Puzzles of the Standard Model

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Outline

- Introduction to the Standard Model.
- Outstanding problems of the high-energy physics (signals of beyond the SM physics)
- Possible solutions - extensions of the SM
- Summary

The Standard Model: Theory

♠ Gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{L} \supset -\frac{1}{4}\underbrace{F_a^{\mu\nu}F_{a\,\mu\nu}}_{SU(3)_C} -\frac{1}{4}\underbrace{W_i^{\mu\nu}W_{i\,\mu\nu}}_{SU(2)_L} -\frac{1}{4}\underbrace{B^{\mu\nu}B_{\mu\nu}}_{U(1)_Y}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$G_a^\mu|_{a=1,\ldots 8} \qquad\qquad W_\mu^\pm, Z_\mu, A_\mu$$

♠ The Higgs sector:

- The minimal choice $H = \begin{pmatrix} G^+ \\ (H + iG^0)/\sqrt{2} \end{pmatrix}$ necessary for $SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

$$\mathcal{L} \supset (D_\mu H)^\dagger D^\mu H - V(H)$$

for $D_\mu \equiv \partial_\mu + igW^i_\mu T^i + ig'\frac{1}{2}YB_\mu$ and

$$V(H) = \mu^2 |H|^2 + \lambda |H|^4 \qquad \text{with} \qquad Y_H = \frac{1}{2}$$

- If $\mu^2 < 0$ then $\langle 0||H|^2|0\rangle = -\frac{1}{2}\frac{\mu^2}{\lambda} \equiv \frac{v^2}{2}$ (spontaneous symmetry breaking, the origin of mass)
- Boson masses: $m_h = \sqrt{2\lambda}v$, $m_{W^\pm} = \frac{1}{2}gv$ and $m_Z = m_W/c_W$, for $c_W \equiv \cos\theta_W = g/(g^2 + g'^2)^{1/2}$

♠ Fermions

fermion	T	T_3	$\frac{1}{2}Y$	Q
$\nu_{i\,L}$	$\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0
$l_{i\,L}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
$u_{i\,L}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
$d_{i\,L}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
$l_{i\,R}$	0	0	-1	-1
$u_{i\,R}$	0	0	$\frac{2}{3}$	$\frac{2}{3}$
$d_{i\,R}$	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$\nu_{i\,R}$	0	0	0	0

$i = 1, \ldots, N_f = 3,\ \psi_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)\psi$ (parity violation), $Q = T_3 + \frac{1}{2}Y$

Neutrino masses:

- Dirac mass: $f_{ij}\ \bar{L}_{i\,L}\nu_{j\,R}\ \tilde{H} +$ H.c. for $\tilde{H} \equiv i\tau_2 H^\star$
- Majorana mass: $\frac{1}{2}M_{ij}\ \nu_{i\,R}\ \mathbf{C}\ \nu_{j\,R} +$ H.c.

Gauge transformations:

$$\psi(x) \to \exp\left\{-igT^i\theta_i(x) - ig'\frac{1}{2}Y\beta(x)\right\}\psi(x)$$

Gauge interactions:

$$\mathcal{L} \supset \sum_{\psi} \bar{\psi} i\gamma^\mu D_\mu \psi \qquad \text{for} \qquad D_\mu \equiv \partial_\mu + igW^i_\mu T^i + ig'\frac{1}{2}YB_\mu$$

Yukawa interactions:

$$\mathcal{L} \supset -\sum_{i,j=1}^{3}\left(\tilde{\Gamma}_{ij}\bar{u}_{i\,R}\tilde{H}^\dagger Q_{j\,L} + \Gamma_{ij}\bar{d}_{i\,R}H^\dagger Q_{j\,L} + \text{H.c.}\right)$$

if $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ then

$$\mathcal{L}_{\text{q mass}} = -\sum_{i,j=1}^{3}\left(\bar{u}_{i\,R}\mathcal{M}^u_{ij}u_{j\,L} + \bar{d}_{i\,R}\mathcal{M}^d_{ij}d_{j\,L} + \text{H.c.}\right)$$

for $\mathcal{M}^u_{ij} = \frac{v}{\sqrt{2}}\tilde{\Gamma}_{ij}$ $\mathcal{M}^d_{ij} = \frac{v}{\sqrt{2}}\Gamma_{ij}$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}$$

$$U_R^\dagger \mathcal{M}^u U_L = \mathrm{diag}(m_u, m_c, m_t) \qquad D_R^\dagger \mathcal{M}^d D_L = \mathrm{diag}(m_d, m_s, m_b)$$

$$\Downarrow$$

$$\tilde{\Gamma}, \Gamma \quad \text{diagonal} \quad (g_f = \sqrt{2}\frac{m_f}{v}) \quad \Rightarrow \quad \text{no FCNC}$$

- charged currents: $\sum_{i=1}^{N_f} \bar{u}_{i\,L} \gamma^\mu d_{i\,L} = (\bar{u}, \bar{c}, \bar{t})_L \underbrace{U_L^\dagger D_L}_{U_{CKM}} \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$
- neutral currents: $\sum_{i=1}^{N_f} \bar{u}_{i\,L} \gamma^\mu u_{i\,L}$, $\sum_{i=1}^{N_f} \bar{d}_{i\,L} \gamma^\mu d_{i\,L}$ remain unchanged upon $U_{L,R}$, $D_{L,R}$ transformations

U_{CKM}:

- unitary complex $N_f \times N_f$ matrix,
  $q_{i\,L} \to e^{i\alpha_i} q_{i\,L} \;\Rightarrow\; \frac{1}{2}(N_f - 1)(N_f - 2)$ phases in U_{CKM}
- $N_f \geq 3$ $\Rightarrow$ CP violation in charged currents

♠ Masses in the SM: $m_V \propto gv$ $m_h \propto \lambda^{1/2} v$ $m_f \propto g_f v$

Leptons:

$$m_{\nu_e} \lesssim 2.2 \text{ eV} \quad m_{\nu_\mu} \lesssim 2.2 \text{ MeV} \quad m_{\nu_\tau} \lesssim 18 \text{ MeV}$$
$$m_e = 0.5 \text{ MeV} \quad m_\mu = 105.5 \text{ MeV} \quad m_\tau = 1.78 \text{ GeV}$$

Quarks:

$$m_u \simeq 2 \text{ MeV} \quad m_c \simeq 1.2 \text{ GeV} \quad m_t \simeq 174 \text{ GeV}$$
$$m_d = 5 \text{ MeV} \quad m_s = 0.1 \text{ GeV} \quad m_b = 4.3 \text{ GeV}$$

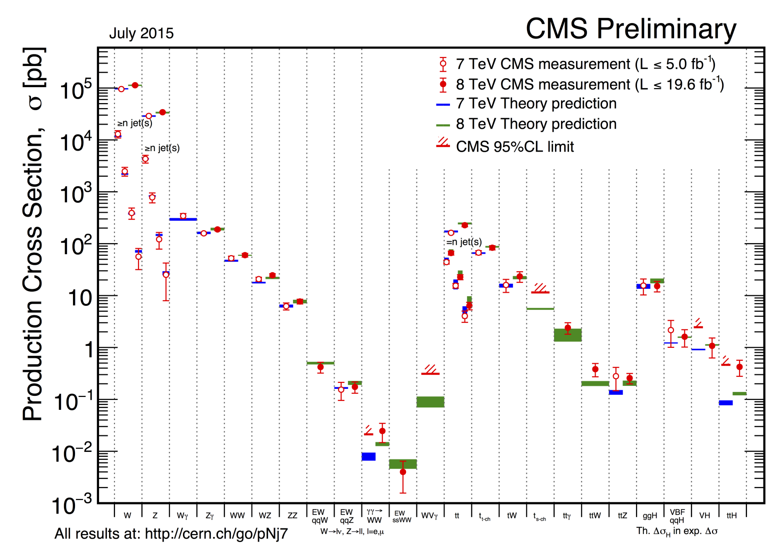
Bosons:

$$m_{W^\pm} = 80.4 \text{ GeV} \qquad m_Z = 91.2 \text{ GeV} \qquad m_h = 125 \text{ GeV}$$

Fine tuning:

$$\frac{m_{\nu_e}}{m_t} \lesssim \cdot 10^{-11} \qquad \Rightarrow \qquad \frac{g_{\nu_e}}{g_t} \lesssim \cdot 10^{-11}$$

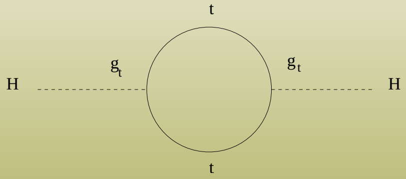
Very good agreement between the SM and the existing data



Outstanding problems of the SM

♠ Gauge-Higgs sector:

- ~~The hierarchy problem~~ - Radiative corrections to the Higgs mass



$$\delta m_h^2 \propto -g_t^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left\{ \frac{1}{(\not{k} - m_t)^2} \right\} \propto \Lambda^2$$

$$\Downarrow$$

$$m_h^2 = m_h^{(\mathrm{tree})\,2} - c \cdot \Lambda^2$$

~~The problem~~: If $\Lambda \gg v$ fine tuning between $m_h^{(\mathrm{tree})\,2}$ and $c\Lambda^2$ is needed

- Why is the scale of electroweak physics (10^2 GeV) so low compared to the Planck mass (10^{19} GeV) - the scale of gravity?
- The strong CP problem
  - symmetries of the SM allow for

$$\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(F_{\mu\nu}F_{\alpha\beta}\right) \xrightarrow{P} -\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

  - odd under CP

$$\mathcal{L}_\theta = \theta\frac{g_s^2}{32\pi^2}F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu} \quad \Rightarrow \quad \mathrm{neutron-EDM} \quad D_n \simeq 2.7\cdot 10^{-16}\theta \text{ e cm}$$

$$\Downarrow$$

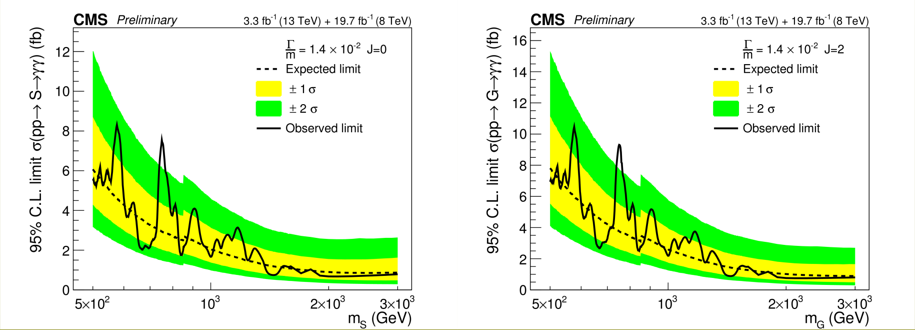
data: $D_n \lesssim 1.1\cdot 10^{-25}$ e cm $\quad\Rightarrow\quad \theta \lesssim 3\cdot 10^{-10}$

The strong CP problem: why is θ so small?

- What is the 750 GeV state observed at the LHC as a di-photon excess?
  - ATLAS Collaboration, "Search for resonances decaying to photon pairs in 3.2 fb^{-1} of pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector", Tech. Rep. ATLAS-CONF-2015-081,
  - CMS Collaboration, "Search for new physics in high mass diphoton events in proton-proton collisions at $\sqrt{s} = 13$ TeV", Tech. Rep. CMS-PAS-EXO-15-004,
  - CMS Collaboration, "Search for new physics in high mass diphoton events in 3.3 fb^{-1} of proton-proton collisions at $\sqrt{s} = 13$ TeV and combined interpretation of searches at 8 TeV and 13 TeV", Tech. Rep. CMS-PAS-EXO-16-018,
  - R. Franceschini *et al.*, "What is the gamma gamma resonance at 750 GeV?", JHEP **1603**, 144 (2016), arXiv:1512.04933.

Table: Summary of the LHC di-photon excess at the invariant mass of ~ 750 GeV from the ALTAS and CMS collaborations at energy $\sqrt{s} = 13$ TeV (σ is the local statistical significance).

	ATLAS @ $\sqrt{s} = 13$ TeV	CMS @ $\sqrt{s} = 8$ & 13 TeV
Excess	3.9σ	3.4σ
$\sigma(pp \to \gamma\gamma)$	(10 ± 3) fb	(6 ± 3) fb



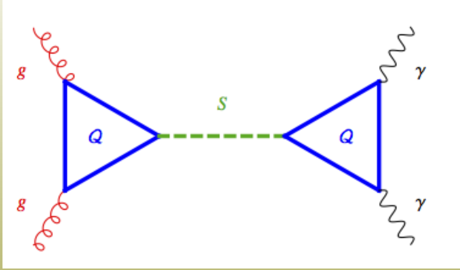


Figure from http://resonaances.blogspot.com/

$$\mathcal{L} \supset c_{s\gamma\gamma}\frac{e^2}{4v} S F_{\mu\nu}F^{\mu\nu} + c_{sgg}\frac{g_s^2}{4v} S G^a_{\mu\nu} G^{a\,\mu\nu}$$

♠ The flavour sector:

- parity violation:

$$W^{+\,\mu}\ \bar{u}_i\gamma_\mu(1-\gamma_5)d_j \quad \xrightarrow{P} \quad W^{+\,\mu}\ \bar{u}_i\gamma_\mu(1+\gamma_5)d_j$$

  Maximal parity violation, why?

- Charge quantization, why $q_u = \frac{2}{3}$, $q_d = -\frac{1}{3}$ and $q_l = -1$?
- Number of generations, why $N_f = 3$?
- Why is the top quark so heavy ($m_t \simeq 174$ GeV while $m_b \simeq 4.3$ GeV) ?

$$m_t \simeq v = \langle 0|H|0\rangle \simeq 246 \text{ GeV}$$

$$\Downarrow$$

top quark is very different (possibly sensitive to the spontaneous symmetry breaking)

- Mixing angles and fermion masses - the flavour problem

$$\mathcal{L} \supset -\sum_{i,j=1}^{3} \left(\tilde{\Gamma}_{ij} \bar{u}_{i\,R} \tilde{H}^\dagger Q_{j\,L} + \Gamma_{ij} \bar{d}_{i\,R} H^\dagger Q_{j\,L} + \text{H.c.} \right)$$

$$\Downarrow$$

$$\mathcal{L}_{\text{q mass}} = -\sum_{i,j=1}^{3} \left(\bar{u}_{i\,R} \mathcal{M}^u_{ij} Q_{j\,L} + \bar{d}_{i\,R} \mathcal{M}^d_{ij} Q_{j\,L} + \text{H.c.} \right) \quad \text{for} \quad \mathcal{M}^u_{ij} = \frac{v}{\sqrt{2}} \tilde{\Gamma}_{ij},$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}$$

$$U_R^\dagger \mathcal{M}^u U_L = \text{diag}(m_u, m_c, m_t) \qquad D_R^\dagger \mathcal{M}^d D_L = \text{diag}(m_d, m_s, m_b)$$

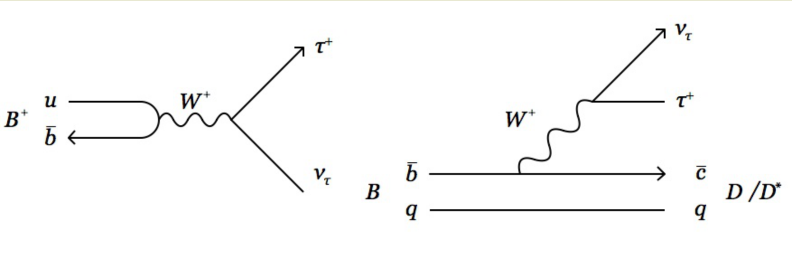
$$\Downarrow$$

$$\sum \bar{u}_{i\,L} \gamma^\mu d_{i\,L} = (\bar{u}, \bar{c}, \bar{t})_L \underbrace{U_L^\dagger D_L}_{U_{CKM}} \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

It is natural to expect that $U_{CKM} = U_{CKM}(m_q/m'_q)$.

- B-meson anomalies

$$B^{(\star)} \equiv |\bar{b}\ldots\rangle,\ D^{(\star)} \equiv |c\ldots\rangle,\ K^{(\star)} \equiv |s\cdots\rangle$$



from http://www.quantumdiaries.org/tag/b-physics/

- B-meson anomalies

$$R(X) = \frac{BR(\bar{B} \to X\,\tau\,\bar{\nu})}{BR(\bar{B} \to X\,\ell\,\bar{\nu})}$$

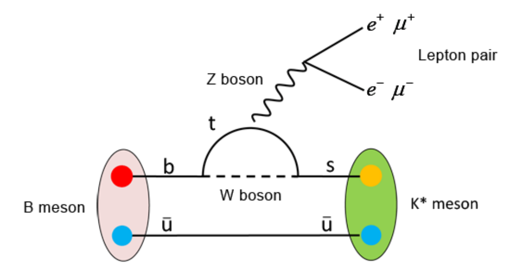
for $\ell = e, \mu$, $B^{(\star)} \equiv |\bar{b} \ldots\rangle$, $D^{(\star)} \equiv |c \ldots\rangle$

-	$R(D)$	$R(D^*)$
SM	0.297 $\pm$ 0.017	0.252 $\pm$ 0.005
Belle	0.375 $\pm 0.064 \pm 0.026$	0.293 $\pm 0.038 \pm 0.015$
BaBar	0.440 $\pm 0.058 \pm 0.042$	0.332 $\pm 0.024 \pm 0.018$
LHCb		0.336 $\pm 0.027 \pm 0.030$
Expt. avg.:	0.408 $\pm$ 0.050	0.321 $\pm$ 0.021

Table: $R(D)$ and $R(D^*)$ anomalies.

- B-meson anomalies

$$B^{(\star)} \equiv |\bar{b}\ldots\rangle,\ K^{(\star)} \equiv |s\cdots\rangle$$



http://phys.org/news/2009-09-belle-hint-physics-extremely-rare.html

$$R_K \equiv \frac{BR(B\to K\mu^+\mu^-)}{BR(B\to Ke^+e^-)} = \begin{cases} \text{SM} & 1.0003 \pm 0.0001 \\ \text{LHCb} & 0.745\,^{+0.090}_{-0.074} \pm 0.036 \end{cases}$$

- Flavour-changing Higgs decays

$$H \to \tau^+\mu^- \qquad H \to \tau^+ e^-$$

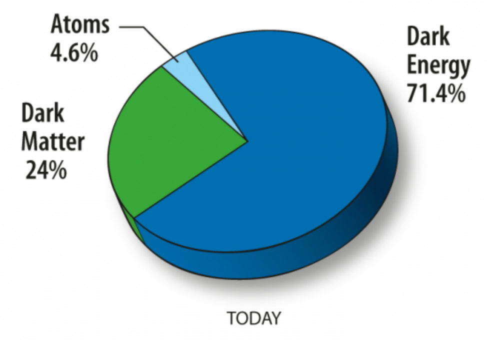
	ATLAS	CMS
$BR(H \to \tau^+\mu^-)$	(0.53 ± 0.51)%	$\left(0.84^{+0.39}_{-0.37}\right)$%
$BR(H \to \tau^+ e^-)$	(-0.3 ± 0.6)%	-

♠ Cosmology

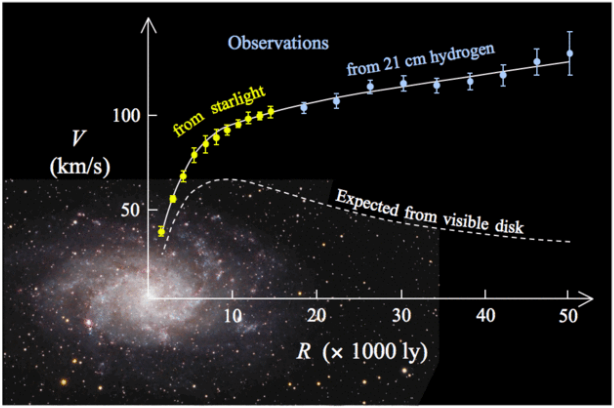
Dark matter evidence:

- Galaxy rotation curves
- Galaxy clusters and gravitational lensing
- Cosmic microwave background
- Structure formation

SM has no candidate for dark matter

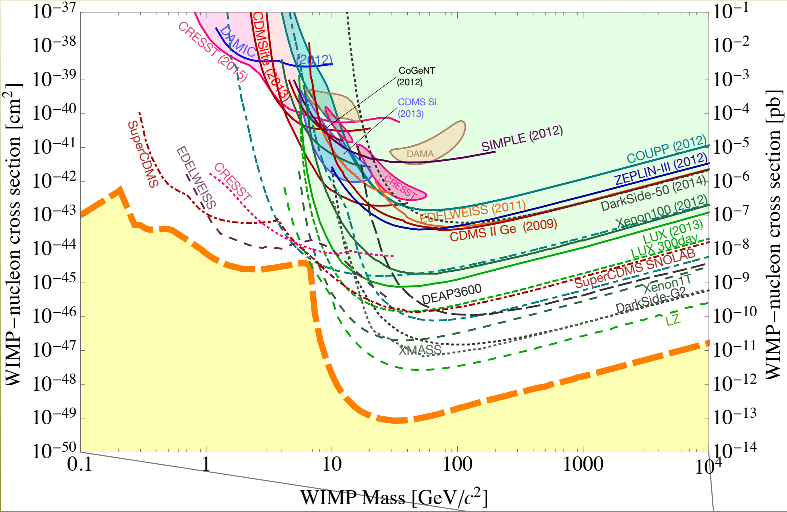


- Galaxy rotation curves



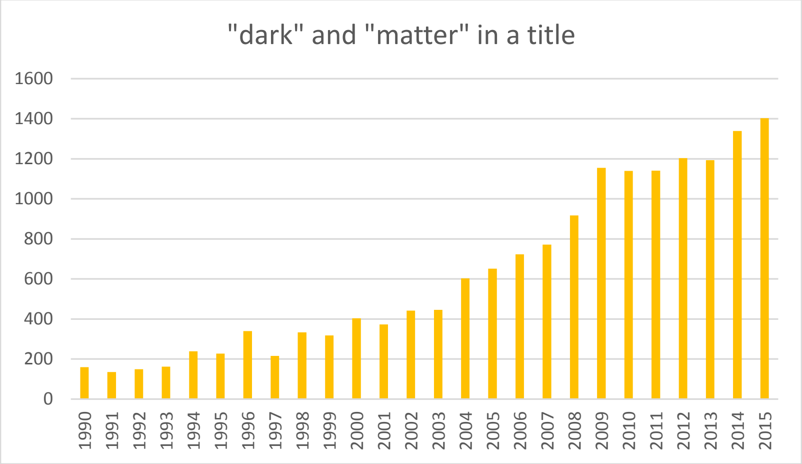
from E. Corbelli; P. Salucci (2000), "The extended rotation curve and the dark matter halo of M33", Monthly Notices of the Royal Astronomical Society 311 (2), 441

■ Dark matter direct detection



from P. Cushman et al., arXiv:1310.8327 [hep-ex]

- Dark matter publications



- $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \simeq 0.7 \qquad \Rightarrow \qquad \rho_\Lambda \simeq 10^{-120} M_{Pl}^4 = (10^{-3}\ \text{eV})^4$ while typical mass scale of the SM is $\mathcal{O}(100\ \text{GeV})$! Explanation needed!

- Inflation: period of fast expansion of the very early Universe,
  $a(t) \simeq \exp\left(\sqrt{\frac{\Lambda}{3}}\, t\right)$
  Again the SM has no means to explain the inflation (no inflaton in the SM). For a typical inflaton $m_\varphi \sim 10^{13}$ GeV and $\lambda \sim 10^{-13}$, so the SM Higgs boson is not an inflaton.

- Baryogenesis and SM CP violation $\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq \frac{n_b}{n_\gamma} \simeq 6 \cdot 10^{-10}$
The Sakharov's necessary conditions for baryogenesis:
  - B number violation
  - C and CP violation
  - Departure from thermal equilibrium

SM:

  - B number violation: **OK**
  - C and CP violation: too weak CP violation $\propto \Im Q$, for $Q \equiv U_{ud} U_{cb} U^\star_{ub} U^\star_{cd}$ (re-phasing invariant)
  - Departure from thermal equilibrium: no electroweak phase transition for $m_h \gtrsim 73$ GeV

Conclusion: the SM doesn't explain the baryogenesis

♠ Parameters of the SM:

$$
\begin{array}{cccccc}
m_e & m_\mu & m_\tau & m_u & m_c & m_t \\
m_{\nu_e} & m_{\nu_\mu} & m_{\nu_\tau} & m_d & m_s & m_b
\end{array}
$$

$$
\underbrace{g\ ,\ g'}_{(\alpha_{QED},\sin\theta_W)}\ ,\ \underbrace{g_s}_{(\alpha_{QCD})}\ ,\ \underbrace{m_h,\ \lambda}_{(\mu,\lambda)}\ ,\ \underbrace{U_{CKM}}_{\theta_{1,2,3},\delta_{CP}}\ ,\ \underbrace{U_{PMNS}}_{\theta_{12,23,13},\delta_{CP}^{(l)}}
$$

25 parameters for Dirac neutrinos!

♠ Summary of the SM puzzles

- lack of DM candidate
- mechanism of baryogenesis unknown (more CPV needed)
- the strong CP problem
- B-meson anomalies, the 750 GeV state, FCNC in Higgs decays, etc.

On the way beyond the SM

- The scalar sector weakly constrained
  - Higgs-boson representation:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W}, \qquad \text{SM} \qquad \Rightarrow \qquad \rho = 1 + \mathcal{O}(\alpha)$$

    for general Higgs multiplets: $\rho = \frac{\sum_i \left[T_i(T_i+1) - T_{i\,3}^2\right] v_i^2}{\sum_i 2T_{i\,3}^2 v_i^2}$

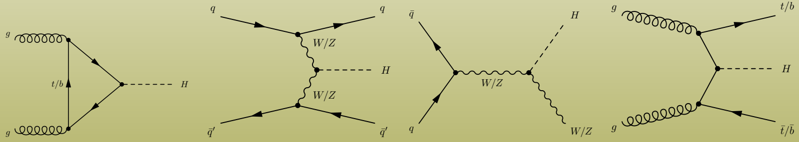
    data:

$$\rho = 1.0002 \begin{cases} +0.0024 \\ -0.0009 \end{cases} \qquad \Rightarrow \qquad T = \tfrac{1}{2} \qquad (\text{doublets are favored})$$

  - Higgs-boson discovery by ATLAS and CMS at the LHC announced on 4 July 2012

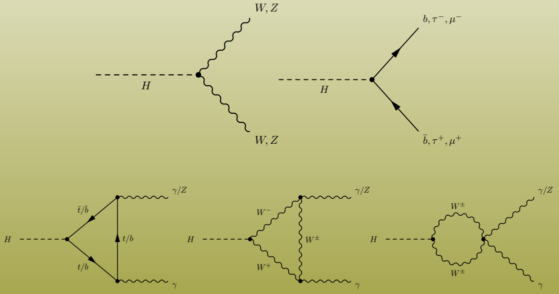
$$m_h = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\ \text{GeV}$$

Higgs boson production



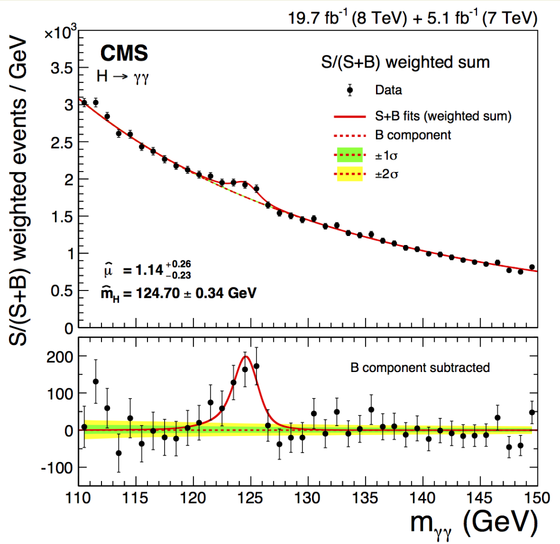
from G. Aad *et al.* [ATLAS Collaboration], Eur. Phys. J. C **76**, no. 1, 6 (2016)

Higgs boson decays

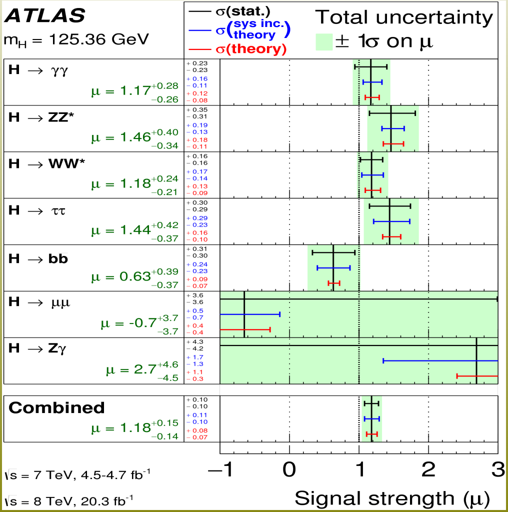


from G. Aad *et al.* [ATLAS Collaboration], Eur. Phys. J. C **76**, no. 1, 6 (2016)

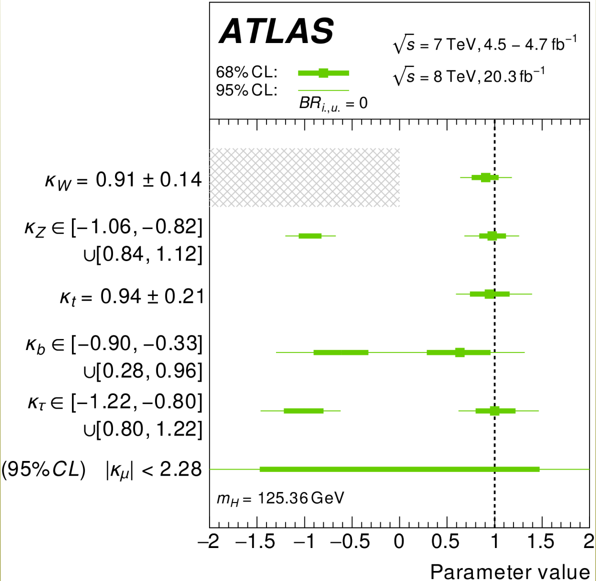
$H \to \gamma\gamma$



$$\mu_f \equiv \frac{\sigma(pp \to H) \times BR(H \to f)}{\sigma(pp \to H) \times BR(H \to f)|_{SM}}$$



$$\kappa_i \equiv \frac{g_{Hii}}{g_{Hii}|_{SM}}$$



♠ The simplest model of DM: Higgs portal interactions

$$O(N) : \vec{\varphi} \to \mathcal{O}\vec{\varphi}$$

$$\mathcal{L}_{scalar} = \frac{1}{2}\partial_\mu\vec{\varphi}\partial^\mu\vec{\varphi} + D_\mu H^\dagger D^\mu H - V(H,\vec{\varphi})$$

$$V(H,\vec{\varphi}) = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \frac{1}{2}\mu_\varphi^2 \vec{\varphi}^2 + \frac{1}{4!}\lambda_\varphi \left(\vec{\varphi}^2\right)^2 + \lambda_x H^\dagger H \vec{\varphi}^2$$

After the symmetry breaking the physical scalars have masses

$$m_h^2 = -\mu_H^2 + 3\lambda_H v^2 = 2\mu_H^2\,, \qquad m_\varphi^2 = \mu_\varphi^2 + \lambda_x v^2$$

Vacuum stability:

$$\lambda_H\,,\ \lambda_\varphi > 0\,; \quad \lambda_x > -\sqrt{\frac{\lambda_\varphi \lambda_H}{6}} = -\frac{m_h}{2v}\sqrt{\frac{\lambda_\varphi}{3}}$$

Tree-level unitarity constraints emerge from the SM condition for $V_L V_L$ scattering and from the requirement that all possible scalar-scalar scattering amplitudes are consistent with unitarity of the S matrix

$$m_h^2 < \frac{8\pi}{3} v^2, \quad \lambda_\varphi < 8\pi \ \text{ and } \ |\lambda_x| < 4\pi$$

Finally, the condition that the global $O(N)$ symmetry remains unbroken requires $\mu_\varphi^2 > 0$ which leads to the very useful inequality:

$$m_\varphi^2 > \lambda_x v^2$$

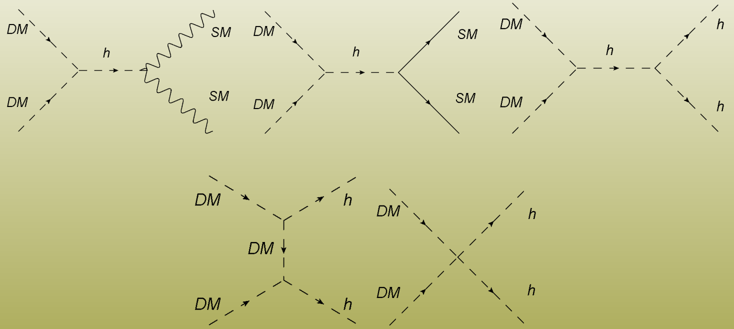
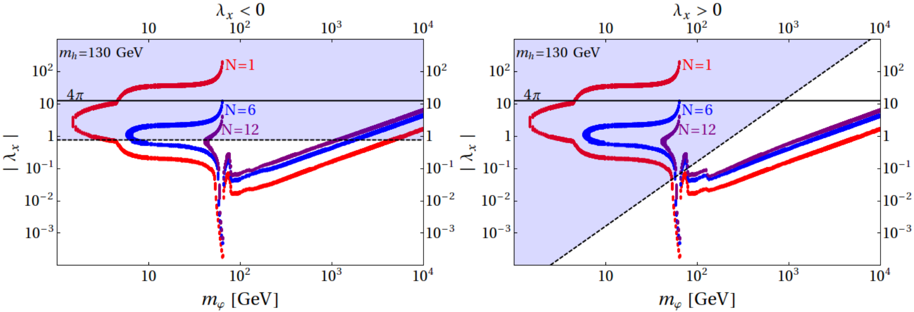


Figure: Dark matter annihilation.

$$\Omega_{\rm DM}^N h^2 = N \frac{\rho_{\rm DM}^1}{\rho_{\rm crit}} = 1.06 \times 10^9 \frac{N x_f}{\sqrt{g_*}\, m_{Pl} \langle \sigma v \rangle} \frac{1}{\rm GeV}$$

where $x_f \equiv m_\varphi / T_f$, and T_f is the freeze-out temperature given in the first approximation by

$$x_f = \log \left(0.038 \frac{\langle \sigma v \rangle\, m_{Pl}\, m_\varphi}{\sqrt{g_*(T_f) x_f}} \right)$$



♠ Extra Higgs bosons

- SM single Higgs doublets quite unnatural, why only one?
- extra sources of CPV from the scalar sector (for baryogenesis)
- explanation of weak mixing angles through horizontal symmetries

$$\mathcal{L} \supset -\sum_{\alpha=1}^{N_H} \sum_{i,j=1}^{3} \left(\tilde{\Gamma}_{ij}^{\alpha} \bar{u}_{i\,R} \tilde{H}^{\alpha\,\dagger} Q_{j\,L} + \Gamma_{ij}^{\alpha} \bar{d}_{i\,R} H^{\alpha\,\dagger} Q_{j\,L} + \text{H.c.} \right)$$

$$H^{\alpha} \to \mathcal{H}_{\beta}^{\alpha} H^{\beta}\,, \quad u_{i\,R} \to \mathcal{U}_{i}^{j} u_{j\,R}\,, \quad d_{i\,R} \to \mathcal{D}_{i}^{j} d_{j\,R}\,, \quad Q_{i\,L} \to \mathcal{Q}_{i}^{j} Q_{j\,L}$$

$$\Downarrow$$

constraints on fermion mass-matrices:

$$\mathcal{M}_{ij}^{u} = \sum_{\alpha=1}^{N_H} \tilde{\Gamma}_{ij}^{\alpha} \frac{v_\alpha}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^{d} = \sum_{\alpha=1}^{N_H} \Gamma_{ij}^{\alpha} \frac{v_\alpha}{\sqrt{2}}$$

$$U_R^{\dagger} \mathcal{M}^u U_L = \mathrm{diag}(m_u, m_c, m_t) \qquad D_R^{\dagger} \mathcal{M}^d D_L = \mathrm{diag}(m_d, m_s, m_b)$$

If $\mathcal{M}^{u,d}$ constrained, then $U_{CKM} \equiv U_L^{\dagger} D_L = U_{CKM}(m_q/m_{q'})$

- Multi-doublet models favored by the ρ measurement
- An example of extra Higgs boson scenario: the 2 Higgs Doublet Model:

$$\begin{aligned} V(\phi_1,\phi_2) \;=\;& m_1^2|\phi_1|^2 + m_2^2|\phi_2|^2 + m_3^2(e^{i\delta_3}\phi_1^\dagger\phi_2 + e^{-i\delta_3}\phi_2^\dagger\phi_1) + \\ & +\lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \\ & +\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda_5\left[e^{i\delta_5}(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right] + \\ & +\lambda_6(\phi_1^\dagger\phi_1)\left[e^{i\delta_6}\phi_1^\dagger\phi_2 + \text{H.c.}\right] + \lambda_7(\phi_2^\dagger\phi_2)\left[e^{i\delta_7}\phi_1^\dagger\phi_2 + \text{H.c.}\right] \end{aligned}$$

where m_i^2 and δ_i real

under CP: $\phi_i(t,\vec{x}) \quad \xrightarrow{CP} \quad e^{i\alpha_i}\phi_i^\star(t,-\vec{x}) \quad \text{for} \quad i=1,2$

- explicit CP violation: $\delta_i \neq 0$

$$\phi_1^\dagger\phi_2 \quad \xrightarrow{CP} \quad e^{i(\alpha_2-\alpha_1)}\phi_2^\dagger\phi_1$$

- spontaneous CP violation ($\delta_i = 0$)

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix} \qquad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ \frac{v_1 e^{i\theta}}{\sqrt{2}} \end{pmatrix}$$

Difficulties/Advantages of extra-Higgs-boson scenarios:

- many new parameters $(m_i^2, \lambda_i, \delta_i)$
- tree-level FCNC to be or not to be suppressed

$$\mathcal{M}_{ij}^u = \sum_{\alpha=1}^{N_H} \tilde{\Gamma}_{ij}^\alpha \frac{v_\alpha}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^d = \sum_{\alpha=1}^{N_H} \Gamma_{ij}^\alpha \frac{v_\alpha}{\sqrt{2}}$$

♠ Extra gauge symmetries

- GUTs, e.g. $SU(5)$: unification of gauge couplings, ...
- $L - R$ symmetry, $SU(2)_L \times SU(2)_R \times U(1)$: spontaneous parity violation
- $SU(2)_L \times U(1) \times U(1)'$: just extra Z'

♠ Extra dimensions (more special dimensions)

- Warped extra dimensions (Randall and Sundrum): solution to the hierarchy problem

RS I, $D = 4 + 1$

$$\begin{aligned} \mathcal{S} &= \int d^4x \int_{-\pi}^{+\pi} d\phi \sqrt{-g}(2M^3 R - \Lambda) + \\ & \int d^4x \sqrt{-g_{\text{vis}}}(\mathcal{L}_{\text{vis}} - V_{\text{vis}}) + \int d^4x \sqrt{-g_{\text{hid}}}(\mathcal{L}_{\text{hid}} - V_{\text{hid}}) \end{aligned}$$

$$\Downarrow$$

$$\sqrt{-g}(R_{MN} - \frac{1}{2}G_{NM}R) = -\frac{1}{4M^3}\left[\Lambda\sqrt{-g}\, g_{MN} + \right.$$
$$\left. +V_{\text{vis}}\sqrt{-g_{\text{vis}}}\, g^{\text{vis}}_{\mu\nu}\delta^\mu_M \delta^\nu_N \delta(\phi - \pi) + V_{\text{hid}}\sqrt{-g_{\text{hid}}}\, g^{\text{hid}}_{\mu\nu}\delta^\mu_M \delta^\nu_N \delta(\phi)\right]$$

$$\Downarrow$$

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2$$

with $\sigma = r_c|\phi|\sqrt{\frac{-\Lambda}{24M^3}}$ $(\Lambda < 0)$ and

$$V_{\text{hid}} = -V_{\text{vis}} = 24M^3 k \qquad \text{for} \qquad \Lambda = -24M^3 k^2$$

$$S_{\text{eff}} \supset 2\int d^4x \left[\int_{-\pi}^{+\pi} d\phi\, M^3 r_c e^{-2kr_c|\phi|}\right]\sqrt{-\bar{g}}\,\bar{R} + \cdots \quad \text{for} \quad \bar{g}_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Downarrow$$

To reproduce GR

$$M_{Pl}^2 = M^3 r_c \int_{-\pi}^{+\pi} d\phi\, e^{-2kr_c|\phi|} = \frac{M^3}{k}(1 - e^{-2kr_c\pi})$$

The hierarchy problem within the RS I

$$\mathcal{S}_{\text{vis}} \supset \int d^4x \sqrt{-g_{\text{vis}}}\left\{g_{\text{vis}}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2\right\} \quad g_{\text{vis}}^{\mu\nu} = g^{\mu\nu}(x,\pi)$$

$$\Downarrow \qquad H \to e^{kr_c\pi} H$$

$$v = e^{-kr_c\pi} v_0 = 246 \text{ GeV}$$

- If $e^{kr_c\pi} \simeq 10^{16}$ ($kr_c\pi \simeq 40$), then v_0 could be $\mathcal{O}(M_{Pl})$
- One can assume that the 5d theory has a single scale $\simeq M_{Pl}$ (so no hierarchy), while the E-W scale is generated through the warping!

♠ Simplest (simplistic) solutions - pragmatic approach

problem	**solution**
lack of DM candidate	many options e.g. extra scalar
extra source of CPV needed for baryogenesis	generic 2HDM, FCNC present
strong CP violations (why is $F\tilde{F}$ small?)	Peccei-Quin axion model
lepton non-universality - B meson anomalies	extra $U(1)$ gauge symmetry with non-universal leptonic charges
the 750 GeV state	Randall-Sundrum like 5-dim models: radion or graviton
FCNC in $h \to \tau\mu$ or τe	extra Higgs doublets, e.g. generic 2HDM with FCNC
why $v \ll M_{Pl}$?	Randall-Sundrum 5-dim model with warped geometry
the flavour problem, why $N_f = 3$?, θ_j, δ, m_f ?	Randall-Sundrum 5-dim model with warped geometry and fermions in the bulk ?

Summary

- SM problems:
  - dark matter
  - baryogenesis
  - the strong CP problem
  - B-anomalies
  - the 750 GeV state
  - dark energy
- Attractive alternatives:
  - The Randall-Sundrum model (warped extra dimensions)
  - Multi-Higgs models e.g 2HDM