Natural 2HDM

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- Summary

- B.G., P. Osland, "A Natural Two-Higgs-Doublet Model", e-Print: arXiv:0910.4068.

The little hierarchy problem

- $$m_h^2 = m_h^{(B)\,2} + \delta^{(SM)} m_h^2 + \cdots$$

$$\delta^{(SM)} m_h^2 \propto \frac{\Lambda^2}{v^2}\left[\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2\right]$$

$$m_h = 130 \text{ GeV} \quad \Rightarrow \quad \delta^{(SM)} m_h^2 \simeq m_h^2 \qquad \text{for} \qquad \Lambda \simeq 600 \text{ GeV}$$

- For $\Lambda \gtrsim 600$ GeV there must be a cancellation between the tree-level Higgs mass^2 $m_h^{(B)\,2}$ and the 1-loop leading correction $\delta^{(SM)} m_h^2$:

$$m_h^{(B)\,2} \sim \delta^{(SM)} m_h^2 > m_h^2$$

$$\Downarrow$$

the perturbative expansion is breaking down.

- The SM cutoff is very low!

Solutions to the little hierarchy problem:

♠ Suppression of corrections growing with Λ^2 at the 1-loop level:

- The Veltman condition, no Λ^2 terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0 \qquad \Longrightarrow \qquad m_h \simeq 310 \text{ GeV}$$

- SUSY:

$$\delta^{(SUSY)} m_h^2 \sim m_{\tilde{t}}^2 \, \frac{3g_t^2}{8\pi^2} \ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right)$$

then for $\Lambda \sim 10^{16-18}$ GeV one gets $m_{\tilde{t}}^2 \lesssim 1$ TeV^2 in order to have $\delta^{(SUSY)} m_h^2 \sim m_h^2$.

♠ Increase of the allowed value of m_h:

- The inert Higgs model by Barbieri, Hall, Rychkov, Phys.Rev.D74:015007,2006, (Ma, Phys.Rev.D73:077301,2006) $\Rightarrow$ $m_h \sim 400 - 600$ GeV, ($\ln m_h$ terms in T parameter canceled by $m_{H^\pm}, m_A, m_S$ contributions).

The Strategy

- The SM 1-loop quadratic divergences are dominated by the top (fermionic) contribution, so to suppress them we are going to introduce extra scalars to suppress δM_i^2 (as the SM Higgs would need to be too heavy to do the job).

- We will look for a model which allows for relatively heavy lightest Higgs boson H_1 in order to suppress $\delta M_i^2/M_i^2$ even more.

- CPV and DM are desirable.

The natural 2 Higgs Doublet Model

B.G., P. Osland, "A Natural Two-Higgs-Doublet Model", e-Print: arXiv:0910.4068

$$
\begin{aligned}
V(\phi_1, \phi_2) &= -\frac{1}{2}\left\{m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + \left[m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.}\right]\right\} \\
&\quad + \frac{1}{2}\lambda_1(\phi_1^\dagger \phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger \phi_2)^2 + \lambda_3(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\
&\quad + \lambda_4(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger \phi_2)^2 + \text{H.c.}\right]
\end{aligned}
$$

The minimization conditions at $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$ and $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$ can be formulated as follows:

$$
\begin{aligned}
m_{11}^2 &= v_1^2 \lambda_1 + v_2^2(\lambda_{345} - 2\nu), \\
m_{22}^2 &= v_2^2 \lambda_2 + v_1^2(\lambda_{345} - 2\nu),
\end{aligned}
$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \Re\lambda_5$ and $\nu \equiv \Re m_{12}^2/(2v_1 v_2)$.

We assume that ϕ_1 and ϕ_2 couple to down- and up-type quarks, respectively (the so-called 2HDM II).

$$
\mathbb{Z}_2 : \qquad \phi_2 \to -\phi_2
$$

$$\phi_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix}$$

Defining $\eta_3 \equiv -s_\beta \chi_1 + c_\beta \chi_2$ orthogonal to the neutral Goldstone boson $G^0 \equiv c_\beta \chi_1 + s_\beta \chi_2$ one gets 3×3 mass matrix $\mathcal{M}^2$ for neutral scalars (η_1, η_2, η_3) that could be diagonalized by the orthogonal rotation matrix R:

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

and

$$R\mathcal{M}^2 R^T = \mathcal{M}^2_{\rm diag} = {\rm diag}({\rm M}_1^2, {\rm M}_2^2, {\rm M}_3^2)$$

with $M_1 \leq M_2 \leq M_3$.

$$R = \begin{pmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -(c_1 s_2 s_3 + s_1 c_3) & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -(c_1 s_3 + s_1 s_2 c_3) & c_2 c_3 \end{pmatrix}$$

where $s_i \equiv \sin\alpha_i$, $c_i \equiv \cos\alpha_i$ for $i = 1, 2, 3$.

1-Loop Corrections

Cancellation of quadratic divergences for ϕ_1 and ϕ_2 (Newton & Wu, 1994):

$$G^{(1)}_{11} \equiv \frac{\Lambda^2}{v^2}\left[\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) - 3\frac{m_b^2}{c_\beta^2}\right] = 0,$$

$$G^{(1)}_{22} \equiv \frac{\Lambda^2}{v^2}\left[\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) - 3\frac{m_t^2}{s_\beta^2}\right] = 0,$$

where $v^2 \equiv v_1^2 + v_2^2$, $\tan\beta \equiv v_2/v_1$

$$\Downarrow$$

For a given choice of the mixing angles α_i's $(i = 1, 2, 3)$, the neutral-Higgs masses M_1^2, M_2^2 and M_3^2 can be determined from the cancellation conditions in terms of $\tan\beta$, $\mu^2 \equiv \mathbf{Re}(m_{12}^2)/(2s_\beta c_\beta)$ and $M^2_{H^\pm}$.

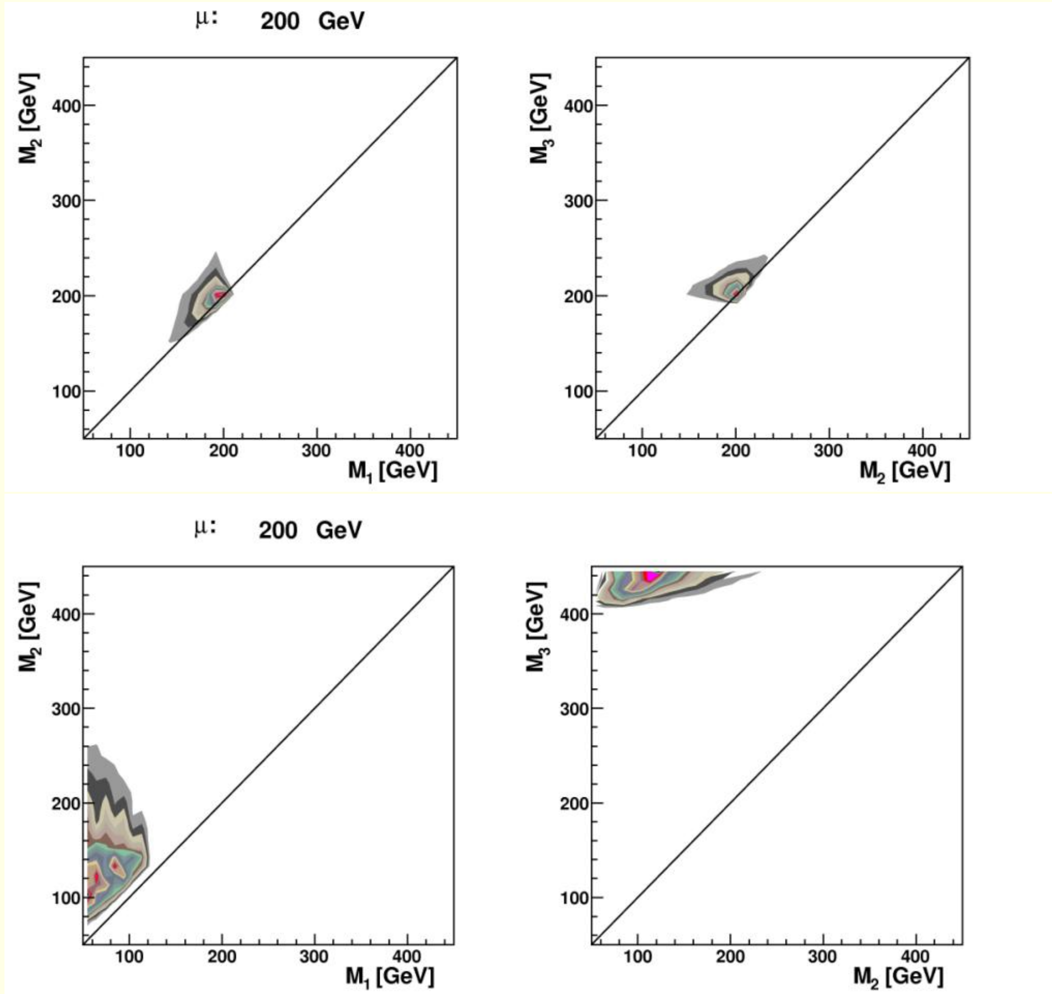

Figure 1: Distributions of allowed masses M_2 vs M_1 (left panels) and M_3 vs M_2 (right) determined at 1 loop, resulting from a scan over the full range of α_i, $\tan\beta \in (0.5, 50)$ and $M_{H^\pm} \in (300, 700)$ GeV, for $\mu = 200$ GeV. No constraints are imposed other than the cancellation of quadratic divergences, $M_i^2 > 0$ and $M_1 < M_2 < M_3$. Two ranges of $\tan\beta$-values are displayed: bottom panels: $0.5 \leq \tan\beta \leq 1$, top panels: $40 \leq \tan\beta \leq 50$. The color coding indicates increasing density of allowed points as one moves inward from the boundary.

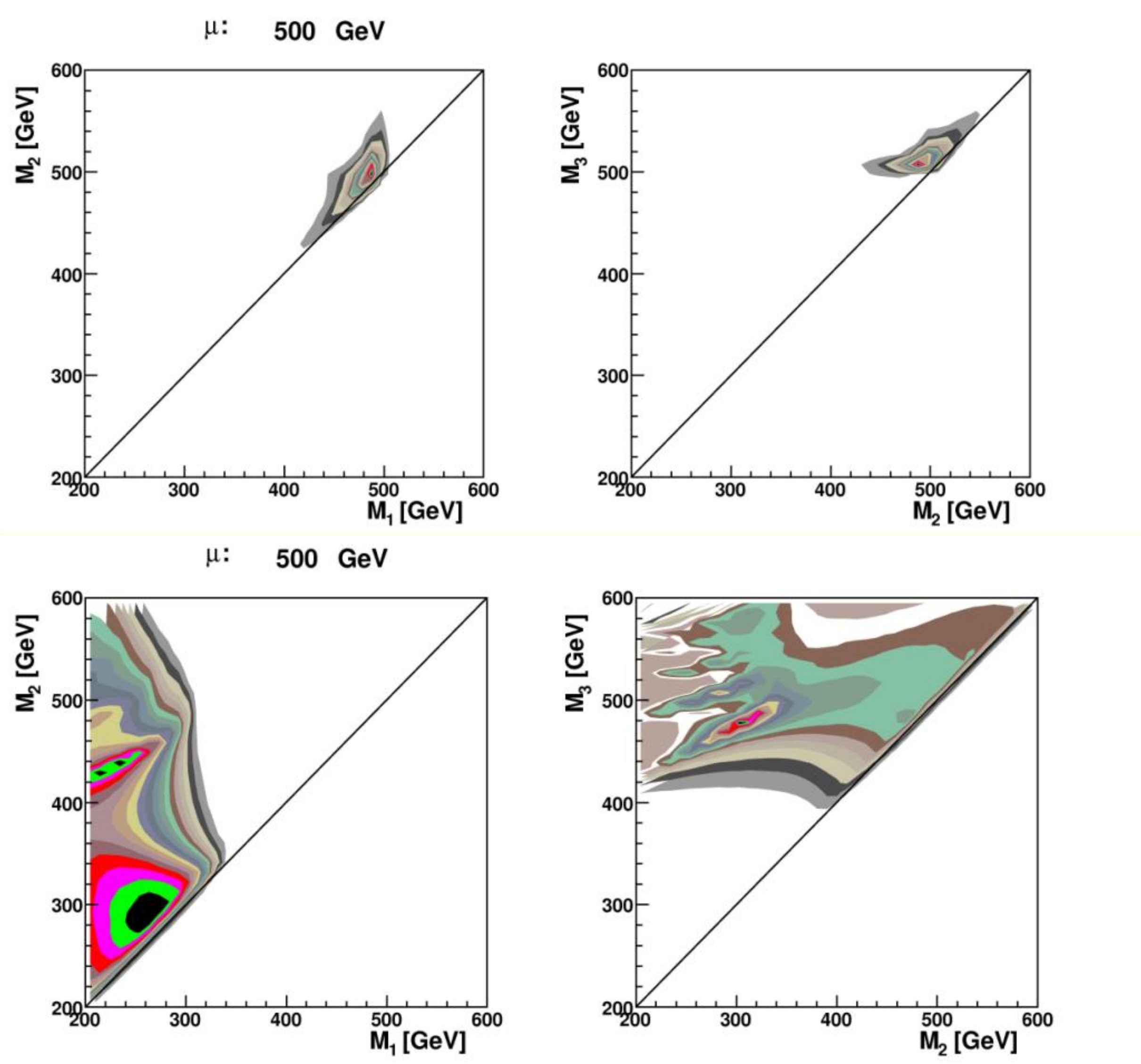

Figure 2: Similar to Fig. 1, for $\mu = 500$ GeV.

$$
\begin{aligned}
M_1^2 - M_2^2 &= \frac{1}{\tan\beta}\frac{R_{33}}{R_{12}R_{22}}\left[-4\bar{m}^2 - 2M_{H^\pm}^2 + 12m_t^2 + \mu^2\right] + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right) \\
M_3^2 &= -\frac{M_1^2 R_{12}R_{13} + M_2^2 R_{22}R_{23}}{R_{32}R_{33}} + \mathcal{O}\left(\frac{1}{\tan\beta}\right).
\end{aligned}
$$

where R_{ij} are elements of the orthogonal rotation matrix for the neutral scalars and $\bar{m}^2 \equiv \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2$.

$$\Downarrow$$

$$\tan\beta \gtrsim 40 \qquad \Longrightarrow \qquad M_1 \simeq M_2 \simeq M_3 \simeq \mu^2 + 4m_b^2$$

2-Loop Leading Corrections

$$G_{11}^{(1)} \equiv \frac{\Lambda^2}{v^2}\left[\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) - 3\frac{m_b^2}{c_\beta^2}\right] = 0,$$

$$G_{22}^{(1)} \equiv \frac{\Lambda^2}{v^2}\left[\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) - 3\frac{m_t^2}{s_\beta^2}\right] = 0,$$

At higher orders (N-loops) leading terms read

$$G_{ii}^{(N)} = \Lambda^2 \sum_{n=0}^{N-1} f_n^{(i)}(\lambda)\,(\ln\Lambda)^n + \cdots\,,$$

where $f_{n-1}^{(i)}$ denotes n-loop results for the ith doublet and λ stands for various couplings that contribute.
Adopting the Einhorn-Jones algorithm one finds at the 2 loop level

$$f_1^{(i)} = \sum_I \frac{\partial f_0^{(i)}(\lambda_I)}{\partial \lambda_I}\beta_I$$

Then the 2-loop conditions for the cancellation of quadratic divergences read:

$$G_{11}^{(1)} + \delta G_{11} = 0$$
$$G_{22}^{(1)} + \delta G_{22} = 0$$

with

$$\delta G_{11} = \frac{v^2}{8}[9g_2\beta_{g_2} + 3g_1\beta_{g_1} + 6\beta_{\lambda_1} + 4\beta_{\lambda_3} + 2\beta_{\lambda_4}] \ln\left(\frac{\Lambda}{\bar{\mu}}\right)$$
$$\delta G_{22} = \frac{v^2}{8}[9g_2\beta_{g_2} + 3g_1\beta_{g_1} + 6\beta_{\lambda_2} + 4\beta_{\lambda_3} + 2\beta_{\lambda_4} - 24g_t\beta_{g_t}] \ln\left(\frac{\Lambda}{\bar{\mu}}\right)$$

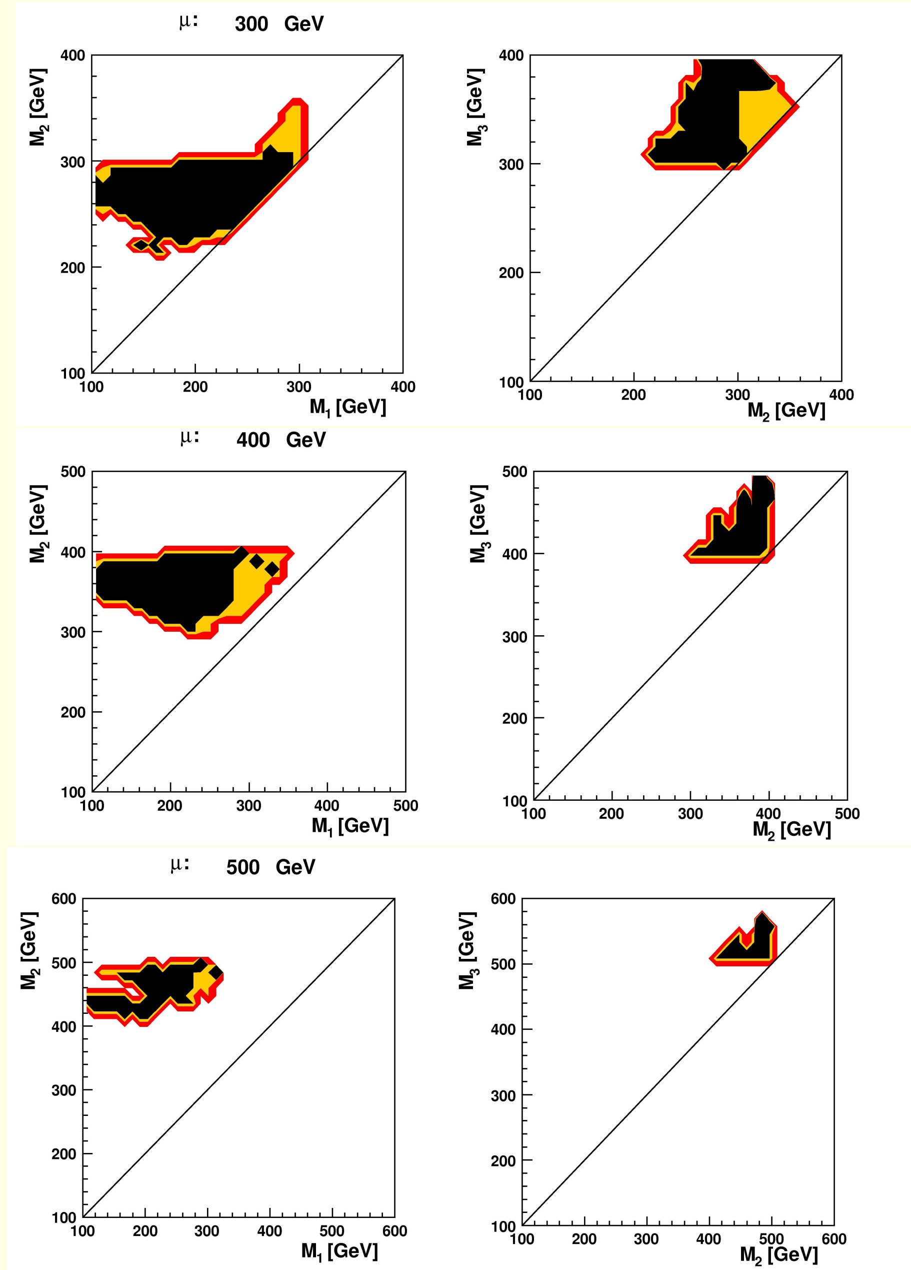

Figure 3: 2-loop masses for $\Lambda = 2.5$ TeV and $\bar{\mu} = v$, scan over α_i, $\tan\beta \in (0.5, 50)$ and $M_{H^\pm} \in (300, 700)$ GeV, for $\mu = 300, 400, 500$ GeV. Red: Positivity is satisfied; yellow: positivity and unitarity satisfied; green: also experimental constraints satisfied.

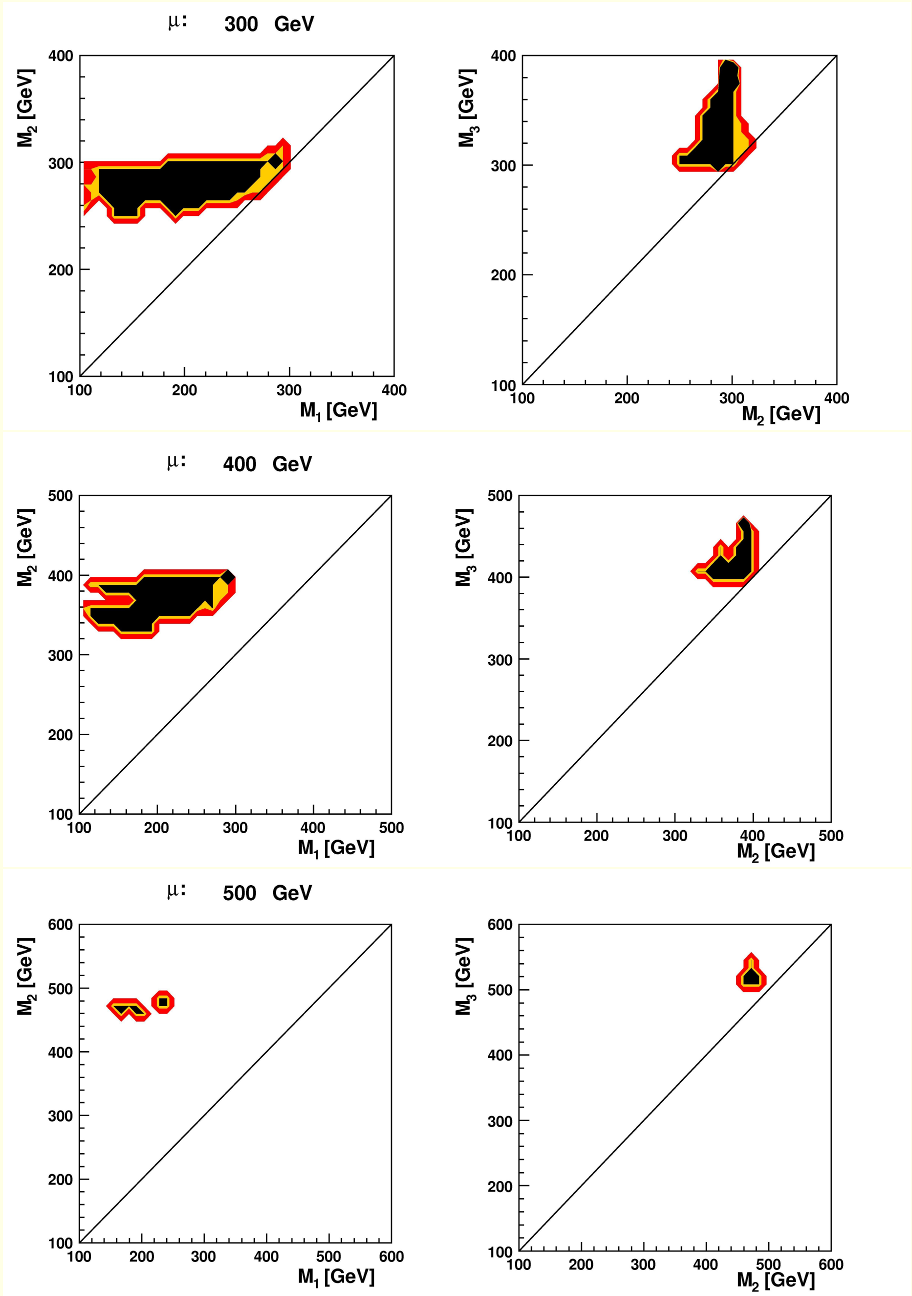

Figure 4: Similar as Fig. 3 for $\Lambda = 6.5$ TeV for $\mu = 300, 400, 500$ GeV.

Advantages:

- No 2-loop (leading) quadratic divergences (so, $\delta M_i^2/M_i^2$ suppressed),
- Large H_1 mass allowed by increased μ (so, $\delta M_i^2/M_i^2$ suppressed),
- A chance for CPV,
- DM candidate easily accommodated by adding singlets φ_i-like.

The following experimental constraints are imposed:

- The oblique parameters T and S
- $B_0 - \bar{B}_0$ mixing
- $B \to X_s \gamma$
- $B \to \tau \bar{\nu}_\tau X$
- $B \to D \tau \bar{\nu}_\tau$
- LEP2 Higgs-boson non-discovery
- R_b
- The muon anomalous magnetic moment
- Electron electric dipole moment

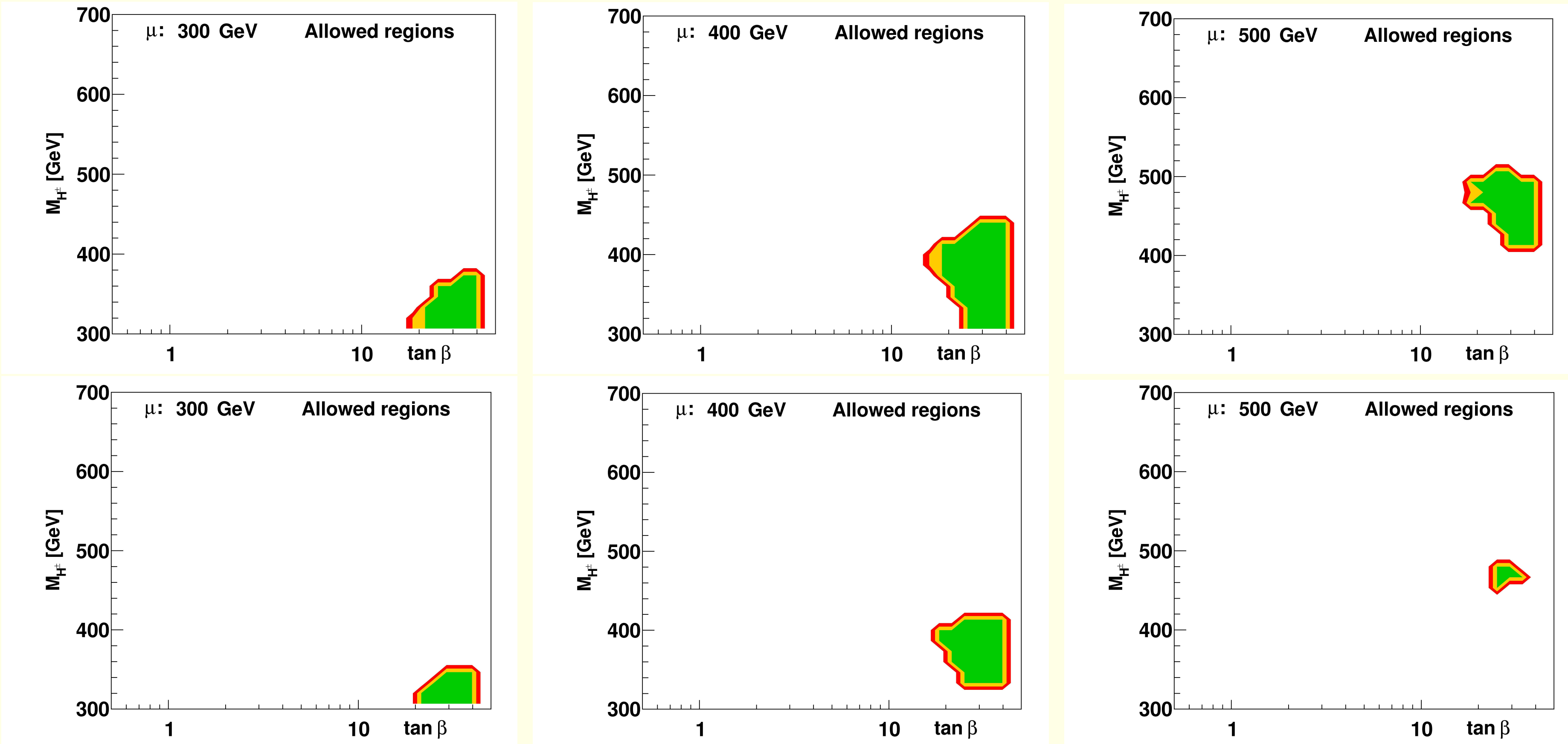

Figure 5: Two-loop allowed regions in the $\tan\beta$–$M_{H^{\pm}}$ plane, for $\Lambda = 2.5$ TeV (top) and $\Lambda = 6.5$ TeV (bottom) with $\bar{\mu} = v$, for $\mu = 300, 400, 500$ GeV (as indicated). Red: positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L..

Violation of CP

$$\begin{aligned}\Im J_1 &= -\frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)\Im\lambda_5,\\ \Im J_2 &= -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_1^4+2(\lambda_1-\lambda_2)\Re\lambda_5 v_1^2 v_2^2\right.\\ &\qquad \left.-\left((\lambda_2-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_2^4\right]\Im\lambda_5,\\ \Im J_3 &= \frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)(\lambda_1+\lambda_2+2\lambda_4)\Im\lambda_5.\end{aligned}$$

For $\tan\beta \gtrsim 40$

$$\Im J_i \sim \frac{\Im\lambda_5}{\tan^2\beta}$$

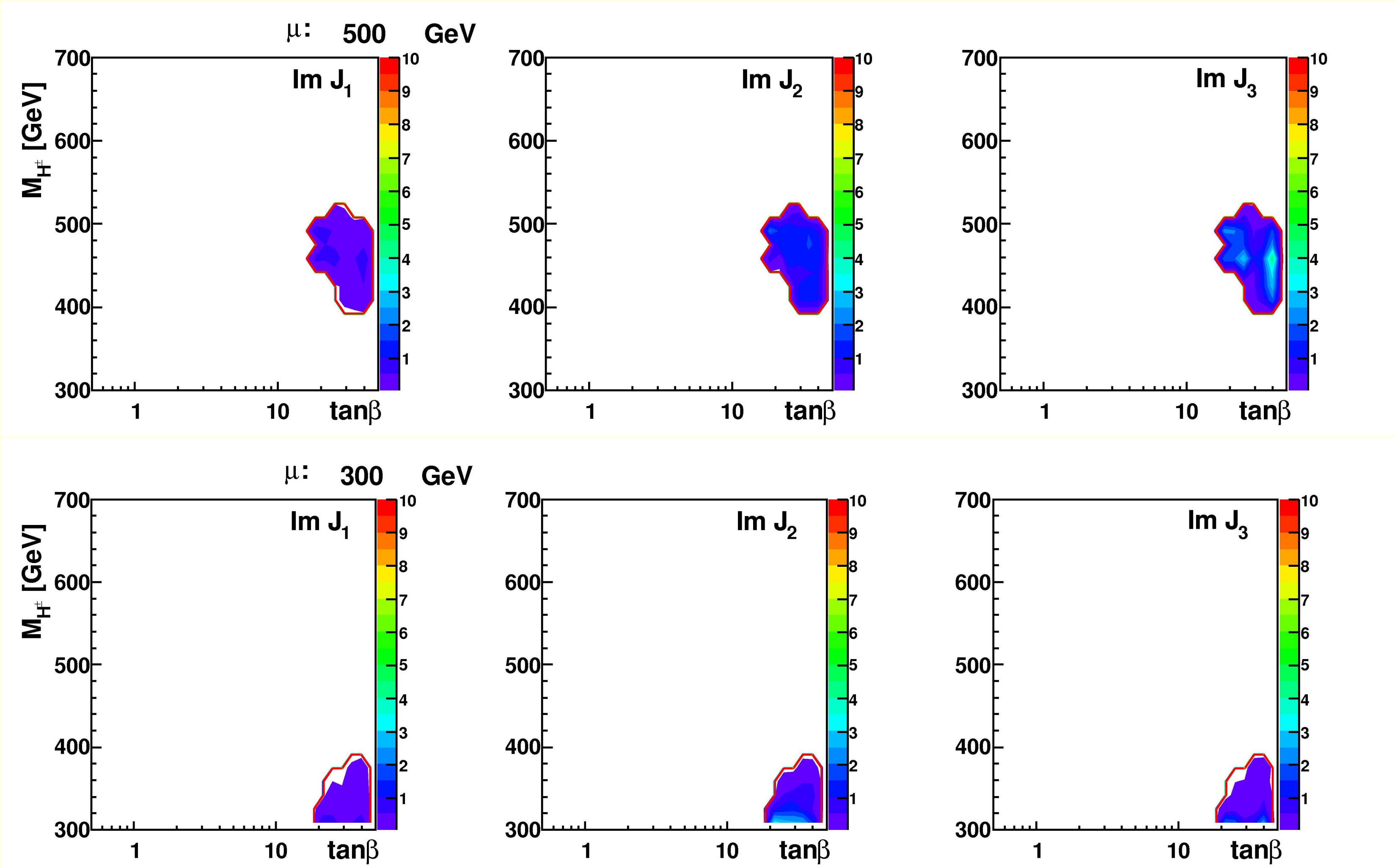

Figure 6: Imaginary parts of the rephasing invariants $|\Im J_i|$, at the 2-loops for $\Lambda = 2.5$ TeV, $\bar{\mu} = v$, $\mu = 500$ GeV (top) and $\mu = 300$ GeV (bottom). The colour coding in units 10^{-3} is given along the right vertical axis.

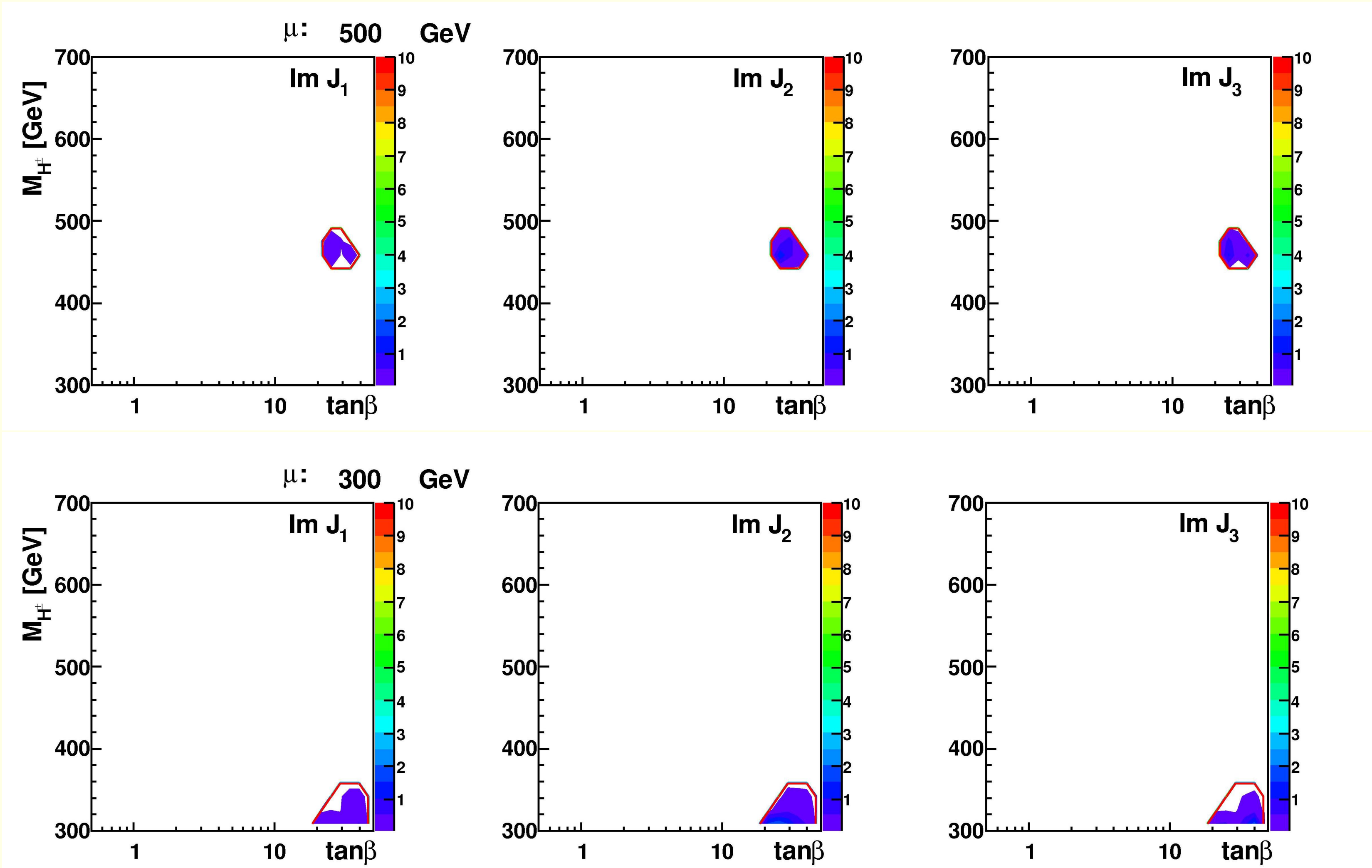

Figure 7: Similar as Fig. 6 for $\Lambda = 6.5$ TeV.

Stability of the cancellation condition

The leading contributions to scalar masses:

$$\delta M_i^2 = \Lambda^2 \sum_{n=0} f_n^{(i)}(\lambda)\,(\ln\Lambda)^n + \cdots\ ,$$

The coefficients $f_n^{(i)}(\lambda)$ can be determined recursively (see Einhorn and Jones), however here a naive estimate is sufficient:

$$f_n^{(i)}(\lambda) \sim \left(\frac{\lambda}{16\pi^2}\right)^{n+1} \sim \left(\frac{4\pi}{16\pi^2}\right)^{n+1} \sim \left(\frac{1}{4\pi}\right)^{n+1}$$

Requiring that the sub-leading ($\propto \Lambda^2 \left[\ln\left(\frac{\Lambda}{v}\right)\right]^0$) 2-loop contribution does not exceed M_1^2 one finds:

$$\Lambda \lesssim 4\pi M_1$$

Then, e.g. for $M_1 = 200(500)$ GeV the cutoff is at $\Lambda \sim 2.5(6.3)$ TeV.

- DM in the Non-Inert Doublet Model with no quadratic divergences

$$
\begin{aligned}
V(\phi_1,\phi_2) &= -\frac{1}{2}\left\{m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \text{H.c.}\right]\right\} \\
&\quad + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\
&\quad + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right] \\
&\quad + \mu_\varphi^2\varphi^2 + \frac{1}{24}\lambda_\varphi\varphi^4 + \varphi^2(\eta_1\phi_1^\dagger\phi_1 + \eta_2\phi_2^\dagger\phi_2)
\end{aligned}
$$

The cancellation conditions:

$$
\begin{aligned}
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{1}{2}\eta_1 + \frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_b^2}{c_\beta^2}, \\
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{1}{2}\eta_2 + \frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_t^2}{s_\beta^2}, \\
\frac{\lambda_\varphi}{2} + 4(\eta_1 + \eta_2) &= 8\mathrm{Tr}\{Y_\varphi Y_\varphi^\dagger\}
\end{aligned}
$$

where $\mathcal{L}_Y = -\varphi\overline{(\nu_R)^c}Y_\varphi\nu_R + \text{H.c.}$.

$$\mathcal{L} = -\varphi^2(\kappa_i v H_i + \lambda_{ij} H_i H_j + \lambda_\pm H^+ H^-)$$

with

$$\kappa_i = \eta_1 R_{i1} c_\beta + \eta_2 R_{i2} s_\beta,$$
$$\lambda_{ij} = \frac{1}{2}\left[\eta_1(R_{i1}R_{j1} + s_\beta^2 R_{i3}R_{j3}) + \eta_2(R_{i2}R_{j2} + c_\beta^2 R_{i3}R_{j3})\right],$$
$$\lambda_\pm = \eta_1 s_\beta^2 + \eta_2 c_\beta^2$$

Assumption: $M_1 \ll M_{2,3}$ so that DM annihilation is dominated by H_1 exchange.

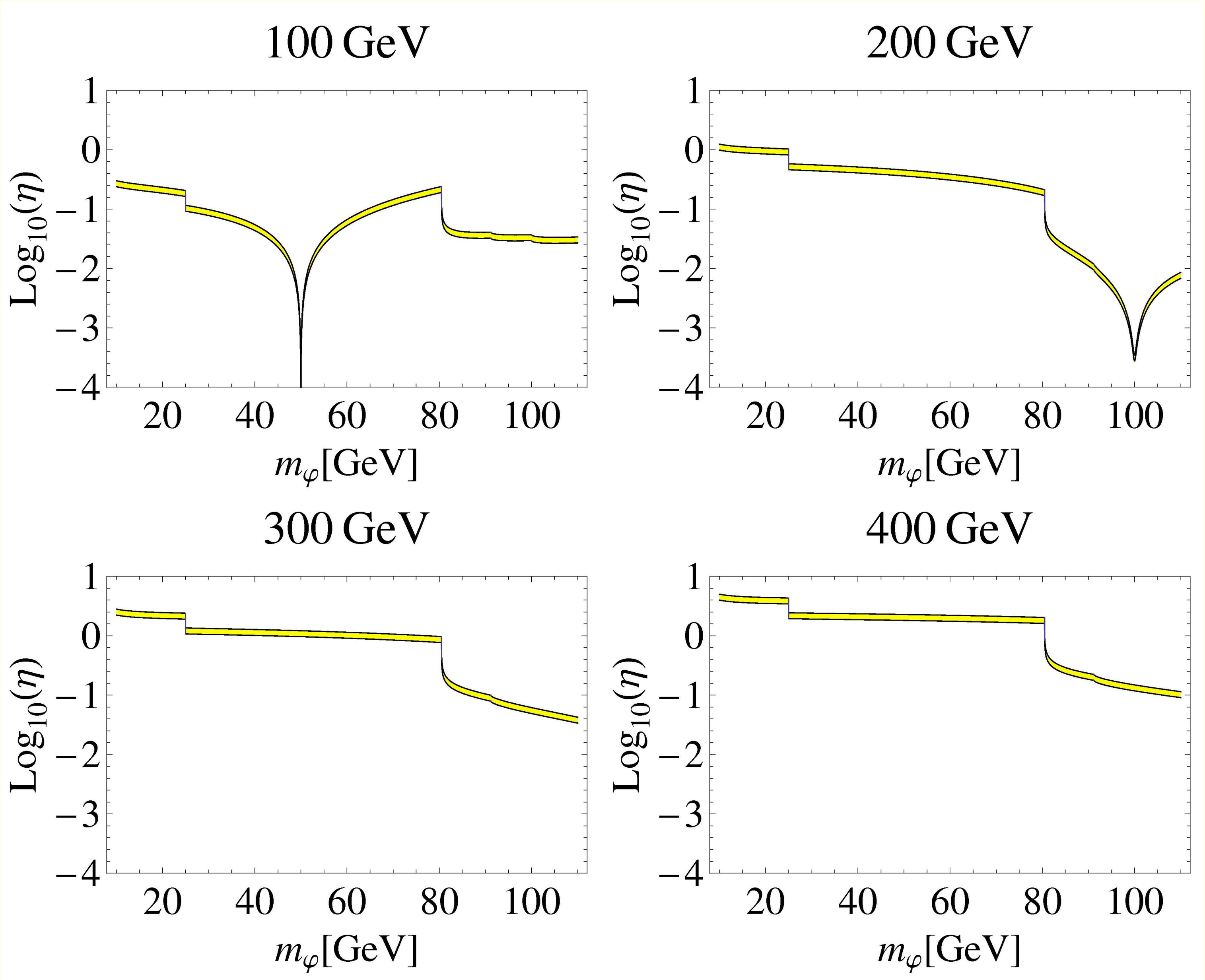

Figure 8: Inert-scalar coupling η (vs m_φ) required by the observed DM abundance $\Omega_{DM}h^2 = 0.106 \pm 0.008$ within a 3-σ band. As indicated above each panel, the lightest Higgs-boson mass ranges from $M_1 = 100$ to 400 GeV . It was assumed that $2\lambda_{11} = \kappa_1 \equiv \eta$.

Summary

- The SM could be easily extended so that the little hierarchy problem is ameliorated, DM candidate is provided and also CP is violated in the extra sector:
  - The addition of N_φ real scalar singlets φ_i to the SM lifts the cutoff Λ to $\sim 4-9$ TeV. It also provides a realistic candidate for DM if $m_\varphi \sim 1-3$ TeV (depending on N_φ).
  - To accommodate CPV in the Higgs potential the SM scalar sector should be replaced by 2 Higgs doublets (non-inert). Cancellation of quadratic divergences could be arranged. Heavy lightest Higgs additionally suppresses $\delta M_i^2/M_i^2$. Adding extra inert scalar singlet or doublet offers a DM candidate.
  - CPV in the Higgs potential with the SM doublet and singlets only?

- Some fine tuning always remains.