Pragmatic approach to the little hierarchy problem
- the case for Dark Matter and neutrino physics -

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- The little hierarchy problem
- Dark Matter
- Neutrino physics
- Summary and comments

B.G., J. Wudka, work in progress and arXiv:0902.0628

The little hierarchy problem

- $$m_h^2 = m_h^{(B)\,2} + \delta^{(SM)} m_h^2 + \cdots$$

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2}\left[\frac{3}{2} m_t^2 - \frac{1}{8}\left(6 m_W^2 + 3 m_Z^2\right) - \frac{3}{8} m_h^2\right]$$

$$m_h = 130 \text{ GeV} \quad \Rightarrow \quad \delta^{(SM)} m_h^2 \simeq m_h^2 \qquad \text{for} \qquad \Lambda \simeq 580 \text{ GeV}$$

- For $\Lambda \gtrsim 580$ GeV there must be a cancellation between the tree-level Higgs mass^2 $m_h^{(B)\,2}$ and the 1-loop leading correction $\delta^{(SM)} m_h^2$:

$$m_h^{(B)\,2} \sim \delta^{(SM)} m_h^2 > m_h^2$$

$$\Downarrow$$

the perturbative expansion is breaking down.

- The SM cutoff is very low!

Solutions to the little hierarchy problem:

♠ Suppression of corrections growing with Λ at the 1-loop level:

- The Veltman condition, no Λ^2 terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0 \qquad \Longrightarrow \qquad m_h \simeq 310 \text{ GeV}$$

- SUSY:

$$\delta^{(SUSY)}m_h^2 \sim m_{\tilde{t}}^2 \, \frac{3g_t^2}{8\pi^2} \ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right)$$

then for $\Lambda \sim 10^{16-18}$ GeV one gets $m_{\tilde{t}}^2 \lesssim 1$ TeV in order to have $\delta^{(SUSY)}m_h^2 \sim m_h^2$.

♠ Increase of the allowed value of m_h:

- The inert Higgs model by Barbieri, Hall, Rychkov, arXiv:hep-ph/0603188, (see also Ma) $\quad \Rightarrow \quad m_h \sim 400 - 600$ GeV, (m_h^2 terms in T parameter canceled by $m_{H^\pm}, m_A, m_S$ contributions).

Motivation: to lift the cutoff to few TeV range in the most economic way

- N_φ extra gauge singlets φ_i with $\langle\varphi_i\rangle = 0$ (no $H \leftrightarrow \varphi_i$ mixing from $\varphi_i^2|H|^2$).

- Symmetries $\mathbb{Z}_2^{(i)}$: $\varphi_i \to -\varphi_i$ (to eliminate $|H|^2\varphi_i$ couplings).

- Gauge singlet neutrinos: $\nu_{R\,j}$ for $j = 1, 2, 3$.

$$V(H,\varphi_i) = -\mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_\varphi^2\sum_{i=1}^{N_\varphi}\varphi_i^2 + \frac{\lambda_\varphi}{24}\sum_{i=1}^{N_\varphi}\varphi_i^4 + \lambda_x|H|^2\sum_{i=1}^{N_\varphi}\varphi_i^2$$

with

$$\langle H\rangle = \frac{v}{\sqrt{2}}, \qquad \langle\varphi_i\rangle = 0 \qquad \text{for} \qquad \mu_\varphi^2 > 0$$

then

$$m_h^2 = 2\mu_H^2 \qquad \text{and} \qquad m^2 = 2\mu_\varphi^2 + \lambda_x v^2$$

- Positivity (stability): $\lambda_H, \lambda_\varphi, \lambda_x > 0$

- Unitarity in the limit $s \gg m_h^2, m^2$: $\lambda_H \leq \frac{4\pi}{3}$ (the SM requirement) and $\lambda_\varphi \leq 8\pi$, $\lambda_x < 4\pi$

$$\delta^{(\varphi)} m_h^2 = -N_\varphi \frac{\lambda_x}{8\pi^2}\left[\Lambda^2 - m^2 \ln\left(1 + \frac{\Lambda^2}{m^2}\right)\right]$$

$$|\delta m_h^2| = |\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2| = D m_h^2$$

$$\Downarrow$$

$$\lambda_x = \lambda_x(m, m_h, D, \Lambda, N_\varphi)$$

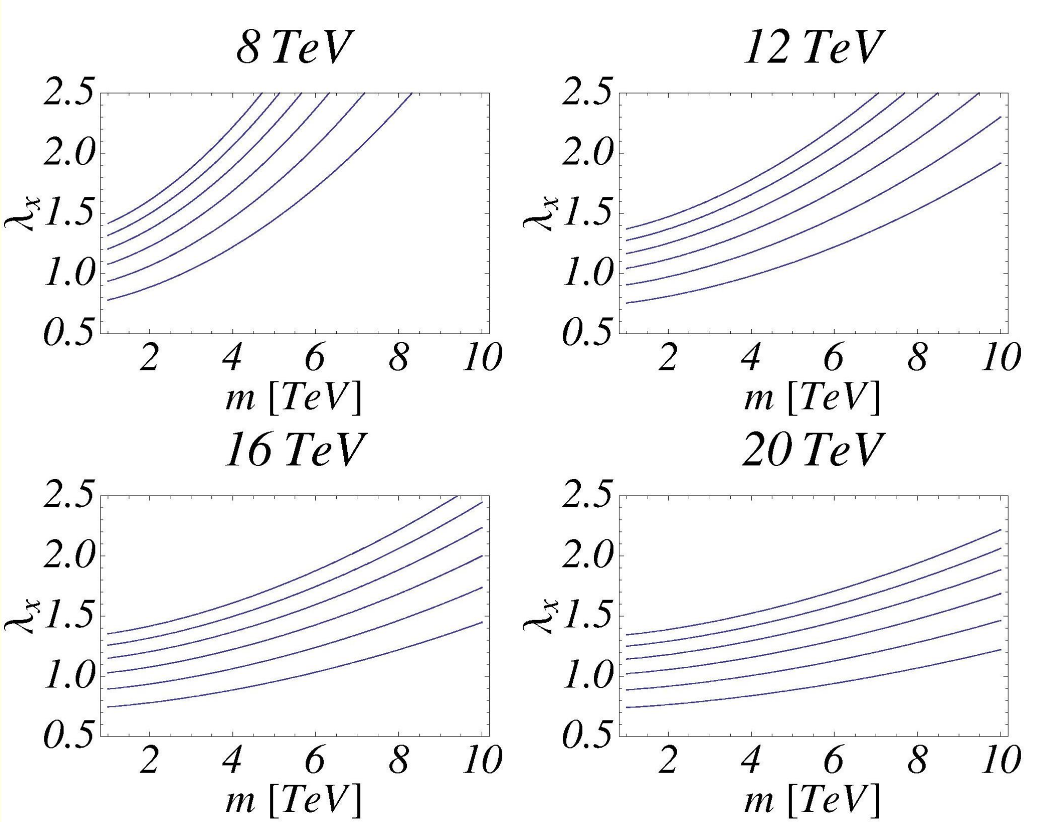

Figure 1: Plots of λ_x as a function of m for $N_\varphi = 3$, $D = 0$ and various choices of $\Lambda = 8, 12, 16$ and 20 TeV shown above each panel. The curves correspond to $m_h = 130,\ 150,\ 170,\ 190,\ 210,\ 230$ GeV (starting with the uppermost curve).

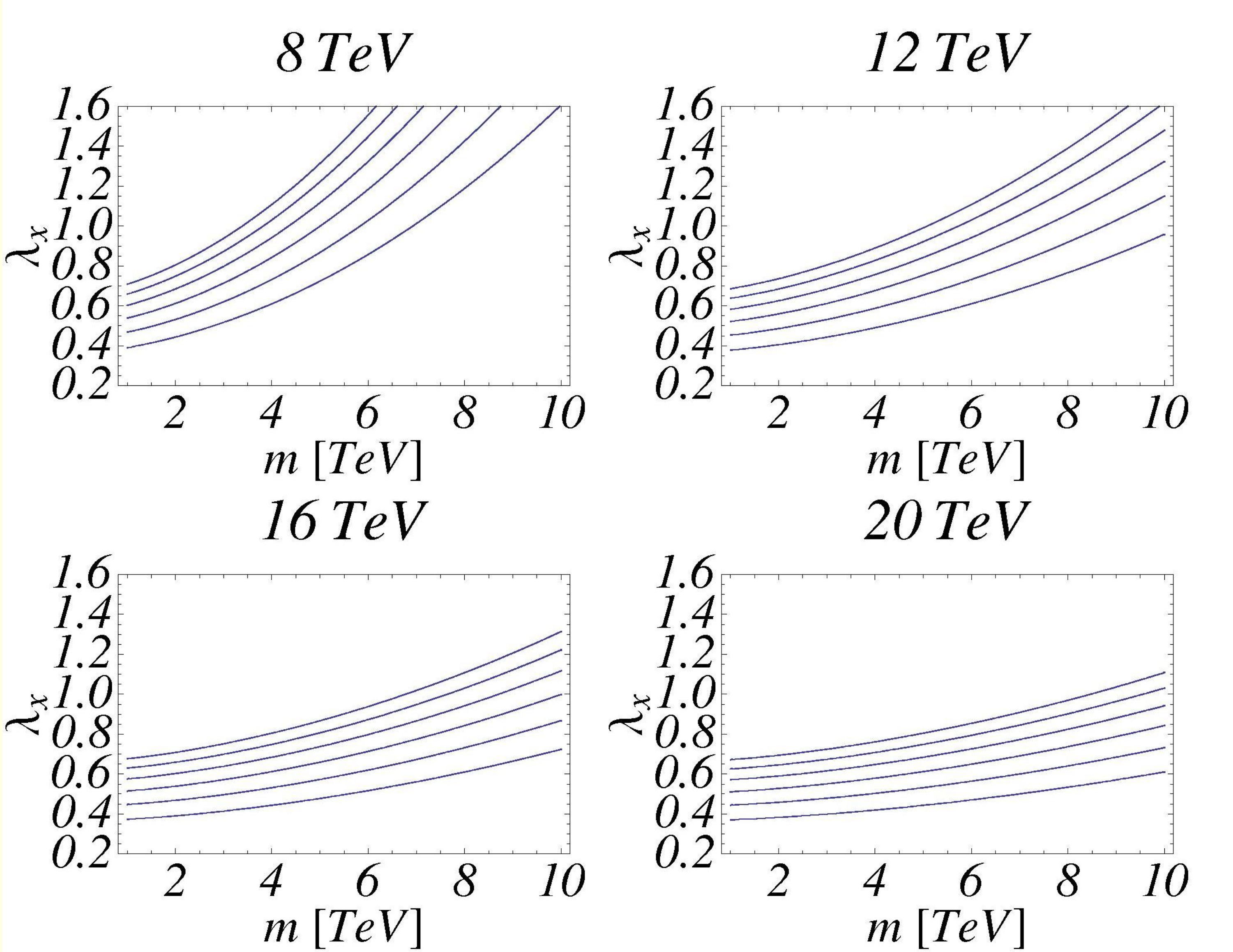

Figure 2: Plots of λ_x as a function of m for $N_\varphi = 6$, $D = 0$ and various choices of $\Lambda = 8, 12, 16$ and 20 TeV shown above each panel. The curves correspond to $m_h = 130,\ 150,\ 170,\ 190,\ 210,\ 230$ GeV (starting with the uppermost curve).

Stability of the fine tunning

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(12 g_t^2 - \frac{9}{2} g_2^2 - \frac{3}{2} g_1^2 - 12\lambda_H\right)$$
$$\delta^{(\varphi)} m_h^2 = -N_\varphi \frac{\lambda_x}{8\pi^2}\left[\Lambda^2 - m^2 \ln\left(1 + \frac{\Lambda^2}{m^2}\right)\right]$$

In general

$$\delta m_h^2 = \underbrace{\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2}_{\simeq 0} + 2\Lambda^2 \sum_{n=1}^{\infty} f_n(\lambda, \dots) \ln^n\left(\frac{\Lambda}{\mu}\right)$$

where

$$(n+1) f_{n+1} = \mu \frac{\partial}{\partial \mu} f_n = \beta_i \frac{\partial}{\partial \lambda_i} f_n \qquad \text{and} \qquad f_n \propto \left(\frac{N_\varphi \lambda_x}{16\pi^2}\right)^{n+1}$$

From 1-loop condition ($n = 0$)

$$\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2 \simeq 0$$

we have

$$\lambda_x = \frac{1}{N_\varphi} \left[4.8 - 3 \left(\frac{m_h}{v} \right)^2 \right] \left[1 - \frac{m^2}{\Lambda^2} \ln \left(\frac{m^2}{\Lambda^2} \right) \right] + \mathcal{O} \left(\frac{m^4}{\Lambda^4} \right) .$$

Therefore at the 2-loop ($n = 1$)

$$D \equiv \frac{\delta m_h^2}{m_h^2} \simeq \left(\frac{N_\varphi \lambda_x}{16\pi^2} \right)^2 \frac{\Lambda^2}{m_h^2} \simeq \left(\frac{4}{16\pi^2} \right)^2 \frac{\Lambda^2}{m_h^2}$$

for $D \lesssim 1$

$$\Lambda \lesssim 4\pi^2 m_h$$

Dark Matter

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3. C. P. Burgess, M. Pospelov and T. ter Veldhuis, Nucl. Phys. B **619**, 709 (2001)
4. H. Davoudiasl, R. Kitano, T. Li and H. Murayama, Phys. Lett. B **609**, 117 (2005)
5. J. J. van der Bij, Phys. Lett. B **636**, 56 (2006)
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It is possible to find parameters Λ, λ_x and m such that
both the hierarchy is ameliorated to the prescribed level and
such that $\sum_i \Omega_{\varphi_i} h^2$ is consistent with $\Omega_{DM} h^2$

$$\varphi_i \varphi_i \to hh, W^+W^-, ZZ, l\bar{l}, q\bar{q}, gg, \gamma\gamma \quad \Rightarrow \quad \langle \sigma_i v \rangle = \langle \sigma_i v \rangle (\lambda_x, m)$$

The Boltzmann equation $\quad \Rightarrow \quad x_f \left(\equiv \frac{m}{T_f} \right) \simeq \ln \left[0.038 \frac{m_{Pl} m \langle \sigma v \rangle}{g_\star^{1/2} x_f^{1/2}} \right]$

$$\Omega_{\varphi_i} h^2 \simeq 1.06 \cdot 10^9 \frac{x_f}{g_\star^{1/2} m_{Pl} \langle \sigma v \rangle \text{ GeV}}$$

$$|\delta m_h^2| = D m_h^2 \qquad \text{and} \qquad \sum_{i=1}^{N_\varphi} \Omega_{\varphi_i} h^2 = \Omega_{DM} h^2 = 0.106 \pm 0.008 \quad \Rightarrow \quad m = m(\Lambda)$$

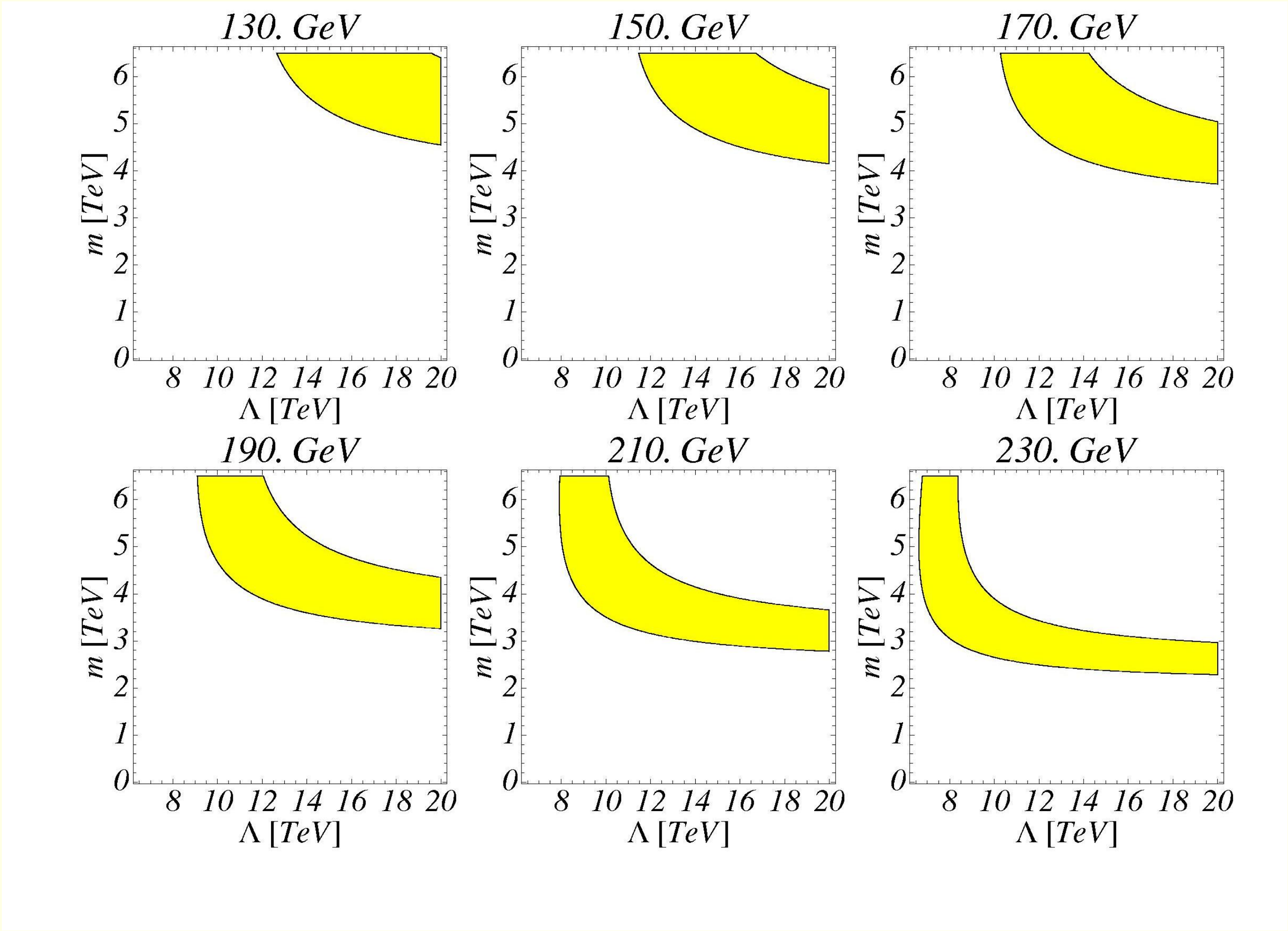

Figure 3: Allowed regions in the space (m, Λ) are plotted for $D(m) = 0$, $N_\varphi = 3$ and assuming that each φ_i contributes the same to the total Ω_{DM} at the 3σ level: $\Omega_\varphi h^2 = 0.106 \pm 0.008$ for the Higgs mass shown above each panel; $m_h = 130, 150, 170, 190, 210, 230$ GeV.

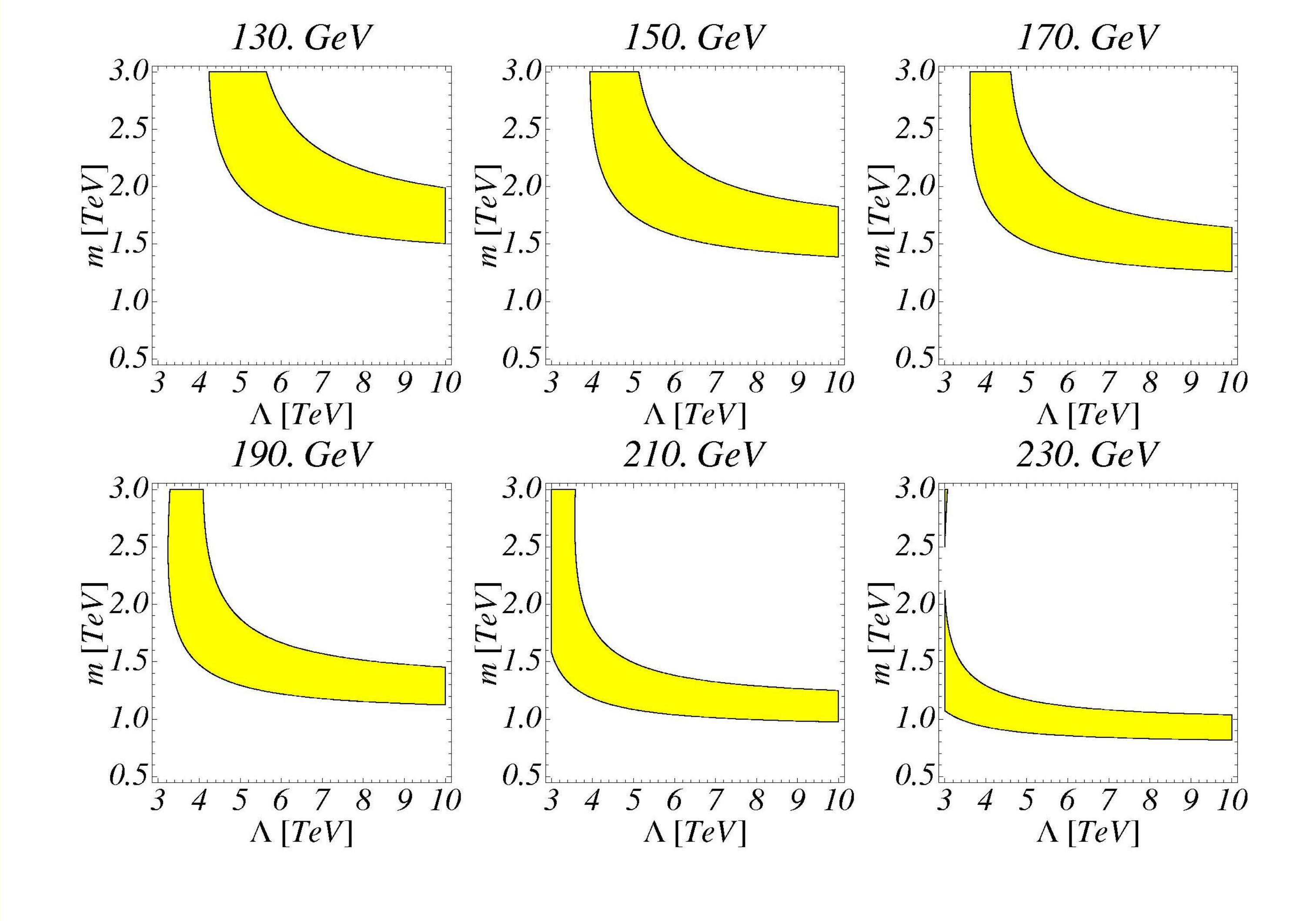

Figure 4: Allowed regions in the space (m, Λ) are plotted for $D(m) = 0$, $N_\varphi = 6$ and assuming that each φ_i contributes the same to the total Ω_{DM} at the 3σ level: $\Omega_\varphi h^2 = 0.106 \pm 0.008$ for the Higgs mass shown above each panel; $m_h = 130, 150, 170, 190, 210, 230$ GeV.

Neutrino physics

$$\mathcal{L}_Y = -\bar{L} Y_l H l_R - \bar{L} Y_\nu \tilde{H} \nu_R - \frac{1}{2} \overline{(\nu_R)^c} M \nu_R - \varphi_i \overline{(\nu_R)^c} Y_{\varphi_i} \nu_R + \text{H.c.}$$

$$\mathbb{Z}_2^{(i)} : \qquad H \to H,\ \varphi_i \to -\varphi_i,\ L \to S_L L,\ l_R \to S_{l_R} l_R,\ \nu_R \to S_{\nu_R} \nu_R$$

The symmetry conditions ($S_i S_i^\dagger = S_i^\dagger S_i = \mathbb{1}$):

$$S_L^\dagger Y_l S_{l_R} = Y_l, \quad S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu, \quad S_{\nu_R}^T M S_{\nu_R} = +M, \quad S_{\nu_R}^T Y_\varphi S_{\nu_R} = -Y_\varphi$$

The implications of the symmetry:

$$S_{\nu_R}^T M S_{\nu_R} = +M \quad \Rightarrow \quad S_{\nu_R} = \pm \mathbb{1}, \qquad S_{\nu_R} = \pm\, \text{diag}(1, 1, -1)$$

$$S_{\nu_R} = \pm \mathbb{1} \quad \Rightarrow \quad Y_\varphi = 0 \text{ or } S_{\nu_R} = \pm\,\mathrm{diag}(1,1,-1) \quad \Rightarrow \quad Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}$$

$$S_L^\dagger Y_l S_{l_R} = Y_l \quad \Rightarrow \quad S_L = S_{l_R} = \mathrm{diag}(s_1, s_2, s_3), \quad |s_i| = 1$$

$$S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu \quad \Rightarrow \qquad \text{10 Dirac neutrino mass textures}$$

For instance the solution corresponding to $s_{1,2,3} = \pm 1$:

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix}$$

$$\mathcal{L}_m = -(\bar{n}M_n n + \bar{N}M_N N)$$

with the see-saw mechanism explaining $M_n \ll M_N$:

$$M_N \sim M \qquad \text{and} \qquad M_n \sim (vY_\nu)\frac{1}{M}(vY_\nu)^T$$

where

$$\nu_L = n_L + M_D\frac{1}{M}N_L \qquad \text{and} \qquad \nu_R = N_R - \frac{1}{M}M_D^T n_R$$

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix} \quad \Rightarrow \quad M_D = Y_\nu \frac{v}{\sqrt{2}} \quad \Rightarrow \quad M_n$$

To compare our results with the data, we use the following approximate lepton mixing matrix (tri-bimaximal lepton mixing) that corresponds to $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\theta_{12} = \arcsin(1/\sqrt{3})$:

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$

Writing the diagonal light neutrino mass matrix as

$$m_{\text{light}} = \mathsf{diag}(m_1, m_2, m_3)$$

we find

$$M_n = U m_{\text{light}} U^T$$

$$\Downarrow$$

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ -\frac{a}{2} & b & 0 \\ -\frac{a}{2} & b & 0 \end{pmatrix} \quad \begin{array}{l} m_1 = -3a^2\frac{v^2}{M_1} \\ m_2 = -6b^2\frac{v^2}{M_2} \\ m_3 = 0 \end{array} \quad \text{and } Y_\nu = \begin{pmatrix} a & b & 0 \\ a & -\frac{b}{2} & 0 \\ a & -\frac{b}{2} & 0 \end{pmatrix} \quad \begin{array}{l} m_1 = -3b^2\frac{v^2}{M_2} \\ m_2 = -6a^2\frac{v^2}{M_1} \\ m_3 = 0 \end{array}$$

Does $Y_\varphi \neq 0$ imply $\varphi \to n_i n_j$ decays?

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix}, \; Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \;\Rightarrow\; \varphi \to N^\star_{1,2} N_3 \to \underbrace{n_{1,2,3}\, h}_{N^\star_{1,2}} \; N_3$$

that can be kinematically forbidden by requiring $M_3 > m$.

Summary and comments

- The addition of N_φ real scalar singlets φ_i to the SM may ameliorate the little hierarchy problem (by lifting the cutoff Λ to $\sim 5 - 10$ TeV range). Some fine tuning remains.

- It also provides a realistic candidate for DM if $m_\varphi \sim 3 - 5$ TeV.

- For appropriate choices of $\mathbb{Z}_2$ charges, the $\mathbb{Z}_2$ symmetry implies one massless neutrino and light-neutrino mass matrix consistent with the tri-bimaximal lepton mixing.