Enhancing dark-matter self-interaction by s-channel resonance

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DSU 2016, Bergen, July 26, 2016

Outline

- Strongly self-interacting dark matter
- Motivation: resonance enhancement of dark-matter self-interaction
- Resonance annihilation
- Early kinetic decoupling
- A model of dark gauged U(1) sector
- Resonant self-interaction of vector dark matter
- Summary

⋆ M. Duch, BG, "Enhancing dark-matter self-interaction by s-channel resonance", in progress

⋆ M. Duch, BG, M. McGarrie, "A stable Higgs portal with vector dark matter", JHEP 1509 (2015) 162, arXiv:1506.08805

Strongly self-interacting dark matter

- Core-cusp problem
- Too big to fail problem
- "Missing satellites" problem

$\Rightarrow$

$$0.1\,\frac{\text{cm}^2}{\text{g}} \lesssim \frac{\sigma_{\text{self}}}{M_{\text{DM}}} \lesssim 10\,\frac{\text{cm}^2}{\text{g}}$$

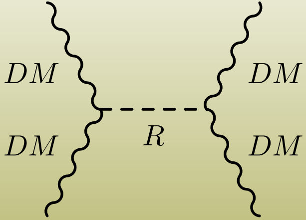
Bullet cluster

$\Rightarrow$

$$\frac{\sigma_{\text{self}}}{M_{\text{DM}}} \lesssim 1\,\frac{\text{cm}^2}{\text{g}}$$

$$0.1\,\frac{\text{cm}^2}{\text{g}} \lesssim \frac{\sigma_{\text{self}}}{M_{\text{DM}}} \lesssim 1\,\frac{\text{cm}^2}{\text{g}} \sim \frac{\text{barn}}{\text{GeV}} \gg \frac{\text{pb}}{\text{GeV}}$$

Resonance enhancement of σ_{self}



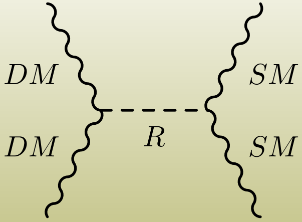
Breit-Wigner resonance ($2M_{DM} \approx M$) *DM self-interaction.*

$$\sigma_{\text{self}} = \frac{32\pi\omega}{s\beta^2} \frac{M^2\Gamma^2 B^2}{(s-M^2)^2 + \Gamma^2 M^2},$$

$$\left.\frac{\sigma_{\text{self}}}{M_{DM}}\right|_{v_{\text{rel}}\approx 0} \simeq \frac{8\pi\omega}{M_{DM}^3} \frac{\eta^2}{\delta^2+\gamma^2}$$

$$\eta \equiv \frac{\Gamma B}{M\beta}, \quad \delta \equiv \frac{4M_{DM}^2}{M^2} - 1, \quad \gamma \equiv \frac{\Gamma}{M} \text{ and } \omega = \frac{(2J+1)}{(2S+1)^2}$$

Resonance annihilation



Breit-Wigner resonance ($2M_{DM} \approx M$) *annihilation.*

$$\sigma v_{\rm rel} = \frac{64\pi\omega}{M^2\beta_i}\frac{\gamma^2}{(\delta + v_{\rm rel}^2/4)^2 + \gamma^2}B_i B_f$$

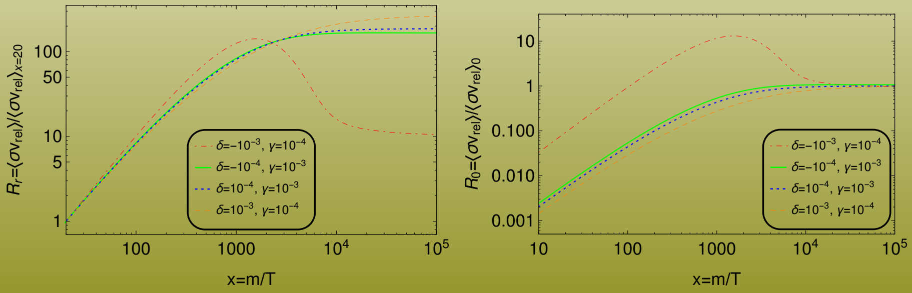
$$\langle \sigma v_{\rm rel}\rangle(x) = \frac{x^{3/2}}{2\sqrt{\pi}}\int_0^\infty dv v^2 e^{-xv^2/4}\sigma v$$

P. Gondolo and G. Gelmini, Nucl. Phys. B 360, 145 (1991),
K. Griest and D. Seckel, Phys. Rev. D 43, 3191 (1991),
M. Ibe, H. Murayama and T. T. Yanagida, Phys. Rev. D 79, 095009 (2009)

Resonance annihilation

$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x)(Y^2 - Y_{EQ}^2)$$

$$R(x) = \frac{\langle \sigma v_{\rm rel}\rangle(x)}{\langle \sigma v_{\rm rel}\rangle_0} = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 e^{-xv^2/4} \frac{\delta^2+\gamma^2}{(\delta+v^2/4)^2+\gamma^2}$$



Thermally averaged annihilation cross section $\langle \sigma v_{\rm rel}\rangle$ normalized to its value at decoupling $\langle \sigma v_{\rm rel}\rangle_{x=20}$ (left panel) and to the low-temperature limit $\langle \sigma v_{\rm rel}\rangle_0$ (right panel).

Resonance annihilation

$$\frac{1}{Y_\infty} \approx \lambda_0 \int_0^\infty dvv \frac{\delta^2 + \gamma^2}{(\delta + v^2/4)^2 + \gamma^2} \mathsf{erfc}\left(\frac{v\sqrt{x_d}}{2}\right)$$

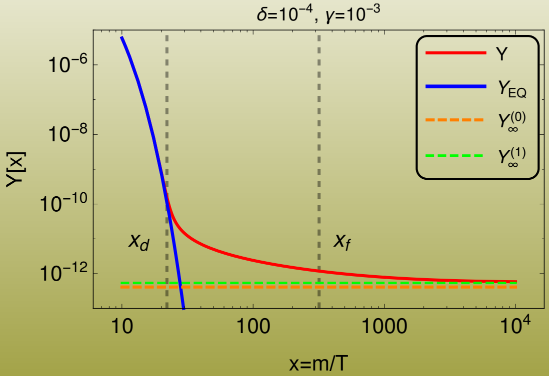
$$\mathsf{erfc}(\epsilon) = \frac{2}{\sqrt{\pi}} \int_\epsilon^\infty e^{-t^2} dt \approx 1 - \frac{2\epsilon}{\sqrt{\pi}} + O(\epsilon^2)$$

$$\frac{1}{Y_\infty^{(0)}} \approx \lambda_0 (\delta^2 + \gamma^2) \frac{\pi - 2\arctan(\delta/\gamma)}{\gamma} \approx \lambda_0 \times \begin{cases} \pi\gamma, & \text{if } \gamma \gg |\delta| \\ 2\delta, & \text{if } \delta \gg \gamma \\ 2\pi\delta^2/\gamma, & \text{if } -\delta \gg \gamma \end{cases}$$

$$\frac{1}{Y_\infty^{(1)}} \approx \lambda_0 \times \begin{cases} \gamma(\pi - 2\sqrt{2\pi x_d \gamma}), & \text{if } \gamma \gg |\delta| \\ \delta(2 - 2\sqrt{\pi x_d \delta}), & \text{if } \delta \gg \gamma \\ \delta^2 \gamma^{-1}(2\pi - 4\sqrt{\pi x_d |\delta|}), & \text{if } -\delta \gg \gamma \end{cases}$$

with $Y(x_d) - Y_{EQ}(x_d) = cY_{EQ}(x_d)$

Resonance annihilation



Evolution of the dark matter yield $Y(x)$ for a wide resonance in unphysical region and a narrow resonance.

Resonance annihilation

$$\frac{1}{Y_\infty} \equiv \frac{\lambda_0}{x_f}$$

$$x_f \approx \left[(\delta^2+\gamma^2)\frac{\pi - 2\arctan(\delta/\gamma)}{\gamma}\right]^{-1} \approx \begin{cases} (\pi\gamma)^{-1}, & \text{if } \gamma \gg |\delta| \\ (2\delta)^{-1}, & \text{if } \delta \gg \gamma \\ \gamma(2\pi\delta^2)^{-1}, & \text{if } -\delta \gg \gamma \end{cases}$$

$$\Omega h^2 = 2.74 \times 10^8 \frac{M_{DM}}{\text{GeV}} Y_\infty = 0.99 \times 10^{-27}\ \text{cm}^3\ \text{s}^{-1} \frac{x_f}{\sqrt{g_*}\langle \sigma v_{\rm rel}\rangle_0}$$

$$\langle \sigma v_{\rm rel}\rangle_0 \approx \frac{x_f}{25}\left(\frac{100}{g_*}\right)^{1/2}\left(\frac{0.12}{\Omega h^2}\right) 2 \times 10^{-26}\ \text{cm}^3\ \text{s}^{-1}$$

$$x_f \sim \frac{1}{\text{max}[|\delta|, \gamma]} \gg x_{f\,\text{WIMP}} \sim 25 \Rightarrow \langle \sigma v_{\rm rel}\rangle_0 \gg \langle \sigma v_{\rm rel}\rangle_{0\,\text{WIMP}}$$

Resonance annihilation

- parameters: $\langle \sigma v_{\rm rel} \rangle_0$ (present annihilation), η (resonance DM coupling), δ (resonance location), γ (resonance width)
- constraints: Ωh^2, $\sigma_{\textsf{self}}/M_{DM}$ and Fermi-LAT upper limits on $\langle \sigma v_{\rm rel} \rangle_0$

the goal: minimize $\langle \sigma v_{\rm rel} \rangle_0$ for a given (large) $\frac{\sigma_{\rm self}}{M_{DM}} \approx \frac{8\pi\omega}{M_{DM}^3} \frac{\eta^2}{\delta^2+\gamma^2}$

$$\frac{\langle \sigma v_{\rm rel} \rangle_0}{2 \times 10^{-26}\ \textsf{cm}^3\ \textsf{s}^{-1}} \gtrsim \frac{560}{\xi \eta \sqrt{\omega}} \left(\frac{M_{DM}}{100\ \textsf{GeV}} \right)^{3/2} \left(\frac{\sigma_{\rm self}/M_{DM}}{1\ \textsf{cm}^2/\textsf{g}} \right)^{1/2} \times \left(\frac{100}{g_*} \right)^{1/2} \left(\frac{0.12}{\Omega h^2} \right)$$

where $2 \leq \xi \leq \pi$ and

$$\eta \equiv \frac{\Gamma\, B(R \to DM\ DM)}{M \beta} \quad \text{and} \quad \omega = \frac{(2J+1)}{(2S+1)^2}$$

Early kinetic decoupling

Dark matter annihilation rate is enhanced by the resonance, therefore coupling of the mediator to the SM particles needs to be suppressed in order to be consistent with the observed abundance.

$\Downarrow$

Temperature of the kinetic decoupling T_{kd} can be, in the resonant case, higher than in the typical WIMP scenario.

$\Downarrow$

If dark matter decouples kinetically when it is non-relativistic, then the DM temperature T_{DM} evolves according to $T_{DM} \propto R^{-2}$, contrary to the radiation-dominated SM thermal bath, for which $T_{SM} \propto R^{-1}$ (X. Chen, M. Kamionkowski and X. Zhang, Phys. Rev. D 64, 021302 (2001))

$$T_{DM} = \begin{cases} T_{SM}, & \text{if } T \geq T_{kd} \\ T_{SM}^2/T_{kd}, & \text{if } T < T_{kd}. \end{cases}$$

$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x_{DM})(Y^2 - Y_{EQ}^2) \ \text{ with } \ x_{DM} = \frac{x^2}{x_{kd}}$$

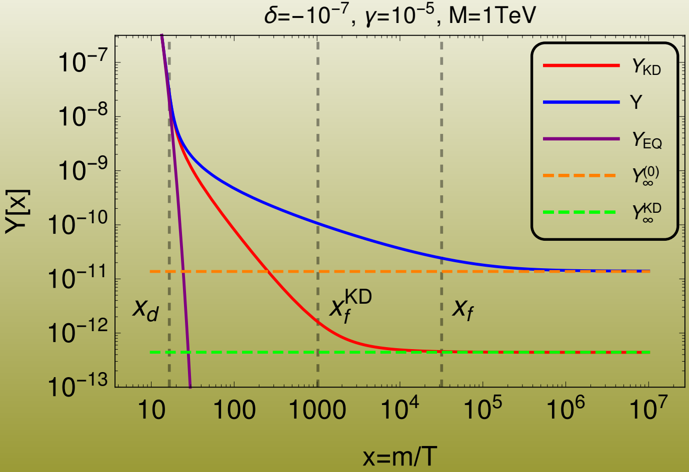
Early kinetic decoupling

$$\frac{1}{Y_\infty} \approx \lambda_0 \int_0^\infty dv \frac{\delta^2+\gamma^2}{(\delta+v^2/4)^2+\gamma^2} \frac{\exp(-x_{kd}v^2/4)}{\sqrt{\pi x_{kd}}}$$

$$\frac{1}{Y_\infty^{(0)}} \approx \lambda_0 \sqrt{\frac{\pi(\delta^2+\gamma^2)(-\delta+\sqrt{\delta^2+\gamma^2})}{2x_{kd}\gamma^2}} \approx \lambda_0 \begin{cases} \left(\frac{\pi\gamma}{2x_{kd}}\right)^{1/2}, & \text{if } \gamma \gg |\delta| \\ \frac{1}{2}\left(\frac{\pi\delta}{x_{kd}}\right)^{1/2}, & \text{if } \delta \gg \gamma \\ \frac{\sqrt{\pi}}{\gamma\sqrt{x_{kd}}}|\delta|^{3/2}, & \text{if } -\delta \gg \gamma \end{cases}$$

$$\frac{1}{Y_\infty^{(1)}} \approx \frac{1}{Y_\infty^{(0)}} - \lambda_0 \begin{cases} \left(\frac{\pi x_{kd}\gamma^3}{2}\right)^{1/2}, & \text{if } \gamma \gg |\delta| \\ \frac{1}{2}(\pi x_{kd}\delta^3)^{1/2}, & \text{if } \delta \gg \gamma \\ \frac{(\pi x_{kd}|\delta|^5)^{1/2}}{\gamma}, & \text{if } -\delta \gg \gamma \end{cases}$$

Early kinetic decoupling



Evolution of dark matter yield $Y(x)$ for dark matter in thermal equilibrium with the SM (blue curve) and in the case of simultaneous chemical and kinetic decoupling $x_{kd} = x_d$ (red curve).

Early kinetic decoupling

$$H(T_{kd}) \sim \Gamma_{\rm scat}(T_{kd}) \Rightarrow x_{kd} \lesssim \left(\frac{\max[\delta,\gamma]^{3/2}}{10^{-6}}\right)^{\frac{1}{4}} \Rightarrow x_{kd} \lesssim 1$$

$$\frac{\langle \sigma v_{\rm rel}\rangle_0}{2\times 10^{-26}\ \mathsf{cm}^3\ \mathsf{s}^{-1}} \gtrsim \frac{3.8}{\sqrt{\eta\sqrt{\omega}}}\left(\frac{M_{DM}}{100\ \mathsf{GeV}}\right)^{3/4}\left(\frac{\sigma_{\rm self}/M_{DM}}{1\ \mathsf{cm}^2/\mathrm{g}}\right)^{1/4} \times \left(\frac{100}{g_*}\right)^{1/2}\left(\frac{0.12}{\Omega h^2}\right)$$

where

$$\eta \equiv \frac{\Gamma\, B(R\to DM\ DM)}{M\beta} \quad \text{and} \quad \omega = \frac{(2J+1)}{(2S+1)^2}$$

A model of dark gauged U(1) sector

The model:

- extra $U(1)$ gauge symmetry (A_X^μ),
- a complex scalar field S, whose vev generates a mass for the $U(1)$'s vector field, $S = (0, \mathbf{1}, \mathbf{1}, 1)$ under $U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)$
- SM fields neutral under $U(1)$,
- to ensure stability of the new vector boson, a $\mathbb{Z}_2$ symmetry is assumed to forbid $U(1)$-kinetic mixing between $U(1)$ and $U(1)_Y$. The extra gauge boson A_X^μ and the scalar S field transform under $\mathbb{Z}_2$ as follows

$$A_X^\mu \to -A_X^\mu \; , \; S \to S^*, \text{ where } S = \phi e^{i\sigma}, \;\; \text{so} \;\; \phi \to \phi, \;\; \sigma \to -\sigma.$$

T. Hambye, "Hidden vector dark matter", JHEP 0901 (2009) 028,
O. Lebedev, H. M. Lee, and Y. Mambrini, "Vector Higgs-portal dark matter and the invisible Higgs", Phys.Lett. B707 (2012) 570, $\cdots$

A model of dark gauged U(1) sector

The scalar potential

$$V = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|S|^2|H|^2.$$

The vector bosons masses:

$$M_W = \frac{1}{2}gv, \qquad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v \quad \text{and} \quad M_{Z'} = g_x v_x,$$

where

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{ and } \langle S \rangle = \frac{v_x}{\sqrt{2}}$$

Positivity of the potential implies

$$\lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2\sqrt{\lambda_H \lambda_S}$$

A model of dark gauged U(1) sector

The mass squared matrix $\mathcal{M}^2$ for the fluctuations (ϕ_H, ϕ_S) and their eigenvalues

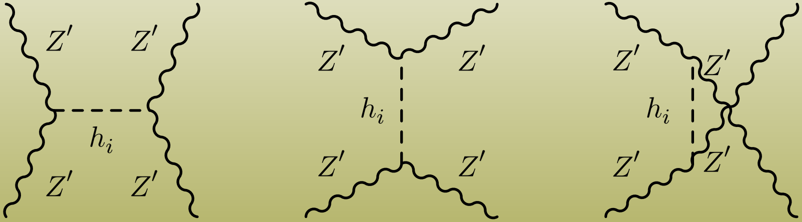
$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_x \\ \kappa v v_x & 2\lambda_S v_x^2 \end{pmatrix}$$

$$M_\pm^2 = \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H \lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}$$

$$\mathcal{M}^2_{\mathsf{diag}} = \begin{pmatrix} M_{h_1}^2 & 0 \\ 0 & M_{h_2}^2 \end{pmatrix}, \quad R = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R^{-1} \begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix}$$

Resonant self-interaction of vector dark matter



Z' self-interaction in different channels.

$$\frac{\sigma_{\rm self}}{M_{Z'}} = g_x^4 \frac{M_{Z'}}{8\pi} \frac{R_{2i}^4}{(4M_{Z'}^2 - M_{h_i}^2)^2 + \Gamma_{h_i}^2 M_{h_i}^2},$$

Resonant self-interaction of vector dark matter

Minimal $\langle \sigma v_{\rm rel} \rangle_0$ in the VDM

$$\eta = \frac{\Gamma_{h_2}}{M_{h_2}} \frac{1}{\bar{\beta}} \lesssim \frac{3}{16}$$

$$\Downarrow$$

$$\frac{\langle \sigma v_{\rm rel} \rangle_0}{2 \times 10^{-26}\ \mathsf{cm}^3\ \mathsf{s}^{-1}} \gtrsim \frac{9 \cdot 10^3}{\xi} \left(\frac{M_{Z'}}{100\ \mathsf{GeV}} \right)^{3/2} \left(\frac{\sigma_{\rm self}/M_{Z'}}{1\ \mathsf{cm}^2/\mathrm{g}} \right)^{1/2} \cdot \left(\frac{100}{g_*} \right)^{1/2} \left(\frac{0.12}{\Omega h^2} \right)$$

where $2 \leq \xi \leq \pi$.

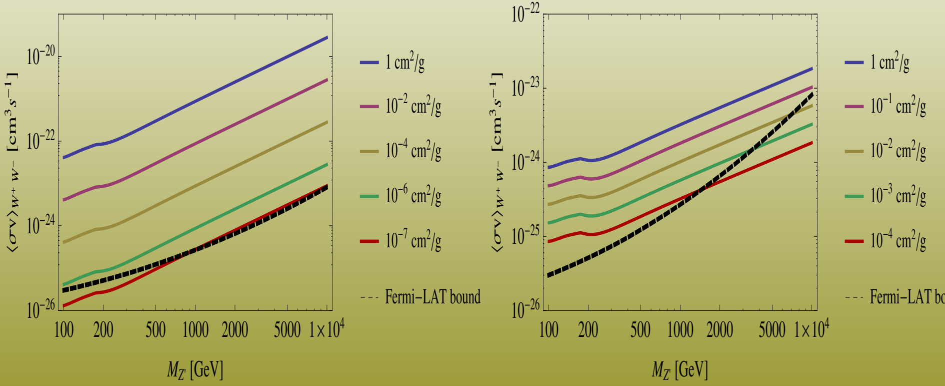
Resonant self-interaction of vector dark matter

With early kinetic decoupling

$$\frac{\langle \sigma v_{\rm rel}\rangle_0}{2\times 10^{-26}\ \mathsf{cm}^3\ \mathsf{s}^{-1}} \gtrsim 15\cdot x_{kd}^{1/2}\left(\frac{M_{Z'}}{100\ \mathsf{GeV}}\right)^{3/4}\left(\frac{\sigma_{\rm self}/M_{Z'}}{1\ \mathsf{cm}^2/\mathrm{g}}\right)^{1/4} \cdot \left(\frac{100}{g_*}\right)^{1/2}\left(\frac{0.12}{\Omega h^2}\right)$$

with $x_{kd} \sim 10 - 20$

Resonant self-interaction of vector dark matter



Dark matter annihilation cross-section in the W^+W^- channel consistent with Ωh^2 and desired $\sigma_{\text{self}}/M_{Z'}$ specified in the legend. Left panel does not take into account the early kinetic decoupling while the right one does for $x_{kd} = 15$.

Summary

- A possibility of enhancing the dark-matter self-interaction cross-section ($\sigma_{\rm self}/M_{DM}$) by s-channel resonance was considered in a model independent way.

- Dark matter annihilation in the vicinity of a resonance was discussed in details. Approximate analytical and exact numerical solutions of the Boltzmann equation were found and analyzed. A possibility of early kinetic decoupling of dark matter was considered, its effects turned out to be crucial for consistency with experimental constraints on annihilation cross section $\langle\sigma v_{\rm rel}\rangle_0$ from indirect dark matter searches.

- The general results were illustrated by $U(1)$ vector-boson dark matter model, which, besides the extra (dark) gauge boson, contains a second neutral Higgs boson h_2.

- For a given $\sigma_{\rm self}/M_{DM}$ a lower limit for the annihilation cross-section $\langle\sigma v_{\rm rel}\rangle_0$ has been derived. The self-interaction cross section $\sigma_{\text{self}}/M_{Z'}$ of the order of $(10^{-2} - 10^{-1})$ cm$^2/g$ could be achieved if dark matter is heavy enough, $M_{Z'} \sim 10^4$ GeV.