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Unparticles

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H. Georgi, PRL **98**, 221601 (2007), cited 142 times:

$$\underbrace{\text{SM} \quad \otimes \quad \text{CFT}(\mathcal{BZ})}$$

$$\Downarrow$$

$$\frac{\mathcal{O}_{\text{SM}}\,\mathcal{O}_{\mathcal{BZ}}}{M_{\mathcal{U}}^k}$$

$$\Downarrow$$

Dimensional transmutation in the $\mathcal{BZ}$ sector
(breaking of the scale invariance at $\mu = \Lambda_{\mathcal{U}}$)

$$\Downarrow$$

IR fixed point (the scale invariance emerges at the loop level)

Scale invariance and conformal transformations

$$x \to x' = sx$$

Is there a corresponding field transformation such that $\mathcal{S} = \int d^4x \mathcal{L}$ is invariant?
Assume

$$\phi(x) \to \phi'(x') = s^{-d}\phi(x)$$

where d (the scaling dimension) is to be determined.

- scalars:

$$\mathcal{S}_\phi = \int d^4x \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4!}\phi^4 \right\} \to \underbrace{s^4 s^{-2} s^{-2d_\phi}}_{1} \mathcal{S}_\phi \Rightarrow d_\phi = 1$$

- fermions:

$$\mathcal{S}_\psi = \int d^4x \bar{\psi} i\gamma^\mu \partial_\mu \psi \to \underbrace{s^4 s^{-1} s^{-2d_\psi}}_{1} \mathcal{S}_\psi \Rightarrow d_\psi = \frac{3}{2}$$

- The scaling dimensions and dimensions coincide.

- Mass terms are not invariant:
  - scalars: $\int d^4x\, m^2\, \phi^2 \to s^4 s^{-2} \int d^4x\, m^2\, \phi^2$
  - fermions: $\int d^4x\, m\, \bar{\psi}\psi \to s^4 s^{-3} \int d^4x\, m\, \bar{\psi}\psi$

The Noether theorem: if $\mathcal{S} = \int d^4x\mathcal{L}$ scale invariant $\Rightarrow \partial^\mu D_\mu = 0$
for a scalar theory

$$D_\mu = \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}(d_\phi + x_\nu\partial^\nu)\phi - x_\mu\mathcal{L}$$

In general (Callan, Coleman, Jackiw (1970)):

$$\partial^\mu D_\mu = T_\mu{}^\mu$$

where

$$T_{\mu\nu} = \underbrace{\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\partial_\nu\phi - g_{\mu\nu}\mathcal{L}}_{\text{the canonical energy-momentum tensor}} - \frac{1}{6}(\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\phi^2$$

$$\text{Scale invariance} \Rightarrow T_\mu{}^\mu = 0$$

For $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$

$$T_\mu{}^\mu = m^2\phi^2$$

Conformal transformations (angle-preserving) are such that

$$\frac{dx^\alpha \, dx_\alpha}{|dx| \, |dx|}$$

remains unchanged, so

$$x^\mu \to x'^\mu = \frac{x^\mu - b^\mu x^2}{1 - 2b \cdot x + b^2 x^2}$$

- conformal invariance $\Rightarrow$ scale invariance
- scale invariance $\Rightarrow$ conformal invariance for all renormalizable field theories of spin ≤ 1

Anomalous breaking of scale invariance and fixed points

The scale invariance implies the following Ward identity for 1PI Green's function $\Gamma^{(n)}$:

$$\partial^\mu D_\mu = 0 \qquad \Rightarrow \qquad \left(-\frac{\partial}{\partial t} + D\right)\Gamma^{(n)}(e^t p_1, \ldots, e^t p_{n-1}) = 0$$

where $D = 4 - nd_\phi$ is the canonical dimension of $\Gamma^{(n)}$. The solution reads

$$\Gamma^{(n)}(sp_i) = s^D \Gamma^{(n)}(p_i) \qquad\qquad (\star)$$

for $s = e^t$.

However, since the loop expansion requires some sort of regularization, (so some scale must be introduced: μ in the dimensional regularization, or Λ in the cutoff regularization), therefore one can expect that classical scale invariance (i.e. invariance of the Lagrangian) would be broken at the quantum (loops) level. One can use the RGE to verify if the canonical scaling $(\star)$ is satisfied:

$$\left[-\frac{\partial}{\partial t} + \beta(\lambda)\frac{\partial}{\partial \lambda} + (\gamma_m - 1)m\frac{\partial}{\partial m} - n\gamma(\lambda) + D\right]\Gamma^{(n)}(sp_i) = 0$$

for

$$\beta = \mu\frac{d\lambda}{d\mu}, \qquad \gamma = \frac{1}{2}\mu\frac{d}{d\mu}Z_3, \qquad \gamma_m = \frac{\mu}{m}\frac{dm}{d\mu}$$

In a massless, non-interacting theory the scaling is canonical!

In general, there is no scaling in a quantum field theory, even if $m = 0$. The solution of the RGE reads:

$$\Gamma^{(n)}(e^t p_i, \lambda, \mu) = (e^t)^D \Gamma^{(n)}(p_i, \bar{\lambda}(t), \mu)\; e^{-n\int_0^t \gamma[\bar{\lambda}(t')]dt'}$$

- Assume there is an IR fixed point at $\lambda = \lambda_{IR}$: so $\beta(\lambda_R) = 0$. Then
-
$$e^{-n\int_0^t \gamma[\bar{\lambda}(t')]dt'} = e^{-n\gamma[\lambda_{IR}]t}$$

  and
- $\Gamma^{(n)}(sp_i) = s^{D-n\gamma(\lambda_{IR})}\Gamma^{(n)}(p_i)$

The Green's functions scale with non-canonical scaling dimension $d = D - n\gamma(\lambda_{IR})$
To check the scale invariance one should find ${T_\mu}^\mu$, e.g. for QCD:

$${T_\mu}^\mu = \frac{\beta(g)}{2g^3} G^a_{\mu\nu}\, G^{a\,\mu\nu}$$

So, if $\beta(g) = 0$ then the theory is scale invariant.

Mass spectrum in scale invariant theories

Operator of the scale transformation: $U(\epsilon) = e^{i\epsilon D}$

$$i[D, \mathbb{P}_\mu] = \mathbb{P}_\mu \quad \Rightarrow \quad [D, \mathbb{P}^2] = -2i\mathbb{P}^2 \quad \Rightarrow \quad e^{i\epsilon D}\mathbb{P}^2 e^{-i\epsilon D} = e^{2\epsilon}\mathbb{P}^2$$

- Let $|p\rangle$ is a state of momentum p_μ: $\mathbb{P}^2|p\rangle = p^2|p\rangle$
- Assume that the scale invariance is not broken spontaneously: $D|0\rangle = 0$, then
- $e^{-i\epsilon D}|p\rangle$ is a state of rescaled momenta: $e^{-i\epsilon D}|p\rangle \propto |e^\epsilon p\rangle$.

Conclusion: varying ϵ, one can construct states of an arbitrary non-negative mass^2: $e^{2\epsilon}p^2$.

$$\Downarrow$$

Mass spectrum is either continuous or particles are massless

Examples of the $\mathcal{BZ}$ sector

- Banks & Zaks (1982): SU(3) YM with n massless fermions in e.g. fundamental representation

$$\beta(g) = -\left(\beta_0 \frac{g^3}{16\pi^2} + \beta_1 \frac{g^5}{(16\pi^2)^2} + 3 \text{ loops} \cdots\right)$$

$$\begin{array}{lll} \beta_0 = 11 - \frac{2}{3}n & \beta_0(n_0) = 0 & n_0 = 16.5 \\ \beta_1 = 102 - \frac{38}{3}n & \beta_1(n_1) = 0 & n_1 \simeq 8.05 \end{array}$$

If $n_1 < n < n_0$ (so $\beta_0 > 0$ & $\beta_1 < 0$) then keeping β_0 and β_1 one gets

$$\beta(g_{IR}) = 0 \quad \text{for} \quad \frac{g_{IR}^2}{16\pi^2} = -\frac{33 - 2n}{306 - 38n}$$

Expanding n around n_0: $n = n_0\left(1 - \frac{\varepsilon}{11}\right)$

$$\frac{g_{IR}^2}{16\pi^2} \simeq \varepsilon \times 10^{-2}$$

Conclusions:

- If $g = g_{IR}$, then the low-energy theory is scale invariant with small anomalous scaling
- For $\varepsilon \ll 1$, the theory remains perturbative, so the continuous spectrum doesn't emerge.

- Massless Abelian Thirring model ($D = 2$)

$$\mathcal{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \lambda (\bar{\psi} \gamma_\mu \psi)^2$$

  - $G(x-y) = i\langle 0 | T\{\psi(x)\bar{\psi}(y)\} | 0\rangle$ - exactly calculable
  - Scaling: $G(x-y) = s^{2d} G[s(x-y)]$ for

$$d = \frac{1}{2} + \frac{\lambda^2}{4\pi^2} \frac{1}{1 - \frac{\lambda^2}{4\pi}}$$

- "Electro-Magnetic" duality (Intriligator-Seiberg): $N = 1$ SUSY YM $SU(N_c)$, N_f massless quarks in the fundamental representation:
  - If $\frac{3}{2} N_c < N_f < 3N_c$ then there is a non-trivial IR fixed point and the dual theory is $SU(N_f - N_c)$, $N = 1$, YM with with N_f quarks.

The scenario again

$$
\begin{array}{llcc}
UV & CFT : \mathcal{BZ}\ \text{fields} & & SM\ \text{fields} \\
\downarrow & \qquad\qquad\searrow & & \swarrow \\
M_{\mathcal{U}} & & \frac{1}{M_{\mathcal{U}}^k}\mathcal{O}_{\mathcal{BZ}}\mathcal{O}_{\rm SM} & \\
\downarrow & & & \\
\Lambda_{\mathcal{U}} & \text{transmutation scale} & \downarrow & \\
\downarrow & & & \\
IR & \text{fixed point} & c_{\mathcal{U}}\frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}\mathcal{O}_{\mathcal{U}}\mathcal{O}_{\rm SM} &
\end{array}
$$

where $k = d_{\rm SM}+d_{\mathcal{BZ}}-4$. $d_{\rm SM}$ and $d_{\mathcal{BZ}}$ are canonical dimensions of $\mathcal{O}_{\rm SM}$ and $\mathcal{O}_{\mathcal{U}}$, respectively, while $d_{\mathcal{U}}$ is the scaling dimension (the same as the mass dimension in this case) of $\mathcal{O}_{\mathcal{U}}$:

$$
\mathcal{O}_{\mathcal{U}}(x) \to \mathcal{O}'_{\mathcal{U}}(x') = s^{-d_{\mathcal{U}}}\mathcal{O}_{\mathcal{U}}(x) \quad \text{with} \quad 1 < d_{\mathcal{U}} < 2 \quad \text{for} \quad x \to x' = sx
$$

An example of matching between $\mathcal{O}_{\mathcal{BZ}}$ and $\mathcal{O}_{\mathcal{U}}$:

- $(\bar{q}q)$ in QCD $\quad\Longleftrightarrow\quad$ $M \propto (\bar{q}q)$ mesons in the chiral non-linear model

Correlators and scaling

$$\langle 0|\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle = \int \frac{d^4p}{(2\pi)^2} e^{-ipx} \rho_{\mathcal{U}}(p^2)$$

for $\rho_{\mathcal{U}}(p^2) = (2\pi)^4 \int d\lambda\, \delta^{(4)}(p - p_\lambda)\, |\langle 0|\mathcal{O}_{\mathcal{U}}(0)|\lambda\rangle|^2$.

- Scaling:

$$\mathcal{O}_{\mathcal{U}}(x) \to \mathcal{O}'_{\mathcal{U}}(x') = s^{-d_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}}(x) \quad \text{with} \quad 1 < d_{\mathcal{U}} < 2 \quad \text{for} \quad x \to x' = sx$$

-

$$\rho_{\mathcal{U}}(p^2) = \int d^4x\, e^{ipx} \langle 0|\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle \quad \Rightarrow \quad \rho_{\mathcal{U}}(p^2) = A_{d_{\mathcal{U}}} \theta(p^0)\theta(p^2)(p^2)^\alpha$$

where $A_{d_{\mathcal{U}}}$ is a normalization constant and α is to be determined.

$$\Downarrow$$

$$\alpha = d_{\mathcal{U}} - 2$$

Phase space for unparticles

- In a free scalar quantum field theory

$$\langle 0|\phi(x)\phi(0)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ipx} = \int \underbrace{\frac{d^4p}{(2\pi)^3}\theta(p^0)\delta(p^2-m^2)}_{d\Phi} e^{-ipx} = \int d\Phi e^{-ipx}$$

- For unparticles

$$\langle 0|\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\rho_{\mathcal{U}}(p^2)e^{-ipx} \quad \text{for} \quad \rho_{\mathcal{U}}(p^2) = A_{d_{\mathcal{U}}}\theta(p^0)\theta(p^2)(p^2)^{d_{\mathcal{U}}-2}$$

$$\Downarrow$$

$$d\Phi_{\mathcal{U}}(p_{\mathcal{U}}) = A_{d_{\mathcal{U}}}\theta(p^0)\theta(p_{\mathcal{U}}^2)(p_{\mathcal{U}}^2)^{d_{\mathcal{U}}-2}\frac{d^4p_{\mathcal{U}}}{(2\pi)^4}$$

Note that integrating the phase space for n massless particles one gets

$$\int (2\pi)^4\delta^{(4)}\left(p - \sum_{i=1}^{n}\right)\prod_{i=1}^{n}\theta(p_i^0)\delta(p_i^2)\frac{d^4p_i}{(2\pi)^3} = A_n\theta(p^0)\theta(p^2)(p^2)^{n-2}$$

for

$$A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}}\frac{\Gamma(n+\frac{1}{2})}{\Gamma(n-1)\Gamma(2n)} \quad \Rightarrow \quad A_{d_{\mathcal{U}}} = A_{n=d_{\mathcal{U}}}$$

The integrating the phase space for n massless particles one gets

$$\int (2\pi)^4 \delta^{(4)} \left(p - \sum_{i=1}^{n} \right) \prod_{i=1}^{n} \theta(p_i^0) \delta(p_i^2) \frac{d^4 p_i}{(2\pi)^3} = A_n \theta(p^0) \theta(p^2) (p^2)^{n-2}$$

for

$$A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n-1)\Gamma(2n)} \qquad \Rightarrow \qquad A_{d_{\mathcal{U}}} = A_{n=d_{\mathcal{U}}}$$

Georgi: **Unparticle stuff with scale dimension $d_{\mathcal{U}}$ looks like a non-integer number $d_{\mathcal{U}}$ of massless particles.**

The limit $n \to 1$ reproduces the single particle phase space:

$$\lim_{\epsilon \to 0^+} \frac{\epsilon \theta(x)}{x^{1-\epsilon}} = \delta(x) \quad \text{for} \quad x = p^2 \text{ and } \quad \epsilon \equiv n - 1$$

Unparticle propagator

The Feynman propagator:

$$i\Delta_F^{\mathcal{U}}(p^2) = \int d^4x e^{ipx} \langle 0| T\left\{\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)\right\}|0\rangle = \int_0^\infty \frac{dm^2}{2\pi}\rho_{\mathcal{U}}(m^2)\frac{i}{p^2 - m^2 + i\varepsilon}$$

From the scaling properties $\rho_{\mathcal{U}}(m^2) = A_{d_{\mathcal{U}}}\theta(m^2)\left(m^2\right)^{d_{\mathcal{U}}-2}$, so

$$\Delta_F^{\mathcal{U}}(p^2) = \frac{A_{d_{\mathcal{U}}}}{2\sin(\pi d_{\mathcal{U}})}\frac{1}{(-p^2 - i\varepsilon)^{2-d_{\mathcal{U}}}}$$

- Non-trivial phase:

$$\mathbf{Im}\left\{\Delta_F^{\mathcal{U}}(p^2)\right\} = -\frac{A_{d_{\mathcal{U}}}}{2}\theta(p^2)(p^2)^{d_{\mathcal{U}}-2}$$

- Interference with the Z-boson:

$$\Delta_Z(p^2) = \frac{1}{p^2 - m_Z^2 + iM_Z\Gamma_Z}$$

- $\mathbf{Im}\left\{\Delta_F^{\mathcal{U}}(p^2)\right\} \neq 0$ doesn't imply unparticle decay!

An example: $t \to u\,\mathcal{O}_{\mathcal{U}}$

$$\mathcal{L}_{\rm int} = i\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\;\bar{u}\gamma_\mu(1-\gamma_5)t\,\partial^\mu\mathcal{O}_{\mathcal{U}} + \text{H.c.} \qquad \text{for} \qquad \lambda = c_{\mathcal{U}}\frac{\Lambda_{\mathcal{U}}^{\mathcal{BZ}}}{M_{\mathcal{U}}^k}$$

The differential decay rate

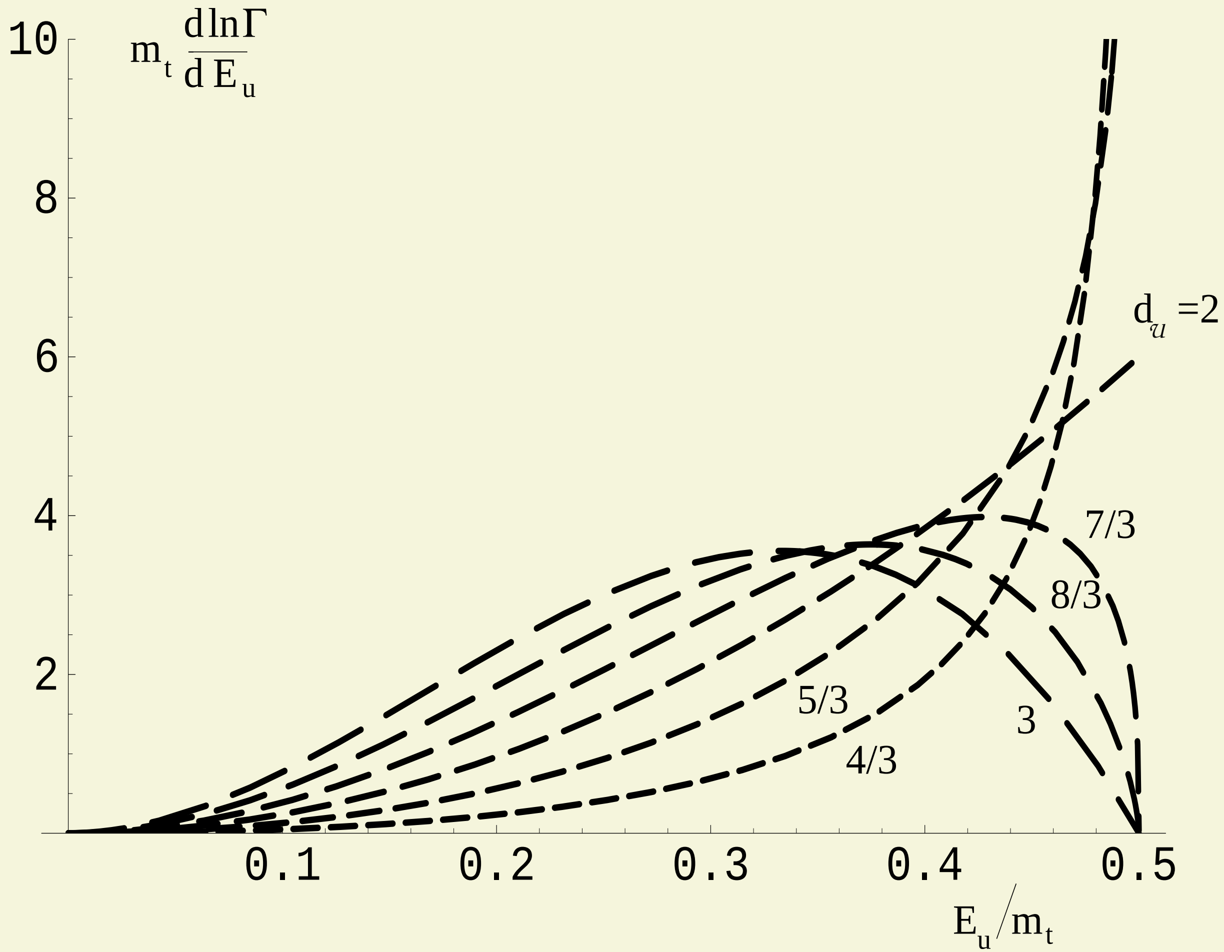
$$d\Gamma = \frac{|\mathcal{M}|^2}{2m_t}\,d\Phi_{\mathcal{U}}\,d\Phi_u$$

$$\Downarrow$$

$$\frac{d\Gamma}{dE_u} = \frac{A_{d_{\mathcal{U}}}m_t^2E_u^2|\lambda|^2}{2\pi^2\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}}}\frac{\theta(m_t-2E_u)}{(m_t^2-2m_tE_u)^{2-d_{\mathcal{U}}}}$$

$$\Downarrow$$

$$m_t\frac{1}{\Gamma}\frac{d\Gamma}{dE_u} = 4d_{\mathcal{U}}(d_{\mathcal{U}}^2-1)\left(1-2\frac{E_u}{m_t}\right)^{d_{\mathcal{U}}-2}\left(\frac{E_u}{m_t}\right)^2$$



$m_t \frac{d\ln\Gamma}{d E_u}$
10
8
6
4
2
0.1
0.2
0.3
0.4
0.5
E_u/m_t
$d_{\mathcal{U}}=2$
7/3
8/3
5/3
3
4/3

Couplings of unparticles to the SM

Assumptions:

- $\mathcal{O}_{\mathcal{U}}$ in neutral under the SM gauge group
- $\dim(\mathcal{O}_{\mathrm{SM}}) \leq 4$

$$\mathcal{L}_{\mathrm{int}} = c_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{k}} \mathcal{O}_{\mathcal{U}} \mathcal{O}_{\mathrm{SM}} \quad \text{for} \quad k = d_{\mathrm{SM}} + d_{\mathcal{BZ}} - 4$$

- Scalar unparticles $\mathcal{O}_{\mathcal{U}}$
  - Couplings with gauge bosons

$$\lambda_{gg}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} G^{\mu\nu}G_{\mu\nu}O_{\mathcal{U}},\ \lambda_{ww}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} W^{\mu\nu}W_{\mu\nu}O_{\mathcal{U}},\ \lambda_{bb}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} B^{\mu\nu}B_{\mu\nu}O_{\mathcal{U}},$$
$$\tilde{\lambda}_{gg}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \tilde{G}^{\mu\nu}G_{\mu\nu}O_{\mathcal{U}},\ \tilde{\lambda}_{ww}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \tilde{W}^{\mu\nu}W_{\mu\nu}O_{\mathcal{U}},\ \tilde{\lambda}_{bb}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \tilde{B}^{\mu\nu}B_{\mu\nu}O_{\mathcal{U}},$$

  - Coupling with Higgs and Gauge bosons

$$\lambda_{hh}\Lambda_{\mathcal{U}}^{2-d_{\mathcal{U}}} H^{\dagger}H\mathcal{O}_{\mathcal{U}},\ \tilde{\lambda}_{hh}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} (H^{\dagger}D_{\mu}H)\partial^{\mu}O_{\mathcal{U}}\ ,$$
$$\lambda_{4h}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} (H^{\dagger}H)^{2}O_{\mathcal{U}},\ \lambda_{dh}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} (D_{\mu}H)^{\dagger}(D^{\mu}H)O_{\mathcal{U}},$$

  - Couplings with fermions and gauge bosons

$$\lambda_{QQ}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{Q}_{L}\gamma_{\mu}D^{\mu}Q_{L}O_{\mathcal{U}},\ \lambda_{UU}\Lambda_{U}^{-d_{\mathcal{U}}} \bar{U}_{R}\gamma_{\mu}D^{\mu}U_{R}O_{\mathcal{U}},\ \lambda_{DD}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{D}_{R}\gamma_{\mu}D^{\mu}D_{R}O_{\mathcal{U}},$$
$$\lambda_{LL}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{L}_{L}\gamma_{\mu}D^{\mu}L_{L}O_{\mathcal{U}},\ \lambda_{EE}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{E}_{R}\gamma_{\mu}D^{\mu}E_{R}O_{\mathcal{U}},\ \lambda_{\nu\nu}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{\nu}_{R}\gamma_{\mu}D^{\mu}\nu_{R}O_{\mathcal{U}},$$
$$\tilde{\lambda}_{QQ}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{Q}_{L}\gamma_{\mu}Q_{L}\partial^{\mu}O_{\mathcal{U}},\ \tilde{\lambda}_{UU}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{U}_{R}\gamma_{\mu}U_{R}\partial^{\mu}O_{\mathcal{U}},\ \tilde{\lambda}_{DD}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{D}_{R}\gamma_{\mu}D_{R}\partial^{\mu}O_{\mathcal{U}},$$

$$\tilde{\lambda}_{LL}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\,\bar{L}_L\gamma_\mu L_L\partial^\mu O_{\mathcal{U}},\ \tilde{\lambda}_{EE}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\,\bar{E}_R\gamma_\mu E_R\partial^\mu O_{\mathcal{U}},\ \tilde{\lambda}_{RR}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\,\bar{\nu}_R\gamma_\mu \nu_R\partial^\mu O_{\mathcal{U}},$$
$$\lambda_{YR}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,\bar{\nu}_R^C\nu_R O_{\mathcal{U}},$$

– Couplings with fermions and Higgs boson

$$\lambda_{YU}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\,\bar{Q}_L H U_R O_{\mathcal{U}},\ \lambda_{YD}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\,\bar{Q}_L\tilde{H}D_R O_{\mathcal{U}},$$
$$\lambda_{Y\nu}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\,\bar{L}_L H\nu_R O_{\mathcal{U}},\ \lambda_{YE}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\,\bar{L}_L\tilde{H}E_R O_{\mathcal{U}},$$

- Vector unparticles $\mathcal{O}_{\mathcal{U}}^\mu$
  – Couplings with fermions

$$\lambda'_{QQ}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,\bar{Q}_L\gamma_\mu Q_L O_{\mathcal{U}}^\mu,\ \lambda'_{UU}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,\bar{U}_R\gamma_\mu U_R O_{\mathcal{U}}^\mu,\ \lambda'_{DD}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,\bar{D}_R\gamma_\mu D_R O_{\mathcal{U}}^\mu,$$
$$\lambda'_{LL}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,\bar{L}_L\gamma_\mu L_L O_{\mathcal{U}}^\mu,\ \lambda'_{EE}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,\bar{E}_R\gamma_\mu E_R O_{\mathcal{U}}^\mu,\ \lambda'_{RR}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,\bar{\nu}_R\gamma_\mu \nu_R O_{\mathcal{U}}^\mu,$$

  – Couplings with Higgs boson and Gauge bosons

$$\lambda'_{hh}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,(H^\dagger D_\mu H)O_{\mathcal{U}}^\mu\ ,\ \lambda'_{bO}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\,B_{\mu\nu}\partial^\mu O^\nu\ .$$

- Spinor unparticles $O_{\mathcal{U}}^s$

$$\lambda_{s\nu}\Lambda_{\mathcal{U}}^{5/2-d_{\mathcal{U}}}\,\bar{\nu}_R O_{\mathcal{U}}^s,\ \lambda_s\Lambda_{\mathcal{U}}^{3/2-d_{\mathcal{U}}}\,\bar{O}_{\mathcal{U}}^s L_L^i\epsilon_{ij}H^j\ .$$

Unitarity constraints

1. G. Mack, "All Unitary Ray Representations Of The Conformal Group SU(2,2) With Positive Energy," Commun. Math. Phys. **55**, 1 (1977).
2. B. Grinstein, K. Intriligator and I. Z. Rothstein, "Comments on Unparticles," arXiv:0801.1140 [hep-ph].

$$\Downarrow$$

- $\mathbf{Im}\,\{\mathcal{A}(i \to i)\} > 0 \quad \Longrightarrow \quad d_V > 3$ for gauge invariant vector unparticle operators
- Corrected vector and tensor propagators for unparticles.

Deconstruction of unparticles

Källen-Lehman representation of the Feynman propagator:

$$i\Delta_F^{\mathcal{U}}(p^2) = \int d^4x e^{ipx} \langle 0|T\{\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)\}|0\rangle = \int_0^\infty \frac{dm^2}{2\pi}\rho(m^2)\frac{i}{p^2 - m^2 + i\varepsilon}$$

with $\rho_{\mathcal{U}}(m^2) = A_{d_{\mathcal{U}}}\theta(m^2)(m^2)^{d_{\mathcal{U}}-2}$. Deconstruction (Stephanov'07):

$$\mathcal{O}_{\mathcal{U}} \to \sum_{n=1}^{\infty} F_n \varphi_n \quad \text{with} \quad m_n^2 = \Delta^2 n$$

Then

$$i\Delta_F^{\mathcal{U}}(p^2) = \int d^4x e^{ipx} \langle 0|T\{\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)\}|0\rangle = \sum_{n=1}^{\infty} \frac{iF_n^2}{p^2 - m_n^2 + i\varepsilon}$$

if $F_n^2 = \frac{A_{d_{\mathcal{U}}}}{2\pi}\Delta^2 (m_n^2)^{d_{\mathcal{U}}-2}$ then

$$i\frac{A_{d_{\mathcal{U}}}}{2\pi}\sum_{n=1}^{\infty}\frac{(m_n^2)^{d_{\mathcal{U}}-2}}{p^2 - m_n^2 + i\varepsilon}\Delta^2 \underset{\Delta\to 0}{\longrightarrow} i\frac{A_{d_{\mathcal{U}}}}{2\pi}\int \frac{(m^2)^{d_{\mathcal{U}}-2}dm^2}{p^2 - m^2 + i\varepsilon} = \int \frac{dm^2}{2\pi}\rho(m^2)\frac{i}{p^2 - m^2 + i\varepsilon}$$

So, the undeconstructed result has been confirmed. Now, let's focus on the non-trivial phase:

$$\mathbf{Im}\left\{\sum_{n=1}^{\infty}\frac{F_n^2}{p^2 - m_n^2 + i\varepsilon}\right\} = -\sum_n F_n^2 \pi \delta(p^2 - m_n^2) \underset{\Delta\to 0}{\longrightarrow} -\frac{A_{d_{\mathcal{U}}}}{2}\theta(p^2)(p^2)^{d_{\mathcal{U}}-2}$$

So, each peak becomes lower as $F_n^2 \sim \Delta^2 \to 0$, but their density increases.

- Each mode φ_n breaks the scale invariance.
- In the limit

$$\lim_{N\to\infty}\sum_{n=1}^{N}$$

  the scale invariance is recovered.

The deconstruction for $t \to u\mathcal{O}_{\mathcal{U}}$ decay

$$i\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,\bar{u}\gamma_\mu(1-\gamma_5)t\,\partial^\mu\mathcal{O}_{\mathcal{U}} \longrightarrow i\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,\bar{u}\gamma_\mu(1-\gamma_5)t\,\sum_{n=1}^{\infty}F_n\partial^\mu\varphi_n$$

$$\Downarrow$$

$$\Gamma(t\to u\varphi_n)=\frac{\lambda^2}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}}}\frac{m_t E_u^2}{2\pi}F_n^2 \quad \text{with} \quad E_u=\frac{m_t^2-m_n^2}{2m_t} \;\text{ and }\; F_n^2=\frac{A_{d_{\mathcal{U}}}}{2\pi}\Delta^2(m_n^2)^{d_{\mathcal{U}}-2}$$

Number of states $|\varphi_n\rangle$ in the interval (E_u, E_u+dE_u): $dN = dE_u\frac{2m_t}{\Delta^2}$

$$\Downarrow$$

$$\frac{d\Gamma}{dE_u}=\frac{2m_t}{\Delta^2}\Gamma(t\to u+\varphi_n)=\frac{\lambda^2}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}}}A_{d_{\mathcal{U}}}\frac{m_t^2}{2\pi^2}\,E_u^2(m_t^2-2m_tE_u)\theta(m_t-2E_u)$$

The same as the Georgi's result!

Brief history of the Universe in the presence of unparticles

- For $T \gg M_{\mathcal{U}}$ the excitations in the new sector are in their $\mathcal{BZ}$ phase; they are also in thermal equilibrium with the SM so that $T = T_{\mathcal{BZ}} = T_{SM}$.
- For $T \lesssim M_{\mathcal{U}}$: the $\mathcal{BZ}$ sector starts to decouple, as the average energy is no longer sufficient to create mediators, however, the thermal equilibrium may still be maintained ($T \simeq T_{\mathcal{BZ}} \simeq T_{SM}$) depending on the strength of effective couplings between the SM and the $\mathcal{BZ}$ sector.
- Let T_f be the temperature such that

$$\Gamma(T_f) \simeq H(T_f)$$

  where Γ is the reaction rate responsible for maintaining the equilibrium between the SM and the new sectors, while H is the Hubble parameter. For $T_f > \Lambda_{\mathcal{U}}$ Γ is determined by reactions of the form SM$\leftrightarrow \mathcal{BZ}$ between the SM and excitations in the $\mathcal{BZ}$ phase. If $T_f < \Lambda_{\mathcal{U}}$ then Γ is determined by processes of the form $SM \leftrightarrow \mathcal{U}$ between the SM and the excitations in the $\mathcal{U}$ phase.

Freeze-out and thaw-in

We assume $\mathcal{O}_{\rm SM} = \bar\psi_{SM}\psi_{SM}$ and $\mathcal{O}_{\mathcal{BZ}} = \bar\psi\psi$, so $k = 2$.

- $\mathcal{BZ}$ phase:

$$1 > \frac{T_{\mathcal{BZ}\text{-}f}}{M_{\mathcal{U}}} = \left[\frac{2\sqrt{\pi^9 g_{\rm tot}(T_{\mathcal{BZ}\text{-}f})/5}}{9 n_c n_f}\frac{M_{\mathcal{U}}}{M_P}\right]^{1/3} > \frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}} \tag{1}$$

  where $g_{\rm tot} \equiv g_{NP} + g_{\rm SM} = g_{\mathcal{BZ}} + g_{\rm SM}$; $g_{\mathcal{BZ}} = 2[n_c^2 - 1 + (7/8)n_c n_f]$, and we imposed the consistency conditions $M_{\mathcal{U}} > T_{\mathcal{BZ}\text{-}f} > \Lambda_{\mathcal{U}}$.

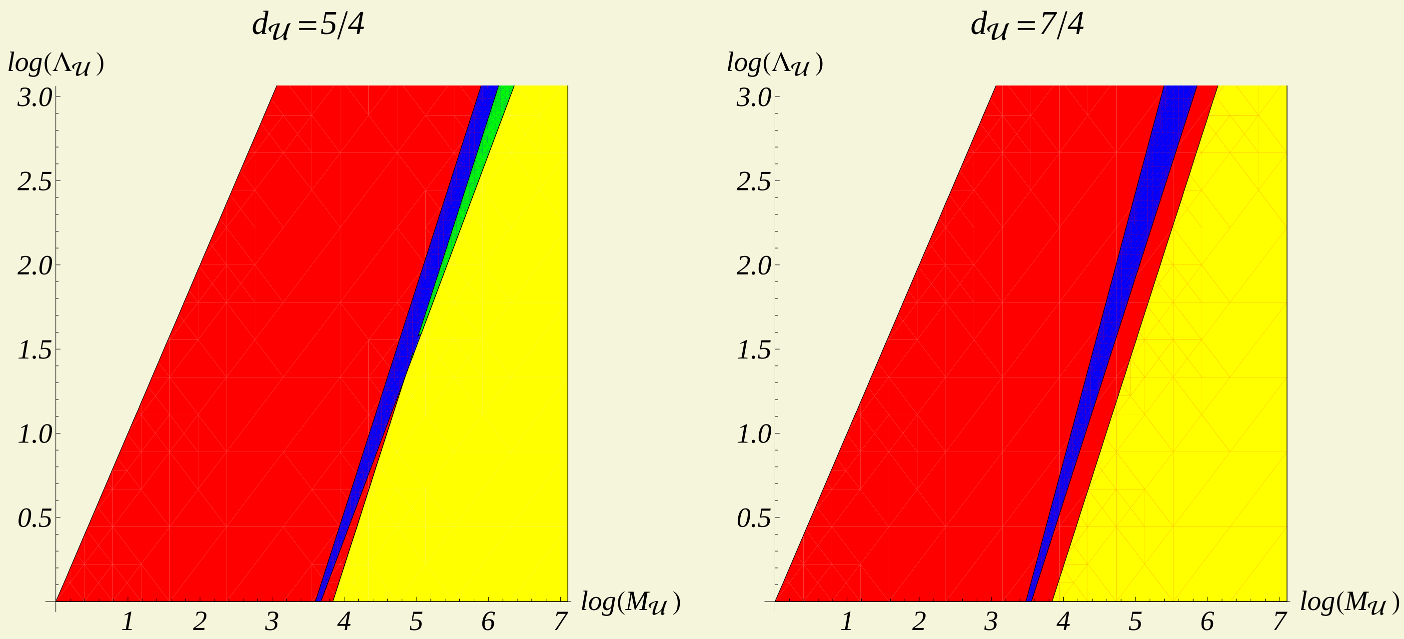
- $\mathcal{U}$ phase:

$$\frac{T_{\mathcal{U}\text{-}f}}{\Lambda_{\mathcal{U}}} = \left[\frac{\Lambda_{\mathcal{U}}}{V M_P}\sqrt{\frac{(2\pi)^3}{90}\frac{g_{\rm tot}(T_{\mathcal{U}\text{-}f})}{g_{\mathcal{U}}^2(T_{\mathcal{U}\text{-}f})}}\right]^{1/(2d_{\mathcal{U}}-3)} < 1 \tag{2}$$

  where now $g_{\rm tot} = g_{\rm SM} + g_{\mathcal{U}} \simeq g_{\rm SM} + g_{\rm IR}$, we also imposed the consistency condition $T_{\mathcal{U}\text{-}f} < \Lambda_{\mathcal{U}}$.

Regions in the $M_{\mathcal{U}} - \Lambda_{\mathcal{U}}$ plane:

1. $\mathcal{R}_1$: (2) holds but (1) does not,
2. $\mathcal{R}_2$: (1) holds but (2) does not,
3. $\mathcal{R}_3$: neither (1) nor (2) hold,
4. $\mathcal{R}_4$: both (1) and (2) hold.



Regions of the $M_{\mathcal{U}} - \Lambda_{\mathcal{U}}$ plane. Blue: $\mathcal{R}_1$; yellow: $\mathcal{R}_2$; red $\mathcal{R}_3$; green: $\mathcal{R}_4$ (see text) We took $n_c = 3$, $n_f = 16$, $k = 2$, $g_{\rm IR} = 100$ and two representative values of $d_{\mathcal{U}}$ (we also required $M_{\mathcal{U}} > \Lambda_{\mathcal{U}}$, $T_{\mathcal{U}\text{-}f} > v = 0.246$ TeV). $M_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$ are in TeV units.

	$d_{\mathcal{U}} > 3/2$	$d_{\mathcal{U}} < 3/2$
$\mathcal{R}_1$	$\Lambda_{\mathcal{U}} > T > T_{\mathcal{U}\text{-}f}$	$T_{\mathcal{U}\text{-}f} > T$
$\mathcal{R}_2$	$M_{\mathcal{U}} > T > T_{\mathcal{BZ}\text{-}f}$	$M_{\mathcal{U}} > T > T_{\mathcal{BZ}\text{-}f}$ $\Lambda_{\mathcal{U}} > T$
$\mathcal{R}_3$	–	$\Lambda_{\mathcal{U}} > T$
$\mathcal{R}_4$	$M_{\mathcal{U}} > T > T_{\mathcal{BZ}\text{-}f}$ $\Lambda_{\mathcal{U}} > T > T_{\mathcal{U}\text{-}f}$	$M_{\mathcal{U}} > T > T_{\mathcal{BZ}\text{-}f}$ $T_{\mathcal{U}\text{-}f} > T$

Temperature ranges where the SM and NP will be in equilibrium. Note that $T_{\mathcal{U}\text{-}f} > \Lambda_{\mathcal{U}}$ in regions $\mathcal{R}_{2,3}$ so in this case there is no transition (freeze-out or thaw-in) in the $\mathcal{U}$ phase; similarly there is transition in the $\mathcal{BZ}$ phase for $\mathcal{R}_{1,4}$. We have assumed $g_{\text{IR}} \simeq g_{\mathcal{BZ}}$ for simplicity.

$$\Downarrow$$

Severe constraints from the Big Bang Nucleosynthesis

Breaking of the scale invariance through $\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{U}}$

(Fox, Rajaraman, Shirman'07)

$$UV: \qquad \frac{1}{M_{\mathcal{U}}^{d_{\mathcal{BZ}}-2}}|H|^2\,\mathcal{O}_{\mathcal{BZ}}$$

$$\Downarrow$$

$$IR: \quad c_{\mathcal{U}}\left(\frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{\mathcal{BZ}}-2}}|H|^2\right)\,\mathcal{O}_{\mathcal{U}}$$

$v^2 \equiv \langle |H|^2\rangle \neq 0 \quad \Rightarrow \quad$ scaling violation at $\Lambda_{\not{\mathcal{U}}}$:

$$\Lambda_{\not{\mathcal{U}}}^{4-d_{\mathcal{U}}} = \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{d_{\mathcal{BZ}}-d_{\mathcal{U}}} M_{\mathcal{U}}^{2-d_{\mathcal{U}}}\,v^2$$

- For $Q < \Lambda_{\not{\mathcal{U}}}$ unparticles become particles (we assume $\Lambda_{\not{\mathcal{U}}} < \Lambda_{\mathcal{U}}$).
- For $Q > \Lambda_{\not{\mathcal{U}}}$ unparticle effects could possibly be seen.
- Low energy experiment may not be sensitive to unparticles.

For the interaction

$$c_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{\rm SM}+d_{\mathcal{BZ}}-4}} \mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{U}}$$

to cause observable effects, the quantity

$$\epsilon \propto \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{2(d_{\mathcal{BZ}}-d_{\mathcal{U}})} \left(\frac{Q}{M_{\mathcal{U}}}\right)^{2(d_{\rm SM}+d_{\mathcal{U}}-4)}$$

must be large enough. However, $Q > \Lambda_{\not{\mathcal{U}}}$ implies

$$\epsilon < \left(\frac{Q}{M_{\mathcal{U}}}\right)^{2d_{\rm SM}} \left(\frac{M_{\mathcal{U}}}{v}\right)^{4}$$

No $\Lambda_{\mathcal{U}}$ dependence!

- For $(g-2)_e$, $\mathcal{O}_{\rm SM} = \bar{e}e$, $Q \simeq m_e$, so

$$\epsilon < \frac{m_e^6}{M_{\mathcal{U}}^2 v^4} < 10^{-28} \quad \text{for} \quad M_{\mathcal{U}} \geq 100\ \text{GeV}$$

  negligible for the present experimental constraint $\epsilon < 10^{-11}$.

- $\epsilon \simeq 1\%$ requires $M_{\mathcal{U}} \lesssim 10^5$ GeV to see unparticle effects at the LHC!

Spontaneous symmetry breaking with unparticles and Higgs boson physics

(Delgado, Espinosa, Quiros'07)

$$UV: \qquad \frac{1}{M_{\mathcal{U}}^{d_{\mathcal{BZ}}-2}}|H|^2\,\mathcal{O}_{\mathcal{BZ}}$$

$$\Downarrow$$

$$IR: \quad c_{\mathcal{U}}\left(\frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{\mathcal{BZ}}-2}}|H|^2\right)\mathcal{O}_{\mathcal{U}} \equiv \kappa_{\mathcal{U}}|H|^2\,\mathcal{O}_{\mathcal{U}}$$

Deconstruction ($\mathcal{O}_{\mathcal{U}} \to \sum_n F_n\varphi_n$, $m_n^2 = \Delta^2 n$) $\Rightarrow$

$$V_{\rm tot} = m^2|H|^2 + \lambda|H|^4 + \delta\,V$$

for

$$\delta\,V = \frac{1}{2}\sum_{n=1}^{\infty} m_n^2\varphi^2 + \kappa_{\mathcal{U}}|H|^2\sum_{n=1}^{\infty}F_n\varphi_n$$

$$\langle\varphi_n\rangle = -\frac{\kappa_{\mathcal{U}}v^2F_n}{m_n^2} \quad \text{for} \quad \langle|H|^2\rangle = v^2, \qquad F_n^2 = \frac{A_{d_{\mathcal{U}}}}{2\pi}\Delta^2(m_n^2)^{d_{\mathcal{U}}-2}$$

So,

$$\langle\mathcal{O}_{\mathcal{U}}\rangle = \sum_{n=1}^{\infty}F_n\langle\varphi_n\rangle \longrightarrow -\kappa_{\mathcal{U}}v^2\frac{A_{d_{\mathcal{U}}}}{2\pi}\int_0^{\infty}\frac{dm^2}{(m^2)^{3-d_{\mathcal{U}}}} = -\infty$$

- The IR divergence!
- A possible regularization $\delta\,V' = \zeta|H|^2\sum\varphi_n^2$ is not scale invariant.

Since the scaling invariance is anyway violated by $\langle H\rangle \neq 0$ through $|H|^2\mathcal{O}_{\mathcal{U}}$ so we adopt

$$\delta\, V' = \zeta |H|^2 \sum_n \varphi_n^2$$

as the IR regulator. Then

$$v_n = \langle \varphi_n \rangle = -\frac{\kappa_{\mathcal{U}} v^2}{2(m_n^2 + \zeta v^2)} F_n$$

The minimization for H reads:

$$m^2 + \lambda v^2 + \kappa_{\mathcal{U}} \sum_n F_n v_n + \zeta \sum_n v_n^2 = 0$$

Inserting v_n one gets in the continuum limit ($\Delta \to 0$):

$$m^2 + \lambda v^2 - \lambda_{\mathcal{U}} (\mu^2)^{2-d_{\mathcal{U}}} v^{2(d_{\mathcal{U}}-1)} = 0$$

$$\text{for} \quad \lambda_{\mathcal{U}} \equiv \frac{d_{\mathcal{U}}}{4} \zeta^{d_{\mathcal{U}}-2} \Gamma(d_{\mathcal{U}}-1)\Gamma(2-d_{\mathcal{U}}) \quad \text{and} \quad (\mu_{\mathcal{U}}^2)^{2-d_{\mathcal{U}}} \equiv \kappa_{\mathcal{U}}^2 \frac{A_{d_{\mathcal{U}}}}{2\pi}$$

$$V_{\text{eff}} = m^2 |H|^2 + \frac{2^{d_{\mathcal{U}}-1}}{d_{\mathcal{U}}} \lambda_{\mathcal{U}} (\mu_{\mathcal{U}}^2)^{2-d_{\mathcal{U}}} |H|^{2d_{\mathcal{U}}} + \lambda |H|^4$$

Even if $m^2 = 0$ one can get $\langle H\rangle \neq 0$ ($\Lambda_{\mathcal{U}}$ provides the scale):

$$v^2 = \left(\frac{\lambda_{\mathcal{U}}}{\lambda}\right)^{\frac{1}{2-d_{\mathcal{U}}}} \mu_{\mathcal{U}}^2 \quad \text{for} \quad \mu_{\mathcal{U}}^2 = \left(\frac{A_{d_{\mathcal{U}}}}{2\pi}\right)^{\frac{1}{2-d_{\mathcal{U}}}} \left(\frac{\Lambda_{\mathcal{U}}^2}{M_{\mathcal{U}}^2}\right)^{\frac{d_{\text{SM}}-2}{2-d_{\mathcal{U}}}} \Lambda_{\mathcal{U}}^2$$

• Unparticle - Higgs mixing, $\kappa_{\mathcal{U}}|H|^2\,\mathcal{O}_{\mathcal{U}}$, implies

$$iP(p^2)^{-1} = p^2 - m_{h_0}^2 + v^2(\mu_{\mathcal{U}}^2)^{2-d_{\mathcal{U}}} \int_0^\infty \frac{(m^2)^{d_{\mathcal{U}}-2}}{m_{\mathcal{U}}^2(m^2) - p^2 + i\varepsilon} r(m^2)\, dm^2$$

for $m_{\mathcal{U}}^2(m^2) = m^2 + \zeta v^2$ and $r(m^2) = \left(\frac{m^2}{m^2+\zeta v^2}\right)^2$

The pole Higgs mass m_h (lower curve) and the unresummed Higgs mass m_{h_0} (upper curve) as a function of $d_{\mathcal{U}}$ for $\kappa_{\mathcal{U}} = v^{2-d_{\mathcal{U}}}$ and $\zeta = 1$. The straight line is the $m_{\text{gap}} = \zeta^{1/2} v$. Figure from Delgado, Espinosa, Quiros'07.

• Unparticle - Higgs mixing, $\kappa_{\mathcal{U}}|H|^2\,\mathcal{O}_{\mathcal{U}}$, implies invisible Higgs decays

Experimental constraints

From A. Freitas and D. Wyler,"Astro Unparticle Physics", arXiv:0708.4339 [hep-ph].

$$
\begin{aligned}
\mathcal{L}_{\mathcal{U}ff} = & \frac{\mathcal{C}_{\rm V}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\,\bar{f}\gamma_\mu f\,O_{\mathcal{U}}^\mu + \frac{\mathcal{C}_{\rm A}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\,\bar{f}\gamma_\mu\gamma_5 f\,O_{\mathcal{U}}^\mu + \frac{\mathcal{C}_{\rm S1}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,\bar{f}\not{D} f\,O_{\mathcal{U}} + \frac{\mathcal{C}_{\rm S2}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,\bar{f}\gamma_\mu f\,\partial^\mu O_{\mathcal{U}} \\
& + \frac{\mathcal{C}_{\rm P1}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,\bar{f}\not{D}\gamma_5 f\,O_{\mathcal{U}} + \frac{\mathcal{C}_{\rm P2}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,\bar{f}\gamma_\mu\gamma_5 f\,\partial^\mu O_{\mathcal{U}}
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
\equiv & \frac{c_{\rm V}}{M_{\rm Z}^{d_{\mathcal{U}}-1}}\,\bar{f}\gamma_\mu f\,O_{\mathcal{U}}^\mu + \frac{c_{\rm A}}{M_{\rm Z}^{d_{\mathcal{U}}-1}}\,\bar{f}\gamma_\mu\gamma_5 f\,O_{\mathcal{U}}^\mu + \frac{c_{\rm S1}}{M_{\rm Z}^{d_{\mathcal{U}}}}\,\bar{f}\not{D} f\,O_{\mathcal{U}} + \frac{c_{\rm S2}}{M_{\rm Z}^{d_{\mathcal{U}}}}\,\bar{f}\gamma_\mu f\,\partial^\mu O_{\mathcal{U}} \\
& + \frac{c_{\rm P1}}{M_{\rm Z}^{d_{\mathcal{U}}}}\,\bar{f}\not{D}\gamma_5 f\,O_{\mathcal{U}} + \frac{c_{\rm P2}}{M_{\rm Z}^{d_{\mathcal{U}}}}\,\bar{f}\gamma_\mu\gamma_5 f\,\partial^\mu O_{\mathcal{U}}.
\end{aligned}
\tag{4}
$$

Here the coefficients have been scaled to a common mass, chosen as the Z-boson mass $M_{\rm Z}$, so that the only unknown quantities are the dimensionless coupling constants $c_{\rm X}$.

For photons we have

$$
\mathcal{L}_{\mathcal{U}\gamma\gamma} = -\frac{\mathcal{C}_{\gamma\gamma}}{4\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,F_{\mu\nu}F^{\mu\nu}\,O_{\mathcal{U}} - \frac{\mathcal{C}_{\tilde{\gamma}\tilde{\gamma}}}{4\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\,O_{\mathcal{U}} \tag{5}
$$

$$
\equiv -\frac{c_{\gamma\gamma}}{4M_{\rm Z}^{d_{\mathcal{U}}}}\,F_{\mu\nu}F^{\mu\nu}\,O_{\mathcal{U}} - \frac{c_{\tilde{\gamma}\tilde{\gamma}}}{4M_{\rm Z}^{d_{\mathcal{U}}}}\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\,O_{\mathcal{U}} \tag{6}
$$

Coupling	c_{V}				c_{A}		
$d_{\mathcal{U}}$	1	4/3	5/3	2	1	4/3	5/3
5th force	$7 \cdot 10^{-24}$	$1.4 \cdot 10^{-15}$	$1.8 \cdot 10^{-10}$	$2 \cdot 10^{-5}$	$4 \cdot 10^{-24}$	$8 \cdot 10^{-16}$	$1 \cdot 10^{-10}$
Star cooling	$5 \cdot 10^{-15}$	$2.5 \cdot 10^{-12}$	$1 \cdot 10^{-9}$	$3.5 \cdot 10^{-7}$	$6.3 \cdot 10^{-15}$	$2 \cdot 10^{-12}$	$7.3 \cdot 10^{-10}$
SN 1987A	$1 \cdot 10^{-9}$	$3.5 \cdot 10^{-8}$	$1 \cdot 10^{-6}$	$3 \cdot 10^{-5}$	$2 \cdot 10^{-11}$	$5.5 \cdot 10^{-10}$	$1.5 \cdot 10^{-8}$
LEP	0.005	0.045	0.04	0.01	0.1	0.045	0.04
Tevatron		0.4	0.05				
ILC	$1.6 \cdot 10^{-4}$	$1.4 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$3.2 \cdot 10^{-4}$	$3.2 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$
LHC		0.25	0.02				
Precision	1	0.2	0.025		1	0.15	0.01
Quarkonia		0.01	0.1	0.45			
Positronium		0.25				$2 \cdot 10^{-13}$	$2 \cdot 10^{-8}$

Coupling	c_{S1}				c_{P1}, $2c_{\text{P2}}$		
$d_{\mathcal{U}}$	1	4/3	5/3	2	1	4/3	5/3
5th force	$6.5 \cdot 10^{-22}$	$1.2 \cdot 10^{-13}$	$1.6 \cdot 10^{-8}$	$1.7 \cdot 10^{-3}$	—	—	—
Star cooling	$1.3 \cdot 10^{-9}$	$7 \cdot 10^{-7}$	$3 \cdot 10^{-4}$	0.13	$4 \cdot 10^{-8}$	$1.1 \cdot 10^{-5}$	$3.3 \cdot 10^{-3}$
SN 1987A	$8 \cdot 10^{-8}$	$2.4 \cdot 10^{-6}$	$6.6 \cdot 10^{-5}$	$2 \cdot 10^{-3}$	$5.5 \cdot 10^{-8}$	$1.3 \cdot 10^{-6}$	$3.5 \cdot 10^{-5}$
LEP	> 1	> 1	> 1	> 1	> 1	> 1	> 1
ILC	> 1	> 1	> 1	> 1	> 1	> 1	> 1

From A. Freitas and D. Wyler,“Astro Unparticle Physics”, arXiv:0708.4339 [hep-ph].

Summary

- Intensive activity on unparticles (142 citations of the first Georgi's paper)
- Interesting and exotic phenomenology
- Unparticles could be deconstructed
- Scale invariance breaking by interactions with the SM ($Q > \Lambda_{\not{\mathcal{U}}}$ to see unparticles)
- Troubles with IR divergences
- Important cosmological consequences
- So far, no fundamental problem of particle physics has been solved by the assumption of unparticles - Is this a drawback?

Jochum van der Bij and S. Dilcher:

1. J. J. van der Bij and S. Dilcher, "HEIDI and the unparticle," Phys. Lett. B **655**, 183 (2007) [arXiv:0707.1817 [hep-ph]].
2. J. J. van der Bij and S. Dilcher, "A higher dimensional explanation of the excess of Higgs-like events at CERN LEP," Phys. Lett. B **638**, 234 (2006) [arXiv:hep-ph/0605008].
3. J. J. van der Bij, "The minimal non-minimal standard model," Phys. Lett. B **636**, 56 (2006) [arXiv:hep-ph/0603082].

The model:

- Extra-dimensional (δ) scalars neutral under the SM gauge group

$$\phi(x,y) = \frac{1}{\sqrt{2}L^{\delta/2}} \sum_{\vec{k}} \phi_{\vec{k}}(x) e^{i\frac{2\pi}{L}\vec{k}\vec{y}}$$

- Extra terms in the scalar potential

$$V(H,\phi) = \cdots - \frac{\lambda_1}{8}(2f_1\phi - |H|^2)$$

Similarities:

- The continuous mass spectrum e.g. for $s \to \infty$: $\rho(s) \sim s^{-3+\delta/2}$

Differences

- In HEIDI only scalars, while unparticles could have any spin
- Van der Bij and Dilcher don't assume scale invariance of the extra sector
- In HEIDI interactions between the SM and the extra scalars assumed to be renormalizable
- Van der Bij and Dilcher claim that only for $0 < \delta < 1$ there is no tachyons in the scalar spectrum, so the potential is stable $(1 < d_{\mathcal{U}} < 2)$