{(-n)} \bar{c}^b_{(r)} c^c_{(s)} \\ + \tilde{h}_{rns} \Sigma^a_{(n)} \bar{c}^b_{(-r)} c^c_{(s)} + \tilde{h}_{snr} \Sigma^a_{(n)} \bar{c}^b_{(r)} c^c_{(-s)} \} \end{aligned} \tag{B.48}$$

B.4 Non-linear Sigma Model

In the non-linear sigma model we choose the action for the links

$$\mathcal{L} = \mathrm{Tr} \sum_{p=1}^{N} \left\{ v^2 D_\mu \Phi_p^\dagger D_\mu \Phi_p \right\}, \tag{B.49}$$

where as before the covariant derivative is

$$D_\mu \Phi_p = \partial \Phi_p + i g A_p \Phi_p - i g \Phi_p A_{p+1}. \tag{B.50}$$

However working in the non-linear sigma model we decouple the massive physical degrees of freedom Σ and keep only the Goldstone bosons G in the spectrum. This consists in substituting

$$\Phi_p = \exp\left(\frac{i}{v} G_p\right) = 1 + \frac{i}{v} G_p - \frac{1}{2v^2} G_p^2 + \ldots \tag{B.51}$$

to eq. (B.49). The resulting action is of course non-renormalizable and contains an infinite number of interaction terms. Below we concentrate only on these interactions that are not suppressed by powers of v, as only these are relevant at energies much below v.

The quadratic terms are the same as in the linear sigma model (except for Σ of course),

$$\mathcal{L}_q = \sum_{p=1}^{N} \left\{ \frac{1}{2}\partial_\mu G_p^a \partial_\mu G_p^a + \frac{1}{2} g^2 v^2 (A_p^a - A_{p+1}^a)(A_p^a - A_{p+1}^a) - gv(A_{p+1}^a - A_p^a)\partial_\mu G_p^a \right\}, \tag{B.52}$$

and so the mode decomposition which diagonalizes the gauge bosons mass terms and the Goldstone-gauge boson kinetic mixing is

$$A_p = \sqrt{\frac{2}{N}} \sum_{n=0}^{(N-1)/2} \left\{ \eta_{(n)} \cos\frac{2n\pi p}{N} A^{(n)} + \sin\frac{2n\pi p}{N} A^{(-n)} \right\}, \tag{B.53}$$

$$G_p = \sqrt{\frac{2}{N}} \sum_{n=0}^{(N-1)/2} \left\{ -\sin\frac{2n\pi(p+1/2)}{N} G^{(n)} + \eta_{(n)} \cos\frac{2n\pi(p+1/2)}{N} G^{(-n)} \right\}. \tag{B.54}$$

The kinetic terms become

$$\mathcal{L}_q = \sum_{n=-(N-1)/2}^{(N-1)/2} \left\{ \frac{1}{2}\partial_\mu G_{(n)}^a \partial_\mu G_{(n)}^a + \frac{1}{2} m_n^2 A_{(n)}^a A_{(n)}^a - m_n A_{(n)}^a \partial_\mu G_{(n)}^a \right\} \tag{B.55}$$

Furthermore, it turns out that also the trilinear terms are analogous as in the linear sigma model with the Σ field removed,

$$\mathcal{L}_{GGA} = -\frac{1}{2} g \sum_{p=1}^{N} \left\{ f^{abc} (A_{p+1}^a + A_p^a) G_p^b \partial_\mu G_p^c \right\}, \tag{B.56}$$

$$\mathcal{L}_{GAA} = \frac{1}{2} g^2 v \sum_{p=1}^{N} \left\{ f^{abc} (A_{p+1}^a - A_p^a)(A_{p+1}^b + A_p^b) G_p^c) \right\}. \tag{B.57}$$

In terms of the eigenvectors these trilinear terms read

$$\begin{gathered} \mathcal{L}_{GGA} = -g_0 f^{abc} \cos\left(\tfrac{n\pi}{N}\right) \textstyle\sum_{n,r,s=0}^{(N-1)/2} \{ \\ h_{nrs} A_{(n)}^a G_{(-r)}^b \partial_\mu G_{(-s)}^c + \tilde{h}_{nrs} A_{(n)}^a G_{(r)}^b \partial_\mu G_{(s)}^c \\ -\tilde{h}_{rns} A_{(-n)}^a G_{(-r)}^b \partial_\mu G_{(s)}^c - \tilde{h}_{snr} A_{(-n)}^a G_{(r)}^b \partial_\mu G_{(-s)}^c \}, \end{gathered} \tag{B.58}$$

$$\begin{gathered} \mathcal{L}_{GAA} = g_0 f^{abc} m_n \cos\left(\tfrac{r\pi}{N}\right) \textstyle\sum_{n,r,s=0}^{(N-1)/2} \{ \\ h_{nrs} A_{(-n)}^a A_{(r)}^b G_{(-s)}^c - \tilde{h}_{nrs} A_{(-n)}^a A_{(-r)}^b G_{(s)}^c \\ +\tilde{h}_{rns} A_{(n)}^a A_{(r)}^b G_{(s)}^c - \tilde{h}_{snr} A_{(n)}^a A_{(-r)}^b G_{(-s)}^c \}. \end{gathered} \tag{B.59}$$

However, the quartic terms are not the same as the corresponding terms in the linear sigma model . They read

$$\mathcal{L}_{GGAA} = \frac{1}{8} g^2 f^{ace} f^{bde} \sum_{p=1}^{N} \left\{ \left((A^a_{p+1} + A^a_p)(A^b_{p+1} + A^b_p) - (A^a_{p+1} - A^a_p)(A^b_{p+1} - A^b_p) \right) G^c_p G^d_p \right\} \tag{B.60}$$

We insert the mode decomposition and write down only those terms in which both gauge fields and both links have the same KK index,

$$\begin{gathered}
\mathcal{L}_{GGAA} \subset \tfrac{1}{2} g_0^2 f^{ace} f^{bde} \textstyle\sum_{n,r=0}^{(N-1)/2} \{ \\
\cos^2\left(\tfrac{n\pi}{N}\right) [k_{nnrr} A^a_{(n)} A^b_{(n)} G^c_{(-r)} G^d_{(-r)} + \tilde{k}_{nnrr} A^a_{(n)} A^b_{(n)} G^c_{(r)} G^d_{(r)} \\
+ \tilde{k}_{rrnn} A^a_{(-n)} A^b_{(-n)} G^c_{(-r)} G^d_{(-r)} + \hat{k}_{nnrr} A^a_{(-n)} A^b_{(-n)} G^c_{(r)} G^d_{(r)}] \\
- \sin^2\left(\tfrac{n\pi}{N}\right) [k_{nnrr} A^a_{(-n)} A^b_{(-n)} G^c_{(-r)} G^d_{(-r)} + \tilde{k}_{nnrr} A^a_{(-n)} A^b_{(-n)} G^c_{(r)} G^d_{(r)} \\
+ \tilde{k}_{rrnn} A^a_{(n)} A^b_{(n)} G^c_{(-r)} G^d_{(-r)} + \hat{k}_{nnrr} A^a_{(n)} A^b_{(n)} G^c_{(r)} G^d_{(r)}] \Big\}
\end{gathered} \tag{B.61}$$

Appendix C

Notation and conventions

We use the 'mostly minus' metric both in 4d, $(+,-,-,-)$ and in 5d $(+,-,-,-,-)$. The 4d coordinates are denoted by x^μ, $\mu = 0,1,2,3$ while the 5d ones are denoted by x^α, $\alpha = 0,1,2,3,5$. Consequently μ, ν stand for 4d Lorentz indices and α, β stand for 5d Lorentz indices. The fifth coordinate x^5 is often denoted as y.

We shall assume that gauge groups we deal with are $SU(K)$ or $U(K)$. Then the gauge fields A_μ (or A_α in 5d) can be written in the matrix form, that is, $A_\mu = A_\mu^a T^a$, where T^a are the hermitian generators of the gauge group. The same notation is used for matter fields in the adjoint representation. The generators are normalized as $\mathrm{Tr}(T^a T^b) = 1/2$. The structure constants f are defined as $[T^a, T^b] = i f^{abc} T^c$. We also define the tensor d as $\{T^a, T^b\} = d^{abc} T^c$.

The KK index in 5d compactified theories is always in brackets, $(n), (m), \ldots$. The values ranged by the KK index depend on the compactification.

In deconstruction, the number of gauge groups is called the replication number and denoted by N. The groups are labeled by the index $p, q, \ldots$ and $p = 1, 2 \ldots N$. In the deconstruction phase the analogue of the 5d KK index is denoted by $(n), (m), \ldots$, analogously as in 5d.

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