Natural extensions of the Standard Model

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- The little hierarchy problem
- The strategy
- Models
- Summary

- B.G., P. Osland, "A Natural Two-Higgs-Doublet Model", e-Print: arXiv:0910.4068.
- B.G., O.M. Ogreid, P. Osland, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys.Rev.D80:055013,2009.
- B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", Phys.Rev.Lett.103:091802,2009.

The little hierarchy problem

- $$m_h^2 = m_h^{(B)\,2} + \delta^{(SM)} m_h^2 + \cdots$$

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2}\left[\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2\right]$$

$$m_h = 130 \text{ GeV} \quad \Rightarrow \quad \delta^{(SM)} m_h^2 \simeq m_h^2 \qquad \text{for} \qquad \Lambda \simeq 600 \text{ GeV}$$

• For $\Lambda \gtrsim 600$ GeV there must be a cancellation between the tree-level Higgs mass^2 $m_h^{(B)\,2}$ and the 1-loop leading correction $\delta^{(SM)} m_h^2$:

$$m_h^{(B)\,2} \sim \delta^{(SM)} m_h^2 > m_h^2$$

$$\Downarrow$$

the perturbative expansion is breaking down.

- The SM cutoff is very low!

Solutions to the little hierarchy problem:

♠ Suppression of corrections growing with Λ^2 at the 1-loop level:

- The Veltman condition, no Λ^2 terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0 \qquad \Longrightarrow \qquad m_h \simeq 310 \text{ GeV}$$

- SUSY:

$$\delta^{(SUSY)} m_h^2 \sim m_{\tilde{t}}^2 \, \frac{3g_t^2}{8\pi^2} \ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right)$$

then for $\Lambda \sim 10^{16-18}$ GeV one gets $m_{\tilde{t}}^2 \lesssim 1$ TeV^2 in order to have $\delta^{(SUSY)} m_h^2 \sim m_h^2$.

♠ Increase of the allowed value of m_h:

- The inert Higgs model by Barbieri, Hall, Rychkov, Phys.Rev.D74:015007,2006, (Ma, Phys.Rev.D73:077301,2006) $\Rightarrow$ $m_h \sim 400 - 600$ GeV, ($\ln m_h$ terms in T parameter canceled by $m_{H^\pm}, m_A, m_S$ contributions).

The Strategy

- The SM 1-loop quadratic divergences are dominated by the top (fermionic) contribution, so to suppress them we are going to introduce extra scalars (as the SM Higgs would need to be too heavy to do the job).

- We will look for a model which allows for relatively heavy lightest Higgs boson (in order to suppress $\delta M_i^2/M_i^2$ even more). Note also that within the SM fit to the precision data there is a tension caused by the lightness of the Higgs.

- DM candidate is mandatory.

- CPV will be desirable.

Natural Models

♠ Less divergence + DM ⇒ SM + N_φ scalar singlets

B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", Phys.Rev.Lett.103:091802,2009.

- N_φ extra gauge singlets φ_i with $\langle \varphi_i \rangle = 0$,

- Symmetries $\mathbb{Z}_2^{(i)}$: $\varphi_i \to -\varphi_i$ (to eliminate $|H|^2\varphi_i$ couplings).

$$V(H,\varphi_i) = -\mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_\varphi^2 \sum_{i=1}^{N_\varphi} \varphi_i^2 + \frac{\lambda_\varphi}{24} \sum_{i=1}^{N_\varphi} \varphi_i^4 + \lambda_x |H|^2 \sum_{i=1}^{N_\varphi} \varphi_i^2$$

with $O(N_\varphi)$ symmetry imposed

$$\langle H \rangle = \frac{v}{\sqrt{2}}, \qquad \langle \varphi_i \rangle = 0 \qquad \text{for} \qquad \mu_\varphi^2 > 0$$

then $m_h^2 = 2\mu_H^2$ and $m^2 = 2\mu_\varphi^2 + \lambda_x v^2$

$$\delta^{(\varphi)} m_h^2 = -N_\varphi \frac{\lambda_x}{8\pi^2}\left[\Lambda^2 - m^2 \ln\left(1 + \frac{\Lambda^2}{m^2}\right)\right]$$

$$|\delta m_h^2| = |\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2| = D_t m_h^2$$

$$\Downarrow$$

$$\lambda_x = \lambda_x(m, m_h, D_t, \Lambda, N_\varphi)$$

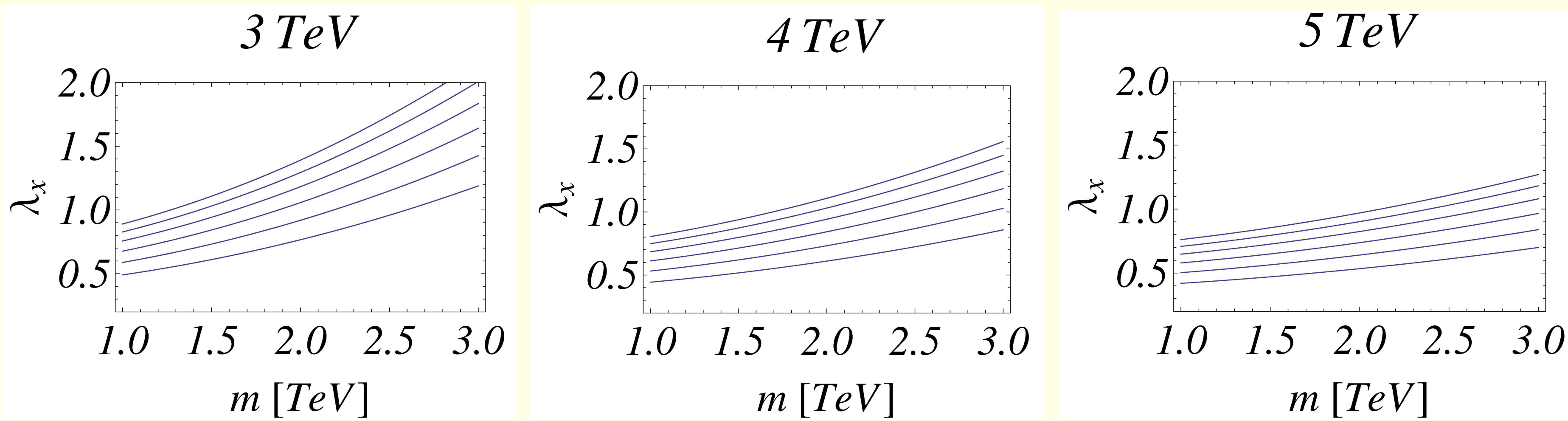

Figure 1: Plots of λ_x as a function of m for $N_\varphi = 6$, $D_t = 0$ and various choices of $\Lambda = 3, 4, 5$ shown above each panel. The curves correspond to $m_h = 130,\ 150,\ 170,\ 190,\ 210,\ 230$ GeV (starting with the uppermost curve).

Stability of the fine tunning

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(12 g_t^2 - \frac{9}{2} g_2^2 - \frac{3}{2} g_1^2 - 12\lambda_H\right)$$
$$\delta^{(\varphi)} m_h^2 = -N_\varphi \frac{\lambda_x}{8\pi^2}\left[\Lambda^2 - m^2 \ln\left(1 + \frac{\Lambda^2}{m^2}\right)\right]$$

In general

$$\delta m_h^2 = \underbrace{\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2}_{\simeq 0} + 2\Lambda^2 \sum_{n=1}^{\infty} f_n(\lambda, \dots) \ln^n\left(\frac{\Lambda}{\mu}\right)$$

where (Jack & Jones,1990)

$$(n+1) f_{n+1} = \mu \frac{\partial}{\partial \mu} f_n = \beta_i \frac{\partial}{\partial \lambda_i} f_n \qquad \text{and} \qquad f_n \propto \left(\frac{N_\varphi \lambda_x}{16\pi^2}\right)^{n+1}$$

From 1-loop condition ($n = 0$)

$$\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2 \simeq 0$$

we have

$$\lambda_x = \frac{1}{N_\varphi} \left\{ 4.8 - 3 \left(\frac{m_h}{v} \right)^2 + 2 D_t \left[\frac{2\pi}{\Lambda / \text{ TeV}} \right]^2 \right\} \left[1 - \frac{m^2}{\Lambda^2} \ln \left(\frac{m^2}{\Lambda^2} \right) \right] + \mathcal{O}\left(\frac{m^4}{\Lambda^4} \right) .$$

Therefore at the 2-loop ($n = 1$)

$$D_t \equiv \frac{\delta m_h^2}{m_h^2} \simeq \left(\frac{N_\varphi \lambda_x}{16\pi^2} \right)^2 \frac{\Lambda^2}{m_h^2} \ln \left(\frac{\Lambda}{m_h} \right) \simeq \left(\frac{4}{16\pi^2} \right)^2 \frac{\Lambda^2}{m_h^2} \ln \left(\frac{\Lambda}{m_h} \right)$$

for $D_t \lesssim 1$

$$\Lambda \lesssim 3 - 5 \text{ TeV} \qquad \text{for} \qquad m_h = 130 - 230 \text{ GeV}$$

Singlet DM

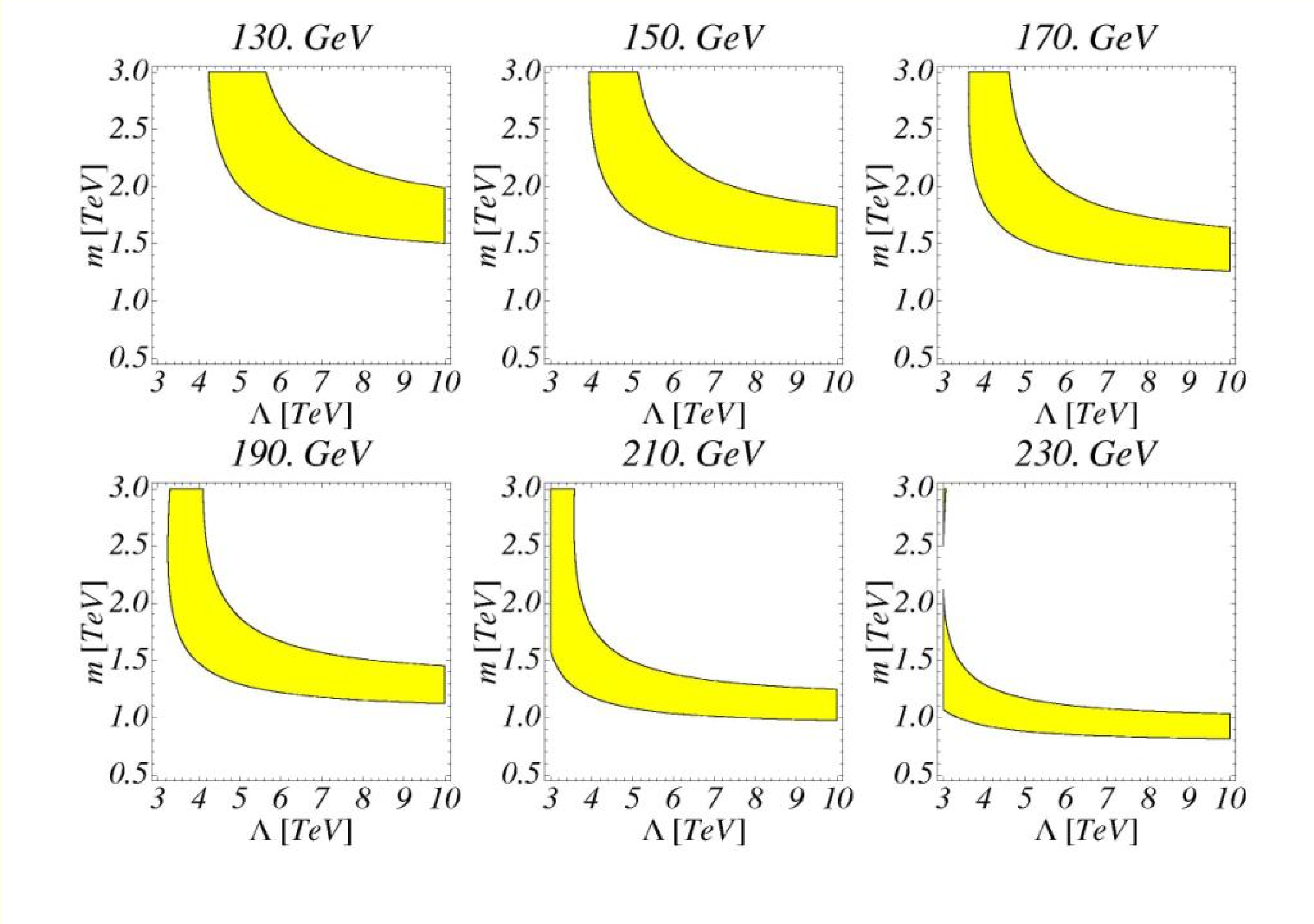

Figure 2: Allowed regions in the space (m, Λ) are plotted for $D_t(m) = 0$, $N_\varphi = 6$ and assuming that each φ_i contributes the same to the total Ω_{DM} at the 3σ level: $\Omega_\varphi h^2 = 0.106 \pm 0.008$ for the Higgs mass shown above each panel; $m_h = 130, 150, 170, 190, 210, 230$ GeV.

♠ Less divergence + DM + CPV $\Rightarrow \begin{cases} \text{2HDM (CPV) + Inert singlet (DM)} \\ \text{2HDM (CPV) + Inert doublet (DM)} \end{cases}$

- The Inert Doublet Model with no quadratic divergences

$$\mathbb{Z}_2 : \qquad \phi_2 \to -\phi_2$$

$$\begin{aligned} V(\phi_1, \phi_2) &= -\frac{1}{2}m_{11}^2\phi_1^\dagger\phi_1 - \frac{1}{2}m_{22}^2\phi_2^\dagger\phi_2 + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 \\ &\quad +\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right]. \end{aligned}$$

$$m_{22}^2 < 0 \quad \text{and} \quad m_{11}^2 > 0 \quad \Longrightarrow \quad \langle\phi_1\rangle = v/\sqrt{2} \quad \text{and} \quad \langle\phi_2\rangle = 0$$

Cancellation of quadratic divergences for ϕ_1 and ϕ_2 (Newton & Wu, 1994):

$$\begin{aligned} \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3m_t^2, \\ \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 0. \end{aligned}$$

Comments on the inert 2HDM:

- Motivations:
  - To allow for heavy SM-like Higgs boson in order to weaken the little hierarchy problem,
  - To provide a candidate for DM.

- No CPV (as implied by exact $\mathbb{Z}_2$).

- The vacuum stability conditions turn out to be inconsistent with the cancellation of quadratic divergences for realistic top mass.

$$\Downarrow$$

- Allow for $m_{12}^2 \phi_1^\dagger \phi_2 +$ H.c. (CPV),
- Allow for $\langle \phi_2 \rangle \neq 0$,
- The price: no DM candidate!

- The Non-Inert Doublet Model with no quadratic divergences

B.G., P. Osland, "A Natural Two-Higgs-Doublet Model", e-Print: arXiv:0910.4068

The cancellation of quadratic divergences

$$\begin{aligned} \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_b^2}{c_\beta^2}, \\ \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_t^2}{s_\beta^2}, \end{aligned}$$

where $v^2 \equiv v_1^2 + v_2^2$, $\tan\beta \equiv v_2/v_1$

$$\Downarrow$$

For a given choice of the mixing angles α_i's $(i = 1, 2, 3)$, the neutral-Higgs masses M_1^2, M_2^2 and M_3^2 can be determined from the cancellation conditions in terms of $\tan\beta$, μ^2 and $M_{H^\pm}^2$.

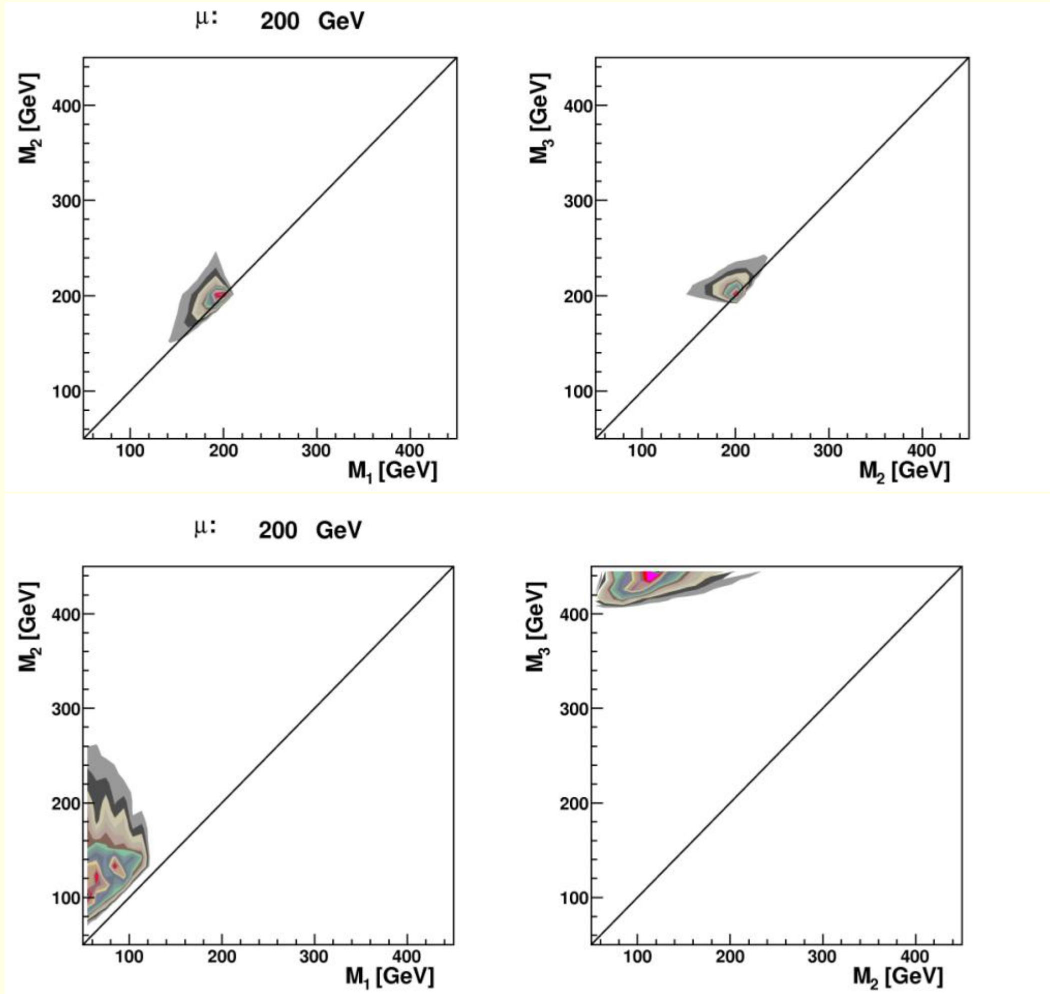

Figure 3: Distributions of allowed masses M_2 vs M_1 (left panels) and M_3 vs M_2 (right), resulting from a scan over the full range of α_i, $\tan\beta \in (0.5, 50)$ and $M_{H^\pm} \in (300, 700)$ GeV, for $\mu = 200$ GeV. No constraints are imposed other than the cancellation of quadratic divergences, $M_i^2 > 0$ and $M_1 < M_2 < M_3$. Two ranges of $\tan\beta$-values are displayed: bottom panels: $0.5 \le \tan\beta \le 1$, top panels: $40 \le \tan\beta \le 50$. The color coding indicates increasing density of allowed points as one moves inward from the boundary.

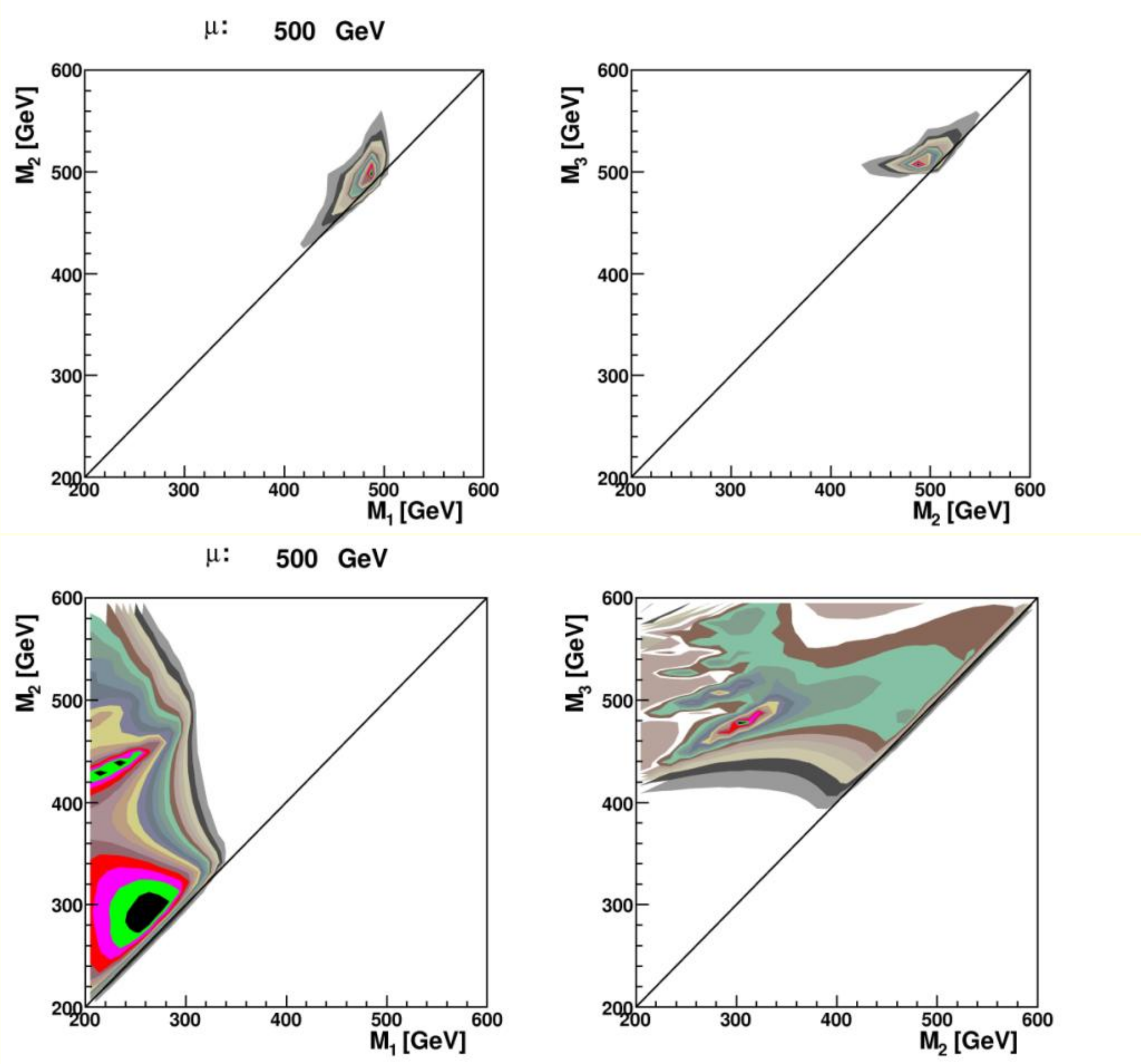

Figure 4: Similar to Fig. 3, for $\mu = 500$ GeV.

$$\begin{aligned} M_1^2 - M_2^2 &= \frac{1}{\tan\beta}\frac{R_{33}}{R_{12}R_{22}}\left[-4\bar{m}^2 - 2M_{H^\pm}^2 + 12m_t^2 + \mu^2\right] + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right) \\ M_3^2 &= -\frac{M_1^2 R_{12}R_{13} + M_2^2 R_{22}R_{23}}{R_{32}R_{33}} + \mathcal{O}\left(\frac{1}{\tan\beta}\right). \end{aligned}$$

where R_{ij} are elements of the orthogonal rotation matrix for the neutral scalars.

$$\Downarrow$$

$$\tan\beta \gtrsim 40 \qquad \Longrightarrow \qquad M_1 \simeq M_2 \simeq M_3 \simeq \mu^2 + 4m_b^2$$

Advantages:

- No 1-loop quadratic divergences (so, $\delta M_i^2/M_i^2$ suppressed),
- Large H_1 mass allowed (so, $\delta M_i^2/M_i^2$ suppressed),
- A chance for substantial CPV,
- DM candidate easily accommodated by adding singlets φ_i-like.

The following experimental constraints are imposed:

- The oblique parameters T and S
- $B_0 - \bar{B}_0$ mixing
- $B \to X_s \gamma$
- $B \to \tau \bar{\nu}_\tau X$
- $B \to D \tau \bar{\nu}_\tau$
- LEP2 Higgs-boson non-discovery
- R_b
- The muon anomalous magnetic moment
- Electron electric dipole moment

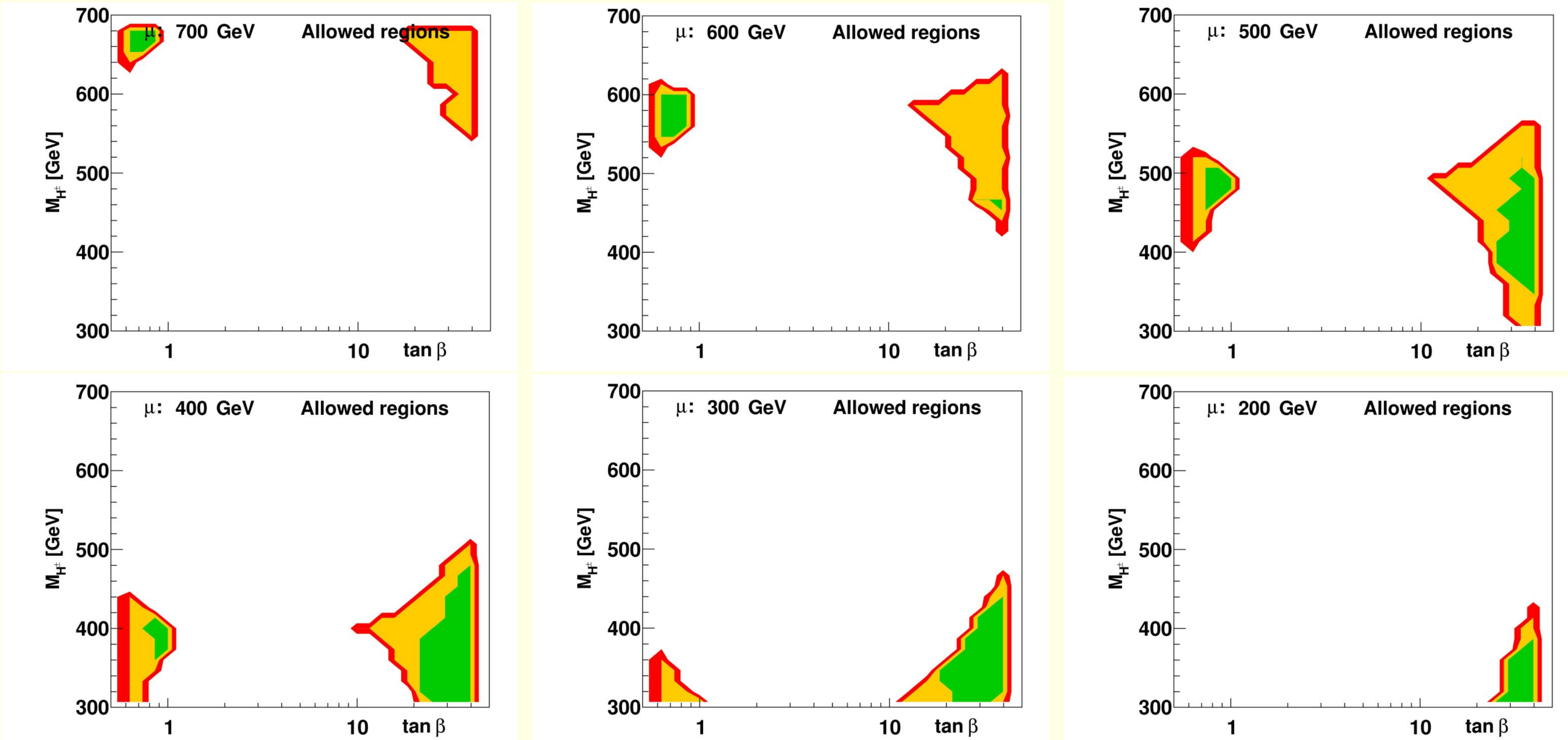

Figure 5: Allowed regions in the $\tan\beta$–$M_{H\pm}$ plane, for $\mu = 200, 300, 400, 500, 600$ and 700 GeV (as indicated). Red: Positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L., as specified in the text.

Violation of CP

$$
\begin{aligned}
\Im J_1 &= -\frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)\Im\lambda_5, \\
\Im J_2 &= -\frac{v_1^2 v_2^2}{v^8}\Big[\big((\lambda_1-\lambda_3-\lambda_4)^2-|\lambda_5|^2\big)\,v_1^4+2(\lambda_1-\lambda_2)\Re\lambda_5 v_1^2 v_2^2 \\
&\qquad -\big((\lambda_2-\lambda_3-\lambda_4)^2-|\lambda_5|^2\big)\,v_2^4\Big]\,\Im\lambda_5, \\
\Im J_3 &= \frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)(\lambda_1+\lambda_2+2\lambda_4)\Im\lambda_5.
\end{aligned}
$$

For $\tan\beta \gtrsim 40$

$$
\Im J_i \sim \frac{\Im\lambda_5}{\tan^2\beta}
$$

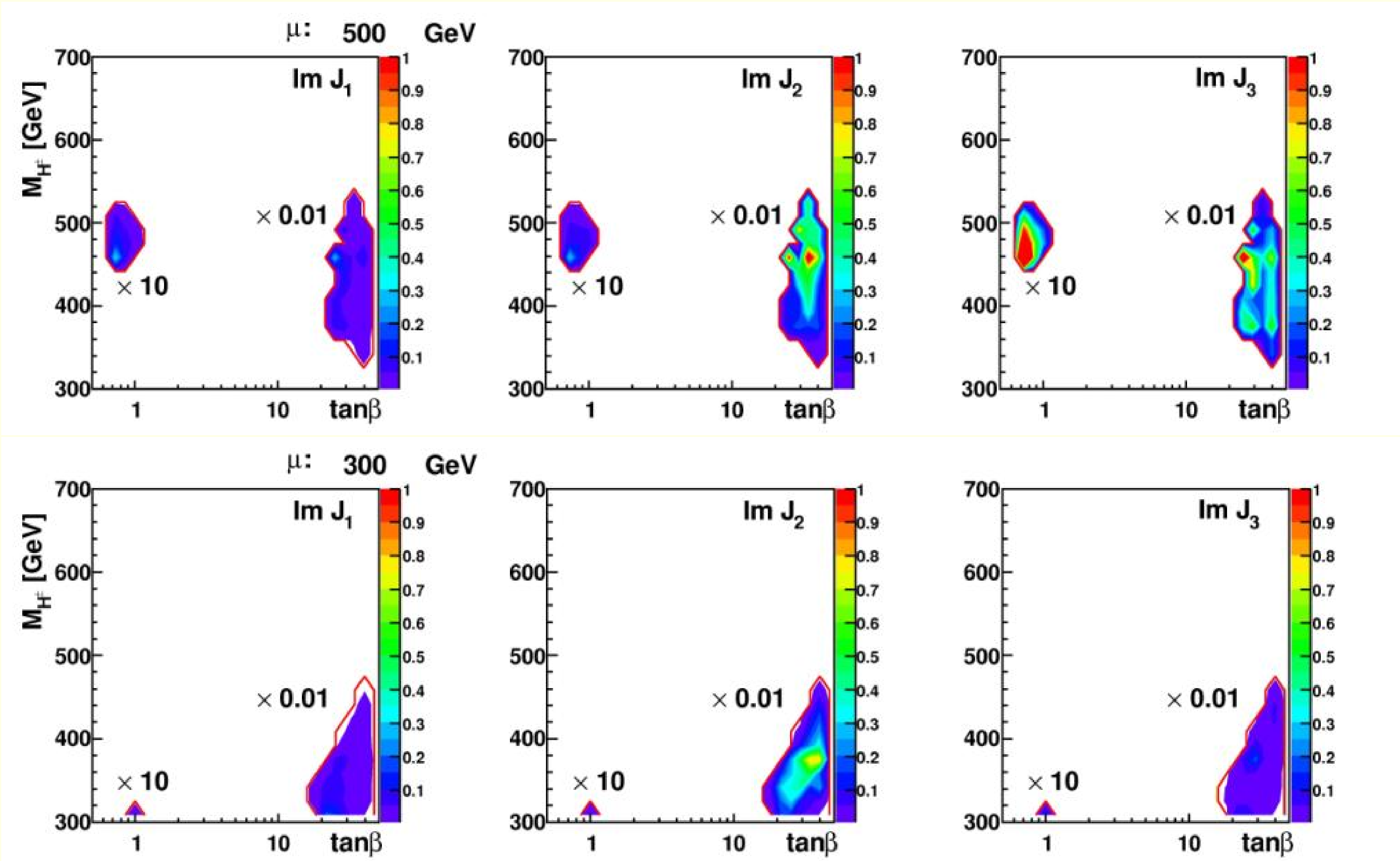

Figure 6: Imaginary parts of the rephasing invariants $|\Im J_i|$, for $\mu = 500$ GeV (top) and $\mu = 300$ GeV (bottom). The colour coding is given along the right vertical axis. At low $\tan\beta$ the values should be rescaled by a factor of 10, at high $\tan\beta$ by a factor 0.01.

Stability of the cancellation condition

$$\delta M_i^2 = \Lambda^2 \sum_{n=0} f_n^{(i)}(\lambda) \left[\ln\left(\frac{\Lambda}{v}\right)\right]^n ,$$

The coefficients $f_n^{(i)}(\lambda)$ can be determined recursively, however here a simple estimate is sufficient:

$$f_n^{(i)}(\lambda) \sim \left(\frac{\lambda}{16\pi^2}\right)^{n+1} \sim \left(\frac{4\pi}{16\pi^2}\right)^{n+1} \sim \left(\frac{1}{4\pi}\right)^{n+1}$$

Requiring that the 2-loop contribution does not exceed M_1^2 one finds:

$$\Lambda^2 \ln\left(\frac{\Lambda}{v}\right) \sim (4\pi M_1)^2$$

Then, e.g. for $M_1 = 200(500)$ GeV the cutoff is at $\Lambda \sim 1.8(3.8)$ TeV.

- DM in the Non-Inert Doublet Model with no quadratic divergences

$$
\begin{aligned}
V(\phi_1,\phi_2) &= -\frac{1}{2}\left\{m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \text{H.c.}\right]\right\} \\
&+\frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\
&+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right] \\
&+\mu_\varphi^2\varphi^2 + \frac{1}{24}\lambda_\varphi\varphi^4 + \varphi^2(\eta_1\phi_1^\dagger\phi_1 + \eta_2\phi_2^\dagger\phi_2)
\end{aligned}
$$

The cancellation conditions:

$$
\begin{aligned}
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{1}{2}\eta_1 + \frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_b^2}{c_\beta^2}, \\
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{1}{2}\eta_2 + \frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) &= 3\frac{m_t^2}{s_\beta^2}, \\
\frac{\lambda_\varphi}{2} + 4(\eta_1 + \eta_2) &= 8\text{Tr}\{Y_\varphi Y_\varphi^\dagger\}
\end{aligned}
$$

where $\mathcal{L}_Y = -\varphi\overline{(\nu_R)^c}Y_\varphi\nu_R + \text{H.c.}$.

$$\mathcal{L} = -\varphi^2(\kappa_i v H_i + \lambda_{ij} H_i H_j + \lambda_\pm H^+ H^-)$$

with

$$\kappa_i = \eta_1 R_{i1} c_\beta + \eta_2 R_{i2} s_\beta,$$
$$\lambda_{ij} = \frac{1}{2}\left[\eta_1 (R_{i1} R_{j1} + s_\beta^2 R_{i3} R_{j3}) + \eta_2 (R_{i2} R_{j2} + c_\beta^2 R_{i3} R_{j3})\right],$$
$$\lambda_\pm = \eta_1 s_\beta^2 + \eta_2 c_\beta^2$$

Assumption: $M_1 \ll M_{2,3}$ so that DM annihilation is dominated by H_1 exchange.

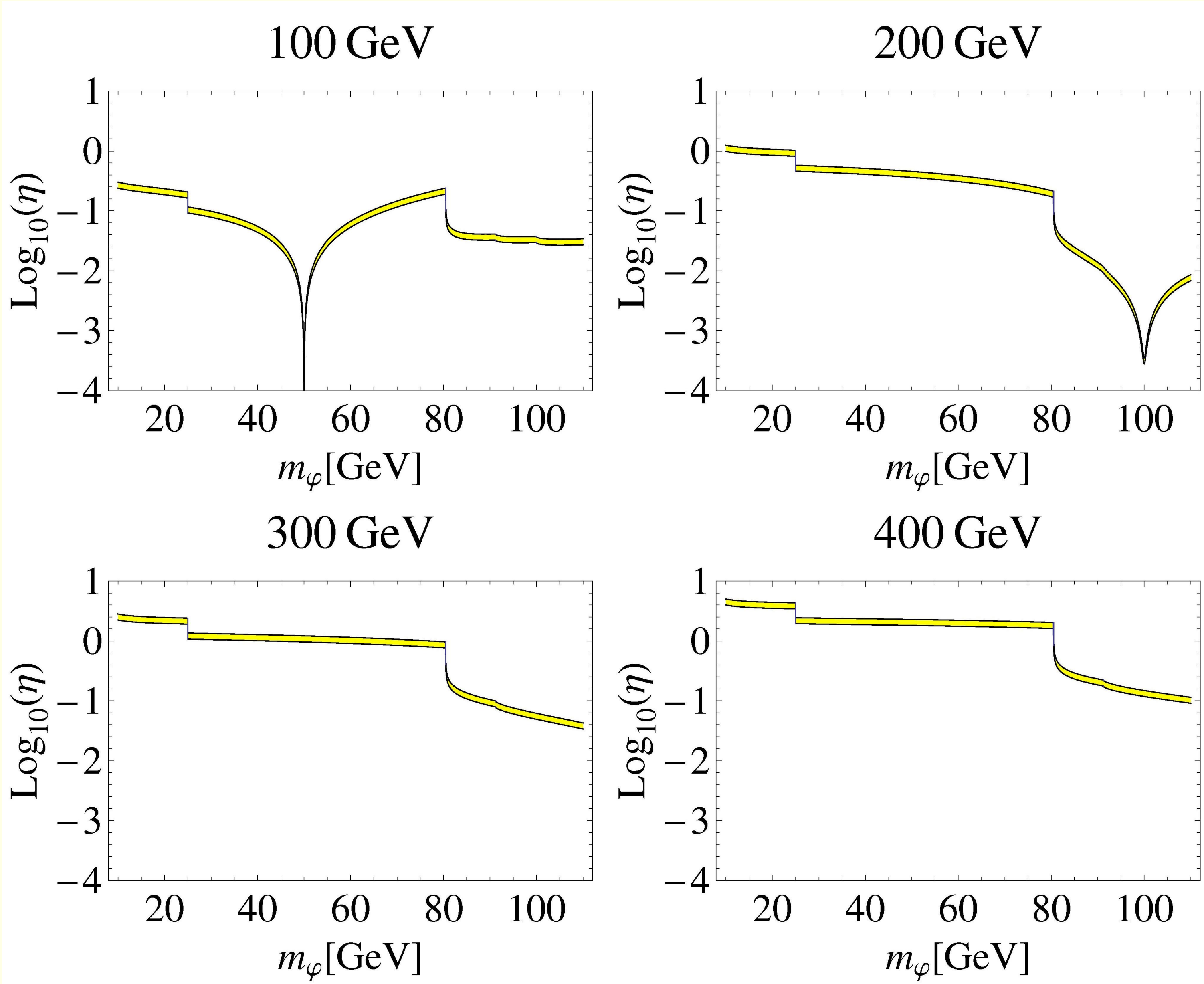

Figure 7: Inert-scalar coupling η (vs m_φ) required by the observed DM abundance $\Omega_{DM}h^2 = 0.106 \pm 0.008$ within a 3-σ band. As indicated above each panel, the lightest Higgs-boson mass ranges from $M_1 = 100$ to 400 GeV . It was assumed that $2\lambda_{11} = \kappa_1 \equiv \eta$.

- 2HDM (CPV) + Inert doublet (DM)

B.G., O.M. Ogreid, P. Osland, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys.Rev.D80:055013,2009.

$$V(\Phi_1, \Phi_2, \eta) = V_{12}(\Phi_1, \Phi_2) + V_3(\eta) + V_{123}(\Phi_1, \Phi_2, \eta)$$

where

$$\begin{aligned}
V_{12}(\Phi_1, \Phi_2) &= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}\right]\right\} \\
&+ \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&+ \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{H.c.}\right], \\
V_3(\eta) &= m_\eta^2\eta^\dagger\eta + \frac{\lambda_\eta}{2}(\eta^\dagger\eta)^2, \\
V_{123}(\Phi_1, \Phi_2, \eta) &= \lambda_{1133}(\Phi_1^\dagger\Phi_1)(\eta^\dagger\eta) + \lambda_{2233}(\Phi_2^\dagger\Phi_2)(\eta^\dagger\eta) \\
&+ \lambda_{1331}(\Phi_1^\dagger\eta)(\eta^\dagger\Phi_1) + \lambda_{2332}(\Phi_2^\dagger\eta)(\eta^\dagger\Phi_2) \\
&+ \frac{1}{2}\left[\lambda_{1313}(\Phi_1^\dagger\eta)^2 + \text{H.c.}\right] + \frac{1}{2}\left[\lambda_{2323}(\Phi_2^\dagger\eta)^2 + \text{H.c.}\right]
\end{aligned}$$

Summary

- The SM could be easily extended so that the little hierarchy problem is ameliorated, DM candidate is provided and also CP is violated in the extra sector:
  - The addition of N_φ real scalar singlets φ_i to the SM lifts the cutoff Λ to $\sim 4-9$ TeV. It also provides a realistic candidate for DM if $m_\varphi \sim 1-3$ TeV (depending on N_φ).
  - To accommodate CPV in the Higgs potential the SM scalar sector should be replaced by 2 Higgs doublets (non-inert). Cancellation of quadratic divergences could be arranged within the 2HDM. Heavy lightest Higgs additionally suppresses $\delta M_i^2/M_i^2$. Adding extra inert scalar singlet or doublet offers a DM candidate.
  - CPV in the Higgs potential with the SM doublet and singlets only?

- Some fine tuning always remains.