University of Warsaw, Warsaw, December 13th 2007

Looking for new physics

Bohdan GRZADKOWSKI

Institute of Theoretical Physics, University of Warsaw
Hoża 69, PL-00-681 Warsaw, POLAND

1. Introduction to the Standard Model
   - Theory (gauge and Higgs sectors, fermions, mixing, . . .)
   - Experiments (LEP, . . .)
2. Outstanding problems of the high-energy physics (why should we go beyond the SM?)
3. Possible extensions of the SM
4. Future perspectives with the LHC
5. Summary

Introduction to the Standard Model: Theory

♠ Gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{L} \supset -\frac{1}{4} \underbrace{F_a^{\mu\nu} F_{a\,\mu\nu}}_{SU(3)_C} - \frac{1}{4} \underbrace{W_i^{\mu\nu} W_{i\,\mu\nu}}_{SU(2)_L} - \frac{1}{4} \underbrace{B^{\mu\nu} B_{\mu\nu}}_{U(1)_Y}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$G_a^\mu|_{a=1,\ldots 8} \qquad\qquad W_\mu^\pm, Z_\mu, A_\mu$$

♠ The Higgs sector:

- The minimal choice $H = \begin{pmatrix} G^+ \\ (H + iG^0)/\sqrt{2} \end{pmatrix}$ necessary for $SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

$$\mathcal{L} \supset (D_\mu H)^\dagger D^\mu H - V(H)$$

for $D_\mu \equiv \partial_\mu + igW_\mu^i T^i + ig' \frac{1}{2} Y B_\mu$ and $V(H) = \mu^2 |H|^2 + \lambda |H|^4$ with $Y_H = \frac{1}{2}$

- If $\mu^2 < 0$ then $\langle 0 || H|^2 | 0 \rangle = -\frac{1}{2} \frac{\mu^2}{\lambda} \equiv \frac{v^2}{2}$ (spontaneous symmetry breaking, the origin of mass)

- Boson masses: $m_h = \sqrt{2\lambda} v$, $m_{W^\pm} = \frac{1}{2} g v$ and $m_Z = m_W / c_W$, for $c_W \equiv \cos\theta_W = g/(g^2 + g'^{\,2})^{1/2}$

♠ Fermions

fermion	T	T_3	$\frac{1}{2}Y$	Q
$\nu_{i\ L}$	$\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0
$l_{i\ L}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
$u_{i\ L}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
$d_{i\ L}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
$l_{i\ R}$	0	0	-1	-1
$u_{i\ R}$	0	0	$\frac{2}{3}$	$\frac{2}{3}$
$d_{i\ R}$	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$\nu_{i\ R}$	0	0	0	0

$i = 1, \ldots, N_f = 3$, $\psi_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)\psi$ (parity violation), $Q = T_3 + \frac{1}{2}Y$

Neutrino masses:

- Dirac mass: $f_{ij}\ \bar{L}_{i\ L}\nu_{j\ R}\ \tilde{H} + \text{H.c.}$ for $\tilde{H} \equiv i\tau_2 H^\star$
- Majorana mass: $\frac{1}{2}M_{ij}\ \nu_{i\ R}\ \mathbf{C}\ \nu_{j\ R} + \text{H.c.}$

Gauge transformations:

$$\psi(x) \to \exp\left\{-igT^i\theta_i(x) - ig'\frac{1}{2}Y\beta(x)\right\}\psi(x)$$

Gauge interactions:

$$\mathcal{L} \supset \sum_\psi \bar{\psi}i\gamma^\mu D_\mu\psi \quad \text{for} \quad D_\mu \equiv \partial_\mu + igW^i_\mu T^i + ig'\frac{1}{2}YB_\mu$$

Yukawa interactions:

$$\mathcal{L} \supset -\sum_{i,j=1}^{3}\left(\tilde{\Gamma}_{ij}\bar{u}_{i\,R}\tilde{H}^\dagger Q_{j\,L} + \Gamma_{ij}\bar{d}_{i\,R}H^\dagger Q_{j\,L} + \text{H.c.}\right)$$

$$\Downarrow$$

$$\text{if } \langle H\rangle \neq 0 \text{ then } m_q \neq 0$$

$$\mathcal{L}_{\text{q mass}} = -\sum_{i,j=1}^{3}\left(\bar{u}_{i\,R}\mathcal{M}^u_{ij}Q_{j\,L} + \bar{d}_{i\,R}\mathcal{M}^d_{ij}Q_{j\,L} + \text{H.c.}\right)$$

for

$$\mathcal{M}^u_{ij} = \frac{v}{\sqrt{2}}\tilde{\Gamma}_{ij} \quad \mathcal{M}^d_{ij} = \frac{v}{\sqrt{2}}\Gamma_{ij} \quad \Rightarrow \quad \text{no FCNC for one Higgs boson doublet}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}$$

$$U_R^\dagger \mathcal{M}^u U_L = \mathrm{diag}(m_u, m_c, m_t) \qquad D_R^\dagger \mathcal{M}^d D_L = \mathrm{diag}(m_d, m_s, m_b)$$

$$\Downarrow$$

$$\tilde{\Gamma}, \Gamma \quad \text{diagonal} \quad (g_f = \sqrt{2}\frac{m_f}{v}) \quad \Rightarrow \quad \text{no FCNC}$$

- charged currents: $\sum \bar{u}_{i\,L} \gamma^\mu d_{i\,L} = (\bar{u}, \bar{c}, \bar{t})_L \underbrace{U_L^\dagger D_L}_{U_{CKM}} \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$
- neutral currents: $\sum \bar{u}_{i\,L} \gamma^\mu u_{i\,L}$, $\sum \bar{d}_{i\,L} \gamma^\mu d_{i\,L}$ remain unchanged upon $U_{L,R}$, $D_{L,R}$ transformations

U_{CKM}:

- unitary complex $N \times N$ matrix, $q_{i\,L} \to e^{i\alpha_i} q_{i\,L} \;\Rightarrow\; \frac{1}{2}(N-1)(N-2)$ phases in U_{CKM}
- $N \geq 3 \quad \Rightarrow \quad$ CP violation in charged currents

♠ Masses in the SM: $\quad m_V \propto g v \quad m_h \propto \lambda^{1/2} v \quad m_f \propto g_f v$

Leptons:

$$m_{\nu_e} \lesssim 3 \text{ eV} \qquad m_{\nu_\mu} \lesssim 0.2 \text{ MeV} \qquad m_{\nu_\tau} \lesssim 18 \text{ MeV}$$
$$m_e = 0.5 \text{ MeV} \qquad m_\mu = 105.5 \text{ MeV} \qquad m_\tau = 1.78 \text{ GeV}$$

Quarks:

$$m_u \simeq 2 \text{ MeV} \qquad m_c \simeq 1.2 \text{ GeV} \qquad m_t \simeq 174 \text{ GeV}$$
$$m_d = 5 \text{ MeV} \qquad m_s = 0.1 \text{ GeV} \qquad m_b = 4.3 \text{ GeV}$$

Bosons:

$$m_{W^\pm} = 80.4 \text{ GeV} \qquad m_Z = 91.2 \text{ GeV} \qquad m_h \geq 115 \text{ GeV}$$

$$\Downarrow$$

Fine tuning:

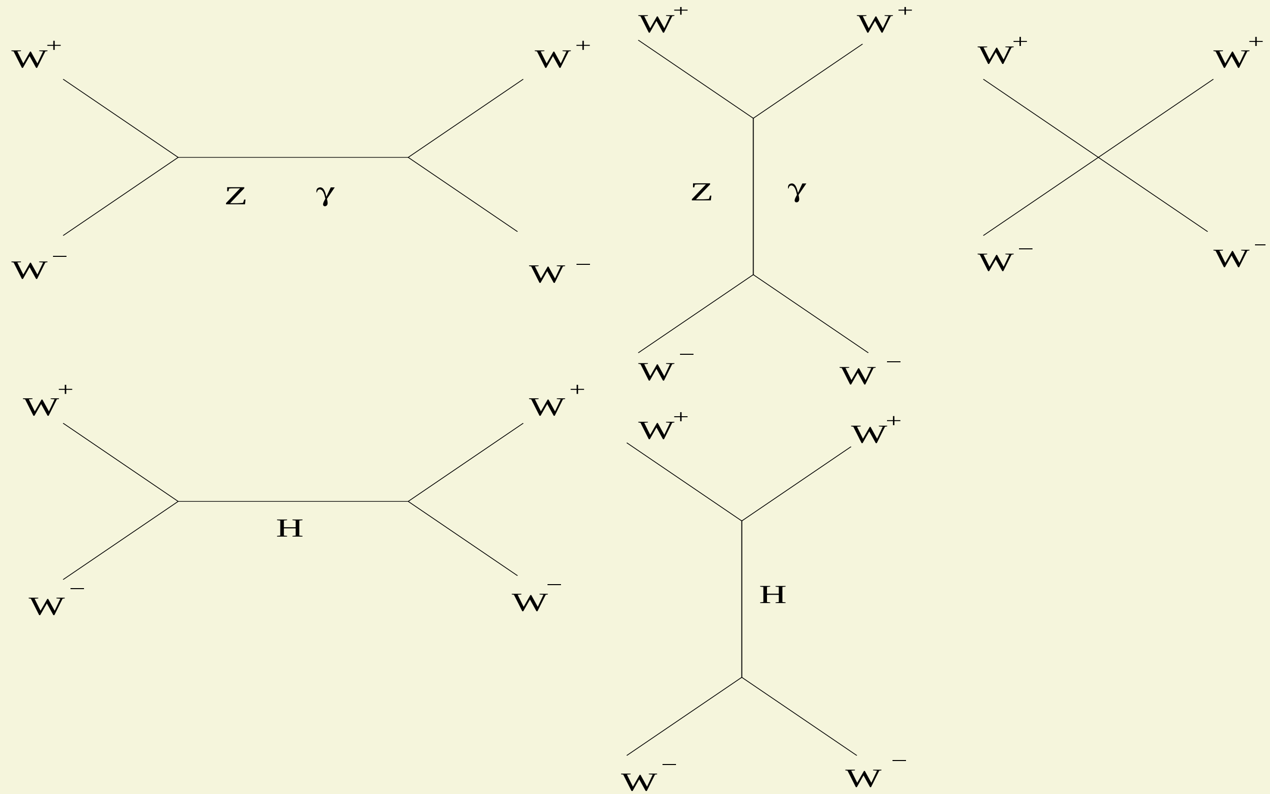
$$\frac{m_{\nu_e}}{m_t} \lesssim 0.5 \cdot 10^{-11} \qquad \Rightarrow \qquad \frac{g_{\nu_e}}{g_t} \lesssim 0.5 \cdot 10^{-11}$$

♠ Perturbative unitarity:

$$A \propto \frac{E^2}{m_W^2} \text{ for } E^2 \gg m_W^2 \qquad \Leftarrow$$

$$\Downarrow$$

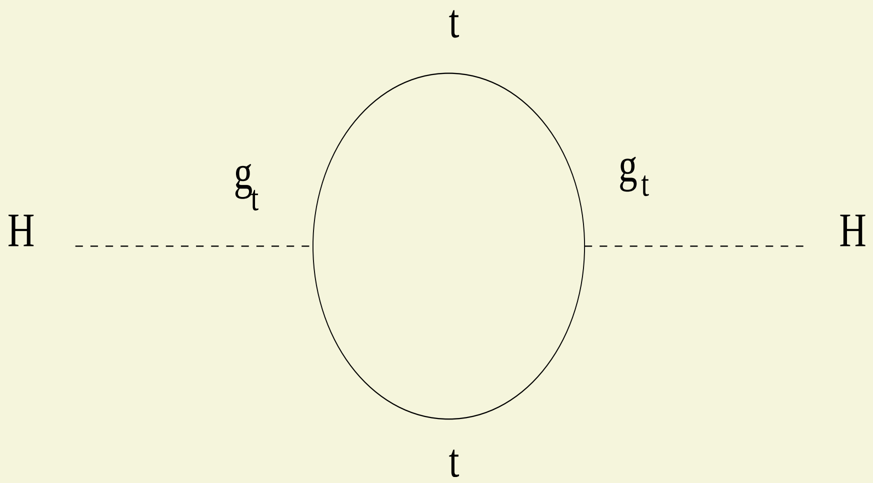
non-renormalizability



$$\Downarrow$$

$$A \propto \text{const.} \propto \frac{m_h^2}{v^2} = 2\lambda$$

♠ Radiative corrections to the Higgs mass:



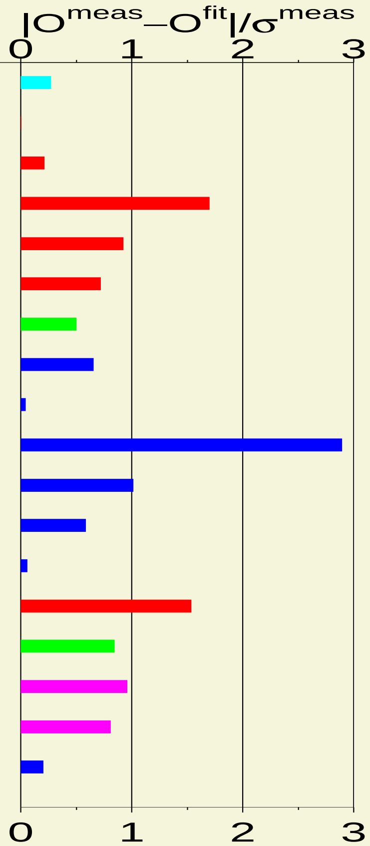
$$\Rightarrow \quad \delta m_h^2 \propto -g_t^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left\{ \frac{1}{(\not{k} - m_t)^2} \right\} \propto \Lambda^2$$

$$\Downarrow$$

$$m_h^2 = m_h^{(\mathrm{tree})\,2} - c \cdot \Lambda^2$$

- Perfect agreement with the existing data

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ_{had}^{0} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21586
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.374
Γ_W [GeV]	2.140 ± 0.060	2.091
m_t [GeV]	170.9 ± 1.8	171.3



- The scalar sector weakly constrained
  - Higgs-boson representation:

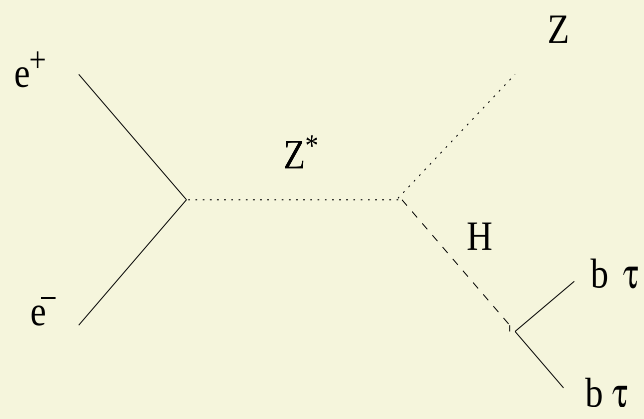
$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W}, \qquad \text{SM} \qquad \Rightarrow \qquad \rho = 1 + \mathcal{O}(\alpha)$$

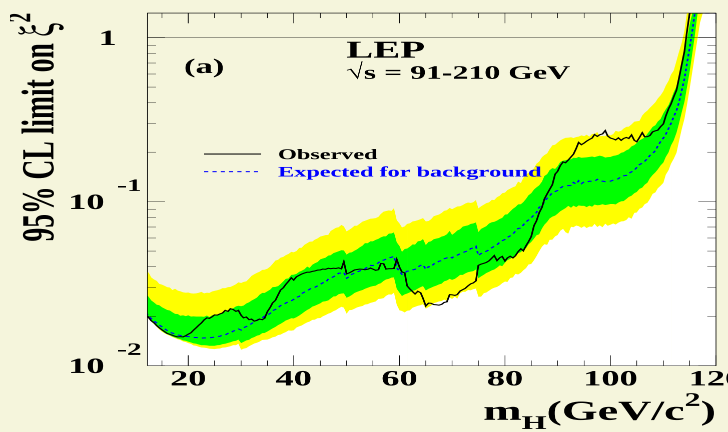
for general Higgs multiplets: $\rho = \frac{\sum_i [T_i(T_i+1) - T_{i\,3}^2] v_i^2}{\sum_i 2T_{i\,3}^2 v_i^2}$

data: $\rho = 1.0002 \begin{cases} +0.0024 \\ -0.0009 \end{cases} \quad \Rightarrow \quad T = \frac{1}{2}$ (doublets are favored)

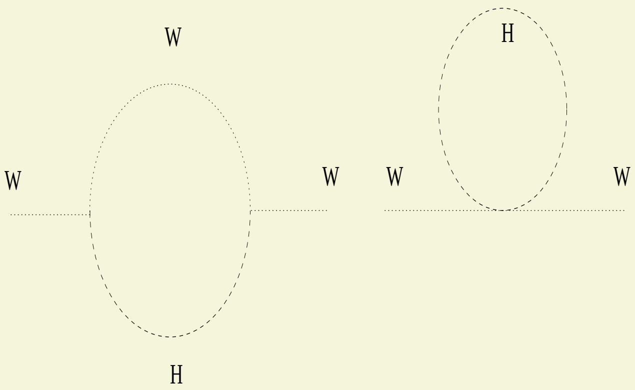
  - Higgs-boson interactions: no direct tests of the scalar potential

- Higgs-boson production at LEP:
  $m_h \gtrsim 115$ GeV

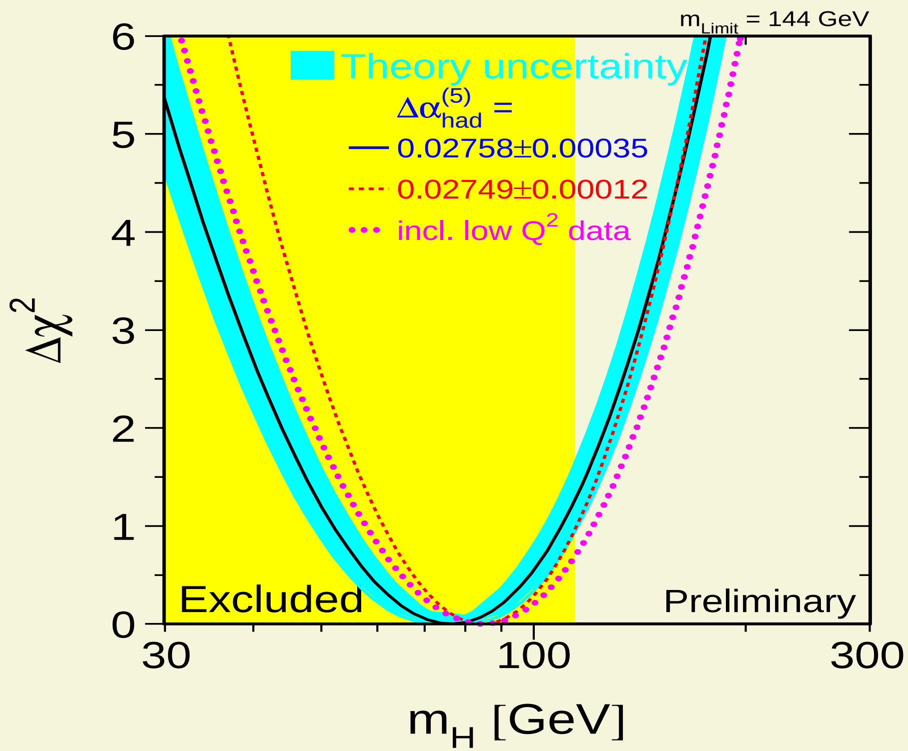




- Indirect Higgs-boson-mass limits (through radiative corrections):



$+\cdots \quad \propto \quad \ln\left(\frac{m_h^2}{v^2}\right)$ (Veltman's screening)

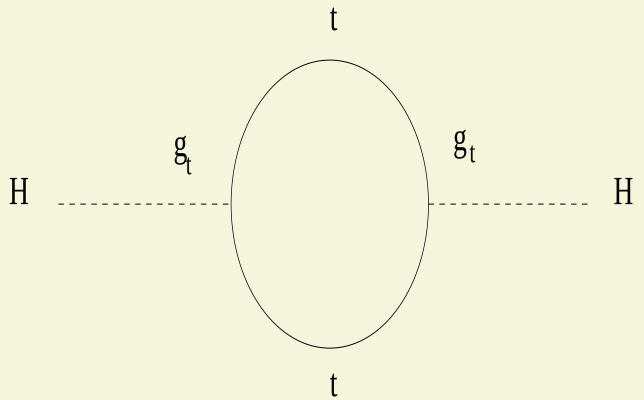


$\Rightarrow \quad m_h \lesssim 200$ GeV

Outstanding problems of the SM

♠ Gauge-Higgs sector:

- The hierarchy problem



$$\Rightarrow \qquad m_h^2 = m_h^{(\mathrm{tree})\,2} - c\cdot\Lambda^2$$

if $m_h \simeq 100$ GeV and $c \simeq \frac{g_t^2}{4\pi^2} \simeq \frac{1}{40}$ then

$$1 = \left(\frac{10 m_h^{(\mathrm{tree})}}{1\ \mathrm{TeV}}\right)^2 - \left(\frac{1.6\Lambda}{1\ \mathrm{TeV}}\right)^2$$

If $\Lambda \gg 1$ TeV then a fine tuned cancellation is needed to obtain $m_h \simeq 100$ GeV, e.g. for $\Lambda = M_{Pl} = 10^{18}$ GeV one has

$$1 = \left(\frac{10 m_h^{(\mathrm{tree})}}{1\ \mathrm{TeV}}\right)^2 - 2.5\cdot 10^{30} \qquad \Rightarrow \qquad \text{to avoid fine tuning} \quad \Lambda \lesssim 5 m_h \lesssim 1\ \mathrm{TeV}$$

$$\Downarrow$$

The New Physics is expected at $E \simeq 1$ TeV

- Why is there only one Higgs boson?
  - The Higgs field was introduced just to make the model renormalizable (unitary)
  - There exist many fermions and vector bosons, so why only one scalar? Why, for instance, not a dedicated scalar for each fermion?

- The strong CP problem:
  - symmetries of the SM allow for

$$\mathrm{tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(F_{\mu\nu}F_{\alpha\beta}\right) \xrightarrow{P} -\mathrm{tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

  - odd under CP

$$\mathcal{L}_\theta = \theta\frac{g_s^2}{32\pi^2}F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu} \quad \Rightarrow \quad \text{neutron} - \text{EDM} \quad D_n \simeq 2.7 \cdot 10^{-16}\theta \text{ e cm}$$

$$\Downarrow$$

$$\text{data: } D_n \lesssim 1.1 \cdot 10^{-25} \text{ e cm} \quad \Rightarrow \quad \theta \lesssim 3 \cdot 10^{-10}$$

The strong CP problem: why is θ so small?

♠ The flavor sector:

- parity violation:

$$W^{+\,\mu}\ \bar{u}_i\gamma_\mu(1-\gamma_5)d_j \quad \xrightarrow{P} \quad W^{+\,\mu}\ \bar{u}_i\gamma_\mu(1+\gamma_5)d_j$$

  Maximal parity violation, why?

- Charge quantization, why $q_u = \frac{2}{3}$, $q_d = -\frac{1}{3}$ and $q_l = -1$?

- Number of generations, why $N = 3$?

- Why is the top quark so heavy ($m_t \simeq 174$ GeV while $m_b \simeq 4.3$ GeV) ?

$$m_t \simeq v = \langle 0|H|0\rangle \simeq 246 \text{ GeV}$$

$$\Downarrow$$

top quark is very different (possibly sensitive to the spontaneous symmetry breaking)

- Mixing angles and fermion masses:

$$\mathcal{L} \supset -\sum_{i,j=1}^{3} \left(\tilde{\Gamma}_{ij} \bar{u}_{i\,R} \tilde{H}^\dagger Q_{j\,L} + \Gamma_{ij} \bar{d}_{i\,R} H^\dagger Q_{j\,L} + \text{H.c.} \right)$$

$$\Downarrow$$

$$\mathcal{L}_{\text{q mass}} = -\sum_{i,j=1}^{3} \left(\bar{u}_{i\,R} \mathcal{M}^u_{ij} Q_{j\,L} + \bar{d}_{i\,R} \mathcal{M}^d_{ij} Q_{j\,L} + \text{H.c.} \right) \quad \text{for} \quad \mathcal{M}^u_{ij} = \frac{v}{\sqrt{2}} \tilde{\Gamma}_{ij}, \ \mathcal{M}^d_{ij} = \frac{v}{\sqrt{2}} \Gamma_{ij}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}$$

$$U_R^\dagger \mathcal{M}^u U_L = \text{diag}(m_u, m_c, m_t) \qquad D_R^\dagger \mathcal{M}^d D_L = \text{diag}(m_d, m_s, m_b)$$

$$\Downarrow$$

$$\sum \bar{u}_{i\,L} \gamma^\mu d_{i\,L} = (\bar{u}, \bar{c}, \bar{t})_L \underbrace{U_L^\dagger D_L}_{U_{CKM}} \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

It is natural to expect that $U_{CKM} = U_{CKM}(m_q/m'_q)$.

♠ Parameters of the SM:

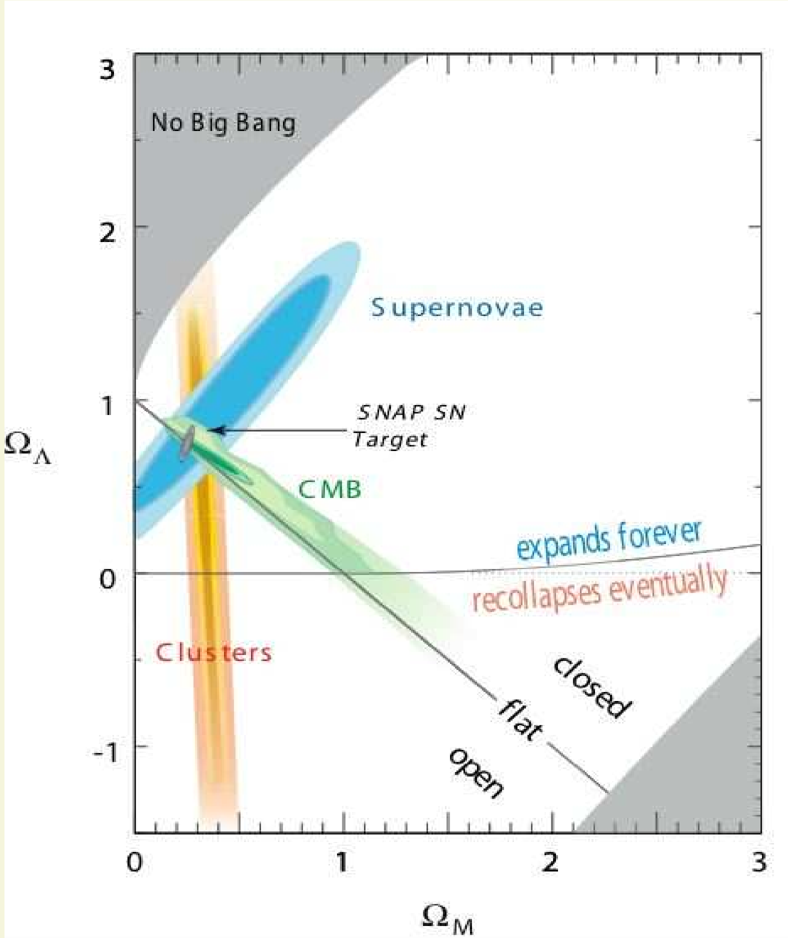
$$\begin{array}{cccccc} m_e & m_\mu & m_\tau & m_u & m_c & m_t \\ m_{\nu_e} & m_{\nu_\mu} & m_{\nu_\tau} & m_d & m_s & m_b \end{array}$$

$$\underbrace{g \quad , \quad g'}_{(\alpha_{QED}, \sin\theta_W)} \; , \; \underbrace{g_s}_{(\alpha_{QCD})} \; , \underbrace{m_h, \; \lambda}_{(\mu, \lambda)} , \; \underbrace{U_{CKM}}_{\theta_{1,2,3}, \delta_{CP}}$$

21 parameters !

♠ Cosmology:

- Dark matter and dark energy



$$\Omega_i \equiv \frac{\rho_i}{\rho_c} \quad \text{for} \quad \rho_c = \frac{3H_0^2}{8\pi G_N} = 11 h^2 m_p/m^3 \quad \text{for} \quad h \simeq 0.7$$

$$\text{data} \quad \Rightarrow \quad \Omega_\Lambda = \frac{\Lambda}{3H_0^2} \simeq 70\%,\ \Omega_{DM} \simeq 27\% \text{ and } \Omega_B \simeq 3\%$$

- SM has no candidate for dark matter
- $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \simeq 0.7 \quad \Rightarrow \quad \rho_\Lambda \simeq 10^{-120} M_{Pl}^4 = (10^{-3}\ \text{eV})^4$ while typical scale of the SM is $\mathcal{O}(100\ \text{GeV})$! Fine tuning again!
- Inflation: period of fast expansion of the very early Universe, $a(t) \simeq \exp\left(\sqrt{\frac{\Lambda}{3}}\, t\right)$

  Again the SM has no means to explain the inflation (no inflaton in the SM). For a typical inflaton $m_\varphi \sim 10^{13}$ GeV and $\lambda \sim 10^{-13}$, so the SM Higgs boson is not an inflaton.
- Baryogenesis and SM CP violation $\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq \frac{n_b}{n_\gamma} \simeq 6 \cdot 10^{-10}$

  The Sakharov's necessary conditions for baryogenesis:
  - B number violation
  - C and CP violation
  - Departure from thermal equilibrium

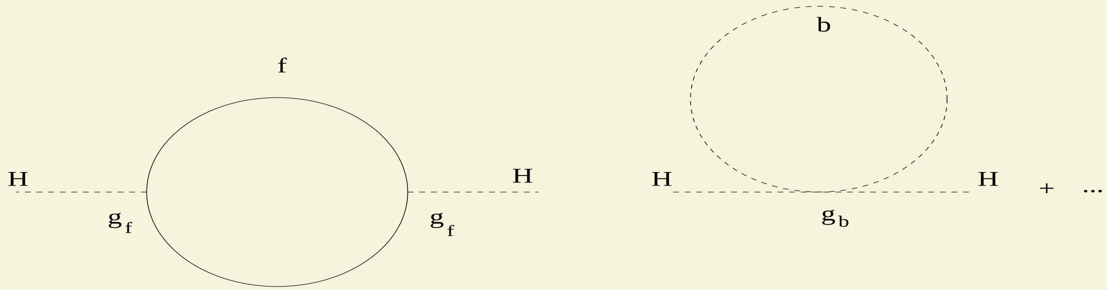
  SM:
  - B number violation: **OK**
  - C and CP violation: too weak CP violation $\propto \mathbf{Im}Q$, for $Q \equiv U_{ud} U_{cb} U_{ub}^\star U_{cd}^\star$ (re-phasing invariant)
  - Departure from thermal equilibrium: no electroweak phase transition for $m_h \gtrsim$ 73 GeV

  Conclusion: the SM doesn't explain the baryogenesis
- Why is gravity so weak? Or, why $M_{Pl} = 10^{18}$ GeV $\gg v = 246$ GeV?

Possible extensions of the SM

♠ SUSY

- SUSY is motivated by the hierarchy problem:



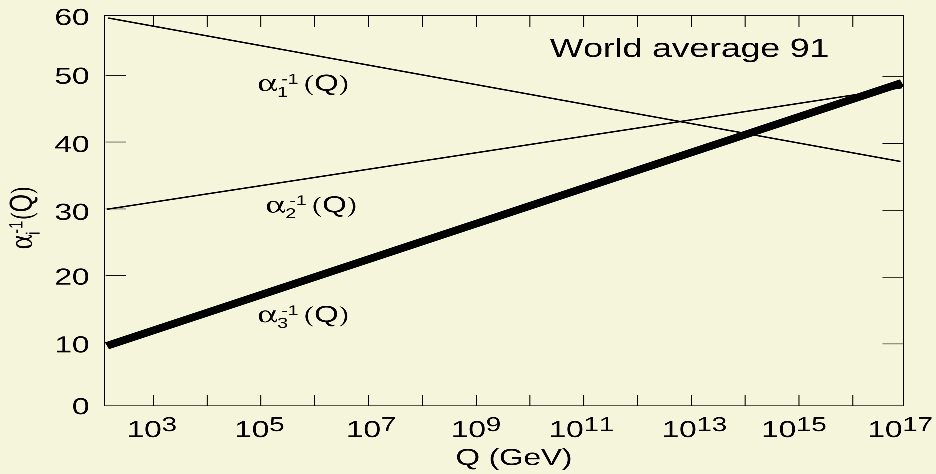
$$\delta m_h^{(f)\,2} = -c g_f^2 \Lambda^2 + \cdots \qquad\qquad \delta m_h^{(b)\,2} = +c g_b \Lambda^2 + \cdots$$

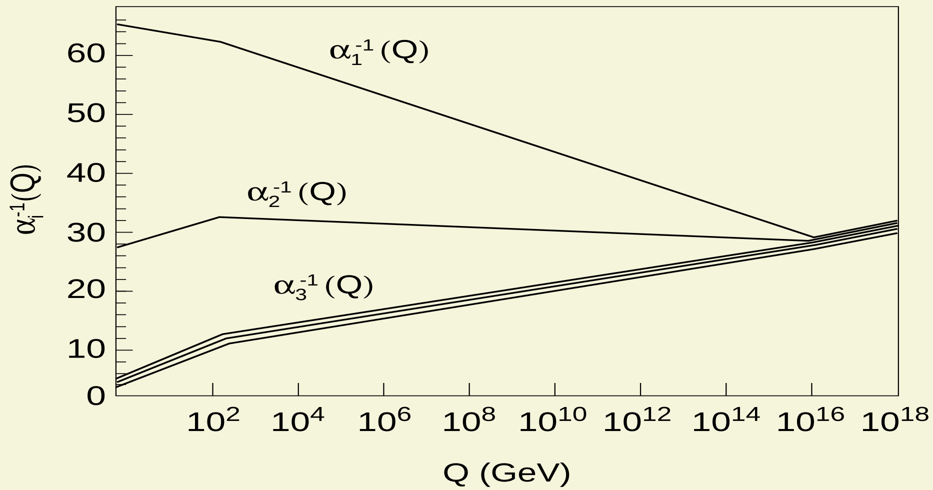
$$\delta m_h^2 = \delta m_h^{(f)\,2} + \delta m_h^{(b)\,2} = c\Lambda^2 \underbrace{(g_b - g_f^2)}_{=0 \text{ for SUSY}} + d(m_f^2 - m_b^2)\ln\Lambda + \cdots$$

– $\delta m_h^2 \propto (m_f^2 - m_b^2)\ln\Lambda$

– c, d independent of m_f and m_b

- to solve the hierarchy problem: $m_f^2 - m_b^2 \simeq 1\ \mathrm{TeV}^2$
- FCNC $\Rightarrow$ little hierarchy problem, as to suppress unwanted FCNC one needs $m_f^2 - m_b^2 \gtrsim$ few TeV^2
- SUSY provides a candidate fro CDM: lightest neutralino
- gauge coupling unification within SUSY





♠ Extra Higgs bosons

- SM single Higgs doublets quite unnatural, why only one?
- extra sources of CP violation from the scalar sector (needed for baryogenesis)
- hope for an explanation of weak mixing angles through horizontal symmetries

$$\mathcal{L} \supset -\sum_{\alpha=1}^{N_H} \sum_{i,j=1}^{3} \left(\tilde{\Gamma}_{ij}{}^{\alpha} \bar{u}_{i\,R} \tilde{H}^{\alpha\,\dagger} Q_{j\,L} + \Gamma_{ij}{}^{\alpha} \bar{d}_{i\,R} H^{\alpha\,\dagger} Q_{j\,L} + \text{H.c.} \right)$$

$$H^\alpha \to \mathcal{H}^\alpha_\beta H^\beta\,, \qquad u_{i\,R} \to \mathcal{U}^j_i u_{j\,R}\,, \qquad d_{i\,R} \to \mathcal{D}^j_i u_{j\,R}\,, \qquad Q_{i\,L} \to \mathcal{Q}^j_i Q_{j\,L}$$

$$\Downarrow$$

constraints on fermion mass-matrices:

$$\mathcal{M}^u_{ij} = \sum_{\alpha=1}^{N_H} \tilde{\Gamma}^\alpha_{ij} \frac{v_\alpha}{\sqrt{2}}, \qquad \mathcal{M}^d_{ij} = \sum_{\alpha=1}^{N_H} \Gamma^\alpha_{ij} \frac{v_\alpha}{\sqrt{2}}$$

$$U_R^\dagger \mathcal{M}^u U_L = \text{diag}(m_u, m_c, m_t) \qquad D_R^\dagger \mathcal{M}^d D_L = \text{diag}(m_d, m_s, m_b)$$

If $\mathcal{M}^{u,d}$ sufficiently constraints, then $U_{CKM} \equiv U_L^\dagger D_L = U_{CKM}(m_q/m_{q'})$

- Multi-doublet models favored by the ρ measurement
- An example of extra Higgs boson scenario: the 2 Higgs Doublet Model:

$$
\begin{aligned}
V(\phi_1,\phi_2) \quad = \quad & m_1^2|\phi_1|^2 + m_2^2|\phi_2|^2 + m_3^2(e^{i\delta_3}\phi_1^\dagger\phi_2 + e^{-i\delta_3}\phi_2^\dagger\phi_1) + \\
& +\lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \\
& +\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda_5\left[e^{i\delta_5}(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right] + \\
& +\lambda_6(\phi_1^\dagger\phi_1)\left[e^{i\delta_6}\phi_1^\dagger\phi_2 + \text{H.c.}\right] + \lambda_7(\phi_2^\dagger\phi_2)\left[e^{i\delta_7}\phi_1^\dagger\phi_2 + \text{H.c.}\right]
\end{aligned}
$$

where m_i^2 and δ_i real

$$
\text{under CP: } \phi_i(t,\vec{x}) \quad \xrightarrow{CP} \quad e^{i\alpha_i}\phi_i^\star(t,-\vec{x}) \quad \text{for} \quad i=1,2
$$

- explicit CP violation: $\delta_i \neq 0$

$$
\phi_1^\dagger\phi_2 \quad \xrightarrow{CP} \quad e^{i(\alpha_2-\alpha_1)}\phi_2^\dagger\phi_1
$$

- spontaneous CP violation ($\delta_i = 0$)

$$
\langle\phi_1\rangle = \left(\begin{array}{c} 0 \\ \frac{v_1}{\sqrt{2}} \end{array}\right) \qquad \langle\phi_2\rangle = \left(\begin{array}{c} 0 \\ \frac{v_1 e^{i\theta}}{\sqrt{2}} \end{array}\right)
$$

$\cos\theta = \frac{2m_3^2 - \lambda_6 v_1^2 - \lambda_7 v_2^2}{4\lambda_5 v_1 v_2} \quad \Rightarrow \quad$ SCPV if $\theta \neq 0, \pi$

Difficulties of extra-Higgs-boson scenarios:

- many new parameters $(m_i^2, \lambda_i, \delta_i)$
- tree-level FCNC to be suppressed

$$\mathcal{M}_{ij}^u = \sum_{\alpha=1}^{N_H} \tilde{\Gamma}_{ij}^\alpha \frac{v_\alpha}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^d = \sum_{\alpha=1}^{N_H} \Gamma_{ij}^\alpha \frac{v_\alpha}{\sqrt{2}}$$

♠ Extra gauge symmetries

- GUTs, e.g. $SU(5)$: unification of gauge couplings, ...
- $L-R$ symmetry, $SU(2)_L \times SU(2)_R \times U(1)$: spontaneous parity violation
- $SU(2)_L \times U(1) \times U(1)'$: just extra Z'

♠ Extra dimensions (more special dimensions)

Motivations:

- Unification of gravity and gauge interactions in g_{AB} (Kaluza & Klein)
- Quantization of gravity (strings)
- Solution (amelioration) of the hierarchy problem

⋆ Large extra dimensions - ADD (Arkani-Hamed, Dimopoulos and Dvali)

$$S_{ADD} = \frac{M^{2+\delta}}{2} \int d^4x \underbrace{\int_0^{2\pi L} \cdots \int_0^{2\pi L}}_{\delta} d^\delta y \sqrt{-g} R_{4+\delta} + \int d^4x \sqrt{-g_{\rm ind}} \mathcal{L}_{SM}$$

- SM localized on the 3-brane (M_4)
- Brane width to be zero
- Brane fluctuations are neglected
- All (δ) extra dimensions of size L
- Only gravity propagates in the bulk

$$\frac{M^{2+\delta}}{2} \int d^4x \underbrace{\int_0^{2\pi L} \cdots \int_0^{2\pi L}}_{\delta} d^\delta y \sqrt{-g} R_{4+\delta} \quad \longrightarrow \quad \frac{M^{2+\delta}(2\pi L)^\delta}{2} \int d^4x \sqrt{-g_{\rm ind}} R_4 \; + \; \cdots$$

$$\Downarrow$$

$$M_{Pl}^2 = M^{2+\delta}(2\pi L)^\delta$$

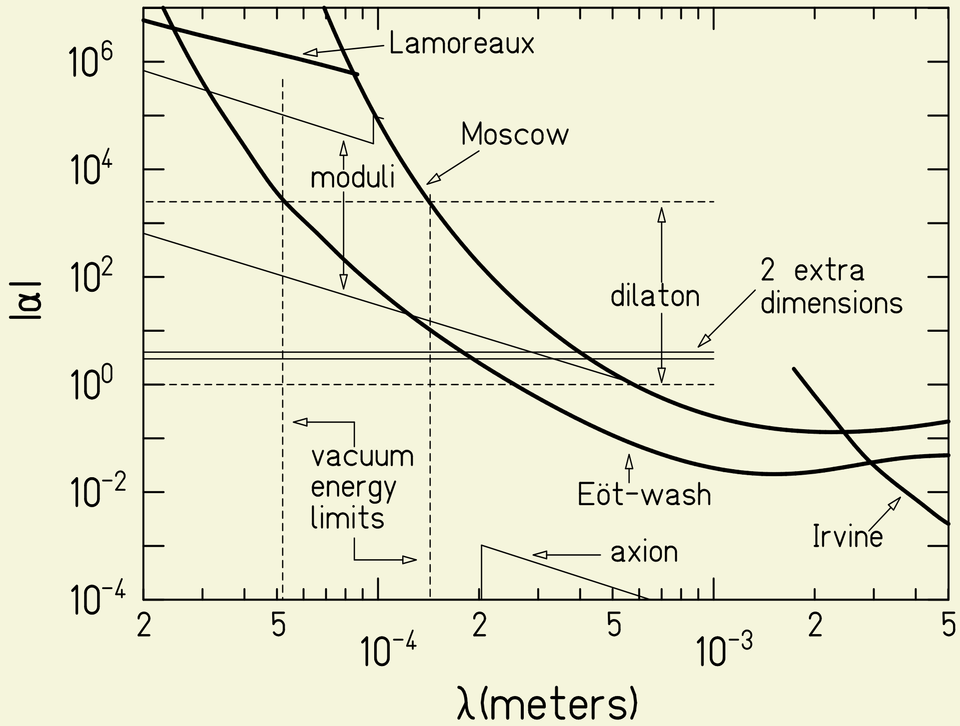
Trade: the hierarchy for the volume

The $4 + \delta$ gravity scale M, can be as low as 1 TeV, then $L = 10^{-17+30/\delta}$ cm

$$L = \begin{cases} 10^{13}\text{cm} & \delta = 1 \quad \text{excluded} \\ 10^{-2}\text{cm} & \delta = 2 \quad \text{allowed} \end{cases}$$

Gravity modified at distances $\simeq L$

$$V(r) = \begin{cases} -\frac{G_N m_1 m_2}{r} & \text{for} \quad r \gg L \\ -\frac{m_1 m_2}{M^{2+\delta} r^{1+\delta}} & \text{for} \quad r \ll L \end{cases}$$



The Eot-Wash Group: submillimeter tests of gravity

$$V(r) = -G_N \frac{m_1 m_2}{r} \left(1 + \alpha\, e^{-\frac{r}{\lambda}}\right)$$

ADD with $\delta = 2 \quad \Rightarrow \quad \lambda = L\,, \quad \alpha = 4.$

An example of Kaluza-Klein expansion: scalar field for $\delta = 1$:

$$\mathcal{L} = \frac{1}{2}\eta^{AB}\partial_A\phi\partial_B\phi \quad \text{for} \quad A = 0,1,2,3,5$$

$$\phi(t,\vec{x},y) = \phi(x,y) \quad \text{space time} \quad M_4 \times S^1 \quad \Rightarrow \quad \phi(x,y+2\pi L) = \phi(x,y)$$

$$\Downarrow$$

$$\phi(x,y) = \sum_{n=-\infty}^{+\infty} \phi_n(x) e^{in\frac{y}{L}}$$

$$\mathcal{L} = \frac{1}{2}\sum_{n,m=-\infty}^{+\infty}\left[\partial_\mu\phi_n\partial^\nu\phi_m + \partial_5\phi_n\partial^5\phi_m\right]e^{i(n+m)\frac{y}{L}} = \frac{1}{2}\sum_{n,m=-\infty}^{+\infty}\left[\partial_\mu\phi_n\partial^\nu\phi_m + \frac{nm}{L^2}\phi_n\phi_m\right]e^{i(n+m)\frac{y}{L}}$$

$$\mathcal{S} = \int d^4x \int_0^{2\pi L} dy\,\mathcal{L} = \frac{2\pi L}{2}\int d^4x \sum_{n=-\infty}^{\infty}\left[\partial_\mu\phi_n\partial^\nu\phi_n^\star - \left(\frac{n}{L}\right)^2\phi_n\phi_n^\star\right] =$$

$$\int d^4x \left\{\left[\frac{1}{2}\partial_\mu\varphi_0\partial^\mu\varphi_0\right] + \sum_{n=0}^{\infty}\left[\partial_\mu\varphi_n\partial^\mu\varphi_n^\star - m_n^2\varphi_n\varphi_n^\star\right]\right\}$$

for $\varphi_n \equiv \sqrt{2\pi L}\phi_n$

- Real massless scalar φ_0
- Complex massive scalars (the KK tower) φ with $m_n = \frac{n}{L}$
- At low energies ($E \ll L^{-1}$) only the zero mode φ_0 is important
- The candidate for DM: φ_1 (stable)

⋆ Warped extra dimensions (Randall and Sundrum): solution to the hierarchy problem

RS II, $D = 4 + 1$

$$\begin{aligned}\mathcal{S} &= \int d^4x \int_{-\pi}^{+\pi} d\phi \sqrt{-g}(2M^3 R - \Lambda) + \\ &\int d^4x \sqrt{-g_{\mathrm{vis}}}(\mathcal{L}_{\mathrm{vis}} - V_{\mathrm{vis}}) + \int d^4x \sqrt{-g_{\mathrm{hid}}}(\mathcal{L}_{\mathrm{hid}} - V_{\mathrm{hid}})\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned}\sqrt{-g}(R_{MN} - \frac{1}{2} G_{NM} R) &= -\frac{1}{4M^3}\left[\Lambda \sqrt{-g}\, g_{MN} + \right. \\ &\left. + V_{\mathrm{vis}} \sqrt{-g_{\mathrm{vis}}}\, g^{\mathrm{vis}}_{\mu\nu} \delta^\mu_M \delta^\nu_N \delta(\phi - \pi) + V_{\mathrm{hid}} \sqrt{-g_{\mathrm{hid}}}\, g^{\mathrm{hid}}_{\mu\nu} \delta^\mu_M \delta^\nu_N \delta(\phi)\right]\end{aligned}$$

$$\Downarrow$$

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2$$

with $\sigma = r_c|\phi|\sqrt{\frac{-\Lambda}{24M^3}}$ $(\Lambda < 0)$ $\quad \Rightarrow \quad$ $g_{MN} = \left(\begin{array}{c|c} e^{-kr_c|\phi|}\eta_{\mu\nu} & \\ \hline & r_c^2 \end{array}\right)$ and

$$V_{\mathrm{hid}} = -V_{\mathrm{vis}} = 24M^3 k \quad \text{for} \quad \Lambda = -24M^3 k^2$$

$$S_{\text{eff}} \supset 2 \int d^4x \left[\int_{-\pi}^{+\pi} d\phi \, M^3 r_c e^{-2kr_c|\phi|} \right] \sqrt{-\bar{g}} \, \bar{R} + \cdots \qquad \text{for} \qquad \bar{g}_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$\Downarrow$$

To reproduce GR

$$M_{Pl}^2 = M^2 r_c \int_{-\pi}^{+\pi} d\phi \, e^{-2kr_c|\phi|} = \frac{M^3}{k}(1 - e^{-2kr_c\pi})$$

The hierarchy problem within the RS II

$$\mathcal{S}_{\text{vis}} \supset \int d^4x \sqrt{-g_{\text{vis}}} \left\{ g_{\text{vis}}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2 \right\} \qquad \text{for} \qquad g_{\text{vis}}^{\mu\nu} = g^{\mu\nu}(x, \phi = \pi)$$

$$\Downarrow \qquad H \to e^{kr_c\pi} H$$

$$v = e^{-kr_c\pi} v_0 = 246 \text{ GeV}$$

- If $e^{kr_c\pi} \simeq 10^{16}$ ($kr_c\pi \simeq 40$), then v_0 could be $\mathcal{O}(M_{Pl})$
- One can assume that the 5d theory has a single scale $\simeq M_{Pl}$ (so no hierarchy), while the E-W scale is generated through the warping!

⋆ The gauge-Higgs unification

The strategy for $D = 4 + 1$:

- SM Higgs in the fundamental representation of $SU(2)$. For A_5 (adjoint) to have iso-doublet components at least $G = SU(3)_w$ is required (a chance for unification with $SU(3)_c$) :

$$A_M = \left(\begin{array}{cc|c} \multicolumn{2}{c|}{A_M^a} & A_M^{\hat{a}} \\ \hline \multicolumn{2}{c|}{A_M^{\hat{a}}} & A_M^a \end{array} \right)$$

- The initial gauge group G broken to $SU(2)_L \times U(1)_Y$ by the Scherk-Schwarz mechanism. Periodicity:

$$A_M(x, y + 2\pi R) = T A_M(x, y) T^\dagger$$

  Orbifold boundary conditions:

$$A_\mu(x, -y) = +P A_\mu(x, y) P^\dagger \qquad A_4(x, -y) = -P A_4(x, y) P^\dagger \,,$$

  where T and P are elements of a global symmetry group (e.g. gauge).
  For $SU(3) \to SU(2)_L \times U(1)_Y$: $P = T = \exp(i\pi\lambda_3) = \mathrm{diag}(-1, -1, 1)$

- $SU(2)_L \times U(1)_y \to U(1)_{\mathrm{EM}}$ by $\langle A_5^{(0)} \rangle$ through 1-loop effective potential (the Hosotani mechanism).

Advantages:

- Solution to the hierarchy problem ($m_h^2 \propto \Lambda^2$):
  The $U(1)$ gauge symmetry:

$$A_\mu(x,y) \to A_\mu(x,y) + \partial_\mu \lambda(x,y) \quad \text{and} \quad A_5(x,y) \to A_5(x,y) + \partial_y \lambda(x,y)$$

  forbids a mass term for A_5, however vanishing of the mass for the zero mode of A_5 is not protected: $A_5^{(0)}(x) \to A_5^{(0)}(x)$, so after the compactification $m_{A_5}^2$ could be be generated in the perturbation expansion:

$$m_{A_5}^2 \propto \frac{1}{L^2}$$

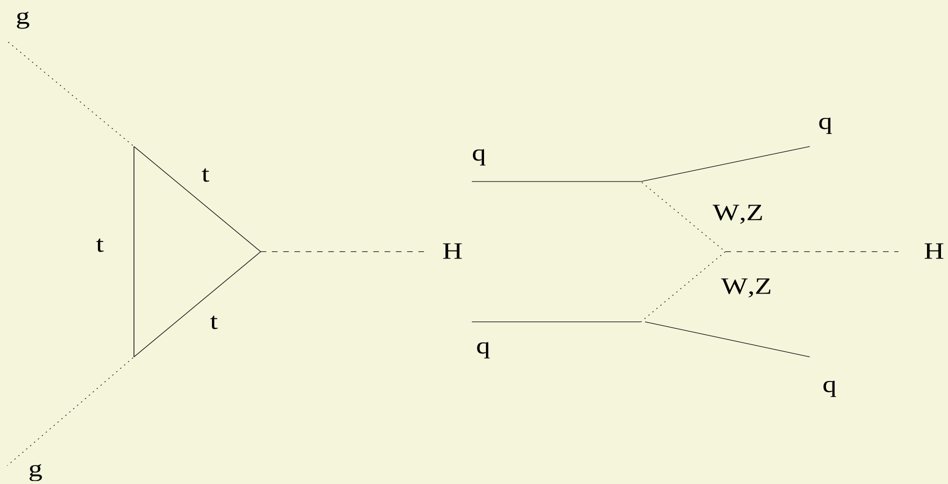
  - No hierarchy problem: the Higgs boson mass is calculable and finite (1- and 2-loop confirmed).

- Chance for an extra source of CP violation: 5D QED compactified on a circle spontaneously breaks CP if at least two fermions are present.

Future perspectives with the LHC

CERN (Geneva), 7 TeV + 7 TeV ($\sqrt{s} = 14$ TeV), pp collider, detectors: ATLAS, CMS, LHCb, ALICE

♠ Higgs search

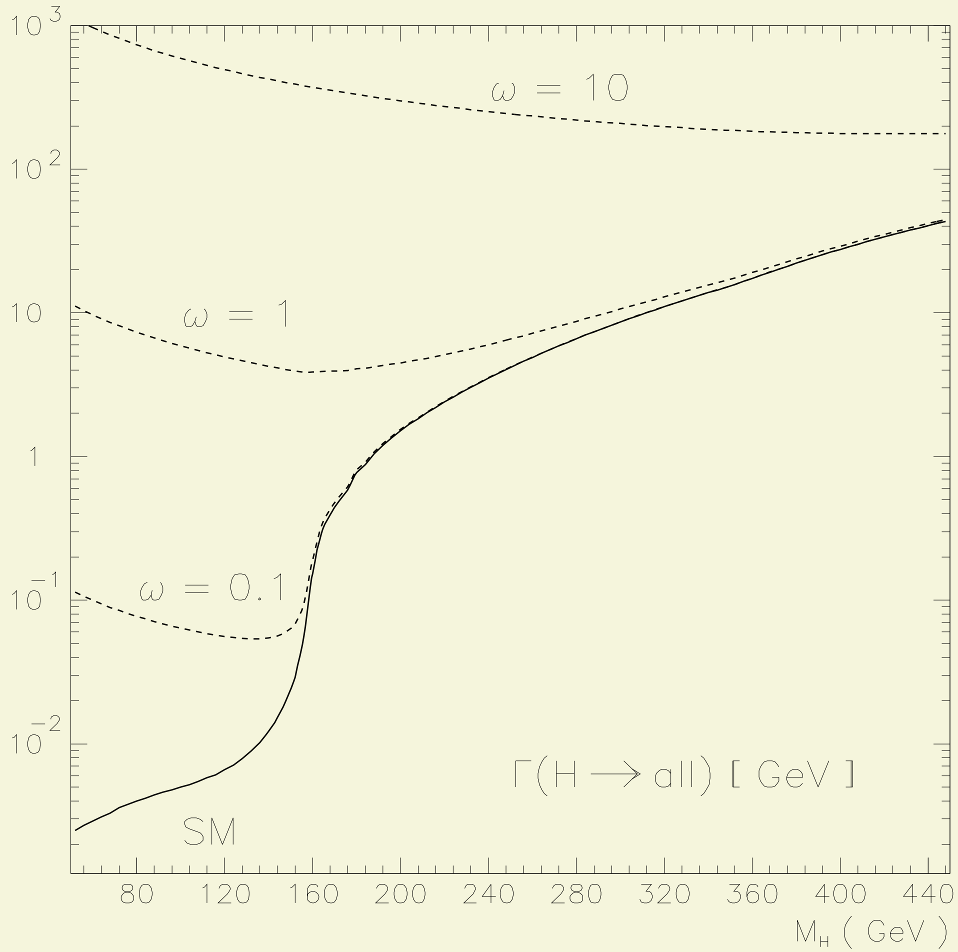


The most pessymistic scenarios:

- SM Higgs boson observed with 115 GeV $\leq m_h \leq$ 200 GeV (production rates and couplings (BR) as in the SM).

- No Higgs boson observed:
  - N extra singlets of $SU(2) \times U(1)$ (Binoth & Van der Bij) $O(N)$ model with $\vec{\varphi}$:

$$\mathcal{L} = -\frac{\omega_0}{2\sqrt{N}} \vec{\varphi}^2 \, |H|^2$$



- No Higgs boson observed:
  - Curvature-Higgs mixing (Giudice, Rattazzi and Wells) and large extra dimensions (ADD)

$$\mathcal{S} = \frac{M^{2+\delta}}{2} \int d^4x d^\delta y \sqrt{-g} R + \int d^4x \sqrt{-g_{\rm ind}} \mathcal{L}_{SM} - \xi \int d^4x \sqrt{-g_{\rm ind}} R(g_{\rm ind}) |H|^2$$

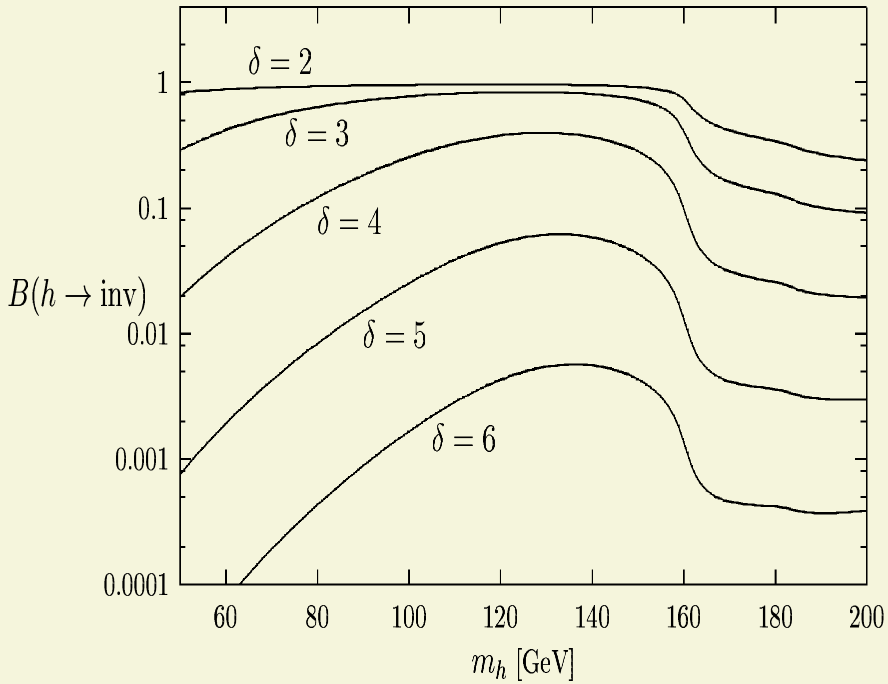
KK expansion:

$$g_{AB} = \eta_{AB} + h_{AB} \quad \text{for} \quad h_{AB} = \sum_{n_1=-\infty}^{+\infty} \cdots \sum_{n_\delta=-\infty}^{+\infty} \frac{h_{AB}^{(n)}(x)}{V_\delta^1/2} e^{i n^j y_j / L}$$

$$\Downarrow$$

* There are scalar modes in $h_{AB}^{(n)}$: $\quad H^{(n)}$
* Higgs-curvature mixing $\Longrightarrow \mathcal{L}_{\rm mix} = -m_{\rm mix}^2 h \sum_{\vec{n}} H^{(n)}$ for $m_{\rm mix}^2 \propto \xi$

For $\xi = 1$ and $M = 2$ TeV



$$B(h \to \text{inv}) \equiv \frac{\Gamma(h \to \text{inv})}{\Gamma(h \to \text{all})}$$

- Other possibilities:
  - Higgs + superparticles
  - 4th generation of quarks and leptons
  - Z', W_R, $\cdots$
  - More Higgs bosons
  - KK resonances
  - radion, ...

Summary

- SM problems:
  - the hierarchy
  - dark matter
  - dark energy
  - baryogenesis
- Attractive alternatives:
  - The Randall-Sundrum model (warped extra dimensions)
  - The gauge-Higgs unification (warped?)