Self-interacting dark matter in $U(1)$-extended Standard Model

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Outline

- A model of vector dark matter
- Problems of ΛCDM vs. self-interacting dark matter
- Model independent resonance enhancement of σ_{self}
- Model independent resonance annihilation
- Early kinetic decoupling
- Resonant self-interaction of vector dark matter
- Summary

⋆ M. Duch, BG, "Enhancing dark-matter self-interaction by s-channel resonance", in progress

⋆ M. Duch, BG, M. McGarrie, "A stable Higgs portal with vector dark matter", JHEP 1509 (2015) 162, arXiv:1506.08805

A model of vector dark matter

The model:

- extra $U(1)$ gauge symmetry (A_X^μ),
- a complex scalar field S, whose vev generates a mass for the $U(1)$'s vector field, $S = (0, \mathbf{1}, \mathbf{1}, 1)$ under $U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)$
- SM fields neutral under $U(1)$,
- to ensure stability of the new vector boson, a $\mathbb{Z}_2$ symmetry is assumed to forbid $U(1)$-kinetic mixing between $U(1)$ and $U(1)_Y$. The extra gauge boson A_X^μ and the scalar S field transform under $\mathbb{Z}_2$ as follows

$$A_X^\mu \to -A_X^\mu\ ,\ S \to S^*,\ \text{where } S = \phi e^{i\sigma},\ \ \text{so}\ \ \phi \to \phi,\ \ \sigma \to -\sigma.$$

T. Hambye, JHEP 0901 (2009) 028,
O. Lebedev, H. M. Lee, and Y. Mambrini, Phys.Lett. B707 (2012) 570,
A. Falkowski, C. Gross and O. Lebedev, JHEP 05 (2015) 057
...

A model of vector dark matter

The scalar potential

$$V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2.$$

The vector bosons masses:

$$M_W = \frac{1}{2} g v, \qquad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v \quad \text{and} \quad M_{Z'} = g_x v_x,$$

where

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{ and } \langle S \rangle = \frac{v_x}{\sqrt{2}}$$

Positivity of the potential implies

$$\lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2\sqrt{\lambda_H \lambda_S}$$

A model of vector dark matter

The mass squared matrix $\mathcal{M}^2$ for the fluctuations (ϕ_H, ϕ_S) and their eigenvalues

$$\begin{aligned}
\mathcal{M}^2 &= \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_x \\ \kappa v v_x & 2\lambda_S v_x^2 \end{pmatrix} \\
M_\pm^2 &= \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H \lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}
\end{aligned}$$

$$\mathcal{M}^2_{\mathsf{diag}} = \begin{pmatrix} M_{h_1}^2 & 0 \\ 0 & M_{h_2}^2 \end{pmatrix}, \quad R = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R^{-1} \begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix}$$

where $M_{h_1} = 125.7$ GeV is the mass of the observed Higgs particle.

A model of vector dark matter

$$\sin 2\alpha = \frac{\mathsf{sign}(\lambda_{SM}-\lambda_H)\, 2\mathcal{M}_{12}^2}{\sqrt{(\mathcal{M}_{11}^2-\mathcal{M}_{22}^2)^2+4(\mathcal{M}_{12}^2)^2}}, \quad \cos 2\alpha = \frac{\mathsf{sign}(\lambda_{SM}-\lambda_H)(\mathcal{M}_{11}^2 -}{\sqrt{(\mathcal{M}_{11}^2-\mathcal{M}_{22}^2)^2+4(\mathcal{M}}}$$

There are 5 real parameters in the potential: μ_H, μ_S, λ_H, λ_S and κ. Adopting the minimization conditions μ_H, μ_S could be replaced by v and v_x. The SM vev is fixed at $v = 246.22$ GeV. Using the condition $M_{h_1} = 125.7$ GeV, v_x^2 could be eliminated in terms of $v^2, \lambda_H, \kappa, \lambda_S, \lambda_{SM} = M_{h_1}^2/(2v^2)$:

$$v_x^2 = v^2 \frac{4\lambda_{SM}(\lambda_H - \lambda_{SM})}{4\lambda_S(\lambda_H - \lambda_{SM}) - \kappa^2}$$

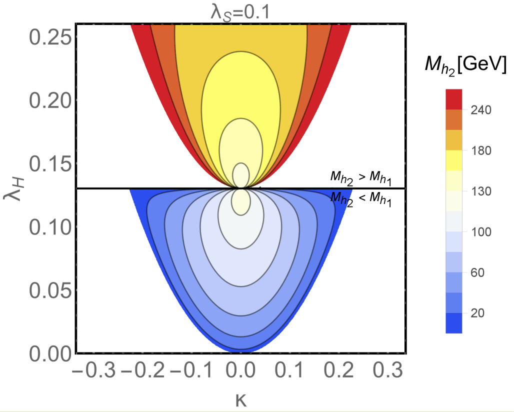
Eventually there are 4 independent parameters:

$$(\lambda_H, \kappa, \lambda_S, g_x),$$

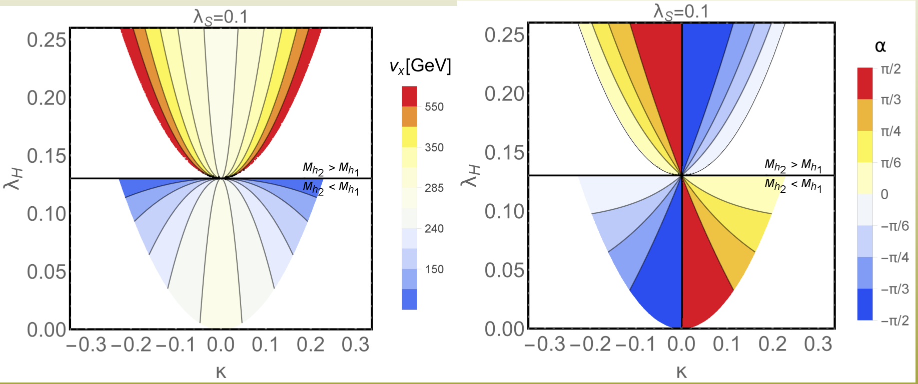
where g_x is the $U(1)$ coupling constant.

A model of vector dark matter

- Bottom part of the plot ($\lambda_H < \lambda_{SM} = M_{h_1}^2/(2v^2) = 0.13$): the heavier Higgs is the currently observed one.
- Upper part ($\lambda_H > \lambda_{SM}$) the lighter state is the observed one.
- White regions in the upper and lower parts are disallowed by the positivity conditions for v_x^2 and $M_{h_2}^2$, respectively.



A model of vector dark matter



Contour plots for the vacuum expectation value of the extra scalar $v_x \equiv \sqrt{2}\langle S \rangle$ *(left panel) and of the mixing angle* α *(right panel) in the plane* (λ_H, κ).

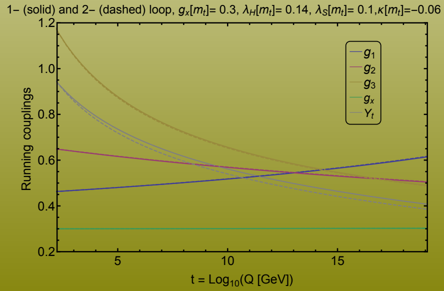
A model of vector dark matter

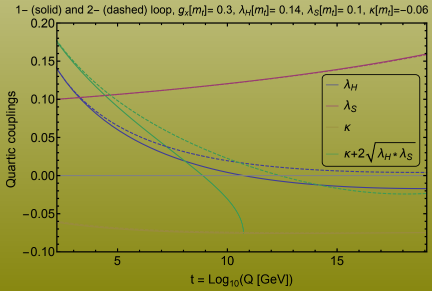
Vacuum stability

$$V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2$$

2-loop running of parameters adopted

$$\lambda_H(Q) > 0, \quad \lambda_S(Q) > 0, \quad \kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0$$





A model of vector dark matter

The mass of the Higgs boson is known experimentally therefore within *the SM* the initial condition for running of $\lambda_H(Q)$ is fixed

$$\lambda_H(m_t) = M_{h_1}^2/(2v^2) = \lambda_{SM} = 0.13$$

For VDM this is not necessarily the case:

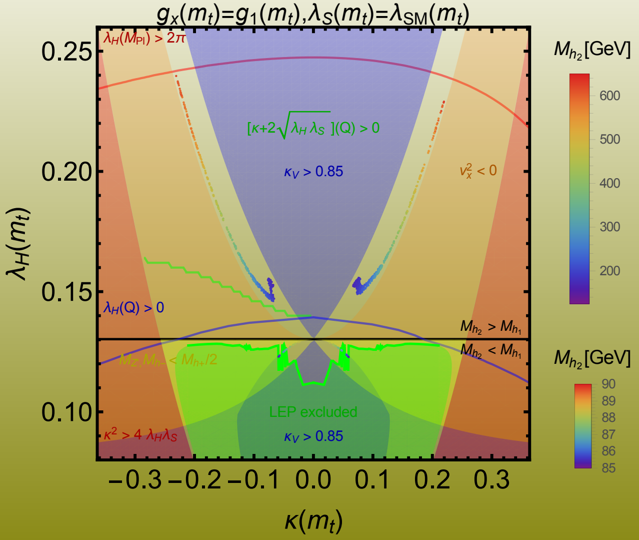
$$M_{h_1}^2 = \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H \lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}.$$

VDM:

- Larger initial values of λ_H such that $\lambda_H(m_t) > \lambda_{SM}$ are allowed delaying the instability (by shifting up the scale at which $\lambda_H(Q) < 0$).
- Even if the initial λ_H is smaller than its SM value, $\lambda_H(m_t) < \lambda_{SM}$, still there is a chance to lift the instability scale if appropriate initial value of the portal coupling $\kappa(m_t)$ is chosen.

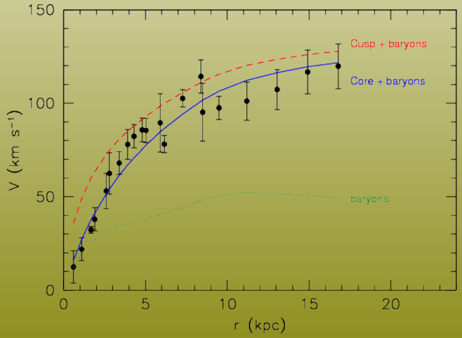
$$\beta_{\lambda_H}^{(1)} = \beta_{\lambda_H}^{SM\ (1)} + \kappa^2$$

A model of vector dark matter



Problems of ΛCDM vs. self-interacting dark matter

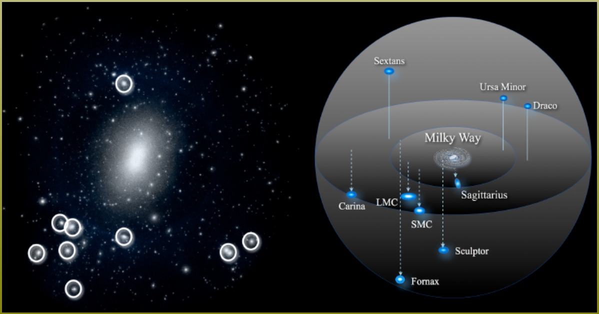
"The core-cusp problem" (also known as the cuspy halo problem) e.g. de Block et al.2001: There is a discrepancy between the observed dark matter density profiles of low-mass galaxies and the density profiles predicted by cosmological N-body simulations.



The measured rotation curve of F568-3 compared to model predictions with cored (blue solid curve) or a cuspy dark matter halo with an NFW profile.

Problems of ΛCDM vs. self-interacting dark matter

- "The too big to fail problem" Bolyan-Kolchin al. 2013: Simulations of galaxies show that satellite galaxies (e.g. Large and Small Magellanic Clouds) are too dense compared to what we observe around the MW.
- "The missing satellites problem" e.g. Klypin et al.1999: The number of Dark Matter sub-halos in Milky Way sized haloes is over-predicted by roughly one order magnitude.



Problems of ΛCDM vs. self-interacting dark matter

Problems

- "The core-cusp problem"
- "The too big to fail problem"
- "The missing satellites problem"

$\Rightarrow$

Solutions

$$\frac{\sigma_{\mathsf{self}}}{M_{\mathsf{DM}}} \neq 0$$

D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. 84, 3760 (2000)

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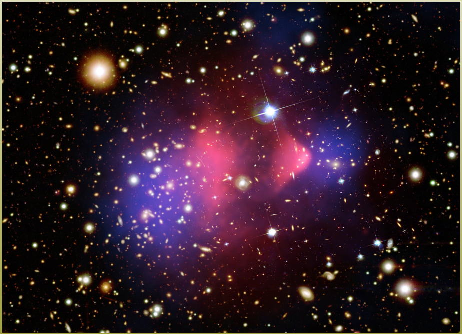
Problems of ΛCDM vs. self-interacting dark matter



$\Downarrow$

Upper bounds on self-interaction cross-section: $\frac{\sigma_{\text{self}}}{M_{\text{DM}}} \lesssim 1\,\frac{\text{cm}^2}{\text{g}}$

Problems of ΛCDM vs. self-interacting dark matter

Problems

- "The core-cusp problem"
- "The too big to fail problem"
- "The missing satellites problem"

$\Rightarrow$

Solutions

$$0.1\,\frac{\mathrm{cm}^2}{\mathrm{g}} \lesssim \frac{\sigma_{\mathrm{self}}}{M_{\mathrm{DM}}} \lesssim 10\,\frac{\mathrm{cm}^2}{\mathrm{g}}$$

Observation

Bullet cluster

$\Rightarrow$

Limit

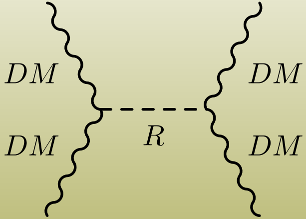
$$\frac{\sigma_{\mathrm{self}}}{M_{\mathrm{DM}}} \lesssim 1\,\frac{\mathrm{cm}^2}{\mathrm{g}}$$

Problems of ΛCDM vs. self-interacting dark matter

Large cross-section

$$0.1\,\frac{\mathrm{cm}^2}{\mathrm{g}} \lesssim \frac{\sigma_{\mathrm{self}}}{M_{\mathrm{DM}}} \lesssim 1\,\frac{\mathrm{cm}^2}{\mathrm{g}} \sim \frac{\mathrm{barn}}{\mathrm{GeV}} \gg \frac{\mathrm{pb}}{\mathrm{GeV}}$$

Model independent resonance enhancement of $\sigma_{\rm self}$



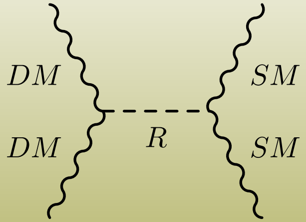
Breit-Wigner resonance ($2M_{DM} \approx M$) DM self-interaction.

$$\sigma_{\rm self} = \frac{32\pi\omega}{s\beta^2}\frac{M^2\Gamma^2B^2}{(s-M^2)^2+\Gamma^2M^2},$$

$$\left.\frac{\sigma_{\rm self}}{M_{DM}}\right|_{v_{\rm rel}\approx 0} \simeq \frac{8\pi\omega}{M_{DM}^3}\frac{\eta^2}{\delta^2+\gamma^2}$$

$$\eta \equiv \frac{\Gamma B}{M\beta}, \quad \delta \equiv \frac{4M_{DM}^2}{M^2}-1, \quad \gamma \equiv \frac{\Gamma}{M} \text{ and } \omega = \frac{(2J+1)}{(2S+1)^2}$$

Model independent resonance annihilation



Breit-Wigner resonance ($2M_{DM} \approx M$) annihilation.

$$\sigma v_{\rm rel} = \frac{64\pi\omega}{M^2\beta_i}\frac{\gamma^2}{(\delta + v_{\rm rel}^2/4)^2 + \gamma^2}B_iB_f$$

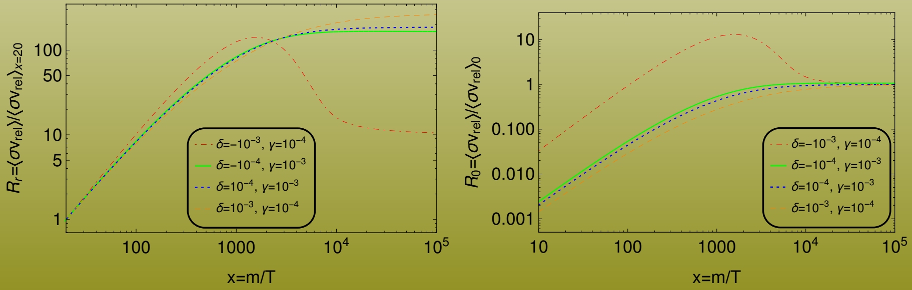
$$\langle\sigma v_{\rm rel}\rangle(x) = \frac{x^{3/2}}{2\sqrt{\pi}}\int_0^\infty dvv^2e^{-xv^2/4}\sigma v$$

P. Gondolo and G. Gelmini, Nucl. Phys. B 360, 145 (1991),
K. Griest and D. Seckel, Phys. Rev. D 43, 3191 (1991),
M. Ibe, H. Murayama and T. Yanagida, Phys. Rev. D 79, 095009 (2009)

Model independent resonance annihilation

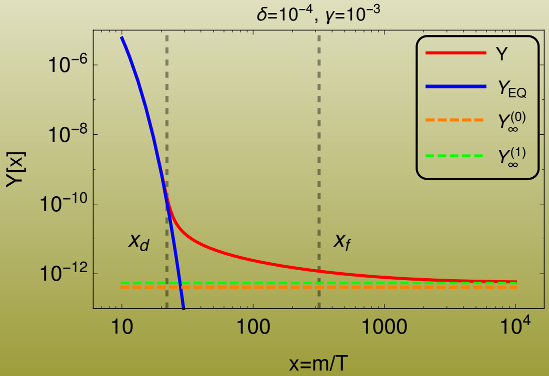
$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x)(Y^2 - Y_{EQ}^2)$$

$$R(x) = \frac{\langle \sigma v_{\rm rel}\rangle(x)}{\langle \sigma v_{\rm rel}\rangle_0} = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 e^{-xv^2/4} \frac{\delta^2+\gamma^2}{(\delta+v^2/4)^2+\gamma^2}$$



Thermally averaged annihilation cross section $\langle \sigma v_{\rm rel}\rangle$ normalized to its value at decoupling $\langle \sigma v_{\rm rel}\rangle_{x=20}$ (left) and to the low-temperature limit $\langle \sigma v_{\rm rel}\rangle_0$ (right).

Model independent resonance annihilation



Evolution of the dark matter yield $Y(x)$ for a wide resonance in unphysical region and a narrow resonance.

Model independent resonance annihilation

$$\frac{1}{Y_\infty} \equiv \frac{\lambda_0}{x_f}$$

$$x_f \approx \left[(\delta^2+\gamma^2)\frac{\pi - 2\arctan(\delta/\gamma)}{\gamma}\right]^{-1} \approx \begin{cases} (\pi\gamma)^{-1}, & \text{if } \gamma \gg |\delta| \\ (2\delta)^{-1}, & \text{if } \delta \gg \gamma \\ \gamma(2\pi\delta^2)^{-1}, & \text{if } -\delta \gg \gamma \end{cases}$$

$$\Omega h^2 = 2.74 \times 10^8 \frac{M_{DM}}{\text{GeV}} Y_\infty = 0.99 \times 10^{-27}\ \text{cm}^3\ \text{s}^{-1} \frac{x_f}{\sqrt{g_*}\langle\sigma v_{\rm rel}\rangle_0}$$

$$\langle\sigma v_{\rm rel}\rangle_0 \approx \frac{x_f}{25}\left(\frac{100}{g_*}\right)^{1/2}\left(\frac{0.12}{\Omega h^2}\right) 2 \times 10^{-26}\ \text{cm}^3\ \text{s}^{-1}$$

Model independent resonance annihilation

- parameters: $\langle \sigma v_{\rm rel} \rangle_0$ (present annihilation), η (resonance DM coupling), δ (resonance location), γ (resonance width)
- constraints: Ωh^2, $\sigma_{\sf self}/M_{DM}$ and Fermi-LAT upper limits on $\langle \sigma v_{\rm rel} \rangle_0$

the goal: minimize $\langle \sigma v_{\rm rel} \rangle_0$ for a given (large) $\frac{\sigma_{\rm self}}{M_{DM}} \approx \frac{8\pi\omega}{M_{DM}^3} \frac{\eta^2}{\delta^2+\gamma^2}$

$$\frac{\langle \sigma v_{\rm rel} \rangle_0}{2 \times 10^{-26}\ \text{cm}^3\ \text{s}^{-1}} \gtrsim \frac{560}{\xi \eta \sqrt{\omega}} \left(\frac{M_{DM}}{100\ \text{GeV}} \right)^{3/2} \left(\frac{\sigma_{\rm self}/M_{DM}}{1\ \text{cm}^2/\text{g}} \right)^{1/2} \times \left(\frac{100}{g_*} \right)^{1/2} \left(\frac{0.12}{\Omega h^2} \right)$$

where $2 \leq \xi \leq \pi$, $\eta \equiv \frac{\Gamma\, B(R \to DM\ DM)}{M\beta}$ and $\omega = \frac{(2J+1)}{(2S+1)^2}$

Early kinetic decoupling

Dark matter annihilation rate is enhanced by the resonance, therefore coupling of the mediator to the SM particles needs to be suppressed in order to be consistent with the observed abundance.

$\Downarrow$

Temperature of the kinetic decoupling T_{kd} (too weak DM-SM elastic scattering in order to maintain equilibrium) can be, in the resonant case, higher than in the typical WIMP scenario.

$\Downarrow$

If dark matter decouples kinetically when it is non-relativistic, then the DM temperature T_{DM} evolves according to $T_{DM} \propto a^{-2}$, contrary to the radiation-dominated SM thermal bath, for which $T_{SM} \propto a^{-1}$

Early kinetic decoupling

$$T_{DM} = \begin{cases} T_{SM}, & \text{if } T \geq T_{kd} \\ T_{SM}^2/T_{kd}, & \text{if } T < T_{kd}. \end{cases}$$

$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x_{DM})(Y^2 - Y_{EQ}^2) \quad \text{with} \quad x_{DM} = \frac{x^2}{x_{kd}}$$

X. Chen, M. Kamionkowski and X. Zhang, Phys. Rev. D 64, 021302 (2001))
T. Bringmann and S. Hofmann, JCAP 0704 (2007) 016
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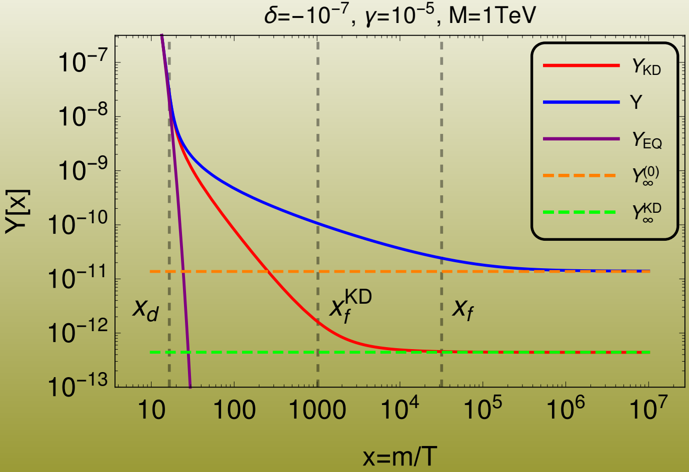
Early kinetic decoupling



Evolution of dark matter yield $Y(x)$ for dark matter in thermal equilibrium with the SM (blue curve) and in the case of simultaneous chemical and kinetic decoupling $x_{kd} = x_d$ (red curve).

Early kinetic decoupling

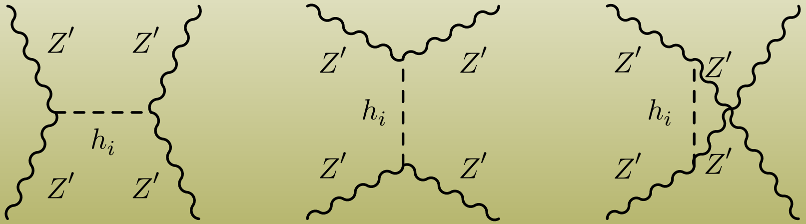
$$H(T_{kd}) \sim \Gamma_{\rm scat}(T_{kd}) \Rightarrow x_{kd}$$

$$\frac{\langle \sigma v_{\rm rel} \rangle_0}{2 \times 10^{-26}\ \mathsf{cm}^3\ \mathsf{s}^{-1}} \gtrsim \frac{3.8}{\sqrt{\eta\sqrt{\omega}}} \left(\frac{M_{DM}}{100\ \mathsf{GeV}} \right)^{3/4} \left(\frac{\sigma_{\rm self}/M_{DM}}{1\ \mathsf{cm}^2/\mathrm{g}} \right)^{1/4} \times \left(\frac{100}{g_*} \right)^{1/2} \left(\frac{0.12}{\Omega h^2} \right)$$

where

$$\eta \equiv \frac{\Gamma\, B(R \to DM\ DM)}{M\beta} \quad \text{and} \quad \omega = \frac{(2J+1)}{(2S+1)^2}$$

Resonant self-interaction of vector dark matter



Z' self-interaction in different channels.

$$\frac{\sigma_{\rm self}}{M_{Z'}} = g_x^4 \frac{M_{Z'}}{8\pi} \frac{R_{2i}^4}{(4M_{Z'}^2 - M_{h_i}^2)^2 + \Gamma_{h_i}^2 M_{h_i}^2},$$

Resonant self-interaction of vector dark matter

Minimal $\langle \sigma v_{\rm rel} \rangle_0$ in the VDM

$$\eta = \frac{\Gamma_{h_2}}{M_{h_2}} \frac{1}{\bar{\beta}} \lesssim \frac{3}{16}$$

$$\Downarrow$$

$$\frac{\langle \sigma v_{\rm rel} \rangle_0}{2 \times 10^{-26}\ \mathrm{cm}^3\ \mathrm{s}^{-1}} \gtrsim \frac{9 \cdot 10^3}{\xi} \left(\frac{M_{Z'}}{100\ \mathrm{GeV}} \right)^{3/2} \left(\frac{\sigma_{\rm self}/M_{Z'}}{1\ \mathrm{cm}^2/\mathrm{g}} \right)^{1/2} \cdot \left(\frac{100}{g_*} \right)^{1/2} \left(\frac{0.12}{\Omega h^2} \right)$$

where $2 \leq \xi \leq \pi$.

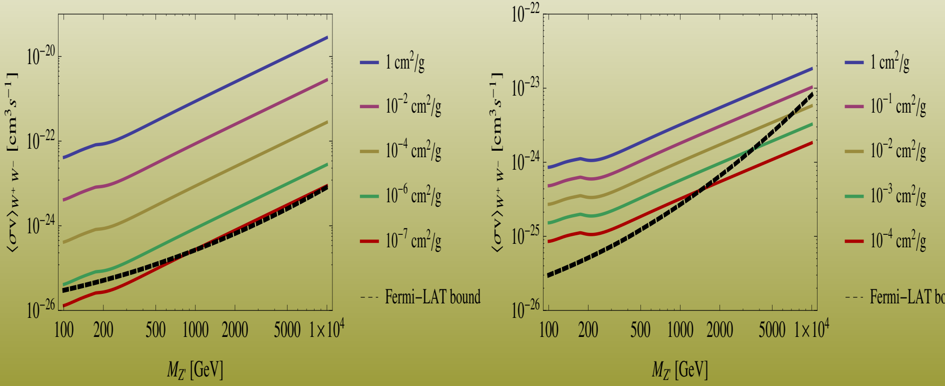
Resonant self-interaction of vector dark matter

With early kinetic decoupling

$$\frac{\langle \sigma v_{\rm rel} \rangle_0}{2 \times 10^{-26}\ \mathsf{cm}^3\ \mathsf{s}^{-1}} \gtrsim 15 \cdot x_{kd}^{1/2} \left(\frac{M_{Z'}}{100\ \mathsf{GeV}} \right)^{3/4} \left(\frac{\sigma_{\rm self}/M_{Z'}}{1\ \mathsf{cm}^2/\mathrm{g}} \right)^{1/4} \cdot \left(\frac{100}{g_*} \right)^{1/2} \left(\frac{0.12}{\Omega h^2} \right)$$

with $x_{kd} \sim 10 - 20$

Resonant self-interaction of vector dark matter



Dark matter annihilation cross-section in the W^+W^- channel consistent with Ωh^2 and desired $\sigma_{\text{self}}/M_{Z'}$ specified in the legend. Left panel does not take into account the early kinetic decoupling while the right one does for $x_{kd} = 15$.

Summary

- A model of vector $U(1)$ dark matter (VDM) was introduced and discussed. The model contains a second neutral Higgs boson h_2.
- Problems of ΛCDM were reviewed.
- A possibility of enhancing the dark-matter self-interaction cross-section ($\sigma_{\rm self}/M_{DM}$) by s-channel resonance was considered in a model independent way.
- Dark matter annihilation in the vicinity of a resonance was discussed in details. Approximate analytical and exact numerical solutions of the Boltzmann equation were found. Early kinetic decoupling of dark matter was considered.
- For a given $\sigma_{\rm self}/M_{DM}$ a lower limit for the annihilation cross-section $\langle \sigma v_{\rm rel} \rangle_0$ has been derived. In the VDM model the self-interaction cross section $\sigma_{\sf self}/M_{Z'}$ of the order of $(10^{-2} - 10^{-1})$ cm$^2/g$ could be achieved if dark matter was heavy enough, $M_{Z'} \sim 10^4$ GeV.