Resonance enhancement of dark matter interactions

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Outline

- $U(1)$ vector dark matter (VDM) model
- Resonance beyond the Breit-Wigner (BW) approximation
- Early kinetic decoupling of DM and coupled Boltzmann equations
- Generic conclusions on the BW approximation
- Gauge dependance of the re-summed propagator
- Self-interacting dark matter
- Numerical results confronted with Fermi-LAT data
- Summary

⋆ M. Duch, BG and A. Pilaftsis, "Pinch technique in dark matter interactions", in progress,

⋆ M. Duch, BG, "Resonance enhancement of dark matter interactions: the case for early kinetic decoupling and velocity dependent resonance width", JHEP 1709 (2017) 159, arXiv:1705.10777.

$U(1)$ VDM model

The model:

- extra $U(1)_X$ gauge symmetry (A^μ_X),
- a complex scalar field S, whose vev generates a mass for the $U(1)_X$'s vector field,
  $S = (0, \mathbf{1}, \mathbf{1}, 1)$ under $U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)_X$
- SM fields neutral under $U(1)_X$,
- to ensure stability of the new vector boson, a $\mathbb{Z}_2$ symmetry is assumed to forbid $U(1)$-kinetic mixing between $U(1)_X$ and $U(1)_Y$: ~~$B_{\mu\nu} V^{\mu\nu}$~~.
  A^μ_X and the scalar S field transform under $\mathbb{Z}_2$ as follows

$$A^\mu_X \to -A^\mu_X \ , \ S \to S^*, \text{ where } S = \phi e^{i\sigma}, \ \ \text{so} \ \ \phi \to \phi, \ \ \sigma \to -\sigma.$$

T. Hambye, JHEP 0901 (2009) 028,
O. Lebedev, H. M. Lee, and Y. Mambrini, Phys.Lett. B707 (2012) 570,
S. Baek, P. Ko, W.-I. Park, E. Senaha, JHEP 1305 (2013) 036
A. Falkowski, C. Gross and O. Lebedev, JHEP 05 (2015) 057
. . .

$U(1)$ VDM model

The scalar potential

$$V = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|S|^2|H|^2.$$

The vector bosons masses:

$$M_W = \frac{1}{2}gv, \quad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v \quad \text{and} \quad M_X = g_X v_X,$$

where

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{ and } \langle S \rangle = \frac{v_X}{\sqrt{2}}$$

Positivity of the potential implies

$$\lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2\sqrt{\lambda_H \lambda_S}$$

$U(1)$ VDM model

The scalar fields shall be expanded around corresponding vev's as follows

$$S = \frac{1}{\sqrt{2}}(v_X + \phi_S + i\sigma_S) \ , \ \ H^0 = \frac{1}{\sqrt{2}}(v + \phi_H + i\sigma_H) \ \text{where} \ \ H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}.$$

The mass squared matrix $\mathcal{M}^2$ for the fluctuations (ϕ_H, ϕ_S) and their eigenvalues

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_X \\ \kappa v v_X & 2\lambda_S v_X^2 \end{pmatrix}$$

$$m_\pm^2 = \lambda_H v^2 + \lambda_S v_X^2 \pm \sqrt{\lambda_S^2 v_X^4 - 2\lambda_H \lambda_S v^2 v_X^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_X^4}$$

$$\mathcal{M}^2_{\text{diag}} = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}, \quad R = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R^{-1} \begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix}$$

where $m_1 = 125.7$ GeV is the mass of the observed Higgs particle.

$U(1)$ VDM model

There are 5 real parameters in the potential: μ_H, μ_S, λ_H, λ_S and κ. Adopting the minimization conditions μ_H, μ_S could be replaced by v and v_X. The SM vev is fixed at $v = 246.22$ GeV. Using the condition $m_1 = 125.7$ GeV, v_X^2 could be eliminated in terms of $v^2, \lambda_H, \kappa, \lambda_S, \lambda_{SM} \equiv m_1^2/(2v^2)$:

$$v_X^2 = v^2 \frac{4\lambda_{SM}(\lambda_H - \lambda_{SM})}{4\lambda_S(\lambda_H - \lambda_{SM}) - \kappa^2}$$

Eventually there are 4 independent parameters:

$$(\lambda_H, \kappa, \lambda_S, g_X),$$

where g_X is the $U(1)_X$ coupling constant. Another choice:

$$(M_X, m_2, \sin\alpha, g_X),$$

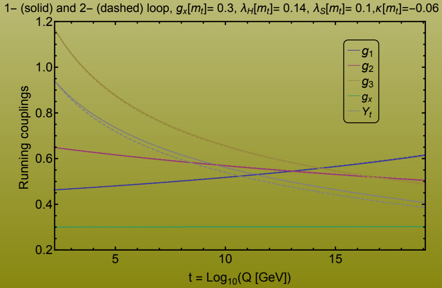
$U(1)$ VDM model

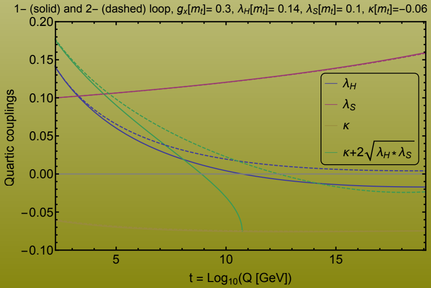
Vacuum stability

$$V = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|S|^2|H|^2$$

2-loop running of parameters adopted

$$\lambda_H(Q) > 0, \quad \lambda_S(Q) > 0, \quad \kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0$$





$U(1)$ VDM model

The mass of the Higgs boson is known experimentally therefore within *the SM* the initial condition for running of $\lambda_H(Q)$ is fixed

$$\lambda_H(m_t) = m_1^2/(2v^2) = \lambda_{SM} = 0.13$$

For VDM this is not necessarily the case:

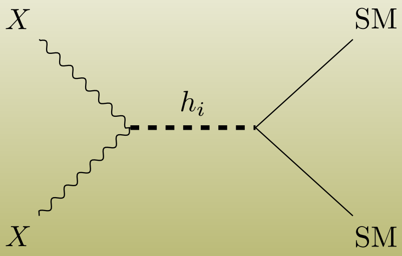
$$m_1^2 = \lambda_H v^2 + \lambda_S v_X^2 - \sqrt{\lambda_S^2 v_X^4 - 2\lambda_H \lambda_S v^2 v_X^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_X^2}.$$

VDM:

- Larger initial values of λ_H such that $\lambda_H(m_t) > \lambda_{SM}$ are allowed delaying the instability (by shifting up the scale at which $\lambda_H(Q) < 0$).
- Even if the initial λ_H is smaller than its SM value, $\lambda_H(m_t) < \lambda_{SM}$, still there is a chance to lift the instability scale if appropriate initial value of the portal coupling $\kappa(m_t)$ is chosen.

$$\beta_{\lambda_H}^{(1)} = \beta_{\lambda_H}^{SM\ (1)} + \kappa^2$$

Resonance beyond the B-W

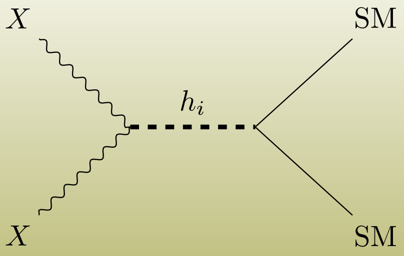


Breit-Wigner resonance ($s \approx M^2$) annihilation.

$$\sigma v \simeq \sum_{f \neq i} \frac{64\pi\omega}{s\beta_i} \frac{M^2\Gamma_i\Gamma_f}{(s-M^2)^2+\Gamma^2 M^2} \simeq \sum_{f \neq i} \frac{64\pi\omega}{M^2} \frac{\eta_i\eta_f\beta_f}{(\delta+v^2/4)^2+\gamma^2}$$

$$\eta_{i,f} \equiv \frac{\Gamma_{i,f}}{M\beta_{i,f}},\ \delta \equiv \frac{4m^2}{M^2}-1,\ \gamma \equiv \frac{\Gamma}{M} \text{ and } \omega = \frac{(2J+1)}{(2S+1)^2}$$

Resonance beyond the B-W



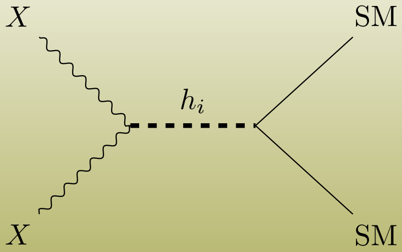
Breit-Wigner resonance ($s \approx M^2$) annihilation.

$$\sigma v = \sum_{f \neq i} \frac{64\pi\omega}{M^2} \frac{\eta_i \eta_f \beta_f}{(\delta + v^2/4)^2 + \gamma^2}$$

$$\langle \sigma v \rangle (x) = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 e^{-xv^2/4} \sigma v, \qquad x \equiv \frac{m}{T}$$

P. Gondolo and G. Gelmini, Nucl. Phys. B 360, 145 (1991),
K. Griest and D. Seckel, Phys. Rev. D 43, 3191 (1991),
M. Ibe, H. Murayama and T. Yanagida, Phys. Rev. D 79, 095009 (2009)

Resonance beyond the B-W



Is the BW approximation applicable ?

$$\sigma \propto \frac{1}{|(s-M^2)+i\Gamma M|^2}$$

$v \ll 1$ and $2m \approx M \implies s \approx 4m^2 + m^2v^2 \approx M^2$

Resonance beyond the B-W

$$\frac{M^2\Gamma_i\Gamma_f}{|s-M^2|^2} \xrightarrow{\text{naïve B-W}} \frac{M^2\Gamma_i\Gamma_f}{|s-M^2+i\Gamma M|^2}$$

$$\frac{M^2\Gamma_i\Gamma_f}{|s-M^2|^2} \xrightarrow{\text{Dyson re-summation of } \Pi(s)} \frac{M^2\Gamma_i\Gamma_f}{|s-M^2+i\Im\Pi(s)|^2}$$

$$\Im\Pi(s) = \frac{1}{2}\sum_f \int d\Phi_f |\mathcal{M}(R\to f)|^2 (2\pi)^4 \delta^{(4)}(k_R - \sum q_f)$$

$$\Downarrow$$

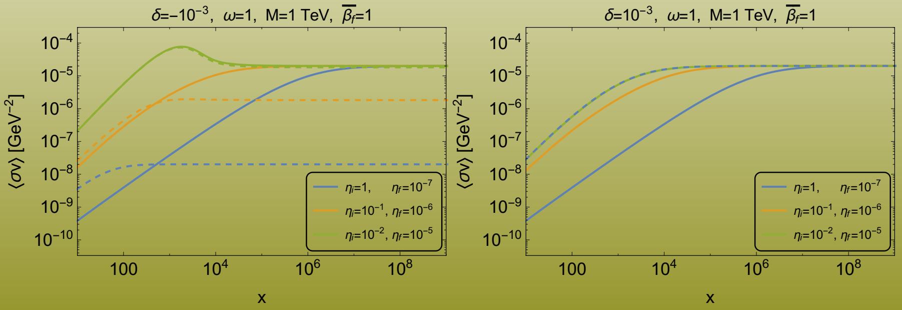
$$\sigma v \propto \frac{\gamma_i\gamma_f}{(\delta+v^2/4)^2+[\gamma_{\rm SM}+\gamma_{\rm DM}(v)]^2} \overset{\gamma_{\rm SM}\ll\gamma_{\rm DM}}{\approx} \frac{\gamma_i\gamma_f}{(\delta+v^2/4)^2+\eta_i^2 v^2/4}$$

where $\eta_i \equiv \frac{\Gamma_i}{M\beta_i}$, $\beta_i \equiv \left(1-\frac{4m^2}{M^2}\right)^{1/2}$ and $\gamma_{i,f} = \frac{\Gamma_{i,f}}{M}$

Resonance beyond the B-W

$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x)(Y^2 - Y_{EQ}^2), \qquad Y \equiv \frac{n_{DM}}{s}, \qquad x \equiv \frac{m}{T}$$

$$R(x) = \frac{\langle \sigma v \rangle (x)}{\langle \sigma v \rangle_0} = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 e^{-xv^2/4} \frac{\delta^2}{(\delta + v^2/4)^2 + \eta_i^2 v^2/4}$$



Thermally averaged annihilation cross-section $\langle \sigma v \rangle (x)$ for negative (left panel) and positive (right panel) value of δ. The solid lines were obtained using the resonance propagator with energy-dependent width and dashed lines refer to constant width approximation. In the right panel all dashed lines coincide.

Resonance beyond the B-W

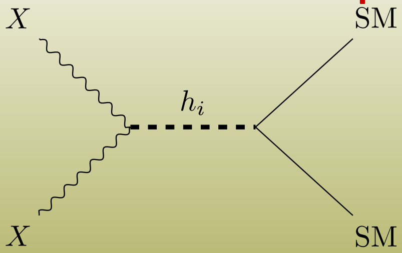
$$\frac{dY}{dx} = -\frac{\lambda_0}{x^2} R(x)(Y^2 - Y_{EQ}^2) \qquad \text{with} \qquad R(x) \propto \langle \sigma v \rangle (x)$$

At low x, $\langle \sigma v \rangle (x)$ for the velocity dependent width is smaller than for the naive constant width $\Gamma(M^2)$.

$\Downarrow$

Velocity dependent width implies higher asymptotic DM yield.

Early kinetic decoupling of DM and coupled Boltzmann equations



Breit-Wigner resonance ($s \approx M^2$) *annihilation.*

$$\Downarrow$$

$g_{h_i XX} \cdot g_{h_i SM} \ll 1$ (to get $\Omega_{DM} \sim 0.1$) and tiny $\sigma(XSM \to XSM)$

$$\Downarrow$$

- Possibility of DM early kinetic decoupling at $T_{kd} \gg T_{kd}^{\mathrm{WIMP}} \sim$ MeV,
- Suppressed cross-sections for direct detection.

Early kinetic decoupling of DM and coupled Boltzmann equations

- If dark matter decouples kinetically, when it is non-relativistic and its thermal distribution is maintained by self-scatterings, then the DM temperature T_{DM} evolves according to $T_{DM} \propto a^{-2}$,
- The temperature of the radiation-dominated SM thermal bath, scales as $T \propto a^{-1}$.

$$T_{DM} = \begin{cases} T, & \text{if } T \geq T_{kd} \\ T^2/T_{kd}, & \text{if } T < T_{kd}, \end{cases}$$

where T stands for the SM temperature.

Early kinetic decoupling of DM and coupled Boltzmann equations

Define DM "temperature":

$$T_{DM} \equiv \frac{2}{3}\left\langle \frac{\vec{p}^{\,2}}{2m} \right\rangle \qquad \text{for} \qquad \langle \mathcal{O}(\vec{p}) \rangle \equiv \frac{1}{n_{DM}} \int \frac{d^3p}{(2\pi)^3} \mathcal{O}(\vec{p}) f(\vec{p})$$

The Boltzmann equation:

$$\hat{L}[f] = C[f]$$

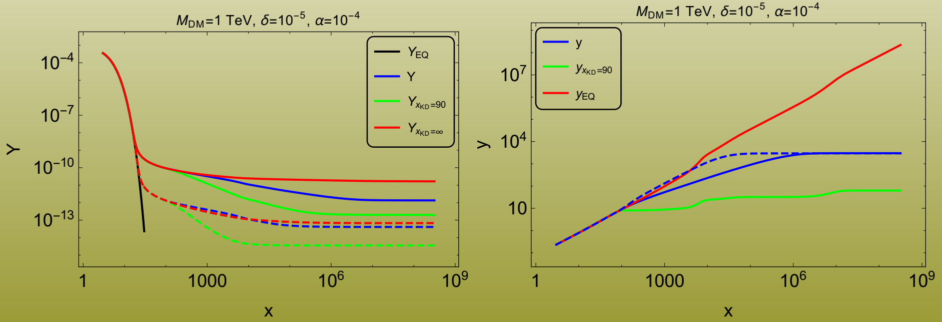
The second moment of the Boltzmann equation:

$$\int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^{\,2}}{p^0} \hat{L}[f] = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^{\,2}}{p^0} C[f]$$

T. Bringmann and S. Hofmann, "Thermal decoupling of WIMPs from first principles," JCAP **0704**, 016 (2007), Erratum: [JCAP **1603**, no. 03, E02 (2016)]

Early kinetic decoupling of DM and coupled Boltzmann equations

$$y \equiv \frac{mT_{DM}}{s^{2/3}} \qquad \text{and} \qquad y_{EQ} \equiv \frac{mT}{s^{2/3}}$$



Dark matter yield Y (left panel) and corresponding DM temperatures (right panel) in different kinetic decoupling scenarios. The blue curves show the solution of the set of BE, whereas the green ones refer to the "sharp splitting" at $x_{kd} = 90$. For the red curves dark matter remains in the kinetic equilibrium during its whole evolution. Dashed curves present the corresponding results for the standard Breit-Wigner approximation (with $\gamma \ll \delta$).

Generic conclusions on the BW approximation

Remarks:

- The presence of velocity-dependent width implies that Y decouples at lower x (as compared to the case with constant width $\Gamma(M^2)$) and the asymptotic DM yield is much larger.
- The asymptotic yield expected in the early decoupling scenario is substantially reduced by more efficient annihilation, $R(x_{DM}) \sim \frac{x}{x_{kd}} R(x) \gg R(x)$.
- Both effects cancel to same extend, so that the increase by the velocity depended width is reduced by $\sim 50\%$.

Gauge dependance of the re-summed propagator

$$\sigma v \propto \frac{M^2 \Gamma_i \Gamma_f}{|s - M^2 + i\Im\Pi(s)|^2}$$ with gauge-dependent $\Im\Pi(s)$

$$\Im\Pi(s) = \frac{1}{2}\sum_f \int d\Phi_f |\mathcal{M}(R \to f)|^2 (2\pi)^4 \delta^{(4)}(k_R - \sum q_f)$$

with $f = XX, XG_X, G_X X, G_X G_X$ in R_ξ gauge that contribute to the $h_i \to XX$ decays.

Gauge dependance of the re-summed propagator

$$\sigma v \propto \frac{M^2 \Gamma_i \Gamma_f}{|s - M^2 + i\Im\Pi(s)|^2} \quad \text{with gauge-dependent } \Im\Pi(s)$$

The pinch technique

- J. Papavassiliou and A. Pilaftsis, Gauge- and renormalization-group-invariant formulation of the Higgs-boson resonance, PR D 58 (1998) 053002,
- J. Papavassiliou, The Pinch technique approach to the physics of unstable particles, PoS Corfu'98 (1998) 076,
- D. Binosi and J. Papavassiliou, Pinch technique: theory and applications, Physics Reports 479 (2009).

allows to calculate resonance amplitudes that are gauge independent, universal and satisfy unitarity.

Gauge dependance of the re-summed propagator

$h_i - h_j$ self-energy:

$$\Pi_{ij}(s) = \frac{g_x^2 R_{2i} R_{2j}}{32\pi^2 M_X^2} \Big[\left(s^2 - 4M_X^2 s + 12 M_X^4 \right) B_0(s, M_X^2, M_X^2) + \\ -(s^2 - m_i^2 m_j^2) B_0(s, \xi_X M_X^2, \xi_X M_X^2) \Big]$$

⇓ "pinching" ⇓

$$\hat{\Pi}_{ij}(s) = \frac{g_x^2 R_{2i} R_{2j}}{8\pi^2} \left[\frac{(m_i m_j)^2}{4M_X^2} + \frac{m_i^2 + m_j^2}{2} - (2s - 3M_X^2) \right] B_0(s, M_X^2, M_X^2)$$

Gauge dependance of the re-summed propagator

The re-summed propagator:

$$\Lambda = P + P\hat{\Pi}P + P(\hat{\Pi}P)^2 + \ldots,$$

where P is a propagator matrix

$$P = \begin{pmatrix} \frac{1}{s-m_1^2} & 0 \\ 0 & \frac{1}{s-m_2^2} \end{pmatrix}$$

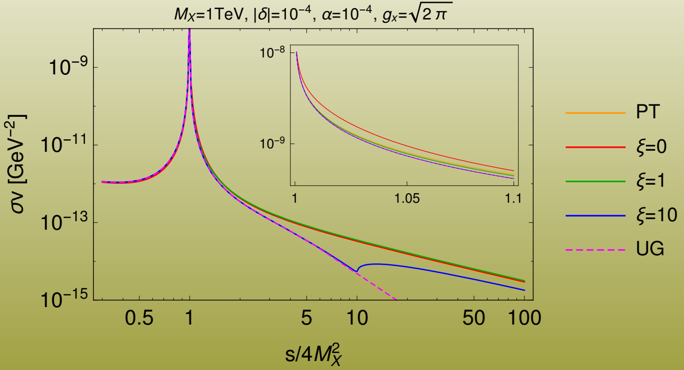
and $\hat{\Pi}_{ij}(s)$ is the pinched self-energy.
Finally the re-summed propagator

$$\Lambda = \frac{1}{D} \begin{pmatrix} s - m_2^2 + \hat{\Pi}_{22} & -\hat{\Pi}_{12} \\ -\hat{\Pi}_{21} & s - m_1^2 + \hat{\Pi}_{11} \end{pmatrix}$$

where D is the determinant of the propagator matrix

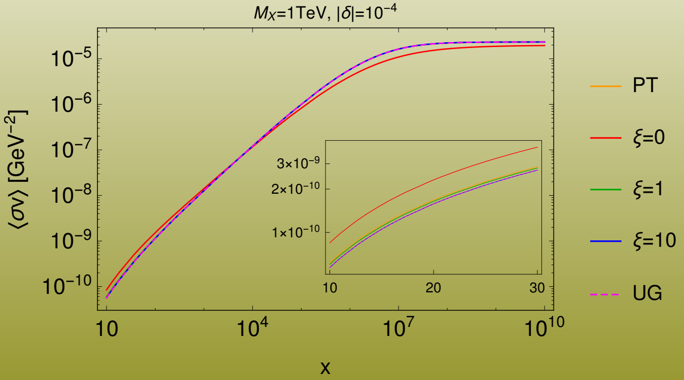
$$D = (s - m_1^2 + \hat{\Pi}_{11})(s - m_2^2 + \hat{\Pi}_{22}) - \hat{\Pi}_{12}\hat{\Pi}_{21}$$

Gauge dependance of the re-summed propagator



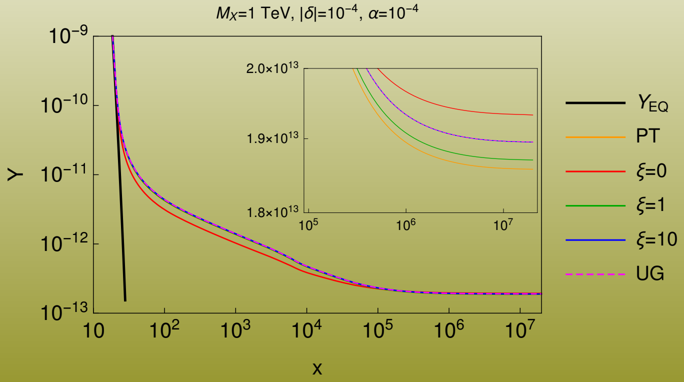
Here we illustrate consequences of gauge dependence of the resonance propagator for $XX \to W^+W^-$ cross-section as a function of $s/(4M_X^2)$. The vicinity of $v_0 = 10$ km/s and 1 km/s is magnified. Results shown correspond to selected values of ξ specified in the legend, "PT" denotes results obtained adopting the pinch technique, the unitary gauge ($\xi \to \infty$) is denoted as "UG".

Gauge dependance of the re-summed propagator



The plot shows the thermal averaged annihilation cross-section for $XX \to SMSM$ *as a function of* $x \equiv M_X/T$.

Gauge dependance of the re-summed propagator



We plot numerical solution of the Boltzmann equations for the dark matter yield $Y(x) \equiv n_{DM}/s$.

Self-interacting dark matter

Small scale WIMP problems:

- ~~Core/cusp problem~~
- Missing satellites
- "Too big to fail"

Solution (Spergel and Steinhardt, 2000):

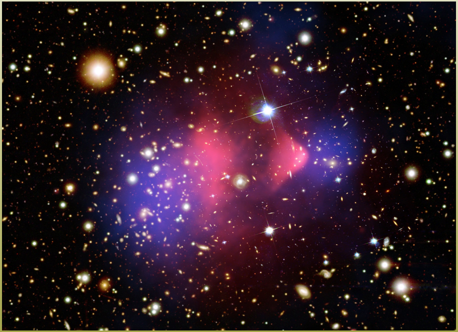
$$\frac{\sigma_{\rm self}}{m_{DM}} \gtrsim 0.1\ \frac{\rm cm^2}{\rm g} \qquad \left(\sim 0.1\ \frac{\rm barn}{\rm GeV} \gg \frac{\rm pb}{\rm GeV}\right)$$

Self-interacting dark matter

Upper bounds on self-interaction cross-section

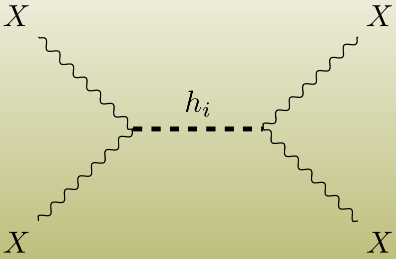
Bullet cluster:

$$\frac{\sigma_{\rm self}}{m_{DM}} \lesssim 1.0\ \frac{\rm cm^2}{\rm g}$$



$$\frac{\sigma_{\rm self}}{m_{DM}} \sim 1.0\ \frac{\rm cm^2}{\rm g} \sim 1\,\frac{\rm barn}{\rm GeV}$$

Self-interacting dark matter



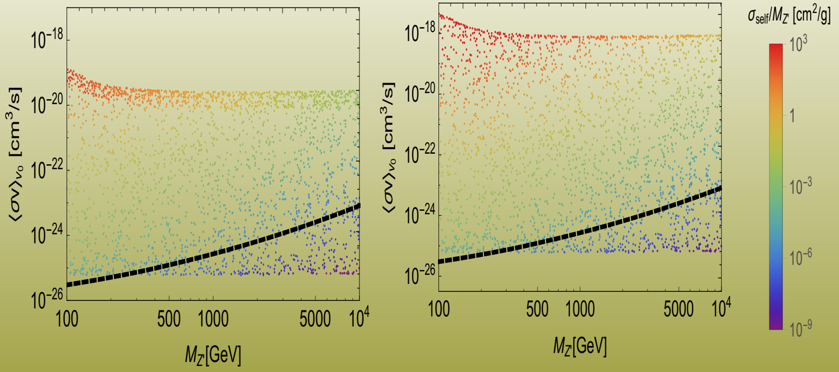
Breit-Wigner resonance ($s \approx M^2$) DM self-interaction.

$$\sigma_{\rm self} \simeq \frac{32\pi\omega}{s\beta_i^2}\frac{M^2\Gamma_i^2}{(s-M^2)^2+\Gamma^2M^2},$$

$$\frac{\sigma_{\rm self}}{m} \simeq \frac{8\pi\omega}{m^3}\frac{\eta_i^2}{(\delta+v^2/4)^2+\gamma^2}$$

$$\eta_i \equiv \frac{\Gamma_i}{M\beta_i},\ \delta \equiv \frac{4m^2}{M^2}-1,\ \gamma \equiv \frac{\Gamma}{M} \text{ and } \omega = \frac{(2J+1)}{(2S+1)^2}$$

Numerical results confronted with Fermi-LAT data



Result of the scan in the parameter space over M_X, δ and $\sin\alpha$. For each point in the plot we fit α to satisfy the relic abundance constraint and then calculate the annihilation $\langle\sigma v\rangle_{v_0}$ and self-interaction $\sigma_{\rm self}/M_X$ cross-section at the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta_i = 3/16$, was chosen.

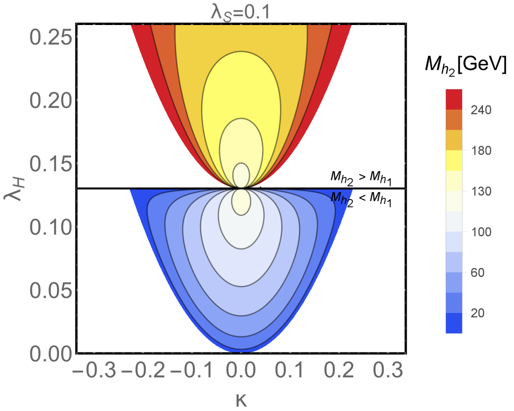
Summary

- The $U(1)$ vector dark matter (VDM) was discussed (extra neutral Higgs boson h_2).
- Breit-Wigner approximation was modified by adopting s-dependent width ($\sim \Im\Pi(s)$), effects are large.
- Correct DM abundance implies early kinetic decoupling of DM with important numerical consequences. Similar effects were discussed in T. Binder, T. Bringmann, M. Gustafsson, A. Hryczuk, "Early kinetic decoupling of dark matter: when the standard way of calculating the thermal relic density fails ", PR D96 (2017) no.11, 115010,
- The pinch technique was adopted to obtain unitarized and gauge-independent Breit-Wigner-like propagator.
- When the Fermi-LAT limits are taken into account, heavy ~ 1 TeV DM is favored and only very limited enhancement of $\sigma_{\rm self}/m \ll 1\ \text{cm}^2/g$ is possible.

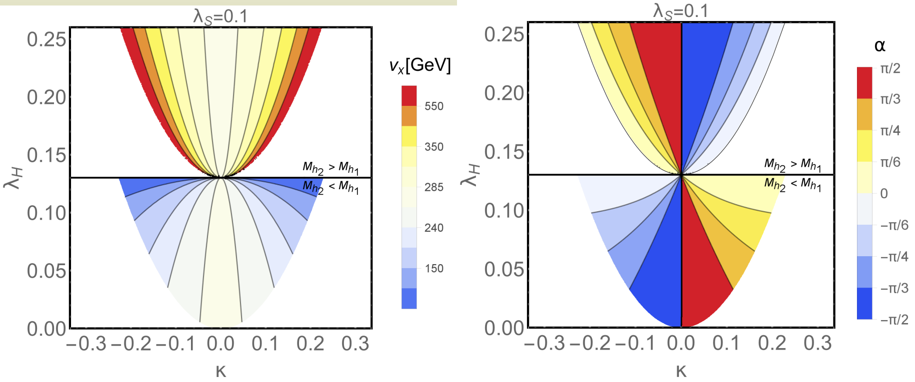
Backup slides

$U(1)$ VDM model

- Bottom part of the plot ($\lambda_H < \lambda_{SM} = m_1^2/(2v^2) = 0.13$): the heavier Higgs is the currently observed one.
- Upper part ($\lambda_H > \lambda_{SM}$) the lighter state is the observed one.
- White regions in the upper and lower parts are disallowed by the positivity conditions for v_X^2 and m_2^2, respectively.

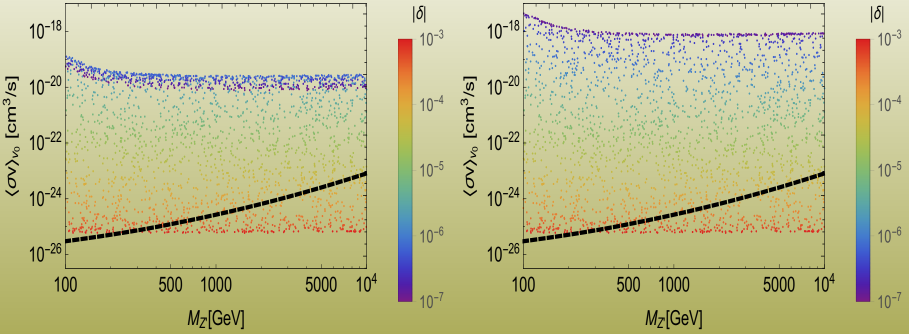


$U(1)$ VDM model



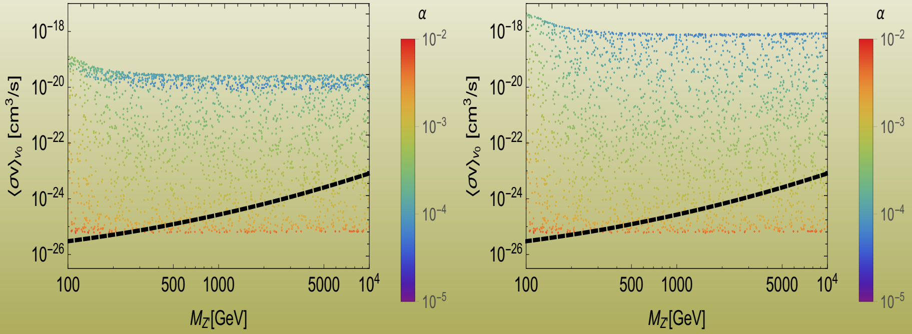
Contour plots for the vacuum expectation value of the extra scalar $v_X \equiv \sqrt{2}\langle S\rangle$ *(left panel) and of the mixing angle* α *(right panel) in the plane* (λ_H, κ).

Numerical results



Result of the scan in the parameter space over M_X, δ and $\sin\alpha$. Colouring with respect to δ the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta = 3/16$, was chosen.

Numerical results



Result of the scan in the parameter space over M_X, δ and $\sin\alpha$. Colouring with respect to α the dispersive velocity v_0 equal to 10 km/s (left panel) and 1 km/s (right panel). The maximal value of η in the VDM model, $\eta = 3/16$, was chosen.

Early kinetic decoupling of DM and coupled Boltzmann equations

$$\frac{dY}{dx} = -\frac{sY^2}{Hx}\left[\langle\sigma v\rangle_{x=m^2/(s^{2/3}y)} - \frac{Y_{EQ}^2}{Y^2}\langle\sigma v\rangle_x\right]$$

$$\frac{dy}{dx} = -\frac{1}{Hx}\Big\{2mc(T)(y-y_{EQ}) + -syY\left[(\langle\sigma v\rangle - \langle\sigma v\rangle_2)_{x=m^2/(s^{2/3}y)} - \frac{Y_{EQ}^2}{Y^2}(\langle\sigma v\rangle - \langle\sigma v\rangle_2)_x\right]\Big\}$$

where the temperature parameter y is defined as

$$y \equiv \frac{mT_{DM}}{s^{2/3}}, \quad \text{for sharp splitting: } y \propto \begin{cases} x, & \text{if } T \geq T_{kd} \\ \frac{m}{T_{kd}} \sim \text{const.}, & \text{if } T < T_{kd}, \end{cases}$$

T. Bringmann and S. Hofmann, "Thermal decoupling of WIMPs from first principles," JCAP **0704**, 016 (2007), Erratum: [JCAP **1603**, no. 03, E02 (2016)]

Early kinetic decoupling of DM and coupled Boltzmann equations

$$\frac{dY}{dx} = -\frac{s}{Hx}\left[Y^2\langle\sigma v\rangle_{x=m^2/(s^{2/3}y)} - Y_{EQ}^2\langle\sigma v\rangle_x\right]$$

$$\frac{dy}{dx} = -\frac{1}{Hx}\Big\{2mc(T)(y-y_{EQ}) +$$

$$-syY\left[\left(\langle\sigma v\rangle - \langle\sigma v\rangle_2\right)_{x=m^2/(s^{2/3}y)} - \frac{Y_{EQ}^2}{Y^2}\left(\langle\sigma v\rangle - \langle\sigma v\rangle_2\right)_x\right]\Big\}$$

where the temperature parameter y is defined as

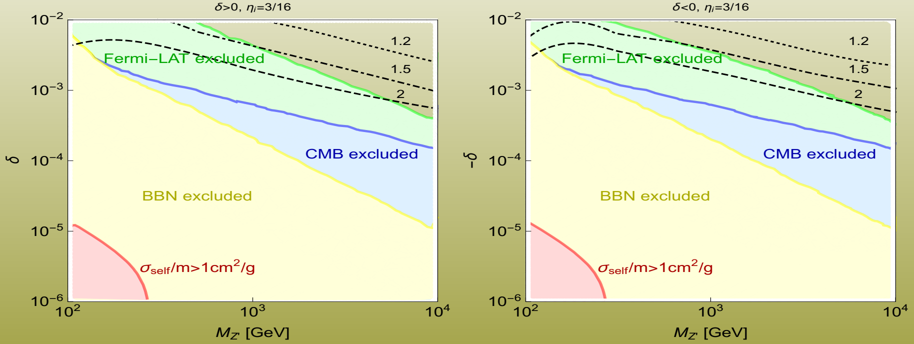
$$y \equiv \frac{mT_{DM}}{s^{2/3}} \qquad \text{and} \qquad y_{EQ} \equiv \frac{mT}{s^{2/3}}$$

the scattering rate $c(T)$ as

$$c(T) = \frac{1}{12(2\pi)^3)m^4T}\sum_f \int dk k^5 \omega^{-1} g|\mathcal{M}|^2_{t=0;s=m^2+2m\omega+M_{SM}^2}$$

$$\langle\sigma v\rangle_2 = \frac{x^{3/2}}{2\sqrt{\pi}}\int_0^\infty dv\sigma v\left(1+\frac{1}{6}v^2x\right)v^2\exp^{-v^2x/4}$$

Numerical results confronted with Fermi-LAT data



Regions in the (δ, M_X) parameter space constrained by Fermi-LAT, CMB and BBN. The self-interaction cross-section needed for the small scale problems is also shown. Below black dotted, dash-dotted or dashed lines relic density without considering kinetic decoupling is larger by factor 1.2, 1.5 or 2 respectively.