Natural Extensions of the Standard Model Scalar Sector

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- The little hierarchy problem
- Strategy
- Natural models
- Summary

◇ A. Drozd, B.G. and J. Wudka, "Multi-Scalar-Singlet Extension of the Standard Model - the Case for Dark Matter and an Invisible Higgs Boson", JHEP 1204, 006 (2012),

◇ B.G., O.M. Ogreid, **P. Osland**, A. Pukhov and M. Purmohammadi, "Exploring the CP-Violating Inert-Doublet Model", JHEP 1106, 003 (2011),

◇ B.G., **P. Osland**, "Tempered Two-Higgs-Doublet Model", Phys. Rev. D82, 125026 (2010),

◇ B.G., O.M. Ogreid and **P. Osland**, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys. Rev. D80, 055013 (2009),

◇ B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", Phys. Rev. Lett. 103, 091802 (2009).

The little hierarchy problem

- $m_h^2 = m_h^{(B)\,2} + \delta^{(SM)} m_h^2 + \cdots$

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2}\left[\frac{3}{2} m_t^2 - \frac{1}{8}\left(6 m_W^2 + 3 m_Z^2\right) - \frac{3}{8} m_h^2\right]$$

$$m_h \simeq 125 \text{ GeV} \quad \Rightarrow \quad \delta^{(SM)} m_h^2 \simeq m_h^2 \qquad \text{for} \qquad \Lambda \simeq 600 \text{ GeV}$$

- For $\Lambda \gtrsim 600$ GeV there must be a cancellation between the tree-level Higgs mass² $m_h^{(B)\,2}$ and the 1-loop leading correction $\delta^{(SM)} m_h^2$:

$$m_h^{(B)\,2} \sim \delta^{(SM)} m_h^2 \gtrsim m_h^2$$

$$\Downarrow$$

the little hierarchy problem.

- The SM cutoff is very low!

Solutions to the little hierarchy problem:

Suppression of corrections growing with Λ^2 at the 1-loop level

- The Veltman condition, no Λ^2 terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0 \qquad \Longrightarrow \qquad m_h \simeq 310 \text{ GeV}$$

P. Osland and T. T. Wu, "Parameters in the electroweak theory: 2. Quadratic divergences", Z. Phys. C 55, 585 (1992).

- SUSY:

$$\delta^{(SUSY)}m_h^2 \sim m_{\tilde{t}}^2 \, \frac{3g_t^2}{8\pi^2} \ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right)$$

then for $\Lambda \sim 10^{16-18}$ GeV one gets $m_{\tilde{t}}^2 \lesssim 1$ TeV^2 in order to have $\delta^{(SUSY)}m_h^2 \sim m_h^2$.

The Strategy

- The SM 1-loop quadratic divergences are dominated by the top (fermionic) contribution, so to suppress them we are going to introduce extra scalars (as the SM Higgs would need to be too heavy to do the job).

- DM candidate is mandatory.

- CPV will be desirable.

Natural Models

♠ Less divergence + DM $\Rightarrow$ SM + N_φ scalar singlets

B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", Phys.Rev.Lett.103:091802,2009,

A. Drozd, B.G. and J. Wudka, "Multi-Scalar-Singlet Extension of the Standard Model - the Case for Dark Matter and an Invisible Higgs Boson", JHEP 1204, 006 (2012),

Assumptions:

- N_φ extra gauge singlets φ_i transforming according to fundamental representation $\vec{\varphi}$ of $O(N_\varphi)$ with $\langle \varphi_i \rangle = 0$,
- $O(N_\varphi)$ symmetry for the potential implies

$$V(H, \varphi_i) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 + \mu_\varphi^2 \vec{\varphi}^2 + \frac{\lambda_\varphi}{24}(\vec{\varphi}^2)^2 + \lambda_x |H|^2 \vec{\varphi}^2$$

with no $|H|^2\varphi_i$ couplings and

$$\langle H \rangle = \frac{v}{\sqrt{2}}, \qquad \langle \varphi_i \rangle = 0 \qquad \text{for} \qquad \mu_\varphi^2 > 0$$

then $m_h^2 = 2\mu_H^2$ and $m^2 = 2\mu_\varphi^2 + \lambda_x v^2$

Vacuum stability: $\lambda_H\ ,\ \lambda_\varphi > 0$ and $\lambda_x > -\sqrt{\frac{\lambda_\varphi \lambda_H}{6}} = -\frac{m_h}{2v}\sqrt{\frac{\lambda_\varphi}{3}}$

Tree-level unitarity: $m_h^2 < \frac{8\pi}{3}v^2,\ \lambda_\varphi < 8\pi$ and $|\lambda_x| < 4\pi$

$$\delta^{(\varphi)} m_h^2 = -N_\varphi \frac{\lambda_x}{8\pi^2} \left[\Lambda^2 - m^2 \ln\left(1 + \frac{\Lambda^2}{m^2}\right)\right]$$

$$|\delta m_h^2| = |\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2| = D_t m_h^2$$

$$\Downarrow$$

$$\lambda_x = \lambda_x(m, m_h, D_t, \Lambda, N_\varphi)$$

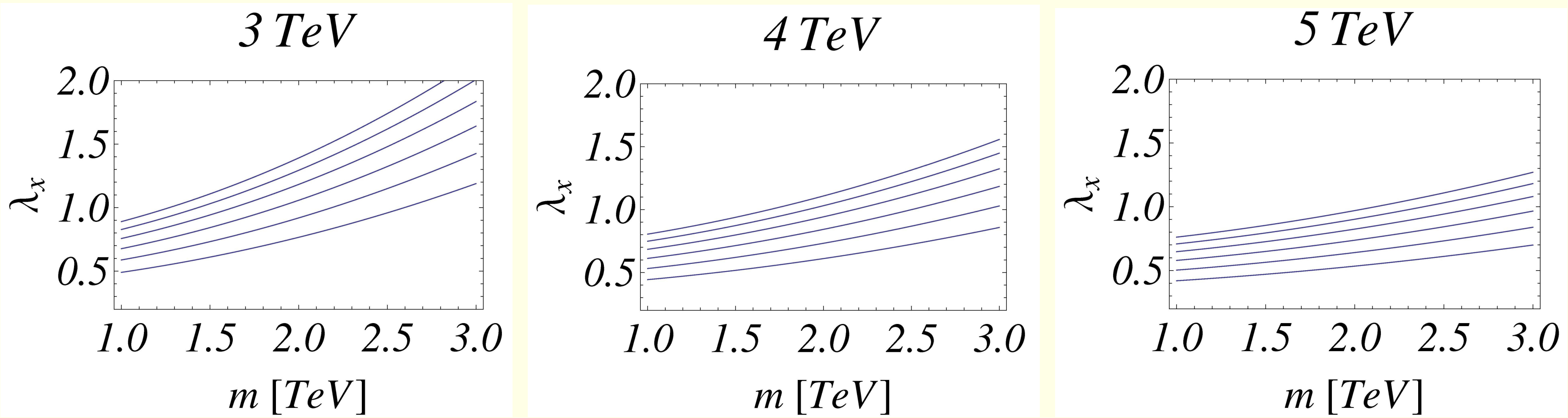

Figure 1: Plots of λ_x as a function of m for $N_\varphi = 6$, $D_t = 0$ and various choices of $\Lambda = 3, 4, 5$ TeV shown above each panel. The curves correspond to $m_h = 130,\ 150,\ 170,\ 190,\ 210,\ 230$ GeV (starting with the uppermost curve).

Singlet DM

It is possible to find parameters Λ, λ_x and m such that both the hierarchy is ameliorated to the prescribed level and such that $\Omega_\varphi h^2$ is consistent with $\Omega_{DM} h^2$.

$$\varphi\varphi \to hh, W_L^+ W_L^-, Z_L Z_L, f\bar{f} \quad \Rightarrow \quad \langle \sigma v \rangle \simeq \frac{1}{8\pi}\frac{\lambda_x^2}{m^2}$$

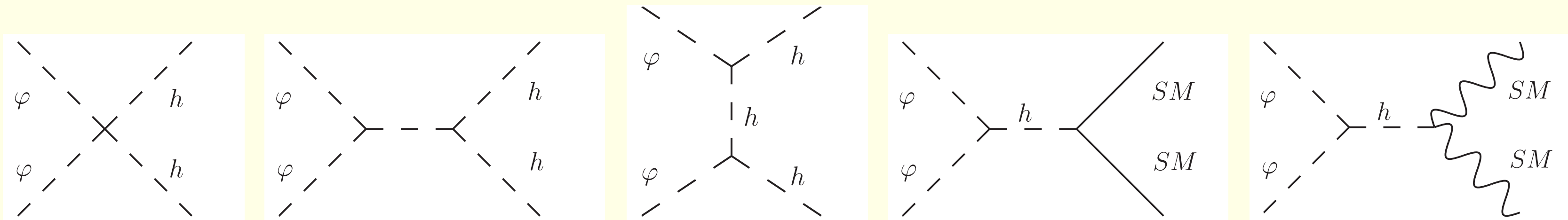

Figure 2: Diagrams contributing to the scalar $\varphi\varphi$ annihilation into SM particles.

The Boltzmann equation $\Rightarrow \quad x_f \left(\equiv \frac{m}{T_f}\right) \simeq \ln\left[0.038 \frac{m_{Pl} m \langle \sigma v \rangle}{g_\star^{1/2} x_f^{1/2}}\right]$

$$\Omega_\varphi h^2 \simeq 1.06 \cdot 10^9 \frac{x_f}{g_\star^{1/2} m_{Pl} \langle \sigma v \rangle \text{ GeV}}$$

Singlet DM

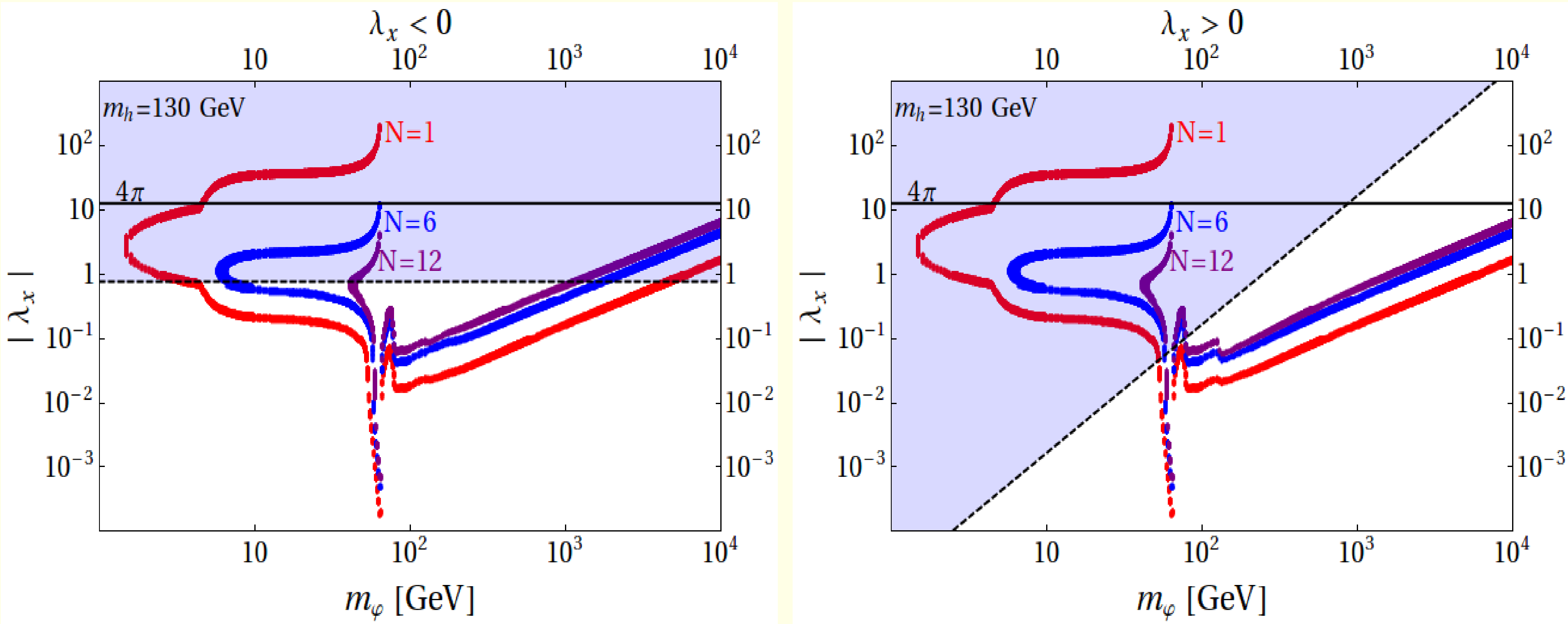

Figure 3: The coupling λ_x as a function of m_φ obtained from the requirement $0.092 < \Omega_{\rm DM}^N < 0.128$; for the Higgs $m_h = 130$ GeV, $N = 1, 6$ and 12 (red, blue and purple bands respectively), and $\lambda_x > 0$ (left panel) or $\lambda_x < 0$ (right panel). The blue areas in the left and right panels correspond to the regions disallowed by consistency condition and stability constraints for $\lambda_\varphi = 8\pi$, respectively. The thick black lines show the unitarity limit saturated by $|\lambda_x| = 4\pi$.

♠ Less divergence + CPV + DM $\Rightarrow \left\{ \begin{array}{l} \text{2HDM (CPV)} + \text{Inert singlet (DM)} \\ \text{2HDM (CPV)} + \text{Inert doublet (DM)} \end{array} \right.$

◇ B.G., O.M. Ogreid and **P. Osland**, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys. Rev. D80, 055013 (2009),
◇ B.G., **P. Osland**, "Tempered Two-Higgs-Doublet Model", Phys. Rev. D82, 125026 (2010),
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$$\begin{aligned} V(\phi_1,\phi_2) &= -\frac{1}{2}\left\{m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \text{H.c.}\right]\right\} + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 \\ &+\frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+\frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right] + \left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right]\left[(\phi_1^\dagger\phi_2) + \text{H.c.}\right] \end{aligned}$$

$$\mathcal{L}_Y^{(q)} = \bar{Q}_L\left(\tilde{\Gamma}_1\tilde{\Phi}_1 + \tilde{\Gamma}_2\tilde{\Phi}_2\right)u_R + \bar{Q}_L\left(\Gamma_1\Phi_1 + \Gamma_2\Phi_2\right)d_R + \text{H.c.}$$

then

$$M_u = -\tilde{\Gamma}_1\langle\tilde{\Phi}_1\rangle - \tilde{\Gamma}_2\langle\tilde{\Phi}_2\rangle \qquad M_d = -\Gamma_1\langle\Phi_1\rangle - \Gamma_2\langle\Phi_2\rangle$$

Type II model, $\mathbb{Z}_2$ softly broken (by $m_{12}^2 \neq 0$): $\Phi_1 \to -\Phi_1$ and $d_R \to -d_R \Rightarrow$ $\lambda_6 = \lambda_7 = 0$, $\tilde{\Gamma}_1 = \Gamma_2 = 0$

$$\phi_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix}$$

Defining $\eta_3 \equiv -s_\beta \chi_1 + c_\beta \chi_2$ orthogonal to the neutral Goldstone boson $G^0 \equiv c_\beta \chi_1 + s_\beta \chi_2$ one gets 3×3 mass matrix $\mathcal{M}^2$ for neutral scalars (η_1, η_2, η_3) that could be diagonalized by the orthogonal rotation matrix R:

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

and

$$R\mathcal{M}^2 R^T = \mathcal{M}^2_{\rm diag} = {\rm diag}({\rm M}_1^2, {\rm M}_2^2, {\rm M}_3^2)$$

with $M_1 \leq M_2 \leq M_3$.

$$R = \begin{pmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -(c_1 s_2 s_3 + s_1 c_3) & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -(c_1 s_3 + s_1 s_2 c_3) & c_2 c_3 \end{pmatrix}$$

where $s_i \equiv \sin\alpha_i$, $c_i \equiv \cos\alpha_i$ for $i = 1, 2, 3$.

1-Loop Corrections

Cancellation of quadratic divergences for ϕ_1 and ϕ_2 (Newton & Wu, 1994):

$$G_{11}^{(1)} \equiv \frac{\Lambda^2}{v^2}\left[\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) - 3\frac{m_b^2}{c_\beta^2}\right] = 0,$$

$$G_{22}^{(1)} \equiv \frac{\Lambda^2}{v^2}\left[\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) - 3\frac{m_t^2}{s_\beta^2}\right] = 0,$$

where $v^2 \equiv v_1^2 + v_2^2$, $\tan\beta \equiv v_2/v_1$

$$\Downarrow$$

For a given choice of the mixing angles α_i's $(i = 1, 2, 3)$, the neutral-Higgs masses M_1^2, M_2^2 and M_3^2 can be determined from the cancellation conditions in terms of $\tan\beta$, $\mu^2 \equiv \mathbf{Re}(m_{12}^2)/(2s_\beta c_\beta)$ and $M_{H^\pm}^2$.

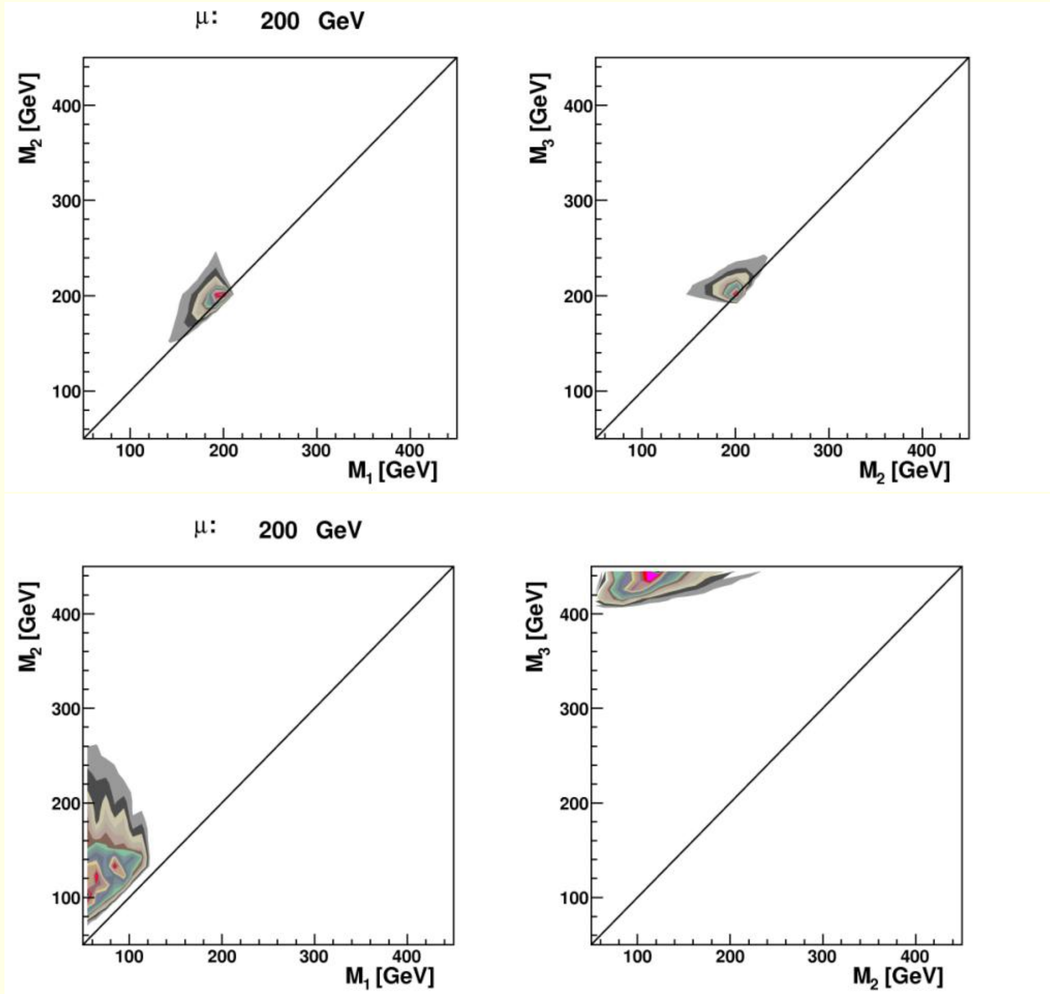

Figure 4: Distributions of allowed masses M_2 vs M_1 (left panels) and M_3 vs M_2 (right), resulting from a scan over the full range of α_i, $\tan\beta \in (0.5, 50)$ and $M_{H^\pm} \in (300, 700)$ GeV, for $\mu = 200$ GeV. No constraints are imposed other than the cancellation of quadratic divergences, $M_i^2 > 0$ and $M_1 < M_2 < M_3$. Two ranges of $\tan\beta$-values are displayed: bottom panels: $0.5 \leq \tan\beta \leq 1$, top panels: $40 \leq \tan\beta \leq 50$. The color coding indicates increasing density of allowed points as one moves inward from the boundary.

$$M_1^2 - M_2^2 = \frac{1}{\tan\beta}\frac{R_{33}}{R_{12}R_{22}}\left[-4\bar{m}^2 - 2M_{H^\pm}^2 + 12m_t^2 + \mu^2\right] + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right)$$

$$M_3^2 = -\frac{M_1^2 R_{12}R_{13} + M_2^2 R_{22}R_{23}}{R_{32}R_{33}} + \mathcal{O}\left(\frac{1}{\tan\beta}\right).$$

where R_{ij} are elements of the orthogonal rotation matrix for the neutral scalars and $\bar{m}^2 \equiv \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2$.

$$\Downarrow$$

$$\tan\beta \gtrsim 40 \qquad \Longrightarrow \qquad M_1 \simeq M_2 \simeq M_3 \simeq \mu^2 + 4m_b^2$$

Advantages:

- No 1-loop quadratic divergences (so, $\delta M_i^2/M_i^2$ suppressed),
- A chance for substantial CPV,
- DM candidate easily accommodated by adding singlets φ-like.

The following experimental constraints are imposed:

- The oblique parameters T and S
- $B_0 - \bar{B}_0$ mixing
- $B \to X_s \gamma$
- $B \to \tau \bar{\nu}_\tau X$
- $B \to D \tau \bar{\nu}_\tau$
- LEP2 Higgs-boson non-discovery
- R_b
- The muon anomalous magnetic moment
- Electron electric dipole moment

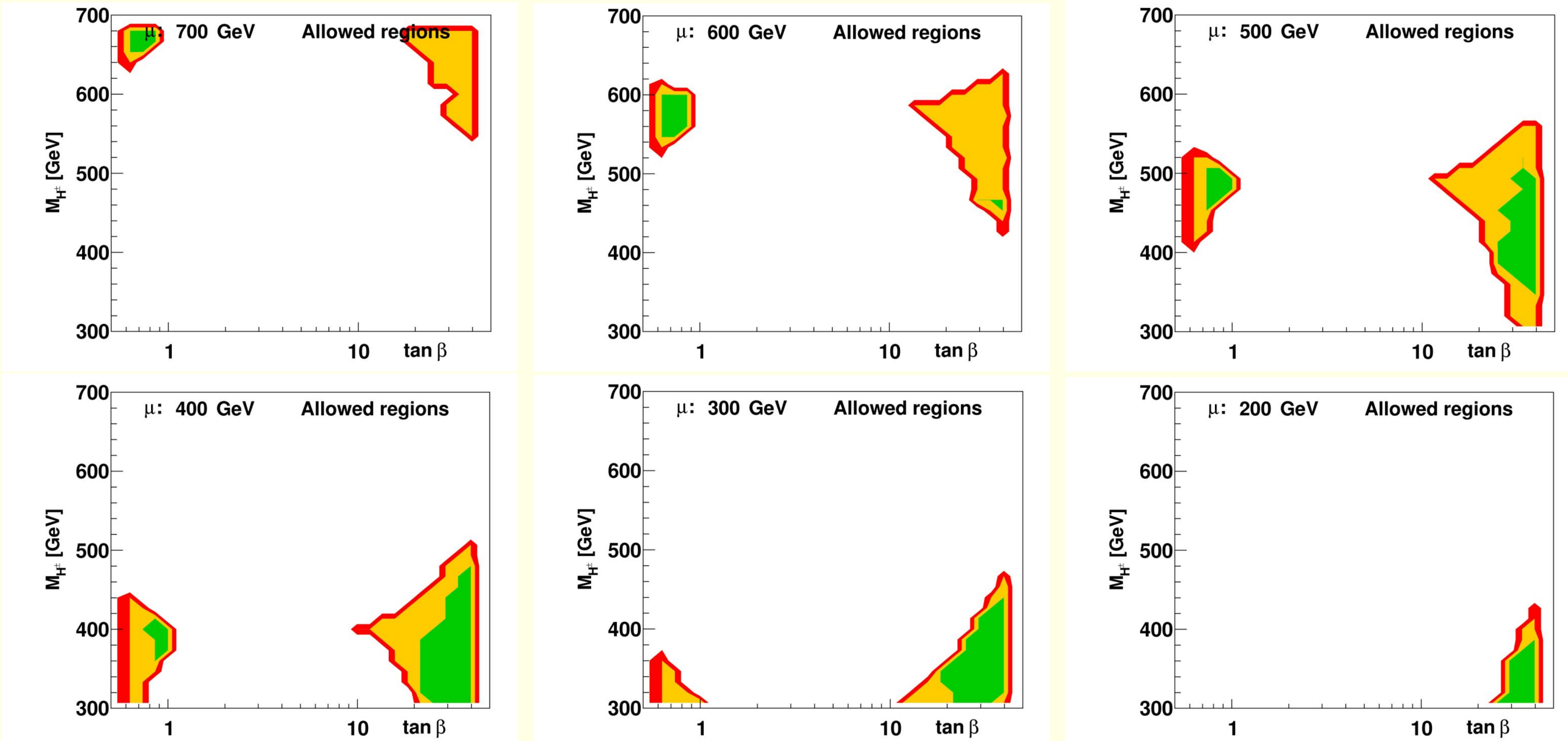

Figure 5: Allowed regions in the $\tan\beta$–$M_{H^\pm}$ plane, for $\mu = 200, 300, 400, 500, 600$ and 700 GeV (as indicated). Red: Positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L..

Violation of CP

$$\begin{aligned}
\mathbf{Im}J_1 &= -\frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)\mathbf{Im}\lambda_5, \\
\mathbf{Im}J_2 &= -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_1^4+2(\lambda_1-\lambda_2)\mathbf{Re}\lambda_5 v_1^2 v_2^2\right. \\
&\qquad \left.-\left((\lambda_2-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_2^4\right]\mathbf{Im}\lambda_5, \\
\mathbf{Im}J_3 &= \frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)(\lambda_1+\lambda_2+2\lambda_4)\mathbf{Im}\lambda_5.
\end{aligned}$$

For $\tan\beta \gtrsim 40$

$$\mathbf{Im}J_i \sim \frac{\mathbf{Im}\lambda_5}{\tan^2\beta}$$

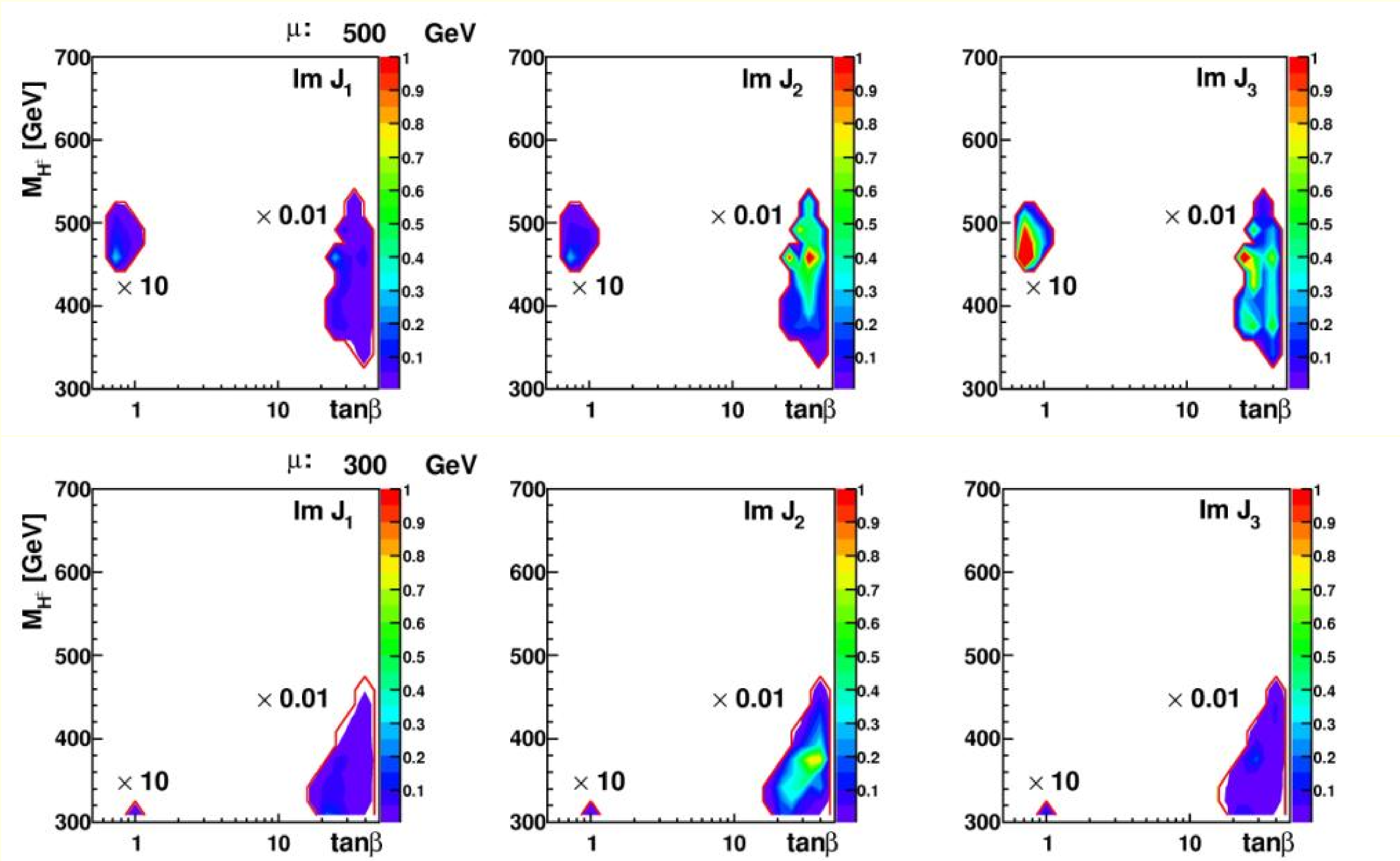

Figure 6: Imaginary parts of the WBI invariants $|\mathbf{Im}J_i|$, for $\mu = 500$ GeV (top) and $\mu = 300$ GeV (bottom). The colour coding is given along the right vertical axis. At low $\tan\beta$ the values should be rescaled by a factor of 10, at high $\tan\beta$ by a factor 0.01.

DM in the 2 Higgs Doublet Model with no quadratic divergences

$$\begin{aligned}
V(\phi_1,\phi_2) &= -\frac{1}{2}\left\{m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \mathsf{H.c.}\right]\right\} \\
&+\frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\
&+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \mathsf{H.c.}\right] \\
&+\mu_\varphi^2\varphi^2 + \frac{1}{24}\lambda_\varphi\varphi^4 + \varphi^2(\eta_1\phi_1^\dagger\phi_1 + \eta_2\phi_2^\dagger\phi_2)
\end{aligned}$$

The cancellation conditions:

$$\begin{aligned}
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{1}{2}\eta_1 + \frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_b^2}{c_\beta^2}, \\
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{1}{2}\eta_2 + \frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_t^2}{s_\beta^2}, \\
\frac{\lambda_\varphi}{2} + 4(\eta_1+\eta_2) &= 8\mathrm{Tr}\{Y_\varphi Y_\varphi^\dagger\}
\end{aligned}$$

where $\mathcal{L}_Y = -\varphi\overline{(\nu_R)^c}Y_\varphi\nu_R + \mathsf{H.c.}$.

$$\mathcal{L} = -\varphi^2(\kappa_i v H_i + \lambda_{ij} H_i H_j + \lambda_\pm H^+ H^-)$$

with

$$\kappa_i = \eta_1 R_{i1} c_\beta + \eta_2 R_{i2} s_\beta,$$

$$\lambda_{ij} = \frac{1}{2}\left[\eta_1(R_{i1}R_{j1} + s_\beta^2 R_{i3}R_{j3}) + \eta_2(R_{i2}R_{j2} + c_\beta^2 R_{i3}R_{j3})\right],$$

$$\lambda_\pm = \eta_1 s_\beta^2 + \eta_2 c_\beta^2$$

Assumption: $M_1 \ll M_{2,3}$ so that DM annihilation is dominated by H_1 exchange.

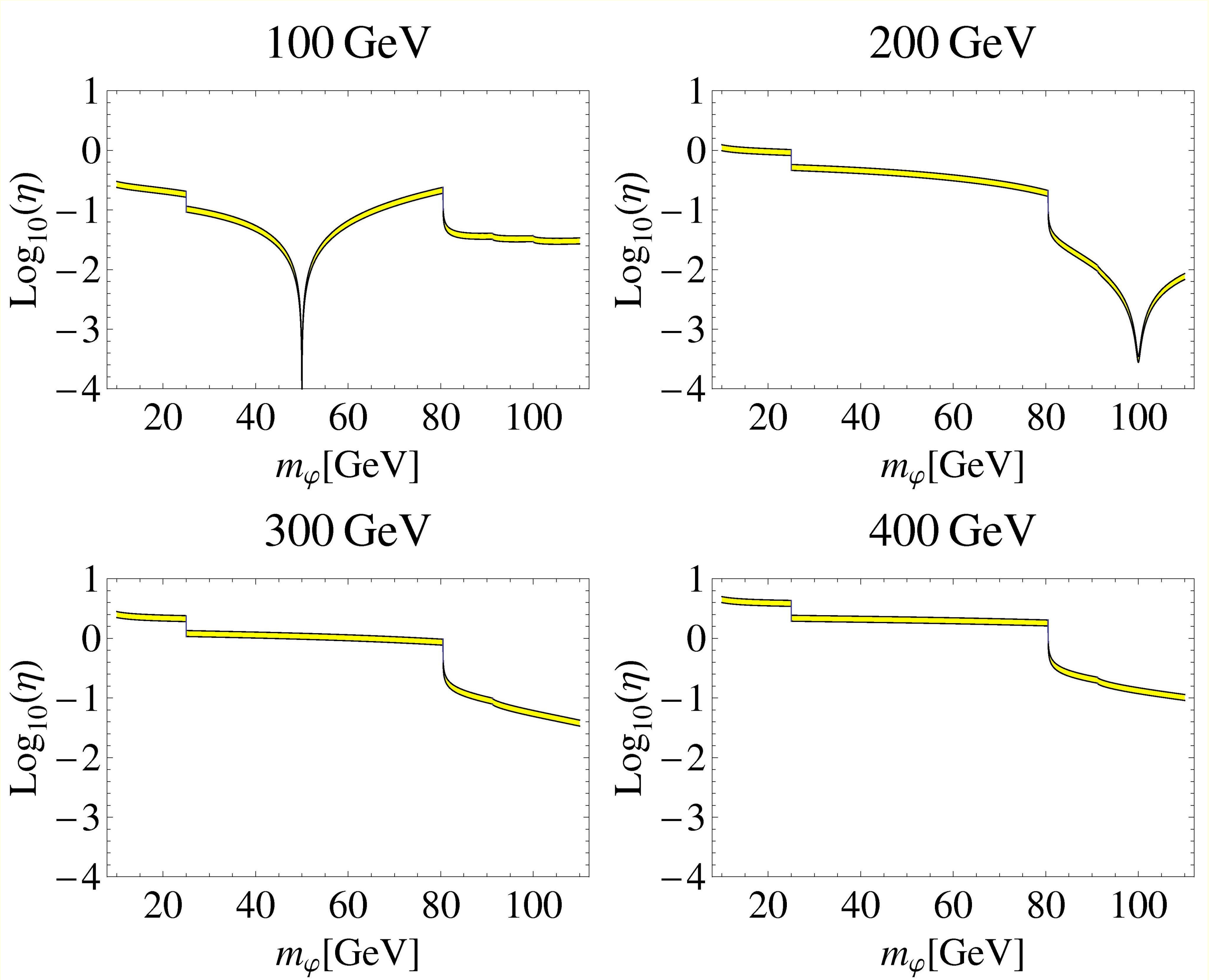

Figure 7: Inert-scalar coupling η (vs m_φ) required by the observed DM abundance $\Omega_{DM}h^2 = 0.106 \pm 0.008$ within a 3-σ band. As indicated above each panel, the lightest Higgs-boson mass ranges from $M_1 = 100$ to 400 GeV . It was assumed that $2\lambda_{11} = \kappa_1 \equiv \eta$.

Summary

- The SM could be easily extended so that the little hierarchy problem is ameliorated, DM candidate is provided and also CP is violated in the extra sector:
  - The addition of N_φ real scalar singlets φ_i to the SM lifts the cutoff Λ to $\sim 4-9$ TeV. It also provides a realistic candidate for DM if $m_\varphi \sim 1-3$ TeV (depending on N_φ).
  - To accommodate CPV in the Higgs potential the SM scalar sector should be replaced by 2 Higgs doublets (non-inert). Cancellation of quadratic divergences could be arranged within the 2HDM. Adding extra inert scalar singlet or doublet offers a DM candidate.

- Some fine tuning always remains.

- The Inert Doublet Model with no quadratic divergences

$$\mathbb{Z}_2 : \qquad \phi_1 \to -\phi_1$$

$$\begin{aligned} V(\phi_1,\phi_2) &= -\frac{1}{2}m_{11}^2\phi_1^\dagger\phi_1 - \frac{1}{2}m_{22}^2\phi_2^\dagger\phi_2 + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 \\ &+\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right]. \end{aligned}$$

$m_{22}^2 > 0$ and $m_{11}^2 < 0$ $\Longrightarrow$ $\langle\phi_1\rangle = 0$ and $\langle\phi_2\rangle = v/\sqrt{2}$

Cancellation of quadratic divergences for ϕ_1 and ϕ_2 (Newton & Wu, 1994):

$$\begin{aligned} \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 0, \\ \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3m_t^2. \end{aligned}$$

Comments on the inert 2HDM:

- Motivation: to provide a candidate for DM.

- No CPV (as implied by exact $\mathbb{Z}_2$).

- The vacuum stability conditions turn out to be inconsistent with the cancellation of quadratic divergences for realistic top mass.

$$\Downarrow$$

- Allow for $m_{12}^2 \phi_1^\dagger \phi_2 +$ H.c. (CPV),
- Allow for $\langle \phi_1 \rangle \neq 0$,
- The price: no DM candidate!

• 2HDM (CPV) + Inert doublet (DM)

B.G., O.M. Ogreid, **P. Osland**, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys.Rev.D80:055013,2009.

$$V(\Phi_1, \Phi_2, \eta) = V_{12}(\Phi_1, \Phi_2) + V_3(\eta) + V_{123}(\Phi_1, \Phi_2, \eta)$$

where

$$\begin{aligned}
V_{12}(\Phi_1, \Phi_2) &= -\frac{1}{2}\left\{m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}\right]\right\} \\
&+ \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
&+ \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}\right], \\
V_3(\eta) &= m_\eta^2 \eta^\dagger \eta + \frac{\lambda_\eta}{2}(\eta^\dagger \eta)^2, \\
V_{123}(\Phi_1, \Phi_2, \eta) &= \lambda_{1133}(\Phi_1^\dagger \Phi_1)(\eta^\dagger \eta) + \lambda_{2233}(\Phi_2^\dagger \Phi_2)(\eta^\dagger \eta) \\
&+ \lambda_{1331}(\Phi_1^\dagger \eta)(\eta^\dagger \Phi_1) + \lambda_{2332}(\Phi_2^\dagger \eta)(\eta^\dagger \Phi_2) \\
&+ \frac{1}{2}\left[\lambda_{1313}(\Phi_1^\dagger \eta)^2 + \text{H.c.}\right] + \frac{1}{2}\left[\lambda_{2323}(\Phi_2^\dagger \eta)^2 + \text{H.c.}\right]
\end{aligned}$$