Resonances in cosmology - revisitation -

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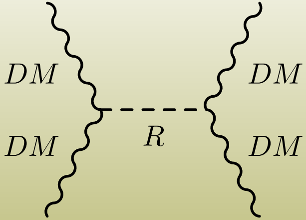
Outline

- Resonance beyond the Breit-Wigner (BW) approximation
- Early kinetic decoupling and coupled Boltzmann equations
- Generic conclusions on the BW approximation
- $U(1)$ vector dark matter (VDM) model
- Resonant self-interaction in VDM model
- Gauge dependance and unitarity violation in VDM annihilation
- Numerical results
- Summary

⋆ M. Duch, BG, "Resonances in cosmology", in progress

⋆ M. Duch, BG, M. McGarrie, "A stable Higgs portal with vector dark matter", JHEP 1509 (2015) 162, arXiv:1506.08805

Resonance beyond the B-W: σ_{self}



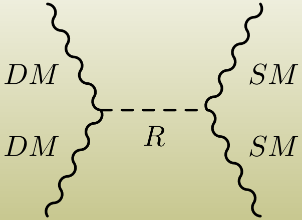
Breit-Wigner resonance ($2M_{DM} \approx M$) DM self-interaction.

$$\sigma_{\text{self}} = \frac{32\pi\omega}{s\beta_i^2} \frac{M^2\Gamma_i^2}{(s-M^2)^2 + \Gamma^2 M^2},$$

$$\left.\frac{\sigma_{\text{self}}}{M_{DM}}\right|_{v_{\text{rel}}\approx 0} \simeq \frac{8\pi\omega}{M_{DM}^3} \frac{\eta^2}{\delta^2 + \gamma^2}$$

$$\eta \equiv \frac{\Gamma_i}{M\beta_i},\ \delta \equiv \frac{4M_{DM}^2}{M^2} - 1,\ \beta_i \equiv (|\delta|)^{1/2},\ \gamma \equiv \frac{\Gamma}{M} \text{ and } \omega = \frac{(2J+1)}{(2S+1)^2}$$

Resonance beyond the B-W: $\langle \sigma v_{\rm rel} \rangle (x)$



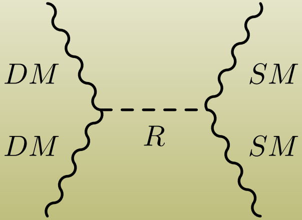
Breit-Wigner resonance ($2M_{DM} \approx M$) annihilation.

$$\sigma v_{\rm rel} = \frac{64\pi\omega}{M^2\beta_i} \frac{\gamma_i\gamma_f}{(\delta + v_{\rm rel}^2/4)^2 + \gamma^2}$$

$$\langle \sigma v_{\rm rel} \rangle (x) = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 e^{-xv^2/4} \sigma v$$

P. Gondolo and G. Gelmini, Nucl. Phys. B 360, 145 (1991),
K. Griest and D. Seckel, Phys. Rev. D 43, 3191 (1991),
M. Ibe, H. Murayama and T. Yanagida, Phys. Rev. D 79, 095009 (2009)

Resonance beyond the B-W: $\langle \sigma v_{\rm rel} \rangle (x)$

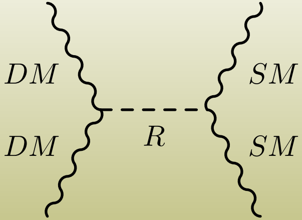


Is the BW approximation applicable ?

$$\sigma \propto \frac{1}{(s - M^2)^2 + \Gamma^2 M^2}$$

$$s \approx M^2$$

Resonance beyond the B-W: $\langle \sigma v_{\rm rel} \rangle (x)$



- $v_{\rm rel} \ll 1$ and $2M_{DM} \approx M \implies s \approx 4M_{DM}^2 + M_{DM}^2 v_{\rm rel}^2 \approx M^2$
- The BW propagator is an approximation that follows from re-summation of an infinite series of 2-point Green's functions, so in general

$$\Gamma M \to \Gamma(s) M \equiv \Im \Sigma(s)$$

$$\Im \Sigma(s) = \frac{1}{2} \sum_f \int d\Pi_f |\mathcal{M}(R \to f)|^2 (2\pi)^4 \delta^{(4)}(k_R - \sum q_f)$$

- Is the BW approximation applicable ?

Resonance beyond the B-W: $\langle \sigma v_{\rm rel} \rangle (x)$

$$\sigma v_{\rm rel} \propto \frac{M^2 \Gamma_i \Gamma_f}{|s - M^2 + i\Gamma M|^2}$$

$$\downarrow$$

$$\sigma v_{\rm rel} \propto \frac{M^2 \Gamma_i \Gamma_f}{|s - M^2 + i\Im\Sigma(s)|^2}$$

$$\sigma v_{\rm rel} \propto \frac{\gamma_i \gamma_f}{(\delta + v_{\rm rel}^2/4)^2 + [\gamma_{\rm non-DM} + \gamma_{\rm DM}(v_{\rm rel})]^2}$$

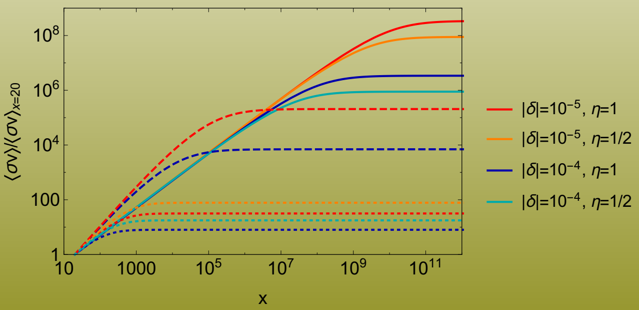
$$\overset{\gamma_{\rm non-DM} \ll \gamma_{\rm DM}}{\approx} \frac{\gamma_i \gamma_f}{(\delta + v_{\rm rel}^2/4)^2 + \eta^2 v_{\rm rel}^2/4}$$

where $\eta \equiv \frac{\Gamma_i}{M\beta_i}$, $\beta_i \equiv \left(1 - \frac{4M_{DM}^2}{M^2}\right)^{1/2}$ and $\gamma_{i,f} = \frac{\Gamma_{i,f}}{M}$

Resonance beyond the B-W: $\langle \sigma v_{\rm rel} \rangle (x)$

$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x)(Y^2 - Y_{EQ}^2)$$

$$R(x) = \frac{\langle \sigma v_{\rm rel}\rangle (x)}{\langle \sigma v_{\rm rel}\rangle_0} = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} \frac{\delta^2}{(\delta + v_{\rm rel}^2/4)^2 + \eta^2 v_{\rm rel}^2/4}$$



Thermally averaged annihilation cross-section $\langle \sigma v_{\rm rel}\rangle / \langle \sigma v_{\rm rel}\rangle_{x=20}$. $\langle \sigma v_{\rm rel}\rangle$ saturates at $x \sim \eta^2/\delta^2$ – for temperatures orders of magnitude smaller than with the naive constant width $\Gamma(M^2)$. The short-dashed lines refer to the constant width approximation $\gamma_{\rm DM}(v_{\rm rel}) = \gamma_{\rm DM}(2|\delta|^{1/2}) = \eta|\delta|^{1/2}$ (corresponding to $\Gamma_{\rm DM}(M^2)$), which can be obtained for $\delta < 0$, whereas for $\delta > 0$ long-dashed curves were obtained for $\gamma_{\rm DM} = 0$.

Resonance beyond the B-W: $\langle \sigma v_{\rm rel} \rangle (x)$

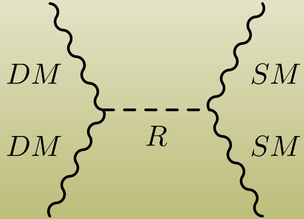
$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x)(Y^2 - Y_{EQ}^2) \qquad \text{with} \qquad R(x) \propto \langle \sigma v_{\rm rel} \rangle (x)$$

At low x, $\langle \sigma v_{\rm rel} \rangle (x)$ is larger for the naive constant width approximation than for velocity dependent one.

$\Downarrow$

Velocity dependent width implies higher asymptotic DM yield.

Early kinetic decoupling and coupled Boltzmann equations



Resonance enhancement of DM annihilation and unsuppressed $\sigma_{\rm self}/M_{DM}$

$\Downarrow$

Suppressed DM-SM interactions (to get $\Omega_{DM} \sim 0.1$)
and tiny $\sigma(DMSM \to DMSM)$

$\Downarrow$

- Possibility of DM early kinetic decoupling at $T_{kd} \ll T_{kd}^{\rm WIMP} \sim$ MeV,
- No problems with direct detection.

Early kinetic decoupling and coupled Boltzmann equations

- If dark matter decouples kinetically, when it is non-relativistic and its thermal distribution is maintained by self-scatterings, then the DM temperature T_{DM} evolves according to $T_{DM} \propto R^{-2}$,
- The temperature of the radiation-dominated SM thermal bath, scales as $T \propto R^{-1}$.

$$T_{DM} = \begin{cases} T, & \text{if } T \geq T_{kd} \\ T^2/T_{kd}, & \text{if } T < T_{kd}, \end{cases}$$

where T stands for the SM temperature.

Early kinetic decoupling and coupled Boltzmann equations

The DM relic density can be obtained by solving the Boltzmann equation

$$\frac{dY(x)}{dx} = -\frac{\lambda_0}{x^2} R(x_{DM}) \left[Y^2(x) - Y_{EQ}^2(x) \right],$$

where $x \equiv \frac{m}{T}$, $x_{DM} \equiv \frac{m}{T_{DM}}$ and

$$R(x_{DM}) = \frac{\langle \sigma v_{\rm rel} \rangle (x_{DM})}{\langle \sigma v_{\rm rel} \rangle_0} = \frac{x_{DM}^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 e^{-x_{DM} v^2/4} \frac{\delta^2 + \gamma(0)^2}{(\delta + v^2/4)^2 + \gamma(v)^2}$$

$$Y_{EQ}(x) = \frac{45}{2\pi^4} \sqrt{\frac{\pi}{8}} \frac{g}{g_*} x^{3/2} e^{-x}$$

The thermal average of the annihilation cross-section is calculated at

$$x_{DM} = \frac{M_{\rm DM}}{T_{DM}} = \frac{x^2}{x_{kd}}$$

with $x_{kd} \equiv m/T_{kd}$.

Early kinetic decoupling and coupled Boltzmann equations

$$x_{DM} = \left(\frac{x}{x_{kd}}\right) \cdot x \qquad \langle \sigma v_{\rm rel} \rangle (x) \propto x$$

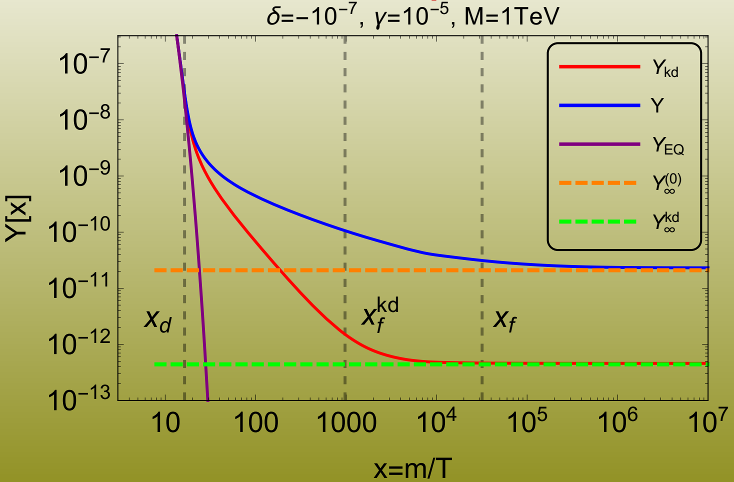
$$\Downarrow$$

$$R(x_{DM}) \sim \left(\frac{x}{x_{kd}}\right) R(x) \gg R(x).$$

$$\Downarrow$$

The asymptotic yield expected in the early decoupling scenario is substantially reduced by more efficient annihilation.

Early kinetic decoupling and coupled Boltzmann equations



Evolution of the dark matter yield $Y(x)$ (blue) for dark matter in kinetic equilibrium with the SM and for simultaneous chemical and kinetic decoupling Y_{kd} at $x_{kd} = x_d$ (red). The decoupling temperature x_d adopted for the $Y_{kd}(x)$ (red) was determined by the decoupling of $Y(x)$ (the blue curve).

Early kinetic decoupling and coupled Boltzmann equations

Define DM "temperature":

$$T_{DM} \equiv \frac{2}{3}\left\langle \frac{\vec{p}^{\,2}}{2M_{DM}} \right\rangle \qquad \text{for} \qquad \langle \mathcal{O}(\vec{p})\rangle \equiv \frac{1}{n_{DM}} \int \frac{d^3p}{(2\pi)^3} \mathcal{O}(\vec{p}) f(\vec{p})$$

$T_{DM} \approx T$ for a situation close to thermal equilibrium and $\epsilon \equiv (T - T_{DM})/T$ is a parameter that measures the deviation of $f(\vec{p})$ from a thermal distribution.

The Boltzmann equation:

$$\hat{L}[f] = C[f]$$

The second moment of the Boltzmann equation:

$$\int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^{\,2}}{p^0} \hat{L}[f] = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^{\,2}}{p^0} C[f]$$

T. Bringmann and S. Hofmann, "Thermal decoupling of WIMPs from first principles," JCAP **0704**, 016 (2007), Erratum: [JCAP **1603**, no. 03, E02 (2016)]

Early kinetic decoupling and coupled Boltzmann equations

$$\frac{dY}{dx} = -\frac{s}{Hx}\left[Y^2\langle\sigma v_{\rm rel}\rangle_{x=M_{\rm DM}^2/(s^{2/3}y)} - Y_{EQ}^2\langle\sigma v_{\rm rel}\rangle_x\right]$$

$$\frac{dy}{dx} = -\frac{1}{Hx}\Big\{2M_{\rm DM}c(T)(y-y_{EQ}) +$$

$$-sy\left[Y\left(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\right)_{x=M_{\rm DM}^2/(s^{2/3}y)} - \frac{Y_{EQ}^2}{Y}\left(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\right)_x\right]\Big\}$$

where the temperature parameter y is defined as

$$y \equiv \frac{M_{\rm DM}T_{DM}}{s^{2/3}}, \quad \text{for sharp splitting: } y \propto \begin{cases} x, & \text{if } T \geq T_{kd} \\ \frac{M_{DM}}{T_{kd}} \sim \text{const.}, & \text{if } T < T_{kd}, \end{cases}$$

T. Bringmann and S. Hofmann, "Thermal decoupling of WIMPs from first principles," JCAP **0704**, 016 (2007), Erratum: [JCAP **1603**, no. 03, E02 (2016)]

Early kinetic decoupling and coupled Boltzmann equations

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where the temperature parameter y is defined as

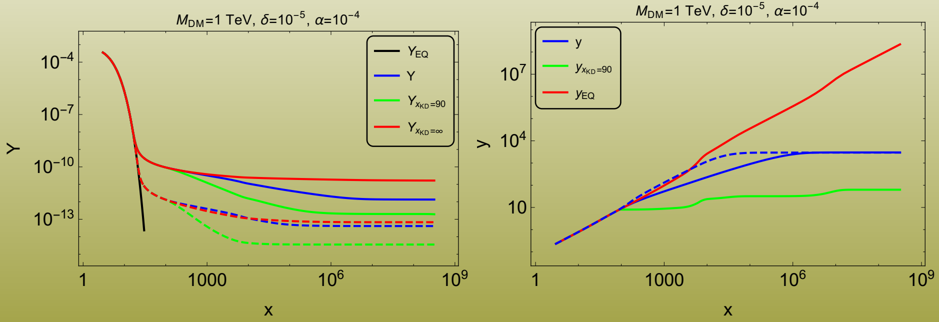
$$y \equiv \frac{M_{\rm DM}T_{DM}}{s^{2/3}},$$

the scattering rate $c(T)$ as

$$c(T) = \frac{1}{12(2\pi)^3)M_{\rm DM}^4 T}\sum_f \int dk k^5 \omega^{-1} g|\mathcal{M}|^2_{t=0;s=M_{\rm DM}^2+2M_{\rm DM}\omega+M_{SM}^2}$$

$$\langle\sigma v_{\rm rel}\rangle_2 = \int_0^\infty dv_{\rm rel}\frac{x^{3/2}}{4\sqrt{\pi}}\sigma v_{\rm rel}\left(1+\frac{1}{6}v_{\rm rel}^2 x\right)v_{\rm rel}^2 \exp^{-v_{\rm rel}^2 x/4}$$

Early kinetic decoupling and coupled Boltzmann equations



Dark matter yield Y (left panel) and corresponding DM temperatures (right panel) in different kinetic decoupling scenarios. The blue curves show the solution of the set of BE, whereas the green ones refer to the "sharp splitting" at $x_{kd} = 90$. For the red curves dark matter remains in the kinetic equilibrium during its whole evolution. Dashed curves present the corresponding results for the standard Breit-Wigner approximation (with $\gamma \ll \delta$).

Generic conclusions on the BW approximation

Remarks:

- The presence of velocity-dependent width implies that Y decouples at lower x (as compared to the case with constant width $\Gamma(M^2)$) and the asymptotic DM yield is much larger.
- The asymptotic yield expected in the early decoupling scenario is substantially reduced by more efficient annihilation, $R(x_{DM}) \sim \frac{x}{x_{kd}} R(x) \gg R(x)$.
- Both effects cancel to same extend, so that the increase by the velocity depended width is reduced by $\sim 50\%$.

$U(1)$ VDM model

The model:

- extra $U(1)$ gauge symmetry (A_X^μ),
- a complex scalar field S, whose vev generates a mass for the $U(1)$'s vector field, $S = (0, \mathbf{1}, \mathbf{1}, 1)$ under $U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)$
- SM fields neutral under $U(1)$,
- to ensure stability of the new vector boson, a $\mathbb{Z}_2$ symmetry is assumed to forbid $U(1)$-kinetic mixing between $U(1)$ and $U(1)_Y$. The extra gauge boson A_X^μ and the scalar S field transform under $\mathbb{Z}_2$ as follows

$$A_X^\mu \to -A_X^\mu\ ,\ S \to S^*, \text{ where } S = \phi e^{i\sigma},\ \ \text{so}\ \ \phi \to \phi,\ \ \sigma \to -\sigma.$$

T. Hambye, JHEP 0901 (2009) 028,
O. Lebedev, H. M. Lee, and Y. Mambrini, Phys.Lett. B707 (2012) 570,
A. Falkowski, C. Gross and O. Lebedev, JHEP 05 (2015) 057
...

$U(1)$ VDM model

The scalar potential

$$V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2.$$

The vector bosons masses:

$$M_W = \frac{1}{2} g v, \qquad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v \quad \text{and} \quad M_{Z'} = g_x v_x,$$

where

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{ and } \langle S \rangle = \frac{v_x}{\sqrt{2}}$$

Positivity of the potential implies

$$\lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2\sqrt{\lambda_H \lambda_S}$$

$U(1)$ VDM model

The mass squared matrix $\mathcal{M}^2$ for the fluctuations (ϕ_H, ϕ_S) and their eigenvalues

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_x \\ \kappa v v_x & 2\lambda_S v_x^2 \end{pmatrix}$$

$$M_\pm^2 = \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H \lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}$$

$$\mathcal{M}^2_{\mathsf{diag}} = \begin{pmatrix} M_{h_1}^2 & 0 \\ 0 & M_{h_2}^2 \end{pmatrix}, \quad R = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R^{-1} \begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix}$$

where $M_{h_1} = 125.7$ GeV is the mass of the observed Higgs particle.

$U(1)$ VDM model

$$\sin 2\alpha = \frac{\mathsf{sign}(\lambda_{SM} - \lambda_H)\, 2\mathcal{M}_{12}^2}{\sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2}}, \quad \cos 2\alpha = \cdots .$$

There are 5 real parameters in the potential: μ_H, μ_S, λ_H, λ_S and κ. Adopting the minimization conditions μ_H, μ_S could be replaced by v and v_x. The SM vev is fixed at $v = 246.22$ GeV. Using the condition $M_{h_1} = 125.7$ GeV, v_x^2 could be eliminated in terms of $v^2, \lambda_H, \kappa, \lambda_S, \lambda_{SM} = M_{h_1}^2/(2v^2)$:

$$v_x^2 = v^2 \frac{4\lambda_{SM}(\lambda_H - \lambda_{SM})}{4\lambda_S(\lambda_H - \lambda_{SM}) - \kappa^2}$$

Eventually there are 4 independent parameters:

$$(\lambda_H, \kappa, \lambda_S, g_x),$$

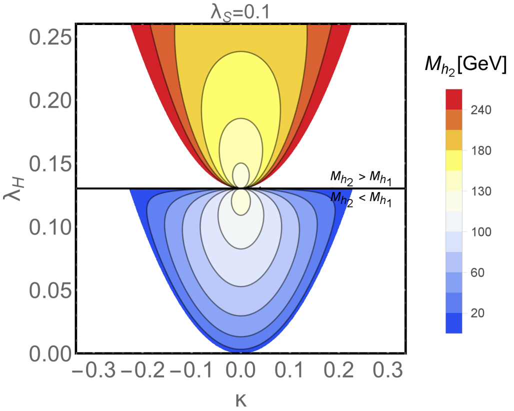
where g_x is the $U(1)$ coupling constant.

$U(1)$ VDM model

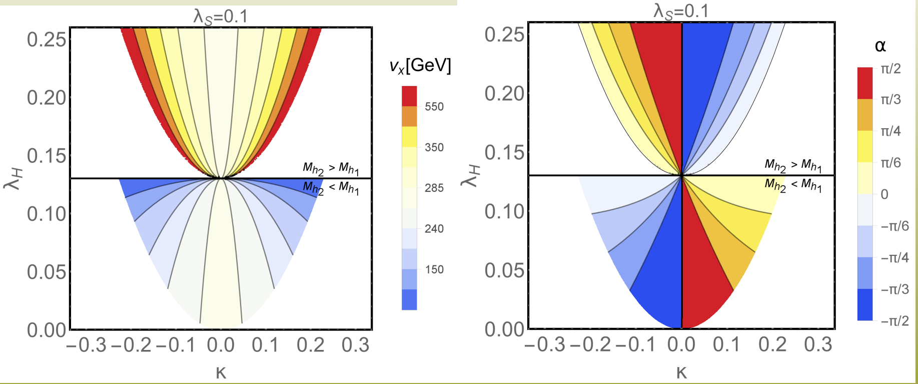
- Bottom part of the plot ($\lambda_H < \lambda_{SM} = M_{h_1}^2/(2v^2) = 0.13$): the heavier Higgs is the currently observed one.

- Upper part ($\lambda_H > \lambda_{SM}$) the lighter state is the observed one.

- White regions in the upper and lower parts are disallowed by the positivity conditions for v_x^2 and $M_{h_2}^2$, respectively.



$U(1)$ VDM model



Contour plots for the vacuum expectation value of the extra scalar $v_x \equiv \sqrt{2}\langle S\rangle$ (left panel) and of the mixing angle α (right panel) in the plane (λ_H, κ).

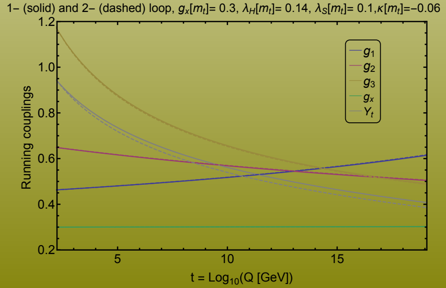
$U(1)$ VDM model

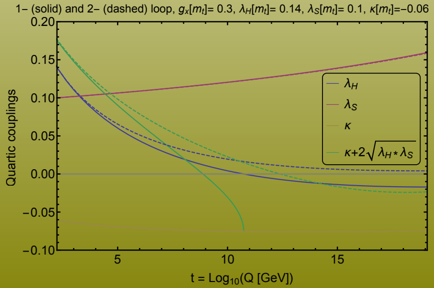
Vacuum stability

$$V = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|S|^2|H|^2$$

2-loop running of parameters adopted

$$\lambda_H(Q) > 0, \quad \lambda_S(Q) > 0, \quad \kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0$$





$U(1)$ VDM model

The mass of the Higgs boson is known experimentally therefore within *the SM* the initial condition for running of $\lambda_H(Q)$ is fixed

$$\lambda_H(m_t) = M_{h_1}^2/(2v^2) = \lambda_{SM} = 0.13$$

For VDM this is not necessarily the case:

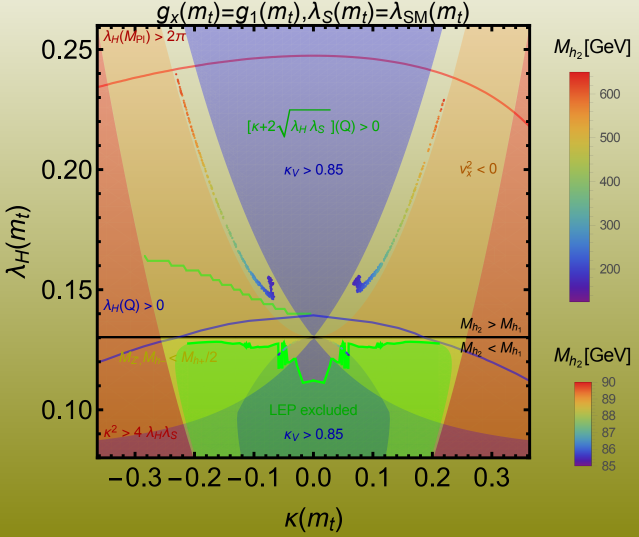
$$M_{h_1}^2 = \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H \lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}.$$

VDM:

- Larger initial values of λ_H such that $\lambda_H(m_t) > \lambda_{SM}$ are allowed delaying the instability (by shifting up the scale at which $\lambda_H(Q) < 0$).
- Even if the initial λ_H is smaller than its SM value, $\lambda_H(m_t) < \lambda_{SM}$, still there is a chance to lift the instability scale if appropriate initial value of the portal coupling $\kappa(m_t)$ is chosen.

$$\beta_{\lambda_H}^{(1)} = \beta_{\lambda_H}^{SM\ (1)} + \kappa^2$$

$U(1)$ VDM model



$g_x(m_t)=g_1(m_t),\lambda_S(m_t)=\lambda_{SM}(m_t)$
$\lambda_H(M_{Pl}) > 2\pi$
$[\kappa+2\sqrt{\lambda_H \lambda_S}](Q) > 0$
$\kappa_V > 0.85$
$v_x^2 < 0$
$\lambda_H(Q) > 0$
$M_{h_2} > M_{h_1}$
$M_{h_2} < M_{h_1}$
$M_Z, M_{h-} < M_{h+}/2$
LEP excluded
$\kappa_V > 0.85$
$\kappa^2 > 4\lambda_H\lambda_S$
$\lambda_H(m_t)$
$\kappa(m_t)$
0.25
0.20
0.15
0.10
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
M_{h_2}[GeV]
600
500
400
300
200
M_{h_2}[GeV]
90
89
88
87
86
85

Resonant self-interaction of VDM

In the VDM model, the self-interaction cross-section ($Z'Z' \to Z'Z'$) at relative velocity v_0 can be written in the vicinity of the resonance as

$$\left.\frac{\sigma_{\rm self}}{M_{Z'}}\right|_{v=v_0} \approx \frac{g_x^4 M_{Z'}}{8\pi} \frac{R_{2i}^4}{[4M_{Z'}^2(1+v_0^2/4) - M_{h_i}^2]^2 + \Gamma_{h_i}^2(v_0) M_{h_i}^2},$$

where R is the scalar mixing matrix ($R_{21} = \sin\alpha$ and $R_{22} = \cos\alpha$). The velocity-dependent width equals

$$\Gamma_{h_i}(v_0) = R_{1i}^2 \Gamma_{SM} + \eta M_{h_i} v_0/2,$$

where Γ_{SM} is the SM value of the Higgs width.

Resonant self-interaction of VDM

$$\left.\frac{\sigma_{\rm self}}{M_{Z'}}\right|_{v=v_0} \approx \frac{g_x^4 M_{Z'}}{8\pi M_{h_2}^4}\frac{\cos^4\alpha}{(\delta+v_0^2/4)^2+\gamma(v_0)^2},$$

where $\delta = (4M_{Z'}^2 - M_{h_2}^2)/M_{h_2}^2$ and $\gamma(v) = \Gamma_{h_2}(v)/M_{h_2}$. On the other hand the annihilation rate into the SM is proportional to $\sin\alpha$.

$$\left.\frac{\sigma_{\rm self}}{M_{Z'}}\right|_{v=v_0} < \frac{g_x^4}{32\pi\eta^2 v_0^2 M_{Z'}^3} = 1.1\times 10^3 \left(\frac{10^{-4}}{v_0}\right)^2 \left(\frac{100\ \text{GeV}}{M_{Z'}}\right)^3\ \text{GeV}^{-3}$$

"cusp-core" problem, $\cdots \to \left.\frac{\sigma_{\rm self}}{M_{Z'}}\right|_{v=v_0} \sim\ 4.6\times 10^{-1} - 4.6\times 10^4\ \text{GeV}^{-3}$

$(10^{-4} - 10\ \text{cm}^2\text{g}^{-1})$

Gauge dependance and unitarity violation in VDM annihilation

Gauge dependance

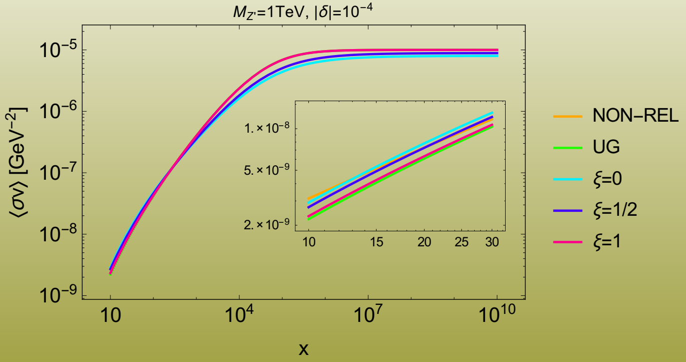
$$\sigma \propto \frac{1}{(\delta + v^2/4)^2 + (\gamma_{\mathrm{non-DM}} + \gamma_{\mathrm{DM}}(v))^2},$$

where $\gamma_{\mathrm{non-DM}} = \Im\Sigma_{\mathrm{non-DM}}(m_{h_2}^2)/m_{h_2}$ (non-DM contributions to the width) and $\gamma_{\mathrm{DM}}(v) = \Im\Sigma_{\mathrm{DM}}(s)/m_{h_2}$ (DM contributions to the imaginary part of the self-energy) with $s \approx 4M_{Z'}^2 + M_{Z'}^2 v^2$.

$$\begin{aligned} \Sigma_{\mathrm{DM}}(s) \quad &= \quad R_{22}^2 \frac{g_x^2}{32\pi^2} \left[\left(\frac{s^2}{4M_{Z'}^2} - s + 3M_{Z'}^2 \right) B_0(s, M_{Z'}^2, M_{Z'}^2) + \right. \\ &+ \quad \left. \frac{m_{h_2}^4 - s^2}{4M_{Z'}^2} B_0(s, \xi M_{Z'}^2, \xi M_{Z'}^2) \right], \end{aligned}$$

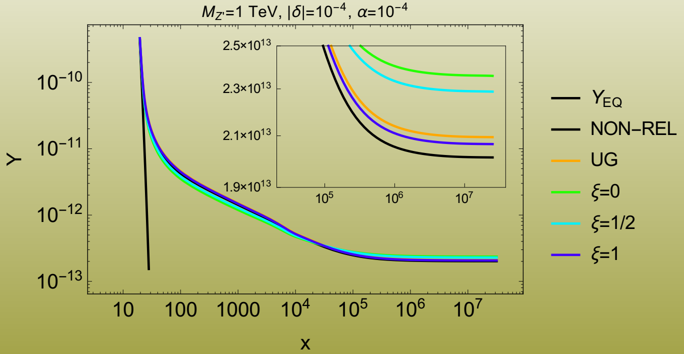
where $B_0(s, m^2, m^2)$ is a Passarino-Veltman function, while ξ is the gauge-fixing parameter.

Gauge dependance and unitarity violation in VDM annihilation



Here we illustrate consequences of gauge dependence of the resonance propagator. Results shown correspond to selected values of ξ specified in the legend. The unitary gauge ($\xi \to \infty$) is denoted as UG, the NON-REL curve shows results obtained within a non-relativistic approximation. We show the thermal averaged annihilation cross-section for $Z'Z' \to SMSM$.

Gauge dependance and unitarity violation in VDM annihilation



Here we illustrate consequences of gauge dependence of the resonance propagator. Results shown correspond to selected values of ξ specified in the legend. The unitary gauge ($\xi \to \infty$) is denoted as UG, the NON-REL curve shows results obtained within a non-relativistic approximation. We plot numerical solution of the Boltzmann equations for the dark matter yield $Y(x)$.

Gauge dependance and unitarity violation in VDM annihilation

Unitarity

$$Z'_L Z'_L \to W^+_L W^-_L \qquad \epsilon^\mu_L(p) \sim \frac{p^\mu}{M_V} + \cdots$$

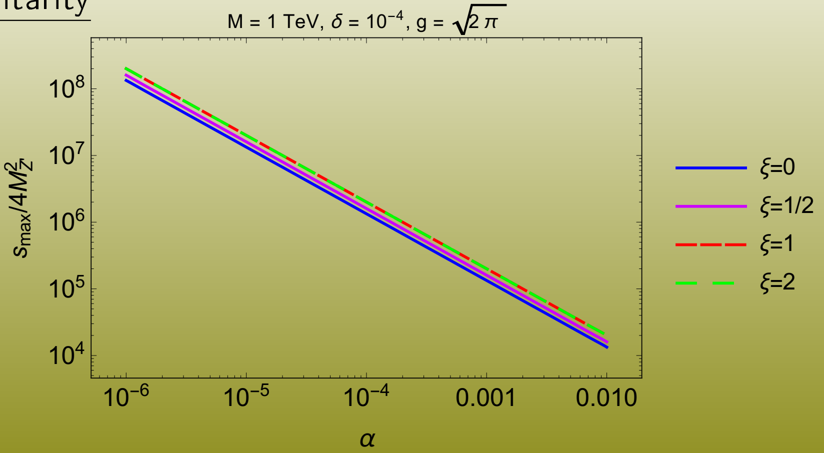
$$M(s,t) \sim \frac{s^2}{s - M^2 + i\Im\Sigma(s)} + \cdots$$

- if $\Im\Sigma(s) = 0$ then $M(s,t) \sim \mathcal{O}(s^2) \times 0 + \mathcal{O}(s) \times 0 + \text{const} + \cdots$
- if $\Im\Sigma(s) \neq 0$ then $M(s,t) \sim \mathcal{O}(s^2) \times 0 + \mathcal{O}(s) + \text{const} + \cdots$

Uniartity is violated: $M \sim \mathcal{O}(s) \cdots$

Gauge dependance and unitarity violation in VDM annihilation

Unitarity



The energy $s_{\max}/(4M^2_{Z'})$ at which amplitude of the process $Z'Z' \to W^+W^-$ violates unitarity presented as a function of the mixing angle α for different gauge parameters ξ.

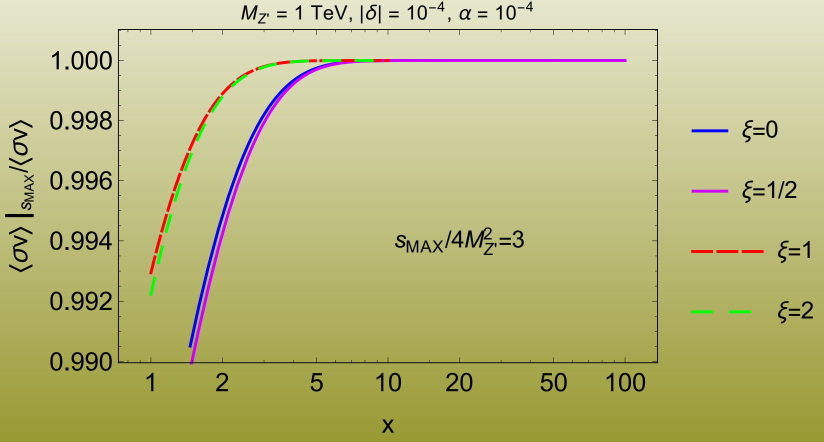
Gauge dependance and unitarity violation in VDM annihilation

Unitarity

$$\langle \sigma v_{\rm rel} \rangle (x) \propto \int_{4M_{DM}^2}^{\infty} ds \sqrt{s} (s - 4M_{DM}^2) K_1 \left(\frac{x\sqrt{s}}{M_{DM}} \right) \sigma(s)$$

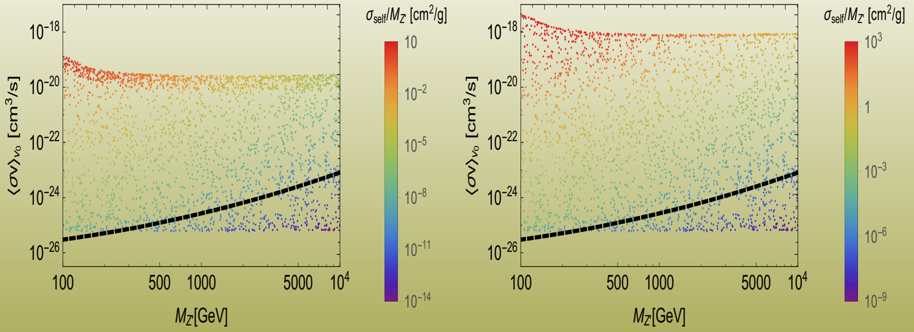
$$\langle \sigma v_{\rm rel} \rangle (x) |_{s_{\mathsf{max}}} \propto \int_{4M_{DM}^2}^{s_{\mathsf{max}}} ds \sqrt{s} (s - 4M_{DM}^2) K_1 \left(\frac{x\sqrt{s}}{M_{DM}} \right) \sigma(s)$$

Gauge dependance and unitarity violation in VDM annihilation



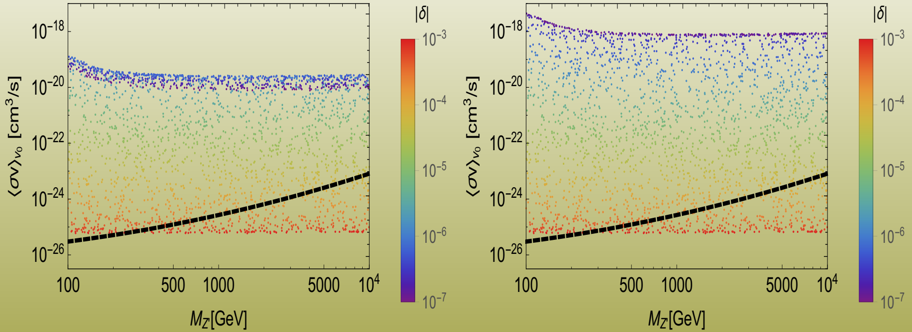
The relative error for the thermally averaged cross-section $\langle \sigma v_{\rm rel} \rangle(x)$, which is implied by choosing $s_{\rm max} = 12 M_{Z'}^2$ as the cut-off.

Numerical results



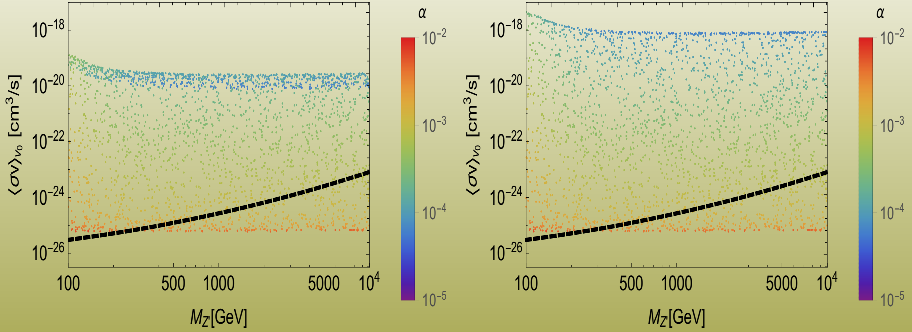
Result of the scan in the parameter space over $M_{Z'}$, δ and $\sin\alpha$. For each point in the plot we fit α to satisfy the relic abundance constraint and then calculate the annihilation $\langle\sigma v_{\rm rel}\rangle_{v_0}$ and self-interaction $\sigma_{\rm self}/M_{Z'}$ cross-section at the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta = 3/16$, was chosen.

Numerical results



Result of the scan in the parameter space over $M_{Z'}$, δ and $\sin\alpha$. Colouring with respect to δ the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta = 3/16$, was chosen.

Numerical results



Result of the scan in the parameter space over $M_{Z'}$, δ and $\sin\alpha$. Colouring with respect to α the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta = 3/16$, was chosen.

Summary

- Breit-Wigner approximation was modified by adopting s-dependent width ($\sim \Im\Sigma(s)$), effects are large.
- If non-DM contribution to the resonance width is non-negligible, then the DM abundance implies early kinetic decoupling with important numerical consequences.
- A possibility of enhancing the dark-matter self-interaction cross-section ($\sigma_{\rm self}/M_{DM}$) by s-channel resonance was considered in a model independent way.
- To illustrate generic results a model of vector $U(1)$ dark matter (VDM) was introduced and discussed (extra neutral Higgs boson h_2).
- Within VDM model a gauge dependence and unitarity violation of the resonance enhancement was discussed.
- When the Fermi-LAT limits are taken into account, heavy ~ 1 TeV DM is favored and only very limited enhancement of $\sigma_{\rm self}/M_{DM}$ $\mathcal{O}(10^{-5})$ GeV^{-3} is possible.