Natural 2HDM

Bohdan GRZADKOWSKI
University of Warsaw

- The little hierarchy problem
- The strategy
- The natural 2 Higgs Doublet Model
- Summary

- B.G., P. Osland, "A Natural Two-Higgs-Doublet Model", e-Print: arXiv:0910.4068.
- B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", Phys.Rev.Lett.103:091802,2009.

The little hierarchy problem

- $$m_h^2 = m_h^{(B)\,2} + \delta^{(SM)} m_h^2 + \cdots$$

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2}\left[\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2\right]$$

$$m_h = 130 \text{ GeV} \quad \Rightarrow \quad \delta^{(SM)} m_h^2 \simeq m_h^2 \qquad \text{for} \qquad \Lambda \simeq 600 \text{ GeV}$$

• For $\Lambda \gtrsim 600$ GeV there must be a cancellation between the tree-level Higgs mass² $m_h^{(B)\,2}$ and the 1-loop leading correction $\delta^{(SM)} m_h^2$:

$$m_h^{(B)\,2} \sim \delta^{(SM)} m_h^2 > m_h^2$$

$$\Downarrow$$

the perturbative expansion is breaking down.

- The SM cutoff is very low!

Solutions to the little hierarchy problem:

♠ Suppression of corrections growing with Λ^2 at the 1-loop level:

- The Veltman condition, no Λ^2 terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0 \qquad \Longrightarrow \qquad m_h \simeq 310 \text{ GeV}$$

- SUSY:

$$\delta^{(SUSY)} m_h^2 \sim m_{\tilde{t}}^2 \, \frac{3g_t^2}{8\pi^2} \ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right)$$

then for $\Lambda \sim 10^{16-18}$ GeV one gets $m_{\tilde{t}}^2 \lesssim 1$ TeV^2 in order to have $\delta^{(SUSY)} m_h^2 \sim m_h^2$.

♠ Increase of the allowed value of m_h:

- The inert Higgs model by Barbieri, Hall, Rychkov, Phys.Rev.D74:015007,2006, (Ma, Phys.Rev.D73:077301,2006) $\Rightarrow$ $m_h \sim 400 - 600$ GeV, ($\ln m_h$ terms in T parameter canceled by $m_{H^\pm}, m_A, m_S$ contributions).

The Strategy

- The SM 1-loop quadratic divergences are dominated by the top (fermionic) contribution, so to suppress them we are going to introduce extra scalars (as the SM Higgs would need to be too heavy to do the job).

- We will look for a model which allows for relatively heavy lightest Higgs boson (in order to suppress $\delta M_i^2/M_i^2$ even more). Note also that within the SM fit to the precision data there is a tension caused by the lightness of the Higgs.

- CPV and DM are desirable.

The natural 2 Higgs Doublet Model

B.G., P. Osland, "A Natural Two-Higgs-Doublet Model", e-Print: arXiv:0910.4068

$$
\begin{aligned}
V(\phi_1, \phi_2) \;\; &= \;\; -\frac{1}{2}\left\{m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + \left[m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.}\right]\right\} \\
&+\frac{1}{2}\lambda_1(\phi_1^\dagger \phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger \phi_2)^2 + \lambda_3(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\
&+\lambda_4(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger \phi_2)^2 + \text{H.c.}\right]
\end{aligned}
$$

The minimization conditions at $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$ and $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$ can be formulated as follows:

$$
\begin{aligned}
m_{11}^2 \;\; &= \;\; v_1^2 \lambda_1 + v_2^2(\lambda_{345} - 2\nu), \\
m_{22}^2 \;\; &= \;\; v_2^2 \lambda_2 + v_1^2(\lambda_{345} - 2\nu),
\end{aligned}
$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \Re\lambda_5$ and $\nu \equiv \Re m_{12}^2/(2v_1 v_2)$.

We assume that ϕ_1 and ϕ_2 couple to down- and up-type quarks, respectively (the so-called 2HDM II).

$$\mathbb{Z}_2 : \qquad \phi_2 \to -\phi_2$$

Cancellation of quadratic divergences for ϕ_1 and ϕ_2 (Newton & Wu, 1994):

$$\begin{aligned}
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_b^2}{c_\beta^2}, \\
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_t^2}{s_\beta^2},
\end{aligned}$$

where $v^2 \equiv v_1^2 + v_2^2$, $\tan\beta \equiv v_2/v_1$

$$\Downarrow$$

For a given choice of the mixing angles α_i's $(i = 1, 2, 3)$, the neutral-Higgs masses M_1^2, M_2^2 and M_3^2 can be determined from the cancellation conditions in terms of $\tan\beta$, $\mu^2 \equiv \mathbf{Re}(m_{12}^2)/(2s_\beta c_\beta)$ and $M_{H^\pm}^2$.

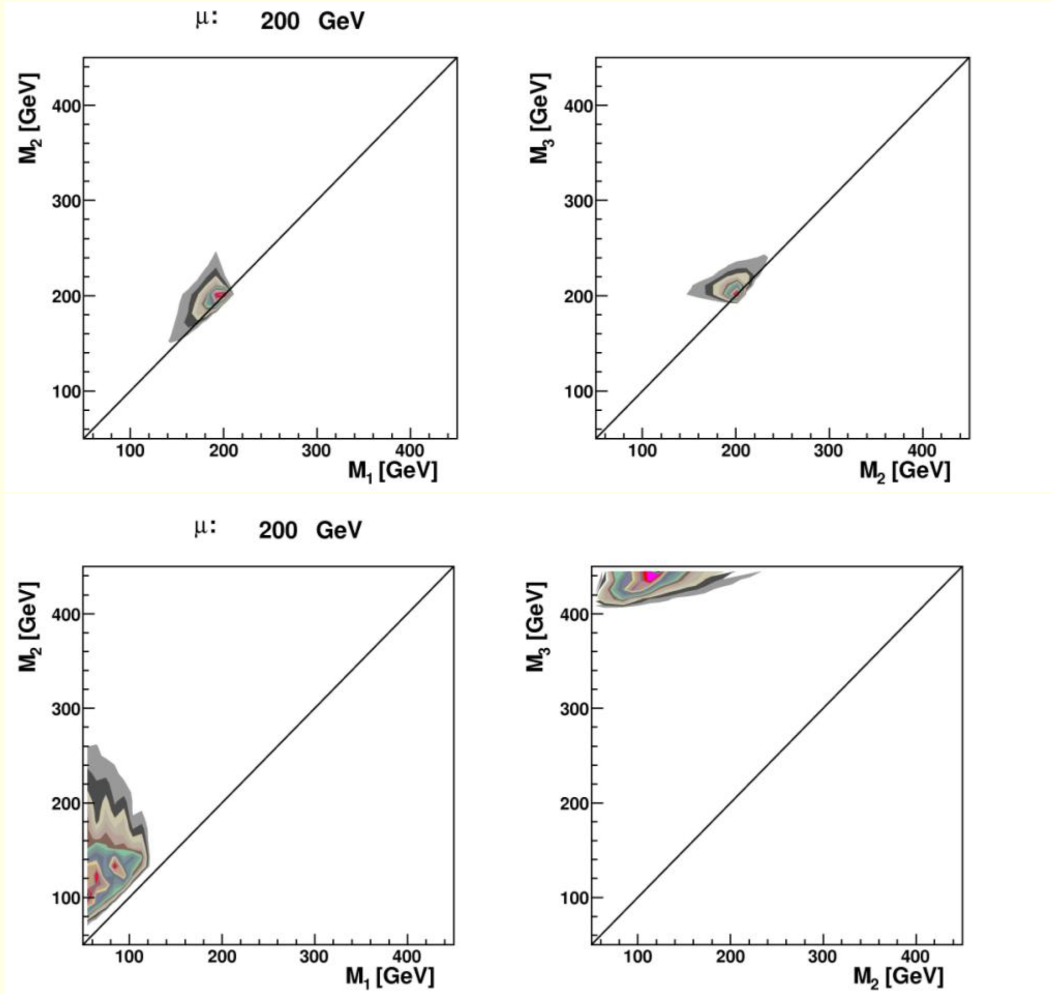

Figure 1: Distributions of allowed masses M_2 vs M_1 (left panels) and M_3 vs M_2 (right), resulting from a scan over the full range of α_i, $\tan\beta \in (0.5, 50)$ and $M_{H^\pm} \in (300, 700)$ GeV, for $\mu = 200$ GeV. No constraints are imposed other than the cancellation of quadratic divergences, $M_i^2 > 0$ and $M_1 < M_2 < M_3$. Two ranges of $\tan\beta$-values are displayed: bottom panels: $0.5 \leq \tan\beta \leq 1$, top panels: $40 \leq \tan\beta \leq 50$. The color coding indicates increasing density of allowed points as one moves inward from the boundary.

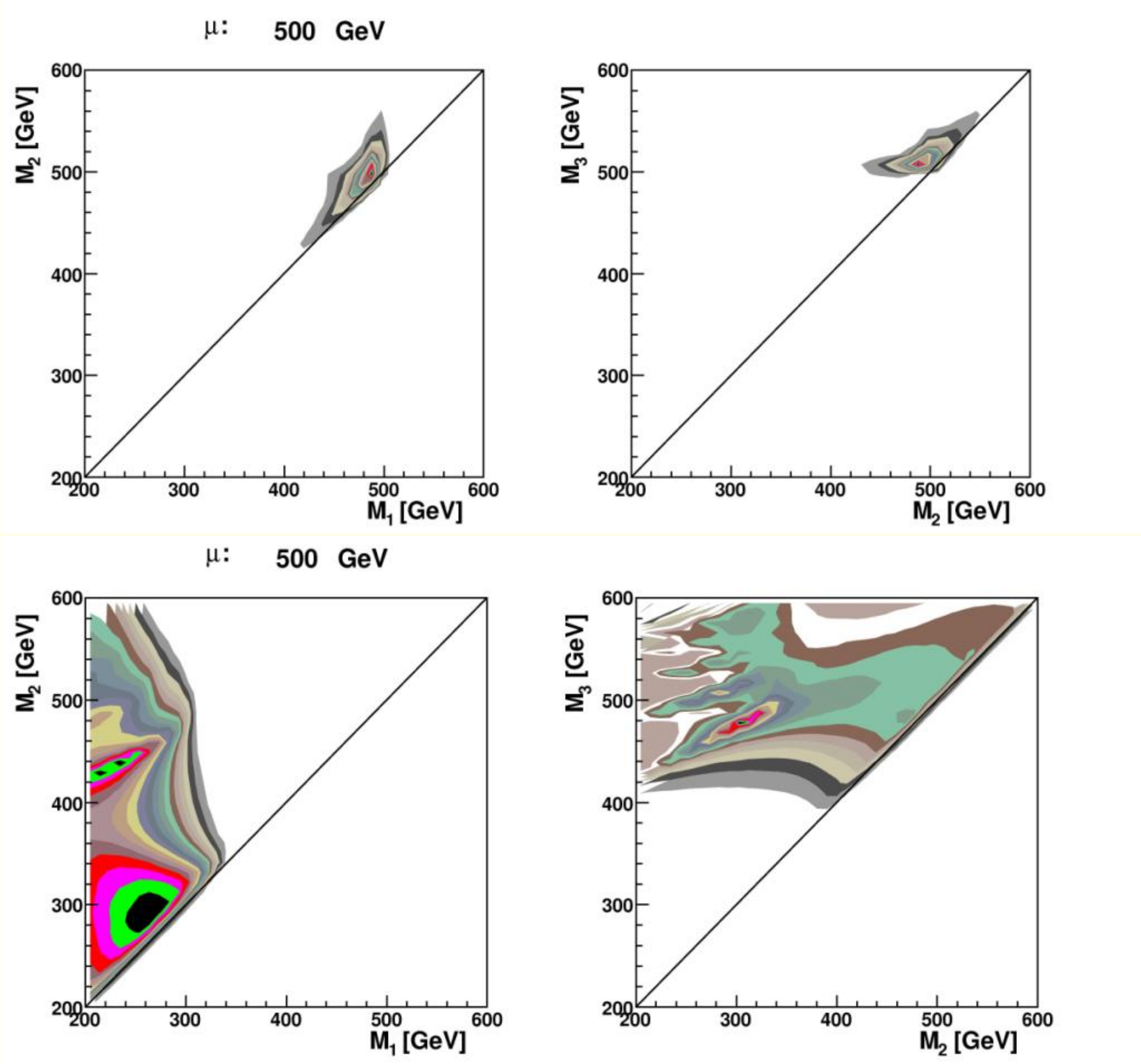

Figure 2: Similar to Fig. 1, for $\mu = 500$ GeV.

$$M_1^2 - M_2^2 = \frac{1}{\tan\beta}\frac{R_{33}}{R_{12}R_{22}}\left[-4\bar{m}^2 - 2M_{H^\pm}^2 + 12m_t^2 + \mu^2\right] + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right)$$

$$M_3^2 = -\frac{M_1^2 R_{12}R_{13} + M_2^2 R_{22}R_{23}}{R_{32}R_{33}} + \mathcal{O}\left(\frac{1}{\tan\beta}\right).$$

where R_{ij} are elements of the orthogonal rotation matrix for the neutral scalars and $\bar{m}^2 \equiv \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2$.

$$\Downarrow$$

$$\tan\beta \gtrsim 40 \qquad \Longrightarrow \qquad M_1 \simeq M_2 \simeq M_3 \simeq \mu^2 + 4m_b^2$$

Advantages:

- No 1-loop quadratic divergences (so, $\delta M_i^2/M_i^2$ suppressed),
- Large H_1 mass allowed (so, $\delta M_i^2/M_i^2$ suppressed),
- A chance for substantial CPV,
- DM candidate easily accommodated by adding singlets φ_i-like.

The following experimental constraints are imposed:

- The oblique parameters T and S
- $B_0 - \bar{B}_0$ mixing
- $B \to X_s \gamma$
- $B \to \tau \bar{\nu}_\tau X$
- $B \to D \tau \bar{\nu}_\tau$
- LEP2 Higgs-boson non-discovery
- R_b
- The muon anomalous magnetic moment
- Electron electric dipole moment

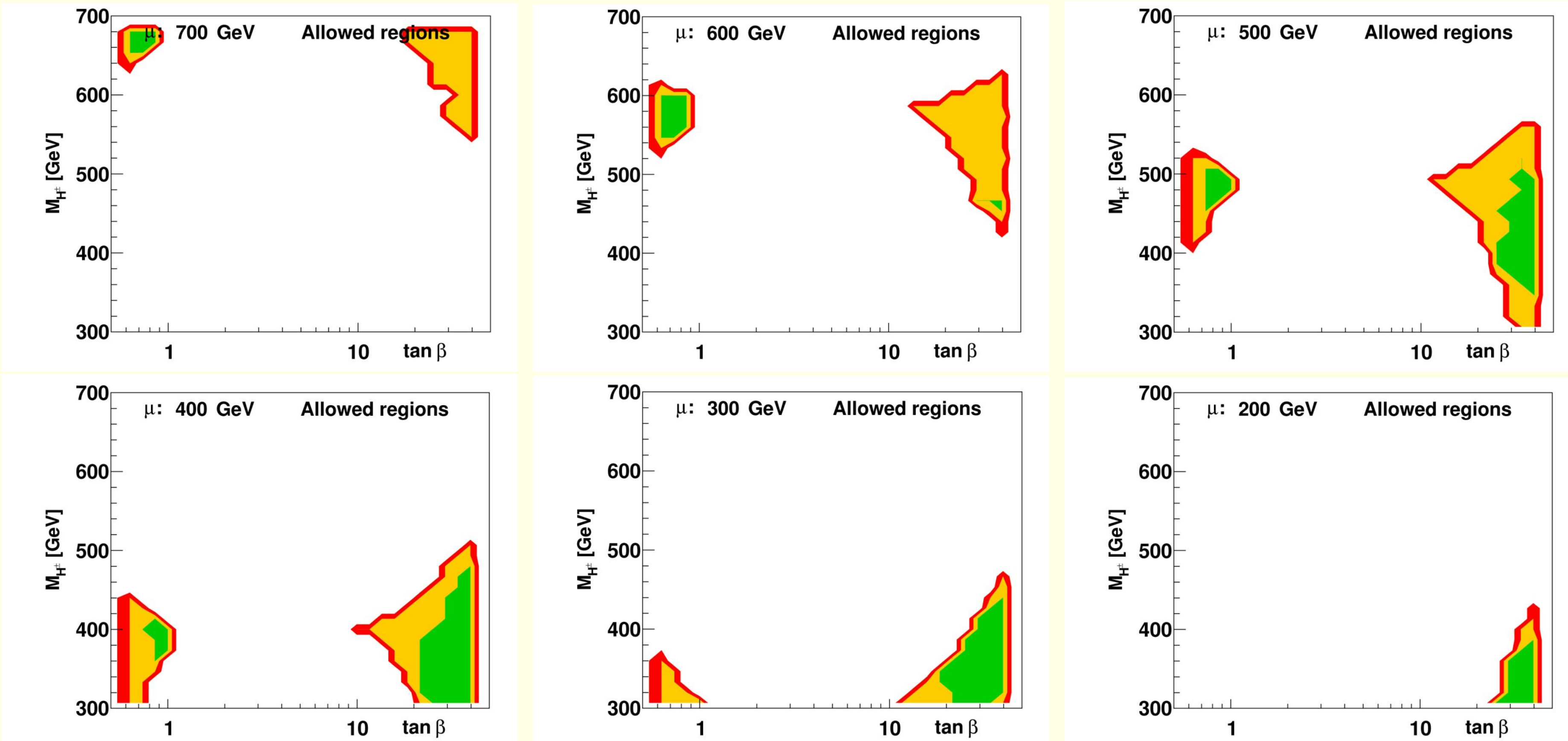

Figure 3: Allowed regions in the $\tan\beta$–$M_{H^\pm}$ plane, for $\mu = 200, 300, 400, 500, 600$ and 700 GeV (as indicated). Red: Positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L., as specified in the text.

Violation of CP

$$
\begin{aligned}
\Im J_1 &= -\frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)\Im\lambda_5, \\
\Im J_2 &= -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2\right) v_1^4 + 2(\lambda_1 - \lambda_2)\Re\lambda_5 v_1^2 v_2^2 \right. \\
&\quad \left. - \left((\lambda_2 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2\right) v_2^4\right] \Im\lambda_5, \\
\Im J_3 &= \frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)(\lambda_1 + \lambda_2 + 2\lambda_4)\Im\lambda_5.
\end{aligned}
$$

For $\tan\beta \gtrsim 40$

$$
\Im J_i \sim \frac{\Im\lambda_5}{\tan^2\beta}
$$

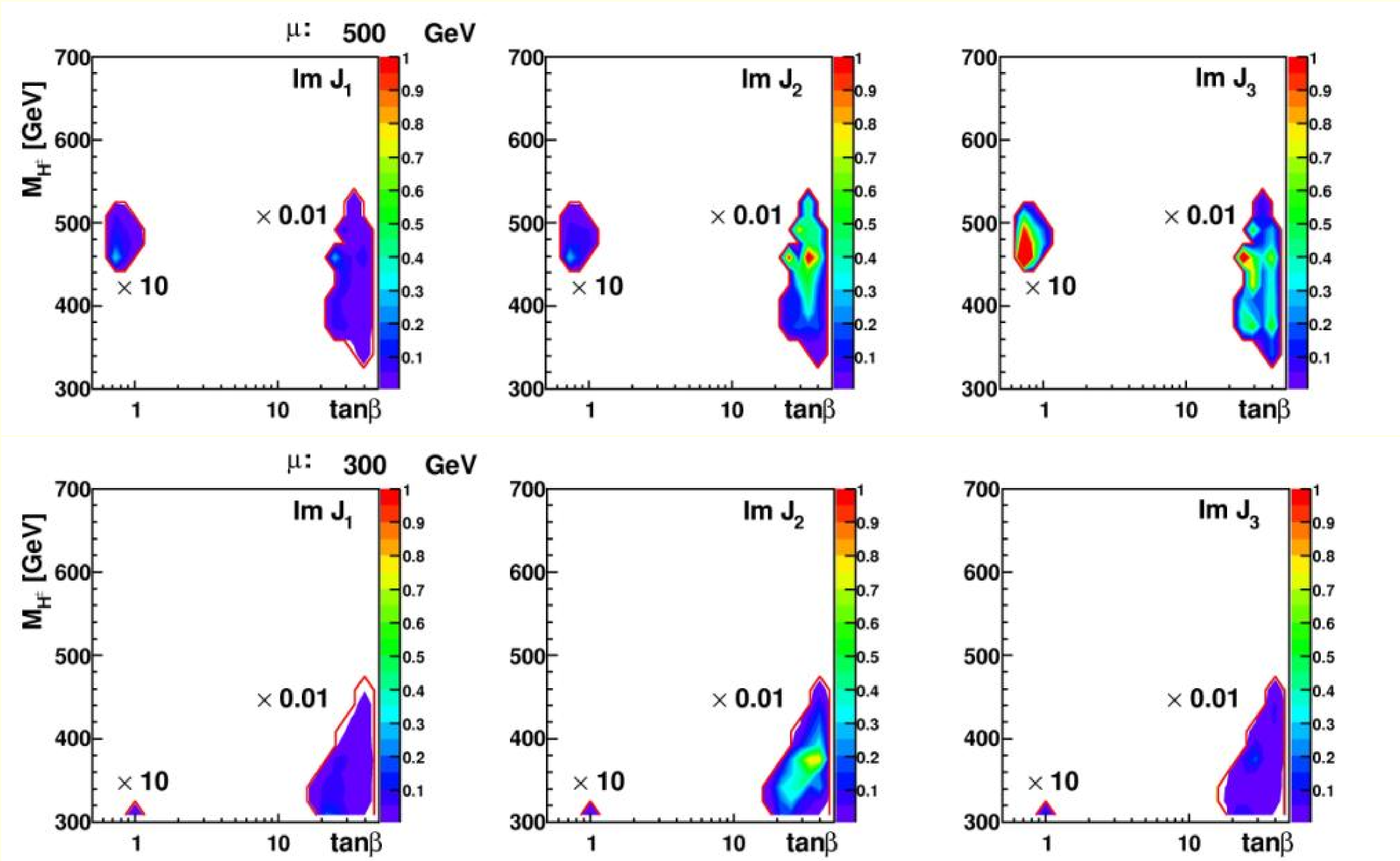

Figure 4: Imaginary parts of the rephasing invariants $|\Im J_i|$, for $\mu = 500$ GeV (top) and $\mu = 300$ GeV (bottom). The colour coding is given along the right vertical axis. At low $\tan\beta$ the values should be rescaled by a factor of 10, at high $\tan\beta$ by a factor 0.01.

Stability of the cancellation condition

$$\delta M_i^2 = \Lambda^2 \sum_{n=0} f_n^{(i)}(\lambda) \left[\ln\left(\frac{\Lambda}{v}\right)\right]^n ,$$

The coefficients $f_n^{(i)}(\lambda)$ can be determined recursively, however here a simple estimate is sufficient:

$$f_n^{(i)}(\lambda) \sim \left(\frac{\lambda}{16\pi^2}\right)^{n+1} \sim \left(\frac{4\pi}{16\pi^2}\right)^{n+1} \sim \left(\frac{1}{4\pi}\right)^{n+1}$$

Requiring that the 2-loop contribution does not exceed M_1^2 one finds:

$$\Lambda^2 \ln\left(\frac{\Lambda}{v}\right) \lesssim (4\pi M_1)^2$$

Then, e.g. for $M_1 = 200(500)$ GeV the cutoff is at $\Lambda \sim 1.8(3.8)$ TeV.

- DM in the Non-Inert Doublet Model with no quadratic divergences

$$
\begin{aligned}
V(\phi_1,\phi_2) &= -\frac{1}{2}\left\{m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \mathsf{H.c.}\right]\right\} \\
&+\frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\
&+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \mathsf{H.c.}\right] \\
&+\mu_\varphi^2\varphi^2 + \frac{1}{24}\lambda_\varphi\varphi^4 + \varphi^2(\eta_1\phi_1^\dagger\phi_1 + \eta_2\phi_2^\dagger\phi_2)
\end{aligned}
$$

The cancellation conditions:

$$
\begin{aligned}
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{1}{2}\eta_1 + \frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_b^2}{c_\beta^2}, \\
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{1}{2}\eta_2 + \frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_t^2}{s_\beta^2}, \\
\frac{\lambda_\varphi}{2} + 4(\eta_1 + \eta_2) &= 8\mathrm{Tr}\{Y_\varphi Y_\varphi^\dagger\}
\end{aligned}
$$

where $\mathcal{L}_Y = -\varphi\overline{(\nu_R)^c}Y_\varphi\nu_R + \mathsf{H.c.}$.

$$\mathcal{L} = -\varphi^2(\kappa_i v H_i + \lambda_{ij} H_i H_j + \lambda_\pm H^+ H^-)$$

with

$$\begin{aligned}
\kappa_i &= \eta_1 R_{i1} c_\beta + \eta_2 R_{i2} s_\beta, \\
\lambda_{ij} &= \frac{1}{2}\left[\eta_1(R_{i1}R_{j1} + s_\beta^2 R_{i3}R_{j3}) + \eta_2(R_{i2}R_{j2} + c_\beta^2 R_{i3}R_{j3})\right], \\
\lambda_\pm &= \eta_1 s_\beta^2 + \eta_2 c_\beta^2
\end{aligned}$$

Assumption: $M_1 \ll M_{2,3}$ so that DM annihilation is dominated by H_1 exchange.

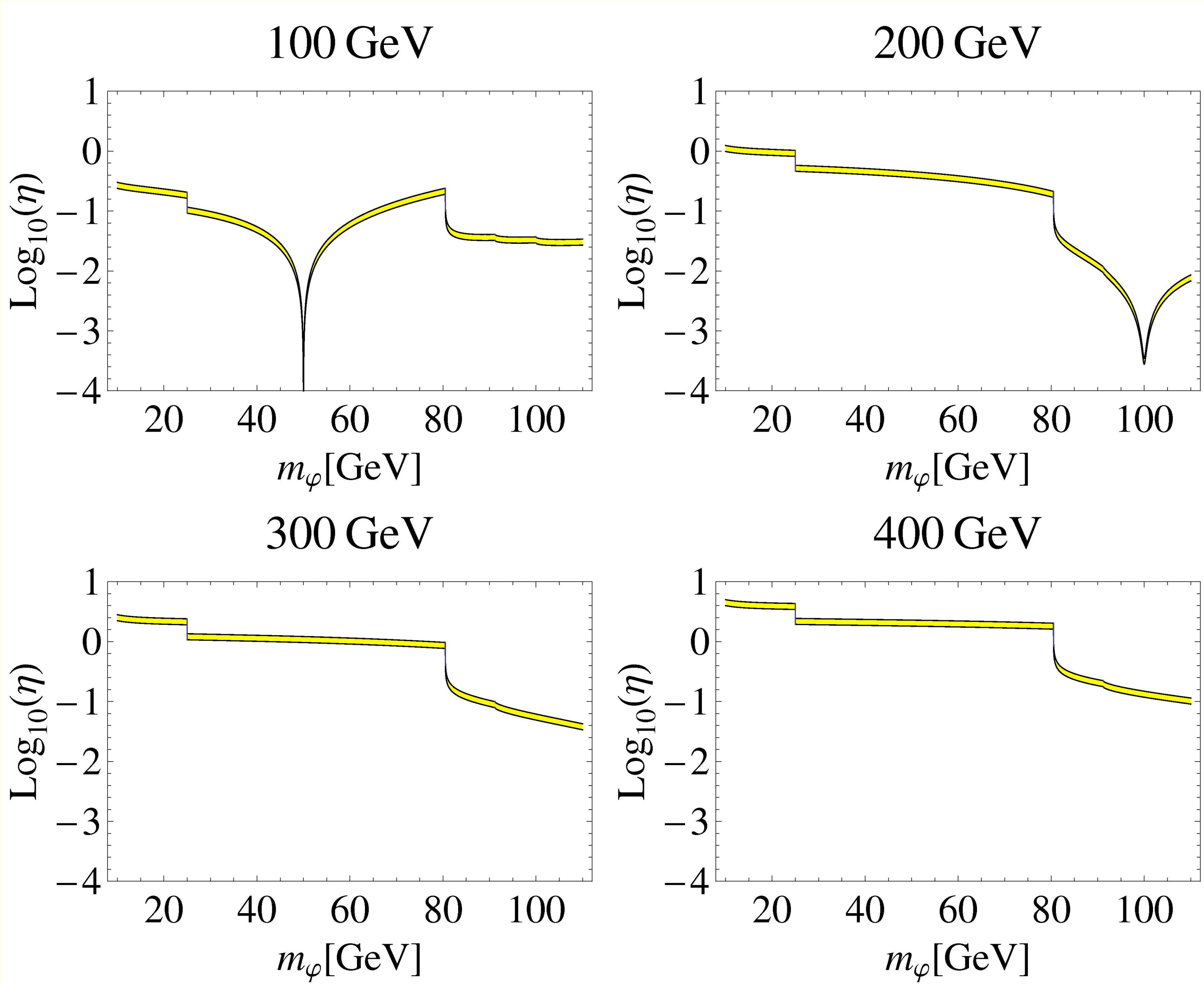

Figure 5: Inert-scalar coupling η (vs m_φ) required by the observed DM abundance $\Omega_{DM}h^2 = 0.106 \pm 0.008$ within a 3-σ band. As indicated above each panel, the lightest Higgs-boson mass ranges from $M_1 = 100$ to 400 GeV . It was assumed that $2\lambda_{11} = \kappa_1 \equiv \eta$.

Summary

- The SM could be easily extended so that the little hierarchy problem is ameliorated, DM candidate is provided and also CP is violated in the extra sector:
  - The addition of N_φ real scalar singlets φ_i to the SM lifts the cutoff Λ to $\sim 4-9$ TeV. It also provides a realistic candidate for DM if $m_\varphi \sim 1-3$ TeV (depending on N_φ), see talk by Jose Wudka.
  - To accommodate CPV in the Higgs potential the SM scalar sector should be replaced by 2 Higgs doublets (non-inert). Cancellation of quadratic divergences could be arranged. Heavy lightest Higgs additionally suppresses $\delta M_i^2/M_i^2$. Adding extra inert scalar singlet or doublet offers a DM candidate.
  - CPV in the Higgs potential with the SM doublet and singlets only?

- Some fine tuning always remains.