# The Minimal Extension of the Standard Model
# - the case for Dark Matter and neutrino physics -

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- The little hierarchy problem
- The model and the little hierarchy problem
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- Summary and comments

B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", arXiv:0902.0628

# The little hierarchy problem

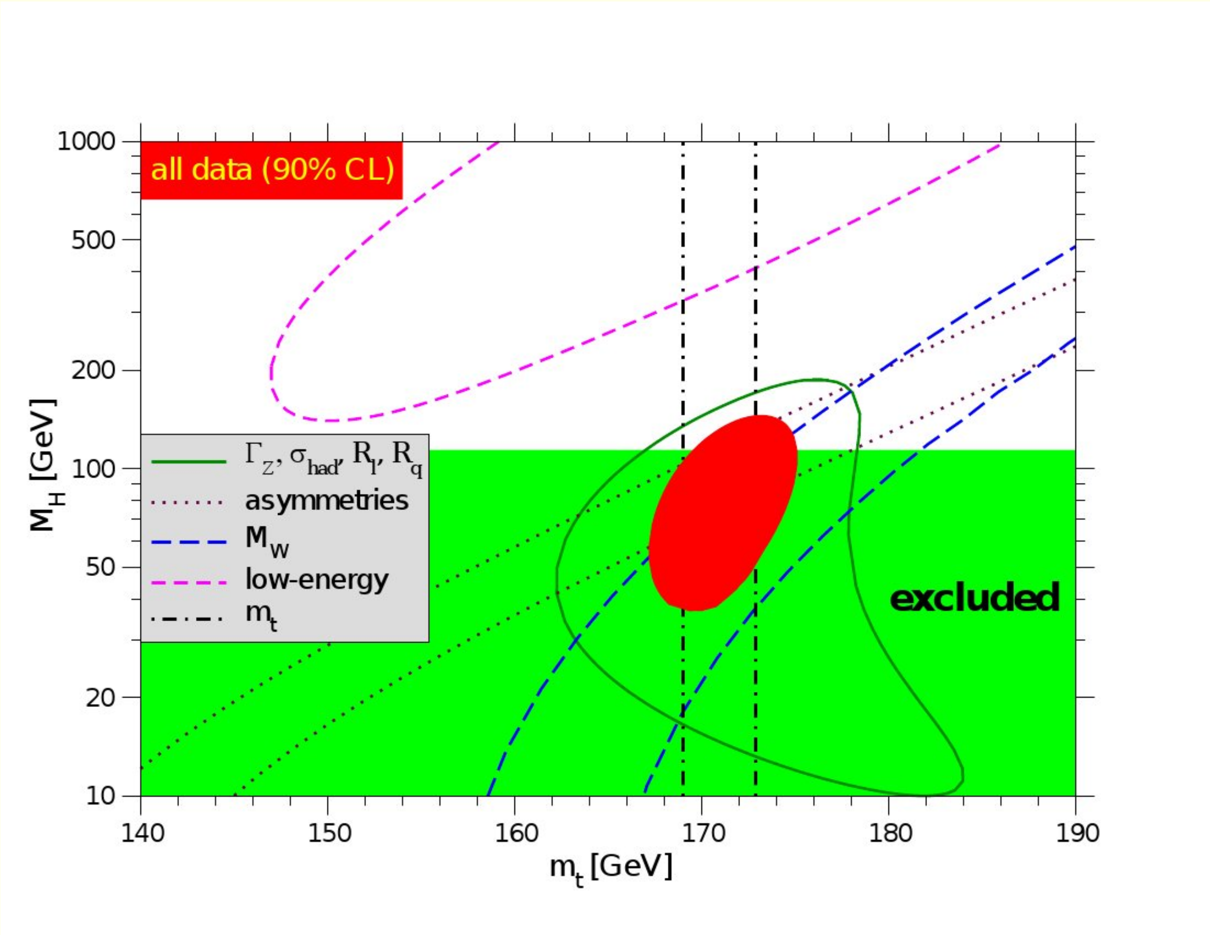


Figure 1: Red is the $90\%$ CL allowed range, from PDG 2008. $m_h < 161$ GeV at the $95\%$ CL.

## The little hierarchy problem:

- $$m_h^2 = m_h^{(B)\ 2} + \delta^{(SM)} m_h^2 + \cdots$$

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2}\left[\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2\right]$$

$$m_h = 130 \text{ GeV} \quad \Rightarrow \quad \delta^{(SM)} m_h^2 \simeq m_h^2 \qquad \text{for} \qquad \Lambda \simeq 580 \text{ GeV}$$

- For $\Lambda \gtrsim 580$ GeV there must be a cancellation between the tree-level Higgs mass$^2$ $m_h^{(B)\ 2}$ and the 1-loop leading correction $\delta^{(SM)} m_h^2$:

$$m_h^{(B)\ 2} \sim \delta^{(SM)} m_h^2 > m_h^2$$

$$\Downarrow$$

the perturbative expansion is breaking down.

- The SM cutoff is very low!

## Solutions to the little hierarchy problem:

♠ Suppression of corrections growing with $\Lambda$ at the 1-loop level:

$\Rightarrow$ The Veltman condition, no $\Lambda^2$ terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0 \qquad \Longrightarrow \qquad m_h \simeq 310 \text{ GeV}$$

In general

$$m_h^2 = m_h^{(B)\,2} + 2\Lambda^2 \sum_{n=0}^{\infty} f_n(\lambda, \dots) \ln^n\left(\frac{\Lambda}{\mu}\right)$$

where

$$(n+1) f_{n+1} = \mu \frac{\partial}{\partial \mu} f_n = \beta_i \frac{\partial}{\partial \lambda_i} f_n$$

with

$$f_0 = \frac{1}{\pi^2 v^2}\left[\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2\right]$$

and

$$f_n \propto \frac{1}{(16\pi^2)^{n+1}}$$

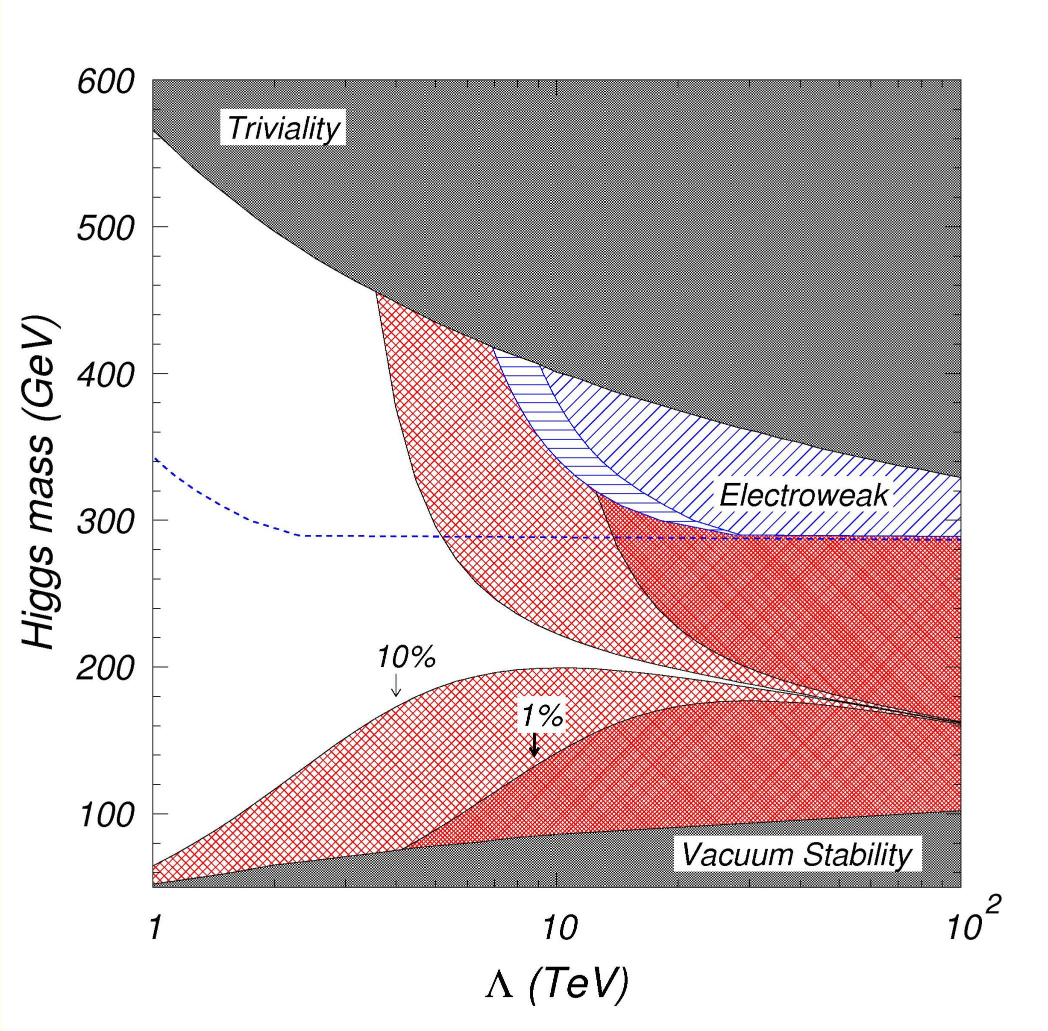


Figure 2: Contour plots of $D_t$ corresponding to $D_t = 10$ $(10\%)$ and $100$ $(1\%)$ for $n \leq 2$, from Kolda & Murayama hep-ph/0003170.

$$D_t \equiv \frac{\delta^{(SM)} m_h^2}{m_h^2} = \frac{2\Lambda^2}{m_h^2} \sum_{n=0}^{\infty} f_n(\lambda, \dots) \ln^n \left(\frac{\Lambda}{\mu}\right)$$

To understand the region allowed by $D_t \leq 10, 100$ in the SM:

- Assume $m_h$ is such that the Veltman condition is satisfied:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0\,,$$

- then at the 1-loop level $\Lambda$ could be arbitrarily large, however

- higher loops limit $\Lambda$ since the Veltman condition implies no $\Lambda^2$ only at the 1-loop level, while higher loops grow with $\Lambda^2$.

$\Rightarrow$ SUSY

$$\delta^{(SUSY)} m_h^2 \sim m_{\tilde{t}}^2 \,\frac{3\lambda_t^2}{8\pi^2} \ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right)$$

then for $\Lambda \sim 10^{16-18}$ GeV one gets $m_{\tilde{t}}^2 \lesssim 1$ TeV in order to have $\delta^{(SUSY)} m_h^2 \sim m_h^2$.

♠ Increase of the allowed value of $m_h$: the inert Higgs model by Barbieri, Hall, Rychkov, arXiv:hep-ph/0603188, (see also Ma) $\quad\Rightarrow\quad m_h \sim 400 - 600$ GeV, ($m_h^2$ terms in $T$ parameter canceled by $m_{H^\pm}, m_A, m_S$ contributions).

Our goal: to lift the cutoff to multi TeV range preserving $\delta^{(SM)} m_h^2 \leq m_h^2$.

- Extra gauge singlet $\varphi$ with $\langle\varphi\rangle = 0$ (to prevent $H \leftrightarrow \varphi$ mixing from $\varphi^2|H|^2$).
- Symmetry $\mathbb{Z}_2$: $\varphi \to -\varphi$ (to eliminate $|H|^2\varphi$ couplings).
- Gauge singlet neutrinos: $\nu_{R\,i}$ for $i = 1, 2, 3$.

$$V(H,\varphi) = -\mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_\varphi^2\varphi^2 + \frac{1}{24}\lambda_\varphi\varphi^4 + \lambda_x|H|^2\varphi^2$$

with

$$\langle H\rangle = \frac{v}{\sqrt{2}}, \qquad \langle\varphi\rangle = 0 \qquad \text{for} \qquad \mu_\varphi^2 > 0$$

then

$$m_h^2 = 2\mu_H^2 \qquad \text{and} \qquad m^2 = 2\mu_\varphi^2 + \lambda_x v^2$$

- Positivity (stability) in the limit $h, \varphi \to \infty$: $\lambda_H\lambda_\varphi > 6\lambda_x^2$
- Unitarity in the limit $s \gg m_h^2, m^2$: $\lambda_H \leq \frac{4\pi}{3}$ (the SM requirement) and $\lambda_\varphi \leq 8\pi$, $\lambda_x < 4\pi$

$$\delta^{(\varphi)} m_h^2 = -\frac{\lambda_x}{8\pi^2}\left[\Lambda^2 - m^2 \log\left(c + \frac{\Lambda^2}{m^2}\right)\right]$$

$$|\delta m_h^2| = |\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2| = D_t m_h^2$$

$$\Downarrow$$

$$\lambda_x = \lambda_x(m, m_h, D_t, \Lambda)$$

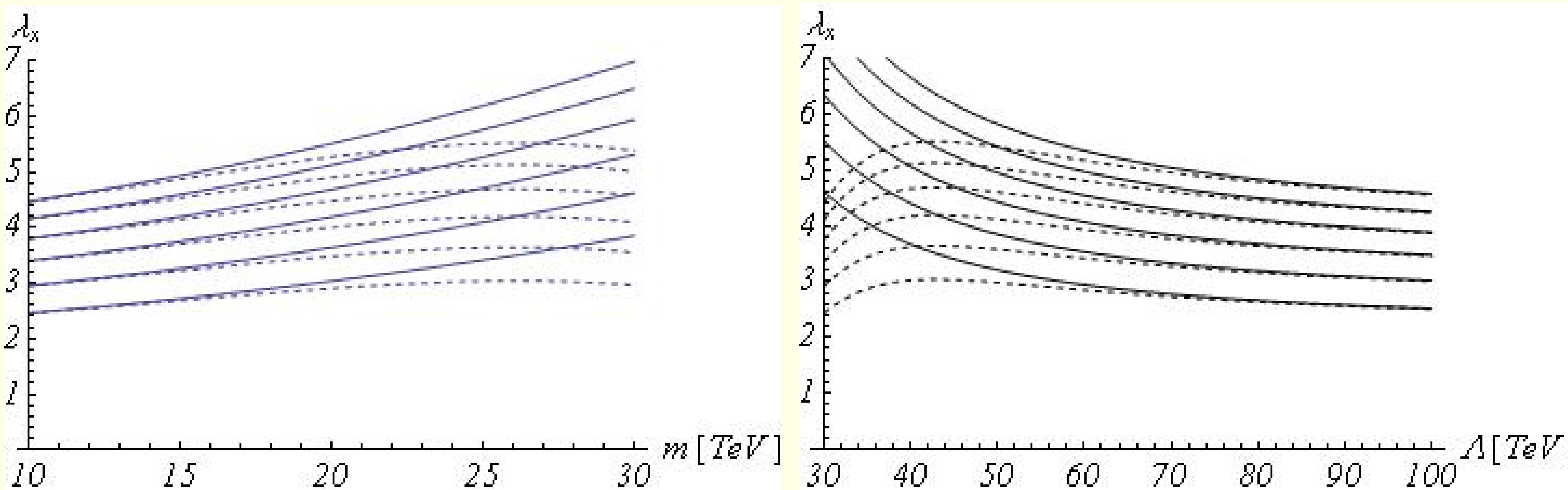


Figure 3: Plot of $\lambda_x$ corresponding to $\delta m_h^2 > 0$ as a function of $m$ for $D_t = 1$, $\Lambda = 56$ TeV (left panel) and $\lambda_x$ as a function of $\Lambda$ for $D_t = 1$, $m = 20$ TeV (right panel). The various curves correspond to $m_h = 130,\ 150,\ 170,\ 190,\ 210,\ 230$ GeV (starting with the uppermost curve). The solid (dashed) lines correspond to $c = +1$ $(c = -1)$. Note that $\lambda_x < 4\pi$.

Comments:

- When $m \ll \Lambda$, the $\lambda_x$ needed for the amelioration of the hierarchy problem is insensitive to $m$, $D_t$ or $\Lambda$:

$$\lambda_x = \left\{ 4.8 - 3\left(\frac{m_h}{v}\right)^2 + 2D_t \left[\frac{2\pi}{(\Lambda/\text{ TeV})}\right]^2 \right\} \left[1 - \frac{m^2}{\Lambda^2} \ln\left(\frac{m^2}{\Lambda^2}\right)\right] + \mathcal{O}\left(\frac{m^4}{\Lambda^4}\right) .$$

- Since we consider $\lambda_x > 1$ higher order corrections could be important. In general

$$\left| \delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2 + \Lambda^2 \sum_{n=1} f_n(\lambda_x, \dots) \left[\ln\left(\frac{\Lambda}{m_h}\right)\right]^n \right| = D_t m_h^2 ,$$

where the coefficients $f_n(\lambda_x, \dots)$ can be determined recursively (see Einhorn & Jones):

$$f_n(\lambda_x, \dots) \sim \left[\frac{\lambda_x}{(16\pi^2)}\right]^{n+1}$$

If $\Lambda = 100$ TeV, $m_h = 120 - 250$ GeV and $m = 10 - 30$ TeV the relative next order correction remains in the range $4 - 12\%$.

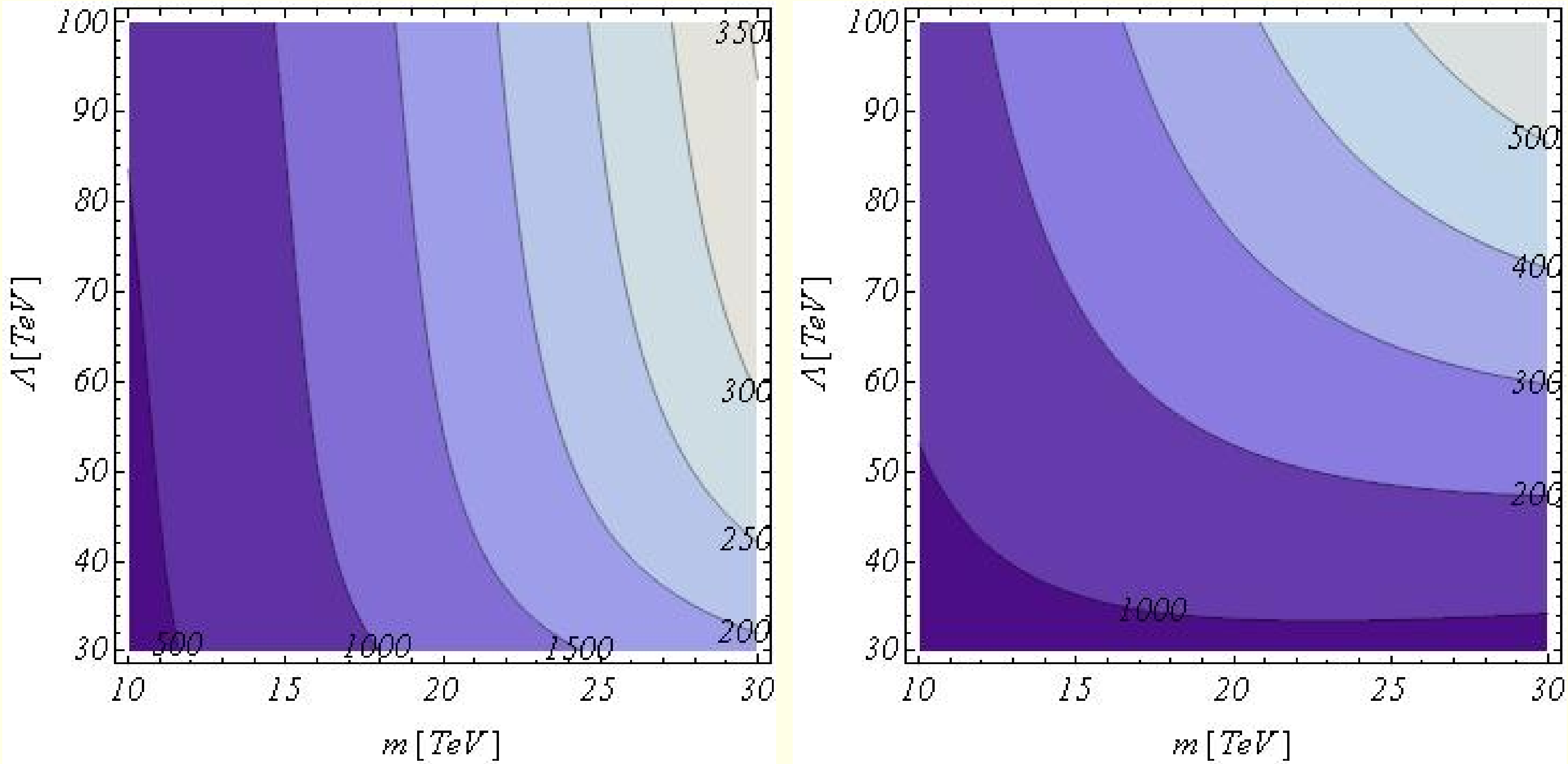


Figure 4: Contour plots of the Barbieri-Giudice parameters $\Delta_\Lambda$ (left panel) and $\Delta_m$ (right panel) for $m_h = 150$ GeV and $\lambda_x = 3.68$.

$$\Delta_\Lambda \equiv \frac{\Lambda}{m_h^2}\frac{\partial m_h^2}{\partial \Lambda} \qquad \Delta_m \equiv \frac{m}{m_h^2}\frac{\partial m_h^2}{\partial m}$$

$$\frac{\delta m_h^2}{m_h^2} = \Delta_\Lambda \frac{\delta \Lambda}{\Lambda} \qquad \frac{\delta m_h^2}{m_h^2} = \Delta_m \frac{\delta m}{m}$$

| model | $\delta m_h^2$ | $\Lambda$ |
|---|---|---|
| SM | $\underbrace{\Lambda^2\left(\frac{3\lambda_t^2}{4\pi^2}+\cdots\right)}_{1-\text{loop }SM}+\underbrace{\Lambda^2 f_1^{(SM)}\ln\left(\frac{\Lambda}{\mu}\right)}_{2-\text{loop }SM}$ | see plots |
| SUSY | $m_{\tilde{t}}^2\,\frac{3\lambda_t^2}{8\pi^2}\ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right)$ | $m_{\tilde{t}} \lesssim 1$ TeV<br>for $\Lambda\sim 10^{16-18}$ GeV |
| SM + $\varphi$ | $\underbrace{\Lambda^2\left(\frac{3\lambda_t^2}{4\pi^2}+\cdots\right)}_{1-\text{loop }SM}\underbrace{-\frac{\lambda_x}{8\pi^2}\left[\Lambda^2-m^2\ln(c+\frac{\Lambda^2}{m^2})\right]}_{1-\text{loop }\varphi}$<br>$+\underbrace{\left(f_1^{(SM)}+f_1^{(\varphi)}\right)}_{2-\text{loop}}\ln\left(\frac{\Lambda}{\mu}\right)$ | For $D_t=1$<br>$\Lambda\sim 60$ TeV, $m\sim 20$ TeV |

For $D_t=1$ (no fine-tuning) and $m_h=130$ GeV:

- SM: $\Lambda\simeq 1$ TeV, while
- SM + $\varphi$: $\Lambda\simeq 60$ TeV for $\lambda_x=\lambda_x(m)$ (fine tuning!) with $m=20$ TeV,
- The range of $(m_h,\Lambda)$ space corresponding to a given $D_t$ is expected to be larger when $\varphi$ is added to the SM, if $\lambda_x=\lambda_x(m,m_h,D_t,\Lambda)$.

## Dark Matter


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4. H. Davoudiasl, R. Kitano, T. Li and H. Murayama, Phys. Lett. B **609**, 117 (2005)
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It is possible to find parameters $\Lambda$, $\lambda_x$ and $m$ such that both the hierarchy is ameliorated to the prescribed level and such that $\Omega_\varphi h^2$ is consistent with $\Omega_{DM}$.

$$\varphi\varphi \to hh, W_L^+ W_L^-, Z_L Z_L \quad \Rightarrow \quad \langle \sigma v \rangle = \frac{1}{8\pi}\frac{\lambda_x^2}{m^2}$$

The Boltzmann equation $\quad \Rightarrow \quad x_f \left(\equiv \frac{m}{T_f}\right) \simeq \ln\left[0.038 \frac{m_{Pl} m \langle \sigma v \rangle}{g_\star^{1/2} x_f^{1/2}}\right]$

$$\Omega_\varphi h^2 \simeq 1.06 \cdot 10^9 \frac{x_f}{g_\star^{1/2} m_{Pl} \langle \sigma v \rangle \text{ GeV}}$$

$$x_f \simeq 30 \quad \Rightarrow \quad m \geq x_f T_c \simeq 8 \text{ TeV}$$

$$\Omega_\varphi = \Omega_{DM} \quad \Rightarrow \quad \lambda_x \sim \frac{1}{4} \frac{m}{\text{TeV}}$$

$$\Downarrow$$

$$|\delta m_h^2| = D_t m_h^2 \quad \Rightarrow \quad m = m(\Lambda)$$

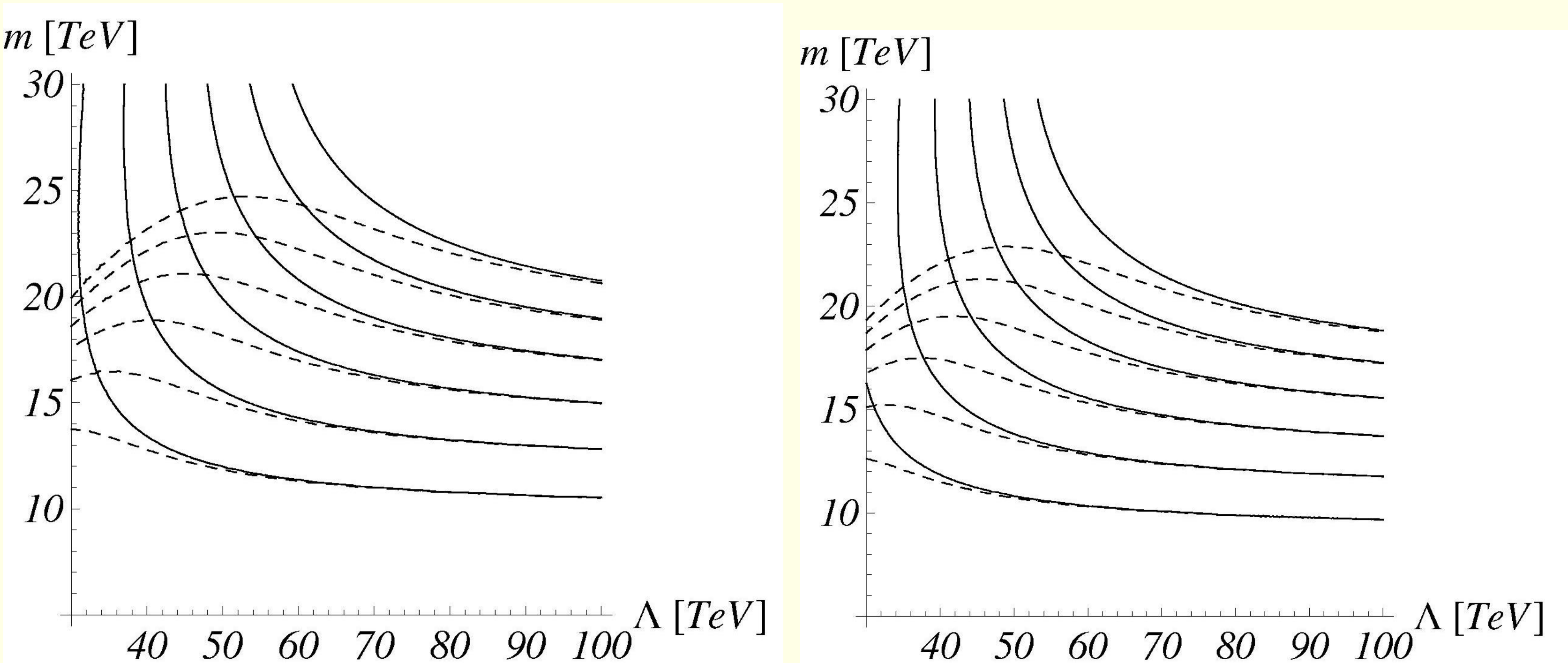


Figure 5: Plot of $m$ as a function of the cutoff $\Lambda$ when $D_t = 1$ and $\Omega_\varphi = \Omega_{DM}$ at the $1\sigma$ level: $\Omega_\varphi h^2 = 0.114$ (left panel) and $\Omega_\varphi h^2 = 0.098$ (right panel); for $m_h = 130, 150, 170, 190, 210, 230$ GeV (starting with the uppermost curve) and for $c = +1$ solid curves and $c = -1$ (dashed curves).

## Neutrino physics

$$\mathcal{L}_Y = -\bar{L} Y_l H l_R - \bar{L} Y_\nu \tilde{H} \nu_R - \frac{1}{2}\overline{(\nu_R)^c} M \nu_R - \varphi \overline{(\nu_R)^c} Y_\varphi \nu_R + \text{H.c.}$$

$$\mathbb{Z}_2: \qquad H \to H,\ \varphi \to -\varphi,\ L \to S_L L,\ l_R \to S_{l_R} l_R,\ \nu_R \to S_{\nu_R} \nu_R$$

The symmetry conditions ($S_i S_i^\dagger = S_i^\dagger S_i = \mathbb{1}$):

$$S_L^\dagger Y_l S_{l_R} = Y_l, \quad S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu, \quad S_{\nu_R}^T M S_{\nu_R} = +M, \quad S_{\nu_R}^T Y_\varphi S_{\nu_R} = -Y_\varphi$$

The implications of the symmetry:

$$S_{\nu_R}^T M S_{\nu_R} = +M \quad \Rightarrow \quad S_{\nu_R} = \pm \mathbb{1}, \qquad S_{\nu_R} = \pm\, \text{diag}(1, 1, -1)$$

$$S_{\nu_R} = \pm \mathbb{1} \quad \Rightarrow \quad Y_\varphi = 0 \;\; \text{or} \;\; S_{\nu_R} = \pm\,\text{diag}(1,1,-1) \quad \Rightarrow \quad Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}$$

$$S_L^\dagger Y_l S_{l_R} = Y_l \quad \Rightarrow \quad S_L = S_{l_R} = \text{diag}(s_1, s_2, s_3), \quad |s_i| = 1$$

$$S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu \quad \Rightarrow \qquad \text{10 Dirac neutrino mass textures}$$

For instance the solution corresponding to $s_{1,2,3} = \pm 1$:

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix}$$

$$\mathcal{L}_m = -(\bar{n} M_n n + \bar{N} M_N N)$$

with the see-saw mechanism explaining $M_n \ll M_N$:

$$M_N \sim M \qquad \text{and} \qquad M_n \sim (vY_\nu)\frac{1}{M}(vY_\nu)^T$$

where

$$\nu_L = n_L + M_D \frac{1}{M} N_L \qquad \text{and} \qquad \nu_R = N_R - \frac{1}{M} M_D^T n_R$$

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix} \quad \Rightarrow \quad M_D = Y_\nu \frac{v}{\sqrt{2}} \quad \Rightarrow \quad M_n$$

To compare our results with the data, we use the following approximate lepton mixing matrix (tri-bimaximal lepton mixing) that corresponds to $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\theta_{12} = \arcsin(1/\sqrt{3})$:

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$

Writing the diagonal light neutrino mass matrix as

$$m_{\text{light}} = \mathsf{diag}(m_1, m_2, m_3)$$

we find

$$M_n = U m_{\text{light}} U^T$$

$$\Downarrow$$

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ -\frac{a}{2} & b & 0 \\ -\frac{a}{2} & b & 0 \end{pmatrix} \quad \begin{array}{l} m_1 = -3a^2 \frac{v^2}{M_1} \\ m_2 = -6b^2 \frac{v^2}{M_2} \\ m_3 = 0 \end{array} \quad \text{and } Y_\nu = \begin{pmatrix} a & b & 0 \\ a & -\frac{b}{2} & 0 \\ a & -\frac{b}{2} & 0 \end{pmatrix} \quad \begin{array}{l} m_1 = -3b^2 \frac{v^2}{M_2} \\ m_2 = -6a^2 \frac{v^2}{M_1} \\ m_3 = 0 \end{array}$$

<u>Does $Y_\varphi \neq 0$ imply $\varphi \to n_i n_j$ decays?</u>

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix}, \; Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \Rightarrow \; \varphi \to N^\star_{1,2} N_3 \to \underbrace{n_{1,2,3}\, h}_{N^\star_{1,2}} \; N_3$$

that can be kinematically forbidden by requiring $M_3 > m$.

## Does $\varphi$ explain the PAMELA data?

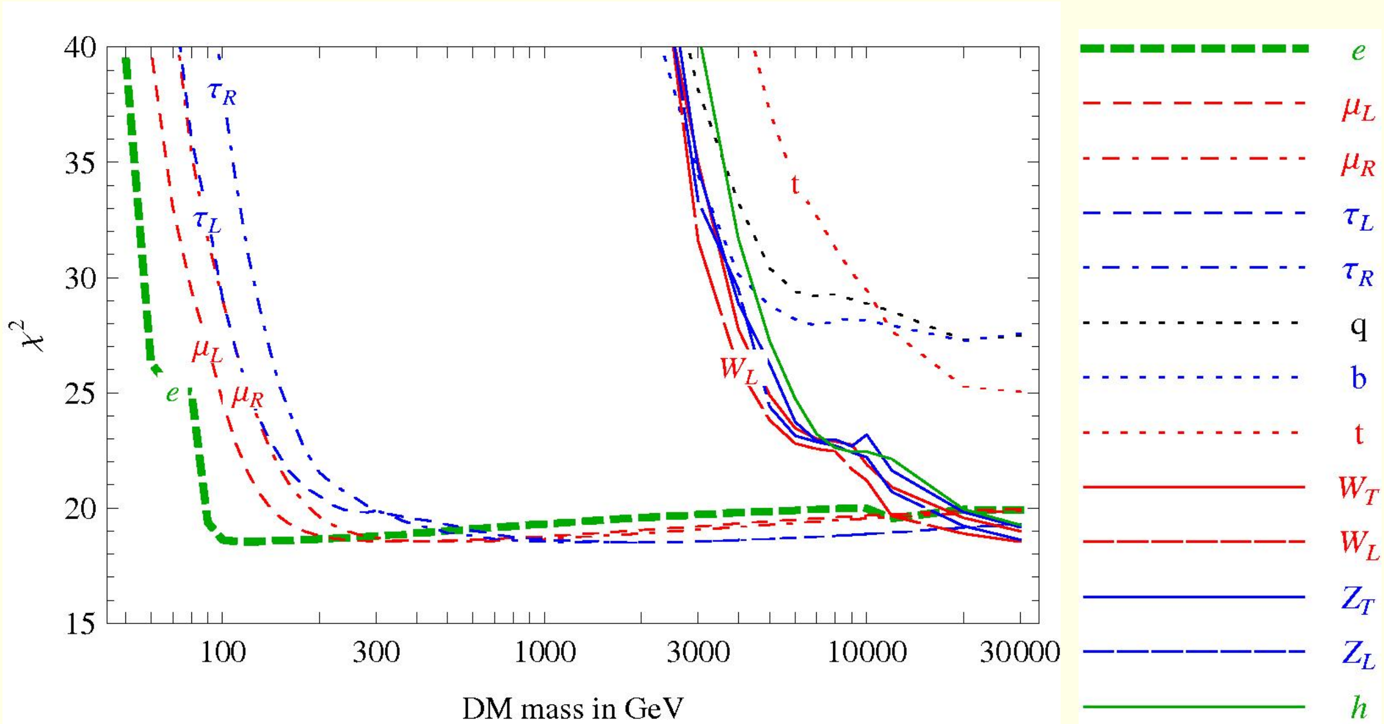


Figure 6: Combined fit of different DM annihilation channels to the PAMELA positron and PAMELA anti-proton data, from Cirelli, Kadastik, Raidal and Strumia, arXiv:0809.2409.

## Summary and comments

- The addition of a real scalar singlet $\varphi$ to the SM may ameliorate the little hierarchy problem (by lifting the cutoff $\Lambda$ to $50 - 100$ TeV range). Fine tuning remains.
- It also provides a realistic candidate for DM.
- Since $m \gtrsim 10$ TeV therefore $\varphi$ can properly describe the PAMELA results both for $e^+$ and $\bar{p}$.
- The $\mathbb{Z}_2$ symmetry implies a realistic texture for the neutrino mass matrix.
- $\varphi$ cannot be assumed to be responsible neither for inflation nor for dark energy.