Cosmology in the Presence of Unparticles

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- Unparticles
- The equation of state - hand-waving arguments
- Freeze-out and thaw-in
- BBN constraints
- Summary

Based on a work being done with J. Wudka.

Unparticles

H. Georgi, PRL **98**, 221601 (2007)

$$\underbrace{\text{SM} \quad \otimes \quad \text{CFT}(\mathcal{BZ})}$$

$$\Downarrow$$

$$\frac{\mathcal{O}_{\text{SM}}\,\mathcal{O}_{\mathcal{BZ}}}{M_{\mathcal{U}}^k}$$

$\mathcal{BZ}$: $SU(3)$, $\quad n_f$ fermions, IR fixed point for $8 \lesssim n_f \lesssim 16$

$$\Downarrow$$

Dimensional transmutation in the BZ sector – unparticles emerge for $\mu \lesssim \Lambda_{\mathcal{U}}$

$$\Downarrow$$

$c_{\mathcal{U}}\frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}\mathcal{O}_{\text{SM}}\mathcal{O}_{\mathcal{U}}$ for $k = d_{\text{SM}} + d_{\mathcal{BZ}} - 4$, scale invariance $\rightarrow$ unparticles

$$\langle 0|\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle = \int \frac{d^4p}{(2\pi)^2} e^{-ipx}\rho_{\mathcal{U}}(p^2)$$

- Scaling:

$$\mathcal{O}_{\mathcal{U}}(x) \to \mathcal{O}'_{\mathcal{U}}(x') = s^{-d_{\mathcal{U}}}\mathcal{O}_{\mathcal{U}}(x) \quad \text{for} \quad x \to x' = sx$$

- The spectral density:

$$\rho_{\mathcal{U}}(p^2) = \int d^4x \, e^{ipx}\langle 0|\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle \quad \Rightarrow \quad \rho_{\mathcal{U}}(p^2) = A_{d_{\mathcal{U}}}\theta(p^0)\theta(p^2)(p^2)^{d_{\mathcal{U}}-2}$$

- The phase space:

$$d\Phi_{\mathcal{U}}(p_{\mathcal{U}}) = A_{d_{\mathcal{U}}}\theta(p^0)\theta(p^2_{\mathcal{U}})(p^2_{\mathcal{U}})^{d_{\mathcal{U}}-2}\frac{d^4p_{\mathcal{U}}}{(2\pi)^4} \quad \text{with} \quad A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+\frac{1}{2})}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}$$

- Unparticles behave as a collection of $d_{\mathcal{U}}$ massless particles $\Rightarrow$ continuous spectrum in $t \to u\mathcal{O}_{\mathcal{U}}$.

The equation of state - hand-waving arguments

The trace anomaly of the energy momentum tensor for a gauge theory with massless fermions:

$$\theta^\mu_\mu = \frac{\beta}{2g} N\left[F^{\mu\nu}_a F_{a\ \mu\nu}\right] \tag{1}$$

where β denotes the beta function and N stands for the normal product.

Non-trivial IR fixed point at $g = g_\star$, so in the IR we assume

$$\beta = \gamma(g - g_\star), \quad \gamma > 0$$

in which case the running coupling reads

$$g(\mu) = g_\star + c\mu^\gamma; \qquad \beta[g(\mu)] = \gamma c \mu^\gamma$$

where c is an integration constant and μ is the renormalization scale.

From the thermal average of (1) choosing the renormalization scale $\mu = T$ and using $\langle \theta^\mu_\mu \rangle = \rho_{\mathcal{U}} - 3p_{\mathcal{U}}$, we get

$$\rho_{\mathcal{U}} - 3p_{\mathcal{U}} = \frac{\beta}{2g_\star} \langle N\left[F^{\mu\nu}_a F_{a\ \mu\nu}\right]\rangle = AT^{4+\gamma}$$

$$\rho_{\mathcal{U}} - 3p_{\mathcal{U}} = AT^{4+\gamma}$$

$$\Downarrow$$

$$\rho_{\mathcal{U}} = \sigma T^4 + A\left(1 + \frac{3}{\gamma}\right) T^{4+\gamma} \quad \text{and} \quad p_{\mathcal{U}} = \sigma \frac{T^4}{3} + \frac{A}{\gamma} T^{4+\gamma}$$

where σ is an integration constant.

$$\Downarrow$$

$$\boxed{p_{\mathcal{U}} = \tfrac{1}{3}\rho_{\mathcal{U}}\left(1 - B\rho_{\mathcal{U}}^{\gamma/4}\right) \quad \text{for} \quad B \equiv \tfrac{A}{\sigma^{1+\gamma/4}}}$$

One can expect that $A \propto \Lambda_{\mathcal{U}}^{-\gamma}$, therefore we obtain

$$\boxed{\rho_{\mathsf{NP}} = \tfrac{\pi^2}{30} T^4 \times \begin{cases} g_{\mathsf{IR}} + f\left(\tfrac{T}{\Lambda_{\mathcal{U}}}\right)^{\gamma} & \text{for} \quad T \ll \Lambda_{\mathcal{U}} \\ g_{\mathcal{BZ}} & \text{for} \quad T \gg \Lambda_{\mathcal{U}} \end{cases}}$$

where $g_{\mathcal{BZ}} = 2(n_c^2 - 1 + \frac{7}{8} n_c n_f)$ for $SU(n_c)$ with n_f flavours in the $\mathcal{BZ}$ sector.

- From the continuity at $T = \Lambda_{\mathcal{U}}$, the constant f could be determined: $f = g_{\mathcal{BZ}} - g_{\mathsf{IR}}$.

- We will assume $g_{\mathcal{BZ}} \sim g_{\mathsf{IR}}$.

$$\boxed{\rho_{\mathsf{NP}} = \frac{\pi^2}{30} T^4 \times \begin{cases} g_{\mathsf{IR}} + f\left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{\gamma} & \textsf{for} \quad T \ll \Lambda_{\mathcal{U}} \\ g_{\mathcal{BZ}} & \textsf{for} \quad T \gg \Lambda_{\mathcal{U}} \end{cases}}$$

Deconstruction (Stephanov'07):

$$\mathcal{O}_{\mathcal{U}} \to \sum_{n=1}^{\infty} F_n \varphi_n \quad \text{with} \quad m_n^2 = \Delta^2 n$$

The above result fits the following guess for the effective number of degrees of freedom:

$$g_{\mathcal{U}}(T) \propto \frac{\int_0^{T^2} dM^2 \rho(M^2) \theta(\Lambda_{\mathcal{U}}^2 - M^2)}{\int_0^{\Lambda_{\mathcal{U}}^2} dM^2 \rho(M^2)}$$

where $\rho(M^2) \propto (M^2)^{(d_{\mathcal{U}}-2)}$. Then

$$g_{\mathcal{U}}(T) \propto \left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{2(d_{\mathcal{U}}-1)}$$

$\Longrightarrow$ In the presence of just one unparticle operator one can argue that $\gamma = 2(d_{\mathcal{U}} - 1)$.

Freeze-out and thaw-in

♣ *Brief history of the Universe in the presence of unparticles (no mass-gap).*

- $T \gg M_{\mathcal{U}}$: the $\mathcal{BZ}$ sector in form of massless particles (no unparticles yet), thermal equilibrium with the SM is maintained (assumption), so $T = T_{\mathcal{BZ}} = T_{\mathsf{SM}}$

- $T \lesssim M_{\mathcal{U}}$:
  - The $\mathcal{BZ}$ sector starts to decouple, as the average energy is no longer sufficient to create mediators.
  - However, the thermal equilibrium may still be maintained ($T = T_{\mathcal{BZ}} = T_{\mathsf{SM}}$) depending on the strength of effective couplings between the SM and the extra sector (which at higher temperature, $T \gtrsim \Lambda_{\mathcal{U}}$, is made of the $\mathcal{BZ}$ matter, while below $\Lambda_{\mathcal{U}}$ of unparticles).

Let's denote by T_f the decoupling temperature at which

$$\Gamma(SM \leftrightarrow NP) \simeq H$$

where H is the Hubble parameter

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho_{\mathsf{tot}}(T) \quad \text{for} \quad \rho_{\mathsf{tot}} = \rho_{\mathsf{SM}} + \rho_{\mathsf{NP}}$$

There are 2 interesting cases:

- $M_{\mathcal{U}} > T_f > \Lambda_{\mathcal{U}}$:
  - T_f is determined by the condition

$$\Gamma(SM \leftrightarrow \mathcal{BZ}) \simeq H$$

  - For $T > T_f$ the SM and the $\mathcal{BZ}$ sectors evolve in thermal equilibrium, but even for $T < T_f$ their temperatures remain equal ($T = T_{\mathcal{BZ}} = T_{\mathsf{SM}}$) since $\Lambda_{\mathcal{U}} > v$.

- $\Lambda_{\mathcal{U}} > T_f$:
  - Till $T = \Lambda_{\mathcal{U}}$ the SM and unparticles still have the same temperature.
  - For $\Lambda_{\mathcal{U}} \gtrsim T \gtrsim T_f$ still the equilibrium is maintained (assumption, in general this depends on $d_{\mathcal{U}}$). The decoupling temperature T_f must be now determined by

$$\Gamma(SM \leftrightarrow \mathcal{O}_{\mathcal{U}}) \simeq H$$

  - Till $T \sim v$ temperatures of SM and unparticles remain equal, at $T \sim v$ they split.

$\Longrightarrow$ The unparticle cosmic background should be there.

♣ *The Banks-Zaks phase.*

$$\mathcal{L}_{\mathcal{BZ}} = \frac{1}{M_{\mathcal{U}}} \left(\phi^\dagger \phi\right) \left(\bar{q}_{\mathcal{BZ}} q_{\mathcal{BZ}}\right)$$

Then

$$\Gamma_{\mathcal{BZ}} \propto \frac{T^3}{M_{\mathcal{U}}^2} \quad \text{and} \quad H \propto \frac{T^2}{M_{Pl}} \quad \Longrightarrow \quad \text{decoupling for} \quad T \lesssim T_{f-\mathcal{BZ}}$$

♣ *The unparticle phase.*

$$\mathcal{L}_{\mathcal{U}} = c_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k} \mathcal{O}_{\mathcal{U}} \mathcal{O}_{\mathsf{SM}} \quad \text{for} \quad k = d_{\mathsf{SM}} + d_{\mathcal{BZ}} - 4$$

The most relevant operators for scalar unparticles are

$$\mathcal{L}_s = c_{\mathcal{U}}^{(s)} \frac{\Lambda^{1-d_{\mathcal{U}}}}{M_{\mathcal{U}}} \left(\phi^\dagger \phi\right) \mathcal{O}_{\mathcal{U}}, \;\; \mathcal{L}_f = c_{\mathcal{U}}^{(f)} \frac{\Lambda^{3-d_{\mathcal{U}}}}{M_{\mathcal{U}}^3} \left(\bar{\ell} \phi e\right) \mathcal{O}_{\mathcal{U}}, \;\; \mathcal{L}_v = c_{\mathcal{U}}^{(v)} \frac{\Lambda^{3-d_{\mathcal{U}}}}{M_{\mathcal{U}}^3} \left(B_{\mu\nu} B^{\mu\nu}\right) \mathcal{O}_{\mathcal{U}}$$

$$\mathcal{L}_s \quad \Longrightarrow \quad \Gamma_{\mathcal{U}} \propto \frac{\Lambda_{\mathcal{U}}^3}{M_{\mathcal{U}}^2} \left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{2d_{\mathcal{U}}-3} \quad \text{and} \quad H \propto \frac{T^2}{M_{Pl}} \quad \Longrightarrow \quad T_{f-\mathcal{U}}$$

$$\frac{\Gamma_{\mathcal{U}}}{H} \propto T^{2d_{\mathcal{U}}-5} \quad \Longrightarrow \quad \begin{cases} d_{\mathcal{U}} > \frac{5}{2} & \text{decoupling for } T < T_{f-\mathcal{U}} & \text{freeze-out} \\ d_{\mathcal{U}} < \frac{5}{2} & \text{decoupling for } T > T_{f-\mathcal{U}} & \text{thaw-in} \end{cases}$$

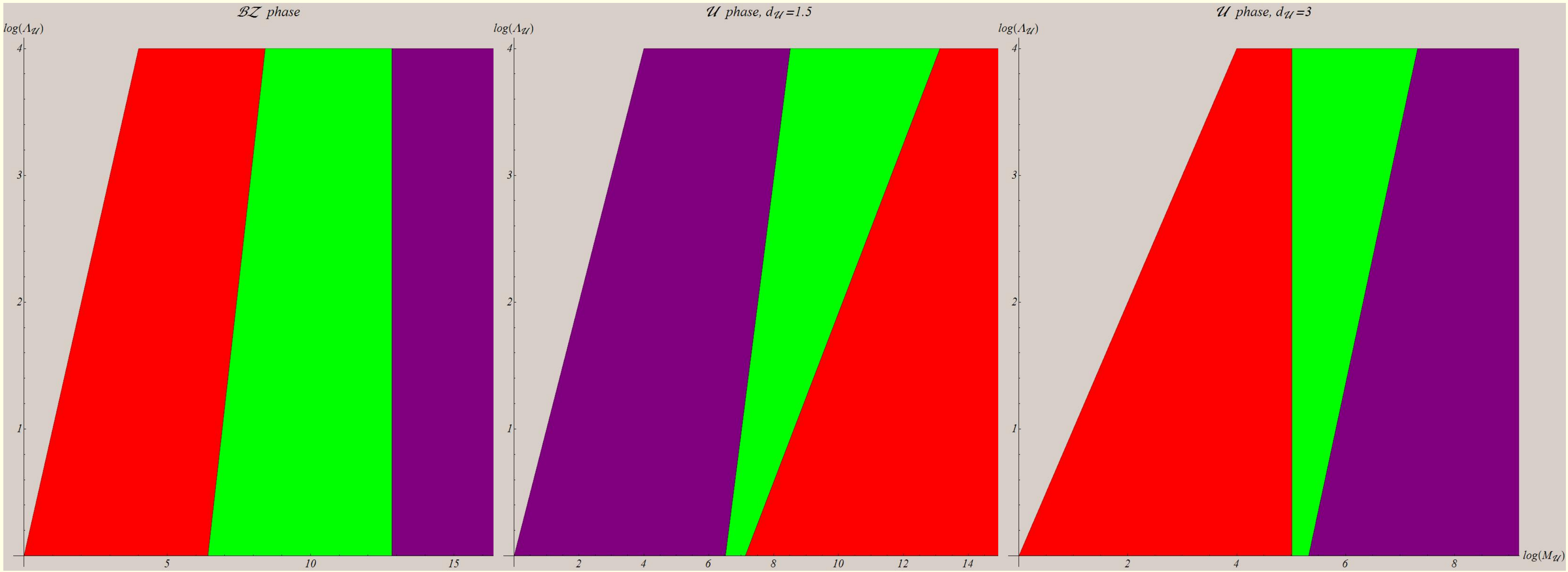

Figure 1: Regions of $(M_{\mathcal{U}}, \Lambda_{\mathcal{U}})$ for decoupling in $\mathcal{BZ}$ phase.

Figure 2: Regions of $(M_{\mathcal{U}}, \Lambda_{\mathcal{U}})$ for decoupling in $\mathcal{U}$ phase for $d_{\mathcal{U}} = \frac{3}{2}$.

Figure 3: Regions of $(M_{\mathcal{U}}, \Lambda_{\mathcal{U}})$ for decoupling in $\mathcal{U}$ phase for $d_{\mathcal{U}} = 3$.

	$\mathcal{BZ}$ - phase	$\mathcal{U}$ - phase
red	$T_{\text{f-}\mathcal{BZ}} < \Lambda_{\mathcal{U}}$	$T_{\text{f-}\mathcal{U}} < v$
green	$\Lambda_{\mathcal{U}} < T_{\text{f-}\mathcal{BZ}} < M_{\mathcal{U}}$	$v < T_{\text{f-}\mathcal{U}} < \Lambda_{\mathcal{U}}$
purple	$T_{\text{f-}\mathcal{BZ}} > M_{\mathcal{U}}$	$T_{\text{f-}\mathcal{U}} > \Lambda_{\mathcal{U}}$

BBN constraints

$$\text{Big-Bang Nucleosynthesis} \Longrightarrow \left.\frac{\Delta\rho_{\text{rad}}}{\rho_{\text{tot}}}\right|_{T=T_{\text{BBN}}} < 7\% \text{ (95\% CL)}$$

$$\Downarrow$$

$$\left.\frac{\Delta\rho_{\mathcal{U}}}{\rho_{\text{tot}}}\right|_{T=T_{\text{BBN}}} < 7\%$$

Assume $\Lambda_{\mathcal{U}} > T_{\text{f-}\mathcal{U}} > v = 246$ GeV.

- $d_{\mathcal{U}}^{(s)} > \frac{5}{2}$ decoupling for $T < T_{\text{f-}\mathcal{U}}$

$$\rho_{\mathcal{U}} = \frac{\pi^2}{30}\, g_{\text{IR}}\, T^4 \underbrace{\left(\frac{g_\gamma}{g_{\text{SM}}}\,\frac{g_\nu + g_{\gamma e}}{g_{\gamma e}}\right)^{4/3}}_{1.2\cdot 10^{-2}} \quad \text{and} \quad \rho_{\text{SM}} = \frac{\pi^2}{30}\, g_{\gamma\nu}\, T^4$$

$$\Downarrow$$

$$\left.\frac{\Delta\rho_{\mathcal{U}}}{\rho_{\text{tot}}}\right|_{T=T_{\text{BBN}}} < 7\% \quad \Longrightarrow \quad g_{\text{IR}} \lesssim 20$$

To be compared with e.g. $g_{\mathcal{BZ}} = 2(n_c^2 - 1 + \frac{7}{8} n_c n_f)$, for $n_c = 3$ and $n_f = 10$, $g_{\mathcal{BZ}} \simeq 60$.

- $d_{\mathcal{U}}^{(s)} < \frac{5}{2}$ decoupling for $T > T_{f-\mathcal{U}}$
  An operator responsible for keeping the equilibrium down (below $T \sim m_H$ where $\mathcal{L}_s$ becomes irrelevant) to T_{BBN} is needed:

$$\mathcal{L}_v = c_{\mathcal{U}}^{(v)} \frac{\Lambda^{3-d_{\mathcal{U}}}}{M_{\mathcal{U}}^3} \left(B_{\mu\nu} B^{\mu\nu}\right) \mathcal{O}_{\mathcal{U}} \quad \text{with} \quad d_{\mathcal{U}}^{(v)} < \frac{1}{2}$$

  – Note that $\mathcal{L}_v$ could be generated radiatively through $\mathcal{O}_{\mathcal{U}} - H$ mixing (from $\mathcal{L}_s$). Assuming the equilibrium down to the BBN temperature $T_{\text{BBN}} \sim 0.1$ MeV we obtain

$$\rho_{\mathcal{U}} = \frac{\pi^2}{30} g_{\text{IR}} T^4 \quad \text{and} \quad \rho_{\text{SM}} = \frac{\pi^2}{30} g_{\gamma\nu} T^4$$

$$\Downarrow$$

$$\left.\frac{\Delta\rho_{\mathcal{U}}}{\rho_{\text{tot}}}\right|_{T=T_{\text{BBN}}} < 7\% \quad \Longrightarrow \quad g_{\text{IR}} \lesssim 0.2$$

Summary

- Rough arguments for the equation of state for unparticles: $p_{\mathcal{U}} = \frac{1}{3}\rho_{\mathcal{U}}\left[1 - B\rho_{\mathcal{U}}^{\delta/4}\right]$
- Rough arguments for the energy density for unparticles "derived":

$$\rho_{\mathsf{NP}} = \frac{\pi^2}{30}T^4 \times \begin{cases} \left[g_{\mathsf{IR}} + (g_{\mathcal{BZ}} - g_{\mathsf{IR}})\left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{\delta}\right] & \text{for} \quad T \ll \Lambda_{\mathcal{U}} \\ g_{\mathcal{BZ}} & \text{for} \quad T \gg \Lambda_{\mathcal{U}} \end{cases}$$

- Unparticles in equilibrium: freeze-out and thaw-in.
- BNN bounds on the number of degrees of freedom for unparticles.

Things to be done:

- Formal (more) derivation of the equation of state.
- Formal (more) derivation of the Boltzmann equation.
- Cosmological consequences of the mass-gap.