GOOFy "symmetries" - a systematic approach

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based on

- P.M. Ferreira, B.G., O.M. Ogreid, and P. Osland, "New Symmetries of the Two-Higgs-Doublet Model", *Eur. Phys. J. C* **84** (2024) 3, 234, e-Print: 2306.02410,
- P.M. Ferreira, B.G. and O.M. Ogreid, "Imaginary scaling invariance of the one-loop effective potential", e-Print: 2506.21145, *Phys. Rev. D* **113**, 075036 – Published 27 April, 2026,
- B.G., O.M. Ogreid, "GOOFy - a systematic approach", e-Print: 2602.20849.

Global symmetries of the 2HDM

The most generic scalar potential for the 2-Higgs-Doublet Model (2HDM):

$$\begin{aligned} V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] + \\ &\quad \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \\ &\quad \left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}, \end{aligned}$$

where m_{12}^2 and $\lambda_{5,6,7}$ might be complex.

Global symmetries of the 2HDM

- *Higgs-family symmetries*, unitary transformations mix both doublets,

$$\Phi_i \;\to\; \Phi'_i = \sum_{j=1}^{2} U_{ij}\,\Phi_j\,, \qquad\qquad U \in U(2)$$

  e.g. Z_2:

$$\Phi_1 \;\to\; \Phi_1 \quad , \quad \Phi_2 \;\to\; -\Phi_2\,,$$

  prevents the occurrence of tree-level flavour-changing neutral currents (FCNC).
- *generalized CP (GCP)*, transformations:

$$\Phi_i \;\to\; \Phi'_i = \sum_{j=1}^{2} X_{ij}\,\Phi^*_j\,, \qquad\qquad X \in U(2)$$

  e.g. “standard" CP transformation (CP1):

$$\Phi_i \;\to\; \Phi^*_i$$

Global symmetries of the 2HDM

S	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			λ_{134}	0	0
$SO(3)$		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 1: Relations between 2HDM scalar potential parameters for each of the six symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

$$
\begin{aligned}
V = \; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \\
& \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\
& \left\{ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} ,
\end{aligned}
$$

Running 2HDM parameters and RG stable relations

$$Z_2 \text{ symmetry} \Rightarrow m_{12}^2 = 0,\ \lambda_6 = \lambda_7 = 0 \Rightarrow \beta_{m_{12}^2} = 0,\ \beta_{\lambda_6} = \beta_{\lambda_7} = 0$$

a symmetry-based conditions on m^2's and λ's are preserved by RG running at the one-loop order.

For the Z_2-symmetric model

$$\beta_{\lambda_5} = \left[\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4 - \frac{3}{2}\left(3g^2 + g'^2\right)\right] \lambda_5$$

$\lambda_5 = 0$ is *a fixed point* of this RG equation: if at any scale $\lambda_5 = 0$, that coupling will remain zero for all renormalization scales. Such fixed points of RG equations are usually fingerprints of symmetries, and indeed that is the case here: if $m_{12}^2 = 0$ and $\lambda_6 = \lambda_7 = 0$, the extra constraint $\lambda_5 = 0$ takes us from Z_2-symmetric model to $U(1)$-symmetric.

Running 2HDM parameters and RG stable relations

We have noticed that

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\; , \;\; \lambda_1 - \lambda_2 = 0 \;\; , \;\; \lambda_6 + \lambda_7 = 0 \right\}$$

- constitutes a fixed point of the 1-loop RG equations,
- are basis transformation invariants.

$$\begin{aligned}
\beta_{m_{11}^2+m_{22}^2} &= 3(\lambda_1 m_{11}^2 + \lambda_2 m_{22}^2) + (2\lambda_3 + \lambda_4)(m_{11}^2 + m_{22}^2) \\
&\quad -3\left[(\lambda_6^* + \lambda_7^*) m_{12}^2 + \text{h.c.}\right] - \frac{1}{4}(9g^2 + 3g'^2)(m_{11}^2 + m_{22}^2) \\
\beta_{\lambda_1-\lambda_2} &= 6\left(\lambda_1^2 - \lambda_2^2\right) + 12\left(|\lambda_6|^2 - |\lambda_7|^2\right) - \frac{3}{2}(\lambda_1 - \lambda_2)(3g^2 + g'^2) \\
\beta_{\lambda_6+\lambda_7} &= 6(\lambda_1\lambda_6 + \lambda_2\lambda_7) + (3\lambda_3 + 2\lambda_4)(\lambda_6 + \lambda_7) + 6\lambda_5(\lambda_6^* + \lambda_7^*) \\
&\quad - \frac{3}{2}(\lambda_6 + \lambda_7)(3g^2 + g'^2)
\end{aligned}$$

Running 2HDM parameters and RG stable relations

It turns out that

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\; , \;\; \lambda_1 - \lambda_2 = 0 \;\; , \;\; \lambda_6 + \lambda_7 = 0 \right\}$$

is also the 2-loop fixed point.

$$\Downarrow$$

Conclusion:
Perhaps there is a symmetry behind the fixed point:

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\; , \;\; \lambda_1 - \lambda_2 = 0 \;\; , \;\; \lambda_6 + \lambda_7 = 0 \right\}$$

The GOOFy symmetry: bosons

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

The transformation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

implies

$$m_{11}^2 + m_{22}^2 = 0 \ , \quad \lambda_1 - \lambda_2 = 0 \ , \quad \lambda_6 + \lambda_7 = 0$$

The GOOFy symmetry: bosons

$$\Phi \equiv \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \to \underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}_{X_\phi} \underbrace{\begin{pmatrix} \Phi_1^\star \\ \Phi_2^\star \end{pmatrix}}_{\Phi^\star}, \qquad \Phi^\dagger = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}^\dagger \to - \underbrace{\begin{pmatrix} \Phi_1, & \Phi_2 \end{pmatrix}}_{\Phi^\dagger} \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{X_\phi^\dagger}$$

- Higgs kinetic terms $\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$,

where $D^\mu = \partial^\mu + \frac{ig}{2}\sigma_i W_i^\mu + i\frac{g'}{2}B^\mu$

$\mathcal{L}_k$ remains invariant if the above transformation of $\Phi_{1,2}$ is supplemented by

$$\partial_\mu \to -i\partial_\mu, \qquad (x^\mu \to i x^\mu),$$

$$B_\mu \to i B_\mu,$$

$$W_{1\mu} \to i W_{1\mu}, \quad W_{2\mu} \to -i W_{2\mu}, \quad W_{3\mu} \to i W_{3\mu}.$$

The GOOFy symmetry: bosons

The GOOFy symmetry is a transformation of scalar, vector and fermionic fields (Φ, V_μ, ψ) that leaves the corresponding kinetic terms invariant if we simultaneously transform coordinates as $x^\mu \to i x^\mu$.

We shall allow for C-inspired transformations of the following form,

$$x^\mu \to i x^\mu, \qquad \Phi \to X_\phi \Phi^\star, \qquad \Phi^\dagger \to \Phi^T \bar{X}_\phi{}^\dagger,$$

where X_ϕ and $\bar{X}_\phi$ are both unitary $N_\phi \times N_\phi$ matrices. Requiring

$$\mathcal{L}_{\text{kin}} = \left(\partial_\mu \Phi^\dagger\right)\left(\partial^\mu \Phi\right)$$

to be invariant under the above transformation implies $\bar{X}_\phi = -X_\phi$. Then

$$x^\mu \to i x^\mu, \qquad \Phi \to X_\phi \Phi^\star, \qquad \Phi^\dagger \to -\Phi^T X_\phi^\dagger$$

will constitute the GOOFy transformation.

The GOOFy symmetry: bosons

Scalar potential

$$V(\Phi) = \Phi^\dagger M^2 \Phi$$

Hermiticity implies that M^2 is a Hermitian $N_\phi \times N_\phi$ matrix.
Goofy-invariance of the $\Phi^\dagger\Phi$ mass terms requires

$$M^2 = -X_\phi^T \left(M^2\right)^T X_\phi^\star$$

Therefore

$$\mathrm{Tr}\, M^2 = 0$$

This requirement forbids $\Phi^\dagger M^2 \Phi$ mass term in the case of a single complex scalar field ($N_\phi = 1$). For $N_\phi \geq 2$, this type of mass terms is allowed.

The GOOFy symmetry: fermions

For fermions we also start with C-inspired transformation:

$$x^\mu \to i x^\mu, \qquad \Psi \to -X_\psi \gamma_0 C \Psi^\star, \qquad \bar{\Psi} \to -\Psi^T C^{-1} \bar{X}_\psi^\dagger \tag{1}$$

where $\Psi^T \equiv (\psi_1, \cdots, \psi_{N_\psi})$ denotes a multiplet of N_ψ fermionic fields while X_ψ and $\bar{X}_\psi$ are unitary $N_\psi \times N_\psi$ matrices. Again, we allow for independent field transformations of ψ and $\bar{\psi}$. As before GOOFy symmetries are defined by demanding invariance of

$$\mathcal{L}_{\text{kin}} = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi$$

under (1), which in turn implies $\bar{X}_\psi = -i X_\psi$ i.e.

$$x^\mu \to i x^\mu, \qquad \Psi \to -X_\psi \gamma_0 C \Psi^\star, \qquad \bar{\Psi} \to -\Psi^T C^{-1} i X_\psi^\dagger$$

which will therefore constitute the GOOFy transformation for fermionic fields.

Yukawa interactions

We are going to consider the following Yukawa interactions:

$$-\mathcal{L}_Y = \bar{\Psi}\Gamma\Phi\Psi + \bar{\Psi}\Phi^\dagger\Gamma^\dagger\Psi \equiv \bar{\psi}_a(\Gamma_i)_{ab}\phi_i\psi_b + \bar{\psi}_a(\phi_i)^\star(\Gamma_i)^\star_{ba}\psi_b.$$

Requiring invariance of $\mathcal{L}_Y$ under GOOFy transformation

$$x^\mu \to ix^\mu, \qquad \Psi \to -X_\psi\gamma_0 C\Psi^\star, \qquad \bar{\Psi} \to -\Psi^T C^{-1} iX_\psi^\dagger,$$

$$\Phi \to X_\phi\Phi^\star, \qquad \Phi^\dagger \to -\Phi^T X_\phi^\dagger,$$

implies the following condition:

$$\Gamma_i^\star = i\, X_\psi^T \Gamma_j X_\psi^\star\, (X_\phi)_{ji}$$

These equations constitute a fixed point under the running of the RG if and only if they are order by order scale invariant, i.e.

$$\beta^{(n)}(\Gamma_i^\star) = i\, X_\psi^T \beta^{(n)}(\Gamma_j)\, X_\psi^\star\, (X_\phi)_{ji}$$

$\beta^{(n)}(\Gamma_j)$ is the n-loop beta function for the Yukawa matrix Γ_j.

GOOFy-invariant Standard Model

$$\mathrm{Tr}\, M^2 = 0$$

$$\Downarrow$$

Concluding, both the scalar potential and Yukawa couplings of the SM are inconsistent with the GOOFy symmetry.

GOOFy-invariant 2HDM

Adopting the generic strategy (described and tested earlier in various contexts), the GOOFy transformation for 2HDM shall be:

$$\Phi \equiv \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

$$\begin{aligned}
\Phi &\to X_\phi \Phi^\star, & \Phi^\dagger &\to -\Phi^T X_\phi^\dagger, \\
q_L &\to -X_\alpha \gamma^0 C q_L^\star, & \bar{q}_L &\to -i q_L^T C^{-1} X_\alpha^\dagger, \\
d_R &\to -X_\beta \gamma^0 C d_R^\star, & \bar{d}_R &\to -i d_R^T C^{-1} X_\beta^\dagger, \\
u_R &\to -X_\gamma \gamma^0 C u_R^\star, & \bar{u}_R &\to -i u_R^T C^{-1} X_\gamma^\dagger.
\end{aligned}$$

GOOFy-invariant 2HDM

The most general scalar potential for the 2HDM reads

$$\begin{aligned} V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h}.c.] \\ &\quad + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 \\ &\quad + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &\quad + \left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)]\Phi_1^\dagger\Phi_2 + \mathrm{h}.c.\right\} \end{aligned}$$

where, other than m_{12}^2 and $\lambda_{5,6,7}$, all parameters are real.

The generic Yukawa Lagrangian for this model is as follows

$$-\mathcal{L}_Y = \bar{q}_L\Gamma_a\Phi_a d_R + \bar{q}_L\Delta_a\tilde{\Phi}_a u_R + \bar{d}_R\Gamma_a^\dagger\Phi_a^\dagger q_L + \bar{u}_R\Delta_a^\dagger(\tilde{\Phi}_a)^\dagger q_L,$$

with $a = 1, 2$.

GOOFy-invariant 2HDM

$$\Phi^\dagger M^2 \Phi - \text{invariance} \quad \Rightarrow \quad X_\phi = \begin{pmatrix} 0 & e^{i\theta_1} \\ e^{i\theta_2} & 0 \end{pmatrix}$$

$$\text{basis choice} \quad \Rightarrow \quad X_\psi = \begin{pmatrix} \cos\theta_\psi & \sin\theta_\psi & 0 \\ -\sin\theta_\psi & \cos\theta_\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

for $\psi = \alpha, \beta$ and γ.

GOOFy-invariant 2HDM

Strategy for 2HDM

- $$\Gamma_1 = -iX_\alpha^T \Gamma_2^\star X_\beta^\star e^{-i\theta_2}, \qquad \Gamma_2 = -iX_\alpha^T \Gamma_1^\star X_\beta^\star e^{-i\theta_1} \quad (\star)$$
- Find the determinant (in terms of $\Delta\theta \equiv \theta_1 - \theta_2, \alpha, \beta, \gamma$) of the resulting coefficient matrix for $(\Gamma_{1,2})_{ij}$ and solve for its zeros (constraints/solutions for $\Delta\theta, \alpha, \beta, \gamma$).
- For $\Delta\theta$ obtained above check invariance of the potential and find relations between parameters that must be satisfied for the invariance. For 2HDM $\Delta\theta$'s obtained from determinants being zero allow for invariant potential.
- Adopt $\Delta\theta, \alpha, \beta, \gamma$ obtained above in the equations for Yukawa's $(*)$ to determine final form of the Yukawa matrices.

GOOFy-invariant 2HDM

The invariance of Yukawa interactions yields for Γ_a

$$\Gamma_1 = -iX_\alpha^T \Gamma_2^\star X_\beta^\star e^{-i\theta_2}, \qquad \Gamma_2 = -iX_\alpha^T \Gamma_1^\star X_\beta^\star e^{-i\theta_1}.$$

For Δ_a terms we find

$$\Delta_1 = \pm iX_\alpha^T \Delta_2^\star X_\gamma^\star e^{i\theta_2}, \qquad \Delta_2 = \pm iX_\alpha^T \Delta_1^\star X_\gamma^\star e^{i\theta_1}.$$

Then we can separate equations for e.g. Γ_1 and Δ_1

$$\Gamma_1 = e^{-i\Delta\theta}(X_\alpha X_\alpha^\star)^\dagger \Gamma_1 (X_\beta X_\beta^\star), \qquad \Delta_1 = e^{+i\Delta\theta}(X_\alpha X_\alpha^\star)^\dagger \Delta_1 (X_\gamma X_\gamma^\star).$$

GOOFy-invariant 2HDM

The determinant D is then the product of the determinants of the four blocks,

$$D = D_1 D_2 D_3 D_4$$

where

$$\begin{aligned}
D_1 &= 256 \sin^2\left(\alpha + \beta - \frac{\Delta\theta}{2}\right) \sin^2\left(\alpha - \beta + \frac{\Delta\theta}{2}\right) \\
&\quad \times \sin^2\left(\alpha - \beta - \frac{\Delta\theta}{2}\right) \sin^2\left(\alpha + \beta + \frac{\Delta\theta}{2}\right), \\
D_2 &= 16 \sin^2\left(\alpha - \frac{\Delta\theta}{2}\right) \sin^2\left(\alpha + \frac{\Delta\theta}{2}\right), \\
D_3 &= 16 \sin^2\left(\beta - \frac{\Delta\theta}{2}\right) \sin^2\left(\beta + \frac{\Delta\theta}{2}\right), \\
D_4 &= 4 \sin^2\left(\frac{\Delta\theta}{2}\right).
\end{aligned} \tag{2}$$

GOOFy-invariant 2HDM

The following solutions allow for all quark masses to be non-zero.

Solution A: $(D_1 = D_2 = D_3 = D_4 = 0), \quad \Delta\theta = \alpha = \beta = 0,$

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{11}^\star & z_{12}^\star & z_{13}^\star \\ z_{21}^\star & z_{22}^\star & z_{23}^\star \\ z_{31}^\star & z_{32}^\star & z_{33}^\star \end{pmatrix}$$

Solution D-1: $(D_2 \neq 0,\ D_3 \neq 0,\ D_1 = D_4 = 0), \quad \Delta\theta = 0,\ \beta = \alpha = \frac{\pi}{2},$

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & 0 \\ z_{21} & z_{22} & 0 \\ 0 & 0 & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{22}^\star & -z_{21}^\star & 0 \\ -z_{12}^\star & z_{11}^\star & 0 \\ 0 & 0 & z_{33}^\star \end{pmatrix}$$

Solution D-2: $(D_2 \neq 0,\ D_3 \neq 0,\ D_1 = D_4 = 0), \quad \Delta\theta = 0,\ \beta = \alpha,\ \alpha \in (0, \frac{\pi}{2}),$

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & 0 \\ -z_{12} & z_{11} & 0 \\ 0 & 0 & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{11}^\star & z_{12}^\star & 0 \\ -z_{12}^\star & z_{11}^\star & 0 \\ 0 & 0 & z_{33}^\star \end{pmatrix}$$

GOOFy-invariant 2HDM

Requiring GOOFy invariance of the 2HDM potential one finds that only four options survive that correspond to different choices of $\Delta\theta \equiv \theta_1 - \theta_2$ and different relations between potential parameters, i.e.:

$$\Delta\theta = 0: \quad m_{11}^2 + m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = \lambda_6,$$

$$\Delta\theta = \pi: \quad m_{11}^2 + m_{22}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = -\lambda_6,$$

$$\Delta\theta = \frac{2\pi}{3}: \quad m_{11}^2 + m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0,$$

$$\Delta\theta = \frac{4\pi}{3}: \quad m_{11}^2 + m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0.$$

GOOFy-invariant 2HDM

Relation set 1 ($\Delta\theta = 0$):

$$m_{11}^2 + m_{22}^2 = 0,\ m_{12}^2 = 0,\ \lambda_1 = \lambda_2,\ \lambda_7 = \lambda_6,$$
$$\Gamma_1 = -iX_\alpha^T \Gamma_2^\star X_\beta^\star e^{-i\theta_2},\ \Gamma_2 = -iX_\alpha^T \Gamma_1^\star X_\beta^\star e^{-i\theta_1},$$
$$\Delta_1 = \pm iX_\alpha^T \Delta_2^\star X_\gamma^\star e^{i\theta_2},\ \Delta_2 = \pm iX_\alpha^T \Delta_1^\star X_\gamma^\star e^{i\theta_1}.$$

- Using PyR@TE 3, we find RG-stability up to two-loop order for *Relation set 1* for Solutions A, D-1 and D-2 that allow for all quark masses to be non-zero.
- All solutions, A-M, have been confirmed RG stable up to two-loop order.
- Relations between potential parameters in Relation set 1, $m_{11}^2 + m_{22}^2 = 0,\ m_{12}^2 = 0,\ \lambda_1 = \lambda_2,\ \lambda_7 = \lambda_6$ (with $\Delta\theta = 0$) differ from the "original" relations for GOOFy/r_0 symmetry (with $\Delta\theta = \pi$) of the scalar potential, $m_{11}^2 + m_{22}^2 = 0, \lambda_1 = \lambda_2, \lambda_7 = -\lambda_6$ that is part of Relation set 2 (which implies at least one massless quark).

Summary and conclusions

- A class of C-inspired GOOFy transformations has been defined and discussed.
- Bosonic version of GOOFy has been extended to fermions.
- In all cases considered, the constraints "implied" by GOOFy were RG stable at least up to 2 loops (adopting PyR@TE 3 code).
- Regardless if GOOFy is a genuine symmetry, resulting relations between parameters are RG-stable.
- Mass term in the SM is vanishing by GOOFy invariance.
- 2HDM might be consistent with GOOFy both in bosonic sector and in Yukawa interactions.

Summary and conclusions

- The constraints could be seen as emerging from the following "GOOFy/ r_0 symmetry":

$$x^\mu \to i x^\mu,$$
$$\Phi \to X_\phi \Phi^\star, \quad \Phi^\dagger \to -\Phi^T X_\phi^\dagger, \quad A_\mu \to i A_\mu$$
$$\Psi \to -X_\psi \gamma_0 C \Psi^\star, \quad \bar{\Psi} \to -\Psi^T C^{-1} i X_\psi^\dagger$$

- Application: finding RG stable relations between parameters.

Backup slides

Goofy is a cartoon character created by the Walt Disney Company.
He is a tall, anthropomorphic dog.

"Goofy" means silly, playful, ridiculous, or charmingly eccentric, often used to describe amusing behavior, or a person. It is generally a lighthearted term for foolishness.

Running 2HDM parameters and RG stable relations

The 1-loop β-functions for the quadratic couplings

$$\beta_{m_{11}^2} = 3\lambda_1 m_{11}^2 + (2\lambda_3 + \lambda_4) m_{22}^2 - 3\left(\lambda_6^* m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}(9g^2 + 3g'^2) m_{11}^2 + \beta^F_{m_{11}^2},$$

$$\beta_{m_{22}^2} = (2\lambda_3 + \lambda_4)\, m_{11}^2 + 3\lambda_2 m_{22}^2 - 3\left(\lambda_7^* m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}(9g^2 + 3g'^2) m_{22}^2 + \beta^F_{m_{22}^2},$$

$$\beta_{m_{12}^2} = -3\left(\lambda_6 m_{11}^2 + \lambda_7 m_{22}^2\right) + (\lambda_3 + 2\lambda_4)\, m_{12}^2 + 3\lambda_5 {m_{12}^2}^* - \frac{1}{4}(9g^2 + 3g'^2) m_{12}^2 + \beta^F_{m_{12}^2},$$

Running 2HDM parameters and RG stable relations

and 1-loop β functions for the quartic ones,

$$
\begin{aligned}
\beta_{\lambda_1} &= 6\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12\,|\lambda_6|^2 \\
&\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2 g'^2) - \frac{3}{2}\lambda_1(3g^2 + g'^2) + \beta^F_{\lambda_1}, \\
\beta_{\lambda_2} &= 6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12\,|\lambda_7|^2 \\
&\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2 g'^2) - \frac{3}{2}\lambda_2(3g^2 + g'^2) + \beta^F_{\lambda_2}, \\
\beta_{\lambda_3} &= (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + |\lambda_5|^2 + 2\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 8\,\mathrm{Re}\left(\lambda_6\lambda_7^*\right) \\
&\quad + \frac{3}{8}(3g^4 + g'^4 - 2g^2 g'^2) - \frac{3}{2}\lambda_3(3g^2 + g'^2) + \beta^F_{\lambda_3}, \\
\beta_{\lambda_4} &= (\lambda_1 + \lambda_2)\lambda_4 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 4\,|\lambda_5|^2 + 5\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 2\,\mathrm{Re}\left(\lambda_6\lambda_7^*\right) \\
&\quad + \frac{3}{2}g^2 g'^2 - \frac{3}{2}\lambda_4(3g^2 + g'^2) + \beta^F_{\lambda_4}, \\
\beta_{\lambda_5} &= (\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\lambda_5 + 5\left(\lambda_6^2 + \lambda_7^2\right) + 2\lambda_6\lambda_7 \\
&\quad - \frac{3}{2}\lambda_5(3g^2 + g'^2) + \beta^F_{\lambda_5}, \\
\beta_{\lambda_6} &= (6\lambda_1 + 3\lambda_3 + 4\lambda_4)\lambda_6 + (3\lambda_3 + 2\lambda_4)\lambda_7 + 5\lambda_5\lambda_6^* + \lambda_5\lambda_7^* \\
&\quad - \frac{3}{2}\lambda_6(3g^2 + g'^2) + \beta^F_{\lambda_6}, \\
\beta_{\lambda_7} &= (6\lambda_2 + 3\lambda_3 + 4\lambda_4)\lambda_7 + (3\lambda_3 + 2\lambda_4)\lambda_6 + 5\lambda_5\lambda_7^* + \lambda_5\lambda_6^* \\
&\quad - \frac{3}{2}\lambda_7(3g^2 + g'^2) + \beta^F_{\lambda_7},
\end{aligned}
$$

where the β_x^F terms contain all contributions coming from fermions.

The GOOFy symmetry: bosons

Example: $N_\phi = 2$

$$\Phi \to X_\phi \Phi^\star, \qquad \Phi^\dagger \to -\Phi^T X_\phi{}^\dagger$$

Then using the invariance condition

$$M^2 = -X_\phi^T \left(M^2\right)^T X_\phi^\star$$

one finds in the real and diagonal basis (where $m_{12}^2 = 0$)

$$X_\phi = \begin{pmatrix} 0 & e^{i\theta_1} \\ e^{i\theta_2} & 0 \end{pmatrix}$$

and the $\Phi^\dagger\Phi$ mass matrix is of the form.

$$M^2 = \begin{pmatrix} m_{11}^2 & 0 \\ 0 & -m_{11}^2 \end{pmatrix}$$

The GOOFy symmetry: bosons

- Gauge kinetic terms

$$\mathcal{L}^B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{i\mu\nu}W_i^{\mu\nu},$$

where $B^{\mu\nu} = \partial^\nu B^\mu - \partial^\mu B^\nu$ and $W_i^{\mu\nu} = \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk}W_j^\mu W_k^\nu$.
Under r_0 transformation

$$B^{\mu\nu} \to B^{\mu\nu},$$

$$W_1^{\mu\nu} \to W_1^{\mu\nu}, \quad W_2^{\mu\nu} \to -W_2^{\mu\nu}, \quad W_3^{\mu\nu} \to W_3^{\mu\nu}$$

The GOOFy symmetry: fermions with Dirac mass

In order to preserve the Dirac fermionic mass term $\mathcal{L}_{\text{D-mass}} = -\bar{\Psi} M_D \Psi$ (with $M_D^\dagger = M_D$) under the GOOFy transformation

$$x^\mu \to i x^\mu, \qquad \Psi \to -X_\psi \gamma_0 C \Psi^\star, \qquad \bar{\Psi} \to -\Psi^T C^{-1} i X_\psi^\dagger,$$

the following condition must hold

$$M_D = i X_\psi^T M_D^\star X_\psi^\star.$$

Since M_D is Hermitian, it turns out, that the above equation implies $M_D = 0$ regardless of N_ψ for any unitary X_ψ. We conclude that fermionic Dirac mass terms are not compatible with GOOFy symmetry.

Phenomenology of the GOOFy symmetry

CP-sensitive invariants in the bilinear notation

$$I_1 = \left(\vec{M} \times \vec{\Lambda}\right) \cdot \left(\Lambda \vec{M}\right)$$
$$I_2 = \left(\vec{M} \times \vec{\Lambda}\right) \cdot \left(\Lambda \vec{\Lambda}\right)$$
$$I_3 = \left[\vec{M} \times \left(\Lambda \vec{M}\right)\right] \cdot \left(\Lambda^2 \vec{M}\right)$$
$$I_4 = \left[\vec{\Lambda} \times \left(\Lambda \vec{\Lambda}\right)\right] \cdot \left(\Lambda^2 \vec{\Lambda}\right)$$

Since the r_0 symmetry implies $\vec{\Lambda} = \vec{0}$ the invariants $I_{1,2,4}$ are automatically zero. However

$$I_3 = -16\lambda_5\, m_{11}^2\, \mathrm{Im}(m_{12}^2)\, \mathrm{Re}(m_{12}^2)\, \left[(\lambda_1 - \lambda_3 - \lambda_4)^2 - \lambda_5^2\right] \neq 0$$

explicit violation of CP

Phenomenology of the GOOFy symmetry

Stationary-point equations:

$$\begin{aligned} m_{11}^2 &= \frac{1}{2}\lambda_1\left(v_2^2 - v_1^2\right), \\ \operatorname{Re} m_{12}^2 &= \frac{1}{2} v_1 v_2 \cos\xi\,(\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5), \\ \operatorname{Im} m_{12}^2 &= -\frac{1}{2} v_1 v_2 \sin\xi\,(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5). \end{aligned}$$

The neutral sector rotation matrix is then given by

$$R = \begin{pmatrix} \frac{v_2\cos\xi}{v} & \frac{v_1\cos\xi}{v} & -\sin\xi \\ -\frac{v_1}{v} & \frac{v_2}{v} & 0 \\ \frac{v_2\sin\xi}{v} & \frac{v_1\sin\xi}{v} & \cos\xi \end{pmatrix},$$

yielding masses

$$M_1^2 = \frac{1}{2} v^2 (\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5), \quad M_2^2 = \lambda_1 v^2,$$

$$M_3^2 = \frac{1}{2} v^2 (\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5), \quad M_{H\pm}^2 = \frac{1}{2}(\lambda_1 + \lambda_3)\, v^2$$

No decoupling limit!

Phenomenology of the GOOFy symmetry

Assuming that M_2 is the SM-like Higgs boson, we obtain from unitarity and boundedness-from-below constraints:

$$M_{H^\pm} \leq 711\ \text{GeV}\,, \quad M_3 \leq 712\ \text{GeV}\,, \quad M_1 \leq 711\ \text{GeV}$$

Input parameters:

$$\mathcal{P} \equiv \{M^2_{H^\pm}, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

Constraints implied by the r_0 symmetry:

$$v^2(e_1 q_2 - e_2 q_1) + e_1 e_2(M_2^2 - M_1^2) = 0, \quad v^2(e_1 q_3 - e_3 q_1) + e_1 e_3(M_3^2 - M_1^2) = 0,$$

$$v^2(e_2 q_3 - e_3 q_2) + e_2 e_3(M_3^2 - M_2^2) = 0, \quad q = \frac{1}{2v^4}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$

$$M^2_{H^\pm} = \frac{1}{2}(e_1 q_1 + e_2 q_2 + e_3 q_3) + \frac{1}{2v^2}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$