

# Various aspects of CPV in 2HDMs

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- CP violation in 2HDM
- Implications of the LHC Higgs signal - *Does any CP-violation remain in the alignment limit?*
- Numerical strategy and illustrations of the alignment limit
- Spontaneous CP violation
- Summary

- B.G., O. M. OGREID and P. OSLAND, "Spontaneous CP violation in the 2HDM: physical conditions and the alignment limit", Phys.Rev. D94 (2016) no.11, 115002
- B.G., O. M. OGREID and P. OSLAND, "CP-Violation in the  $ZZZ$  and  $ZW^+W^-$  vertices at  $e^+e^-$  colliders in Two-Higgs-Doublet Models", JHEP 1605 (2016) 025,
- B.G., O. M. OGREID and P. OSLAND, "Measuring CP violation in Two-Higgs-Doublet models in light of the LHC Higgs data", JHEP 1411 (2014) 084,
- B.G., O. M. OGREID and P. OSLAND, "Diagnosing CP properties of the 2HDM", JHEP **1401** (2014) 105,

## CP violation in 2HDM

Motivations for 2HDM:

- Baryon asymmetry and Sakharov conditions for baryogenesis
  - Baryon number non-conservation,
  - C- and CP-violation,
  - Thermal inequilibrium,

Extra sources of the CP-violation are required

- Possibility of large (tree-level generated) FCNC, e.g.  $t \rightarrow cH$  decays, interesting non-standard flavour physics
- 2HDM provide a framework for light new physics that is easily tolerated by the Higgs boson discovery.  
see e.g.  
B. Dumont, J. F. Gunion, Y. Jiang and S. Kraml, "Constraints on and future prospects for Two-Higgs-Doublet Models in light of the LHC Higgs signal", Phys. Rev. D **90**, 035021 (2014) [arXiv:1405.3584 [hep-ph]].

The 2HDM potential:

$$\begin{aligned}
 V(\phi_1, \phi_2) = & -\frac{1}{2} \left\{ m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + \left[ m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.} \right] \right\} + \frac{1}{2} \lambda_1 (\phi_1^\dagger \phi_1)^2 \\
 & + \frac{1}{2} \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) \\
 & + \frac{1}{2} \left[ \lambda_5 (\phi_1^\dagger \phi_2)^2 + \text{H.c.} \right] + \left[ \lambda_6 (\phi_1^\dagger \phi_1) + \lambda_7 (\phi_2^\dagger \phi_2) \right] \left[ (\phi_1^\dagger \phi_2) + \text{H.c.} \right]
 \end{aligned}$$

Yukawa couplings:

$$\mathcal{L}_Y^{(q)} = \bar{Q}_L \left( \tilde{\Gamma}_1 \tilde{\Phi}_1 + \tilde{\Gamma}_2 \tilde{\Phi}_2 \right) u_R + \bar{Q}_L \left( \Gamma_1 \Phi_1 + \Gamma_2 \Phi_2 \right) d_R + \text{H.c.}$$

then

$$M_u = -\tilde{\Gamma}_1 \langle \tilde{\Phi}_1 \rangle - \tilde{\Gamma}_2 \langle \tilde{\Phi}_2 \rangle \quad M_d = -\Gamma_1 \langle \Phi_1 \rangle - \Gamma_2 \langle \Phi_2 \rangle$$

The type II model:

$\mathbb{Z}_2$  softly broken (by  $m_{12}^2 \neq 0$ ):  $\Phi_1 \rightarrow -\Phi_1$  and  $d_R \rightarrow -d_R \Rightarrow \lambda_6 = \lambda_7 = 0$ ,  
 $\tilde{\Gamma}_1 = \Gamma_2 = 0$



In an arbitrary basis, the vevs may be complex, and the Higgs-doublets can be written as

$$\Phi_j = e^{i\xi_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2.$$

Here  $v_j$  are real numbers, so that  $v_1^2 + v_2^2 = v^2$ . The fields  $\eta_j$  and  $\chi_j$  are real. The phase difference between the two vevs is defined as

$$\xi \equiv \xi_2 - \xi_1.$$

Next, let's define the Goldstone bosons  $G_0$  and  $G^\pm$  by an orthogonal rotation

$$\begin{pmatrix} G_0 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix}$$

where  $s_\beta \equiv \sin \beta$  and  $c_\beta \equiv \cos \beta$  for  $\tan \beta \equiv v_2/v_1$ . Then  $G_0$  and  $G^\pm$  become the massless Goldstone fields.  $H^\pm$  are the charged scalars.

The model contains three neutral scalar mass-eigenstates, which are linear compositions of the  $\eta_i$ ,

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix},$$

with the  $3 \times 3$  orthogonal rotation matrix  $R$  satisfying

$$R\mathcal{M}^2R^T = \mathcal{M}_{\text{diag}}^2 = \text{diag}(M_1^2, M_2^2, M_3^2),$$

and with  $M_1 \leq M_2 \leq M_3$ . A convenient parametrization of the rotation matrix  $R$  is

$$\begin{aligned} R = R_3R_2R_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -(c_1 s_2 s_3 + s_1 c_3) & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -(c_1 s_3 + s_1 s_2 c_3) & c_2 c_3 \end{pmatrix} \end{aligned}$$

The potential contains 14 real parameters, our input parameters are

$$\mathcal{P}_{67} \equiv \{M_{H^\pm}^2, \mu^2, M_1^2, M_2^2, M_3^2, \text{Im}\lambda_5, \text{Re}\lambda_6, \text{Re}\lambda_7, v_1, v_2, \xi, \alpha_1, \alpha_2, \alpha_3\},$$

where  $\mu^2$  is defined as  $\text{Re} m_{12}^2 \equiv \frac{2v_1v_2}{v^2}\mu^2$ . and the extrema conditions are:

$$\begin{aligned} m_{11}^2 &= v_1^2\lambda_1 + v_2^2(\lambda_3 + \lambda_4) + \frac{v_2^2}{c_\xi}(\text{Re}\lambda_5c_\xi - \text{Im}\lambda_5s_\xi) \\ &\quad + \frac{v_1v_2}{c_\xi}[\text{Re}\lambda_6(2 + c_{2\xi}) - \text{Im}\lambda_6s_{2\xi}] + \frac{v_2}{v_1c_\xi}(v_2^2\text{Re}\lambda_7 - \text{Re}m_{12}^2), \end{aligned}$$

$$\begin{aligned} m_{22}^2 &= v_2^2\lambda_2 + v_1^2(\lambda_3 + \lambda_4) + \frac{v_1^2}{c_\xi}(\text{Re}\lambda_5c_\xi - \text{Im}\lambda_5s_\xi) \\ &\quad + \frac{v_1v_2}{c_\xi}[\text{Re}\lambda_7(2 + c_{2\xi}) - \text{Im}\lambda_7s_{2\xi}] + \frac{v_1}{v_2c_\xi}(v_1^2\text{Re}\lambda_6 - \text{Re}m_{12}^2), \end{aligned}$$

$$\begin{aligned} \text{Im} m_{12}^2 &= \frac{v_1v_2}{c_\xi}(\text{Re}\lambda_5s_{2\xi} + \text{Im}\lambda_5c_{2\xi}) + \frac{v_1^2}{c_\xi}(\text{Re}\lambda_6s_\xi + \text{Im}\lambda_6c_\xi) \\ &\quad + \frac{v_2^2}{c_\xi}(\text{Re}\lambda_7s_\xi + \text{Im}\lambda_7c_\xi) - \text{Re}m_{12}^2t_\xi, \end{aligned}$$

with  $c_x = \cos x$ ,  $s_x = \sin x$ , and  $t_x = \tan x$ .

## Couplings:

$$H_i Z_\mu Z_\nu : \frac{ig^2}{2 \cos^2 \theta_W} e_i g_{\mu\nu}, \quad H_i W_\mu^+ W_\nu^- : \frac{ig^2}{2} e_i g_{\mu\nu}$$

where

$$e_i \equiv v_1 R_{i1} + v_2 R_{i2}$$

In terms of the mixing angles

$$\begin{aligned} e_1 &= v \cos \alpha_2 \cos(\beta - \alpha_1) \\ e_2 &= v [\cos \alpha_3 \sin(\beta - \alpha_1) - \sin \alpha_2 \sin \alpha_3 \cos(\beta - \alpha_1)] \\ e_3 &= -v [\sin \alpha_3 \sin(\beta - \alpha_1) + \sin \alpha_2 \cos \alpha_3 \cos(\beta - \alpha_1)] \end{aligned}$$

Note that

$$e_1^2 + e_2^2 + e_3^2 = v^2.$$

## Couplings:

$$(Z^\mu H_i H_j) : \frac{g}{2v \cos \theta_W} \epsilon_{ijk} e_k (p_i - p_j)^\mu,$$

$$H_i H^- H^+ : -iq_i$$

where

$$q_i = \frac{2e_i}{v^2} M_{H^\pm}^2 - \frac{R_{i2}v_1 + R_{i1}v_2 - R_{i3}vt_\xi}{v_1v_2c_\xi} \mu^2 + \frac{g_i - R_{i3}v^3t_\xi}{v^2v_1v_2} M_i^2 + \frac{R_{i3}v^3}{2v_1v_2c_\xi^2} \text{Im } \lambda_5 \\ - \frac{v^2 (R_{i3}vt_\xi - R_{i2}v_1 + R_{i1}v_2)}{2v_2^2c_\xi} \text{Re } \lambda_6 - \frac{v^2 (R_{i3}vt_\xi + R_{i2}v_1 - R_{i1}v_2)}{2v_1^2c_\xi} \text{Re } \lambda_7$$

and  $g_i \equiv v_1^3 R_{i2} + v_2^3 R_{i1}$ .

## CP violation and invariants under $U(2)$ basis rotations

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \bar{\Phi}_1 \\ \bar{\Phi}_2 \end{pmatrix} = e^{i\psi} \begin{pmatrix} \cos \theta & e^{-i\xi} \sin \theta \\ -e^{i\chi} \sin \theta & e^{i(\chi-\xi)} \cos \theta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}.$$

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- C. C. Nishi, "CP violation conditions in N-Higgs-doublet potentials", Phys. Rev. D 74 (2006) 036003 [Erratum-ibid. D 76 (2007) 119901] [hep-ph/0605153].
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- M. Maniatis, A. von Manteuffel and O. Nachtmann, "CP violation in the general two-Higgs-doublet model: A Geometric view", Eur. Phys. J. C 57 (2008) 719 [arXiv:0707.3344 [hep-ph]].
- G. C. Branco, L. Lavoura and J. P. Silva, "CP Violation", Int. Ser. Monogr. Phys. 103, 1 (1999).

## CP conservation:

CP is conserved if and only if:

$$\text{Im } J_1 = \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} M_i^2 e_i e_j q_k = 0$$

$$\text{Im } J_2 = \frac{e_1 e_2 e_3}{v^9} \sum_{i,j,k} \epsilon_{ijk} M_i^4 M_k^2 = 0$$

$$\text{Im } J_{30} = \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} M_i^2 q_i e_j q_k = 0.$$

where  $J_i$  are the weak basis invariants found by Lavoura, Silva and Botella (1994, 1995) and discussed by Branco, Rebelo and Silva-Marcos (2005), Davidson, Gunion and Haber (2005), Ivanov (2006, 2007), Nishi (2006) and M. Maniatis, A. von Manteuffel and O. Nachtmann (2008).

CP conservation: The conditions for CP conservation could be rewritten as:

$$\text{Im } J_1 = \frac{1}{v^5} [M_1^2 e_1 (e_2 q_3 - e_3 q_2) + M_2^2 e_2 (e_3 q_1 - e_1 q_3) + M_3^2 e_3 (e_1 q_2 - e_2 q_1)] = 0$$

$$\text{Im } J_2 = \frac{e_1 e_2 e_3}{v^9} (M_2^2 - M_1^2)(M_3^2 - M_2^2)(M_1^2 - M_3^2) = 0$$

$$\text{Im } J_{30} = \frac{1}{v^5} [M_1^2 q_1 (e_2 q_3 - e_3 q_2) + M_2^2 q_2 (e_3 q_1 - e_1 q_3) + M_3^2 q_3 (e_1 q_2 - e_2 q_1)] = 0$$

$$\text{Im } J_1 = \frac{1}{v^5} \begin{vmatrix} q_1 & q_2 & q_3 \\ e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \end{vmatrix}, \quad \text{Im } J_2 = \frac{2}{v^9} \begin{vmatrix} e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \\ e_1 M_1^4 & e_2 M_2^4 & e_3 M_3^4 \end{vmatrix}$$

$$\text{Im } J_{30} = \frac{1}{v^5} \begin{vmatrix} e_1 & e_2 & e_3 \\ q_1 & q_2 & q_3 \\ q_1 M_1^2 & q_2 M_2^2 & q_3 M_3^2 \end{vmatrix}$$



# Implications of the LHC Higgs signal

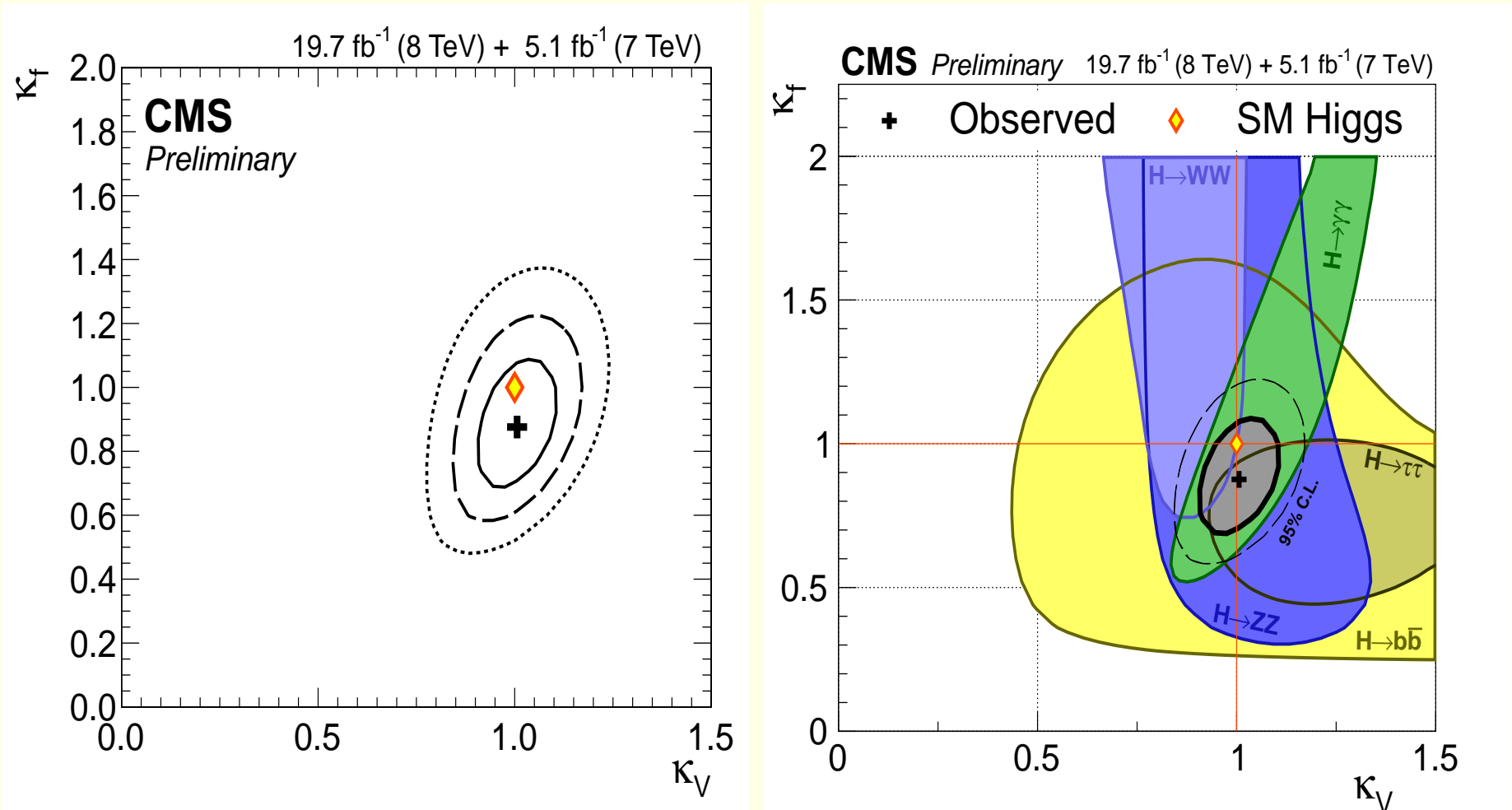


Figure 1: CMS PAS HIG-14-009: Results of 2D likelihood scans for the  $(\kappa_V, \kappa_f)$ . Left: the solid, dashed, and dotted contours show the 68%, 95%, and 99.7% CL regions, respectively. Right: the 68% CL contours for individual channels and for the overall combination (thick curve), the dashed contour bounds the 95% CL region.

The LHC Higgs data suggest that  $HZZ$  and  $HW^+W^-$  couplings are close to the SM prediction.

$$\Downarrow$$
$$H_i Z_\mu Z_\nu : \frac{ig^2}{2\cos^2\theta_W} e_i g_{\mu\nu}, \quad H_i W_\mu^+ W_\nu^- : \frac{ig^2}{2} e_i g_{\mu\nu}$$

We define (within 2HDM) the alignment limit as  $e_1 = v$

Then

$$e_1^2 + e_2^2 + e_3^2 = v^2 \quad \Rightarrow \quad e_2 = e_3 = 0$$

Note that no assumption has been made concerning the mass scale of beyond the SM physics:  $M_2$ ,  $M_3$  and  $\mu^2$  defined as

$$\text{Re } m_{12}^2 = \frac{2v_1 v_2}{v^2} \mu^2.$$

The coupling of  $H_1$  to a pair of vector bosons,  $e_1$ , could be written as follows:

$$e_1 = v \cos(\alpha_2) \cos(\alpha_1 - \beta)$$

The most general solution of the alignment limit (known also as the alignment condition)  $e_1 = v$ ,  $e_2 = 0$ ,  $e_3 = 0$ :

$$\alpha_2 = 0 \quad \alpha_1 = \beta$$

The rotation matrix in this case becomes

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 c_3 & c_1 c_3 & s_3 \\ s_1 s_3 & -c_1 s_3 & c_3 \end{pmatrix}$$

Note that the mixing matrix could be written in this case as

$$R = R_3 R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

## Couplings:

In the alignment limit  $Z$  couples only to  $H_2H_3$ :

$$(Z^\mu H_i H_j) : \frac{g}{2v \cos \theta_W} \epsilon_{ijk} e_k (p_i - p_j)^\mu \neq 0 \text{ only if } i = 2 \text{ and } j = 3$$

Couplings between  $H_i$  and  $H^+H^-$  are given in the alignment limit (for  $\xi = 0$ ) by:

$$\begin{aligned} q_1 &= \frac{1}{v} (2M_{H^\pm}^2 - 2\mu^2 + M_1^2) \\ q_2 &= +c_3 \left[ \frac{(c_\beta^2 - s_\beta^2)}{vc_\beta s_\beta} (M_2^2 - \mu^2) + \frac{v}{2s_\beta^2} \text{Re } \lambda_6 - \frac{v}{2c_\beta^2} \text{Re } \lambda_7 \right] + s_3 \frac{v}{2c_\beta s_\beta} \text{Im } \lambda_5, \\ q_3 &= -s_3 \left[ \frac{(c_\beta^2 - s_\beta^2)}{vc_\beta s_\beta} (M_3^2 - \mu^2) + \frac{v}{2s_\beta^2} \text{Re } \lambda_6 - \frac{v}{2c_\beta^2} \text{Re } \lambda_7 \right] + c_3 \frac{v}{2c_\beta s_\beta} \text{Im } \lambda_5 \end{aligned}$$

## Does the alignment limit allow for CP-violation?

The  $J$  invariants in the alignment limit:

$$\begin{aligned} \text{Im } J_1 &= \frac{1}{v^5} [M_1^2 e_1 (e_2 q_3 - e_3 q_2) + M_2^2 e_2 (e_3 q_1 - e_1 q_3) + M_3^2 e_3 (e_1 q_2 - e_2 q_1)] \rightarrow 0 \\ \text{Im } J_2 &= \frac{e_1 e_2 e_3}{v^9} (M_2^2 - M_1^2)(M_3^2 - M_2^2)(M_1^2 - M_3^2) \rightarrow 0 \\ \text{Im } J_{30} &= \frac{1}{v^5} [M_1^2 q_1 (e_2 q_3 - e_3 q_2) + M_2^2 q_2 (e_3 q_1 - e_1 q_3) + M_3^2 q_3 (e_1 q_2 - e_2 q_1)] \\ &\rightarrow \frac{e_1 q_2 q_3}{v^3} (M_3^2 - M_2^2) \end{aligned}$$

- Note that  $e_1 = v$  implies no CP violation in  $H_i V V$  couplings ( $\text{Im } J_2 = 0$ ), the only possible CP violation may appear in cubic scalar couplings  $H_2 H^+ H^-$  and  $H_3 H^+ H^-$ , proportional to  $q_2$  and  $q_3$ , respectively.
- The necessary condition for CP violation is that both  $H_2 H^+ H^-$  and  $H_3 H^+ H^-$  must exist *together* with non-zero  $Z H_2 H_3$  vertex. The latter implies that for CP invariance either  $H_2$  or  $H_3$  would have to be odd under CP, on the other hand if *both* of them couple to  $H^+ H^-$  (that is CP even), then there is no way to preserve CP.

In the case  $\lambda_6 = \lambda_7 = 0$  the  $(\mathcal{M}^2)_{13}$  and  $(\mathcal{M}^2)_{23}$  are related as follows

$$(\mathcal{M}^2)_{13} = \tan \beta (\mathcal{M}^2)_{23}$$

As a consequence of the above relation there is a constraint that relates mass eigenvalues, mixing angles and  $\tan \beta$  (Khater and Osland, 2003):

$$M_1^2 R_{13}(R_{12} \tan \beta - R_{11}) + M_2^2 R_{23}(R_{22} \tan \beta - R_{21}) + M_3^2 R_{33}(R_{32} \tan \beta - R_{31}) = 0$$

⇓

$$\text{Alignment limit } (\alpha_2 = 0, \alpha_1 = \beta) \Rightarrow \boxed{(M_2^2 - M_3^2) s_3 c_3 s_\beta = 0}$$

⇓

- $M_2 \neq M_3$ , but  $\alpha_3 = 0, \pi/2$ , then  $q_3 = 0, q_2 = 0$ , respectively (since in 2HDM5, in the alignment limit  $\text{Im } \lambda_5 = 0$ ), so no CP violation, or
- $M_2 = M_3$ , therefore  $\text{Im } J_3 = 0$ , so again no CP violation.

## Partial conclusions:

- The observation of the SM-like Higgs boson at the LHC implies (within the 2HDM with  $\mathbb{Z}_2$  softly broken) vanishing CP violation in the scalar potential.
- The above conclusion could be realized either by large masses of the extra Higgs bosons (the decoupling regime) or by alignment with relatively light extra Higgs bosons (the case discussed here). For both possibilities the  $H_1VV$  coupling is SM-like and CP violation disappears (within the 2HDM with  $\mathbb{Z}_2$  softly broken).

If  $\lambda_6 = \lambda_7 = 0$ , so within the  $\mathbb{Z}_2$ -symmetric model (HDM5),  
the alignment implies no CP violation.



In order to have scalar sector CPV in the alignment limit the  $\mathbb{Z}_2$  must be relaxed (2HDM67).



FCNC

## Alignment conditions in terms of the potential parameters

$$\begin{aligned} v_2^2 \text{Im} (e^{i\xi} \lambda_7) + v_2 v_1 \text{Im} (e^{2i\xi} \lambda_5) + v_1^2 \text{Im} (e^{i\xi} \lambda_6) &= 0, \\ v_2^4 \text{Re} (e^{i\xi} \lambda_7) + v_2^3 v_1 (-\lambda_2 + \lambda_{345}) + 3v_2^2 v_1^2 \text{Re} [e^{i\xi} (\lambda_6 - \lambda_7)] + \\ &+ v_2 v_1^3 (\lambda_1 - \lambda_{345}) - v_1^4 \text{Re} (e^{i\xi} \lambda_6) = 0 \end{aligned}$$

where  $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \text{Re} (e^{2i\xi} \lambda_5)$ .

In the CP-conserving limit, with  $\xi = 0$ ,  $\text{Im} \lambda_5 = \text{Im} \lambda_6 = \text{Im} \lambda_7 = 0$ , we reproduce the single alignment condition found by Dev and Pilaftsis, JHEP 1412 (2014) 024,

If one wishes to satisfy the alignment conditions for any value of  $v_1$ ,  $v_2$  and  $\xi$ , the following constraints must be fulfilled:

$$\lambda_1 = \lambda_2 = \lambda_3 + \lambda_4, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0$$

- The above condition is inconsistent with CPV
- In a different context a potential that satisfies the above condition was considered in “Do precision electroweak constraints guarantee  $e^+e^-$  collider discovery of at least one Higgs boson of a two Higgs doublet model?” P. Chankowski, T. Farris, B. Grzadkowski, J. Gunion, J. Kalinowski, M. Krawczyk, Phys.Lett. B496 (2000) 195-205



## Alignment defined in terms of bilinear potential couplings and vevs

Using the minimization conditions the alignment conditions can be formulated as

$$\begin{aligned}\operatorname{Im} m_{12}^2 &= 0, \\ m_{11}^2 - m_{22}^2 &= \operatorname{Re} m_{12}^2 \left( \frac{v_1}{v_2} - \frac{v_2}{v_1} \right).\end{aligned}$$

## Numerical strategy and illustrations

Parameters:

- for 2HDM5 ( $\mathbb{Z}_2$  imposed, so  $\lambda_6 = \lambda_7 = 0$ ):

$$\mathcal{P}_5 \equiv \{M_{H^\pm}^2, \mu^2, M_1^2, M_2^2, v_1, v_2, \xi = 0, \alpha_1, \alpha_2, \alpha_3\}$$

- for 2HDM67 ( $\mathbb{Z}_2$  not imposed, so  $\lambda_6 \neq 0, \lambda_7 \neq 0$ ):

$$\mathcal{P}_{67} \equiv \{M_{H^\pm}^2, \mu^2, M_1^2, M_2^2, M_3^2, \text{Im}\lambda_5, \text{Re}\lambda_6, \text{Re}\lambda_7, v_1, v_2, \xi = 0, \alpha_1, \alpha_2, \alpha_3\}$$

Plots shown in next slides have been obtained adopting the following strategy:

- 2HDM5 ( $\mathbb{Z}_2$  imposed, so  $\lambda_6 = \lambda_7 = 0$ ):
  - $M_{H^\pm}^2, \mu^2, M_2, \tan\beta$  are fixed parameters
  - scan over  $\alpha_1, \alpha_2, \alpha_3$  for chosen maximal deviation  $\delta \equiv |e_1/v - 1|$  and imposing  $M_1 < M_2 < M_3$ , vacuum stability and unitarity.
- for 2HDM67 ( $\mathbb{Z}_2$  not imposed, so  $\lambda_6 \neq 0, \lambda_7 \neq 0$ ):
  - $M_{H^\pm}^2, \mu^2, M_2, M_3$ , and  $\tan\beta$  are fixed parameters
  - scan over  $\alpha_1, \alpha_2, \alpha_3, \text{Im}\lambda_5, \text{Re}\lambda_6, \text{Re}\lambda_7$ , for chosen maximal deviation  $\delta \equiv |e_1/v - 1|$  and imposing  $M_1 < M_2 < M_3$ , vacuum stability and unitarity.

$\tan \beta=2, M_1=125, M_2=400, M_{\text{ch}}=500, \mu=400$

$\tan \beta=2, M_1=125, M_2=400, M_3=500, M_{\text{ch}}=500, \mu=400$

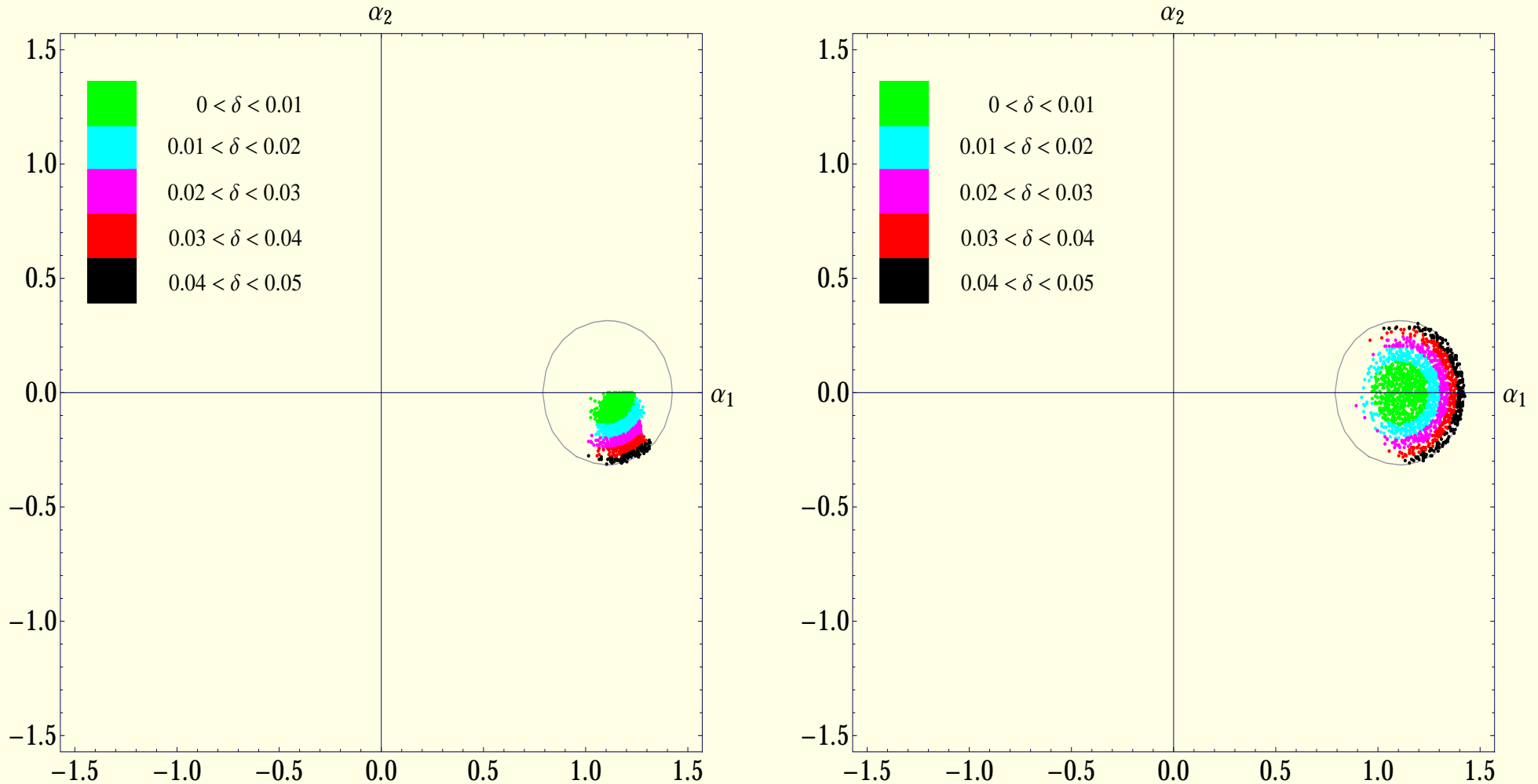


Figure 2: Allowed regions in the  $(\alpha_1, \alpha_2)$  space for  $\tan \beta = 2$  corresponding to maximal deviation  $\delta \equiv |e_1/v - 1| = 0.05$  within 2HDM5 ( $\mathbb{Z}_2$  imposed) and 2HDM67, are shown in the left and right panels, respectively. Coloring corresponds to ranges of  $\delta$  shown in the legend. Vacuum stability and unitarity constraints are satisfied. Parameters adopted are shown in the plot.

The goal is to see

**How much CP violation remain  
for a given maximal deviation  $\delta \equiv |e_1/v - 1|$  from the alignment limit?**



For points inside "circles" we calculate  $\text{Im } J_1$ ,  $\text{Im } J_2$  and  $\text{Im } J_{30}$

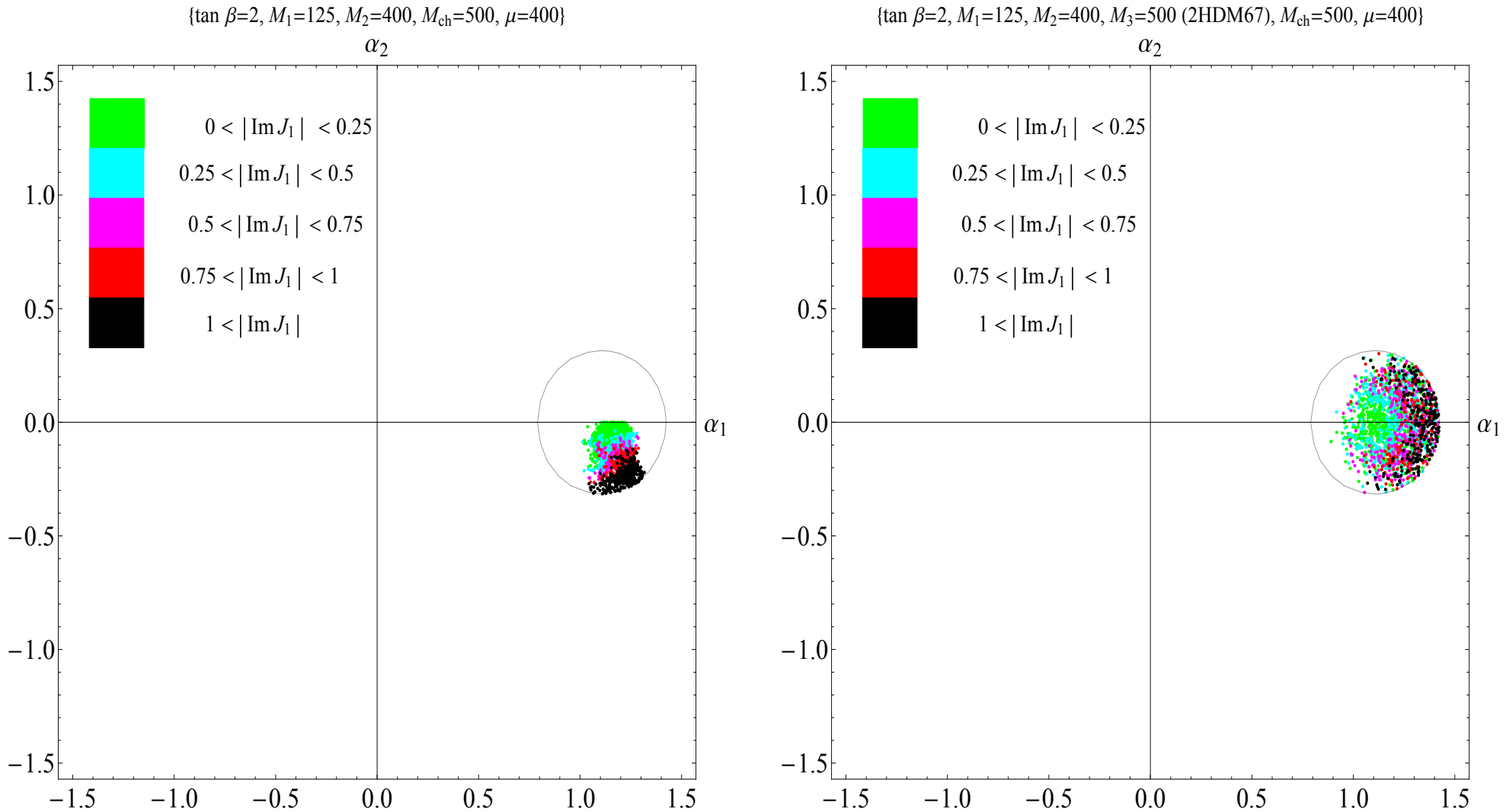


Figure 3: Allowed regions in the  $(\alpha_1, \alpha_2)$  space for  $\tan \beta = 2$  corresponding to maximal deviation  $\delta \equiv |e_1/v - 1| = 0.05$  within 2HDM5 ( $\mathbb{Z}_2$  imposed) and 2HDM67, are shown in the left and right panels, respectively. Coloring corresponds to ranges of  $|\text{Im } J_1|$  shown in the legend. Vacuum stability and unitarity constraints are satisfied. Parameters adopted are shown in the plot.

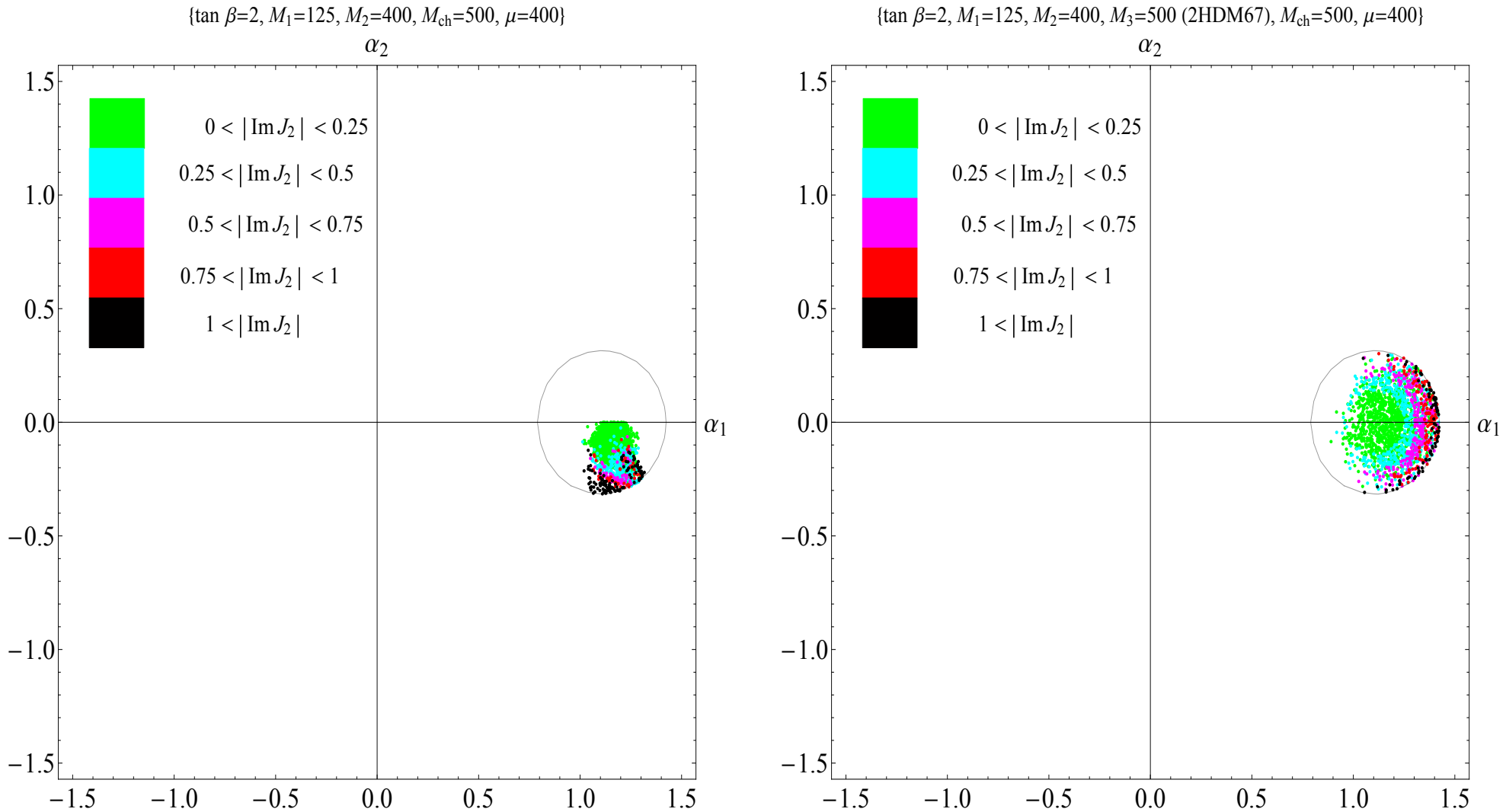


Figure 4: Allowed regions in the  $(\alpha_1, \alpha_2)$  space for  $\tan \beta = 2$  corresponding to maximal deviation  $\delta \equiv |e_1/v - 1| = 0.05$  within 2HDM5 ( $\mathbb{Z}_2$  imposed) and 2HDM67, are shown in the left and right panels, respectively. Coloring corresponds to ranges of  $|\text{Im } J_2|$  shown in the legend. Vacuum stability and unitarity constraints are satisfied. Parameters adopted are shown in the plot.

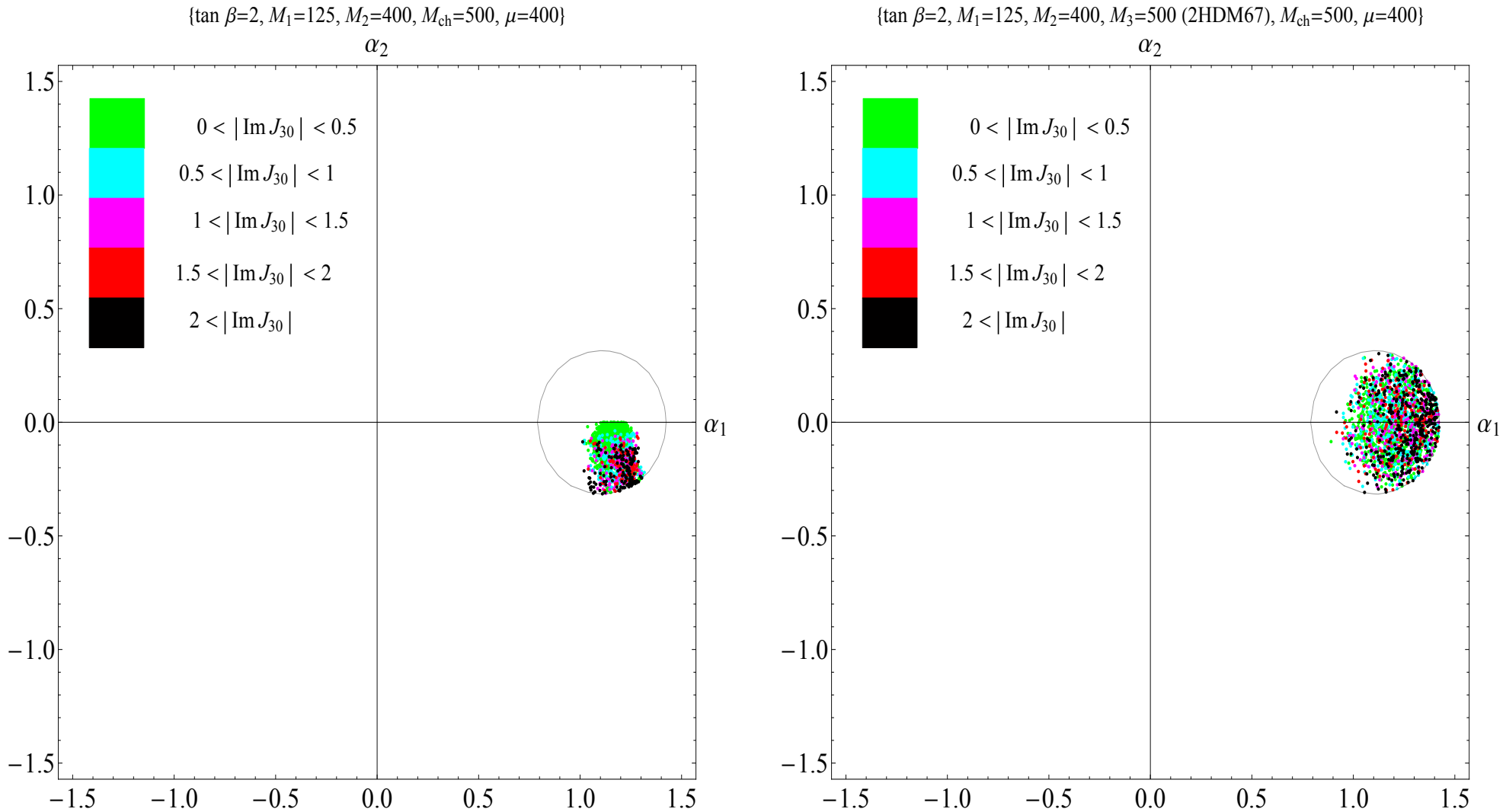


Figure 5: Allowed regions in the  $(\alpha_1, \alpha_2)$  space for  $\tan \beta = 2$  corresponding to maximal deviation  $\delta \equiv |e_1/v - 1| = 0.05$  within 2HDM5 ( $\mathbb{Z}_2$  imposed) and 2HDM67, are shown in the left and right panels, respectively. Coloring corresponds to ranges of  $|\text{Im } J_3|$  shown in the legend. Vacuum stability and unitarity constraints are satisfied. Parameters adopted are shown in the plot.

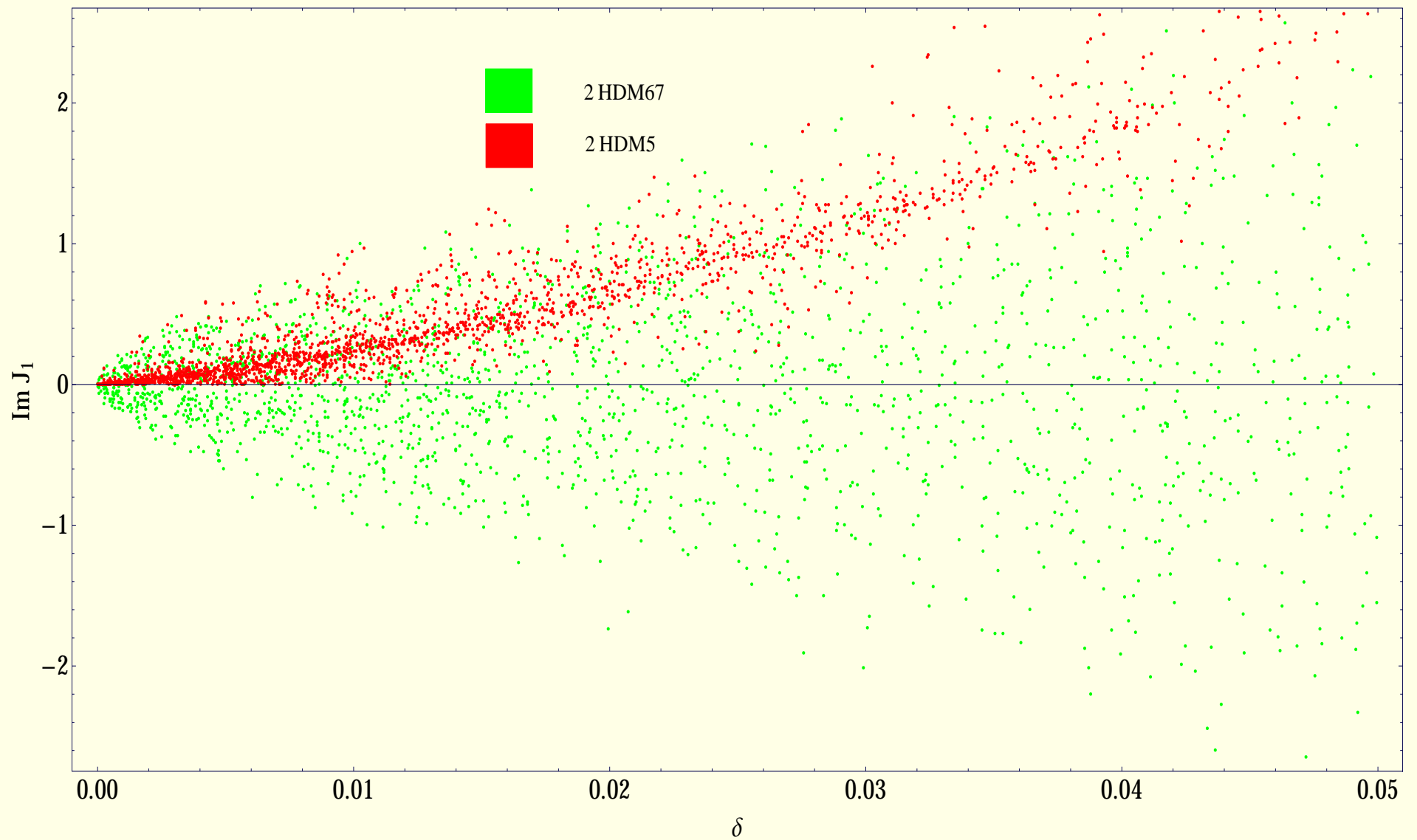


Figure 6: Correlation between  $\text{Im } J_1$  and the maximal deviation  $\delta \equiv |e_1/v - 1| = 0.05$ . Green and red dots correspond to 2HDM67 and 2HDM5, respectively.



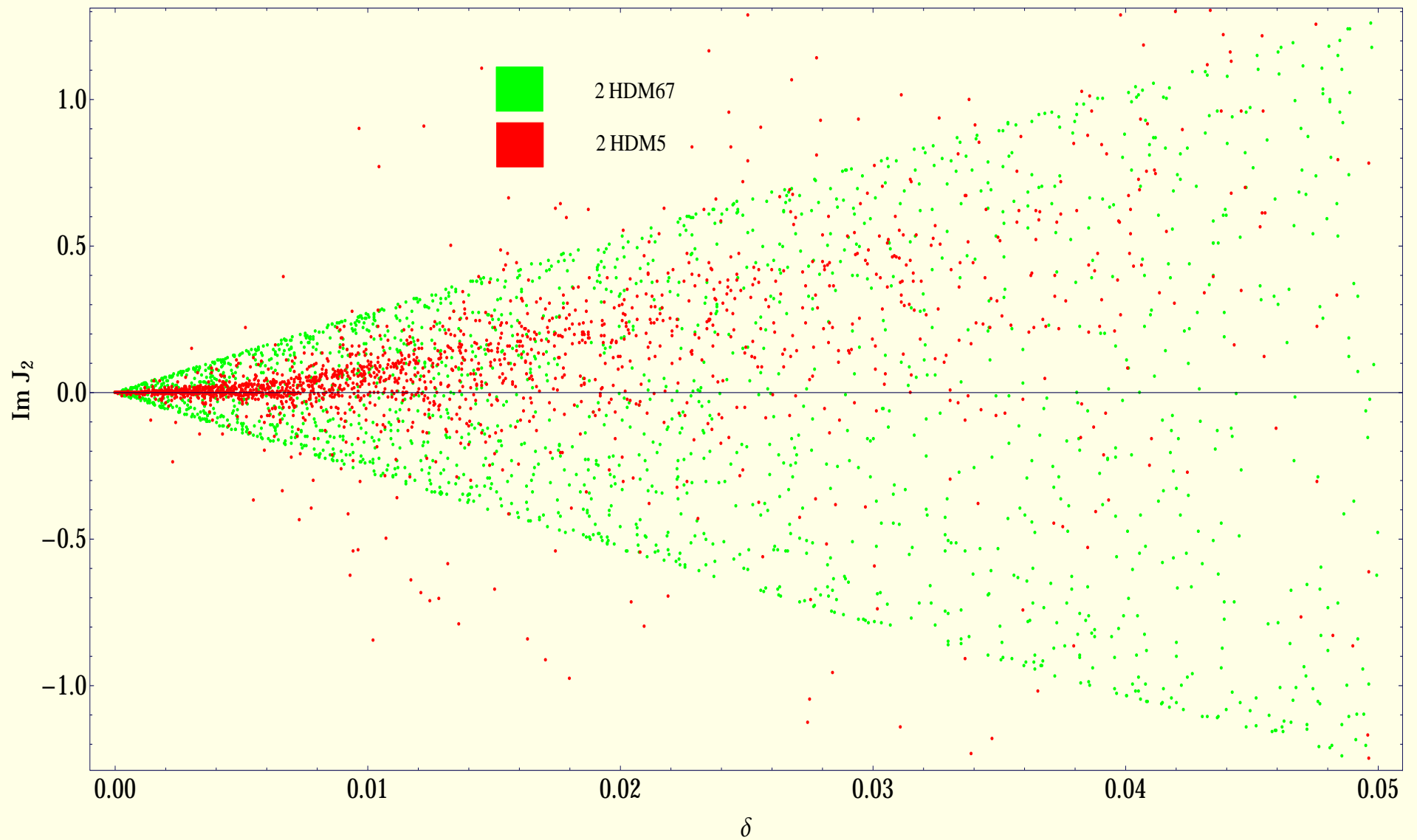


Figure 7: Correlation between  $\text{Im } J_2$  and the maximal deviation  $\delta \equiv |e_1/v - 1| = 0.05$ . Green and red dots correspond to 2HDM67 and 2HDM5, respectively.

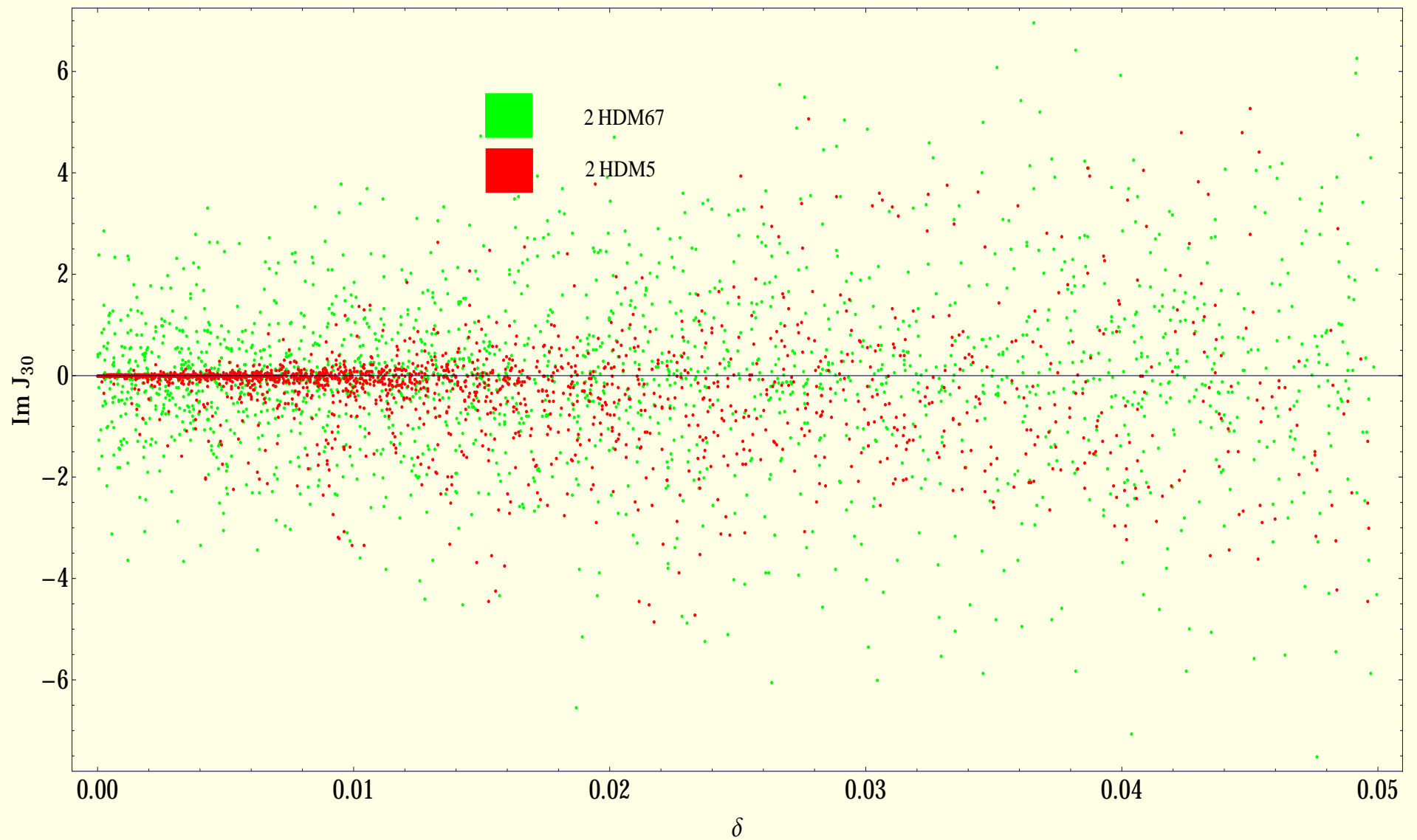


Figure 8: Correlation between  $\text{Im } J_{30}$  and the maximal deviation  $\delta \equiv |e_1/v - 1| = 0.05$ . Green and red dots correspond to 2HDM67 and 2HDM5, respectively.

## Spontaneous CP violation

The goal: to formulate conditions for SCPV in terms of observables

$$\begin{aligned}
 \text{Im } J_1 &= \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} M_i^2 e_i e_k q_j \\
 &= \frac{1}{v^5} [e_1 e_2 q_3 (M_2^2 - M_1^2) - e_1 e_3 q_2 (M_3^2 - M_1^2) + e_2 e_3 q_1 (M_3^2 - M_2^2)], \\
 \text{Im } J_2 &= \frac{2}{v^9} \sum_{i,j,k} \epsilon_{ijk} e_i e_j e_k M_i^4 M_k^2 = \frac{2e_1 e_2 e_3}{v^9} \sum_{i,j,k} \epsilon_{ijk} M_i^4 M_k^2 \\
 &= \frac{2e_1 e_2 e_3}{v^9} (M_2^2 - M_1^2)(M_3^2 - M_2^2)(M_3^2 - M_1^2), \\
 \text{Im } J_{30} &\equiv \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} q_i M_i^2 e_j q_k, \\
 &= \frac{1}{v^5} [q_1 q_2 e_3 (M_2^2 - M_1^2) - q_1 q_3 e_2 (M_3^2 - M_1^2) + q_2 q_3 e_1 (M_3^2 - M_2^2)].
 \end{aligned}$$

**Theorem: CP is conserved if and only if  $\text{Im } J_1 = \text{Im } J_2 = \text{Im } J_{30} = 0$ .**

$$\begin{aligned}
V(\Phi_1, \Phi_2) &= -\frac{1}{2} \left\{ m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right] \right\} \\
&+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
&+ \frac{1}{2} \left[ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.} \right] + \left\{ \left[ \lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right] (\Phi_1^\dagger \Phi_2) + \text{H.c.} \right\} \\
&\equiv Y_{a\bar{b}} \Phi_{\bar{a}}^\dagger \Phi_b + \frac{1}{2} Z_{a\bar{b}c\bar{d}} (\Phi_{\bar{a}}^\dagger \Phi_b) (\Phi_{\bar{c}}^\dagger \Phi_d).
\end{aligned}$$

**Theorem:** In order for CP violation to be spontaneous, at least one of the  $\text{Im } J_i$  invariants must be non-zero, while four other weak-basis  $I$  invariants constructed from the coefficients of the potential, must vanish.

$$I_{Y3Z} = \text{Im} \left[ Z_{a\bar{c}}^{(1)} Z_{e\bar{b}}^{(1)} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}} \right],$$

$$I_{2Y2Z} = \text{Im} \left[ Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z_{f\bar{c}}^{(1)} \right],$$

$$I_{3Y3Z} = \text{Im} \left[ Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}} \right],$$

$$I_{6Z} = \text{Im} \left[ Z_{a\bar{b}c\bar{d}} Z_{b\bar{f}}^{(1)} Z_{d\bar{h}}^{(1)} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}} \right].$$

## Theorem:

Let us assume that the quantity

$$D \equiv e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2$$

is non-zero. Then, in a charge-conserving general 2HDM, CP is violated spontaneously if and only if the following three statements are satisfied simultaneously:

- At least one of the three invariants  $\text{Im } J_1$ ,  $\text{Im } J_2$ ,  $\text{Im } J_{30}$  is nonzero.
- $M_{H^\pm}^2 = \frac{v^2}{2D} [e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2]$ ,
- $q = \frac{1}{2D} [(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2]$ .

# Spontaneous CP violation in the alignment limit in terms of physical couplings

$$e_1 = v, e_2 = e_3 = 0$$

⇓

$$M_{H^\pm}^2 = \frac{vq_1 - M_1^2}{2},$$
$$q = \frac{1}{2} \left( \frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2} + \frac{M_1^2}{v^2} \right).$$

## Summary

- 2HDM allows for extra sources of CP-violation that might be useful to explain baryon asymmetry.
- We have defined the alignment limit as  $e_1 = v$ , so that  $H_1$  couples to  $VV$  as in the SM.
- In the alignment limit there is no CP-violation if  $\lambda_6 = \lambda_7 = 0$  ( $\mathbb{Z}_2$  imposed).
- The requirement of extra sources of CP-violation in the presence of light extra scalars favours the most general 2HDM with  $\lambda_6 \neq 0$  and  $\lambda_7 \neq 0$  (no  $\mathbb{Z}_2$  symmetry).
- The requirement of extra sources of CP-violation in the presence of light extra scalars implies an interesting possibility of large FCNC that couple to Higgs bosons (in progress).
- In order to disprove SCPV a minimal set of measurements consists of  $M_{H^\pm}$  and  $q_1$ , if they do not satisfy  $q_1 = (2M_{H^\pm}^2 + M_1^2) / v$ , then CP is not violated spontaneously.

- To prove SCPV is strictly speaking impossible since one would need to show that the conditions

$$M_{H^\pm}^2 = \frac{v^2}{2D} [e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2],$$

$$q = \frac{1}{2D} [(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2].$$

hold exactly. Since measurements are always subject to experimental (and theoretical) uncertainties, indeed, the above equations could at best only hold within some confidence level. Note, however, that the verification of the above constraints requires a determination of 11 parameters.  $M_1$  and  $v$  are already known, so 9 new measurements should be performed in order to test these constraints. Therefore we conclude that in order to test SCPV, all potential parameters must be known.



Stationary points of the potential

By demanding that the derivatives of the potential with respect to the fields should vanish in the vacuum, we end up with the following stationary-point equations:

$$\begin{aligned}
 m_{11}^2 &= v_1^2 \lambda_1 + v_2^2 (\lambda_3 + \lambda_4) + \frac{v_2^2}{c_\xi} (\text{Re } \lambda_5 c_\xi - \text{Im } \lambda_5 s_\xi) \\
 &\quad + \frac{v_1 v_2}{c_\xi} \left[ \text{Re } \lambda_6 (2 + c_{2\xi}) - \text{Im } \lambda_6 s_{2\xi} \right] + \frac{v_2}{v_1 c_\xi} (v_2^2 \text{Re } \lambda_7 - \text{Re } m_{12}^2), \\
 m_{22}^2 &= v_2^2 \lambda_2 + v_1^2 (\lambda_3 + \lambda_4) + \frac{v_1^2}{c_\xi} (\text{Re } \lambda_5 c_\xi - \text{Im } \lambda_5 s_\xi) \\
 &\quad + \frac{v_1 v_2}{c_\xi} \left[ \text{Re } \lambda_7 (2 + c_{2\xi}) - \text{Im } \lambda_7 s_{2\xi} \right] + \frac{v_1}{v_2 c_\xi} (v_1^2 \text{Re } \lambda_6 - \text{Re } m_{12}^2), \\
 \text{Im } m_{12}^2 &= \frac{v_1 v_2}{c_\xi} (\text{Re } \lambda_5 s_{2\xi} + \text{Im } \lambda_5 c_{2\xi}) + \frac{v_1^2}{c_\xi} (\text{Re } \lambda_6 s_\xi + \text{Im } \lambda_6 c_\xi) \\
 &\quad + \frac{v_2^2}{c_\xi} (\text{Re } \lambda_7 s_\xi + \text{Im } \lambda_7 c_\xi) - \text{Re } m_{12}^2 t_\xi,
 \end{aligned}$$

with  $c_x = \cos x$ ,  $s_x = \sin x$ , and  $t_x = \tan x$ . Thus, we may eliminate  $m_{11}^2$ ,  $m_{22}^2$  and  $\text{Im } m_{12}^2$  from the potential by these substitutions, thereby reducing the number of parameters of the model.

$$\begin{aligned} \mathcal{M}_{11}^2 &= v_1^2 \lambda_1 - v_2^2 s_\xi^2 \text{Re } \lambda_5 - \frac{v_2^2}{2c_\xi} s_\xi c_{2\xi} \text{Im } \lambda_5 + \frac{v_1 v_2}{2c_\xi} (1 + 2c_{2\xi}) \text{Re } \lambda_6 \\ &\quad - 2v_1 v_2 s_\xi \text{Im } \lambda_6 - \frac{v_2^3}{2v_1 c_\xi} \text{Re } \lambda_7 + \frac{v_2}{2v_1 c_\xi} \text{Re } m_{12}^2, \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{22}^2 &= v_2^2 \lambda_2 - v_1^2 s_\xi^2 \text{Re } \lambda_5 - \frac{v_1^2}{2c_\xi} s_\xi c_{2\xi} \text{Im } \lambda_5 + \frac{v_1 v_2}{2c_\xi} (1 + 2c_{2\xi}) \text{Re } \lambda_7 \\ &\quad - 2v_1 v_2 s_\xi \text{Im } \lambda_7 - \frac{v_1^3}{2v_2 c_\xi} \text{Re } \lambda_6 + \frac{v_1}{2v_2 c_\xi} \text{Re } m_{12}^2, \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{33}^2 &= -v^2 c_\xi^2 \text{Re } \lambda_5 - \frac{v^2}{2c_\xi} (2s_\xi^3 - 3s_\xi) \text{Im } \lambda_5 \\ &\quad - \frac{v^2 v_1}{2v_2 c_\xi} \text{Re } \lambda_6 - \frac{v^2 v_2}{2v_1 c_\xi} \text{Re } \lambda_7 + \frac{v^2}{2v_1 v_2 c_\xi} \text{Re } m_{12}^2, \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{12}^2 &= v_1 v_2 (\lambda_3 + \lambda_4) + v_1 v_2 c_\xi^2 \text{Re } \lambda_5 + \frac{v_1 v_2}{2c_\xi} (2s_\xi^3 - 3s_\xi) \text{Im } \lambda_5 + \frac{v_1^2}{2c_\xi} (2 + c_{2\xi}) \text{Re } \lambda_6 \\ &\quad - v_1^2 s_\xi \text{Im } \lambda_6 + \frac{v_2^2}{2c_\xi} (2 + c_{2\xi}) \text{Re } \lambda_7 - v_2^2 s_\xi \text{Im } \lambda_7 - \frac{1}{2c_\xi} \text{Re } m_{12}^2, \end{aligned}$$

$$\mathcal{M}_{13}^2 = -\frac{1}{2} v v_2 s_{2\xi} \text{Re } \lambda_5 - \frac{1}{2} v v_2 c_{2\xi} \text{Im } \lambda_5 - v v_1 s_\xi \text{Re } \lambda_6 - v v_1 c_\xi \text{Im } \lambda_6,$$

$$\mathcal{M}_{23}^2 = -\frac{1}{2} v v_1 s_{2\xi} \text{Re } \lambda_5 - \frac{1}{2} v v_1 c_{2\xi} \text{Im } \lambda_5 - v v_2 s_\xi \text{Re } \lambda_7 - v v_2 c_\xi \text{Im } \lambda_7.$$

The charge boson mass is given as follows

$$M_{H^\pm}^2 = \frac{v^2}{2v_1v_2c_\xi} \text{Re} \left( m_{12}^2 - v_1^2\lambda_6 - v_2^2\lambda_7 - v_1v_2e^{i\xi} [\lambda_4 + \lambda_5] \right)$$

The eigenvalues of this matrix will be the masses of the three neutral scalars. In order to find these, a cubic equation needs to be solved. For our purposes, a different approach will suffice. We may rewrite the elements of the mass matrix  $\mathcal{M}_{ij}^2$  in terms of the eigenvalues  $M_i^2$  and elements of the rotation matrix  $R_{ij}$  as six equations:

$$\mathcal{M}_{11}^2 = M_1^2 R_{11}^2 + M_2^2 R_{21}^2 + M_3^2 R_{31}^2, \quad (3)$$

$$\mathcal{M}_{22}^2 = M_1^2 R_{12}^2 + M_2^2 R_{22}^2 + M_3^2 R_{32}^2, \quad (4)$$

$$\mathcal{M}_{33}^2 = M_1^2 R_{13}^2 + M_2^2 R_{23}^2 + M_3^2 R_{33}^2, \quad (5)$$

$$\mathcal{M}_{12}^2 = M_1^2 R_{11}R_{12} + M_2^2 R_{21}R_{22} + M_3^2 R_{31}R_{32}, \quad (6)$$

$$\mathcal{M}_{13}^2 = M_1^2 R_{11}R_{13} + M_2^2 R_{21}R_{23} + M_3^2 R_{31}R_{33}, \quad (7)$$

$$\mathcal{M}_{23}^2 = M_1^2 R_{12}R_{13} + M_2^2 R_{22}R_{23} + M_3^2 R_{32}R_{33}. \quad (8)$$

The above seven equations are linear in the  $\lambda_i$ -parameters of the potential. We have 10 such parameters (counting both real and imaginary parts of  $\lambda_5$ ,  $\lambda_6$  and  $\lambda_7$ ) and may now solve this set of seven equations for seven of the  $\lambda_i$ -parameters, thus expressing them in terms of the other parameters we have introduced. It is convenient to solve for the following set of parameters:  $(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \text{Re}\lambda_5, \text{Im}\lambda_6, \text{Im}\lambda_7)$ . We also introduce the more convenient parameter  $\mu^2$  by putting

$$\text{Re } m_{12}^2 = \frac{2v_1v_2}{v^2} \mu^2.$$

Solving the set of equations, we arrive at

$$\begin{aligned}
\lambda_1 &= -\frac{v_2^2}{v^2 v_1^2 c_\xi^3} \mu^2 + \frac{(R_{11}v - R_{13}v_2 t_\xi)^2}{v^2 v_1^2} M_1^2 + \frac{(R_{21}v - R_{23}v_2 t_\xi)^2}{v^2 v_1^2} M_2^2 \\
&\quad + \frac{(R_{31}v - R_{33}v_2 t_\xi)^2}{v^2 v_1^2} M_3^2 - \frac{v_2^2 t_\xi}{2v_1^2 c_\xi^2} \text{Im}\lambda_5 - \frac{v_2(2c_2\xi + 1)}{2v_1 c_\xi^3} \text{Re}\lambda_6 + \frac{v_2^3}{2v_1^3 c_\xi^3} \text{Re}\lambda_7, \\
\lambda_2 &= -\frac{v_1^2}{v^2 v_2^2 c_\xi^3} \mu^2 + \frac{(R_{12}v - R_{13}v_1 t_\xi)^2}{v^2 v_2^2} M_1^2 + \frac{(R_{22}v - R_{23}v_1 t_\xi)^2}{v^2 v_2^2} M_2^2 \\
&\quad + \frac{(R_{32}v - R_{33}v_1 t_\xi)^2}{v^2 v_2^2} M_3^2 - \frac{v_1^2 t_\xi}{2v_2^2 c_\xi^2} \text{Im}\lambda_5 + \frac{v_1^3}{2v_2^3 c_\xi^3} \text{Re}\lambda_6 - \frac{v_1(2c_2\xi + 1)}{2v_2 c_\xi^3} \text{Re}\lambda_7, \\
\lambda_3 &= \frac{2}{v^2} M_{H^\pm}^2 - \frac{1}{v^2 c_\xi^3} \mu^2 + \frac{(R_{12}v - R_{13}v_1 t_\xi)(R_{11}v - R_{13}v_2 t_\xi)}{v^2 v_1 v_2} M_1^2 \\
&\quad + \frac{(R_{22}v - R_{23}v_1 t_\xi)(R_{21}v - R_{23}v_2 t_\xi)}{v^2 v_1 v_2} M_2^2 \\
&\quad + \frac{(R_{32}v - R_{33}v_1 t_\xi)(R_{31}v - R_{33}v_2 t_\xi)}{v^2 v_1 v_2} M_3^2 \\
&\quad - \frac{1}{2c_\xi^2} t_\xi \text{Im}\lambda_5 - \frac{v_1 c_2 \xi}{2v_2 c_\xi^3} \text{Re}\lambda_6 - \frac{v_2 c_2 \xi}{2v_1 c_\xi^3} \text{Re}\lambda_7,
\end{aligned}$$

$$\begin{aligned}
\lambda_4 &= -\frac{2}{v^2}M_{H^\pm}^2 + \frac{c_2\xi}{v^2c_\xi^3}\mu^2 + \frac{R_{13}^2}{v^2c_\xi^2}M_1^2 + \frac{R_{23}^2}{v^2c_\xi^2}M_2^2 + \frac{R_{33}^2}{v^2c_\xi^2}M_3^2 \\
&\quad - \frac{1}{2c_\xi^2}t_\xi\text{Im}\lambda_5 - \frac{v_1c_2\xi}{2v_2c_\xi^3}\text{Re}\lambda_6 - \frac{v_2c_2\xi}{2v_1c_\xi^3}\text{Re}\lambda_7, \\
\text{Re}\lambda_5 &= \frac{1}{v^2c_\xi^3}\mu^2 - \frac{R_{13}^2}{v^2c_\xi^2}M_1^2 - \frac{R_{23}^2}{v^2c_\xi^2}M_2^2 - \frac{R_{33}^2}{v^2c_\xi^2}M_3^2 \\
&\quad + \frac{1}{4c_\xi^3}(3s_\xi + s_3\xi)\text{Im}\lambda_5 - \frac{v_1}{2v_2c_\xi^3}\text{Re}\lambda_6 - \frac{v_2}{2v_1c_\xi^3}\text{Re}\lambda_7, \\
\text{Im}\lambda_6 &= -\frac{v_2t_\xi}{v^2v_1c_\xi^2}\mu^2 + \frac{R_{13}(R_{13}v_2t_\xi - R_{11}v)}{v^2v_1c_\xi}M_1^2 + \frac{R_{23}(R_{23}v_2t_\xi - R_{21}v)}{v^2v_1c_\xi}M_2^2 \\
&\quad + \frac{R_{33}(R_{33}v_2t_\xi - R_{31}v)}{v^2v_1c_\xi}M_3^2 - \frac{v_2}{2v_1c_\xi^3}\text{Im}\lambda_5 - \frac{1}{2c_\xi^2}t_\xi c_2\xi\text{Re}\lambda_6 + \frac{v_2^2t_\xi}{2v_1^2c_\xi^2}\text{Re}\lambda_7, \\
\text{Im}\lambda_7 &= -\frac{v_1t_\xi}{v^2v_2c_\xi^2}\mu^2 + \frac{R_{13}(R_{13}v_1t_\xi - R_{12}v)}{v^2v_2c_\xi}M_1^2 + \frac{R_{23}(R_{23}v_1t_\xi - R_{22}v)}{v^2v_2c_\xi}M_2^2 \\
&\quad + \frac{R_{33}(R_{33}v_1t_\xi - R_{32}v)}{v^2v_2c_\xi}M_3^2 - \frac{v_1}{2v_2c_\xi^3}\text{Im}\lambda_5 + \frac{v_1^2t_\xi}{2v_2^2c_\xi^2}\text{Re}\lambda_6 - \frac{1}{2c_\xi^2}t_\xi c_2\xi\text{Re}\lambda_7.
\end{aligned}$$