Semisymmetries of Two-Higgs-doublet models

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based on

- P.M. Ferreira , B.G., O.M. Ogreid, P. Osland, "New Symmetries of the Two-Higgs-Doublet Model", *Eur.Phys.J.C* 84 (2024) 3, 234, e-Print: 2306.02410
- work in progress

The Two-Higgs Doublet Model (2HDM) in the bilinear notation

$$\begin{aligned} V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \mathrm{h.c.}] + \\ &\quad \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\ &\quad \left\{ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)]\Phi_1^\dagger \Phi_2 + \mathrm{h.c.} \right\} , \end{aligned}$$

where m_{12}^2 and $\lambda_{5,6,7}$ might be complex.

An alternative notation uses four gauge-invariant bilinears constructed from the doublets (Velhinho 1994, Nagel 2004, Ivanov 2005, Maniatis 2006, Nishi 2006):

$$\begin{aligned} r_0 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2\right), \\ r_1 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) = \mathrm{Re}\left(\Phi_1^\dagger \Phi_2\right), \\ r_2 &\equiv -\tfrac{i}{2}\left(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1\right) = \mathrm{Im}\left(\Phi_1^\dagger \Phi_2\right), \\ r_3 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2\right). \end{aligned}$$

The Two-Higgs Doublet Model (2HDM) in the bilinear notation

The potential of may be written as

$$V = M_\mu \, r^\mu + \Lambda_{\mu\nu} \, r^\mu \, r^\nu \,,$$

where

$$r^\mu \equiv (r_0\,,\, r_1\,,\, r_2\,,\, r_3) = (r_0\,,\, \vec{r})\,,$$

$$M^\mu \equiv \left(m_{11}^2 + m_{22}^2\,,\, 2\mathrm{Re}(m_{12}^2)\,,\, -2\mathrm{Im}(m_{12}^2)\,,\, m_{22}^2 - m_{11}^2\right) = (M_0\,,\, \vec{M})\,,$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 & -\mathrm{Re}\,(\lambda_6 + \lambda_7) & \mathrm{Im}\,(\lambda_6 + \lambda_7) & \frac{1}{2}(\lambda_2 - \lambda_1) \\ -\mathrm{Re}\,(\lambda_6 + \lambda_7) & \lambda_4 + \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6 - \lambda_7) \\ \mathrm{Im}\,(\lambda_6 + \lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4 - \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) \\ \frac{1}{2}(\lambda_2 - \lambda_1) & \mathrm{Re}\,(\lambda_6 - \lambda_7) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix}$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

Basis transformations

Weak-basis transformation, U(2):

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \underbrace{e^{i\psi} \begin{pmatrix} \cos\theta & e^{-i\tilde{\xi}} \sin\theta \\ -e^{i\chi} \sin\theta & e^{i(\chi-\tilde{\xi})} \cos\theta \end{pmatrix}}_{U(2)} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

The Higgs kinetic terms remain invariant

Basis transformations

$$V = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

The basis rotation matrix

$$R_{ij}(U) \equiv \frac{1}{2}\mathrm{Tr}\left(U^\dagger \sigma_i U \sigma_j\right),$$

where σ_i $(i = 1, 2, 3)$ are the Pauli matrices.

The basis transformations:

$$\vec{r} \to \vec{r}\,' = R\,\vec{r}$$
$$\vec{M} \to \vec{M}' = R\,\vec{M}$$
$$\vec{\Lambda} \to \vec{\Lambda}' = R\,\vec{\Lambda}$$
$$\Lambda \to \Lambda' = R\,\Lambda\,R^T$$

whereas r_0, M_0 and Λ_{00} do not change under basis transformations – they are *basis invariants*.

Global symmetries of 2HDM

- *Higgs-family symmetries*, unitary transformations mix both doublets,

$$\Phi_i \to \Phi_i' = \sum_{j=1}^{2} U_{ij}\,\Phi_j\,, \qquad U \in U(2)$$

e.g. Z_2:

$$\Phi_1 \to \Phi_1 \quad , \quad \Phi_2 \to -\Phi_2\,,$$

prevents the occurrence of tree-level flavour-changing neutral currents (FCNC).

- *generalized CP (GCP)*, transformations:

$$\Phi_i \to \Phi_i' = \sum_{j=1}^{2} X_{ij}\,\Phi_j^*\,, \qquad X \in U(2)$$

e.g. “standard" CP transformation (CP1):

$$\Phi_i \to \Phi_i^*$$

Global symmetries of 2HDM

In the bilinear formalism, both Higgs-family and GCP field transformations are represented by rotations in the 3-dimensional space defined by the vector $\vec{r}$, namely

$$\vec{r} \to \vec{r}\,' = S\,\vec{r},$$

where $S \in O(3)$ defines a rotation of $\vec{r}$.

$$S_{Z_2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad , \quad S_{CP1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Global symmetries of 2HDM

CP2: $\Phi_1 \to \Phi_2^*, \qquad \Phi_2 \to -\Phi_1^*$

$$S_{CP2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

A parity transformation about the three axes.

S	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			λ_{134}	0	0
$SO(3)$		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 1: Relations between 2HDM scalar potential parameters for each of the six symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

Running of parameters of 2HDM

The 1-loop β-functions for the quadratic couplings

$$\beta_{m_{11}^2} = 3\lambda_1 m_{11}^2 + (2\lambda_3 + \lambda_4)\, m_{22}^2 - 3\left(\lambda_6^*\, m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}(9g^2 + 3g'^2)\, m_{11}^2 + \beta^F_{m_{11}^2},$$

$$\beta_{m_{22}^2} = (2\lambda_3 + \lambda_4)\, m_{11}^2 + 3\lambda_2\, m_{22}^2 - 3\left(\lambda_7^*\, m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}(9g^2 + 3g'^2)\, m_{22}^2 + \beta^F_{m_{22}^2},$$

$$\beta_{m_{12}^2} = -3\left(\lambda_6\, m_{11}^2 + \lambda_7\, m_{22}^2\right) + (\lambda_3 + 2\lambda_4)\, m_{12}^2 + 3\lambda_5\, m_{12}^{2\,*} - \frac{1}{4}(9g^2 + 3g'^2) m_{12}^2 + \beta^F_{m_{12}^2},$$

Running of parameters of 2HDM

and 1-loop β functions for the quartic ones,

$$\beta_{\lambda_1} = 6\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12|\lambda_6|^2 + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_1(3g^2 + g'^2) + \beta^F_{\lambda_1},$$

$$\beta_{\lambda_2} = 6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12|\lambda_7|^2 + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_2(3g^2 + g'^2) + \beta^F_{\lambda_2},$$

$$\beta_{\lambda_3} = (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + |\lambda_5|^2 + 2\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 8\,\mathrm{Re}\left(\lambda_6\lambda_7^*\right) + \frac{3}{8}(3g^4 + g'^4 - 2g^2g'^2) - \frac{3}{2}\lambda_3(3g^2 + g'^2) + \beta^F_{\lambda_3},$$

$$\beta_{\lambda_4} = (\lambda_1 + \lambda_2)\lambda_4 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 4|\lambda_5|^2 + 5\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 2\,\mathrm{Re}\left(\lambda_6\lambda_7^*\right) + \frac{3}{2}g^2g'^2 - \frac{3}{2}\lambda_4(3g^2 + g'^2) + \beta^F_{\lambda_4},$$

$$\beta_{\lambda_5} = (\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\lambda_5 + 5\left(\lambda_6^2 + \lambda_7^2\right) + 2\lambda_6\lambda_7 - \frac{3}{2}\lambda_5(3g^2 + g'^2) + \beta^F_{\lambda_5},$$

$$\beta_{\lambda_6} = (6\lambda_1 + 3\lambda_3 + 4\lambda_4)\lambda_6 + (3\lambda_3 + 2\lambda_4)\lambda_7 + 5\lambda_5\lambda_6^* + \lambda_5\lambda_7^* - \frac{3}{2}\lambda_6(3g^2 + g'^2) + \beta^F_{\lambda_6},$$

$$\beta_{\lambda_7} = (6\lambda_2 + 3\lambda_3 + 4\lambda_4)\lambda_7 + (3\lambda_3 + 2\lambda_4)\lambda_6 + 5\lambda_5\lambda_7^* + \lambda_5\lambda_6^* - \frac{3}{2}\lambda_7(3g^2 + g'^2) + \beta^F_{\lambda_7},$$

where the β^F_x terms contain all contributions coming from fermions.

Running of parameters of 2HDM

- If one imposes a Z_2 symmetry so that $\lambda_6 = \lambda_7 = 0$ one immediately obtains $\beta_{\lambda_6} = \beta_{\lambda_7} = 0$, confirming that the symmetry-based condition on λ's are preserved under radiative corrections at the one-loop order.
- For the Z_2 model

$$\beta_{\lambda_5} = \left[\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4 - \frac{3}{2}\left(3g^2 + g'^2\right)\right] \lambda_5$$

A fixed point of this RG equation – if at any scale $\lambda_5 = 0$, that coupling will remain equal to zero for all renormalization scales. Such fixed points of RG equations are usually fingerprints of symmetries, and indeed that is the case here: if $\lambda_6 = \lambda_7 = 0$, the extra constraint $\lambda_5 = 0$ takes us from a Z_2-symmetric model to a $U(1)$-symmetric.

Running of parameters of 2HDM

We have noticed that

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\;,\;\; \lambda_1 - \lambda_2 = 0 \;\;,\;\; \lambda_6 + \lambda_7 = 0 \right\}$$

- constitutes a fixed point of the 1-loop RG equations,
- are basis transformation invariants.

$$\beta_{\lambda_1 - \lambda_2} = 6\left(\lambda_1^2 - \lambda_2^2\right) + 12\left(|\lambda_6|^2 - |\lambda_7|^2\right) - \frac{3}{2}(\lambda_1 - \lambda_2)(3g^2 + g'^2)$$

$$\beta_{\lambda_6 + \lambda_7} = 6(\lambda_1\lambda_6 + \lambda_2\lambda_7) + (3\lambda_3 + 2\lambda_4)(\lambda_6 + \lambda_7) + 6\lambda_5(\lambda_6^* + \lambda_7^*) - \frac{3}{2}(\lambda_6 + \lambda_7)(3g^2 + g'^2)$$

$$\beta_{m_{11}^2 + m_{22}^2} = 3(\lambda_1 m_{11}^2 + \lambda_2 m_{22}^2) + (2\lambda_3 + \lambda_4)(m_{11}^2 + m_{22}^2) - 3\left[(\lambda_6^* + \lambda_7^*) m_{12}^2 + \text{h.c.}\right] - \frac{1}{4}(9g^2 + 3g'^2)(m_{11}^2 + m_{22}^2)$$

Running of parameters of 2HDM

It turns out that

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \ , \ \ \lambda_1 - \lambda_2 = 0 \ , \ \ \lambda_6 + \lambda_7 = 0 \right\}$$

is also the 2-loop fixed point.

Conclusion:
Perhaps there is a symmetry behind the fixed point:

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \ , \ \ \lambda_1 - \lambda_2 = 0 \ , \ \ \lambda_6 + \lambda_7 = 0 \right\}$$

New 2HDM symmetries/semisymmetries

$$V = M_\mu \, r^\mu + \Lambda_{\mu\nu} \, r^\mu \, r^\nu$$

The rotation matrix $R_{ij}(U) = \mathrm{Tr}\left(U^\dagger \sigma_i U \sigma_j\right)/2$, and the basis transformations:

$$\begin{aligned} \vec{r} \to \vec{r}\,' &= R\,\vec{r} \\ \vec{M} \to \vec{M}' &= R\,\vec{M} \\ \vec{\Lambda} \to \vec{\Lambda}' &= R\,\vec{\Lambda} \\ \Lambda \to \Lambda' &= R\,\Lambda\,R^T \end{aligned}$$

whereas r_0, M_0 and Λ_{00} do not change under basis transformations – they are *basis invariants*.

$$\Lambda^{\mu\nu} = \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

New 2HDM symmetries/semisymmetries

Basis transformation invariants:

$$I_{1,1} = \Lambda_{00}\,, \qquad I_{1,2} = \mathrm{Tr}\Lambda$$

$$I_{2,1} = \vec{\Lambda}\cdot\vec{\Lambda}\,, \qquad I_{2,2} = \mathrm{Tr}\Lambda^2$$

$$I_{3,1} = \vec{\Lambda}\cdot\Lambda\vec{\Lambda}\,, \qquad I_{3,2} = \mathrm{Tr}\Lambda^3$$

$$I_{4,1} = \vec{\Lambda}\cdot\Lambda^2\vec{\Lambda}\,,$$

To all orders of perturbation theory,

$$\beta_{\vec{\Lambda}} = a_0\,\vec{\Lambda} + a_1\,\Lambda\,\vec{\Lambda} + a_2\,\Lambda^2\,\vec{\Lambda}$$

- $\vec{\Lambda} = \vec{0}$ *is a fixed point to all orders of perturbation theory.*

where the a_i are polynomial expressions involving invariants,

see A.V. Bednyakov, "On three-loop RGE for the Higgs sector of 2HDM", JHEP 11 (2018) 154, e-Print: 1809.04527

New 2HDM symmetries/semisymmetries

$$\beta_{M_0} = b_0\, M_0 + b_1\, \vec{\Lambda}\cdot\vec{M} + b_2\, \vec{\Lambda}\cdot\left(\Lambda\vec{M}\right) + b_3\, \vec{\Lambda}\cdot\left(\Lambda^2\vec{M}\right)$$

- *If $\vec{\Lambda} = \vec{0}$, then $M_0 = 0$ is a fixed point to all orders.*

$$\beta_{\vec{M}} = c_0\,\vec{M} + c_1\,\Lambda\,\vec{M} + c_2\,\Lambda^2\,\vec{M} + c_3\, I_{M3}\,\vec{\Lambda} + c_4\, I_{M4}\,\Lambda\vec{\Lambda} + c_5\, I_{M5}\,\Lambda^2\,\vec{\Lambda}$$

- *If $\vec{\Lambda} = \vec{0}$, then $\vec{M} = \vec{0}$ is a fixed point to all orders*

where the c_i are polynomial expressions involving invariants,

see A.V. Bednyakov

New 2HDM symmetries/semisymmetries

Two all-order fixed points of the 2HDM RG equations:

- $\{\vec{M} = \vec{0}\,,\, \vec{\Lambda} = \vec{0}\}$.

$$m_{11}^2 = m_{22}^2 \quad , \quad m_{12}^2 = 0 \quad , \quad \lambda_1 = \lambda_2 \quad , \quad \lambda_6 = -\lambda_7 \,.$$

These are exactly the CP2 symmetry conditions.

- $\{M_0 = 0\,,\, \vec{\Lambda} = \vec{0}\}$.

$$M_0 \equiv m_{11}^2 + m_{22}^2 = 0 \quad , \quad \lambda_1 = \lambda_2 \quad , \quad \lambda_6 = -\lambda_7 \,.$$

These are the conditions mentioned before and they coincide with the CP2 symmetry conditions for the quartic couplings, but have different conditions for the quadratic ones. The conditions are basis invariant, so they are *not* a basis change of the previous ones.

The r_0 symmetry/semisymmetry

$$V = M_\mu\, r^\mu + \Lambda_{\mu\nu}\, r^\mu\, r^\nu$$

where

$$\begin{aligned}
r_0 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right)\\
r_1 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) = \mathrm{Re}\left(\Phi_1^\dagger\Phi_2\right)\\
r_2 &\equiv -\tfrac{i}{2}\left(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right) = \mathrm{Im}\left(\Phi_1^\dagger\Phi_2\right)\\
r_3 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right)
\end{aligned}$$

$$V = M_0\, r_0 + \Lambda_{00}\, r_0^2 - \vec{M}\cdot\vec{r} - 2\left(\vec{\Lambda}\cdot\vec{r}\right) r_0 + \vec{r}\cdot(\Lambda\,\vec{r})$$

- $\{\vec{M} = \vec{0}\,,\ \vec{\Lambda} = \vec{0}\}$. These are exactly the CP2 ($\vec{r} \to -\vec{r}$).
- $\{M_0 = 0\,,\ \vec{\Lambda} = \vec{0}\}$ These are new, perhaps $r_0 \overset{?}{\to} -r_0$

The r_0 symmetry/semisymmetry

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

The transformation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

implies

$$r_0 \to -r_0 \qquad r_i \to +r_i$$

The r_0 symmetry/semisymmetry

$$\Phi_1 \to -\Phi_2^* \quad \Phi_1^\dagger \to \Phi_2^T,$$
$$\Phi_2 \to \Phi_1^*, \quad \Phi_2^\dagger \to -\Phi_1^T.$$

- Higgs kinetic terms

$$\mathcal{L}_k = (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2),$$

where

$$D^\mu = \partial^\mu + \frac{ig}{2}\sigma_i W_i^\mu + i\frac{g'}{2}B^\mu,$$

$\mathcal{L}_k$ remains invariant if the above transformation of $\Phi_{1,2}$ is supplemented by

$$\partial_\mu \to -i\partial_\mu,$$
$$B_\mu \to iB_\mu,$$
$$W_{1\mu} \to iW_{1\mu}, \quad W_{2\mu} \to -iW_{2\mu}, \quad W_{3\mu} \to iW_{3\mu}.$$

The r_0 symmetry/semisymmetry

- Gauge kinetic terms

$$\mathcal{L}^B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{i\mu\nu}W_i^{\mu\nu},$$

where $B^{\mu\nu} = \partial^\nu B^\mu - \partial^\mu B^\nu$ and $W_i^{\mu\nu} = \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk}W_j^\mu W_k^\nu$.
Under r_0 transformation

$$B^{\mu\nu} \to B^{\mu\nu},$$
$$W_1^{\mu\nu} \to W_1^{\mu\nu}, \quad W_2^{\mu\nu} \to -W_2^{\mu\nu}, \quad W_3^{\mu\nu} \to W_3^{\mu\nu}$$

Symmetries/Semisymmetries and 1-loop CW effective potential

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}$$

$$V_{\text{CW}}^{(1-loop)}(\phi_a) = \frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4} \text{Tr}\left[\ln(p_E^2 + M_S^2)\right] = -\frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4} \left[\text{Tr}\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\frac{\text{M}_S^2}{p_E^2}\right)^n\right]$$

$$\left(\text{M}_S^2\right)_{ab} \equiv \frac{\partial^2 V}{\partial \phi_a \partial \phi_b}$$

$a, b = 1, \cdots, 8$

Q.-H. Cao, K. Cheng, and C. Xu, "Global Symmetries and Effective Potential of 2HDM in Orbit Space", Phys.Rev.D 108 (2023) 055036, arXiv:2305.12764 [hep-ph].

Symmetries/Semisymmetries and 1-loop CW effective potential

$$r_0 \to -r_0 \qquad r_i \to +r_i$$

Is $V_{\rm CW}^{(1-loop)}(\phi_a)$ invariant under the r_0 transformation?

At the new fixed point $M_0 = 0$ and $\vec{\Lambda} = 0$ ($m_{11} + m_{22} = 0$, $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$):

$$n = 1: \qquad \mathrm{Tr}(M_S^2) = 4[5\Lambda_{00} + \mathrm{tr}(\Lambda)]r_0 \xrightarrow{r_0} -\mathrm{Tr}(M_S^2) = -4[5\Lambda_{00} + \mathrm{tr}(\Lambda)]r_0$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & -\mathrm{Re}(\lambda_6+\lambda_7) & \mathrm{Im}(\lambda_6+\lambda_7) & \frac{1}{2}(\lambda_2-\lambda_1) \\ -\mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) \\ \mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) \\ \frac{1}{2}(\lambda_2-\lambda_1) & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \frac{1}{2}(\lambda_1+\lambda_2)-\lambda_3 \end{pmatrix}$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

Symmetries/Semisymmetries and 1-loop CW effective potential

$$r_0 \to -r_0 \qquad r_i \to +r_i$$

$$V_{\rm CW}^{(1-loop)}(\phi_a) = -\frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4}\left[\mathrm{Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{\mathrm{M}_S^2}{p_E^2}\right)^n\right]$$

For $M_0 = 0$ and $\vec{\Lambda} = 0$ ($m_{11}^2 + m_{22}^2 = 0$, $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$):

$n = 1:$	$\mathrm{Tr}\left[M_S^2\right]$	odd
$n = 2:$	$\mathrm{Tr}\left[(M_S^2)^2\right]$	even
$\vdots$		
$n = 2k:$	$\mathrm{Tr}\left[(M_S^2)^{2k}\right]$	even
$n = 2k+1:$	$\mathrm{Tr}\left[(M_S^2)^{2k+1}\right]$	odd
$\vdots$		

Conclusion: The r_0 symmetry is explicitly broken by $n = 2k + 1$ contributions to $V_{\rm CW}^{(1-loop)}(\phi_a)$.

The toy model - 2RSM

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2) - V(\phi_1,\phi_2)\,,$$

with

$$V(\phi_1\,\phi_2) = \frac{1}{2}m_1^2(\phi_1^2 - \phi_2^2) + m_{12}^2\phi_1\phi_2 + \frac{1}{2}\lambda_1(\phi_1^4 + \phi_2^4) + \lambda_3(\phi_1\phi_2)^2 + \lambda_6(\phi_1^2 - \phi_2^2)\phi_1\phi_2\,.$$

The model is invariant under the following r_0-like transformation

$$\boldsymbol{x^\mu \to i x^\mu}, \qquad \boldsymbol{\phi_1 \to i\phi_2}, \qquad \boldsymbol{\phi_2 \to -i\phi_1}$$

It is possible to choose a (ϕ_1, ϕ_2) basis such that $\lambda_6 = 0$.

The mass^2 matrix

$$\left(M_S^2\right)_{ij} = \begin{pmatrix} m_1^2 + 6\lambda_1\phi_1^2 + 2\lambda_3\phi_2^2 & m_{12}^2 + 4\lambda_3\phi_1\phi_2 \\ & -m_1^2 + 6\lambda_1\phi_2^2 + 2\lambda_3\phi_1^2 \end{pmatrix}$$

The toy model - 2RSM

One can express the potential in terms of bilinear variables:

$$r_0 \equiv \frac{1}{2}(\phi_1^2 + \phi_2^2)$$
$$r_1 \equiv \phi_1 \phi_2$$
$$r_2 \equiv \frac{1}{2}(\phi_1^2 - \phi_2^2)\,.$$

Upon the r_0 transformation

$$(r_0, r_1, r_2) \xrightarrow{r_0} (-r_0, r_1, r_2)$$

The potential could be written as

$$V(r^\mu) = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

for $\mu, \nu = 0, 1, 2$ with $M_\mu = (0, m_{12}^2, m_1^2)$ and

$$\Lambda_{\mu\nu} = \begin{pmatrix} \Lambda_{00} & 0 & 0 \\ 0 & \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{21} & \Lambda_{22} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_1 \end{pmatrix}\,.$$

The toy model - 2RSM

$M_0 = 0$ and $\vec{\Lambda} = 0$ are implied by the r_0 symmetry
$(m_1^2 + m_2^2 = 0,\ \lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7)$.

$$\mathrm{Tr}\left(M_S^2\right) = 4(3\lambda_1 + \lambda_3)r^0$$

Under the r_0 transformation the trace is odd:

$$\mathrm{Tr}\left(M_S^2\right) \xrightarrow{r_0} -\mathrm{Tr}\left(M_S^2\right) \ ,$$

Two local minima:

$$(v_1^2 - v_2^2) = \frac{-m_1^2}{\lambda_1}, \qquad v_1 v_2 = \frac{-m_{12}^2}{(\lambda_1 + \lambda_3)}$$

where $\langle\phi_{1,2}\rangle \equiv v_{1,2}/\sqrt{2}$.

The toy model - 2RSM

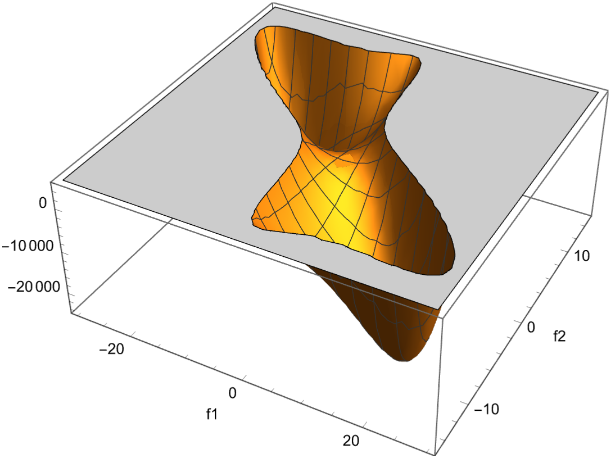

Figure 1: Scalar potential of the toy model, $m_1 = 10$, $m_{12} = 20$, $\lambda_1 = 1$, $\lambda_3 = 2$.

The toy model - 2RSM

The eigenvalues of M_S^2 could be expressed through bilinears

$$M_1^2(r_\mu) = 2(3\lambda_1 + \lambda_3)r_0 + \sqrt{\Delta}$$
$$M_2^2(r_\mu) = 2(3\lambda_1 + \lambda_3)r_0 - \sqrt{\Delta}\,,$$

where

$$\Delta = m_1^4 + m_{12}^4 + 4m_1^2(3\lambda_1 - \lambda_3)r_2 + 8m_{12}^2\lambda_3 r_1 + 16\lambda_3^2 r_0^2 + 12(3\lambda_1 + \lambda_3)(\lambda_1 - \lambda_3)r_2^2$$

$$M_1^2 \xrightarrow{r_0} -M_2^2$$
$$M_2^2 \xrightarrow{r_0} -M_1^2$$

The 1-loop effective potential

$$V_{CW}^{\text{1-loop}}(r_\mu) = \frac{1}{64\pi^2} \sum_{i=1,2} M_i^4(r_\mu) \left[\log \frac{M_i^2(r_\mu)}{\mu^2} - \frac{3}{2} \right] .$$

$$V_{CW}^{\text{1-loop}}(r_0) \xrightarrow{r_0} V_{CW}^{\text{1-loop}}(-r_0) = V_{CW}^{\text{1-loop}}(r_0) - i\pi \left(M_1^4 + M_2^4 \right) , \qquad (2)$$

for

$$M_1^4 + M_2^4 = 2 \left\{ [2(3\lambda_1 + \lambda_3) r_0]^2 + \Delta \right\} .$$

The 1-loop effective potential is not invariant under the r_0 transformation.

The model considered in this section indeed is stable under 1-loop RGE running.

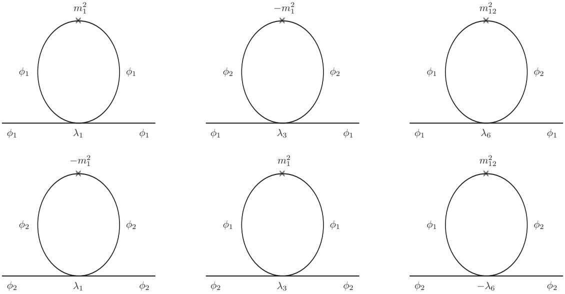

Figure 2: Diagrams which generate mass^2 beta functions: $\beta_{m_1^2}$ and $\beta_{m_2^2}$.

Symmetries/Semisymmetries and 1-loop CW effective potential

$$r_0 \to -r_0 \qquad r_i \to +r_i$$

$$V_{\mathrm{CW}}^{(1-loop)}(\phi_a) = -\frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4}\left[\mathrm{Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{\mathrm{M}_S^2}{p_E^2}\right)^n\right]$$

For $M_0 = 0$ and $\vec{\Lambda} = 0$ $\left(m_{11}^2 + m_{22}^2 = 0, \lambda_1 = \lambda_2 \text{ and } \lambda_6 = -\lambda_7\right)$:

$$\begin{aligned}
n = 1: &\qquad \mathrm{Tr}\left[M_S^2\right] &\qquad \text{odd}\\
n = 2: &\qquad \mathrm{Tr}\left[(M_S^2)^2\right] &\qquad \text{even}\\
&\vdots\\
n = 2k: &\qquad \mathrm{Tr}\left[(M_S^2)^{2k}\right] &\qquad \text{even}\\
n = 2k+1: &\qquad \mathrm{Tr}\left[(M_S^2)^{2k+1}\right] &\qquad \text{odd}\\
&\vdots
\end{aligned}$$

Conclusion: The r_0 symmetry is explicitly broken by $n = 2k + 1$ contributions to $V_{\mathrm{CW}}^{(1-loop)}(\phi_a)$.

Summary and conclusions

- A set of constraints on 2HDM scalar parameters which is RG invariant to all orders with bosonic contributions to the β-functions – and which can be invariant to at least two loops if fermions are also included, have been found.
- The constraints are

$$m_{11}^2 + m_{22}^2 = 0 \quad , \quad \lambda_1 = \lambda_2 \quad , \quad \lambda_6 = -\lambda_7 \, ,$$

- The constraints are basis invariant.
- The constraints are fixed points of RGE equations for corresponding quantities, however they do not imply presence of any known symmetry.
- The constraints could be seeing as emerging from the "r_0 symmetry" (semisymmetry): $r_0 \to -r_0$ defined in terms of the bilinears $r_0 \equiv \frac{1}{2}\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right)$.

- The r_0 symmetry can not be obtained in terms of unitary transformation acting upon Higgs doubles, except for an unorthodox transformation (i.e. r_0 transformation) that involves $x_\mu \to i\, x_\mu$ and perhaps $p^\mu \to i\, p^\mu$.
- $$V_{\text{CW}}^{(1-loop)}(\phi_a) = -\frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4}\left[\text{Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{\text{M}_S^2}{p_E^2}\right)^n\right]$$

  If change of sign of massless propagator (i.e. $p_E^2 \to -p_E^2$) is applied while calculating 1-loop CW potential, the r_0 parity of $\text{Tr}\left(\frac{M_S^2}{p_E^2}\right)^n$ changes for $n = 2k + 1$ so that the total effective potential becomes r_0 invariant.

Backup slides

$$\Lambda^3 = (\mathrm{Tr}\Lambda)\Lambda^2 - \frac{1}{2}\left[(\mathrm{Tr}\Lambda)^2 - \mathrm{Tr}\Lambda^2\right]\Lambda + \frac{1}{6}\left[(\mathrm{Tr}\Lambda)^3 - 3\mathrm{Tr}\Lambda\,\mathrm{Tr}\Lambda^2 + 2\mathrm{Tr}\Lambda^3\right]$$

New 2HDM symmetries/semisymmetries

Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
r_0		$-m_{11}^2$			λ_1					$-\lambda_6$
0CP1		$-m_{11}^2$	real		λ_1			real	real	$-\lambda_6$
$0Z_2$		$-m_{11}^2$	0		λ_1				0	0
0U(1)		$-m_{11}^2$	0		λ_1			0	0	0
0CP2	0	0	0		λ_1					$-\lambda_6$
0CP3	0	0	0		λ_1			λ_{134}	0	0
$0SO(3)$	0	0	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 2: Relations between 2HDM scalar potential parameters for each of the new seven symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

Fermionic digression

Remarks:

- The two fixed points
  - $\{\vec{M} = \vec{0}\,,\ \vec{\Lambda} = \vec{0}\}$.
  - $\{M_0 = 0\,,\ \vec{\Lambda} = \vec{0}\}$.

  imply the same quartic scalar couplings, i.e. CP2 invariant.
- Yukawa couplings consistent with CP2 are known, see P. M. Ferreira and J. P. Silva, "A Two-Higgs Doublet Model With Remarkable CP Properties," Eur. Phys. J. C **69** (2010), 45-52, [arXiv:1001.0574 [hep-ph]].
- r_0 transformations of fermions are unknown,
- in the following we will adopt CP2 invariant Yukawas to calculate fermionic contributions to beta functions.

Fermionic digression

$$-\mathcal{L}_Y = \bar{q}_L(\Gamma_1\Phi_1+\Gamma_2\Phi_2)n_R + \bar{q}_L(\Delta_1\tilde{\Phi}_1+\Delta_2\tilde{\Phi}_2)p_R + \bar{l}_L(\Pi_1\Phi_1+\Pi_2\Phi_2)l_R + \mathrm{H.c.}$$

- For the CP2 symmetry:

$$\Gamma_1 = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & -a_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} -a_{12}^* & a_{11}^* & 0 \\ a_{11}^* & a_{12}^* & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

  Similarly for Δ and Π matrices, with different coefficients b_{ij} and c_{ij} instead of a_{ij}.

Fermionic digression

For the most general 2HDM

$$\beta^{F,1L}_{m^2_{11}} = \left[3\,\mathrm{Tr}(\Delta_1\Delta_1^\dagger) + 3\,\mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) + \mathrm{Tr}(\Pi_1\Pi_1^\dagger)\right]\, m^2_{11} - \left\{\left[3\,\mathrm{Tr}(\Delta_1^\dagger\Delta_2) + 3\,\mathrm{Tr}(\Gamma_1^\dagger\Gamma_2) + \mathrm{Tr}(\Pi_1^\dagger\Pi_2)\right]\, m^2_{12} + \mathrm{h.c.}\right\}\,,$$

$$\beta^{F,1L}_{m^2_{22}} = \left[3\mathrm{Tr}(\Delta_2\Delta_2^\dagger) + 3\mathrm{Tr}(\Gamma_2\Gamma_2^\dagger) + \mathrm{Tr}(\Pi_2\Pi_2^\dagger)\right]\, m^2_{22} - \left\{\left[3\,\mathrm{Tr}(\Delta_1^\dagger\Delta_2) + 3\,\mathrm{Tr}(\Gamma_1^\dagger\Gamma_2) + \mathrm{Tr}(\Pi_1^\dagger\Pi_2)\right]\, m^2_{12} + \mathrm{h.c.}\right\}\,,$$

It turns out that

$$\mathrm{Tr}(\Delta_1\Delta_1^\dagger) = \mathrm{Tr}(\Delta_2\Delta_2^\dagger)\;,\quad \mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) = \mathrm{Tr}(\Gamma_2\Gamma_2^\dagger)\;,\quad \mathrm{Tr}(\Pi_1\Pi_1^\dagger) = \mathrm{Tr}(\Pi_2\Pi_2^\dagger)\,,$$

as well as

$$\mathrm{Tr}(\Delta_1\Delta_2^\dagger) = \mathrm{Tr}(\Gamma_1\Gamma_2^\dagger) = \mathrm{Tr}(\Pi_1\Pi_2^\dagger) = 0.$$

Hence,

$$\beta^{F,1L}_{m^2_{11}+m^2_{22}} = \left[3\,\mathrm{Tr}(\Delta_1\Delta_1^\dagger) + 3\,\mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) + \mathrm{Tr}(\Pi_1\Pi_1^\dagger)\right]\,\left(m^2_{11} + m^2_{22}\right)$$

Fermionic digression

It could be shown that

$$\beta^{F,1-loop}_{m_{11}^2+m_{22}^2} \propto \left(m_{11}^2 + m_{22}^2\right)$$

and

$$\beta^{F,2-loop}_{m_{11}^2+m_{22}^2} \propto \left(m_{11}^2 + m_{22}^2\right)$$

So $m_{11}^2 + m_{22}^2 = 0$ is preserved by fermionic contributions up to 2 loops.

Phenomenology of the r_0 symmetry (semisymmetry)

The set of 11 independent physical parameters of 2HDM:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

The kinetic Lagrangian:

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

$$\mathrm{Coefficient}\left(\mathcal{L}_k, Z^\mu \left[H_j \overleftrightarrow{\partial_\mu} H_i\right]\right) = \frac{g}{2v\cos\theta_W}\epsilon_{ijk} e_k$$

$$\mathrm{Coefficient}\left(\mathcal{L}_k, H_i Z^\mu Z^\nu\right) = \frac{g^2}{4\cos^2\theta_W} e_i\, g_{\mu\nu}$$

$$\mathrm{Coefficient}\left(\mathcal{L}_k, H_i W^{+\mu} W^{-\nu}\right) = \frac{g^2}{2} e_i\, g_{\mu\nu}$$

$$q_i \equiv \mathrm{Coefficient}(V, H_i H^+ H^-)$$

$$q \equiv \mathrm{Coefficient}(V, H^+ H^+ H^- H^-)$$

Phenomenology of the r_0 symmetry (semisymmetry)

CP-sensitive invariants in the bilinear notation

$$I_1 = \left(\vec{M} \times \vec{\Lambda}\right) \cdot \left(\Lambda \vec{M}\right)$$

$$I_2 = \left(\vec{M} \times \vec{\Lambda}\right) \cdot \left(\Lambda \vec{\Lambda}\right)$$

$$I_3 = \left[\vec{M} \times \left(\Lambda \vec{M}\right)\right] \cdot \left(\Lambda^2 \vec{M}\right)$$

$$I_4 = \left[\vec{\Lambda} \times \left(\Lambda \vec{\Lambda}\right)\right] \cdot \left(\Lambda^2 \vec{\Lambda}\right)$$

Since the r_0 symmetry implies $\vec{\Lambda} = \vec{0}$ the invariants $I_{1,2,4}$ are automatically zero. However

$$I_3 = -16\lambda_5\, m_{11}^2 \,\mathrm{Im}(m_{12}^2)\,\mathrm{Re}(m_{12}^2)\, \left[(\lambda_1 - \lambda_3 - \lambda_4)^2 - \lambda_5^2\right] \neq 0$$

explicit violation of CP

Phenomenology of the r_0 symmetry (semisymmetry)

Stationary-point equations:

$$\begin{aligned} m_{11}^2 &= \frac{1}{2}\lambda_1 \left(v_2^2 - v_1^2\right), \\ \mathrm{Re}\, m_{12}^2 &= \frac{1}{2} v_1 v_2 \cos\xi \,(\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5), \\ \mathrm{Im}\, m_{12}^2 &= -\frac{1}{2} v_1 v_2 \sin\xi \,(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5). \end{aligned}$$

The neutral sector rotation matrix is then given by

$$R = \begin{pmatrix} \frac{v_2 \cos\xi}{v} & \frac{v_1 \cos\xi}{v} & -\sin\xi \\ -\frac{v_1}{v} & \frac{v_2}{v} & 0 \\ \frac{v_2 \sin\xi}{v} & \frac{v_1 \sin\xi}{v} & \cos\xi \end{pmatrix},$$

yielding masses

$$M_1^2 = \frac{1}{2} v^2 (\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5), \quad M_2^2 = \lambda_1 v^2,$$

$$M_3^2 = \frac{1}{2} v^2 (\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5), \quad M_{H\pm}^2 = \frac{1}{2} (\lambda_1 + \lambda_3) v^2$$

No decoupling limit!

Phenomenology of the r_0 symmetry (semisymmetry)

Assuming that M_2 is the SM-like Higgs boson, we obtain from unitarity and boundedness-from-below constraints:

$$\begin{aligned} M_{H^\pm} &\leq 711\ \text{GeV}\,, \\ M_3 &\leq 712\ \text{GeV}\,, \\ M_1 &\leq 711\ \text{GeV} \end{aligned}$$

Input parameters:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

Constraints implied by the r_0 symmetry:

$$v^2(e_1 q_2 - e_2 q_1) + e_1 e_2(M_2^2 - M_1^2) = 0, \quad v^2(e_1 q_3 - e_3 q_1) + e_1 e_3(M_3^2 - M_1^2) = 0,$$

$$v^2(e_2 q_3 - e_3 q_2) + e_2 e_3(M_3^2 - M_2^2) = 0, \quad q = \frac{1}{2v^4}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$

$$M_{H^\pm}^2 = \frac{1}{2}(e_1 q_1 + e_2 q_2 + e_3 q_3) + \frac{1}{2v^2}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$