

INTERFERENCYJNY ŚWIADEK SPLATANIA

Krzysztof Wódkiewicz



Symposium IFT, Warszawa, 2-3 XII, 2005

Publikacje

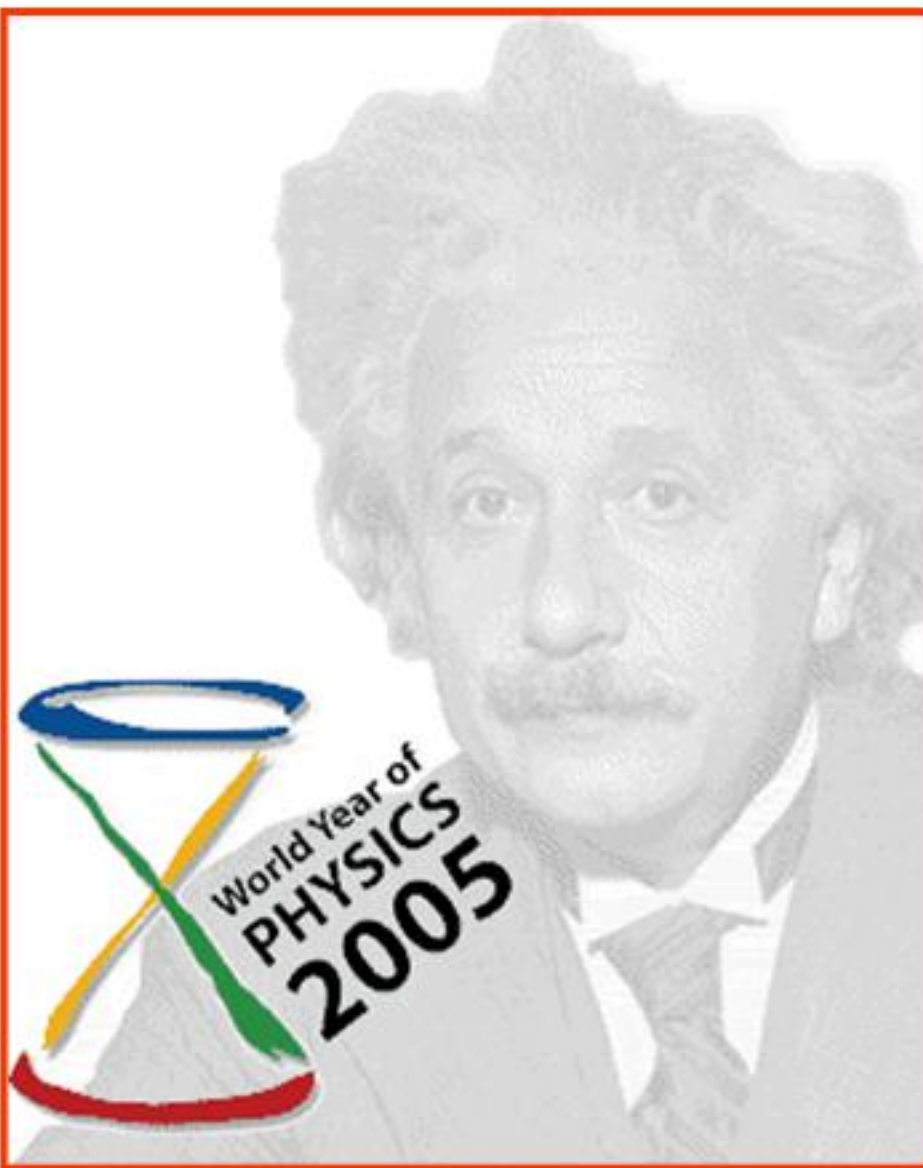
- K. Banaszek and K. W., Phys. Rev. A **58**, 4345 (1998).
- K. Banaszek and K. W., Phys. Rev. Lett. **82**, 2009 (1999).
- K. Banaszek, A. Dragan, K. W. and Cz. Radzewicz, Phys. Rev. A **66**, 043803 (2002).
- B. G. Englert and K. W., Phys. Rev. A **65**, 054303 (2002).
- B.G. Englert and K. W., Int. J. of Q. Info., **1**, 153 (2003).
- S. Daffer, K. W., J. K. McIver, Phys. Rev. A **68**, 012104 (2003).
- C. M. Caves and K. W., Phys. Rev. Lett. **93**, 040506 (2004)
- L. Praxmeyer, B.-G. Englert and K. W., Eu. Phys. J. D **32**, 227 (2005).
- M. Stobińska and K. W., Phys. Rev. A **71**, 032204 (2005).
- M. Stobińska and K. W., <http://xxx.lanl.gov/quant-ph/0508146> (2005)



- 1905
Kwant światła



- 1935
Kwantowe korelacje





1935

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN. *Institute for Advanced Study, Princeton, New Jersey*
 (Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function is

quantum mechanics is not complete; or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2) u_n(x_1), \quad (7)$$

Rozkład Schmidta

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/\hbar)(x_1 - x_2 + x_0)p} p dp, \quad (9)$$

$$\psi_{EPR} \sim \delta(q_a - q_b) \quad \tilde{\psi}_{EPR} \sim \delta(p_a + p_b)$$

Stan spleciony (1935)



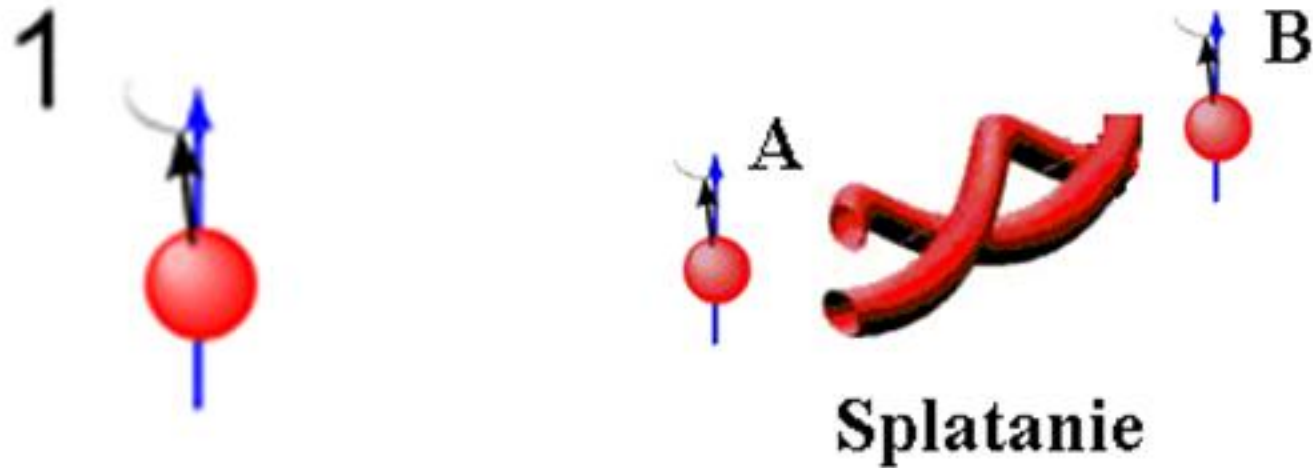
Schrödinger

$$\frac{1}{\sqrt{2}} \left(\left| \begin{array}{c} \text{atom} \\ \text{cat} \end{array} \right\rangle + \left| \begin{array}{c} \text{atom} \\ \text{cat with red X eyes} \end{array} \right\rangle \right)$$

$$|\Psi\rangle \neq |\psi_a\rangle \otimes |\psi_b\rangle$$



Qubit



Stan Bella

Maksymalnie spleciony

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\mathbf{H}, \mathbf{V}\rangle + |\mathbf{V}, \mathbf{H}\rangle)$$



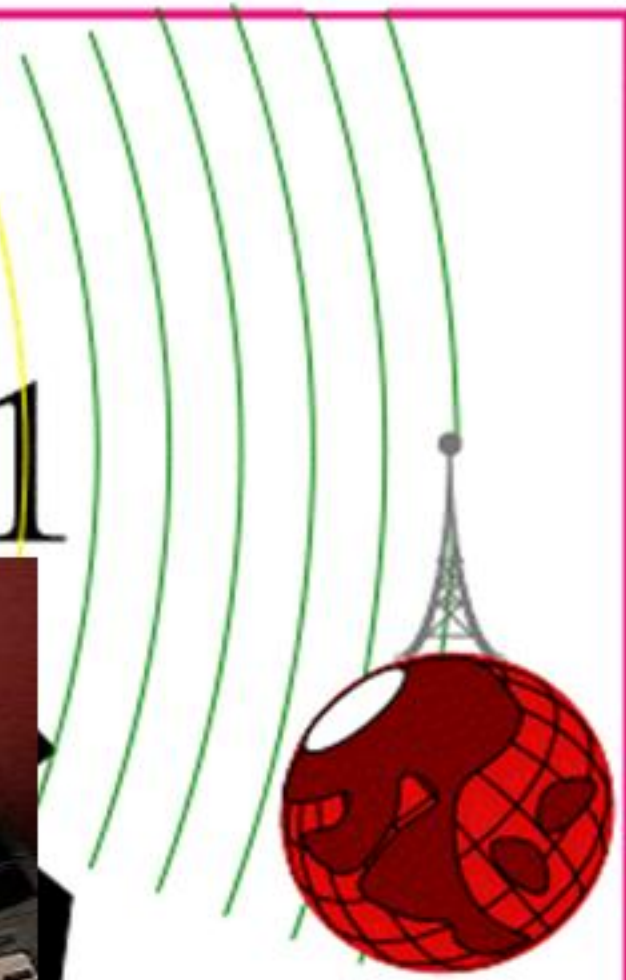
Alice



01



Quantum Information



Bob



**SPLATANIE
10% SALE**



ψ
 ψ
 ψ  qubity

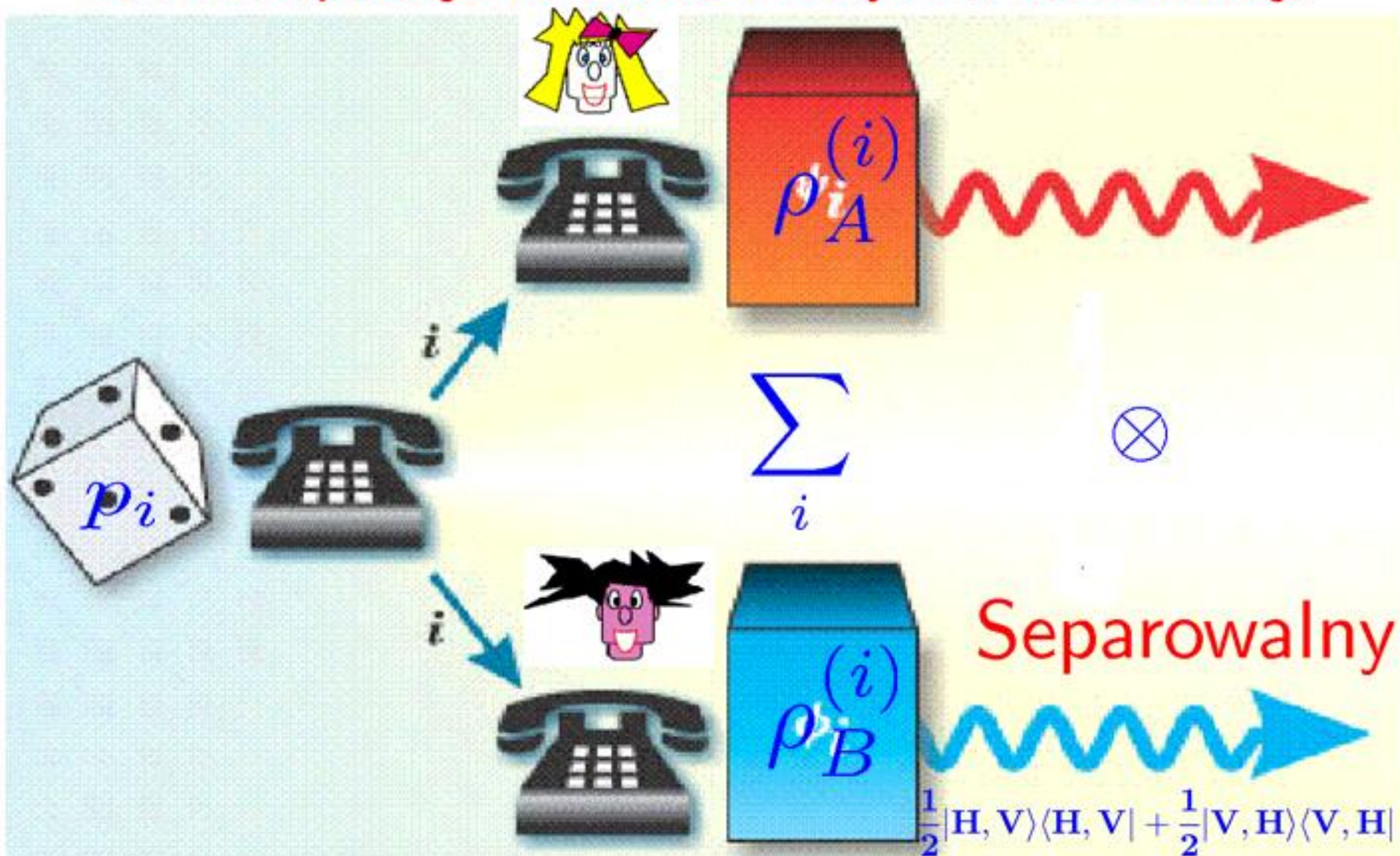
$$\frac{1}{2}|\text{H}, \text{V}\rangle\langle\text{H}, \text{V}| + \frac{1}{2}|\text{V}, \text{H}\rangle\langle\text{V}, \text{H}|$$

$$\frac{1}{\sqrt{2}}(|\text{H}, \text{V}\rangle + |\text{V}, \text{H}\rangle)$$

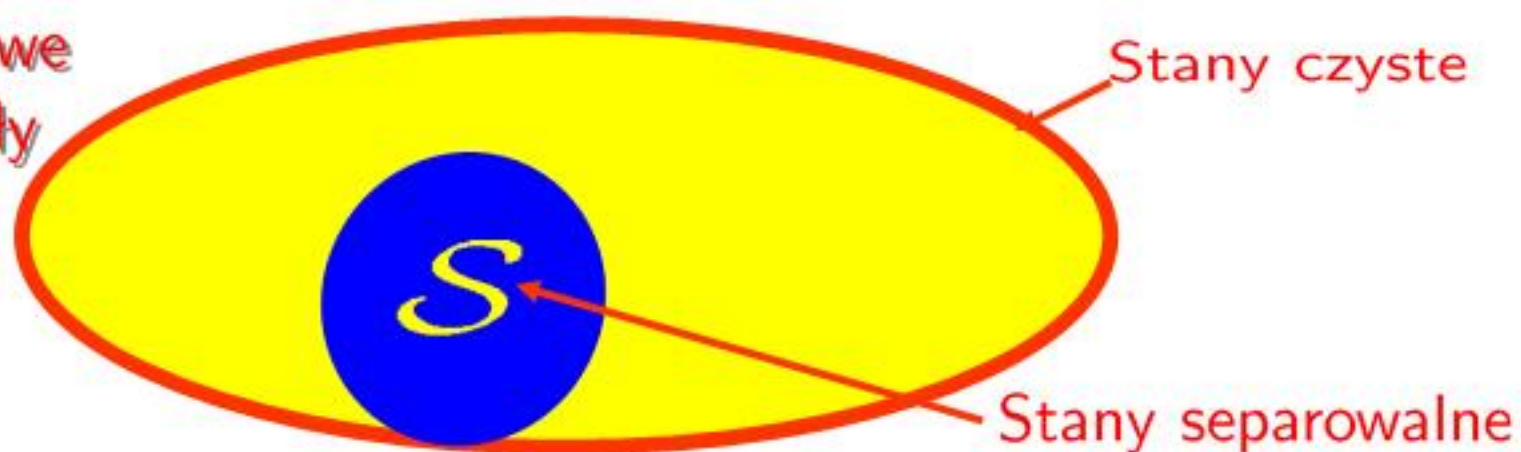


Kwantowa separowalność, Werner (1989)

Lokalne operacje na stanie i klasyczna komunikacja



Stany kwantowe
Zbiór wypukły



$$\rho = p\rho_1 + (1 - p)\rho_2, \quad 0 \leq p \leq 1$$

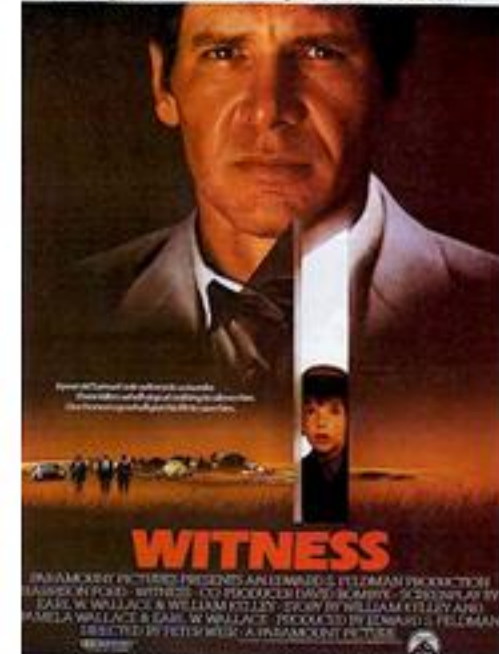
Stany nieseparowalne. Kwantowe.



Stany separowalne. Klasyczne.

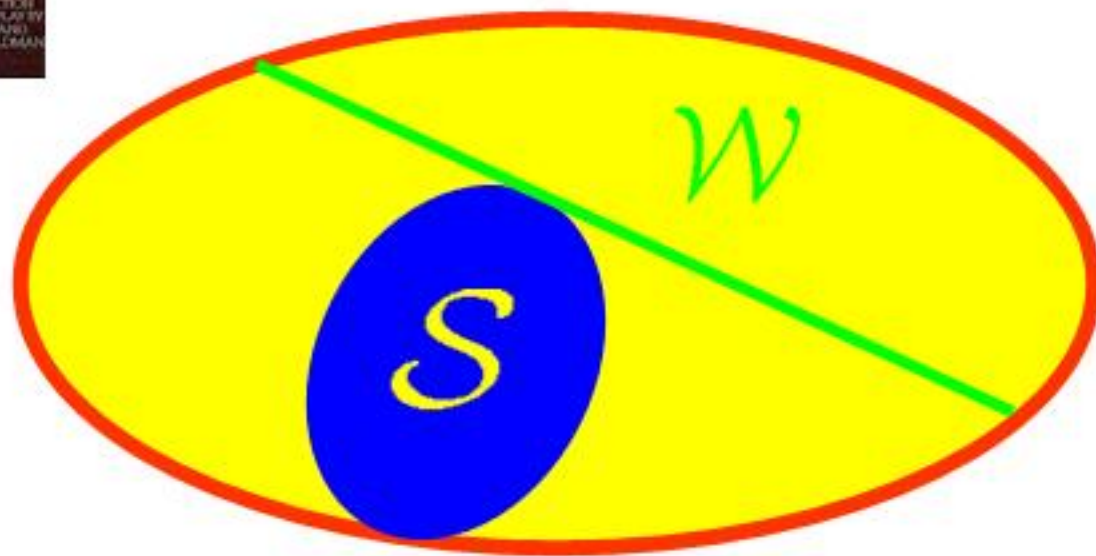
$$\rho = \sum p_i \rho_A^{(i)} \otimes \rho_B^{(i)}$$

Harrison Ford is John Book
A big city cop who knows too much.
His only evidence, a small boy who's seen too much.



Świadek splatania

$$\langle \hat{w} \rangle = \begin{cases} \geq 0, & \rho \in S \\ < 0, & \rho \notin S \end{cases}$$



Twierdzenie Hahna Banach

Świadek Bella dla qubitów



J. Bell

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|H, V\rangle + |V, H\rangle)$$



Nierówność Bella $\mathcal{B} = \langle \hat{B} \rangle \leq 2$

$\hat{\mathcal{W}} = 2 - \hat{B}$ $\langle \hat{\mathcal{W}} \rangle < 0$ jest spleciony

$$\langle \psi | \hat{\mathcal{W}} | \psi \rangle = 2(1 - \sqrt{2}) = -0.8284\dots$$



Stan Wenera

Wypukła kombinacja separowalnego i nieseparowalnego

$$\rho_W = p |\Psi\rangle\langle\Psi| + (1-p) \frac{I_a \otimes I_b}{4}$$

$$\rho_W = \sum p_i \rho_A^{(i)} \otimes \rho_B^{(i)} \iff 0 \leq p \leq \frac{1}{3}$$

$$\text{Tr}\{\hat{\mathcal{W}}\rho_W\} > 0 \iff 0 \leq p \leq \frac{1}{\sqrt{2}} = 0.7071\dots$$

nie jest spleciony

Zły świadek



Świadek Bella kłamie

Splecione struktury w NATURZE

?

Splecione struktury gaussowskie



Splatanie fotonów

$$\vec{\mathbf{E}} \sim \vec{\mathbf{e}}(\mathbf{a}e^{-i\omega\mathbf{t}} + \mathbf{a}^\dagger e^{i\omega\mathbf{t}})$$

$$\vec{\mathbf{E}} = \vec{\mathbf{e}}(\mathbf{q} \cos \omega\mathbf{t} + \mathbf{p} \sin \omega\mathbf{t})$$

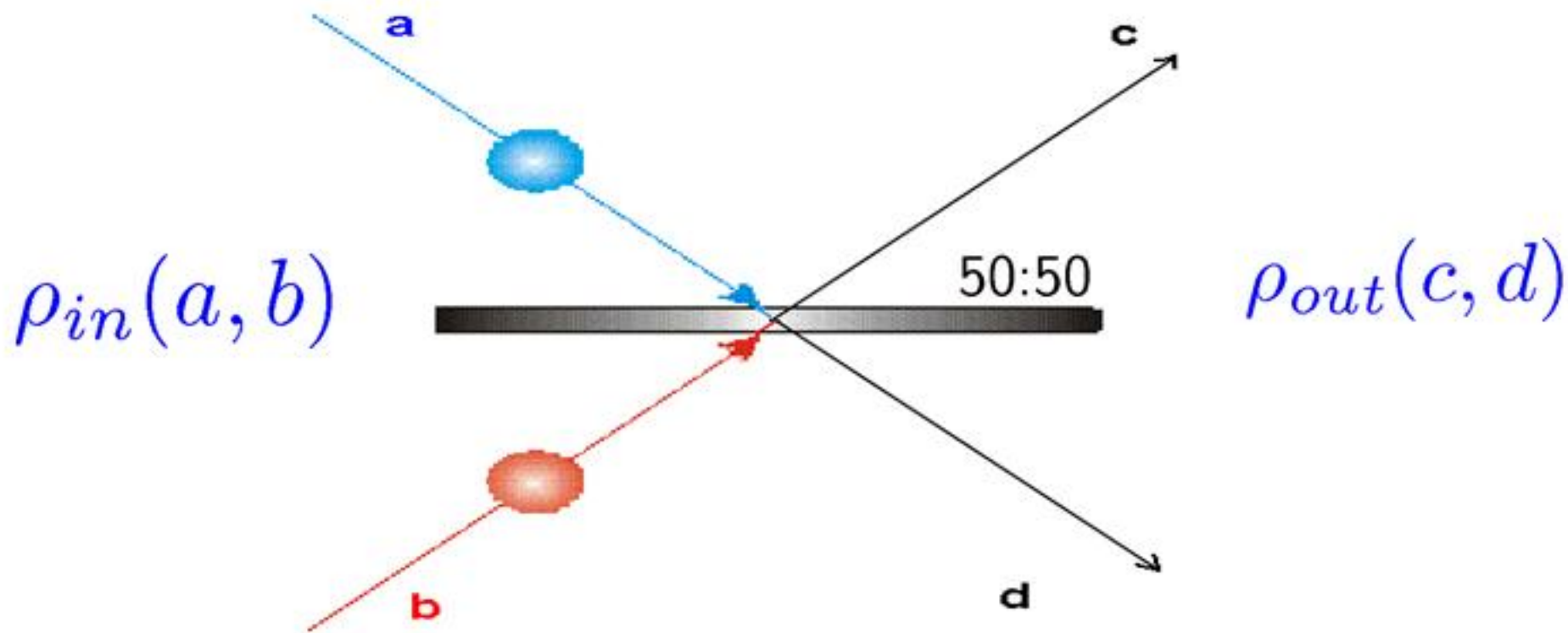
polaryzacja $\vec{\mathbf{e}} = (H, V)$



$$[\mathbf{q}, \mathbf{p}] = i$$

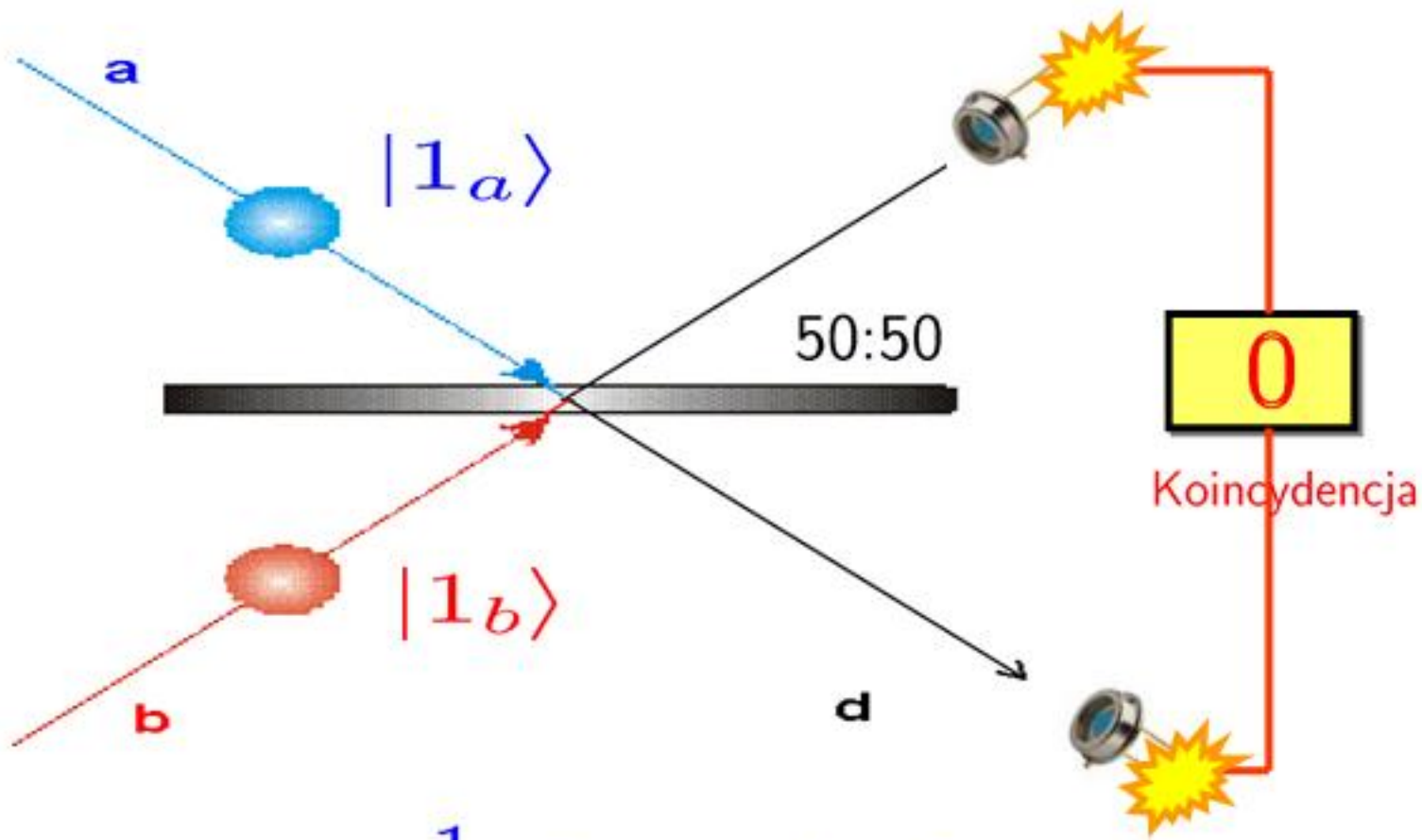
Kwadratury pola

Splatanie fotonów



$$c = \frac{1}{\sqrt{2}}(a + b)$$

$$d = \frac{1}{\sqrt{2}}(a - b)$$



$$a^\dagger b^\dagger |0; 0\rangle \Rightarrow \frac{1}{2}(c^\dagger + d^\dagger)(c^\dagger - d^\dagger)|0; 0\rangle$$

$$\frac{1}{\sqrt{2}}(|2, 0\rangle - |0, 2\rangle)$$

$$\langle \hat{W} \rangle = ?$$

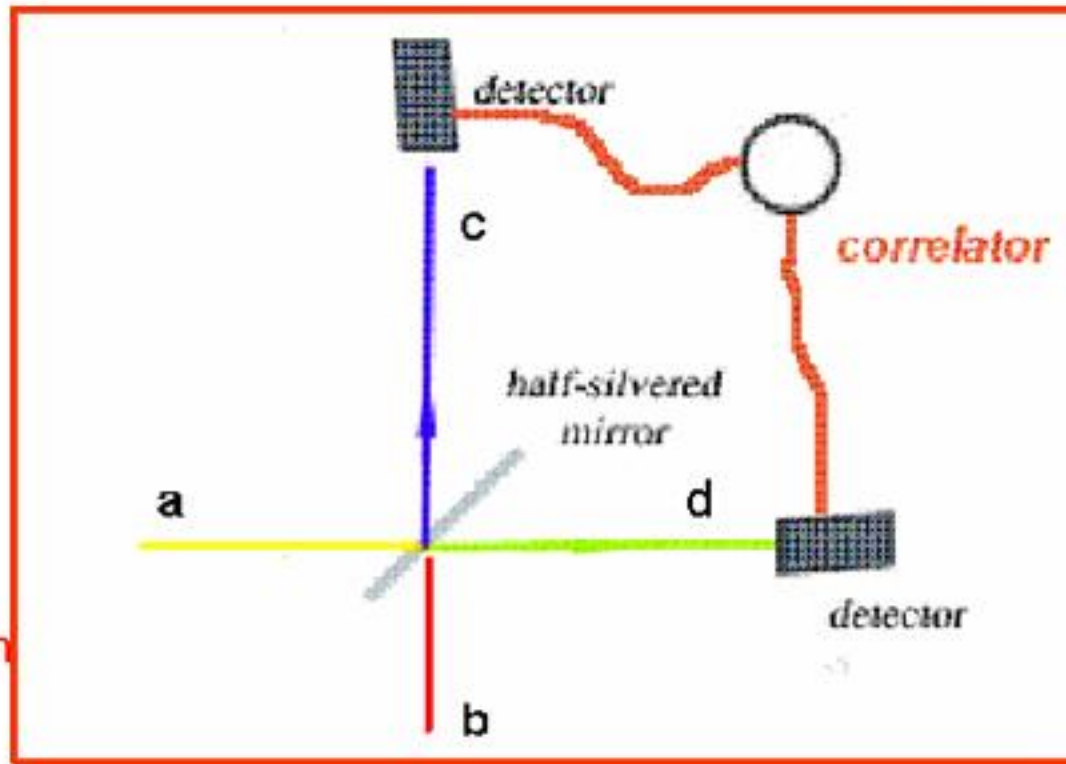


- Interferencja

Świadek interferencyjny



G. Hanbury-Brown
R. Twiss

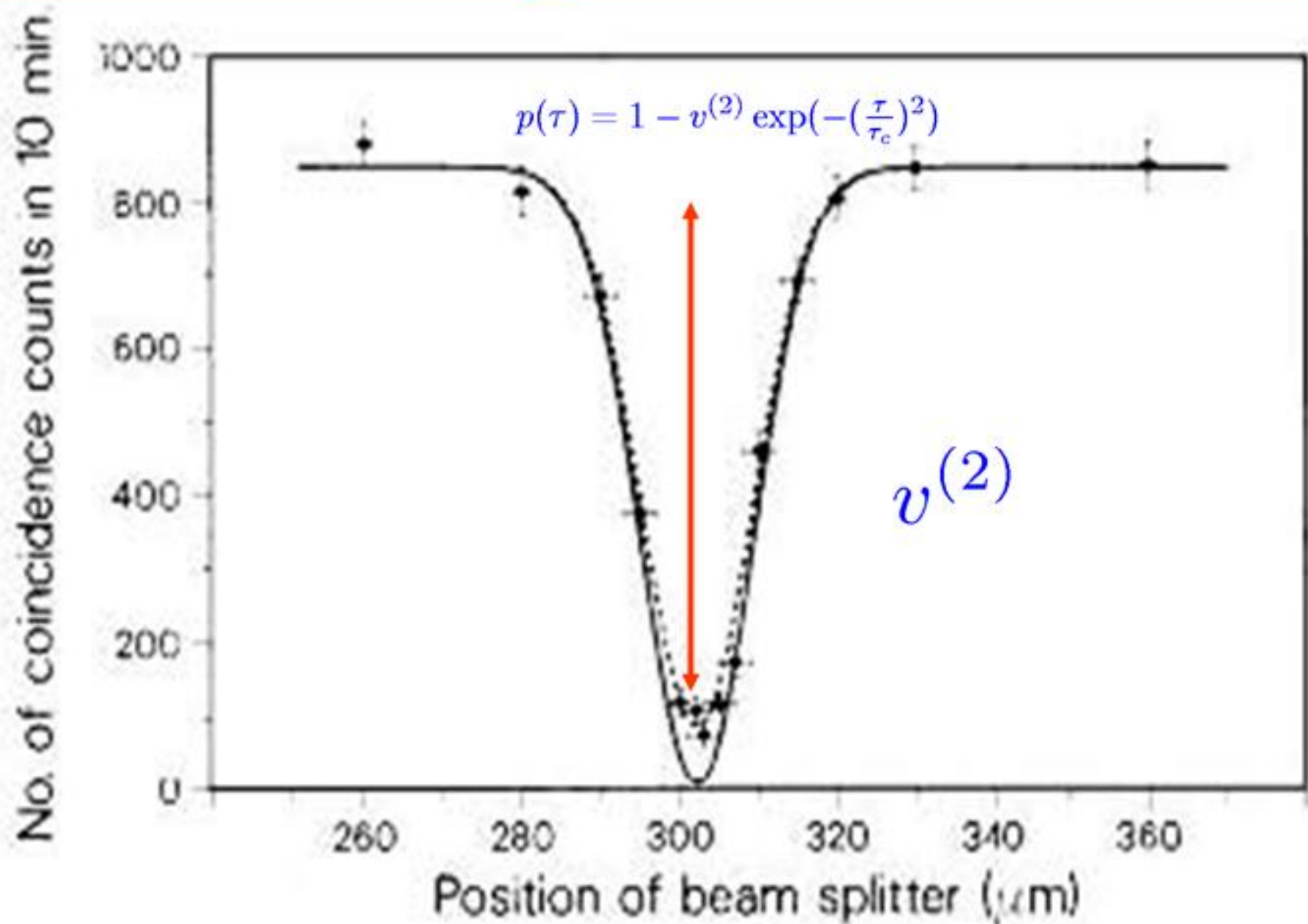


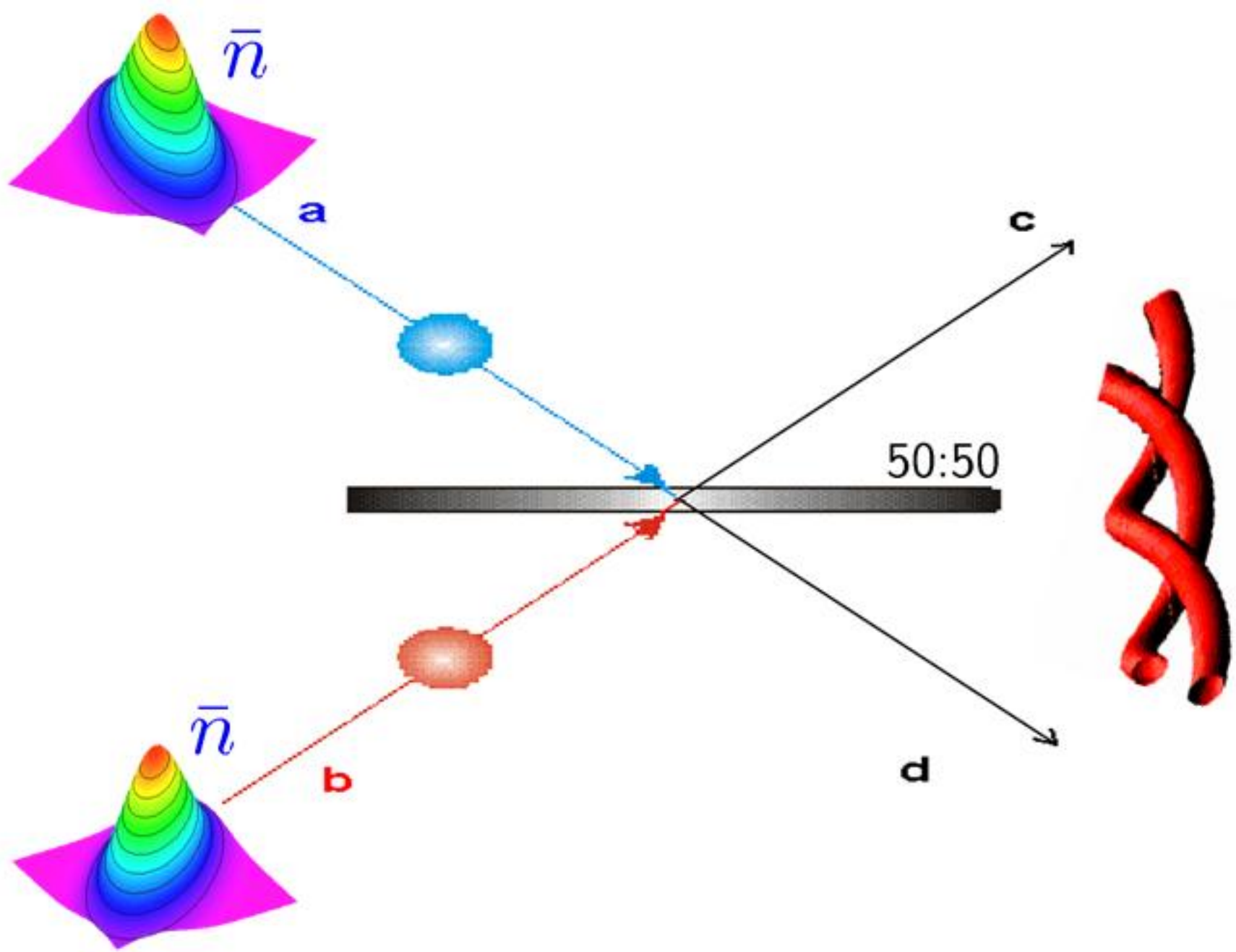
L. Mandel

$$\langle E_d^{(-)}(t) E_c^{(-)}(t + \tau) E_c^{(+)}(t + \tau) E_d^{(+)}(t) \rangle$$

$$p(\tau) = 1 - v^{(2)} \exp\left(-\left(\frac{\tau}{\tau_c}\right)^2\right)$$

Hong Ou Mandel





Termiczne stany ścisnięte

Stan termiczny

$$\Delta p = \Delta q = \sqrt{\bar{n} + \frac{1}{2}}$$

$$\Delta q \cdot \Delta p = \bar{n} + \frac{1}{2}$$

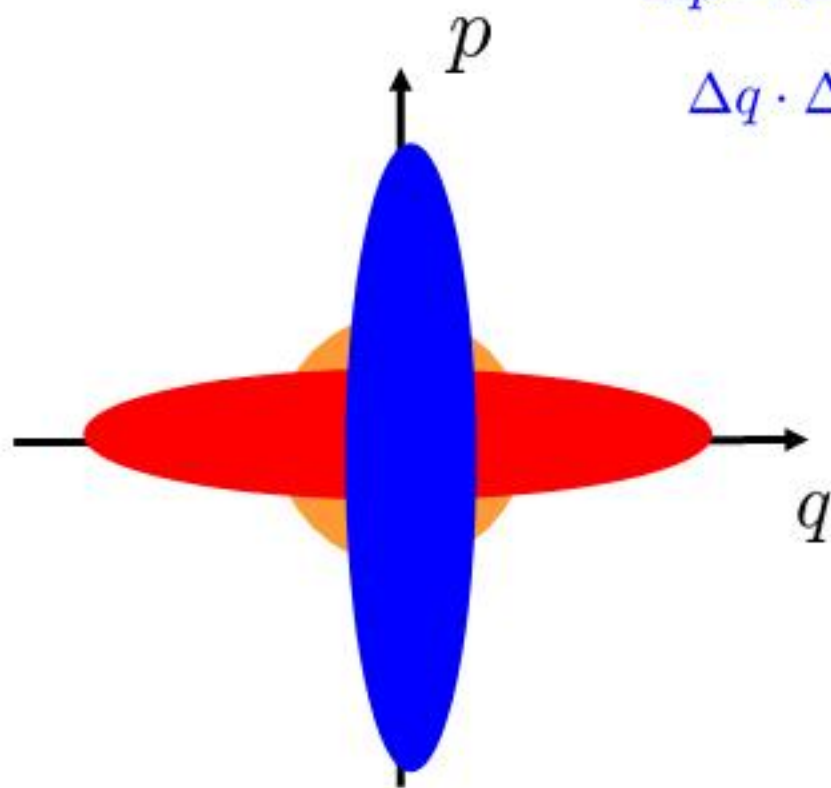
Termiczny ścisnięty

$$\Delta q = \sqrt{\bar{n} + \frac{1}{2} - m}$$

$$\Delta p = \sqrt{\bar{n} + \frac{1}{2} + m}$$

$$\Delta q \cdot \Delta p = \sqrt{(\bar{n} + \frac{1}{2})^2 - m^2}$$

$$0 \leq m \leq \bar{n}(\bar{n} + 1)$$

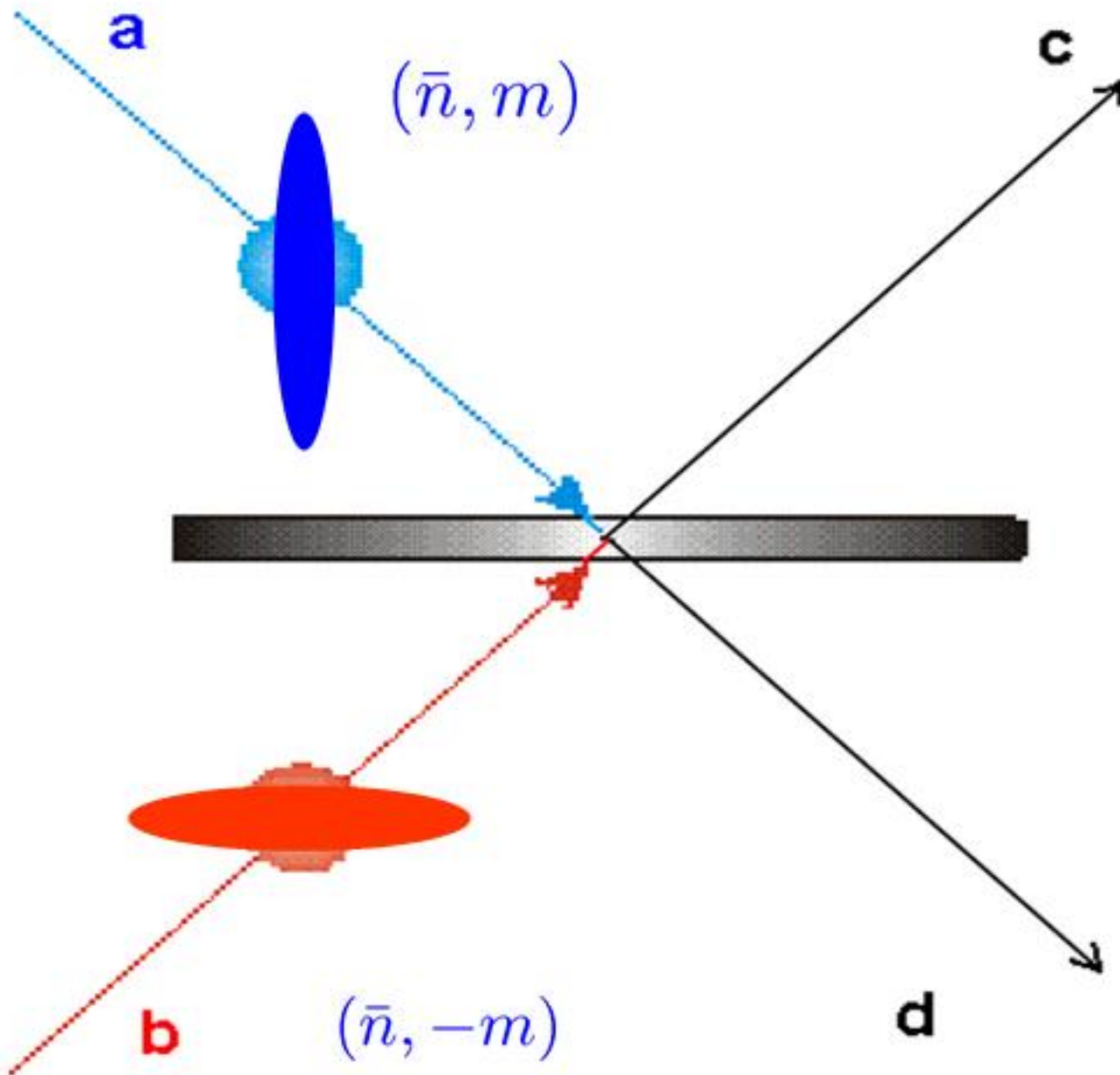


Najbardziej ogólny
stan Gaussa

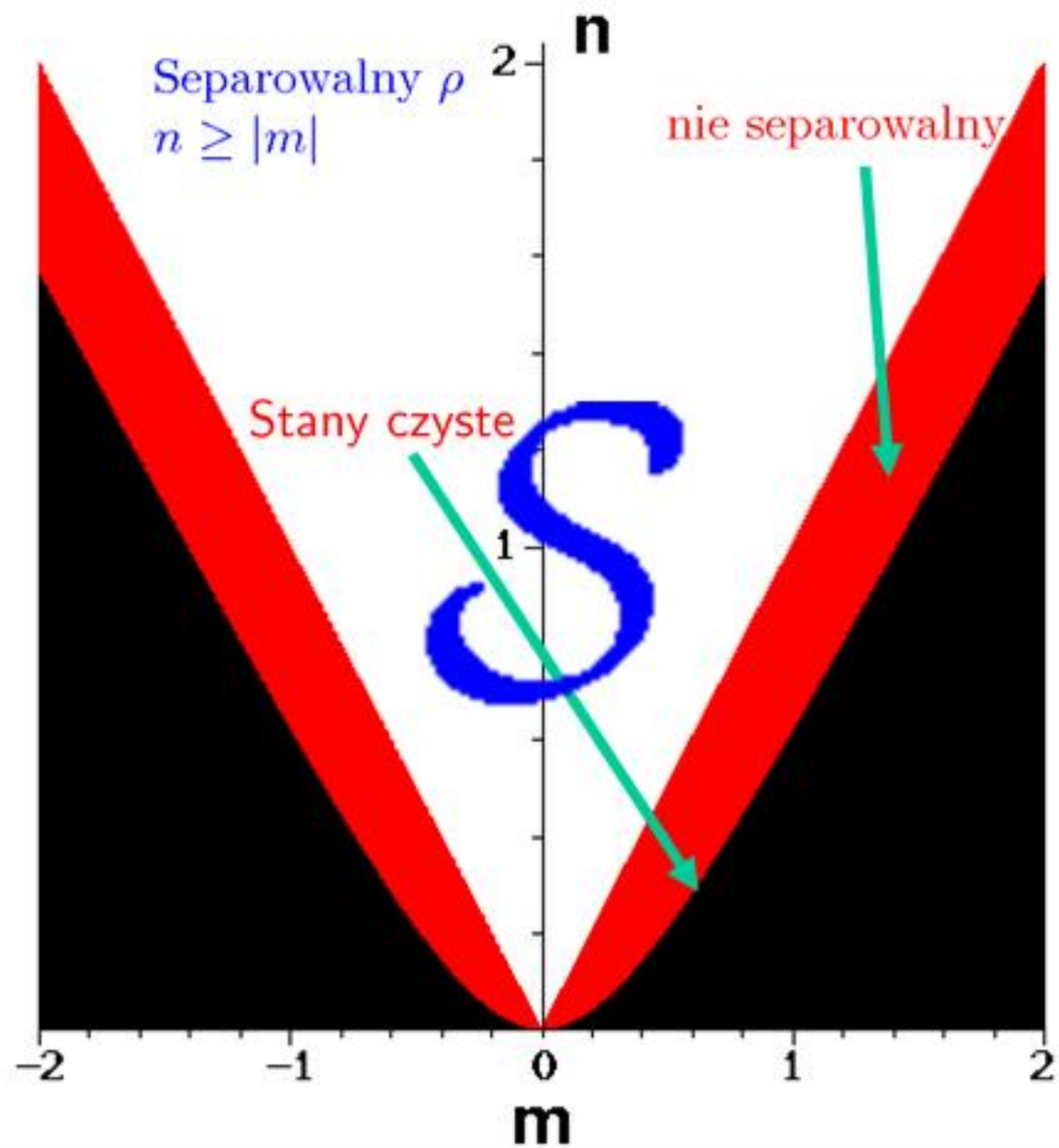
$$\rho_S = \frac{e^{-(\bar{n})a^\dagger a - (m^*)a^2 - (m)a^\dagger{}^2}}{Z}$$

Splatanie stanów ściśniętych

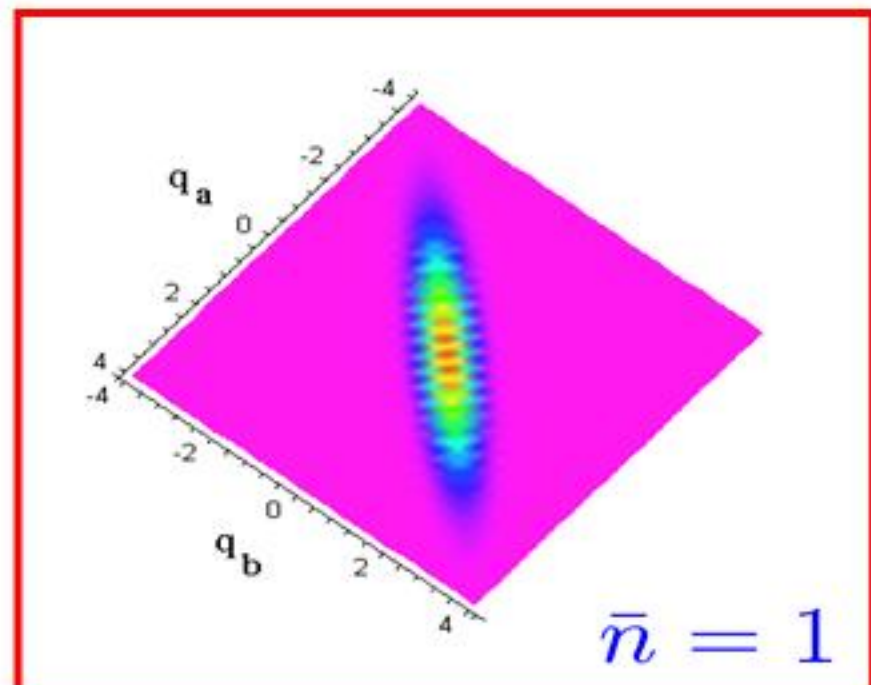
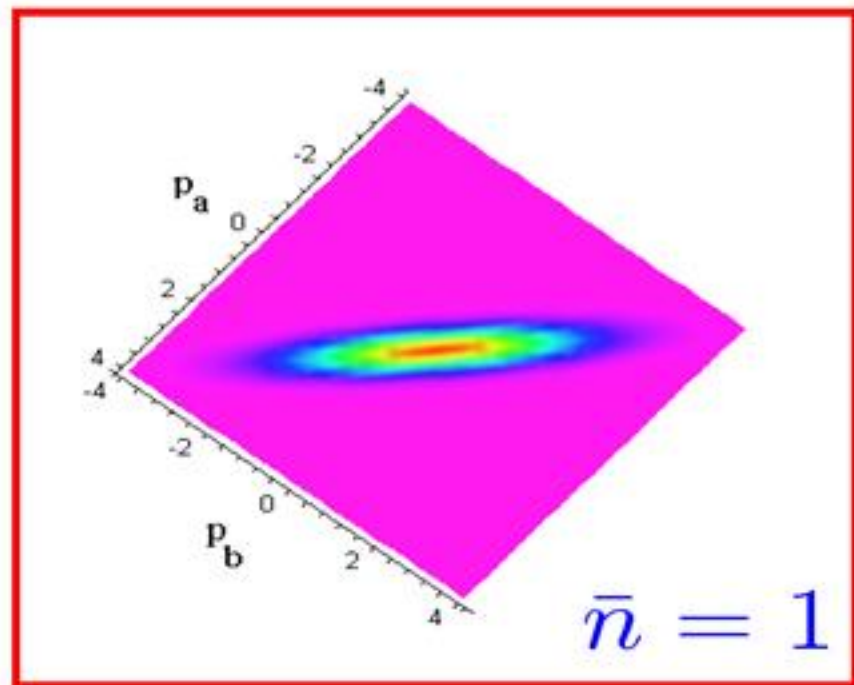
M. Stobińska, K.W.
G. Leuchs



$$\rho_{AB}(\bar{n}, m)$$



Stan czysty

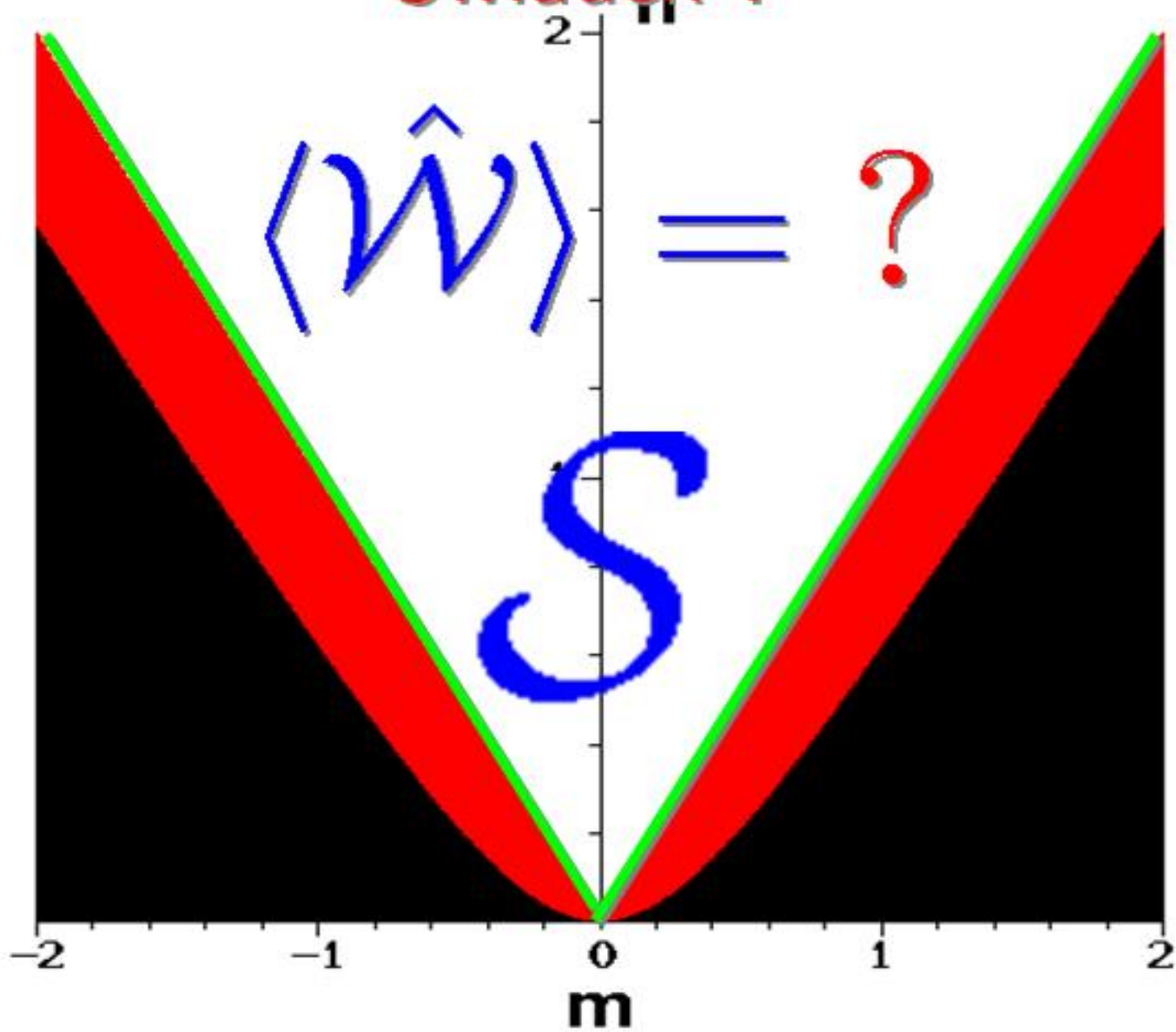


$$\bar{n} \rightarrow \infty$$

$$\Psi_{\text{EPR}} \sim |0, 0\rangle + |1, 1\rangle + |2, 2\rangle \cdots$$

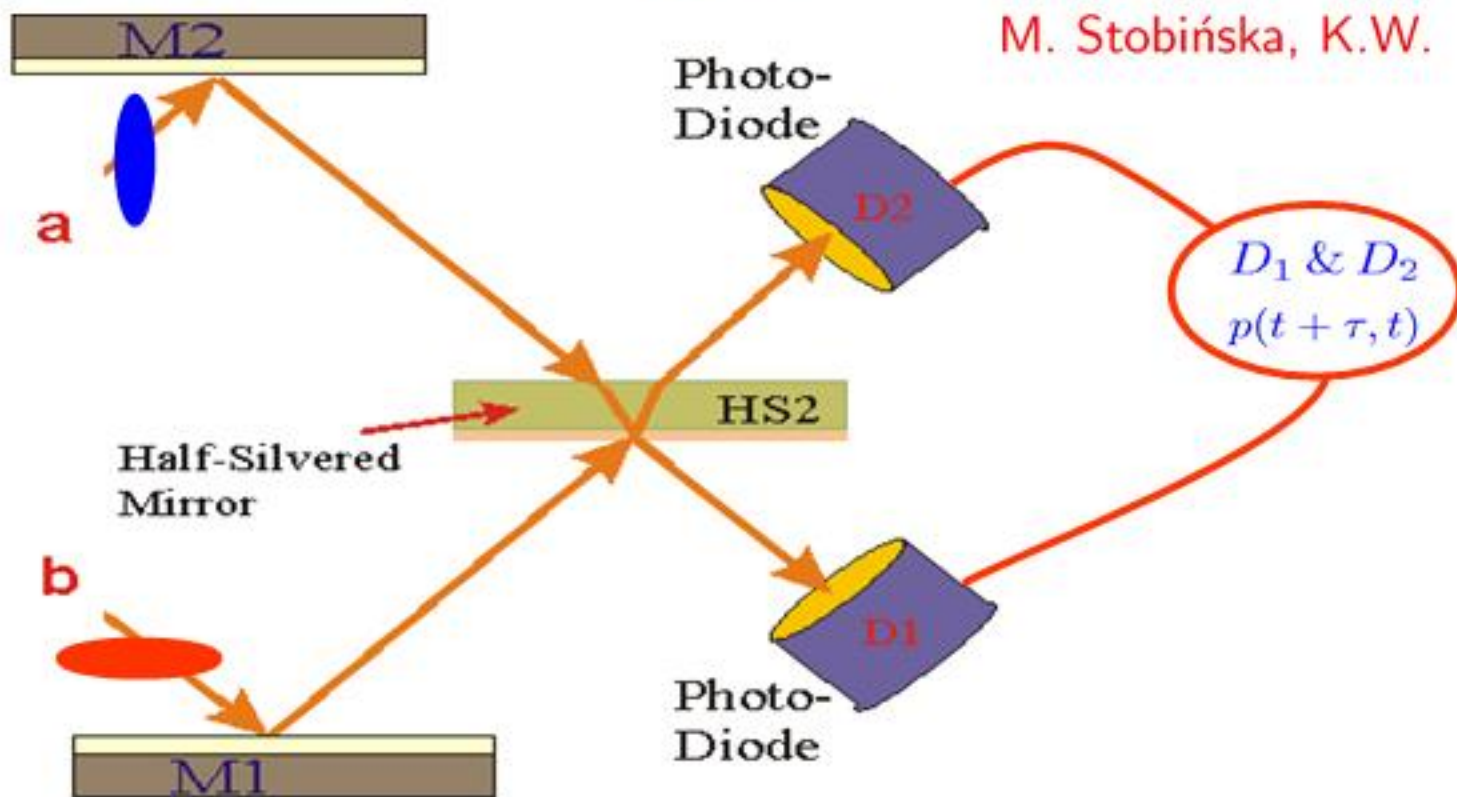
$$\Psi_{\text{EPR}}(q_a, q_b) \sim \delta(q_a - q_b) \quad \tilde{\Psi}_{\text{EPR}}(p_a, p_b) \sim \delta(p_a + p_b)$$

Świadek ?



Interferencyjny Świadek

M. Stobińska, K.W.



$$\mathcal{W}^{(HBT)} = \frac{1}{2} - \frac{2a^\dagger b^\dagger ab + b^{\dagger 2} a^2 + a^{\dagger 2} b^2}{\langle : (a^\dagger a + b^\dagger b)^2 : \rangle}$$

$$\text{Tr}\{\mathcal{W}^{(HBT)} \rho\} = \frac{n^2 - |m|^2}{2(3n^2 + |m|^2)} = \begin{cases} \geq 0, & \text{separowalny} \\ < 0, & \text{nieseparowalny} \end{cases}$$

Interferencja HBT

Dołek Hong-Ou-Mandela

$$\langle E_d^{(-)}(t) E_c^{(-)}(t + \tau) E_c^{(+)}(t + \tau) E_d^{(+)}(t) \rangle \longleftrightarrow \langle \hat{\mathcal{W}} \rangle$$

$$p(\tau) = 1 - v^{(2)} \exp\left(-\left(\frac{\tau}{\tau_c}\right)^2\right)$$

$$v^{(2)} = \frac{n^2 + |m|^2}{3n^2 + |m|^2}$$

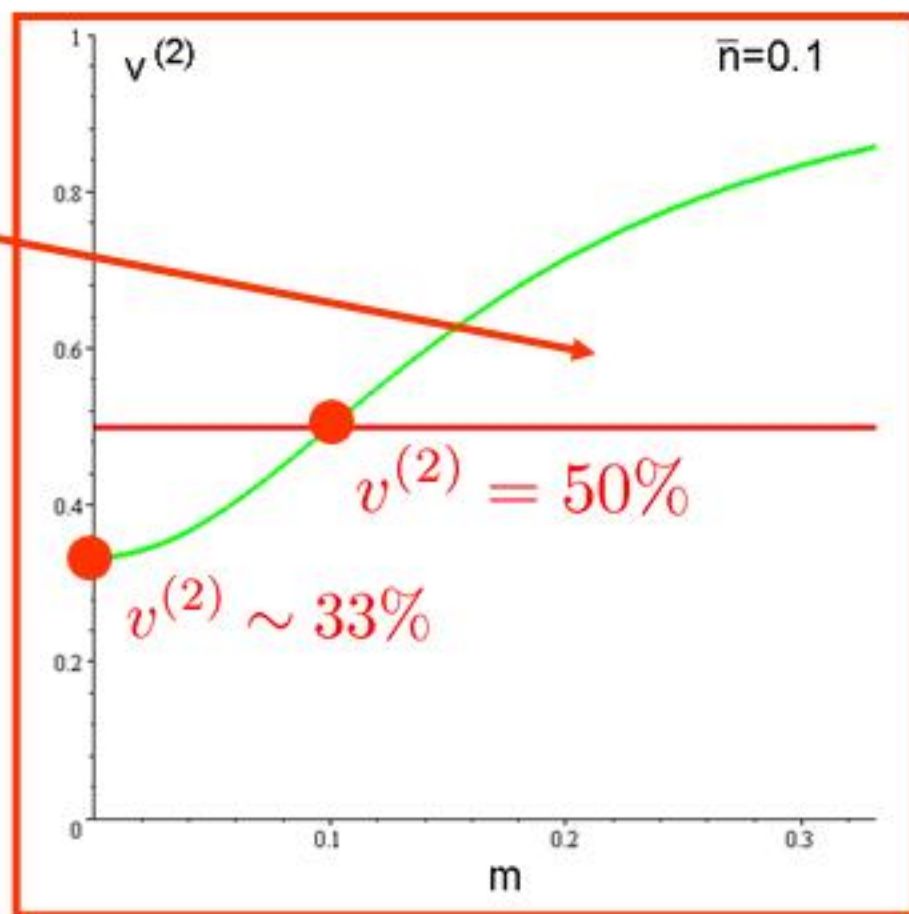
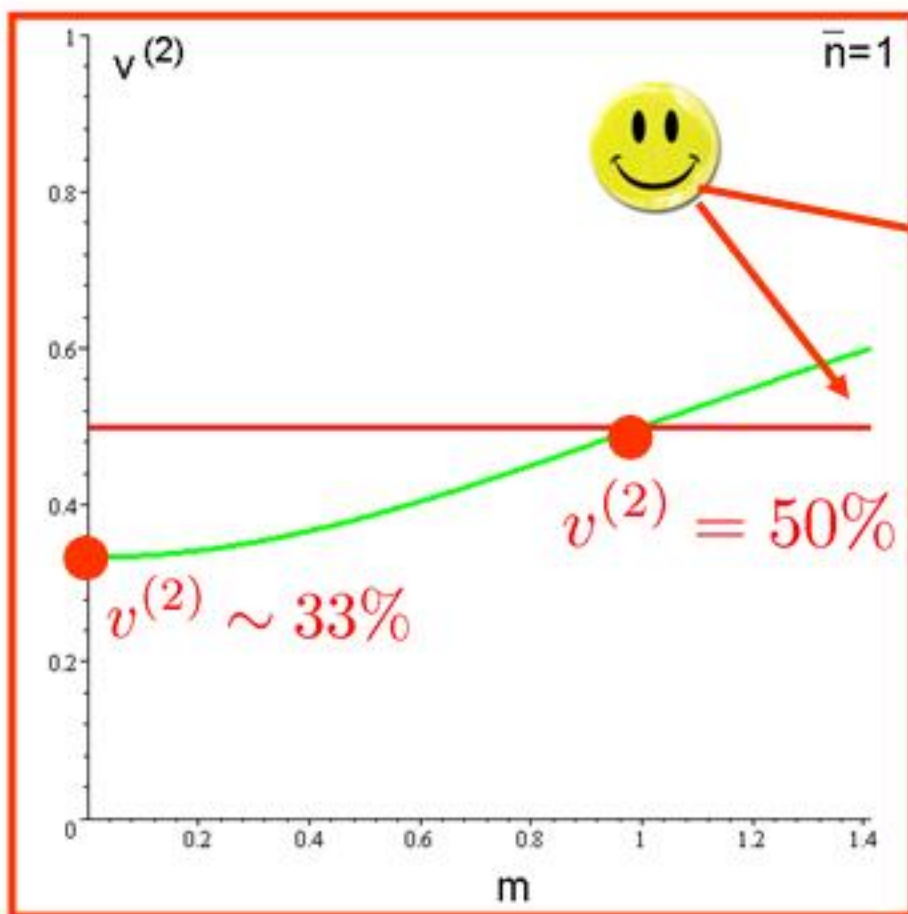
$$\langle \hat{\mathcal{W}} \rangle = \frac{1}{2} - v^{(2)}$$

Widzialność

$$v^{(2)} = \frac{n^2 + |m|^2}{3n^2 + |m|^2}$$

separowalny $\bar{n} = m \rightarrow v^{(2)} = 50\%$

nieseparowalny $\rightarrow v^{(2)} \geq 50\%$

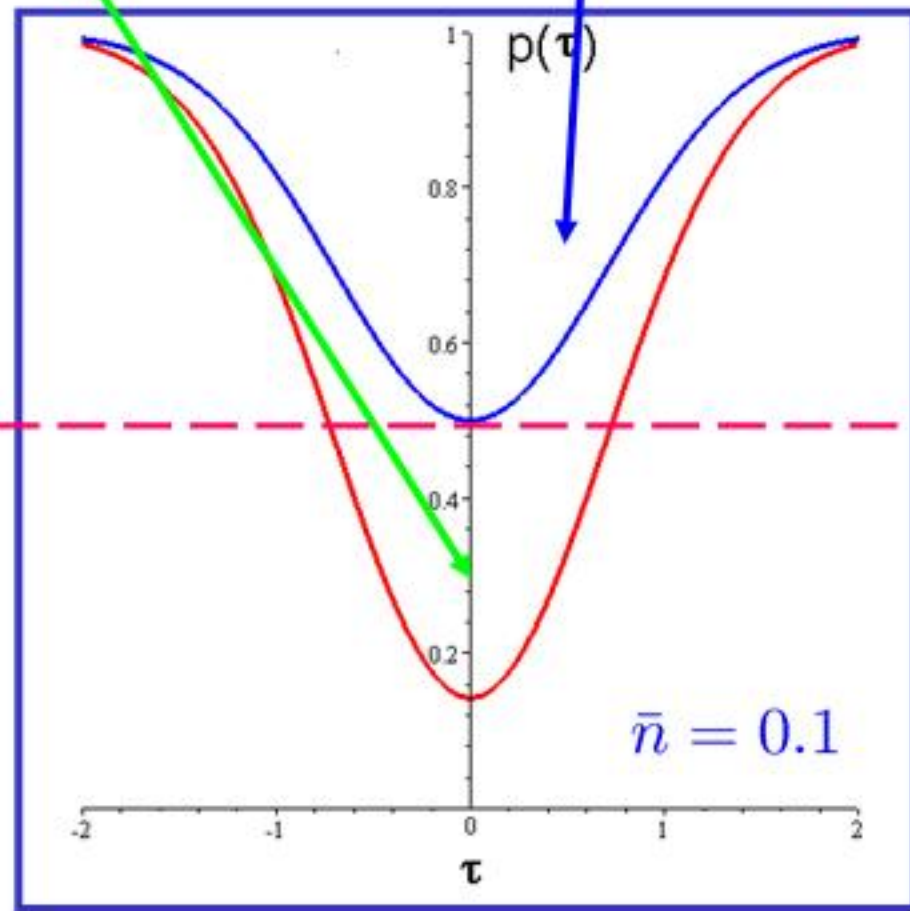
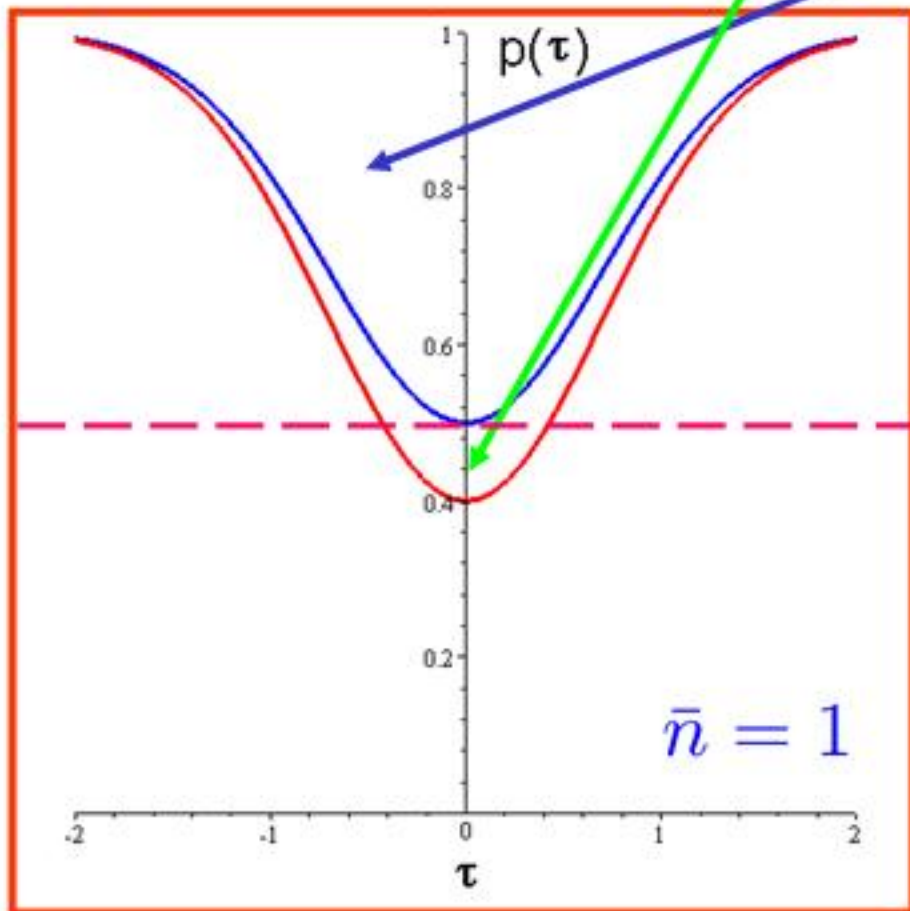


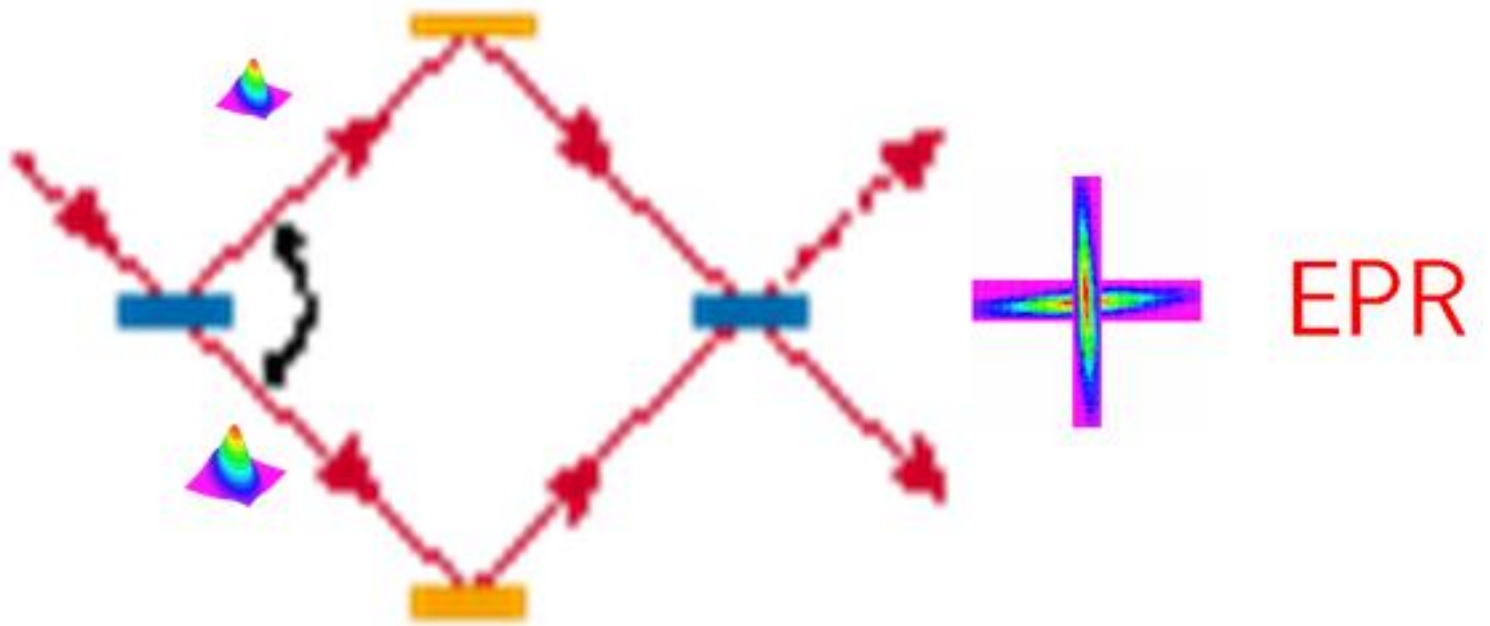
Łączna koincydencja

nieseparowalny



separowalny





$$\mathcal{W}^{(HBT)} = \frac{1}{2} - \frac{2a^\dagger b^\dagger ab + b^{\dagger 2} a^2 + a^{\dagger 2} b^2}{\langle : (a^\dagger a + b^\dagger b)^2 : \rangle}$$

$$\text{Tr}\{\mathcal{W}^{(HBT)} \rho_{EPR}\} = < 0 \quad \text{nieseprawalny}$$