Breit-Wigner resonance in cosmology

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Outline

- $U(1)$ vector dark matter (VDM) model
- Resonance beyond the Breit-Wigner (BW) approximation
- Early kinetic decoupling and coupled Boltzmann equations
- Generic conclusions on the BW approximation
- Numerical results confronted with Fermi-LAT data
- Summary

⋆ M. Duch, BG, “Resonance enhancement of dark matter interactions: the case for early kinetic decoupling and velocity dependent resonance width”, arXiv:1705.10777

⋆ M. Duch, BG, M. McGarrie, “A stable Higgs portal with vector dark matter”, JHEP 1509 (2015) 162, arXiv:1506.08805

$U(1)$ VDM model

The model:

- extra $U(1)$ gauge symmetry (A_X^μ),
- a complex scalar field S, whose vev generates a mass for the $U(1)$'s vector field, $S = (0, \mathbf{1}, \mathbf{1}, 1)$ under $U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)$
- SM fields neutral under $U(1)$,
- to ensure stability of the new vector boson, a $\mathbb{Z}_2$ symmetry is assumed to forbid $U(1)$-kinetic mixing between $U(1)$ and $U(1)_Y$. The extra gauge boson A_X^μ and the scalar S field transform under $\mathbb{Z}_2$ as follows

$$A_X^\mu \to -A_X^\mu \;,\; S \to S^*, \text{ where } S = \phi e^{i\sigma}, \;\; \text{so} \;\; \phi \to \phi, \;\; \sigma \to -\sigma.$$

T. Hambye, JHEP 0901 (2009) 028,

O. Lebedev, H. M. Lee, and Y. Mambrini, Phys.Lett. B707 (2012) 570,

A. Falkowski, C. Gross and O. Lebedev, JHEP 05 (2015) 057

...

$U(1)$ VDM model

The scalar potential

$$V = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|S|^2|H|^2.$$

The vector bosons masses:

$$M_W = \frac{1}{2}gv, \qquad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v \quad \text{and} \quad M_{Z'} = g_x v_x,$$

where

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \;\text{ and }\; \langle S \rangle = \frac{v_x}{\sqrt{2}}$$

Positivity of the potential implies

$$\lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2\sqrt{\lambda_H \lambda_S}$$

$U(1)$ VDM model

The scalar fields shall be expanded around corresponding vev's as follows

$$S = \frac{1}{\sqrt{2}}(v_x + \phi_S + i\sigma_S) \;\;,\;\; H^0 = \frac{1}{\sqrt{2}}(v + \phi_H + i\sigma_H) \;\text{ where }\; H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}.$$

The mass squared matrix $\mathcal{M}^2$ for the fluctuations (ϕ_H, ϕ_S) and their eigenvalues

$$\mathcal{M}^2 = \left(\begin{array}{cc} 2\lambda_H v^2 & \kappa v v_x \\ \kappa v v_x & 2\lambda_S v_x^2 \end{array} \right)$$

$$M_\pm^2 = \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H \lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}$$

$$\mathcal{M}^2_{\mathsf{diag}} = \left(\begin{array}{cc} M_{h_1}^2 & 0 \\ 0 & M_{h_2}^2 \end{array} \right), \quad R = \left(\begin{array}{cc} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{array} \right)$$

$$\left(\begin{array}{c} h_1 \\ h_2 \end{array} \right) = R^{-1} \left(\begin{array}{c} \phi_H \\ \phi_S \end{array} \right)$$

where $M_{h_1} = 125.7$ GeV is the mass of the observed Higgs particle.

$U(1)$ VDM model

There are 5 real parameters in the potential: μ_H, μ_S, λ_H, λ_S and κ. Adopting the minimization conditions μ_H, μ_S could be replaced by v and v_x. The SM vev is fixed at $v = 246.22$ GeV. Using the condition $M_{h_1} = 125.7$ GeV, v_x^2 could be eliminated in terms of $v^2, \lambda_H, \kappa, \lambda_S, \lambda_{SM} = M_{h_1}^2/(2v^2)$:

$$v_x^2 = v^2 \frac{4\lambda_{SM}(\lambda_H - \lambda_{SM})}{4\lambda_S(\lambda_H - \lambda_{SM}) - \kappa^2}$$

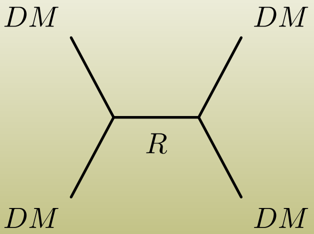
Eventually there are 4 independent parameters:

$$(\lambda_H, \kappa, \lambda_S, g_x),$$

where g_x is the $U(1)$ coupling constant. Another choice:

$$(M_{Z'}, M_{h_2}, \sin\alpha, g_x),$$

Resonance beyond the B-W



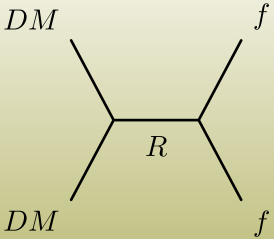
Breit-Wigner resonance ($2m \approx M$) DM self-interaction.

$$\sigma_{\rm self} \simeq \frac{32\pi\omega}{s\beta_i^2}\frac{M^2\Gamma_i^2}{(s-M^2)^2+\Gamma^2 M^2},$$

$$\frac{\sigma_{\rm self}}{m} \simeq \frac{8\pi\omega}{m^3}\frac{\eta^2}{(\delta+v_{\rm rel}^2/4)^2+\gamma^2}$$

$$\eta \equiv \frac{\Gamma_i}{M\beta_i},\ \delta \equiv \frac{4m^2}{M^2}-1,\ \gamma \equiv \frac{\Gamma}{M} \text{ and } \omega = \frac{(2J+1)}{(2S+1)^2}$$

Resonance beyond the B-W



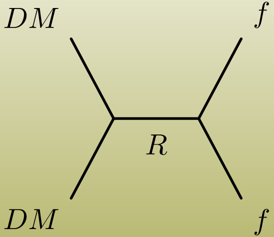
Breit-Wigner resonance ($2m \approx M$) annihilation.

$$\sigma v_{\rm rel} = \frac{64\pi\omega}{M^2}\frac{\eta\gamma_f}{(\delta + v_{\rm rel}^2/4)^2 + \gamma^2}$$

$$\langle\sigma v_{\rm rel}\rangle(x) = \frac{x^{3/2}}{2\sqrt{\pi}}\int_0^\infty dv v^2 e^{-xv^2/4}\sigma v$$

P. Gondolo and G. Gelmini, Nucl. Phys. B 360, 145 (1991),
K. Griest and D. Seckel, Phys. Rev. D 43, 3191 (1991),
M. Ibe, H. Murayama and T. Yanagida, Phys. Rev. D 79, 095009 (2009)

Resonance beyond the B-W

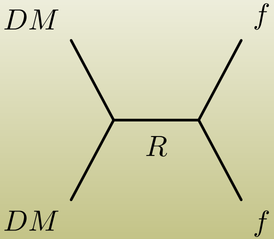


Is the BW approximation applicable ?

$$\sigma \propto \frac{1}{(s - M^2)^2 + \Gamma^2 M^2}$$

$$s \approx M^2$$

Resonance beyond the B-W



- $v_{\mathrm{rel}} \ll 1$ and $2m \approx M \implies s \approx 4m^2 + m^2 v_{\mathrm{rel}}^2 \approx M^2$
- The BW propagator is an approximation that follows from re-summation of an infinite series of 2-point Green's functions, so in general

$$\Gamma M \to \Gamma(s) M \equiv \Im\Sigma(s)$$

$$\Im\Sigma(s) = \frac{1}{2} \sum_f \int d\Pi_f |\mathcal{M}(R \to f)|^2 (2\pi)^4 \delta^{(4)}(k_R - \sum q_f)$$

- Is the BW approximation applicable ?

Resonance beyond the B-W

$$\sigma v_{\rm rel} \propto \frac{M^2 \Gamma_i \Gamma_f}{|s - M^2 + i\Gamma M|^2}$$

$$\downarrow$$

$$\sigma v_{\rm rel} \propto \frac{M^2 \Gamma_i \Gamma_f}{|s - M^2 + i\Im\Sigma(s)|^2}$$

$$\sigma v_{\rm rel} \propto \frac{\gamma_i \gamma_f}{(\delta + v_{\rm rel}^2/4)^2 + [\gamma_{\rm SM} + \gamma_{\rm DM}(v_{\rm rel})]^2}$$

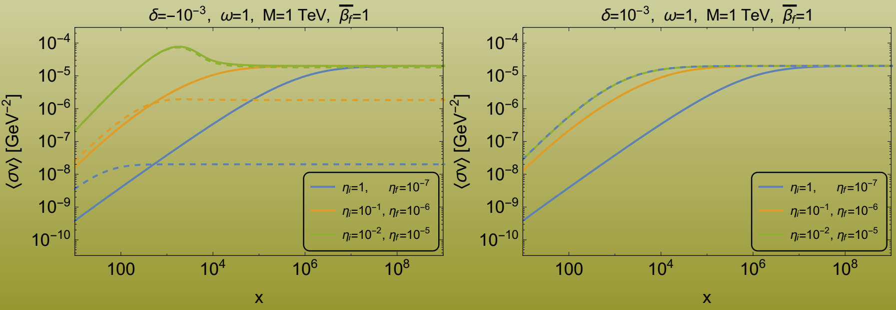
$$\overset{\gamma_{\rm SM} \ll \gamma_{\rm DM}}{\approx} \frac{\gamma_i \gamma_f}{(\delta + v_{\rm rel}^2/4)^2 + \eta^2 v_{\rm rel}^2/4}$$

where $\eta \equiv \frac{\Gamma_i}{M\beta_i}$, $\beta_i \equiv \left(1 - \frac{4m^2}{M^2}\right)^{1/2}$ and $\gamma_{i,f} = \frac{\Gamma_{i,f}}{M}$

Resonance beyond the B-W

$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x)(Y^2 - Y_{EQ}^2) \qquad Y \equiv \frac{n_{DM}}{s} \qquad x \equiv \frac{m}{T}$$

$$R(x) = \frac{\langle \sigma v_{\rm rel} \rangle (x)}{\langle \sigma v_{\rm rel} \rangle_0} = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} \frac{\delta^2}{(\delta + v_{\rm rel}^2/4)^2 + \eta^2 v_{\rm rel}^2/4}$$



Thermally averaged annihilation cross-section $\langle \sigma v_{\rm rel} \rangle (x)$ for negative (left panel) and positive (right panel) value of δ. The solid lines were obtained using the resonance propagator with energy-dependent width and dashed lines refer to constant width approximation. In the right panel all dashed lines coincide.

Resonance beyond the B-W

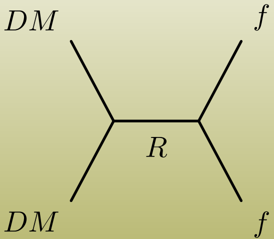
$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x)(Y^2 - Y_{EQ}^2) \qquad \text{with} \qquad R(x) \propto \langle \sigma v_{\mathrm{rel}} \rangle (x)$$

At low x, $\langle \sigma v_{\mathrm{rel}} \rangle (x)$ for the velocity dependent width is smaller than for the naive constant width $\Gamma(M^2)$.

$$\Downarrow$$

Velocity dependent width implies higher asymptotic DM yield.

Early kinetic decoupling and coupled Boltzmann equations



Resonance enhancement of DM annihilation

$\Downarrow$

Suppressed $DM\ DM \to SM\ SM$ resonant annihilation (to get $\Omega_{DM} \sim 0.1$) and tiny $\sigma(DMSM \to DMSM)$

$\Downarrow$

- Possibility of DM early kinetic decoupling at $T_{kd} \gg T_{kd}^{\mathrm{WIMP}} \sim$ MeV,
- Suppressed cross-sections for direct detection.

Early kinetic decoupling and coupled Boltzmann equations

- If dark matter decouples kinetically, when it is non-relativistic and its thermal distribution is maintained by self-scatterings, then the DM temperature T_{DM} evolves according to $T_{DM} \propto a^{-2}$,
- The temperature of the radiation-dominated SM thermal bath, scales as $T \propto a^{-1}$.

$$T_{DM} = \begin{cases} T, & \text{if } T \geq T_{kd} \\ T^2/T_{kd}, & \text{if } T < T_{kd}, \end{cases}$$

where T stands for the SM temperature.

Early kinetic decoupling and coupled Boltzmann equations

Define DM "temperature":

$$T_{DM} \equiv \frac{2}{3}\left\langle \frac{\vec{p}^{\,2}}{2m} \right\rangle \qquad \text{for} \qquad \langle \mathcal{O}(\vec{p}) \rangle \equiv \frac{1}{n_{DM}} \int \frac{d^3p}{(2\pi)^3} \mathcal{O}(\vec{p}) f(\vec{p})$$

The Boltzmann equation:

$$\hat{L}[f] = C[f]$$

The second moment of the Boltzmann equation:

$$\int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^{\,2}}{p^0} \hat{L}[f] = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^{\,2}}{p^0} C[f]$$

T. Bringmann and S. Hofmann, "Thermal decoupling of WIMPs from first principles," JCAP **0704**, 016 (2007), Erratum: [JCAP **1603**, no. 03, E02 (2016)]

Early kinetic decoupling and coupled Boltzmann equations

$$\frac{dY}{dx} = -\frac{sY^2}{Hx}\left[\langle\sigma v_{\rm rel}\rangle_{x=m^2/(s^{2/3}y)} - \frac{Y_{EQ}^2}{Y^2}\langle\sigma v_{\rm rel}\rangle_x\right]$$

$$\frac{dy}{dx} = -\frac{1}{Hx}\Big\{2mc(T)(y-y_{EQ}) +$$
$$-syY\left[\left(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\right)_{x=m^2/(s^{2/3}y)} - \frac{Y_{EQ}^2}{Y^2}\left(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\right)_x\right]\Big\}$$

where the temperature parameter y is defined as

$$y \equiv \frac{mT_{DM}}{s^{2/3}}, \quad \text{for sharp splitting: } y \propto \begin{cases} x, & \text{if } T \geq T_{kd} \\ \frac{m}{T_{kd}} \sim \text{const.}, & \text{if } T < T_{kd}, \end{cases}$$

T. Bringmann and S. Hofmann, "Thermal decoupling of WIMPs from first principles," JCAP **0704**, 016 (2007), Erratum: [JCAP **1603**, no. 03, E02 (2016)]

Early kinetic decoupling and coupled Boltzmann equations

$$\frac{dY}{dx} = -\frac{s}{Hx}\left[Y^2\langle\sigma v_{\rm rel}\rangle_{x=m^2/(s^{2/3}y)} - Y_{EQ}^2\langle\sigma v_{\rm rel}\rangle_x\right]$$

$$\frac{dy}{dx} = -\frac{1}{Hx}\Big\{2mc(T)(y-y_{EQ}) +$$

$$-syY\left[\left(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\right)_{x=m^2/(s^{2/3}y)} - \frac{Y_{EQ}^2}{Y^2}\left(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\right)_x\right]\Big\}$$

where the temperature parameter y is defined as

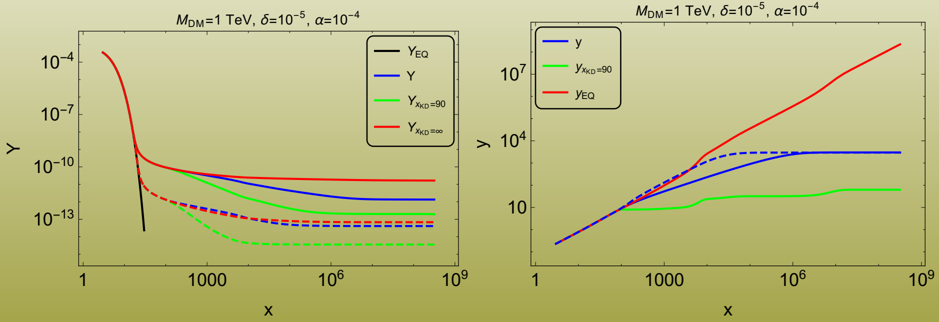
$$y \equiv \frac{mT_{DM}}{s^{2/3}} \qquad \text{and} \qquad y_{EQ} \equiv \frac{mT}{s^{2/3}}$$

the scattering rate $c(T)$ as

$$c(T) = \frac{1}{12(2\pi)^3)m^4T}\sum_f \int dk k^5\omega^{-1} g|\mathcal{M}|^2_{t=0;s=m^2+2m\omega+M_{SM}^2}$$

$$\langle\sigma v_{\rm rel}\rangle_2 = \frac{x^{3/2}}{2\sqrt{\pi}}\int_0^\infty dv_{\rm rel}\sigma v_{\rm rel}\left(1+\frac{1}{6}v_{\rm rel}^2 x\right) v_{\rm rel}^2 \exp^{-v_{\rm rel}^2 x/4}$$

Early kinetic decoupling and coupled Boltzmann equations



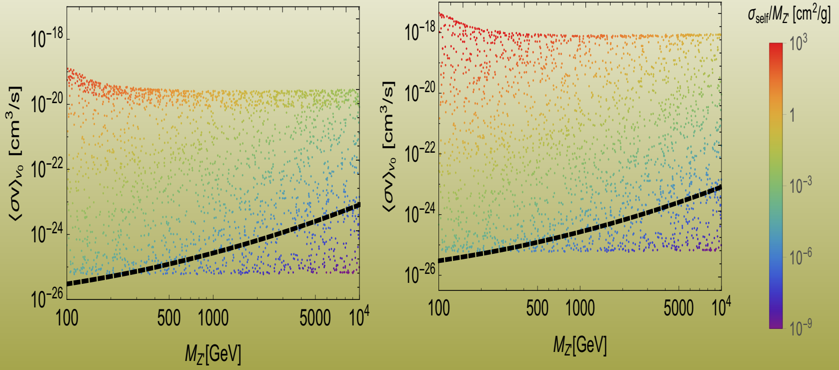
Dark matter yield Y (left panel) and corresponding DM temperatures (right panel) in different kinetic decoupling scenarios. The blue curves show the solution of the set of BE, whereas the green ones refer to the "sharp splitting" at $x_{kd} = 90$. For the red curves dark matter remains in the kinetic equilibrium during its whole evolution. Dashed curves present the corresponding results for the standard Breit-Wigner approximation (with $\gamma \ll \delta$).

Generic conclusions on the BW approximation

Remarks:

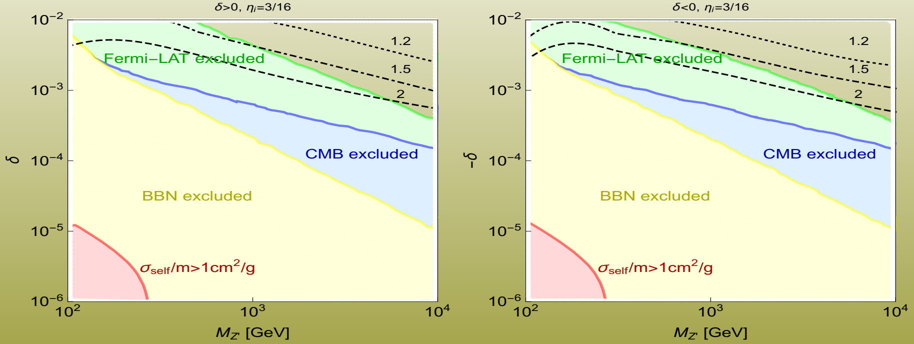
- The presence of velocity-dependent width implies that Y decouples at lower x (as compared to the case with constant width $\Gamma(M^2)$) and the asymptotic DM yield is much larger.
- The asymptotic yield expected in the early decoupling scenario is substantially reduced by more efficient annihilation, $R(x_{DM}) \sim \frac{x}{x_{kd}} R(x) \gg R(x)$.
- Both effects cancel to same extend, so that the increase by the velocity depended width is reduced by $\sim 50\%$.

Numerical results confronted with Fermi-LAT data



Result of the scan in the parameter space over $M_{Z'}$, δ and $\sin\alpha$. For each point in the plot we fit α to satisfy the relic abundance constraint and then calculate the annihilation $\langle\sigma v_{\rm rel}\rangle_{v_0}$ and self-interaction $\sigma_{\rm self}/M_{Z'}$ cross-section at the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta = 3/16$, was chosen.

Numerical results confronted with Fermi-LAT data



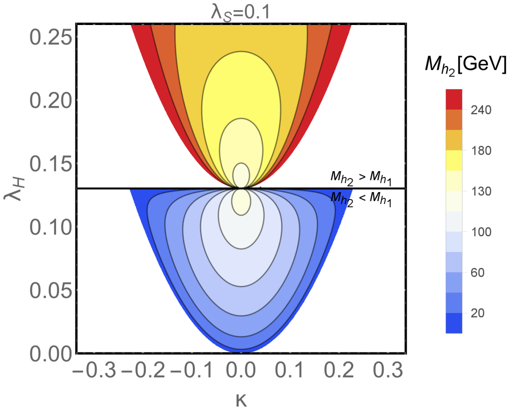
Regions in the $(\delta, M_{Z'})$ parameter space constrained by Fermi-LAT, CMB and BBN. The self-interaction cross-section needed for the small scale problems is also shown. Below black dotted, dash-dotted or dashed lines relic density without considering kinetic decoupling is larger by factor 1.2, 1.5 or 2 respectively.

Summary

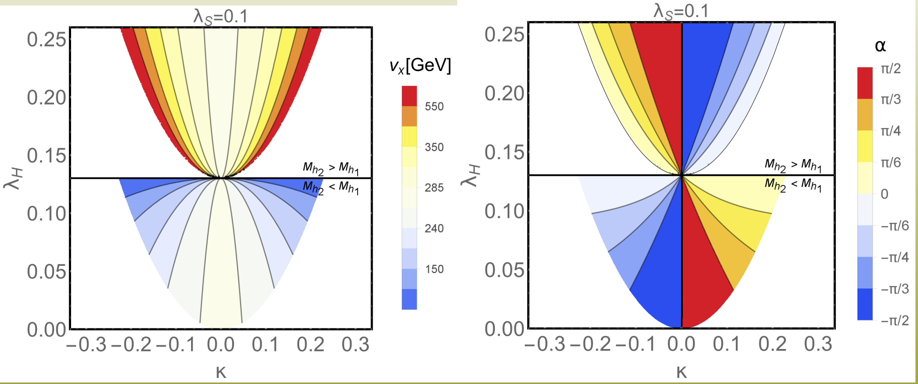
- The $U(1)$ vector dark matter (VDM) was introduced and discussed (extra neutral Higgs boson h_2).
- Breit-Wigner approximation was modified by adopting s-dependent width ($\sim \Im\Sigma(s)$), effects are large.
- Correct DM abundance implies early kinetic decoupling of DM with important numerical consequences. Similar effects are present for the real-scalar DM, see T. Binder, T. Bringmann, M. Gustafsson and A. Hryczuk, presented at Planck 2017 in Warsaw.
- The dark-matter self-interaction cross-section ($\sigma_{\rm self}/m$) would be enhanced if $M_{Z'} \sim 100$ GeV was allowed.
- When the Fermi-LAT limits are taken into account, heavy ~ 1 TeV DM is favored and only very limited enhancement of $\sigma_{\rm self}/m$ $\mathcal{O}(10^{-5})$ GeV^{-3} is possible.

$U(1)$ VDM model

- Bottom part of the plot ($\lambda_H < \lambda_{SM} = M_{h_1}^2/(2v^2) = 0.13$): the heavier Higgs is the currently observed one.
- Upper part ($\lambda_H > \lambda_{SM}$) the lighter state is the observed one.
- White regions in the upper and lower parts are disallowed by the positivity conditions for v_x^2 and $M_{h_2}^2$, respectively.



$U(1)$ VDM model



Contour plots for the vacuum expectation value of the extra scalar $v_x \equiv \sqrt{2}\langle S\rangle$ (left panel) and of the mixing angle α (right panel) in the plane (λ_H, κ).

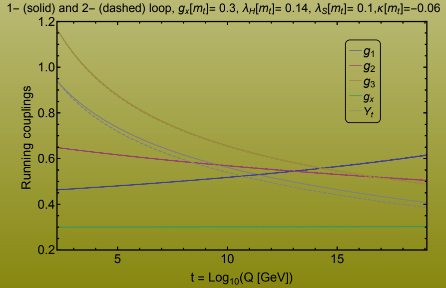
$U(1)$ VDM model

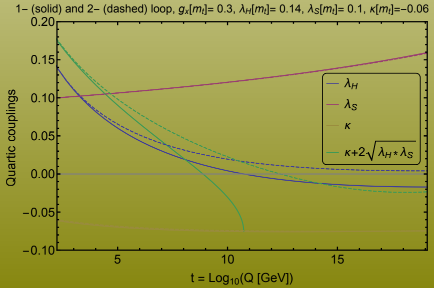
Vacuum stability

$$V = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|S|^2|H|^2$$

2-loop running of parameters adopted

$$\lambda_H(Q) > 0, \quad \lambda_S(Q) > 0, \quad \kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0$$





$U(1)$ VDM model

The mass of the Higgs boson is known experimentally therefore within *the SM* the initial condition for running of $\lambda_H(Q)$ is fixed

$$\lambda_H(m_t) = M_{h_1}^2/(2v^2) = \lambda_{SM} = 0.13$$

For VDM this is not necessarily the case:

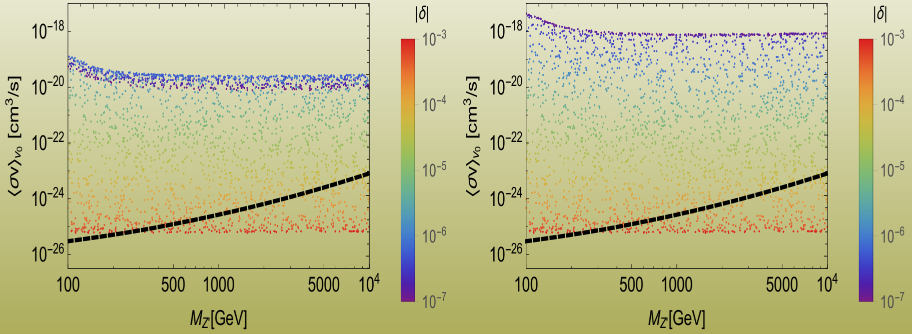
$$M_{h_1}^2 = \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H \lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}.$$

VDM:

- Larger initial values of λ_H such that $\lambda_H(m_t) > \lambda_{SM}$ are allowed delaying the instability (by shifting up the scale at which $\lambda_H(Q) < 0$).
- Even if the initial λ_H is smaller than its SM value, $\lambda_H(m_t) < \lambda_{SM}$, still there is a chance to lift the instability scale if appropriate initial value of the portal coupling $\kappa(m_t)$ is chosen.

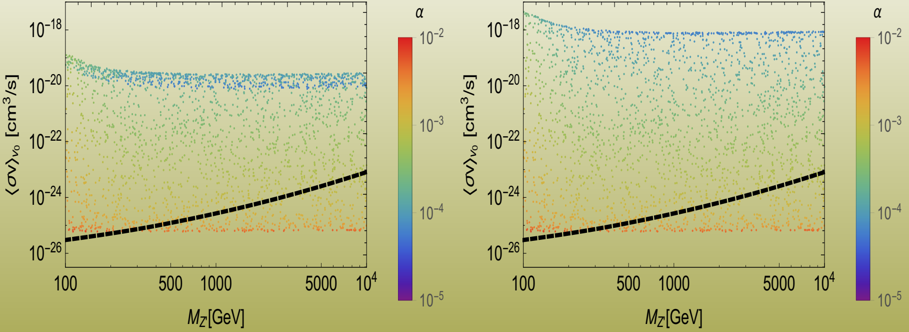
$$\beta_{\lambda_H}^{(1)} = \beta_{\lambda_H}^{SM\ (1)} + \kappa^2$$

Numerical results



Result of the scan in the parameter space over $M_{Z'}$, δ and $\sin\alpha$. Colouring with respect to δ the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta = 3/16$, was chosen.

Numerical results



Result of the scan in the parameter space over $M_{Z'}$, δ and $\sin\alpha$. Colouring with respect to α the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta = 3/16$, was chosen.